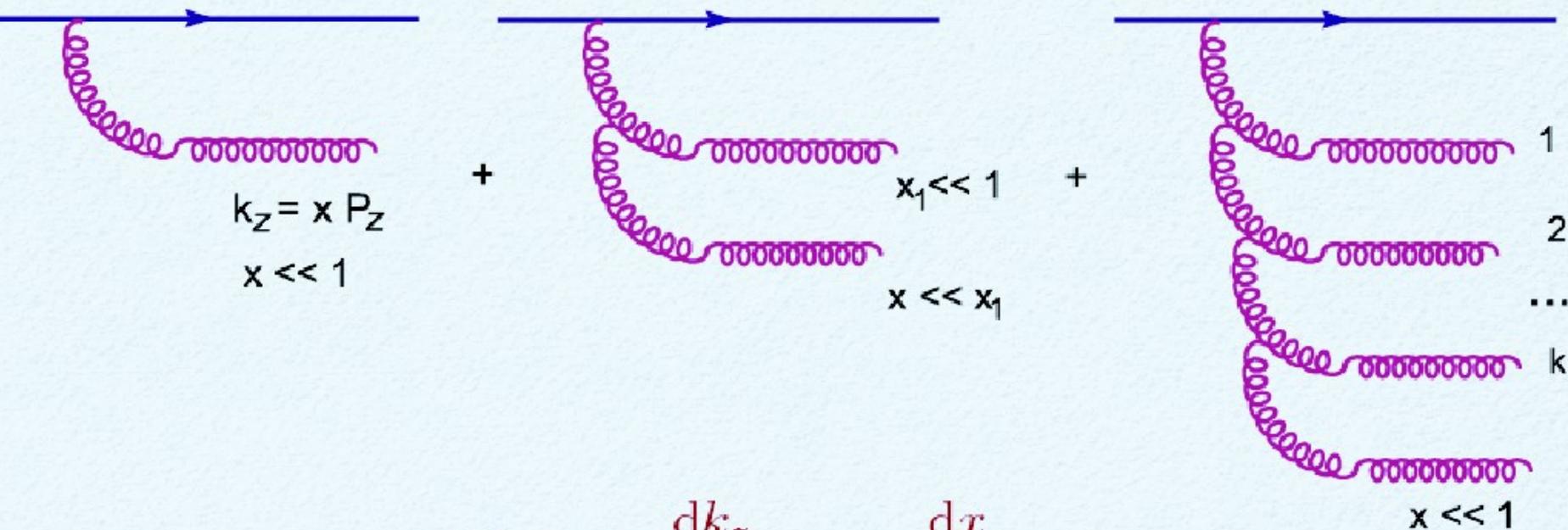


**CGC
and
Two-particle correlations
at the LHC**

*Jamal Jalilian-Marian
Baruch College, New York NY*

gluon radiation at small x : pQCD

The infrared sensitivity of bremsstrahlung favors the emission of 'soft' (= small- x) gluons



$$d\mathcal{P} \propto \alpha_s \frac{dk_z}{k_z} = \alpha_s \frac{dx}{x}$$

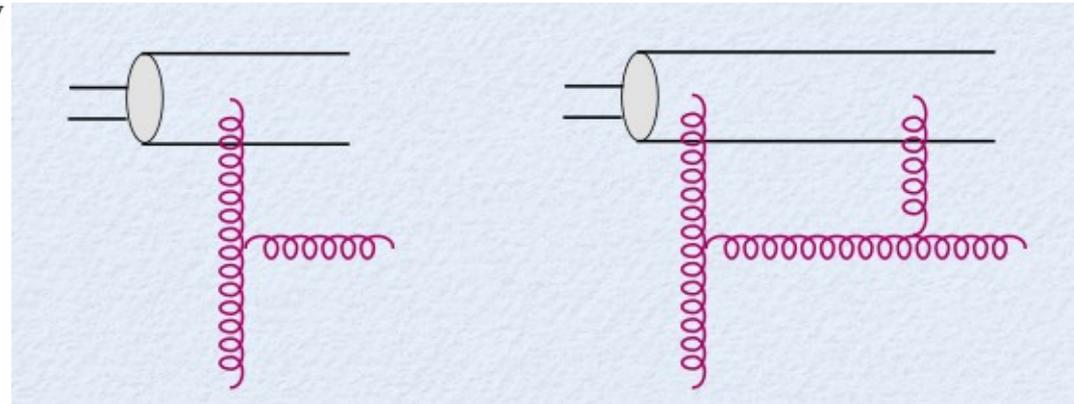
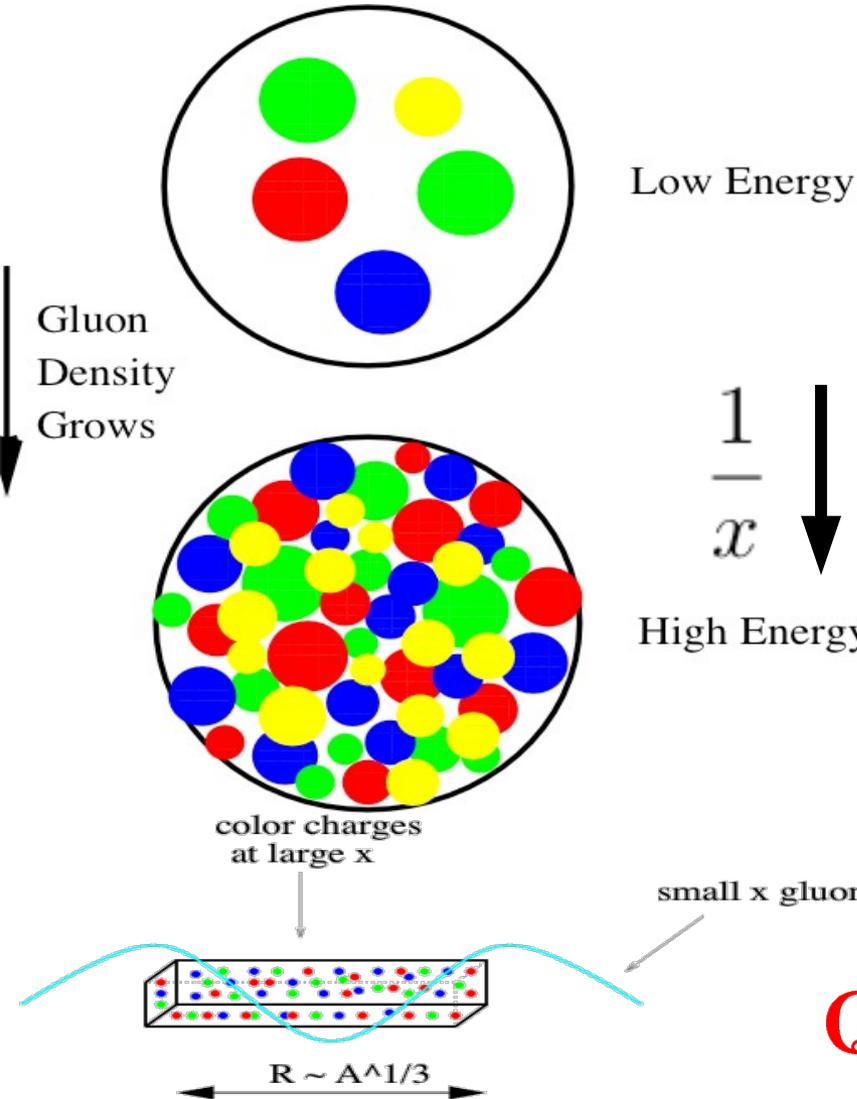
The 'price' of an additional gluon:

$$\mathcal{P}(1) \propto \alpha_s \int_x^1 \frac{dx_1}{x_1} = \alpha_s \ln \frac{1}{x} \quad \text{number of gluons grows fast} \quad n \sim e^{\alpha_s \ln 1/x}$$

Gluon saturation

*Gribov-Levin-Ryskin
Mueller-Qiu*

**“attractive” bremsstrahlung
vs. “repulsive” recombination**



$$\frac{\alpha_s x G(x, b_t, Q^2)}{S_\perp Q^2} \sim 1$$

$$Q_s^2(x, b_t, A) \sim A^{1/3} \left(\frac{1}{x}\right)^{0.3}$$

MV effective Action + RGE

$$S[\mathbf{A}, \rho] = -\frac{1}{4} \int d^4x F_{\mu\nu}^2 + \frac{i}{N_c} \int d^2x_t dx^- \delta(x^-) \text{Tr}[\rho(x_t) \mathbf{U}(\mathbf{A}^-)]$$

Large x: color source ρ **small x: gluon field \mathbf{A}^μ**

$$\mathbf{U}(\mathbf{A}^-) = \hat{\mathbf{P}} \text{Exp} \left[ig \int dx^+ \mathbf{A}_a^- \mathbf{T}_a \right]$$

$$\mathbf{Z}[\mathbf{j}] = \int [\mathbf{D}\rho] \mathbf{W}_{\Lambda^+}[\rho] \left[\frac{\int^{\Lambda^+} [\mathbf{D}\mathbf{A}] \delta(\mathbf{A}^+) e^{iS[\mathbf{A}, \rho] - \int \mathbf{j} \cdot \mathbf{A}}}{\int^{\Lambda^+} [\mathbf{D}\mathbf{A}] \delta(\mathbf{A}^+) e^{iS[\mathbf{A}, \rho]}} \right]$$

weight functional:

**$\mathbf{W}_{\Lambda^+}[\rho]$ probability distribution of color source ρ
at longitudinal scale Λ^+**

invariance under change of $\Lambda^+ \longrightarrow$ RGE for $\mathbf{W}_{\Lambda^+}[\rho]$

The Classical Field

saddle point of effective action \rightarrow Yang-Mills equations

$$\mathbf{D}_\mu \mathbf{F}_a^{\mu\nu} = \delta^\nu + \delta(\mathbf{x}^-) \rho_a(\mathbf{x}_t)$$

solutions are non-Abelian
Weizsäcker-Williams fields

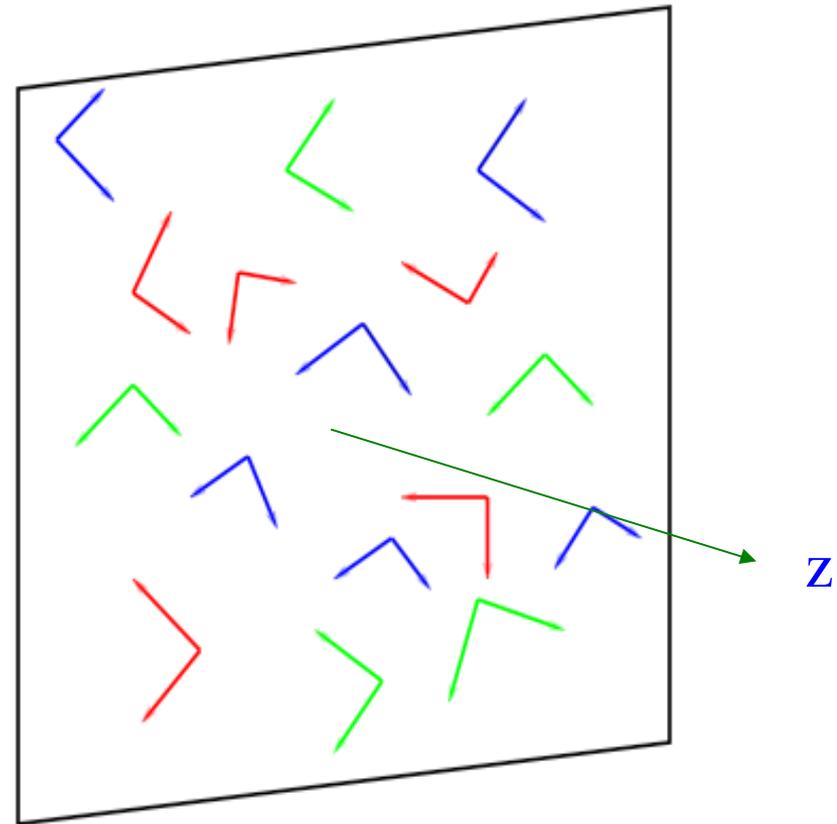
$$\mathbf{A}^+ = \mathbf{0}$$

$$\mathbf{A}^- = \mathbf{0}$$

$$\mathbf{A}_a^i = \theta(\mathbf{x}^-) \alpha_a^i(\mathbf{x}_t)$$

$$\partial^i \alpha_a^i = g \rho_a$$

pure (2d) gauge



color $\mathbf{E}_\perp, \mathbf{B}_\perp$ fields

Energy (\mathbf{x}) dependence

JIMWLK-BK evolution equation

$$\frac{d}{dy} \langle O \rangle = \frac{1}{2} \left\langle \int d^2x d^2y \frac{\delta}{\delta \alpha_x^b} \eta_{xy}^{bd} \frac{\delta}{\delta \alpha_y^d} O \right\rangle$$

$$\eta_{xy}^{bd} = \frac{1}{\pi} \int \frac{d^2z}{(2\pi)^2} \frac{(x-z) \cdot (y-z)}{(x-z)^2 (y-z)^2} [1 + U_x^\dagger U_y - U_x^\dagger U_z - U_z^\dagger U_y]^{bd}$$

