MISSING HIGHER ORDERS AND PARTON DISTRIBUTIONS

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THUTF MEETING

CERN, MAY 20, 2013

STATISTICAL AND PARAMETRIC UNCERTAINTIES

SOME FACTS WE AGREE UPON

- PDF UNCERTAINTIES AS GIVEN BY NNPDF REPLICAS OR CTEQ ERROR SETS FOR FIXED α_s ARE JUST STATISTICAL UNCERTANTIES
  THEY NEEDN'T BE GAUSSIAN (EG BECAUSE OF POSITIVITY BOUNDS) BUT OFTEN ARE TO GOOD APPROXIMATION
- PDF UNCERTAINTIES REFLECT THE UNCERTAINTIES IN THE DATA USED TO DETERMINE THE PDF; THEY ARE THUS USUALLY UNCORRELATED TO OTHER UNCERTAINTIES
  example of (admittedly baroque) exception: one PDF combination (eg $s - \bar{s}$) determined by one single dataset whose normalization is correlated to that of process which is being predicted
  THEY DO INDUCE CORRELATION BETWEEN DIFFERENT PDFS IN SAME SET
- PARAMETRIC UNCERTAINTIES SUCH AS THE UNCERTAINTY ON α_s ARE USUALLY GAUSSIAN
  THEY AFFECT BOTH THE PARTONIC CROSS SECTIONS AND THE PDFS IN A CORRELATED WAY (OFTEN STRONGLY CORRELATED)
- CORRELATIONS BETWEEN PARM UNCERTAINTIES OF XSECT & PDF MUST BE KEPT INTO ACCOUNT,
  E.G. BY USE PDFS DETERMINED USING THE SAME α_s VALUE ADOPTED IN THE MATRIX ELEMENT, AND VARYING BOTH SIMULTANEOUSLY

THEORETICAL UNCERTAINTIES

- DUE TO UNKNOWN HIGHER ORDER TERMS
- CLEARLY PRESENT BOTH IN PROCESSES USED FOR PDF DETERMINATION ⇒ THEORETICAL UNCERTAINTY ON PDFs AND IN PROCESSES COMPUED USING A GIVEN PDF SET ⇒ THEORETICAL UNCERTAINTY ON PROCESS
- THU ON PDF & ON PROCESS GENERALLY UNCORRELATED UNLESS PDF DETERMINATION IS DRIVEN MAINLY OR ENTIRELY BY ONE PROCESS, AND ONE IS LOOKING AT THE SAME (OR CLOSELY RELATED PROCESS) NOT IMPOSSIBLE, BUT UNUSUAL ⇒ UNCORRELATED TREATMENT MIGHT BE JUST VERY SLIGHTLY TOO CONSERVATIVE
- AT PRESENT THE ISSUE IS ACADEMIC: NO AVAILABLE PDF SET INCLUDES THESE UNCERTAINTIES
- WHAT IF WE DID TRY TO INCLUDE THEM?:
  - HOW COULD WE DO IT?
  - HOW LARGE MIGHT THEY BE?

ESTIMATING THEORETICAL UNCERTAINTIES ON PDF

- STANDARD WAY: FACTORIZATION & RENORMALIZATION SCALE VARIATION OF FACTORIZED QCD CROSS-SECTIONS
  $\sigma(M^2,\tau) = \hat\sigma\left(\alpha_s(\mu_R^2), \frac{M^2}{\mu_F^2}, \frac{M^2}{\mu_R^2}, z\right) \otimes f(z,\mu_F^2)$
  $f \to$ PDF; $\hat\sigma \to$ PARTONIC XSECT., $\sigma \to$ HADRONIC XSECT; $\otimes$ CONVOLUTION
- HOW WOULD IT WORK FOR PDFS?

