

# Fixed Order and Resummed Uncertainties with the Stewart-Tackmann Method

---

Jon Walsh, UC Berkeley

based on discussions with Ye Li, Xiaohui Liu, Frank Petriello, Frank Tackmann

# ST Method for Fixed Order Cross Sections

---

Basic ansatz of ST method:

*Uncertainties in inclusive jet bins are uncorrelated*

Using the ansatz, any element of covariance matrices for any set of inclusive, exclusive jet bins can be determined

$$\Delta_{\text{FO}}^2(\geq n, = m) = \begin{cases} \Delta_{\geq n}^2 & \text{if } m = n, \\ -\Delta_{\geq n}^2 & \text{if } m = n - 1, \\ 0 & \text{otherwise.} \end{cases}$$

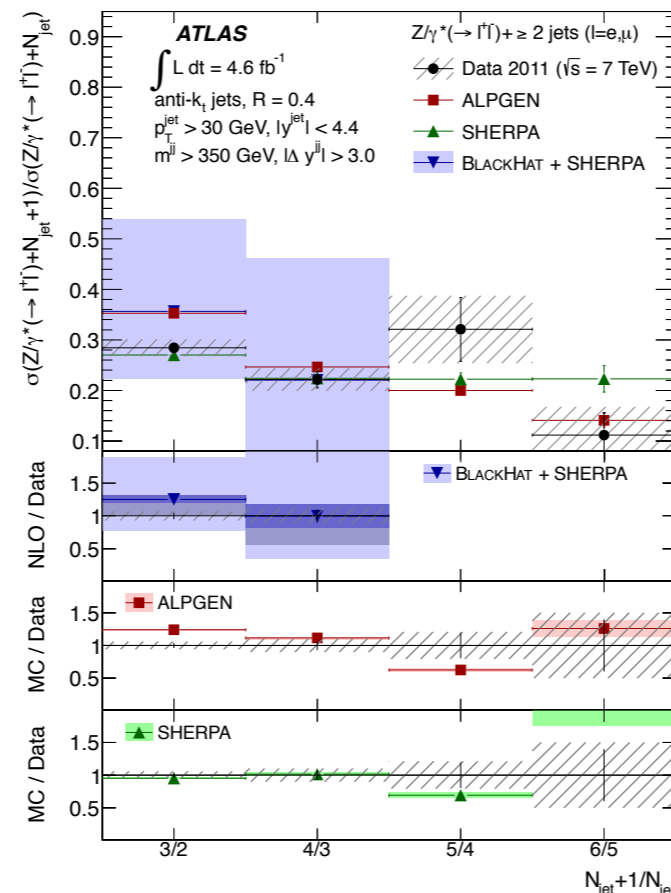
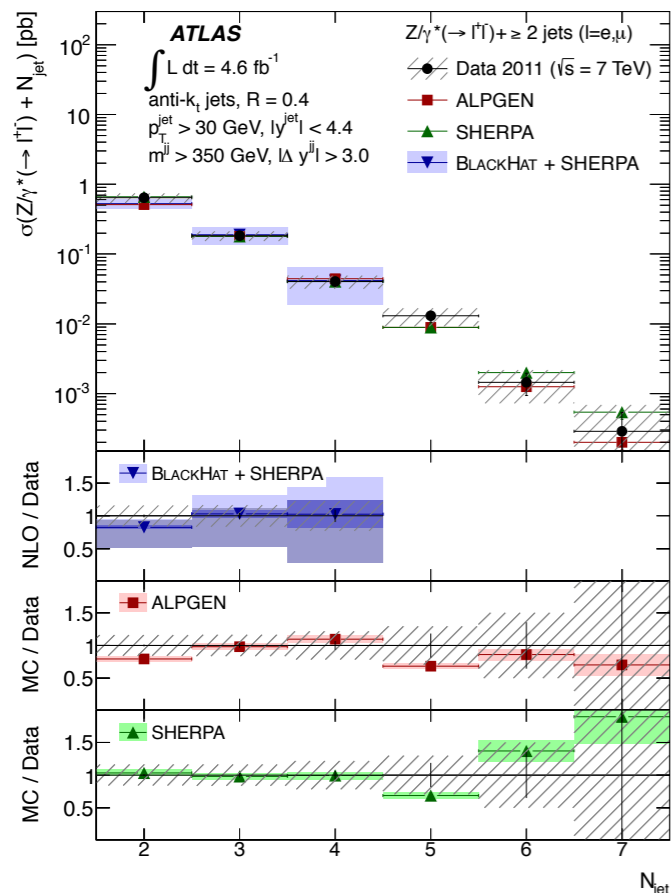
$$\Delta_{\text{FO}}^2(= n, = m) = \begin{cases} \Delta_{\geq n}^2 + \Delta_{\geq n+1}^2 & \text{if } m = n, \\ -\Delta_{\geq n}^2 & \text{if } m = n - 1, \\ -\Delta_{\geq n+1}^2 & \text{if } m = n + 1, \\ 0 & \text{otherwise.} \end{cases}$$

# Ratios of Exclusive Cross Sections

Interesting case is ratios of exclusive cross sections  $R_n = \frac{\sigma_{n+1}}{\sigma_n}$

$$\left( \frac{\Delta_{=n}^2}{\sigma_n^2} + \frac{\Delta_{=n+1}^2}{\sigma_{n+1}^2} \right)^{1/2} < \frac{\Delta R_n}{R_n} < \frac{\Delta_{=n}}{\sigma_n} + \frac{\Delta_{=n+1}}{\sigma_{n+1}}$$

Pretty tight bound on fractional uncertainty of  $R_n$  in terms of fractional uncertainties in exclusive bins - *useful cross check*



The ST-derived uncertainties in these ratios are relatively large

Are these uncertainties reasonable, or is there a better ansatz?

