

# Two historic views on the Pomeron $\frac{d\sigma}{dt} \sim s^{\alpha(t)}$

Prehistoric: Regge, Pomeranchuk, Gribov

intercept +string scale

 $\alpha(t) = \alpha(0) + \alpha' t + \dots$ 

I 960's: Veneziano dual resonance amplitude => appearance of strings

I 970's QCD Gribov,Lipatov => gluon ladders => BFKL

So, do we have two different Pomerons, soft (strongly coupled) and hard (weakly coupled)? small digression: few (more exotic) non-perturbative Pomerons

- Kharzeev-Levin 2000: gluon ladder but with sigma rungs
- Shuryak-Zahed 2000: instanton vertices, sphaleron rungs
- Polchinski-Strassler 2002: AdS/CFT Pomeron, basically graviton exchange  $\alpha(0) = 2 - O(1/\lambda)^2$

## Holographic Pomeron based on AdS/QCD

- the answer to the question is No:
- Only one Pomeron because the gauge description on the boundary is dual to string description in the bulk.Weak and strong coupling are its limits
- Concrete model of this type has been worked out by Zahed et al (Stoffers,Basar, Kharzeev): I will call it **Z+**
- Our main statement: as a function of b there are three distinct regimes: subcritical, near-critical and supercritical!

# AdS/QCD

- Holographic down-to-top model
- 5-th curved coordinate z=1/r
- z=>0, large r is UV=> conformal



reproduces many things including correct Regge trajectories

#### Holographic Pomeron and the Schwinger Mechanism

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Contains the basic calculation of the string amplitude

Euclid-Minkowski relation

Wilson Line correlator

$$\begin{split} & \underbrace{\theta \rightarrow -i\chi}_{.} \\ & \frac{1}{-2is}\mathcal{T}(\theta,q) \approx \int d^{2}b \; e^{iq_{\perp}\cdot b} \\ & \times \Big\langle \left(\mathbf{W}(-\theta/2,-b/2)-\mathbf{1}\right) \left(\mathbf{W}(\theta/2,b/2,)-\mathbf{1}\right) \Big\rangle \\ & = \int d^{2}b \; e^{iq_{\perp}\cdot b} \left\langle \mathbf{W}(-\theta/2,-b/2)\mathbf{W}(\theta/2,b/2)-\mathbf{1} \right\rangle \end{split}$$

holographic setting in 5d



string "tube" quantization by Polyakov action

$$\mathbf{W}\mathbf{W} = g_s^2 \int_0^\infty \frac{dT}{2T} \, \mathbf{K}(T) \,,$$

where

$$\mathbf{K}(T) = \int_{\mathrm{T}} \, d[x] \, e^{-\mathbf{S}[x] + \mathrm{ghosts}} \,,$$

$$\mathbf{S} = \frac{\sigma_T}{2} \int_0^T d\tau \int_0^1 d\sigma \left( \dot{x}^{\mu} \dot{x}_{\mu} + {x'}^{\mu} {x'}_{\mu} \right) \,,$$

### the "tube"



FIG. 1: Dipole-dipole scattering configuration in Euclidean space. The dipoles have size a and are b apart. The dipoles are tilted by  $\pm \theta/2$  (Euclidean rapidity) in the longitudinal  $x_0 x_L$  plane.

If cut horizontally, it describes production of a pair of open strings

 If cut vertically, it describes an exchange by a closed string

string fluctuations are include mode-by-mode

$$\frac{1}{-2is}\mathcal{T}(s,t) \approx \frac{\pi^2 g_s^2 a^2}{2} \sum_{k=1}^{k_{\max}} \sum_{n=0}^{\infty} \frac{(-1)^k}{k} \left(\frac{k\pi}{\ln s}\right)^{D_\perp/2-1} \times d(n) s^{-2n/k+D_\perp/12k+\alpha' t/2k}, \quad (70)$$

k=1 in SU(3), n is excitation

the string is nearly straight, with small effective excitations (small effective T). The meaning of

