A Curious Case of an Effective String

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Gauge Theories, Confinement and "Topology"

There is a general belief that confinement is in some way related to topology of fields.

Variations on this theme: Monopoles, Vortices, Knots...

In 3D things are very clear, and the connection is well established.

In 3D the picture we have unifies confinement with another important element: the massless photon of the Abelian limit is another integral facet of the confinement-topology connection.

This work - is an attempt to generalize this picture to 4D.

Not complete, and in its present form does not quite do what we want. But a cute toy model, and perhaps can lead to something more viable in future.

Confinement in 3D - The Lightning Review

Scalar Electrodynamics:

$$L = -\frac{1}{4}F_{\mu\nu}^2 + |D_\mu H|^2 - V(H^* H)$$

The CONSERVED symmetry current:

$$\tilde{F}_\mu \equiv \frac{1}{2}\epsilon_{\mu\nu\lambda}F^{\nu\lambda}; \qquad \partial^\mu \tilde{F}_\mu = 0$$

The conserved CHARGE of magnetic symmetry $U_\mu(1)$ is the MAGNETIC FLUX : $\Phi = \int d^2xB$

Carriers - magnetic vortices. "Order parameter" - vortex field V.

$$e^{i\alpha\Phi}V(x)e^{-i\alpha\Phi} = e^{i\frac{2\pi\alpha}{g}}V(x)$$

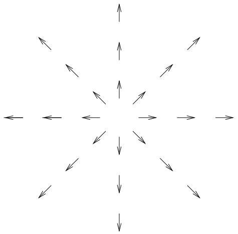
or equivalently

$$V(x)B(y)V^\dagger(x) = B(y) + \frac{2\pi}{g}\delta^2(x-y)$$

COULOMB phase: $U_\mu(1)$ is spontaneously broken: $\langle V\rangle = \frac{g^2}{2\pi} \neq 0$
Massless photon : **Goldstone boson**.
HIGGS phase: $U_\mu(1)$ is the symmetry of the vacuum: $\langle V\rangle = 0$
Magnetic vortices are particles with finite mass.
Effective low energy description:

$$\mathcal{L}_{eff} = \partial_\mu V^* \partial^\mu V - \lambda(V^* V - \mu^2)^2$$

Electric current: $J_\mu = \frac{1}{g}\epsilon_{\mu\nu\lambda}\partial^\nu \tilde{F}^\lambda = \frac{1}{g^2}\epsilon_{\mu\nu\lambda}\partial^\nu V^* \partial^\lambda V$
This is the topological winding current:
$V(x) \propto g e^{i\phi(x)} \quad \rightarrow \quad Q = \int d^2x J_0 \propto \oint_\infty dl_i \partial_i \phi$
Charged states: SOLITONS OF V:



Nonabelian World: the Georgi-Glashow Model

$$L = -\frac{1}{4}F^a_{\mu\nu}F_a^{\mu\nu} + \left[\frac{1}{2}(D_\mu H^a)^2 - \frac{\lambda}{4}(H^a H^a - v^2)^2\right]$$

Perturbatively $\langle H^a \rangle = v\delta^{a3}$.
Perturbatively -QED with heavy charged vector particles $W^\pm$: $M_W^2 = g^2 v^2$
Perturbatively 'tHooft's Abelian magnetic field is conserved:

$$F_{\mu\nu} = \frac{H^a}{|H|}F^a_{\mu\nu} - \frac{1}{H^3}\epsilon_{abc}H^a(D_\mu H)^b(D_\nu H)^c$$

$$\partial_\mu \tilde{F}^\mu = 0$$

Nonerturbatively Polyakov's monopole-instantons ruin the conservation: ρ - monopole density

$$\boxed{\partial_\mu \tilde{F}^\mu = \frac{4\pi}{g}\rho}$$

The current is not conserved, but $Z_\mu(2)$ subgroup survives:
$\int d^3x\rho(x) = \text{integer}$: $e^{i\frac{g}{2}\Phi}$ - is conserved

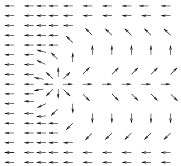
$$V \to -V \qquad \text{for SU(N)} \quad V \to e^{i\frac{2\pi n}{N}} V$$

$$\mathcal{L}_{eff} = \partial_\mu V^* \partial^\mu V - \lambda(V^*V - \mu^2)^2 - \xi(V^2 + V^{*2})$$

Tiny monopole fugacity (Polyakov): $\xi \propto e^{-\frac{4\pi M_W}{g^2}}$
Photon acquires mass: $V(x) = g e^{i\phi(x)}$: the potential is $\sim g^2\xi\cos 2\phi$.

$$m^2_{ph} \sim \xi$$

$W^\pm$ are solitons of winding number ± 1



The width of the string $d \sim 1/m_{ph}$.
Energy is linearly divergent with string tension $\sigma \propto (\Delta V)^2 d^{-1} = g^2 m_{ph}$

Abstract these features.