U is a Wilson line in adjoint representation

CGC: QCD at high gluon density

effective degrees of freedom: Wilson line $V(\mathbf{x}_t)$

CGC observables: $\langle \text{Tr} V \dots V^\dagger \rangle$ with $V(\mathbf{x}_t) = \hat{P} e^{ig \int dx^- A_a^+ t_a}$

$$A_a^\mu(\mathbf{x}_t, \mathbf{x}^-) \sim \delta^{\mu+} \delta(\mathbf{x}^-) \alpha_a(\mathbf{x}_t) \quad \alpha^a(\mathbf{k}_t) = g \rho^a(\mathbf{k}_t) / k_t^2$$

gluon distribution: $xG(x, Q^2) \sim \int^{Q^2} \frac{d^2 k_t}{k_t^2} \phi(x, \mathbf{k}_t)$ with $\phi(x, \mathbf{k}_t) \sim \langle \rho_a^*(\mathbf{k}_t) \rho_a(\mathbf{k}_t) \rangle$

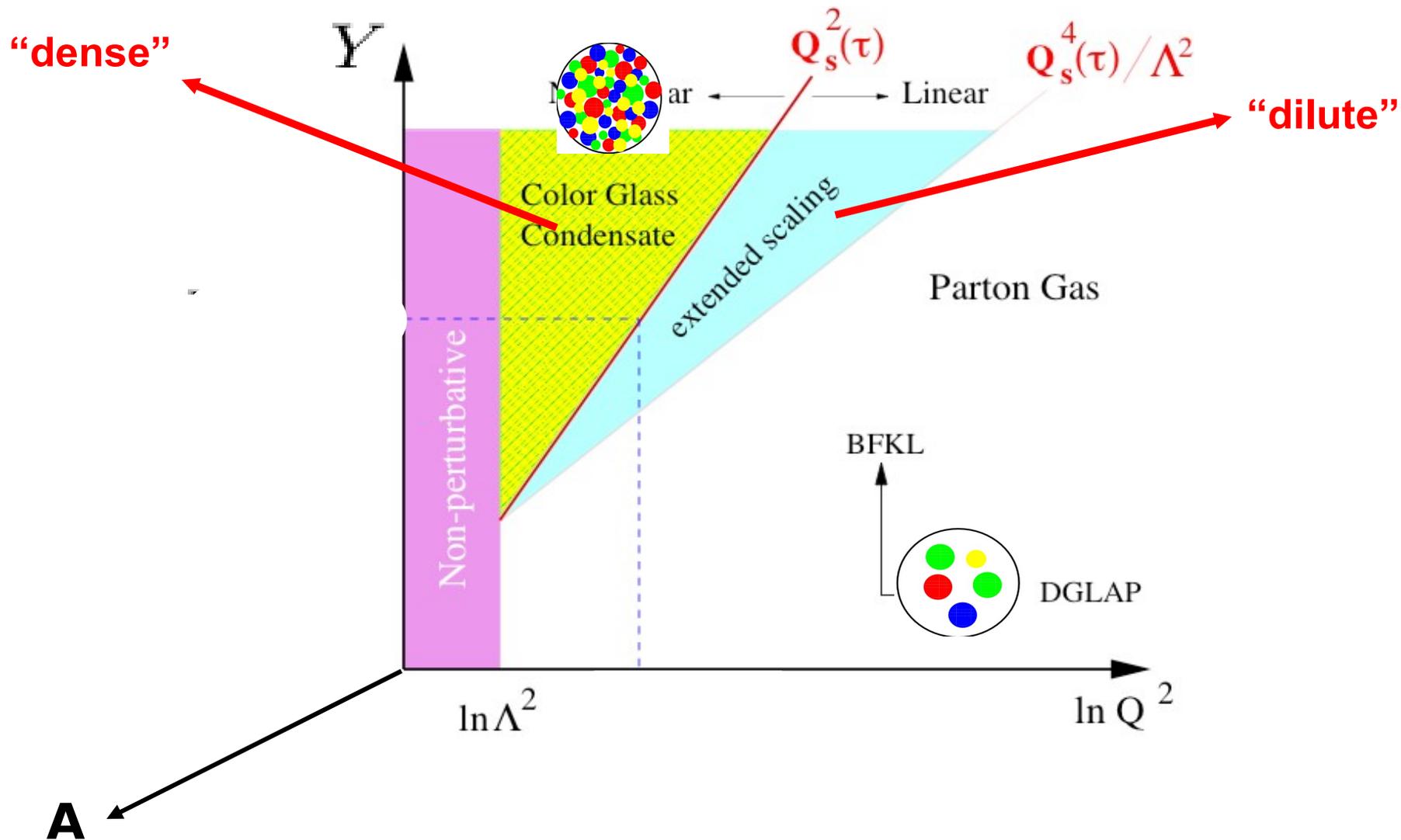
two main effects:

multiple scatterings \longrightarrow p_t broadening, Cronin effect

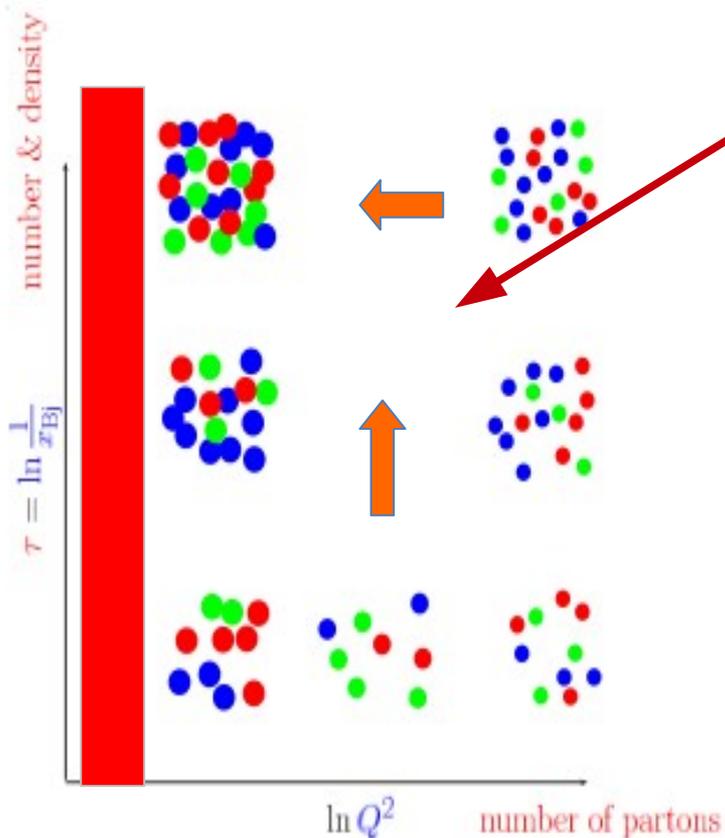
evolution with $\ln(1/x)$ \longrightarrow suppression

“Leading twist” nuclear shadowing

Road Map of QCD Phase Space



Many-body dynamics of universal gluonic matter



How does this happen ?

How do correlation functions of these evolve ?

Is there a universal fixed point for the RG evolution of d.o.f

How does the coupling run ?

Pre-equilibrium stage of HI collisions: thermalization,

Color *Glass* Condensate

Advantages:

A systematic, first-principle approach to high energy scattering in QCD

Controlled approximations

Same formalism can describe a wide range of phenomena

Disadvantages:

Applicable only at low x (high x , Q^2 missing)

Observables

DIS:

structure functions

particle production

dilute-dense (pA, forward pp) collisions:

multiplicities

single inclusive spectra

2-particle correlations

dense-dense (AA, pp) collisions:

multiplicities, spectra

long range rapidity correlations

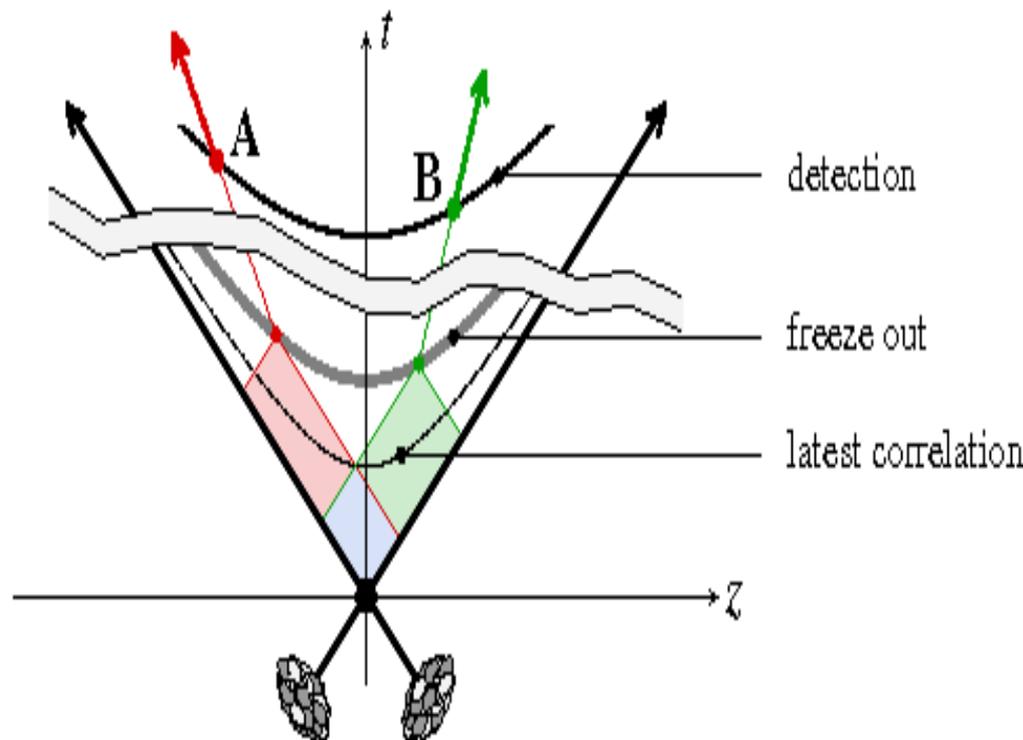
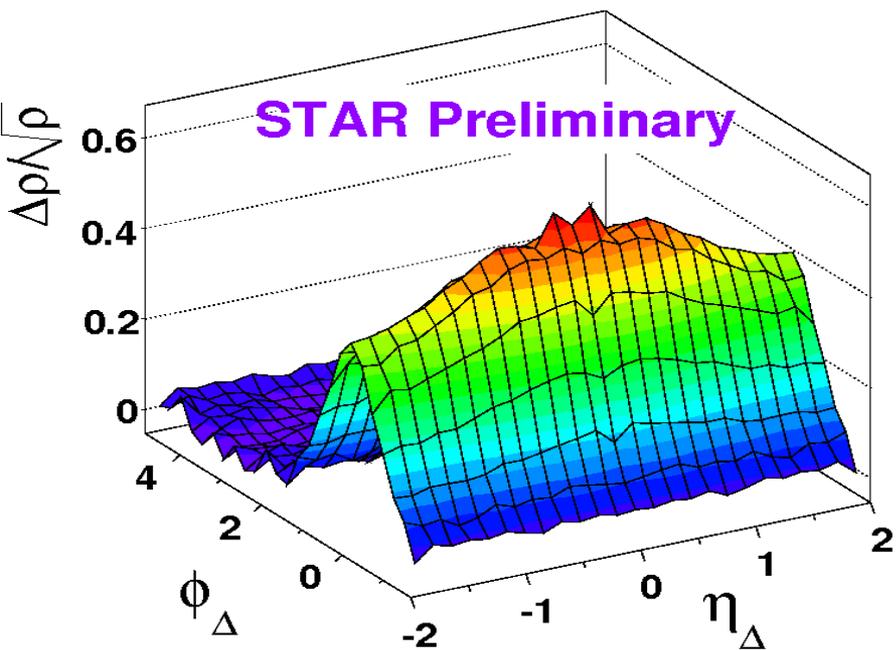
Spin asymmetries