We will now review the Pomeron results in this setting. The amplitude of the elastic dipole-dipole scattering reads [2–4]

$$\frac{1}{-2is}\mathcal{T}(s,t;k) \approx g_s^2 \int d^2 \mathbf{b} \, e^{iq \cdot \mathbf{b}} \, \mathbf{K}_T(\beta,\mathbf{b};k)(15)$$
$$\mathbf{K}_T(\beta,\mathbf{b};1) = \left(\frac{\beta}{4\pi^2 \mathbf{b}}\right)^{D_\perp/2} \text{classical action b^2} \\ \times e^{-\sigma\beta \mathbf{b} \left(1 - (\tilde{\beta}_H/\beta)^2/2\right)} \text{ vibrations b-independent} \\ \times \sum_{n=0..\infty} d(n) exp(-2\chi n)$$

Linear Regge trajectories, daughters shifted by 2 down



As we mentioned, the expression (18) has been derived in [4] from the semiclassical approach to a Polyakov string, but ( to leading order in  $1/\lambda$ ) it can alternatively be derived from a diffusion equation

$$\left(\partial_{\chi} + \mathbf{D}_{k} \left(\mathbf{M}_{0}^{2} - \nabla_{\mathbf{b}}^{2}\right)\right) \mathbf{K}_{T} = 0 \qquad (20)$$

where the rapidity  $\chi$  interval is the time and the diffusion happens in the (curved) transverse space with the diffusion constant  $\mathbf{D}_k = \alpha'/2k = l_s^2/k$ . This diffusion (20) is nothing else but the Gribov diffusion of the Pomeron, leading on average to an impact parameter  $\langle \mathbf{b}^2 \rangle = \mathbf{D}_k \chi$  for close Pomeron strings. If the "mother dipoles"

#### connection to Gribov diffusion



### one and 2 particle spectra: Kancheli-Mueller diagrams



we thank Dima Kharzeev who reminded us of it

# CMS correlation function does contain small rapidity correlation



It was interpreted as jet, but we interpret is as a "string ball" cluster. Note it is seen for medium  $p_t=1-3$  GeV.

### fluctuations of flat membrane and a tube are different

- The (T=0) potential <W>
  is linear at large b and Coulombic (pQCD)
  at small b : yet transition is smooth
- The tube has a periodic variable => quantization formally the same as the (Matsubara) thermal formalism => thus we expect thermal-like behavior with a nontrivial transition to pQCD regime

#### New Regimes of Stringy (Holographic) Pomeron

**1.** A "cold" regime, with low string excitations;

**2.** A "near-critical" or "HPS regime", in which strings indefinitely increase their energy and entropy, but not their free energy / pressure;

**3.** An "explosive regime", in which the string occupies large portion of space and generates sufficient pressure for hydrodynamical explosion.

#### Hagedorn-Polyakov-Susskind regime

stringy excitations grows with an excitation energy exponentially. To see this, imagine a ddimensional lattice with spacing a and draw all possible strings of length L/a making all possible turns (except going backward) at each site, that is

$$N(E) \approx (2d-1)^{L/a} = e^{E(L)/T_H}$$
 (5)

where in the last term we changed length into energy using the string tension  $E(L) = \sigma_T L$ and defined

$$T_H = \frac{\sigma_T a}{\ln(2d - 1)} \tag{6}$$



FIG. 3: (Color on-line) Schematic temperature dependence of the entropy density. The dashed line represents equilibrium gluodynamics with a first order transition at  $T = T_c$ . The solid line between points A and B represents the expected behavior of a single string approaching its Hagedorn temperature  $T_H$ .

As T=>TH the entropy and energy grow, but not free energy (pressure) as F=E-TS and two terms cance



with its highest value at the center or  $T(1/2) \equiv T = \chi/2\pi \mathbf{b}$ . It is instructive to focus on the actual effective temperature values, corresponding to LHC collisions. For that we define a typical impact parameter  $\mathbf{b}_{\text{eff}}$  for pp collisions at energy s as



FIG. 4: (Color on-line) The effective string temperature  $T_{\rm eff}$  (GeV) versus the c.m. beam gamma factor  $\gamma$ , solid for black disc estimate  $F_{\rm gray} = 1$  and dashed for gray factor  $F_{\rm gray} = 0.7$ . As argued in the text, its value is to be compared to the effective Hagedorn temperature  $\tilde{T}_H$ .

$$D_{\perp} \rightarrow \tilde{D}_{\perp} = D_{\perp} \left( 1 - \frac{3(D_{\perp} - 1)^2}{2kD_{\perp}\sqrt{\lambda}} \right)$$

