Common representations for CSE

1. $N_{pt} \times N_{\lambda}$ correlation matrix $\beta_{k\alpha}$ for N_{λ} random nuisance parameters λ_{α}

$$\chi^2 = \sum_{e=\{\text{expt.}\}} \left[\sum_{k=1}^{N_{pt}} \frac{1}{s_k^2} \left(D_k - T_k(\{z\}) - \sum_{\alpha=1}^{N_{\lambda}} \lambda_{\alpha} \beta_{k\alpha} \right)^2 + \sum_{\alpha=1}^{N_{\lambda}} \lambda_{\alpha}^2 \right]$$

- D_k and T_k are data and theory values $(k = 1, ..., N_{pt})$;
- s_k is the stat.+syst. uncorrelated error;
- $\{z\}$ are PDF parameters;$\{z = 0\}$ in the best fit

2. $N_{pt} \times N_{pt}$ covariance matrix C (not used by CTEQ):

$$\chi^2 = \sum_{k,k'} (D_k - T_k) C^{-1}_{kk'} (D_{k'} - T_{k'})$$

Algebraic solution for CSE parameters λ_a

β and C are related by **algebraic minimization** of χ^2 with respect to λ_α.
If $d_i \equiv D_i - T_i$**;** d_i**,** $\beta_{i\alpha}$ **are given in units of** s_i **for each** $i = 1, ..., N_{pt}$; and **for Gaussian** λ_α:

$$\lambda_\alpha\left(\{z\}\right) = \sum_{\alpha'=1}^{N_\lambda} (\mathcal{A}^{-1})_{\alpha\alpha'} B_{\alpha'}(\{z\})$$

$$\mathcal{A}_{\alpha\alpha'} = \delta_{\alpha\alpha'} + \sum_{i=1}^{N_{pt}} \beta_{\alpha i}\beta_{\alpha' i}; \qquad B_\alpha(\{z\}) = \sum_{i=1}^{N_{pt}} \beta_{\alpha i}(D_i - T_i)$$

$$\chi^2(z, \lambda(z)) = \sum_{k,k'} d_k \left[I - \beta\mathcal{A}^{-1}\beta^T\right]_{kk'} d_{k'} \equiv d^T \left[I - \beta\mathcal{A}^{-1}\beta^T\right] d$$

$$\therefore C = \left(I - \beta\mathcal{A}^{-1}\beta^T\right)^{-1} = I + \beta\beta^T$$

Numerical minimization of $\chi^2(z, \lambda(z))$ establishes the region of acceptable $\{z\}$, which includes the largest possible variations of $\{z\}$ allowed by the systematic effects

An estimate of missing higher-order corrections: basic idea

See also Olness, Soper, arXiv:0907.5052; Cacciari, Houdeau, arXiv:1105.5152

For arbitrary $\mu_{R,F}$, the NLO cross sections in the experimental bins i can be written as

$$\sigma_{bin}^{NLO}(\mu_F,\,\mu_R,\,i) = \sigma_{bin}^{NLO}(\mu_F^{(0)},\,\mu_R^{(0)},\,i)\Big\{1+\sum_{j=1}^{5} e_j(\mu_F^{(0)},\mu_R^{(0)},i)x_j + \mathcal{O}(\alpha_s^3(\mu_R^{(0)}))\Big\}$$

with

$$x_1 = \ln(\frac{\mu_F}{\mu_F^{(0)}}),\qquad x_2 = \ln(\frac{\mu_R}{\mu_R^{(0)}}),\qquad x_3 = \ln^2(\frac{\mu_F}{\mu_F^{(0)}}),$$

$$x_4 = \ln^2(\frac{\mu_R}{\mu_R^{(0)}}),\qquad x_5 = \ln(\frac{\mu_F}{\mu_F^{(0)}})\ln(\frac{\mu_R}{\mu_R^{(0)}}),$$

where $\mu_F^{(0)}$ and $\mu_R^{(0)}$ are the reference scales.