Two-hadron correlations

near-side long-range rapidity correlations: the Ridge

away-side correlations in dA: forward rapidity

Ridge in AA



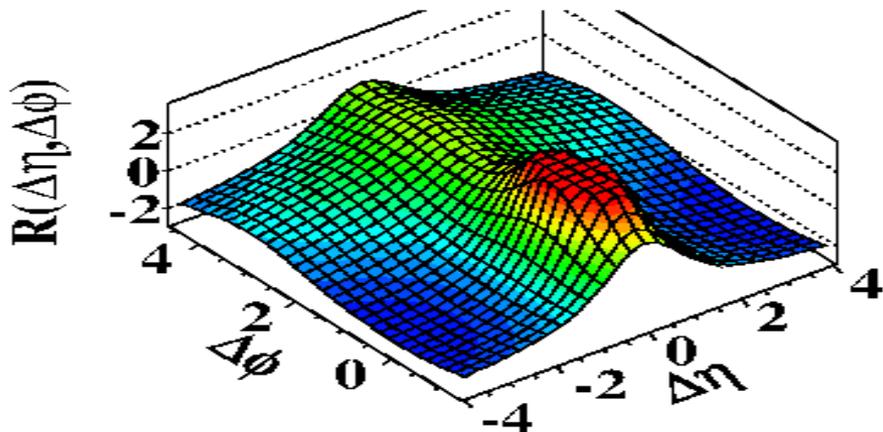
$$\tau \leq \tau_{fo} e^{-\frac{1}{2}|y_A - y_B|}$$

DGMV: NPA810 (2008) 91

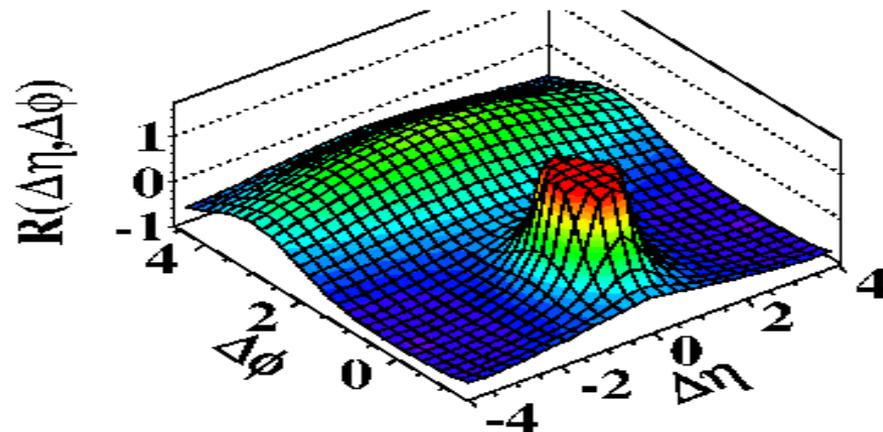
late time interactions can not affect long-range rapidity correlations

The CMS ridge at LHC

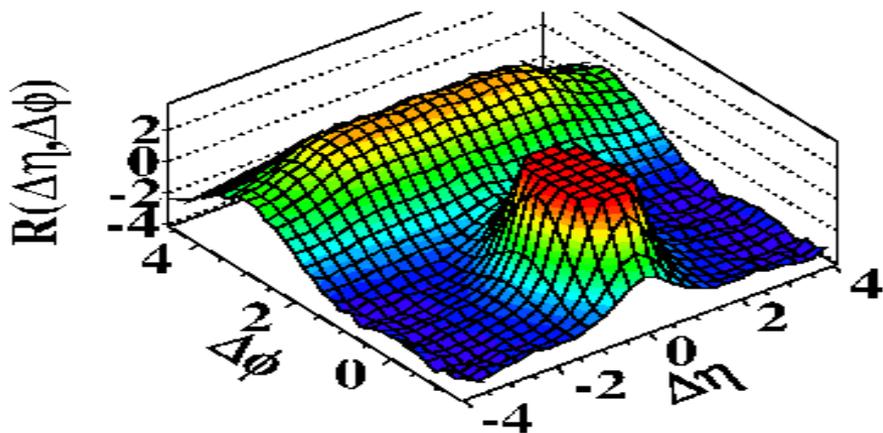
(a) CMS MinBias, $p_T > 0.1 \text{ GeV}/c$



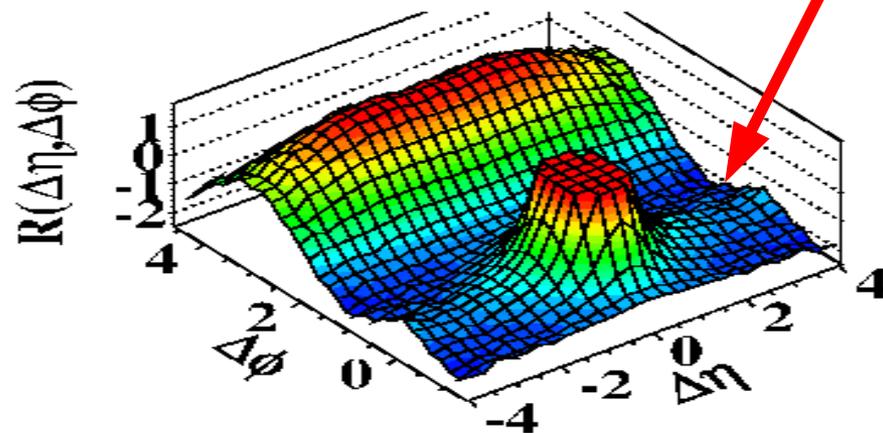
(b) CMS MinBias, $1.0 \text{ GeV}/c < p_T < 3.0 \text{ GeV}/c$



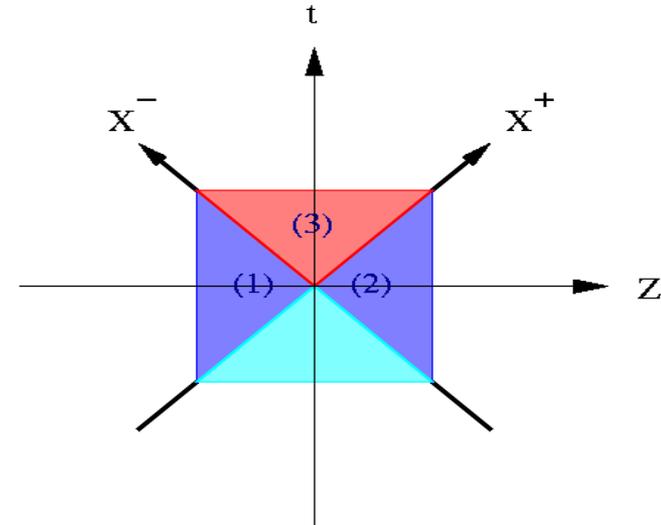
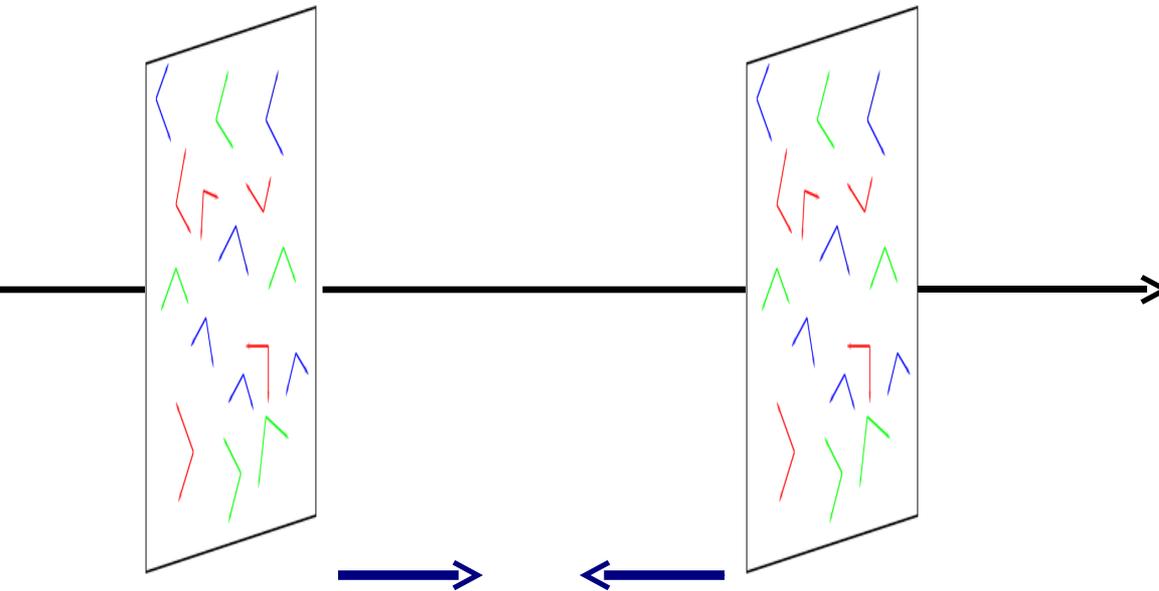
(c) CMS $N \geq 110$, $p_T > 0.1 \text{ GeV}/c$



(d) CMS $N \geq 110$, $1.0 \text{ GeV}/c < p_T < 3.0 \text{ GeV}/c$



Colliding Sheets of Color Glass



before the collision:

$$\mathbf{A}^+ = \mathbf{A}^- = \mathbf{0}$$

$$\mathbf{A}^i = \mathbf{A}_1^i + \mathbf{A}_2^i$$

$$\mathbf{A}_1^i = \theta(x^-)\theta(-x^+)\alpha_1^i$$

$$\mathbf{A}_2^i = \theta(-x^-)\theta(x^+)\alpha_2^i$$

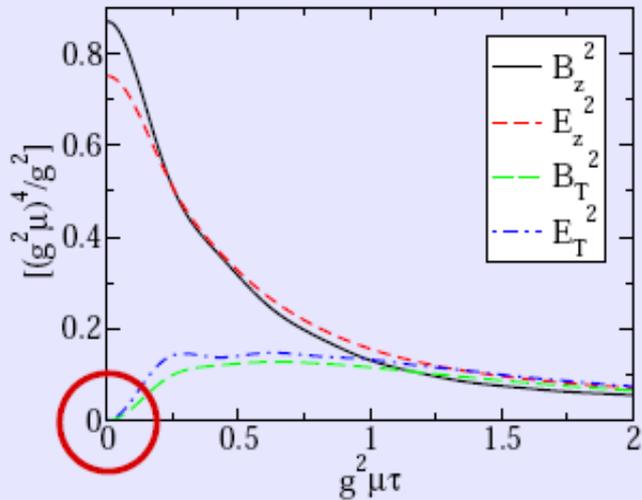
after the collision:

solve for \mathbf{A}_μ

in the forward LC

GLASMA:

gluon fields produced in collision of two sheets of color glass



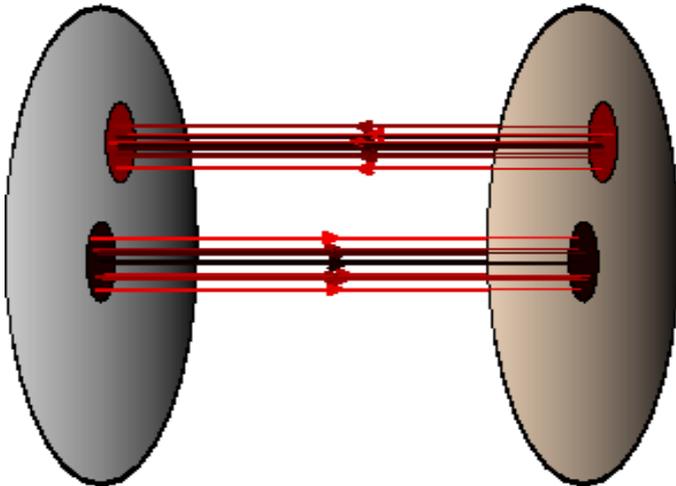
Early on glasma fields (E and B) are longitudinal