This translates to a higher effective Hagedorn temperature  $\tilde{T}_H > T_H$  through (10) with

$$T_H^2 \to \tilde{T}_H^2 = \frac{3}{\tilde{D}_\perp} \frac{\sigma_T}{2\pi} \approx 1.8 \, T_H^2 \qquad (36)$$

# Can new regimes be seen in the elastic amplitude?



After integration over b and dipole sizes, can one still be able to see it in T(t) ? Here is our 1st observation: the middle of a string develops a "string b



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# perhaps observable as high multiplicity cluster near mid-rapidity

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FIG. 3: String exchange between two sources (crosses)ated by the impact parameter **b**: the cold string case  $\beta < \beta_H$  (a); the near-critical string case  $\beta \to \beta_H$  (b).

s be1

ipa(

One can re-sum

lision energies (abereteners" to colliders) at hay approach the the government perature  $T \cong T_{H}$ . At current energies (LHC) it can also happen, as fluctuations, /Wervill argue that in this new regime the string will develop large excitations in the form of a *"string ball"* depicted in Fig.3b. Theis Zamodel Cobrained is based of a based of the A stringexchangesabetweenotheingllithing and enouror ergybtbjectle stuisessentvilithaviltheQCDpervice with a nonzero tension related to QCD of M ment is used, and not the conformal superst which (has; 1)  $\approx (a_{ssless}) spin-2^{\beta b} (1-\beta^2) \delta^2$ tion. There is no supersymmetry and the results get clearly glue witinapplicable i as soon anguments in Pomeron diffusite in the sign However the plitude where  $\mathbf{K}_T$  is the Pomeron propagator for distributed with  $\mathbf{F}_{\mathbf{K}}$  and  $\mathbf{K}_T$  is the Pomeron propagator for distributed where  $\mathbf{K}_T$  is the Pomeron propagator for distributed with  $\mathbf{F}_{\mathbf{K}}$  and  $\mathbf{K}_T$  is the Pomeron propagator for distributed where  $\mathbf{K}_T$  is the Pomeron propagator for the P  $C_{\rm C}$  sources of color N-ality k describing the string coupling a sources of color N-ality k describing the st



#### Compare to "cold" one, parati in which it is a small ige (b) correction

$$\begin{array}{l} \text{Kole} \quad \text{Kole}$$

We will now review the Pomeron results

Can one figure out what to do when T>TH and argument of the sqrt <0?

### from stringy to perturbative world

# String theorists worked out the regime at $T= or > T_{H.}$

Two constant modes of string excitation become a complex scalar field which develops VEV — a disorder parameter

$$m^2 \sim \beta^2 - \beta_H^2 < 0$$

#### strings break



Selfgravitating fundamental strings Gary T. Horowitz , Joseph Polchinski Phys.Rev. D57 (1998) 2557-2563 hep-th/9707170

$$<\chi^+\chi(0)\chi^+\chi(b)>=|\chi|^4+e^{-b\sqrt{|\beta_H^2-\beta^2|}}$$

In string language it means that a black hole is formed which can hide one end of a fundamental string

#### $\sqrt{s} = 23.5 \text{ GeV}$ $\sqrt{s} = 30.7 \text{ GeV}$ Diffinactive and deeply virtual Compton scattering in devioraphic QCD





Figure 3: Differential pp cross section. Black dots: data from the TOTEM experiment at LHC, [59]. Dashed blue line and red dots: holographic result. See text.

#### Supercritical (explosive) regime

The onset is expected when the entropy becomes as large as that of the gluons, which coresponds to

$$s \approx N_c^2 \tilde{T}_H^3 \tag{46}$$

The estimate of the probability of this to happen in LHC pp events can be done using

$$\frac{\Delta\beta}{\tilde{\beta}_H} = \frac{\tilde{T}_H}{T} - 1 = \mathcal{O}\left(\frac{1}{N_c}\right) \tag{47}$$

For near critical strings we have (47) and  $g_s \approx 1/N_c$ . At LHC,  $\chi \approx 10$  so using (45) we find

$$\frac{\sigma_{NC}}{\sigma_{MB}} \approx 10^{-5} \tag{48}$$

which is comparable to the probability of the high multiplicity events in which the CMS collaboration discovered the "ridge" phenomenon.