FACTORIZATION SCALE

- FACTORIZATION SCALE INDEPENDENCE IS A PROPERTY OF PHYSICAL OBSERVABLES: PDF IS μ_f DEPENDENT!
- IMPOSE $\frac{d\sigma}{d\mu_F^2} = 0$ TO FINITE PERTURBATIVE ORDER & USE RESULT TO DETERMINE DEPENDENCE OF $\hat\sigma\left(\alpha_s(\mu_R^2), \frac{M^2}{\mu_F^2}, \frac{M^2}{\mu_R^2}, z\right)$ ON μ_F^2
- $\to \sigma(Q^2,\tau,\mu_F^2)$ ACQUIRES NEW SCALE DEPENDENCE: IT ESTIMATES SIZE OF HIGHER-ORDER CORRECTIONS TO $\hat\sigma$
- PERFORMING VARIATION FOR PROCESSES USED IN PDF EXTRACTION WOULD GIVE UNCERTAINTY ON PDFS DUE TO MISSING HIGHER ORDERS IN THESE PROCESSES PDF ACQUIRE DEPENDENCE ON NEW SCALE $\mu_F^{\rm fit}$: $f(\mu_F,\mu_F^{\rm fit})$ MODELS THU DUE TO MISSING HIGHER ORDERS
- $\mu_F^{\rm fit}$ DEPENDENCE OF PDFS TOTALLY UNCORRELATED TO μ_f DEPENDENCE OF PDFS
- $\mu_F^{\rm fit}$ DEPENDENCE OF PDFS GENERALLY UNCORRELATED TO μ_f DEPENDENCE OF $\sigma(Q^2,\tau,\mu_F^2)$, UNLESS PDF DETERMINATION DRIVEN MOSTLY BY $\sigma(Q^2,\tau,\mu_F^2)$ WHICH IS BEING COMPUTED (SEE ABOVE).

ESTIMATING THEORETICAL UNCERTAINTIES ON PDF

- STANDARD WAY: FACTORIZATION & RENORMALIZATION SCALE VARIATION OF FACTORIZED QCD CROSS-SECTIONS
  $\sigma(M^2,\tau) = \hat\sigma\left(\alpha_s(\mu_R^2), \frac{M^2}{\mu_F^2}, \frac{M^2}{\mu_R^2}, z\right) \otimes f(z,\mu_F^2)$
  $f \to$ PDF; $\hat\sigma \to$ PARTONIC XSECT., $\sigma \to$ HADRONIC XSECT; $\otimes$ CONVOLUTION

- HOW WOULD IT WORK FOR PDFS?

RENORMALIZATION SCALE

- RENORMALIZATION SCALE INDEPENDENCE IS A PROPERTY OF EITHER THE PHYSICAL OBSERVABLE (μ_R^2 INDEP AT FIXED Q^2) OR THE PDF (μ_R^2 INDEP AT FIXED μ_F^2)

- IMPOSE $\frac{d\sigma}{d\mu_R^2} = 0$ TO FINITE PERTURBATIVE ORDER

  & USE RESULT TO DETERMINE DEPENDENCE OF $\hat\sigma\left(\alpha_s(\mu_R^2), \frac{M^2}{\mu_F^2}, \frac{M^2}{\mu_R^2}, z\right)$ ON μ_R^2

- $\to \sigma(Q^2,\tau,\mu_R^2)$ ACQUIRES NEW SCALE DEPENDENCE: IT ESTIMATES SIZE OF HIGHER-ORDER CORRECTIONS TO $\hat\sigma$ BUT ALSO OF HIGHER ORDER CORRECTIONS TO EVOLUTION EQUATIONS

- PERFORMING VARIATION FOR PROCESSES USED IN PDF EXTRACTION WOULD GIVE UNCERTAINTY ON PDFS DUE TO MISSING HIGHER ORDERS IN THESE PROCESSES BUT ALSO DUE TO MISSING HIGHER ORDERS IN PERTURBATIVE EVOLUTION PDF ACQUIRE DEPENDENCE ON NEW SCALE μ_R: $f(\mu_F,\mu_R)$