Is there a general class of observables like this?

# Resummation Uncertainties

---

Goal: have a consistent framework to assess uncertainties  
in exclusive 0-jet, 1-jet, and inclusive 2-jet bins

Banfi, Monni, Salam, Zanderighi  
Becher, Neubert, Rothen  
Stewart, Tackmann, Walsh, Zuberi



exclusive 0-jet cross section  
0-jet efficiency  
inclusive 1-jet cross section

Liu, Petriello



exclusive 1-jet cross section  
1-jet efficiency  
inclusive 2-jet cross section

ATLAS/CMS will use the results of these studies (in some form) for their analysis  
*unified uncertainty framework will make results more applicable*

# Resummation Uncertainties

---

With resummation, the ST ansatz(es) differ and are (subjectively) more robust

*property:* uncertainty comes from two sources: fixed order and resummation, which can be separately estimated,  $C = C_\mu + C_{\text{res}}$  (FO + resum)

$$C_\mu = \begin{pmatrix} \Delta_{\mu\text{tot}}^2 & \Delta_{\mu\text{tot}}\Delta_{\mu 0} & \Delta_{\mu\text{tot}}\Delta_{\mu \geq 1} & \Delta_{\mu\text{tot}}\Delta_{\mu 1} & \Delta_{\mu\text{tot}}\Delta_{\mu \geq 2} \\ \Delta_{\mu\text{tot}}\Delta_{\mu 0} & \Delta_{\mu 0}^2 & \Delta_{\mu 0}\Delta_{\mu \geq 1} & \Delta_{\mu 0}\Delta_{\mu 1} & \Delta_{\mu 0}\Delta_{\mu \geq 2} \\ \Delta_{\mu\text{tot}}\Delta_{\mu \geq 1} & \Delta_{\mu 0}\Delta_{\mu \geq 1} & \Delta_{\mu \geq 1}^2 & \Delta_{\mu \geq 1}\Delta_{\mu 1} & \Delta_{\mu \geq 1}\Delta_{\mu \geq 2} \\ \Delta_{\mu\text{tot}}\Delta_{\mu 1} & \Delta_{\mu 0}\Delta_{\mu 1} & \Delta_{\mu \geq 1}\Delta_{\mu 1} & \Delta_{\mu 1}^2 & \Delta_{\mu 1}\Delta_{\mu \geq 2} \\ \Delta_{\mu\text{tot}}\Delta_{\mu \geq 2} & \Delta_{\mu 0}\Delta_{\mu \geq 2} & \Delta_{\mu \geq 1}\Delta_{\mu \geq 2} & \Delta_{\mu 1}\Delta_{\mu \geq 2} & \Delta_{\mu \geq 2}^2 \end{pmatrix}$$

basis:  $\{\sigma_{\geq 0}, \sigma_{=0}, \sigma_{\geq 1}, \sigma_{=1}, \sigma_{\geq 2}\}$

$$\Delta_{\mu\text{tot}} = \Delta_{\mu 0} + \Delta_{\mu \geq 1},$$

$$\Delta_{\mu\text{tot}} = \Delta_{\mu 0} + \Delta_{\mu 1} + \Delta_{\mu \geq 2}.$$

*ansatz:* fixed order uncertainties in different jet bins are totally correlated

# Resummation Uncertainties

---

basis:  $\{\sigma_{\geq 0}, \sigma_{=0}, \sigma_{\geq 1}, \sigma_{=1}, \sigma_{\geq 2}\}$

$$C_{\text{res}} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & \Delta_{\text{res}=0}^2 & -\Delta_{\text{res}=0}^2 & 0 & -\Delta_{\text{res}=0}^2 \\ 0 & -\Delta_{\text{res}=0}^2 & \Delta_{\text{res}=0}^2 & 0 & \Delta_{\text{res}=0}^2 \\ 0 & 0 & 0 & \Delta_{\text{res}=1}^2 & -\Delta_{\text{res}=1}^2 \\ 0 & -\Delta_{\text{res}=0}^2 & \Delta_{\text{res}=0}^2 & -\Delta_{\text{res}=1}^2 & \Delta_{\text{res}=0}^2 + \Delta_{\text{res}=1}^2 \end{pmatrix}$$