Lappi+McLerran. NPA772 (2006) 200

Classical solutions are boost invariant

Transverse size of these flux tubes is $\sim \frac{1}{Q_s}$

can be solved numerically



Two-gluon correlation: dilute region

DGMV, NPA810 (2008) 91

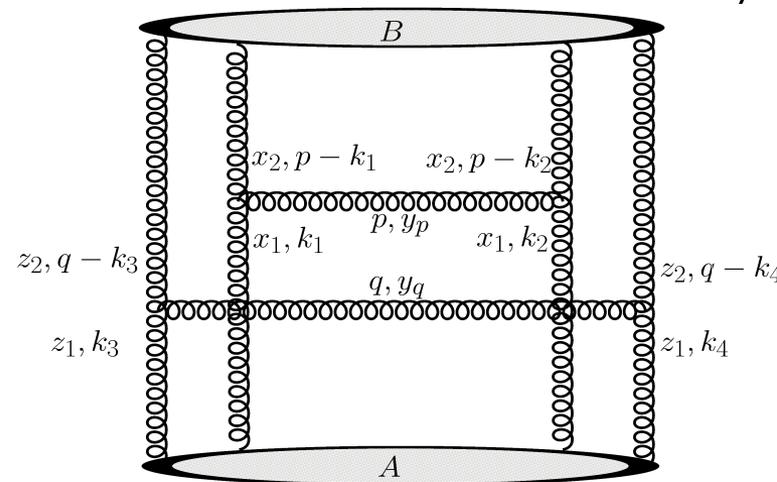
$$\begin{aligned}
 C(p_{\perp}, q_{\perp}) = & \frac{g^{12}}{64(2\pi)^6} (f_{abc} f_{a'\bar{b}\bar{c}} f_{a\hat{b}\hat{c}} f_{a'\tilde{b}\tilde{c}}) \int \prod_{i=1}^4 \frac{d^2 k_{i\perp}}{(2\pi)^2 k_{i\perp}^2} \\
 & \times \frac{L_{\mu}(p_{\perp}, k_{1\perp}) L^{\mu}(p_{\perp}, k_{2\perp}) L_{\nu}(q_{\perp}, k_{3\perp}) L^{\nu}(q_{\perp}, k_{4\perp})}{(p_{\perp} - k_{1\perp})^2 (p_{\perp} - k_{2\perp})^2 (q_{\perp} - k_{3\perp})^2 (q_{\perp} - k_{4\perp})^2} \\
 & \times \left\langle \rho_1^{*\hat{b}}(k_{2\perp}) \rho_1^{*\tilde{b}}(k_{4\perp}) \rho_1^b(k_{1\perp}) \rho_1^{\bar{b}}(k_{3\perp}) \right\rangle \quad \leftarrow \text{Gaussian averaging} \\
 & \times \left\langle \rho_2^{*\hat{c}}(p_{\perp} - k_{2\perp}) \rho_2^{*\tilde{c}}(q_{\perp} - k_{4\perp}) \rho_2^c(p_{\perp} - k_{1\perp}) \rho_2^{\bar{c}}(q_{\perp} - k_{3\perp}) \right\rangle
 \end{aligned}$$

assume Gaussian factorization

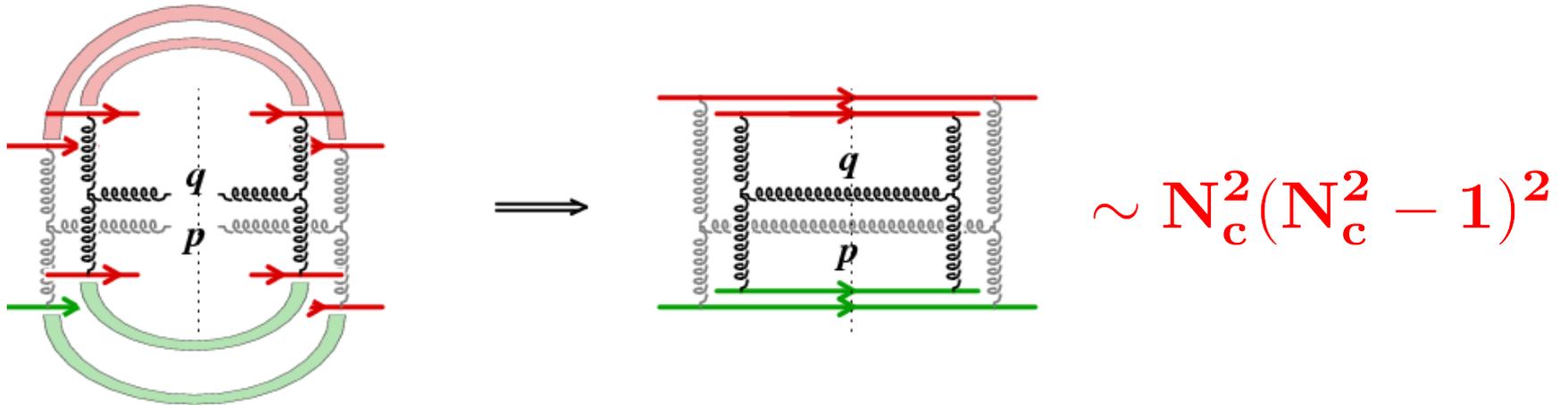
$$\langle \rho \cdots \rho \rangle \sim \langle \rho^2 \rangle \cdots \langle \rho^2 \rangle$$

un-integrated gluon distribution

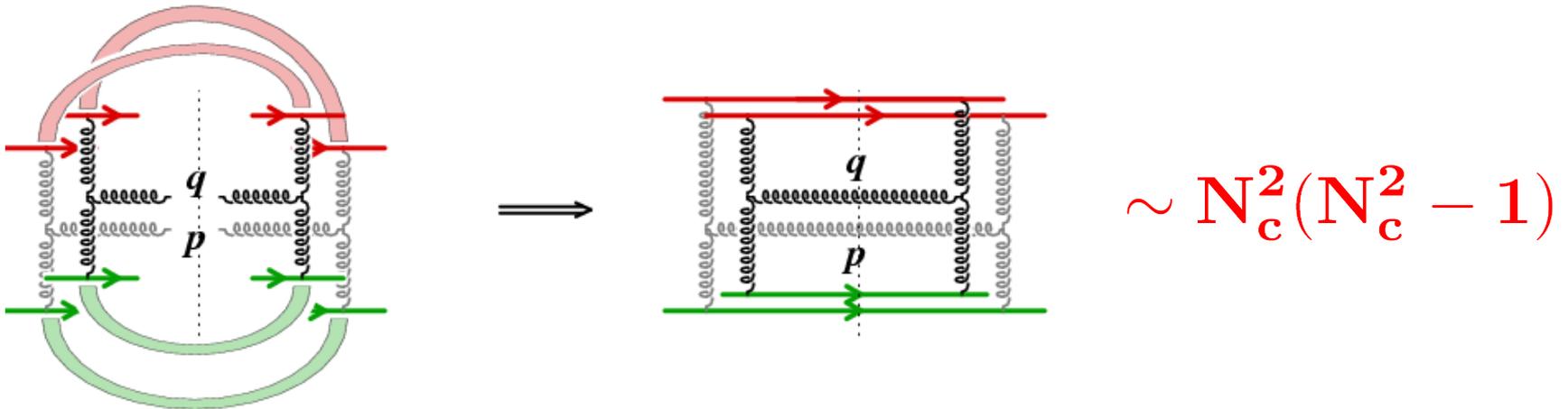
$$\phi(\mathbf{x}, \mathbf{k}_t^2) \sim \langle \rho^2 \rangle$$



Independent production of two gluons (subtracted):



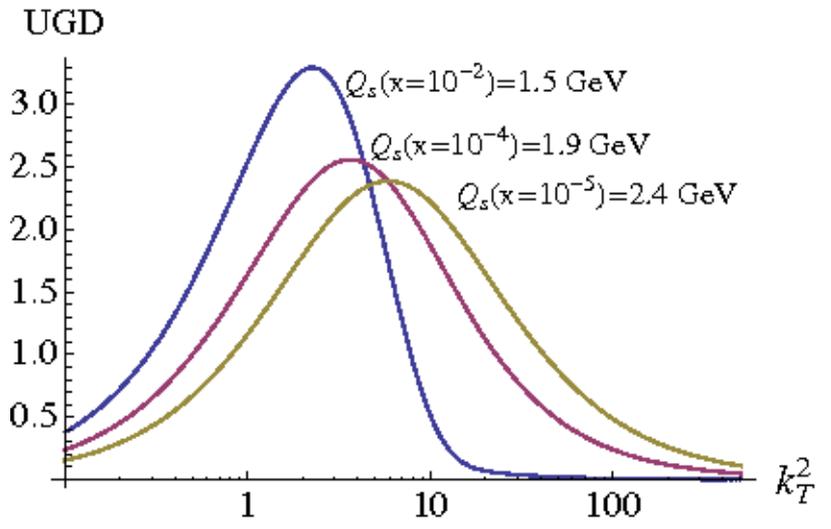
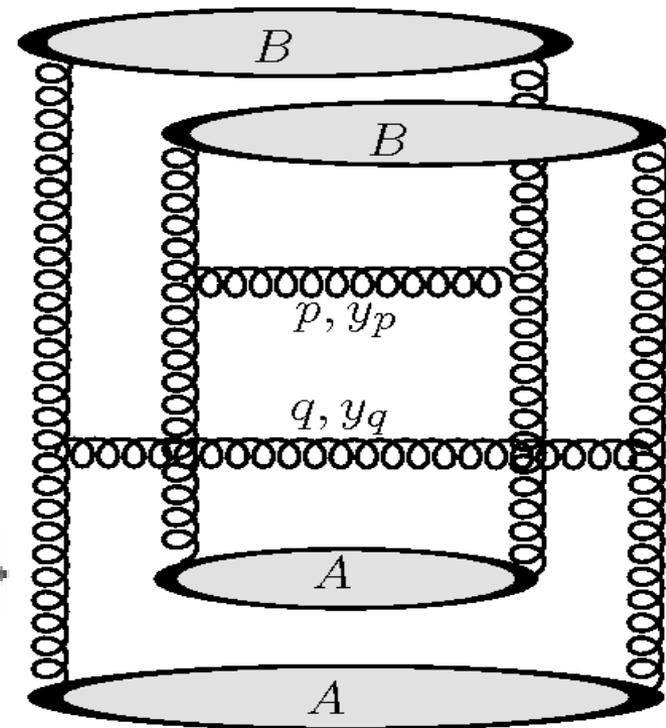
Correlated two-gluon production:



Correlated production is suppressed by N_c^2

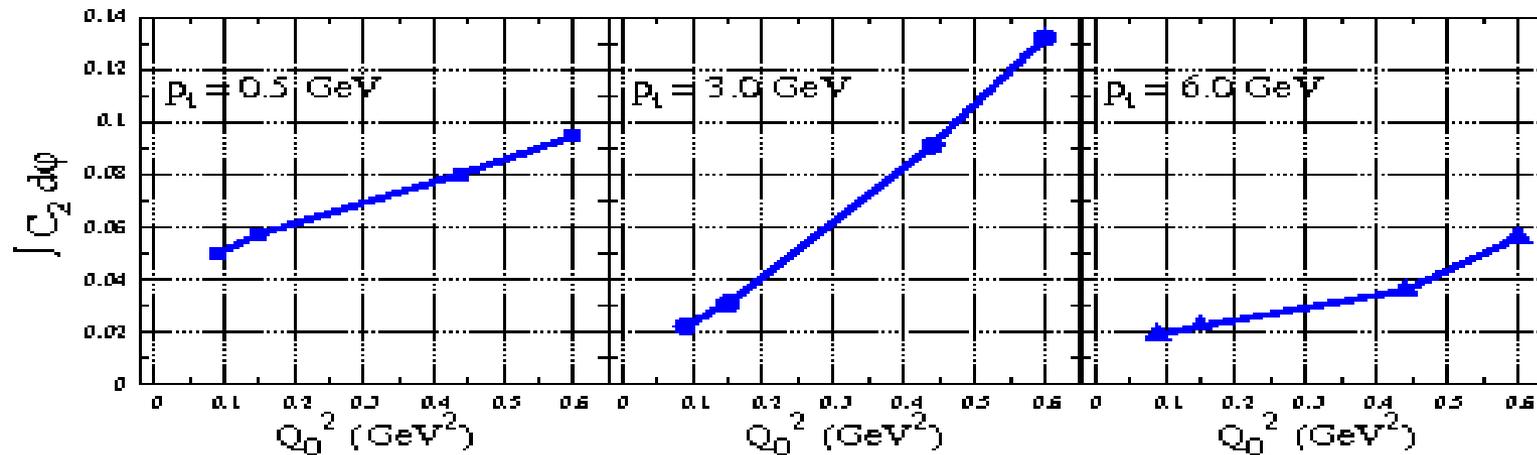
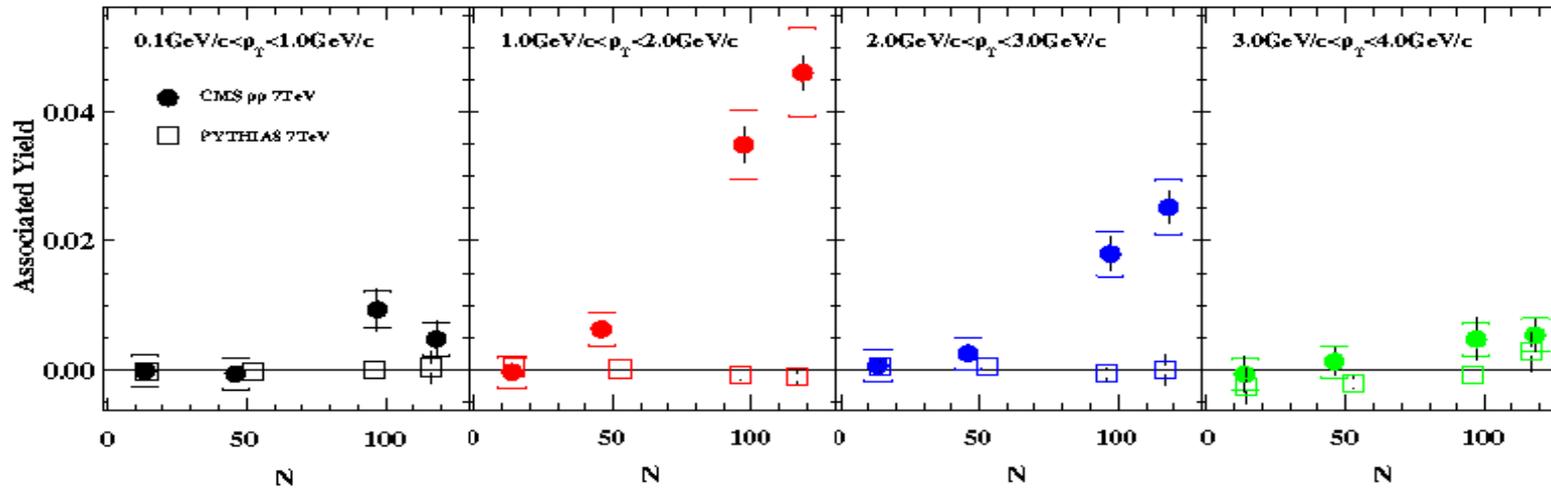
Two-gluon production in AA/pp

