THUTF
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Bayesian approach to missing higher orders, and a few other considerations

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Also with Emanuele Bagnaschi, Alberto Guffanti, Laura Jenniches.
Previous work with Nicolas Houdeau, arXiv: 1105.5152

Origin of uncertainties in a hadronic cross section calculated as a perturbative series in α_s

- **MHOU** = Missing Higher Order Uncertainties
  - They are variously referred to as "scale uncertainties", "perturbative uncertainties", "theory uncertainties", etc...
  - They **could** be estimated using scale variations, but it is not the only way, and it is **not** what I will do.
- **IU** = Inputs Uncertainties

  Composed of (at least):
  - PDFU = Parton Distribution Functions Uncertainties
  - ASU = α_s Uncertainties
- I won't really talk about these, if not to assume that they contain exclusively uncertainties of experimental origin. See Stefano's and Pavel's talk for role of MHOU in PDFU

Glossary

- **pdf** (small case) = probability density function

- **DoB** = Degree of Belief / Credibility

Roughly, bayesian-speak for Confidence Level. Also called credibility of an interval, it is obtained as an integral over a credibility distribution:

$$\mathbb{C}(\eta \in [a,b]) \equiv \int_a^b f(\eta)\mathrm{d}\eta$$

Degree of Belief
(or Credibility)

Credibility distribution
(or density)

↑

Posterior
of the Bayesian model

Topics for discussion

- **How to estimate MHOU** and IU can be up for debate (in fact, it is, especially for MHOU).
  This is what my talk will focus on.

- How to **combine** them should not be.
  Once correlations (or their absence, or our ignorance of them) are known, one expects probability theory to show the way.
  However, I will not address this issue in my talk.

MHOU: the good old way

- Choose a central scale μ_0
- Choose a range ξ
- Vary scales in the range $[\mu_0/\xi, \xi\mu_0]$
- Determine an uncertainty band using the values of the cross sections for $\mu \in [\mu_0/\xi, \xi\mu_0]$

Which is simple, reasonable, and has served us well for 30+ years. But....