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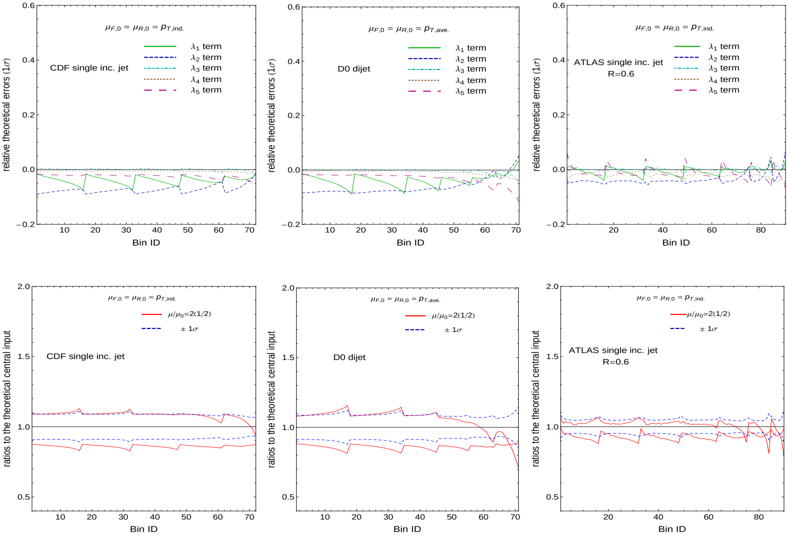
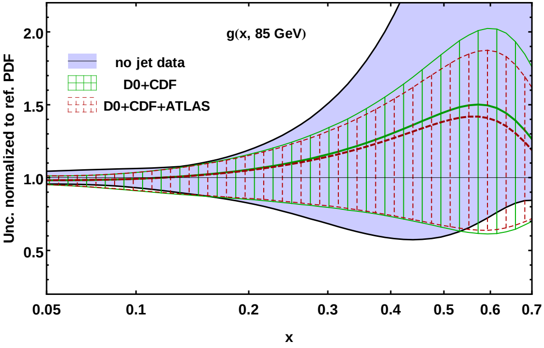
Treat x_i as independent corr. sources with quasi-Gaussian distributions (plausible, but not necessarily true). Assign your favorite confidence level (68% c.l.) to the range $1/2 < \mu_{F,R}/\mu_{F,R}^{(0)} < 2$. Evaluate the variation of $\sigma_{bin}^{NLO}(\mu_F,\, \mu_R,\, i)$ in this scale range. Find $e_j(i)$ numerically and use them to construct the correlation matrix. Reduce the number of principal components to eliminate x_i combinations that have vanishing effect on theory cross sections.

Application of CEMA: correlated theoretical errors for jet cross sections at the Tevatron and LHC



Fit with(out) jet data

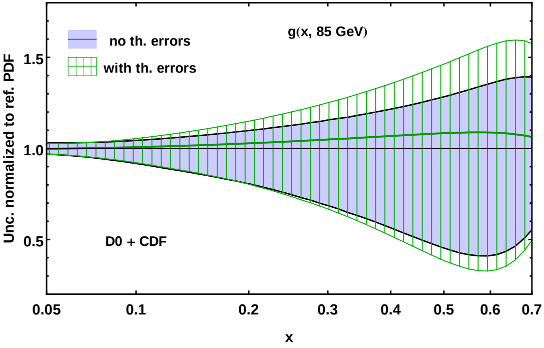
- Tevatron inclusive jet data does impose constraints on the CT10 gluon PDF. The 2010 ATLAS jet data does not strengthen the constraints yet because of large exp. errors.



PRELIMINARY

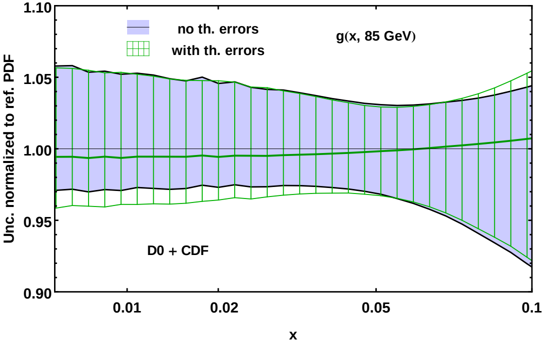
Effects of theoretical errors of jet data

- Gluon PDF uncertainties at 90% C.L. for the fits with and without theoretical errors. Scale dependence of jet cross sections increases the net gluon PDF uncertainty at $x > 0.1$ by about 20%.



PRELIMINARY

- The gluon PDFs in the moderate x region is also affected by the scale dependence errors, as a result of the anti-correlation with the gluon PDF at large x



PRELIMINARY

Conclusions

- We explored a prescription for treating theoretical uncertainties in NLO jet cross sections caused by scale variations as correlated systematic errors. In this approach, scale dependence can increase the gluon PDF uncertainties in the large x region by about 20%, also in the moderate x region by about 10% indirectly.