$$\frac{dN_2}{d^2p_\perp dy_p d^2q_\perp dy_q} = \frac{\alpha_s^2}{16\pi^{10}} \frac{N_c^2 S_\perp}{(N_c^2 - 1)^3 p_\perp^2 q_\perp^2} \times \int d^2k_\perp \left\{ \Phi_A^2(y_p, k_\perp) \Phi_B(y_p, p_\perp - k_\perp) \times [\Phi_B(y_q, q_\perp + k_\perp) + \Phi_B(y_q, q_\perp - k_\perp)] + \Phi_B^2(y_q, k_\perp) \Phi_A(y_p, p_\perp - k_\perp) + \Phi_A^2(y_q, k_\perp) \Phi_A(y_p, p_\perp - k_\perp) \times [\Phi_A(y_q, q_\perp + k_\perp) + \Phi_A(y_q, q_\perp - k_\perp)] \right\}$$



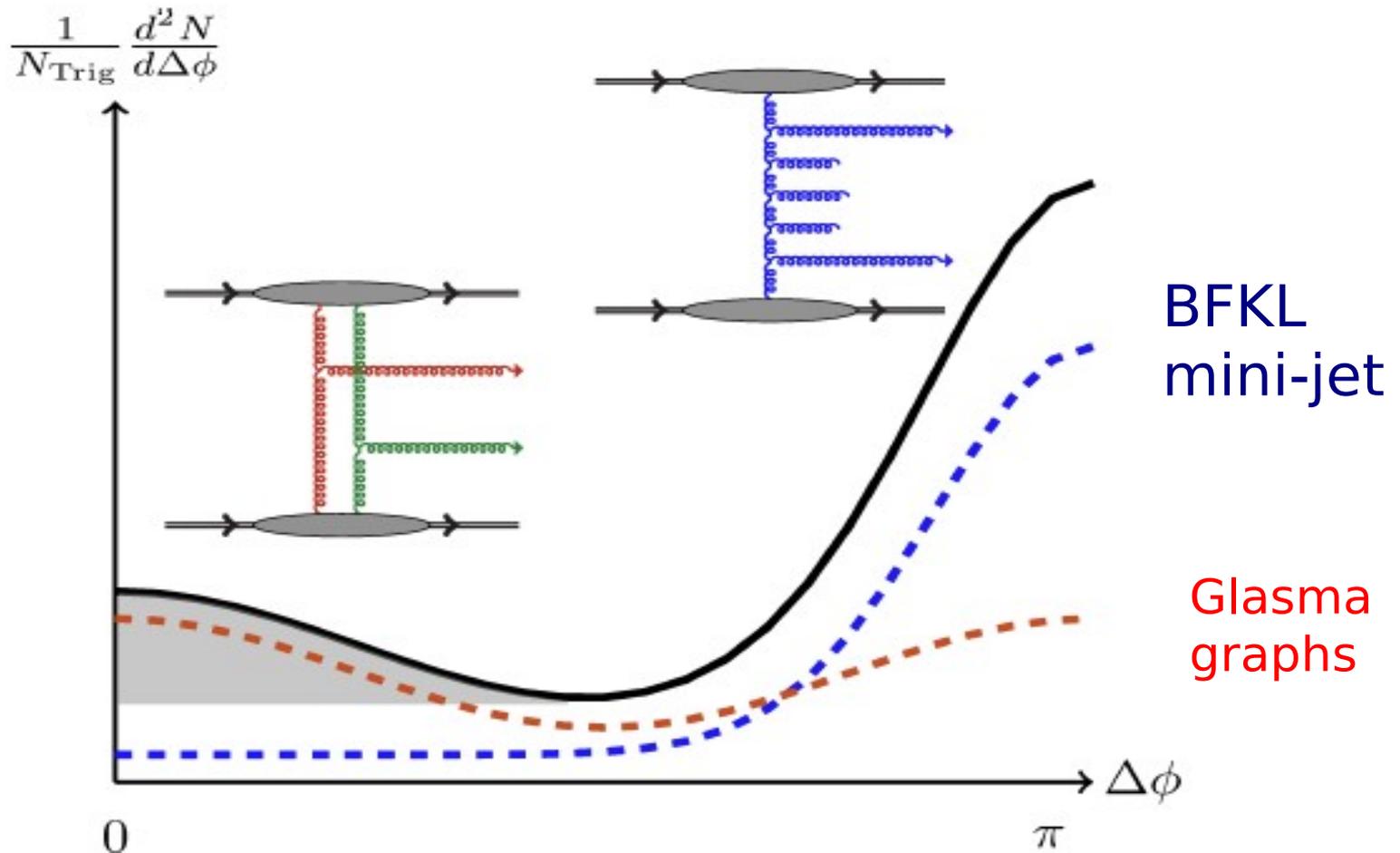
**solutions of rcBK
angular collimation**

The CMS ridge at LHC



Dumitru et al., PLB697 (2011) 21

Anatomy of long range di-hadron collimation



Extensive phenomenology, K. Dusling+R. Venugopalan,
arXiv:1302.7018

CGC vs. hydro

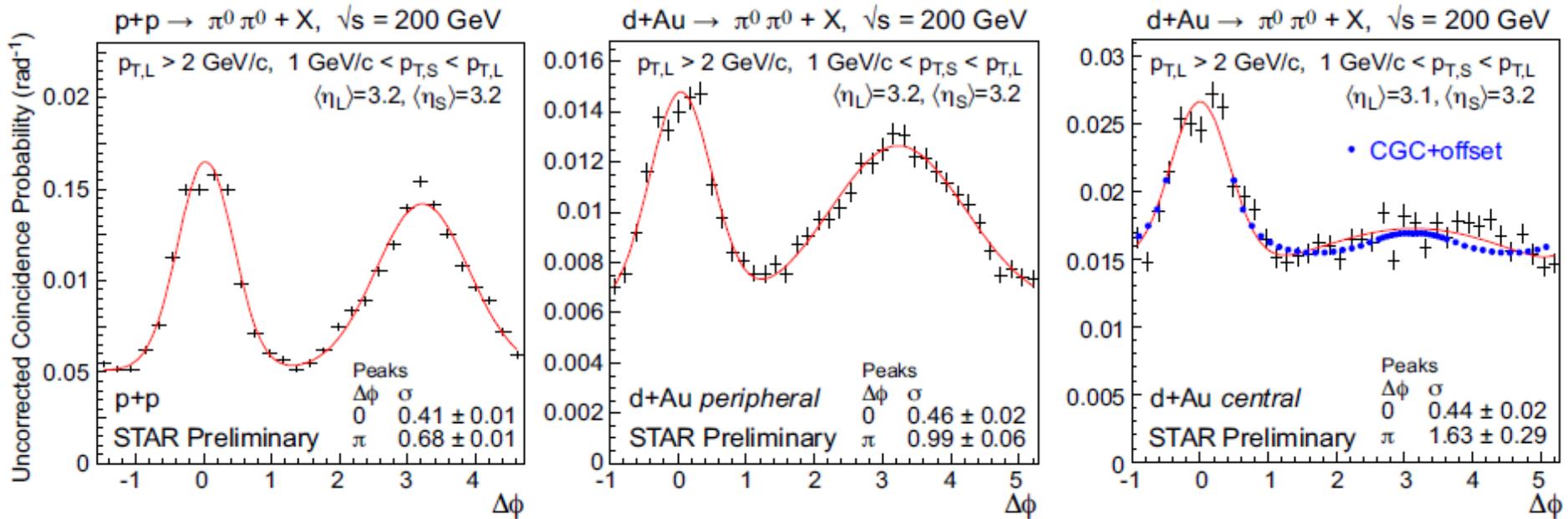
Two-particle correlations in pA (forward-forward)

away-side correlations:
di-hadron
photon-hadron

disappearance of back to back jets

Prediction by C. Marquet using CGC (2007)

Recent STAR measurement (arXiv:1008.3989v1):



CGC fit from Albacete, Marquet, PRL (2010)

K. Tuchin, NPA846 (2010)

A. Stasto, B-W. Xiao, F. Yuan, PLB (2012)

T. Lappi, H. Mantysari, NPA (2013)

another idea: shadowing + energy loss (M. Strikman et al.)

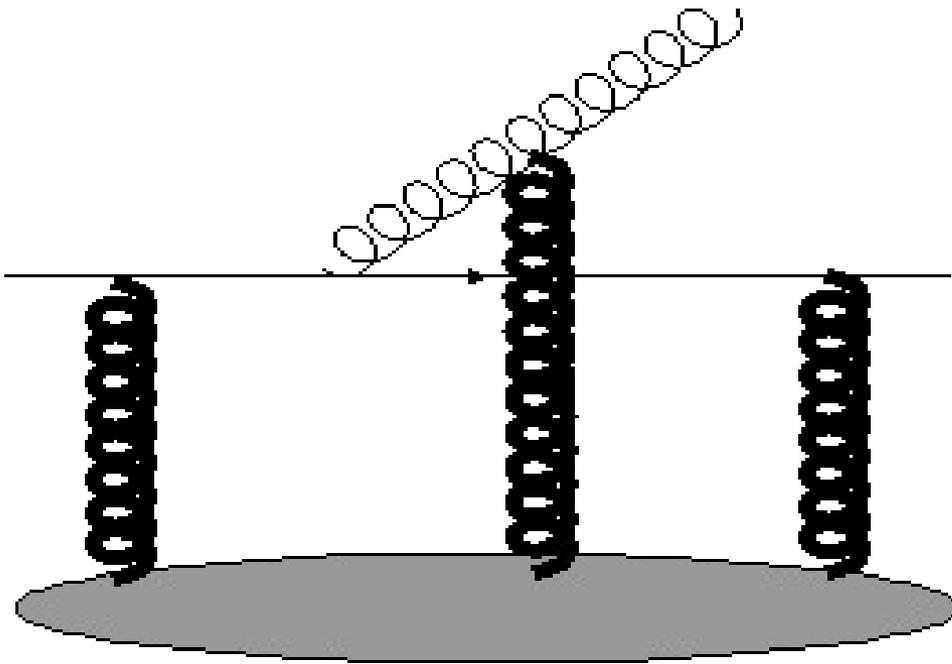
Z. Kang, I. Vitev and H. Xing,

PRD85 (2012) 054024

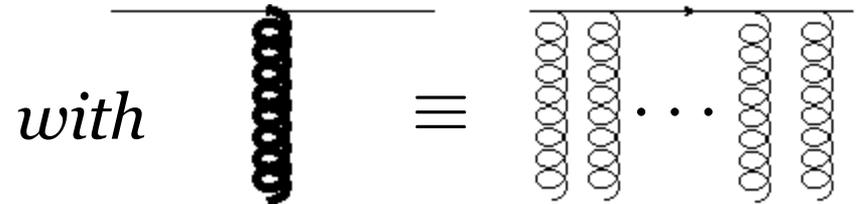
Di-jet production: pA

DIS, single inclusive production in pA: dipoles $\langle \text{Tr } V V^\dagger \rangle$

di-jet production in pA (and DIS): quadrupoles $\langle \text{Tr } V V^\dagger V V^\dagger \rangle$