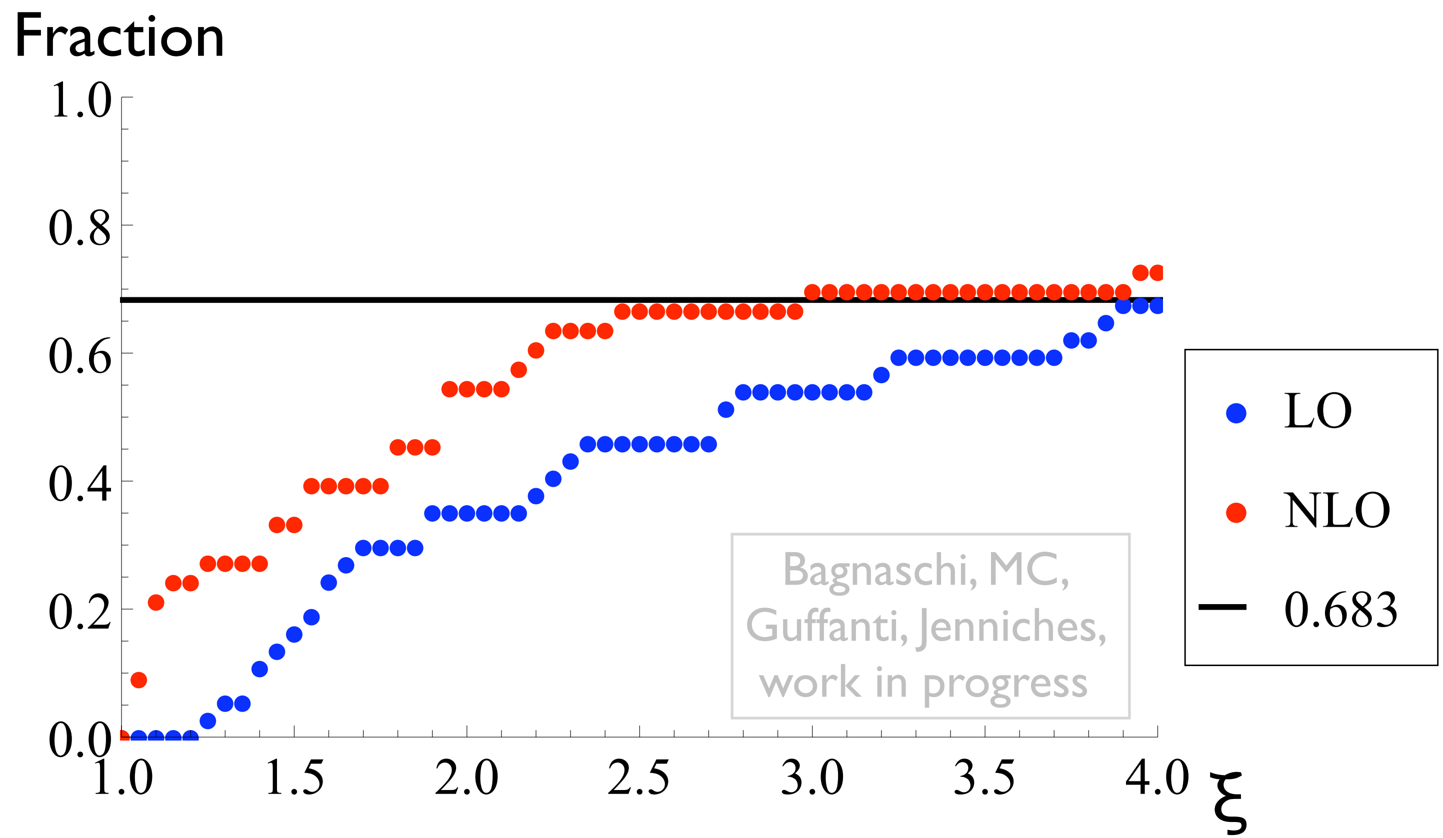
"Uncertainties" of the good old way

- What is an appropriate value for μ_0?
- What is an appropriate value for the range ξ?
- If more than one scale, vary them independently?
- Vary all scales in $[\mu_0/\xi, \xi\mu_0]$, or use a fiducial region such that $\mu_i/\mu_j \in [1/\xi, \xi]$?
- Once you have a band, **what does it mean?**
  - In order to be used in uncertainty estimation and combination, the MHOU must be expressed in the form of a credibility interval and a credibility density
    (Independently of how the MHOU will be estimated, I'm using bayesian terminology here because MHOU are obviously not frequentist entities)

- One can ask (at least) two questions:
  - How well do MHOU estimated with scale variations fare?
  - How are higher orders actually distributed? (and hence how should MHOU be?)

A frequentist study of scale variations

- Select many (20+) non-hadronic observables known at least to NNLO
- Calculate the scale variations band given by $\mu \in [\mu_0/\xi, \xi\mu_0]$
- Find out **how often** the next order is within the band, **as a function of ξ**



At NLO, 68% starts being approached for $\xi > 2.5$

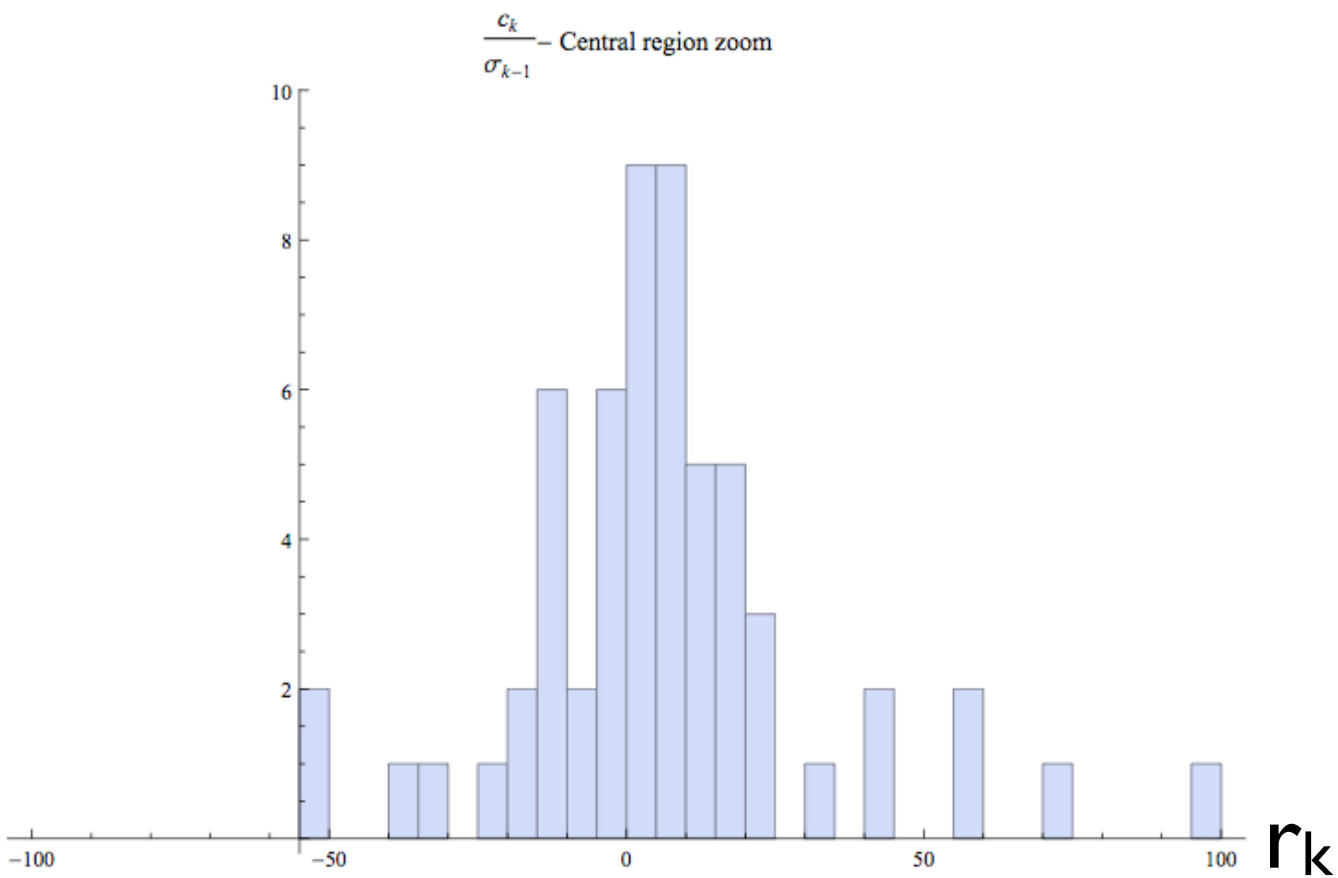
(for this particular class of observables)