- Nothing but scalar fields with a global symmetry.
- Has Abelian regime - symmetry is enhanced to $U(1)$ - no confinement, but there is a Goldstone boson - massless photon.
- In the Abelian regime charges are classical topological solitons with Coulomb energy due to long range nature of the field.
- Non-abelian regime - the potential is perturbed so that the symmetry is discrete Z_N. Confinement appears as confinement of topological defects due to discreteness of symmetry and finite number of degenerate vacua.

Attempt to construct a 4D model

- The model should describe dynamics of scalar fields, and contain no fundamental gauge fields.
- The model should have the "Abelian limit" in which it has two massless Goldstone bosons. The Goldstone bosons $\equiv$ photons.
- In the Abelian regime there must exist classical topological solitons - electrically charged particles.The topological charge should relate to mapping of the spatial infinity onto the manifold of vacua,$\Pi_2(M)$. The energy of the solitons has to be finite in the infrared. The energy density of a soliton solution should decrease as $1/r^4$ far from the soliton core.
- Soliton must become confined in the "Non-abelian regime", when a symmetry breaking perturbation is added. This same perturbation must eliminate massless Goldstones by explicitly breaking the (previously) spontaneously broken symmetry group down to a discrete subgroup. Confinement should be accompanied by formation of string between the solitons.

Abelian model

The simplest choice of phase space that can give nontrivial Π_2 is the $O(3)$ - σ-model: $\Pi_2(S^2) = Z$.

$$\phi^a, \; a = 1, 2, 3; \qquad \phi^2 = 1; \qquad\qquad Q = \frac{e}{4\pi^2} \int d^3x \epsilon_{abc} \epsilon_{ijk} \partial^i \phi^a \partial_j \phi^b \partial_k \phi^c$$

WHAT LAGRANGIAN?

The standard kinetic term will not do.

The soliton configuration

$$\phi^a_h(x) = \frac{r^a}{|r|} f(|r|); \qquad\qquad f(|r|) \to_{r \to \infty} 1$$

must have finite energy. For standard kinetic term energy IR diverges as $\int d^3x \frac{1}{x^2} = L$.

The kinetic term must have four derivatives!

The natural choice is to identify $F^{\mu\nu}$ in terms of the topological current

$$\partial_\mu F^{\mu\nu} \propto j^\nu; \qquad j^\mu = \frac{e}{4\pi^2}\epsilon_{abc}\epsilon^{\mu\nu\lambda\sigma}\partial_\nu\phi^a\partial_\lambda\phi^b\partial_\sigma\phi^c$$

The QED-like Lagrangian (following in 2+1 D footsteps)

$$L = \frac{1}{16e^2}F^{\mu\nu}F_{\mu\nu} + \lambda(\phi^2 - 1)^2; \qquad F^{\mu\nu} = \epsilon^{abc}\epsilon^{\mu\nu\lambda\sigma}\phi^a\partial_\lambda\phi^b\partial_\sigma\phi^c$$

One half of Maxwell's equations are implemented topologically, like in usual duality

$$\partial^\mu F_{\mu\nu} = 0$$

For weaker coupled σ-model, $\phi^2 \neq 0$ and nonvanishing current is allowed.

In the perfect world the other half of Maxwell's equations - EoM.

SOLITON "electric field" is that of a pointlike charge, $E_i \propto \frac{\hat{r}_i}{r^2}$, and the soliton self energy is IR finite and UV renormalized $E \propto \Lambda$.

The Symmetries and Symmetry Breaking

We started with a simple $O(3)$ symmetry, but in fact our model has a much larger symmetry group.
The equations of motion can be written in terms of independent DOF

$$\phi_3 = z, \qquad \psi = \phi_1 + i\phi_2 = \sqrt{1-z^2}e^{i\chi}$$

as

$$\partial^\mu J^G_\mu = 0; \qquad J^G_\nu = G(z,\chi)\partial^\mu\left(\partial_\mu z \partial_\nu \chi - \partial_\nu z \partial_\mu \chi\right)$$

where $G(z,\chi)$- an arbitrary function of two variables.
Conserved currents - symmetry transformations:

$$(z,\chi) \to (z',\chi'); \qquad \frac{\partial(z',\chi')}{\partial(z,\chi)} = 1$$

Infinite dimensional symmetry group: area preserving diffeomorphisms of S^2
(World sheet diffeomorphism invariance of a string? What string??)