*ansatz:*  $\Delta(=0, =1) = 0$

reasonable since the 0-jet, 1-jet exclusive uncertainties are separately estimated

total incl.:  $\Delta_{\text{tot}}^2 = \Delta_{\mu\text{tot}}^2,$

excl. 0-jet:  $\Delta_0^2 = \Delta_{\mu 0}^2 + \Delta_{\text{res}=0}^2,$

excl. 1-jet:  $\Delta_1^2 = \Delta_{\mu 1}^2 + \Delta_{\text{res}=1}^2,$

incl. 1-jet:  $\Delta_{\geq 1}^2 = (\Delta_{\mu\text{tot}} - \Delta_{\mu 0})^2 + \Delta_{\text{res}=0}^2,$

incl. 2-jet:  $\Delta_{\geq 2}^2 = (\Delta_{\mu\text{tot}} - \Delta_{\mu 0} - \Delta_{\mu 1})^2 + \Delta_{\text{res}=0}^2 + \Delta_{\text{res}=1}^2.$

# Estimating Uncertainties

---

need to estimate resummed and fixed order uncertainties for the resummed cross section

$$\sigma_0(\mu_H, \mu_{\text{ns}}, \mu_B, \mu_S, \nu_B, \nu_S)$$

$\Delta_{\mu_0}$  : vary *all* scales up and down collectively (by 2, 1/2)  
also can vary resummation profile shape

$\Delta_{\text{res}=0}$  : vary (groups of) individual scales up and down

} take envelope  
in each case

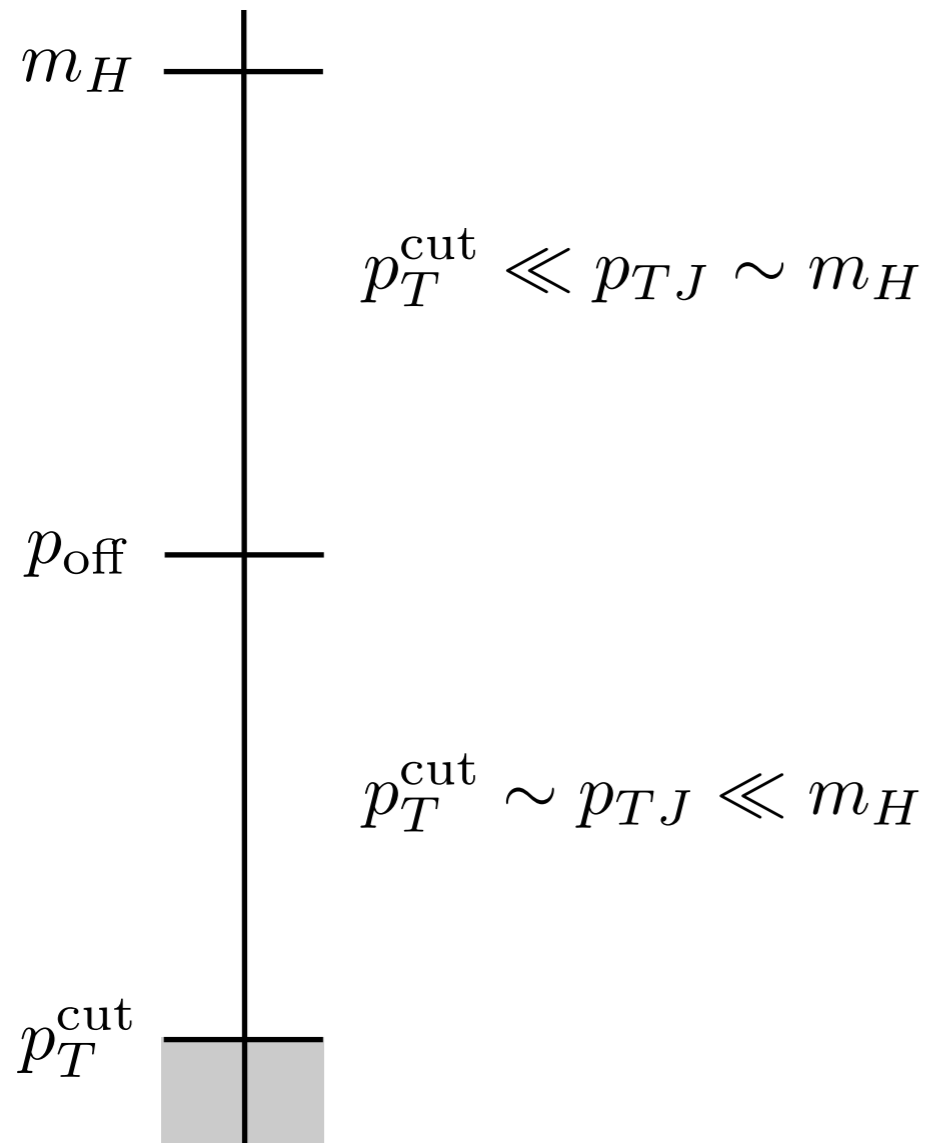
scale variation more subjective than fixed order

same basic ideas can be reapplied to other resummed cross sections (e.g. 1-jet)

# Open Questions: 1-Jet Bin

---

two regions of  $p_{TJ}$  for 1-jet bin



resummation performed by Petriello, Liu  
looking at common uncertainty scheme

currently described with FO

what uncertainty is correct?  
direct scale variation may have pinches  
looking at options