Distributions of higher orders coeffs.

Consider the quantity

Bagnaschi, MC, Guffanti, Jenniches, work in progress

$$r_k \equiv \frac{1}{\alpha_s^k} \frac{\sigma_k - \sigma_{k-1}}{\sigma_{k-1}} = \frac{c_k}{\sigma_{k-1}}$$



Certainly not a box, likely not a Gaussian

A credibility profile for MHOU

- I do **not** expect it to be
  - A box, e.g. with size given by the scale variations band
    An absolute edge does not look reasonable, as there are numerous cases where the next order is outside the scale variations band (ξ=2)
  - A gaussian, with sigma related to the scale variations band
    We have clear evidence that the most probable value for a higher order contribution is not zero, i.e. the next order does not usually sit on top of the previous one
- On the other hand, one can speculate that it could look like a **roughly flat top with falling tails**
  - The flat top is justified by higher order corrections that are often sizable but with no typical preferred value
  - The falling tails account for the fact that huge higher order corrections tend to be rare

- I'm not saying that 'scales are bad'
- I'm not saying that they should not be used
- I'm saying that, by themselves, they only give a **very incomplete answer**
    - To the very least, they must be complemented with (yet another) prescription to give a **credibility value** for the interval, and a **credibility profile**
- Why not then replacing this tower of prescriptions with another (perhaps equally arbitrary, but at least more systematic) approach?

A bayesian model

- Since scale variations effectively takes the last calculated perturbative order and adds an α_s (and some coefficients) to it, one may think of building a bayesian model containing a similar hypothesis, but where **all priors are made explicit**
- This is what Cacciari, Houdeau [1105.5152] did:
  - In practice, **assume that all coefficients c_k in a perturbative expansion of the form $\sigma = \Sigma\, \alpha_s^k c_k$ have similar size**
- This leads to the credibility profile for the uncertainty Δ_k

$$f(\Delta_k|c_l,\ldots,c_k) \simeq \left(\frac{n_c}{n_c+1}\right)\frac{1}{2\alpha_s^{k+1}\bar{c}_k}\begin{cases} 1 & \text{if } |\Delta_k| \le \alpha_s^{k+1}\bar{c}_k \\ \frac{1}{(|\Delta_k|/(\alpha_s^{k+1}\bar{c}_k))^{n_c+1}} & \text{if } |\Delta_k| > \alpha_s^{k+1}\bar{c}_k \end{cases}$$

and to p%-credible intervals given by

$$d_k^{(p)} = \begin{cases} \alpha_s^{k+1}\bar{c}_k \frac{n_c+1}{n_c} p\% & \text{if} \quad p\% \le \frac{n_c+1}{n_c} \\ \alpha_s^{k+1}\bar{c}_k \left[(n_c+1)(1-p\%)\right]^{(-1/n_c)} & \text{if} \quad p\% > \frac{n_c+1}{n_c} \end{cases}$$

Bagnaschi, MC, Guffanti, Jenniches, work in progress

Modified CH: CHbar

- A criticism of CH was that we don't know that a simple α_s is really the appropriate parameter for expansion.
  - We have therefore modified CH a little, allowing for a further parameter λ controlling the expansion parameter, and including a factorial (suggested by perturbative expansions):

$$O = \sum_{k=l}^{\infty} \frac{\alpha_s^k}{\lambda^k}(k-1)!\frac{\lambda^k c_k}{(k-1)!} \equiv \sum_{k=0}^{\infty}\left(\frac{\alpha_s}{\lambda}\right)^k (k-1)!\, c'_k$$