J. Jalilian-Marian, Y. Kovchegov
PRD (2004)



large N_c : quadrupoles at most
Dominguez, Marquet, Stasto, Xiao,
PRD (2013)

quadrupole evolution: limits

$$\langle Q(r, \bar{r}, \bar{s}, s) \rangle \equiv \frac{1}{N_c} \langle \text{Tr} V(r) V^\dagger(\bar{r}) V(\bar{s}) V^\dagger(s) \rangle$$

can be calculated in a Gaussian model DMXY

line config.: $r = \bar{s}, \bar{r} = s, z \equiv r - \bar{r}$

square config.: $r - \bar{s} = \bar{r} - s = r - \bar{r} = \dots \equiv z$

“naive” Gaussian: $Q = S^2$

Gaussian $Q_1(z) \approx \frac{N_c + 1}{2} [S(z)]^{2\frac{N_c+2}{N_c+1}} - \frac{N_c - 1}{2} [S(z)]^{2\frac{N_c-2}{N_c-1}}$

Gaussian + large N_c $Q_1(z) \rightarrow S^2(z)[1 + 2 \log[S(z)]]$

quadrupole evolution: limits

$$\langle Q(r, \bar{r}, \bar{s}, s) \rangle \equiv \frac{1}{N_c} \langle \text{Tr} V(r) V^\dagger(\bar{r}) V(\bar{s}) V^\dagger(s) \rangle$$

Gaussian

$$Q_{sq}(z) = [S(z)]^2 \left[\frac{N_c + 1}{2} \left(\frac{S(z)}{S(\sqrt{2}z)} \right)^{\frac{2}{N_c+1}} - \frac{N_c - 1}{2} \left(\frac{S(\sqrt{2}z)}{S(z)} \right)^{\frac{2}{N_c-1}} \right]$$

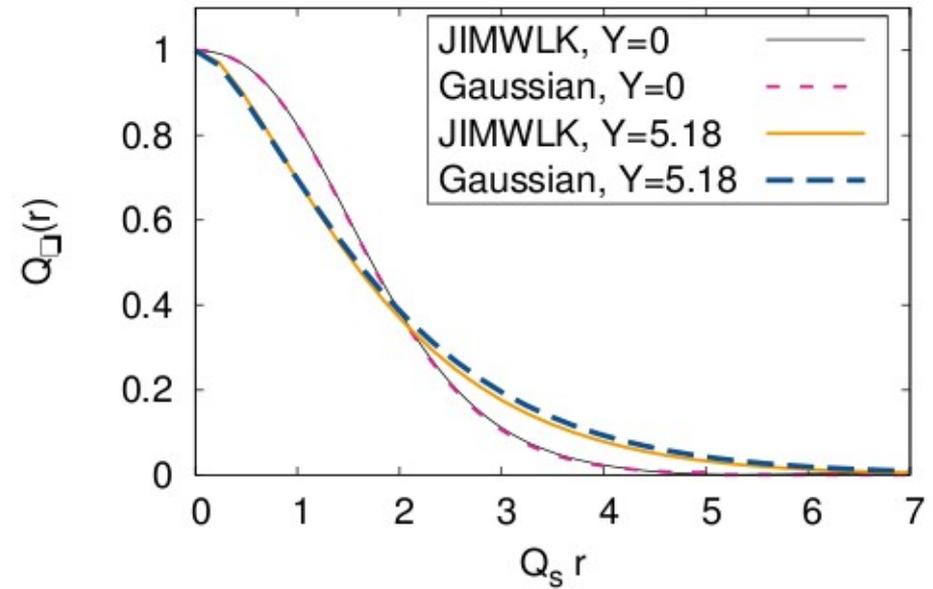
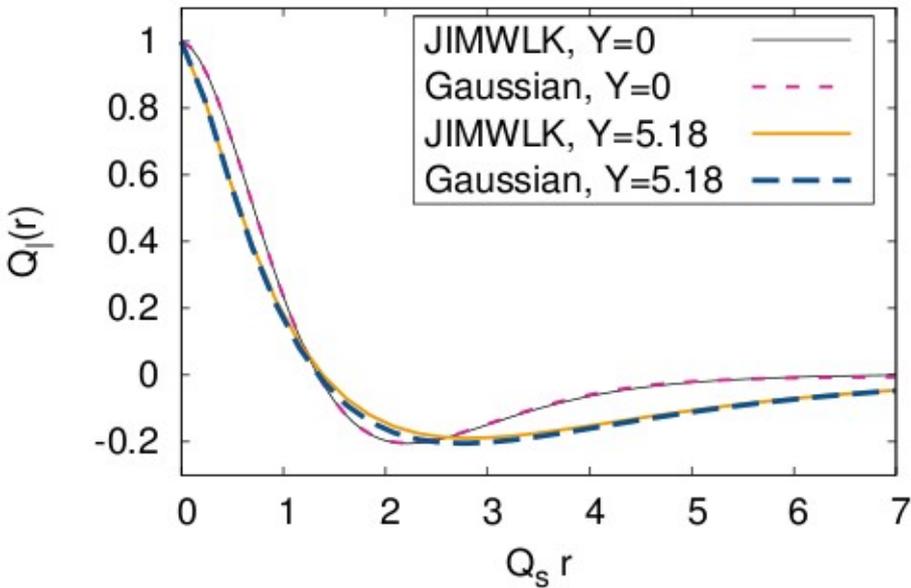
Gaussian + large N_c

$$Q_{sq}(z) = \left[1 + 2 \ln \left(\frac{S(z)}{S(\sqrt{2}z)} \right) \right]$$

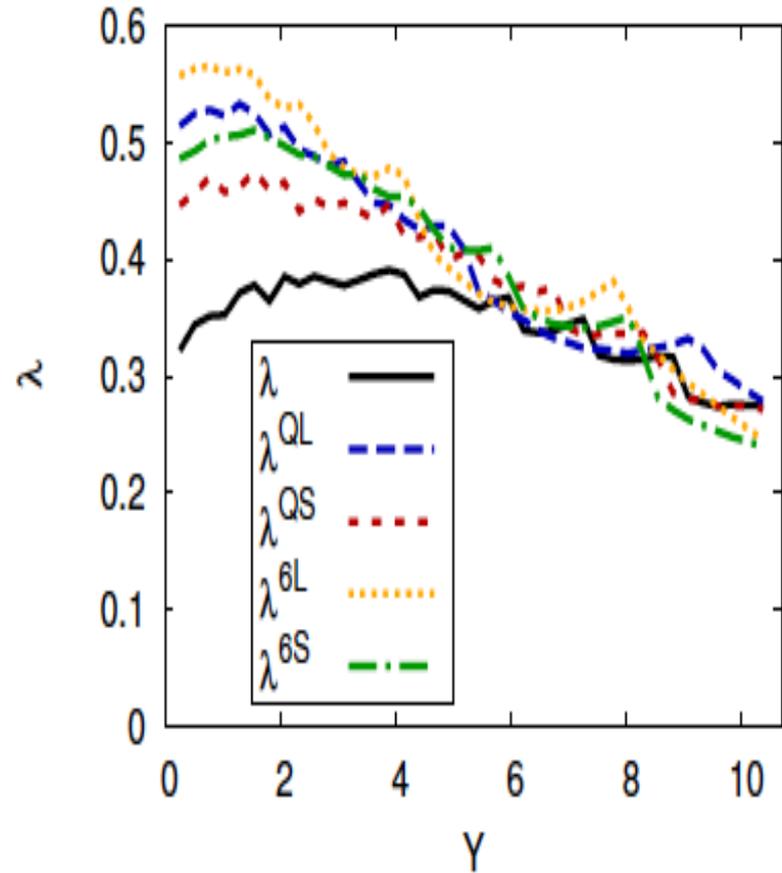
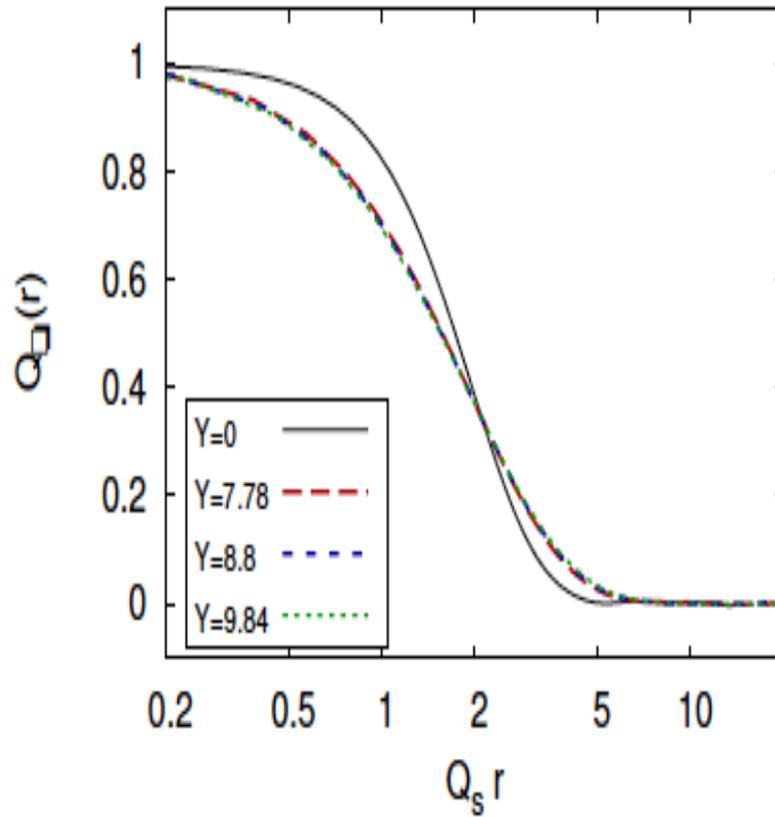
EI+DT, JHEP (2012): Gaussian approx. is good

Quadrupole evolution

comparing with Gaussian



Quadrupole evolution



Geometric scaling also present in quadrupoles

Growth of the saturation scale

di-hadron correlations in the high p_t limit

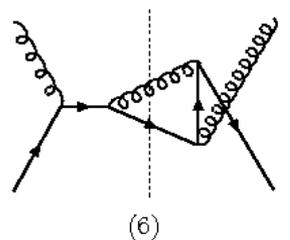
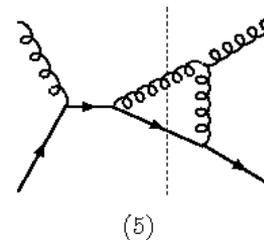
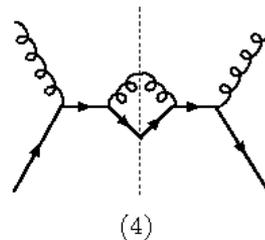
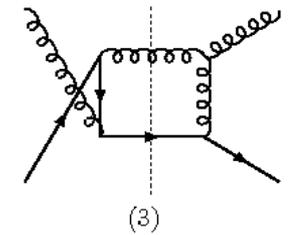
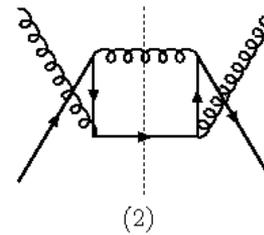
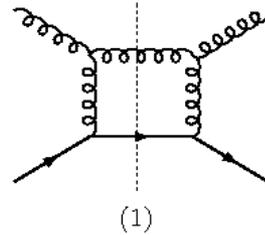
$O(\alpha^2)$

Dominguez, Marquet, Xiao, Yuan (2011)