#### The appearance of QGP pressure leads to hydro explosion, which we discuss in a separate paper

High Multiplicity pp and pA Collisions: Hydrodynamics at its Edge and Edward Shuryak, Ismail Zahed (SUNY, Stony Brook). Jan 2013. Published in Phys.Rev. C88 (2013) 044915 ; arXiv:1301.4470

### the first discovery at LHC

Opportunity of studying novel QCD phenomena opened up by the LHC

Two-particle  $\Delta \eta$ - $\Delta \phi$  correlation

September, 2010

#### 10^10\$ per 10^10 events, but those cost 1000000\$ each

Observation of long-range, near-side angular correlations in proton-proton collisions at the LHC

The CMS collaboration

ABSTRACT: Results on two-particle angular correlations for charged particles emitted in proton-proton collisions at center-of-mass energies of 0.9, 2.36, and 7 TeV are presented, using data collected with the CMS detector over a broad range of pseudorapidity ( $\eta$ ) and azimuthal angle ( $\phi$ ). Short-range correlations in  $\Delta \eta$ , which are studied in minimum bias



#### Unexpected ridge-like correlations in high multiplicity pp!

Not in minimum bias pp or pp MC models

#### CMS pPb: v2 from 2 and 4-particles



#### High-multiplicity *pp* and *pA* collisions: Hydrodynamics at its edge

Edward Shuryak and Ismail Zahed

3

 $m_{\perp}(GeV)$ 

0.7 0.8

the dashed line are for AMTP model.

m(GeV)

0.9

#### We predicted the radial flow $\frac{dN}{dydm_{\perp}^{2}}(y=0)$ 10 in pp/pA to be even stronger 10 than in central AA 10 CMS 0.7 10.5 pPb, $\sqrt{s_{NN}} = 5.02 \text{ TeV}$ , L = 1 $\mu \text{b}^{-1}$ 0.6 235 10-6 (a) 210 185 2 0 0.5 160 135 T' [GeV/c] 0.8 109 0.4 T'(GeV)84 0.7 58 0.3 32 0.6 0.2 8 0.5 0.4 0.1 0.3 0.4 0.8 0 0.2 .6 1 1.2 m [G V/c<sup>2</sup>] 0.4 0.5 0.6 0.2 0.3 FIG. 8. (Color online) The slopes of the $m_{\perp}$ distribution T' (GeV) FIG. 9. (Color online) (a) A sample of spectra calculated for as a function of the particle mass, from [13]. The numbers on the right $\pi, K, p$ , top-to-bottom, versus $m_{\perp}$ (GeV), together with fitted exponents.(b) Comparison of the experimental slopes T'(m) versus No Mt scaling => not a large the particle mass m (GeV). The solid circles are from the highest multiplicity bin data of Fig. 8, compared to the theoretical predictions. Qs in Glasma, The solid and dash-dotted lines are our calculations for freezeout temperatures $T_f = 0.17, 0.12 \text{ GeV}$ , respectively. The asteriskmarked dashed lines are for Epos LHC model, diagonal crosses on

but collective flow: p=m v

#### THE STRING BALL AS AN EFFECTIVE BLACK HOLE

The near-critical string has a propagator (40) that behaves like a thermal ensemble with Unruh temperature  $1/\beta_U$ . Its free energy or pressure  $\mathbf{F} = -\ln \mathbf{K}_T / \beta_U$  [2] are small

$$\mathbf{F}(\beta, \mathbf{b}) \approx k\sigma \, \mathbf{b} \left(1 - \frac{\tilde{\beta}_H^2}{\beta^2}\right)^{1/2}$$
 (52)

but the energy and the entropy are very large

$$\mathbf{E} = \partial_{\beta_U} (\beta_U k \mathbf{F}) \approx k \sigma \mathbf{b} \left( 1 - \frac{\tilde{\beta}_H^2}{\beta^2} \right)^{-1/2} (53)$$
$$\mathbf{S} = \beta_U^2 \partial_{\beta_U} \mathbf{F} \approx (\tilde{\beta}_H^2 / \beta) k \sigma \mathbf{b} \left( 1 - \frac{\tilde{\beta}_H^2}{\beta^2} \right)^{-1/2}$$

For  $\beta \approx \tilde{\beta}_H$  this coincides with the first law of thermodynamics for black-holes in Rindler coordinates as noted by Susskind [40]