This leads to the credibility profile

$$f(\Delta_k|c_l,\ldots,c_k) \simeq \left(\frac{n_c}{n_c+1}\right)\frac{1}{2k!(\alpha_s/\lambda)^{k+1}\bar{c}'_k}\begin{cases} 1 & \text{if } |\Delta_k| \le k!(\alpha_s/\lambda)^{k+1}\bar{c}'_k \\ \frac{1}{(|\Delta_k|/(k!(\alpha_s/\lambda)^{k+1}\bar{c}'_k))^{n_c+1}} & \text{if } |\Delta_k| > k!(\alpha_s/\lambda)^{k+1}\bar{c}'_k \end{cases}$$

  - We have studied λ only in a frequentist way so far, but one could think of including a further prior, e.g. a Jeffreys one for maximal non-informativeness.

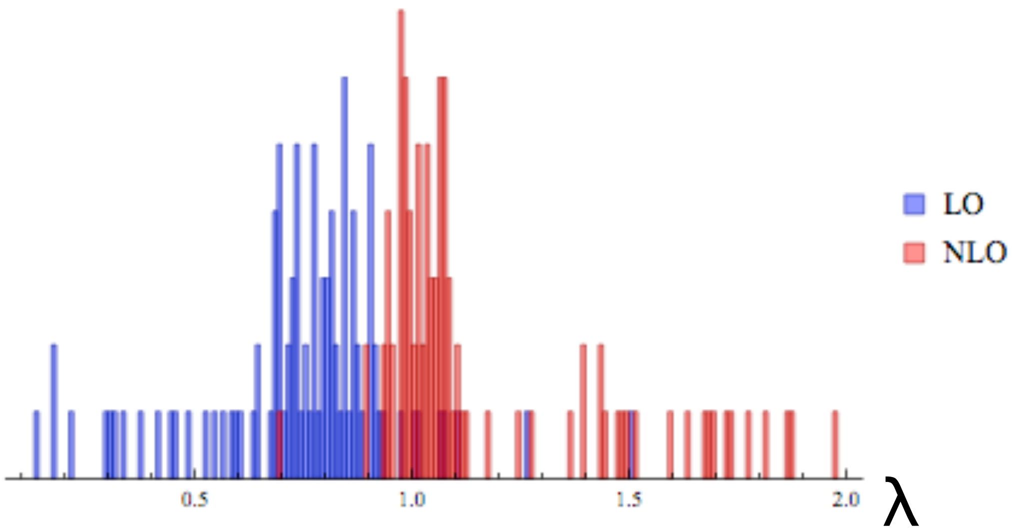
List of observables

Bagnaschi, MC, Guffanti, Jenniches, work in progress

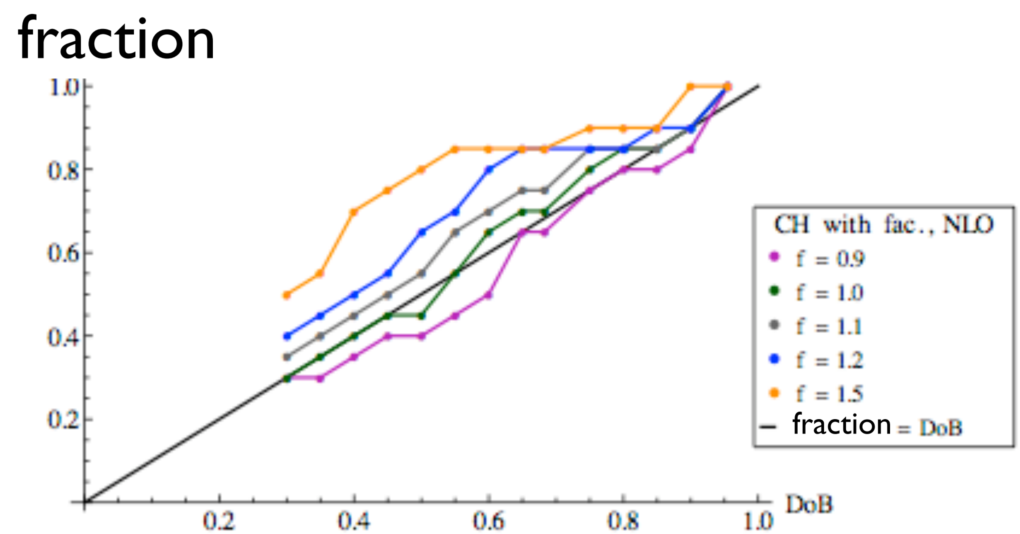
- $e^+e^- \to \gamma \to$ hadr at NNLO (3)
- Bjorken sum rule at NNLO (3)
- GLS sum rule at NNLO (3)
- $b \to ce\bar{\nu}_\tau$ at NNLO (2)
- ratio $\mathcal{R} = \Gamma(b \to c\tau\bar{\nu}_\tau)/\Gamma(b \to ce\bar{\nu}_\tau)$ at NNLO (2)
- ratio $\mathcal{R} = \Gamma(Z \to b\bar{b})/\Gamma(Z \to \text{hadr})$ (3)
- $Z \to$ hadr (4)
- $Z \to b\bar{b}$ (3)
- Thrust, 2nd moment (3)
- Heavy jet mass, 2nd moment (3)
- Wide jet broadening, 2nd moment (3)
- Total jet broadening, 2nd moment (3)
- C parameter, 2nd moment (3)
- three-to-two jet transition, 2nd moment (3)
- Splitting kernel γ^+_{ns}, γ_{qg}, γ_{gg}, 2nd Mellin moment (3)
- Structure functions $c^q_{2,ns}$, $c^g_{2,ns}$, $c^q_{l,ns}$, $c^g_{l,ns}$, 2nd Mellin moment (3).