Photons?

Does it have photons?

We don't really expect that: Goldstone bosons should be scalars, since the broken currents are vectors. But this is an unusual group, so on the outside chance...

EoM:

$$[\partial_\nu G(z,\chi)]\partial_\mu \tilde{F}^{\mu\nu} = 0$$

Not quite the homogeneous Maxwell equation. But solutions of the homogeneous equation are solutions of EoM.

Consider:

$$\chi(x) = A\epsilon^\mu x_\mu; \qquad z(x) = \sin k^\mu x_\mu; \qquad k^2 = 0; \ \epsilon \cdot k = 0$$

On this configuration:

$$\tilde{F}^{\mu\nu} = A(\epsilon^\mu k^\nu - \epsilon^\nu k^\mu) \cos k \cdot x; \qquad \partial^\mu F_{\mu\nu} = 0$$

Looks just like a photon!

Unfortunately we cannot find multiphoton states (superpositions of plane waves) with arbitrary polarization vectors...

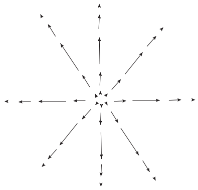
Let us add a perturbation that breaks the $SO(3)$ symmetry, and with it the diffeomorphisms.

$$L = \frac{1}{16e^2} F^{\mu\nu} F_{\mu\nu} - \frac{2}{e^2} \Lambda^2 (z-1)^2$$

The vacuum now is nondegenerate: $z = 1$. For a soliton we do not expect a hedgehog solution, as before, but rather a stringlike object.

The cross section of the string has to be a "baby skyrmion". For two solitons separated in the x_3 direction, the profile in the $x_1 - x_2$ plane is

$$\chi(x) = \theta(x); \qquad z(x) = z(r); \quad z(0) = -1; \quad z(r) \to_{r\to\infty} 1$$



The actual classical solution is

$$z(r^2) = 1 - 2\exp\{-\Lambda r^2\}; \quad \chi(x) = \theta(x)$$

String tension $\sigma = 8\pi\frac{\Lambda}{e^2}$
Fairly unusual profile: approaches vacuum as a Gaussian and not as an exponential as we are used to.

Going "Nonabelian" II

The actual nonabelian perturbation shoud only preseve Z_N symmetry, and not the complete $U(1)$. So let's try this

$$L = \frac{1}{16e^2}F^2 - \frac{2}{e^2}\Lambda^2(z-1)^2\left[1 - \mu(\psi^N + \psi^{\star N})\right]$$

$$= \frac{1}{16e^2}F^2 - \frac{2}{e^2}\Lambda^2(z-1)^2\left[1 - 2\mu(1-z^2)^{N/2}\cos N\chi\right]$$

We expect modulation of the phase χ in the transverse plane of the form

$$\delta\chi \propto \cos(N\theta)$$

so that only Z_N symmetry is left.
We do indeed find such solution. However the surprise is, that there is an infinite number of solutions.

For static, x_3 independent configurations the energy functional is

$$E = \int d^2x \frac{1}{2e^2}(\epsilon_{ij}\partial_i z \partial_j \chi)^2 + \frac{2}{e^2}\Lambda^2(z-1)^2\left[1 - 2\mu(1-z^2)^{N/2}\cos N\chi\right]$$

This, funnily, has a different diffeomorphism invariance:

$$(z(x), \chi(x)) \to (z(x'), \chi(x')); \qquad \frac{\partial(x'^1, x'^2)}{\partial(x^1, x^2)} = 1$$

We only see this invariance for t, x_3 independent configurations - should be lifted for fluctuations.

It is almost "gauge symmetry" - the "electromagnetic field " $F_{\mu\nu}(x)$ is invariant under this diffeomorphism transformation.

But the basic degrees of freedom $z(x), \chi(x)$ are not.

Asymptotically in r all solutions have the same Gaussian behavior.

One can write solutions in terms of some incomplete elliptic integrals, but I will leave it at that.

Conclusions

We have constructed a scalar "effective theory" that satisfies our minimal requirements.

However:

- It does not have enough photons.
- Puzzing diffeomorphism invariances of various kinds.
- Puzzling Gaussian falloff of field profile.

Food for thought...