- PART (THOUGH NOT ALL) OF THIS UNCERTAINTY HIGHLY CORRELATED ACROSS PROCESSES

ESTIMATING THEORETICAL UNCERTAINTIES ON PDF

TENTATIVE CONCLUSIONS

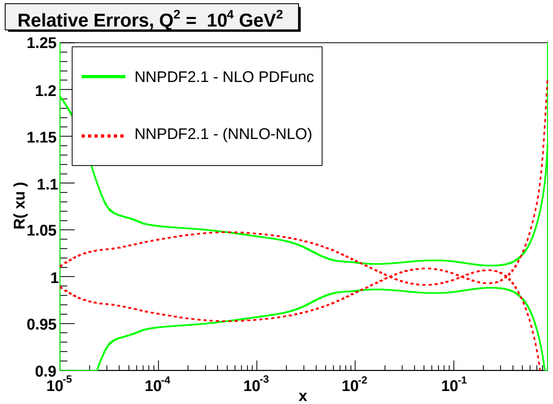
- THEORETICAL UNCERTAINTIES DUE TO MISSING HIGHER ORDERS IN PROCESSES USED FOR PDF EXTRACTION CURRENTLY NOT INCLUDED IN PDF FITS
- IF INCLUDED THEY WOULD BE ALMOST ENTIRELY UNCORRELATED TO THAT IN PROCESSES IN WHICH PDFS ARE SUBSEQUENTLY USED
- SCALE VARIATION APPEARS ESPECIALLY INEFFICIENT FOR THE ESTIMATION OF SUCH UNCERTAINTIES ON PDFS, AS IT INDUCES SPURIOUS CORRELATIONS
- METHODS BASED ON THE PERTURBATIVE BEHAVIOUR (CACCIARI-HOUDEAU) MORE PROMISING
- SUGGEST THEORETICAL UNCERTAINTIES ON PDFS IN COMPARISON TO CURRENT PDF UNCERTAINTIES ARE NEGLIGIBLE AT NNLO BUT NOT AT NLO

THEORETICAL UNCERTAINTIES

NLO PDF UNCERTAINTIES VS NLO-NNLO SHIFT (NNPDF2.1)

NNLO KNOWN $\Rightarrow$ NLO TH. UNCERTAINTIES KNOWN!

UP



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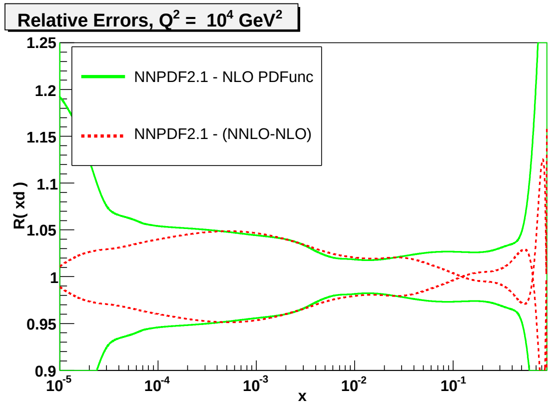

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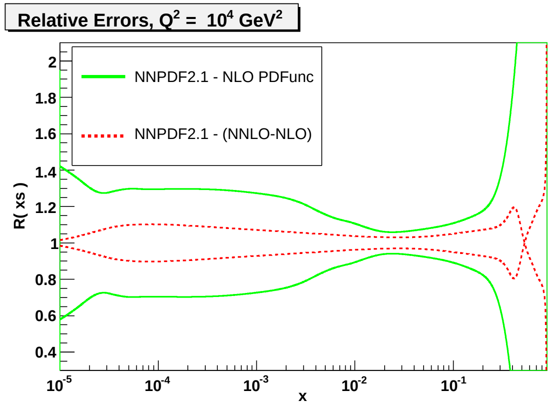

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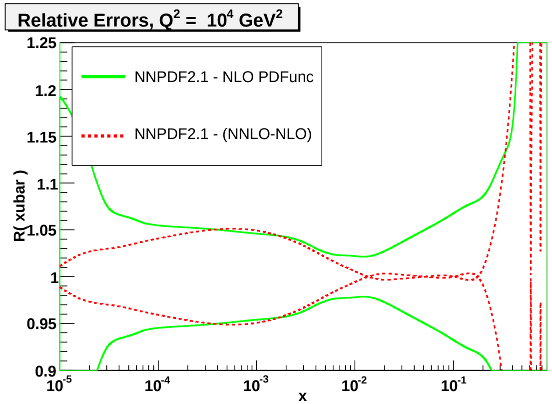

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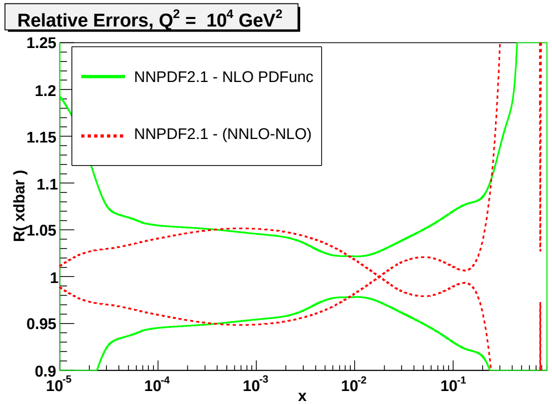