Dominguez, Xiao, Yuan (2011)

factorization of target distribution functions and hard scattering matrix element

$$d\sigma \sim \Phi \otimes \frac{d\sigma^{2 \rightarrow 2}}{dt}$$



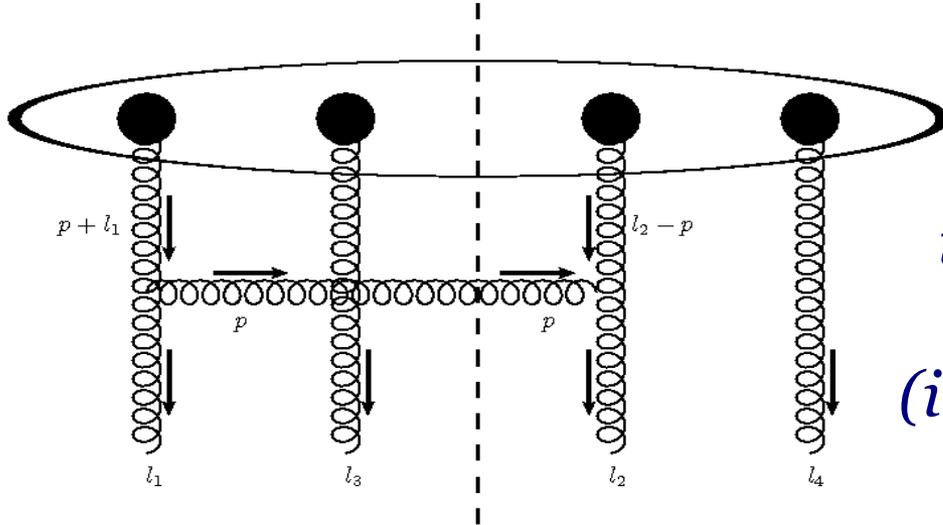
$$\frac{d\sigma^{qg \rightarrow qg}}{dt} \sim \frac{1}{s^2} \left[\frac{4}{9} \frac{s^2 + u^2}{-su} + \frac{s^2 + u^2}{t^2} \right]$$

partons are back to back

quadrupole evolution in the linear regime

BJKP equation

$\mathcal{O}(\alpha^4)$: 4-gluon exchange



J. Jalilian-Marian, PRD85 (2012) 014037

*the color structure is identical
on both sides of this eq.
(independent of color averaging)*

$$\begin{aligned} \frac{d}{dy} \hat{T}_4(l_1, l_2, l_3, l_4) &= \frac{N_c \alpha_s}{\pi^2} \int d^2 p_t \left[\frac{p^i}{p_t^2} - \frac{(p^i - l_1^i)}{(p_t + l_1)^2} \right] \cdot \left[\frac{p^i}{p_t^2} - \frac{(p^i - l_2^i)}{(p_t + l_2)^2} \right] \\ &\quad \hat{T}_4(p_t + l_1, l_2 - p_t, l_3, l_4) + \dots \\ &- \frac{N_c \alpha_s}{(2\pi)^2} \int d^2 p_t \left[\frac{l_1^2}{p_t^2 (l_1 - p_t)^2} + \{l_1 \rightarrow l_2, l_3, l_4\} \right] \hat{T}_4(l_1, l_2, l_3, l_4) \end{aligned}$$

this will de-correlate the produced partons at high $p_t > Q_s$

Two-hadron angular correlations

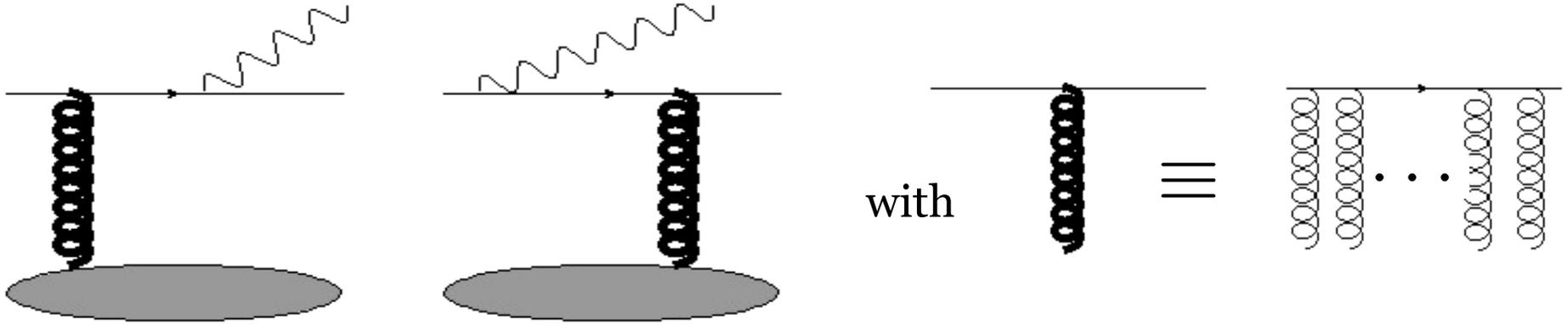
*Quadrupoles: a unique window to
dynamics of high energy QCD*

*Connection to pomerons, odderons,
triple (and more) pomeron vertex*

*A simpler correlation to probe CGC:
photon-hadron azimuthal correlation*

Photon-hadron correlation in p(d)A

$$q(p) \mathbf{T} \rightarrow q(l) \gamma(k) \mathbf{X}$$



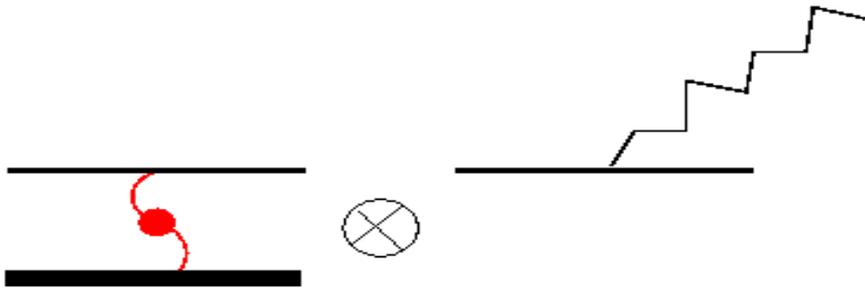
$$\frac{d\sigma^{q(p) \mathbf{T} \rightarrow q(l) \gamma(k) \mathbf{X}}}{d^2\vec{b}_t dk_t^2 dl_t^2 dy_\gamma dy_l d\theta} = \frac{e_q^2 \alpha_{em}}{\sqrt{2}(2\pi)^3} \frac{k^-}{k_t^2 \sqrt{S}} \frac{1 + \left(\frac{l^-}{p^-}\right)^2}{[k^- \vec{l}_t - l^- \vec{k}_t]^2}$$

$$\delta\left[x_q - \frac{l_t}{\sqrt{S}} e^{y_l} - \frac{k_t}{\sqrt{S}} e^{y_\gamma}\right] \left[2l^- k^- \vec{l}_t \cdot \vec{k}_t + k^- (p^- - k^-) l_t^2 + l^- (p^- - l^-) k_t^2\right]$$

$$\int d^2\vec{r}_t e^{i(\vec{l}_t + \vec{k}_t) \cdot \vec{r}_t} N_F(b_t, r_t, x_g)$$

pQCD limit

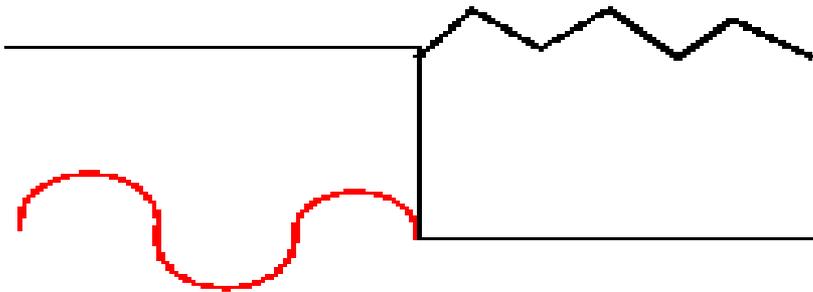
near side: collinear divergence $\theta \rightarrow 0$



$$\mathbf{N}_F \otimes \mathbf{D}_{\gamma/q}$$

away side: $\theta \rightarrow \pi$

Baier, Mueller, Schiff,
NPA (2004)

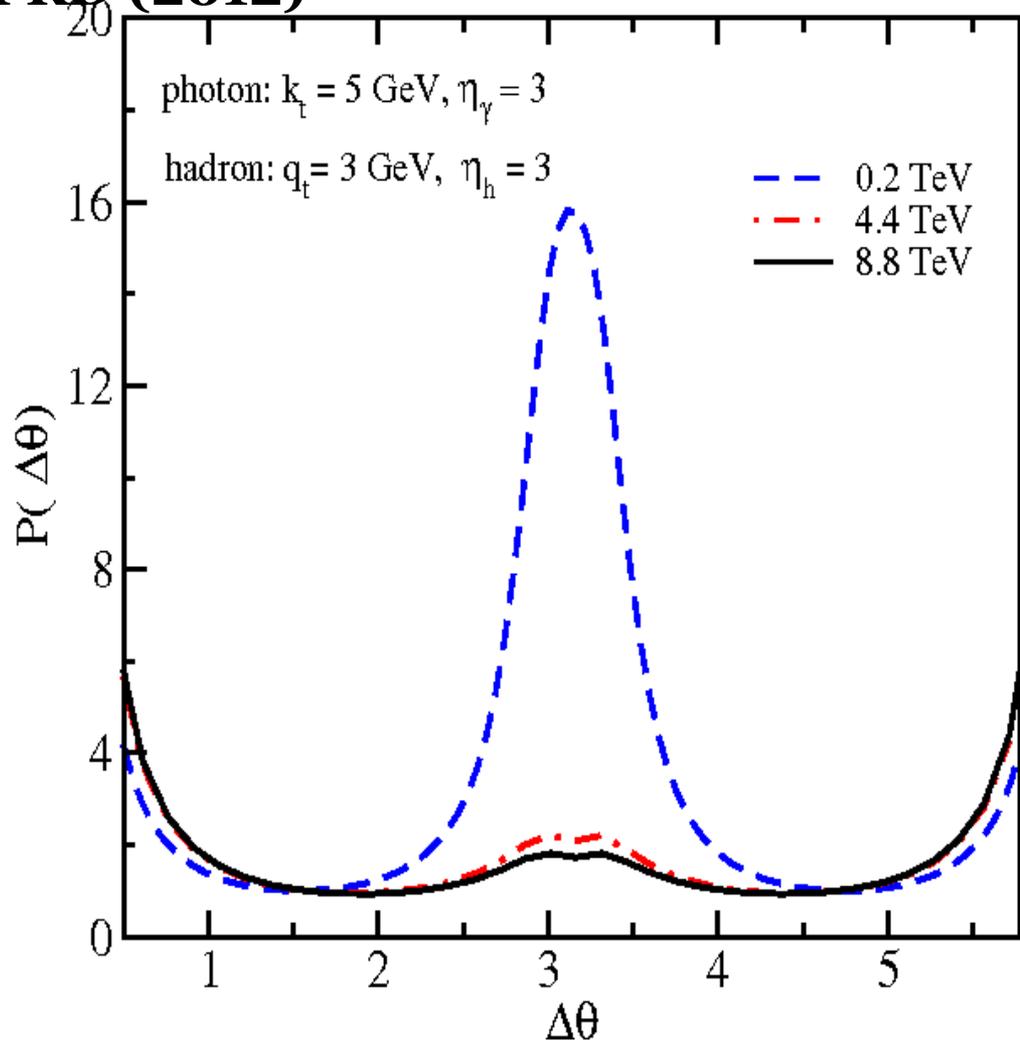
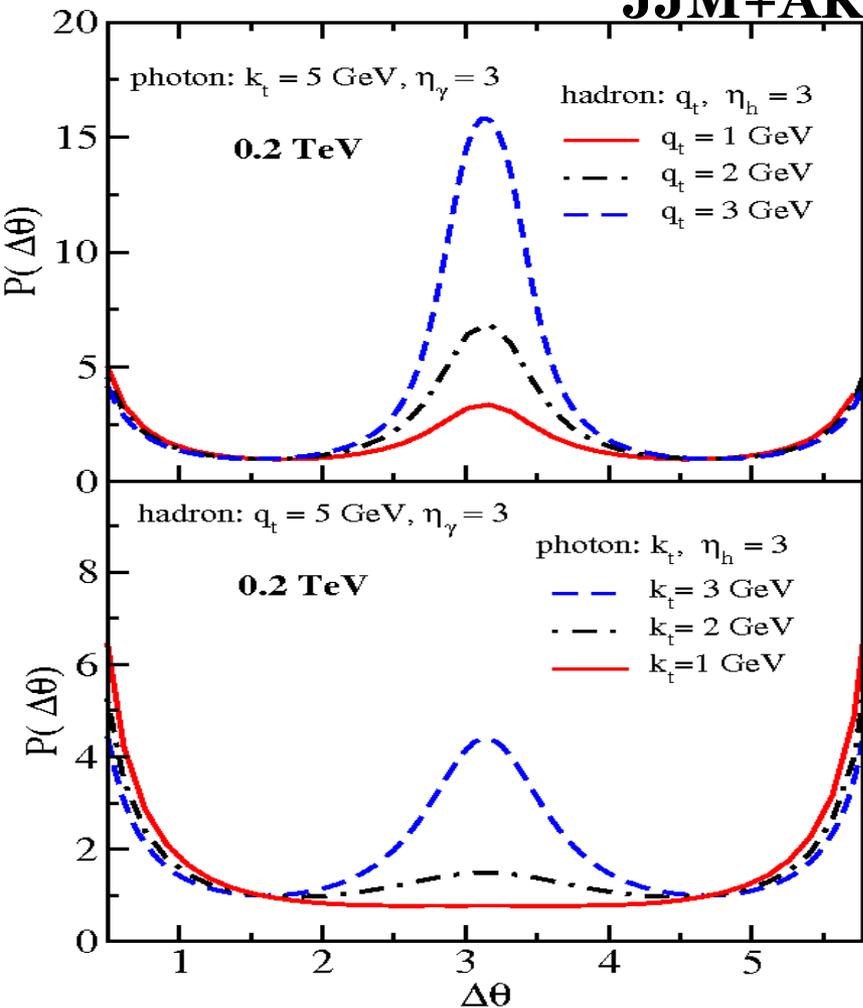