Bagnaschi, MC,
Guffanti, Jenniches,
work in progress

Tests of CHbar



Value of λ that gives the best agreement between the theoretical DoB and the observed fraction of observables whose higher order falls within the predicted uncertainty



As a function of the DoB, and for a few selected values of λ, the comparison between the theoretical DoB and the observed fraction

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Guffanti, Jenniches,
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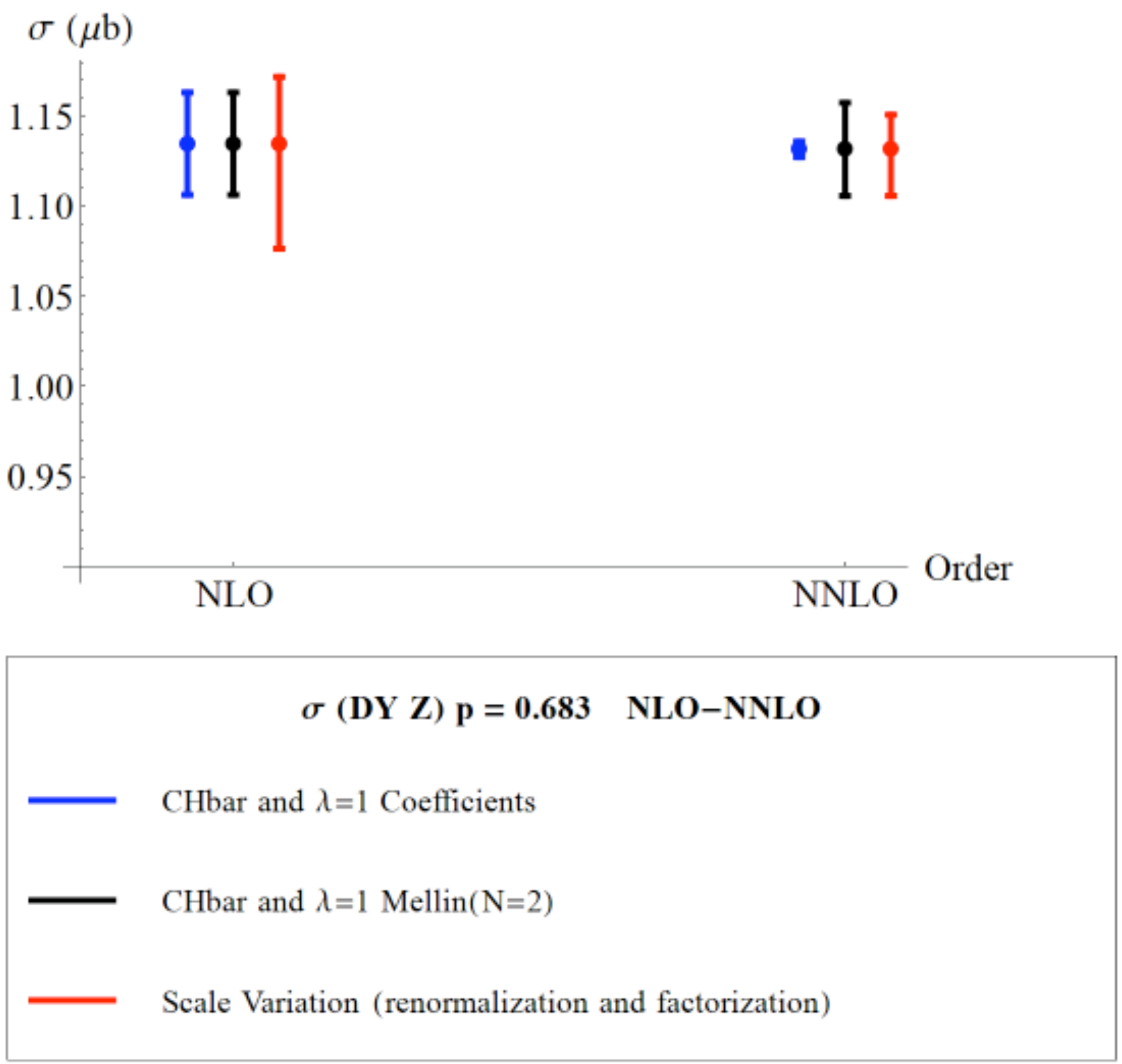
Results for Drell-Yan