F,p are small,

E,S are large

$$\mathbf{S} \approx \beta_H \mathbf{E} = 2\pi \left( \mathbf{E} l_s \right) \tag{54}$$

The transverse area of the black-hole is the area of the diffusing string in rapidity

$$A_{BH} = 2\pi^2 \left(\sqrt{\chi/k} \, l_s\right)^3 \tag{56}$$

in transverse  $D_{\perp} = 3$  provided that the diffusion length in the z-direction is within the confining wall. As a result, we have the Bekenstein-Hawking type relation

$$\frac{\mathbf{S}_{BH}}{A_{BH}} \equiv \frac{1}{4G_5} \tag{57}$$

with an effective Newton constant

$$G_5 = \pi^2 \left( (\chi/k^3) (1 - \tilde{\beta}_H^2/\beta^2) \right)^{1/2} l_s^3 \qquad (58)$$

For a fundamental string, the Planck and string constants are related with  $G_5$  through  $G_5 = l_P^3 = g_s^2 l_s^3$ . We recall in the large  $N_c$  counting  $g_s^2 \approx 1/N_c^2$ .

#### unneling (Euclidean) to Real time

 $ImA_{elastic} \sim |A_{inelastic}|^2$ 



Outlook

The Minkowski part has exp(iS) so the probability=1 <sup>b</sup> The end of E. path becomes the initial condition for the Minkowski part of the path

- at time zero strings are born transverse, in b direction
- then they are stretched along the beam direction



If the cluster — string ball — is formed in their center and it is heavy enough, a trapped surface appears: it creates string breaking, Hawking radiation etc

#### Hydro = falling in z under gravity

A significant leap forward had been done recently by Gubser, Pufu and Yarom [123], who proposed to look at heavy ion collision as a process of head-on collision of two point-like black holes, separated from the boundary by some depth L – tuned to the nuclear size of Au to be about 4 fm, see Fig.??. By using global AdS coordinates, these authors argued that (apart of obvious axial O(2) symmetry) this case has higher – namely O(3)– symmetry with the resulting black hole at the collision moment at its center, thus in certain coordinate

$$q = \frac{\vec{x}_{\perp}^2 + (z - L)^2}{4zL}$$
(91)

the 3-d trapped surface C at the collision moment should be just a 3-sphere, at constant  $q = q_c$ . (Here  $x_{\perp}$  are two coordinates transverse to the collision axes.) The picture of it is shown in Fig.29(b)

If so, one can find the radius at which it is the trapped null-surface and determine its energy and Bekenstein entropy. For large  $q_c$  these expressions are

$$E \approx \frac{4L^2 q_c^3}{G_5}, \ S \approx \frac{4\pi L^3 q_c^2}{G_5},$$
 (92)

from which, eliminating  $q_c$ , the main result of the paper follows, namely that the entropy grows with the collision energy as

$$S \sim E^{2/3}$$
 (93)

Note that this power very much depends on the 5-dimentional gravity and is different from the 1950's prediction of Fermi and Landau (??) in which this power was 1/2 and (accidentally or not) fits the data better.



No time is needed: trapped surface is there at time zero

#### On the critical condition in gravitational shock wave collision and heavy ion collisions Shu Lin (Munich, Max Planck Inst.), Edward Shuryak (SUNY, Stony Brook). Nov 2010. 26 pp.

#### Grazing Collisions of Gravitational Shock Waves and Entropy Production in Heavy Ion Collision

Shu Lin<sup>1</sup>, and Edward Shuryak<sup>2</sup>

arXiv 0902.1508

The shock wave moving in  $+x^3$  direction is given by:

$$ds^{2} = L^{2} \frac{-dudv + (dx^{1})^{2} + (dx^{2})^{2} + dz^{2}}{z^{2}} + L \frac{\Phi(x^{1}, x^{2}, z)}{z} \delta(u) du^{2}$$

with  $\Phi(x^1, x^2, z)$  satisfies the following equation:

$$\left(\Box - \frac{3}{L^2}\right)\Phi = 16\pi G_5 J_{uv}$$

The vanishing of expansion gives the equation:

$$\left(\Box - \frac{3}{L^2}\right)(\Psi_1 - \Phi_1) = 0$$
$$\Psi_1|_{\mathcal{C}} = \Psi_2|_{\mathcal{C}} = 0$$