$$p_t \gg Q_s$$

photon-hadron azimuthal correlations

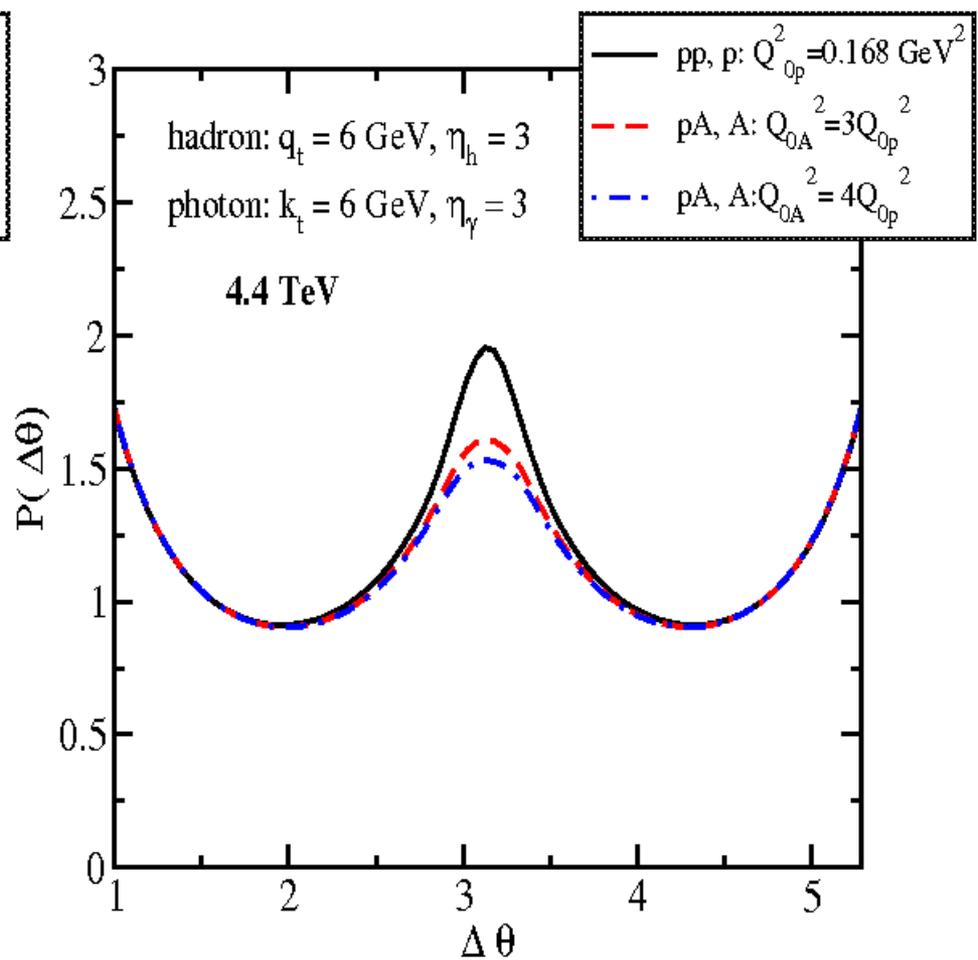
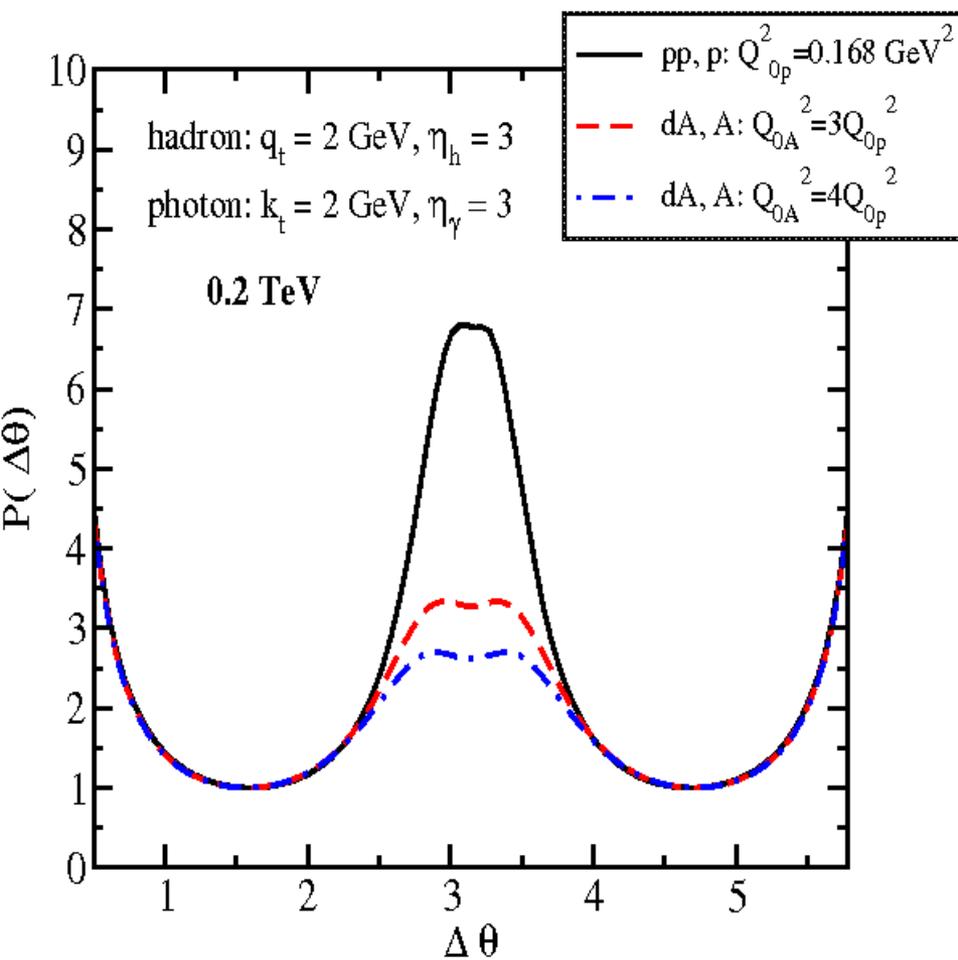
$$P(\Delta\theta) = \frac{d\sigma^{p(d) T \rightarrow h(q) \gamma(k) X}}{d^2\vec{b}_t dk_t^2 dq_t^2 dy_\gamma dy_l d\theta} [\Delta\theta] / \frac{d\sigma^{p(d) T \rightarrow h(q) \gamma(k) X}}{d^2\vec{b}_t dk_t^2 dq_t^2 dy_\gamma dy_l d\theta} [\theta = \theta_c]$$

JJM+AR, PRD (2012)



Centrality dependence

$$P(\Delta\theta) = \frac{d\sigma^{p(d) T \rightarrow h(q) \gamma(k) X}}{d^2\vec{b}_t dk_t^2 dq_t^2 dy_\gamma dy_l d\theta} [\Delta\theta] / \frac{d\sigma^{p(d) T \rightarrow h(q) \gamma(k) X}}{d^2\vec{b}_t dk_t^2 dq_t^2 dy_\gamma dy_l d\theta} [\theta = \theta_c]$$



Photon-hadron correlations

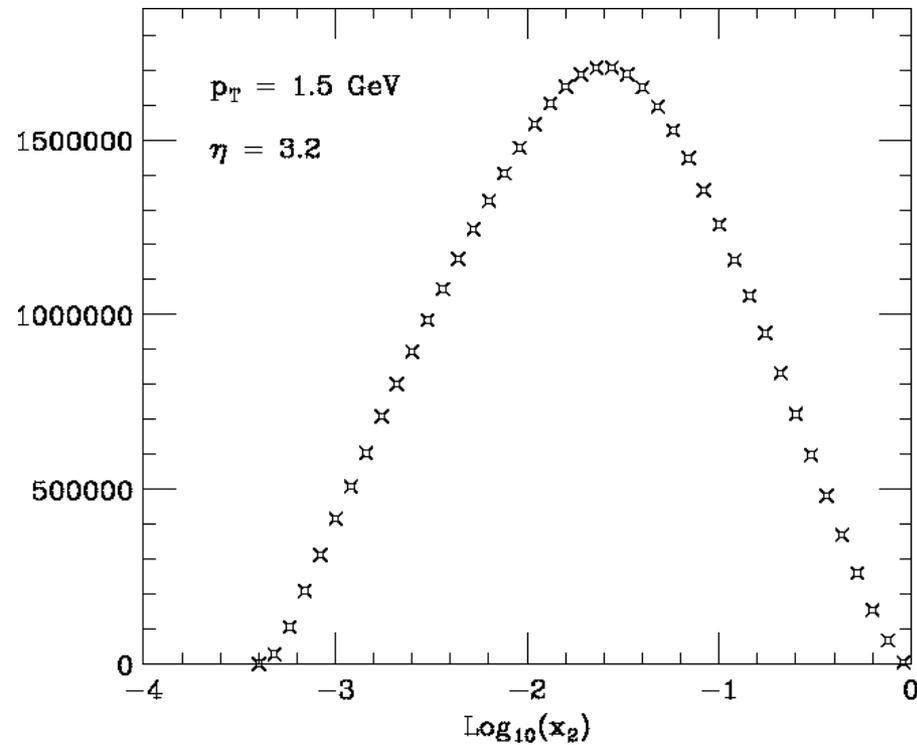
new processes to probe the dynamics of high energy QCD

suppression of prompt photon spectrum
in forward rapidity in $p(d)A$

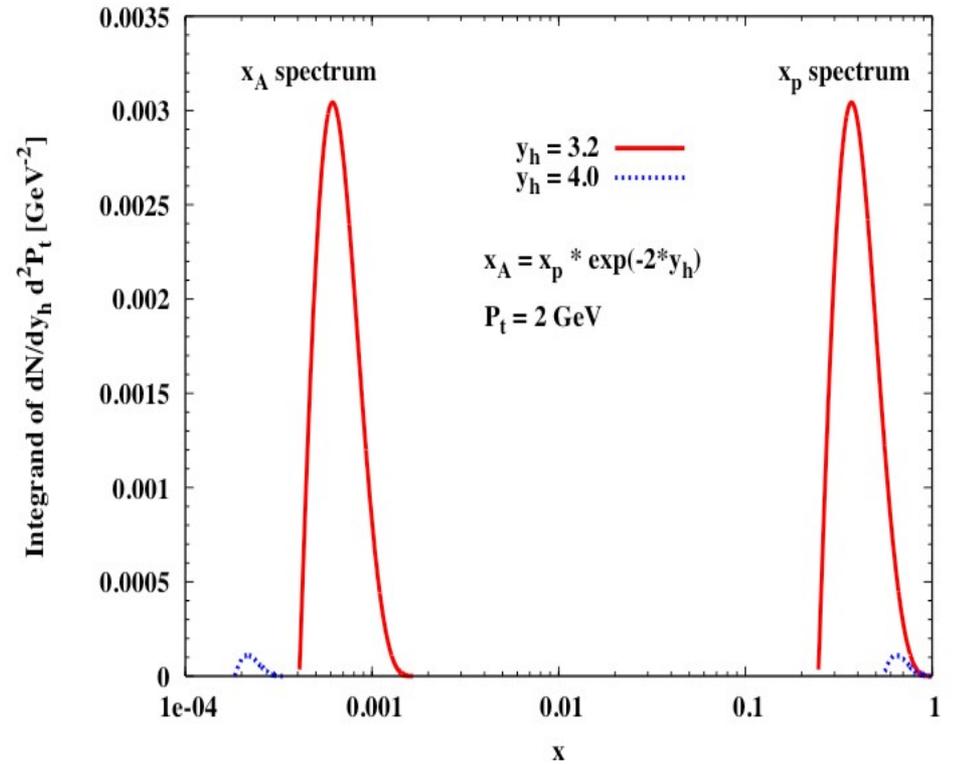
disappearance of the away side peak in
photon-hadron azimuthal correlations
in $p(d)A$

need to measure these at RHIC/LHC

$2 \rightarrow 2$ vs. $2 \rightarrow 1$



GSV



DHJ