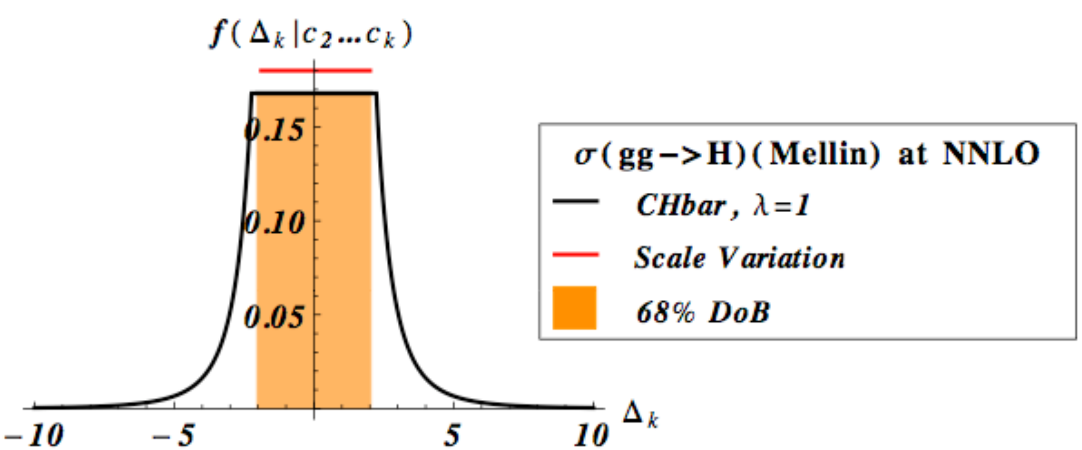


68%-credible uncertainty intervals

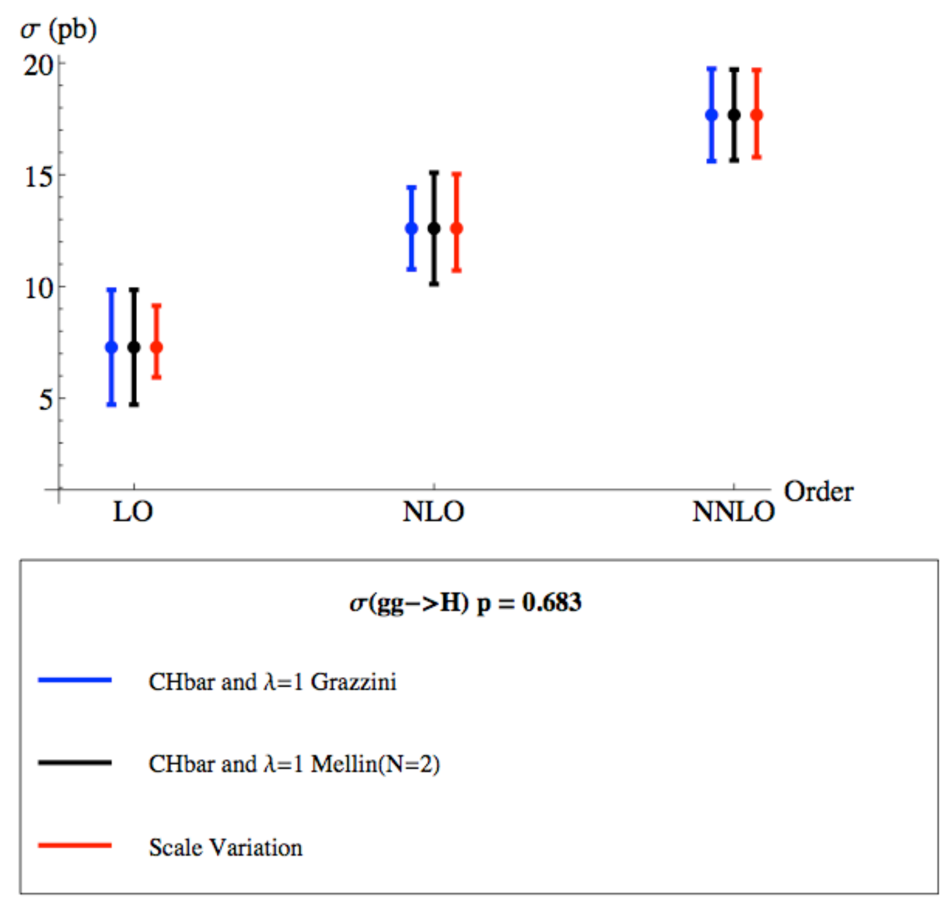
Note: here "Coefficients" means that the coefficients have been taken at **hadron** level (i.e. including the PDFs).
Instead, "Mellin" means that they have been taken at **parton** level using a saddle-point approximation