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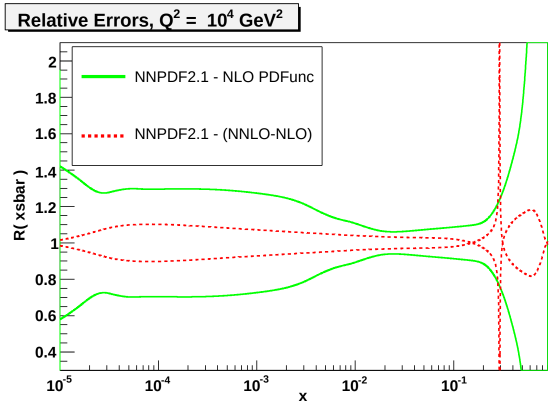

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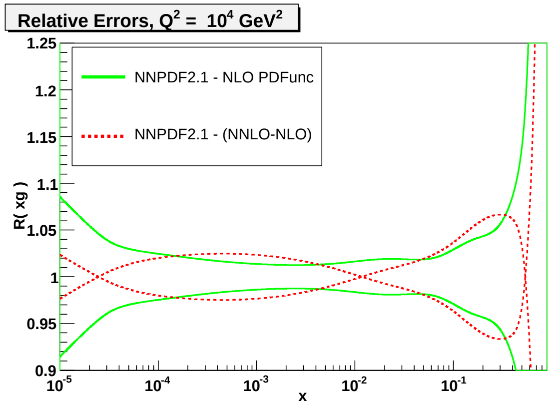
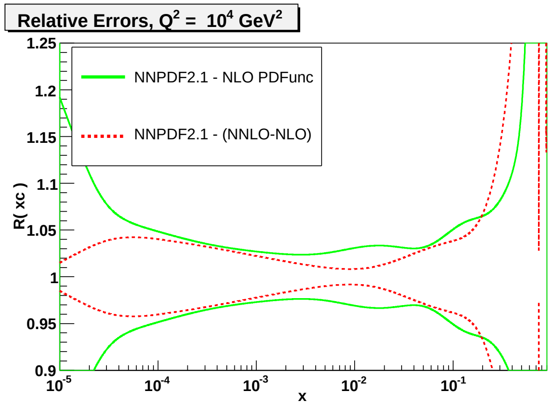
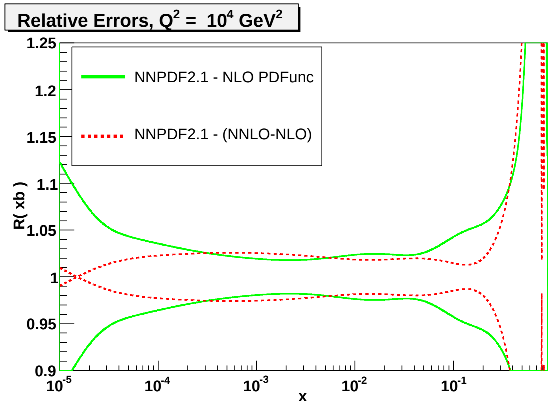


CHARM



BOTTOM



THEORETICAL UNCERTAINTIES

CRUDE IMPLEMENTATION OF THE CACCIARI-HOUDEAU METHOD

- ASSUME COEFFICIENT OF PERTURBATIVE EXPANSION BOUNDED FROM ABOVE AND WITH SOME (UNIFORM?) DISTRIBUTION

- ESTIMATE SIZE OF NEXT ORDER BASED ON KNOWN COEFFICIENTS

series in α_s starting at α_s^0; uncertainty on k-th order:

$$\Delta_k = \begin{cases} \alpha_s^{k+1} \max\{|c_l|, \dots, |c_k|\} \frac{n_c+1}{n_c} p & \text{if } p \leq \frac{n_c}{n_c+1} \\ \alpha_s^{k+1} \max\{|c_l|, \dots, |c_k|\} \left[(n_c+1)(1-p)\right]^{-1/n_c} & \text{if } p > \frac{n_c}{n_c+1} \end{cases}$$

$n_c = k+1$ number of known coefficients; P $\Rightarrow$ c.l. (one $\sigma \leftrightarrow P = 0.68$)