The boundary  $\mathcal{C}$  should be chosen to satisfy the constraint:

 $\nabla \Psi_1 \cdot \nabla \Psi_2|_{\mathcal{C}} = 4$ 

#### The b.h. disappears at a particular b



Figure 1: (Color online.) Comparisons between the numerics of [36] and the analytic formula (58). The black dashed curve represents the leading term in (58); the solid red curve corresponds to the first two terms in (58); the dotted blue curve represents the expression (58), which is correct up to a term of order  $\mathcal{O}(1/\zeta^2)$ ; the green dots represent the numerical evaluations used in figure 3 of [36]; lastly, the vertical green line marks the place where, according to [36], the maximum impact parameter  $b_{\max}/L$  occurs. We thank S. Lin and E. Shuryak for providing us with the results of their numerical evaluations.

See also Shuryak and Lin, arXiv:1011.1918, for collision of two walls and Qs interpreted as the z coordinate of the walls

### summary

- A single Pomeron, but in 3 regimes
- it is stringy at large b and perturbative at small b
- with (crossover near-1st order) phase transition in between
- strongly fluctuating string => high multiplicity clusters, related to Pomeron daughters
- near-critical regime is thermodynamically dual to black hole
- outlook: real-time part of the path.is B.H. really formed? (trapped surfaces...)
- supercritical regime leads to QGP fireball and hydro explosion: hydro description works





#### seen in pA with few % probabiliy

Observation of long-range, near-side angular correlations in pPb collisions at the LHC



Figure 1: 2-D two-particle correlation functions for 5.02 TeV pPb collisions for pairs of charged particles with  $1 < p_T < 3 \text{ GeV}/c$ . Results are shown (a) for low-multiplicity events ( $N_{trk}^{offline} < 35$ ) and (b) for a high-multiplicity selection ( $N_{trk}^{offline} \ge 110$ ). The sharp near-side peaks from jet correlations have been truncated to better illustrate the structure outside that region.

#### a string ball is dual to a black hole: viscosity

**Kubo** 

#### The

#### correspondence is usually derived via the entropy

(=Hawking-Bekenstein)

#### But one can also calculate viscosity, which gives 1/4pi

although the calculation is stringy not BH. (And even if BH it is very different from that in AdS/ CFT, not located in 5-th dimension, so were surprised) tensor. To assess the primordial viscosity, we follow [2] and write the needed expression on the streched horizon for the excited string

$$\eta_R = \lim_{\omega_R \to 0} \frac{A_R}{2\omega_R} \int_0^\infty d\tau e^{i\omega_R \tau} \mathbf{R}_{23,23}(\tau) \quad (87)$$

with  $A_R$  the area of the black-hole and  $\tau$  a dimensionless Rindler time. The retarded commutator of the normal ordered transverse stress tensor for the Polyakov string on the Rindler horizon reads

$$\mathbf{R}_{23,23}(\tau) = \left\langle \left[ T_{\perp}^{23}(\tau), T_{\perp}^{23}(0) \right] \right\rangle$$
(88)

with

$$T_{\perp}^{23}(\tau) = \frac{1}{2A_R} \sum_{n \neq 0} : a_n^2 a_n^3 : e^{-2in\tau}$$
(89)

and the canonical rules  $[a_m^i, a_n^j] = m \delta_{m+n,0} \delta^{ij}$ . The averaging in (88) is carried using the blackbody spectrum as in (84). The result is

$$\eta_R = \lim_{\omega_R \to 0} \frac{A_R}{2\omega_R} \frac{\pi}{2A_R^2} \frac{(\omega_R/2)^2}{e^{\beta_R \omega_R/2} - 1} = \frac{1}{A_R} \frac{\pi}{8\beta_R}$$
(90)

We note the occurrence of the Bekenstein-Hawking or Rindler temperature  $\beta_{BH} = \beta_R$  in the thermal factor.

Combining (86) for the entropy to (90) yields the viscosity on the streched horizon

$$\frac{\eta_R}{\mathbf{S}_{\mathbf{R}}/A_R} = \frac{1}{4\pi} \left(\frac{3}{D_\perp}\right) \equiv \frac{1}{4\pi} \tag{91}$$

### fig for twisted tube