Results for gg→Higgs

Credibility profile



68%-credible uncertainty



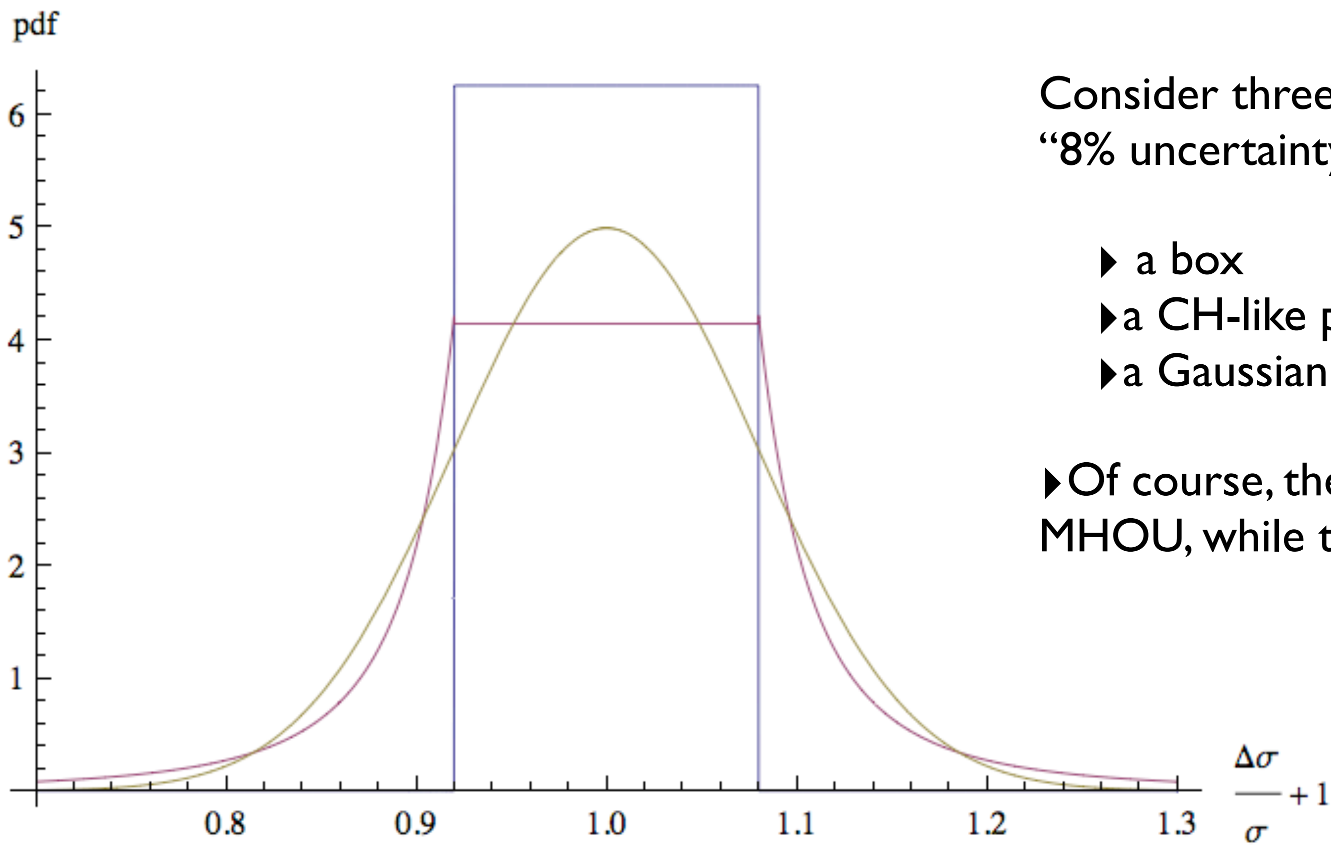
Note: here "Grazzini" means that the coefficients have been taken at **hadron** level (i.e. including the PDFs) using Grazzini's code. Instead, "Mellin" means that they have been taken at **parton** level using a saddle-point approximation