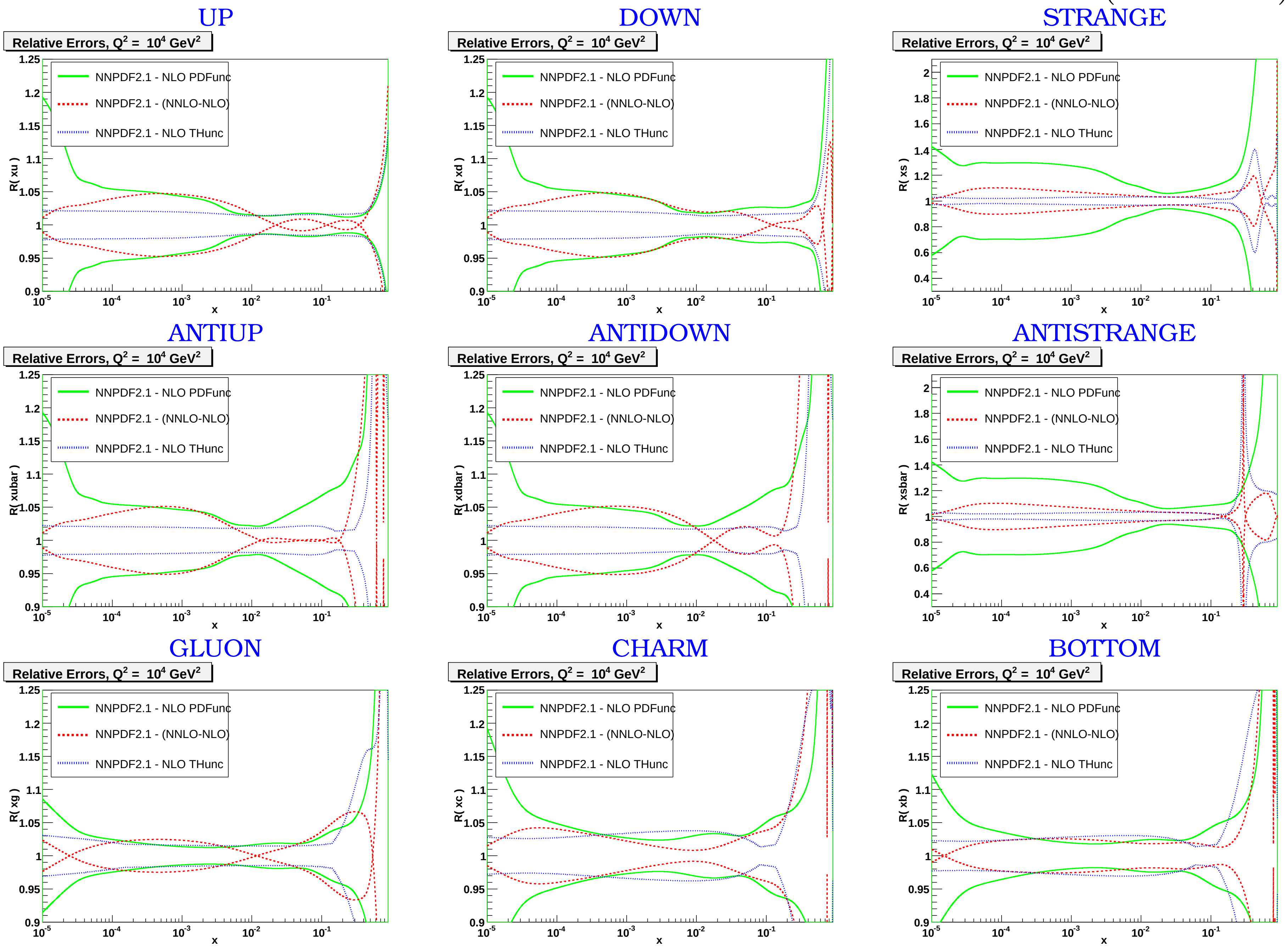
THE CACCIARI-HOUDEAU METHOD

APPLICATION TO PDFs

- CONSIDER PDFs FOR GIVEN x, Q^2 AS A SERIES IN α_s
- AT NLO USE FORMULA WITH $k = 1$ ETC. (HENCEFORTH, $\alpha_s = 0.119$)

THEORETICAL UNCERTAINTIES

NLO PDF unc. vs NLO-NNLO shift vs NLO Cacciari-Houdeau (NNPDF2.1)



THEORETICAL UNCERTAINTIES

NNLO PDF UNC. VS NLO-NNLO SHIFT VS NNLO CACCIARI-HOUDEAU (NNPDF2.1)