Some final considerations

- I think that we need a procedure to estimate the MHOU that provides not only a band, but a full credibility density profile
    - It could be scale variations complemented with something else
    - It could be a bayesian approach along the lines of CH(bar)
    - It could be something completely different
- Whatever the choice is, I'd expect this procedure to produce a credibility profile similar to the one of CH (though all numbers could certainly vary)
- It may well be that, after convolution with other uncertainties (e.g. the the IU), and especially if the latter have a gaussian form, the detailed shape of the credibility profile returned by the estimation of the MHOU won't matter very much (see backup slides)
    - However, the **size** will still matter of course

Backup slides

Convolutions



Consider three functional forms, each with a "8% uncertainty" (whatever this means):

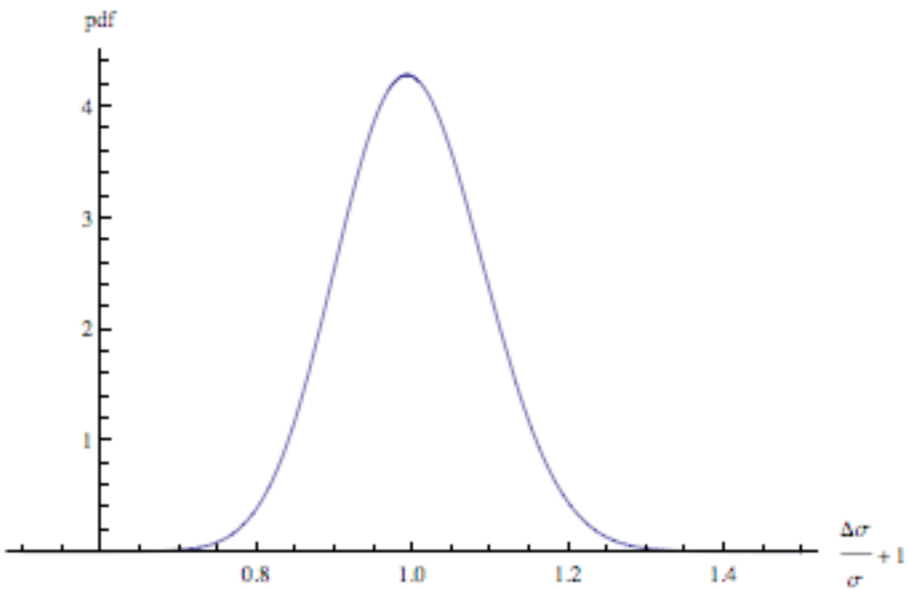
- a box
- a CH-like profile
- a Gaussian

- Of course, the first two are meant to simulate MHOU, while the latter could simulate PDFU

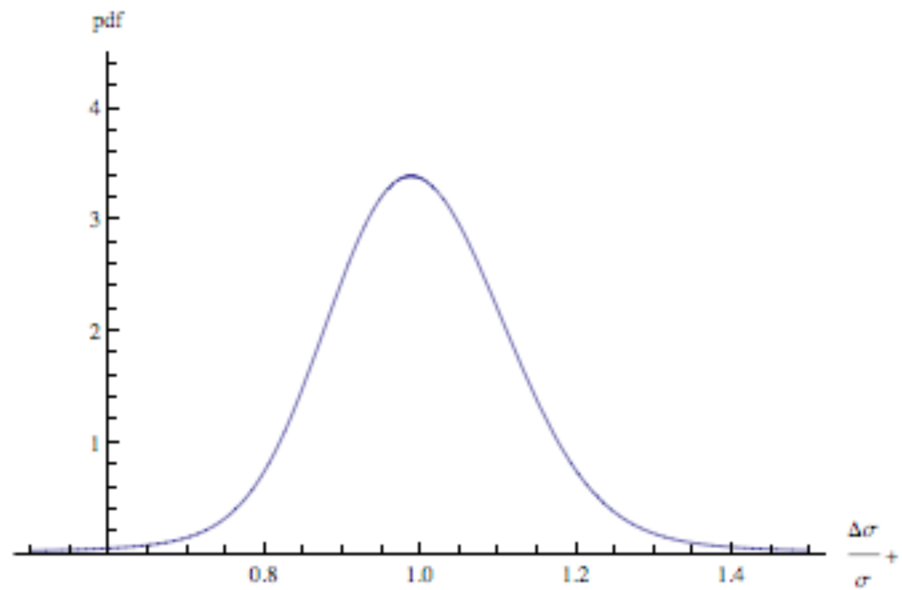
- Consider now alternatively the **convolution** of the box and the CH-like shape with the Gaussian

Convolutions

Convolution of
box and Gaussian



Convolution of
CH-shape and Gaussian



- The box gives a slightly narrower result, but this is only because the box was narrower to begin with, since it had no tails
- Otherwise, **the detailed shape of the MHOU does not seem to have much impact on the final THU shape** (for this choice of shapes and sizes of uncertainties, of course)