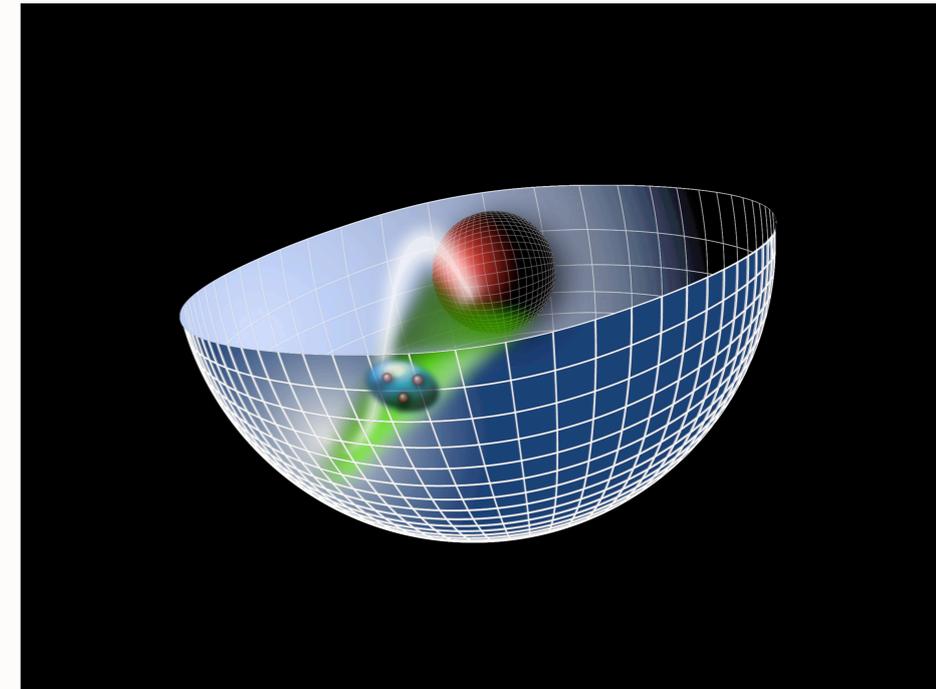
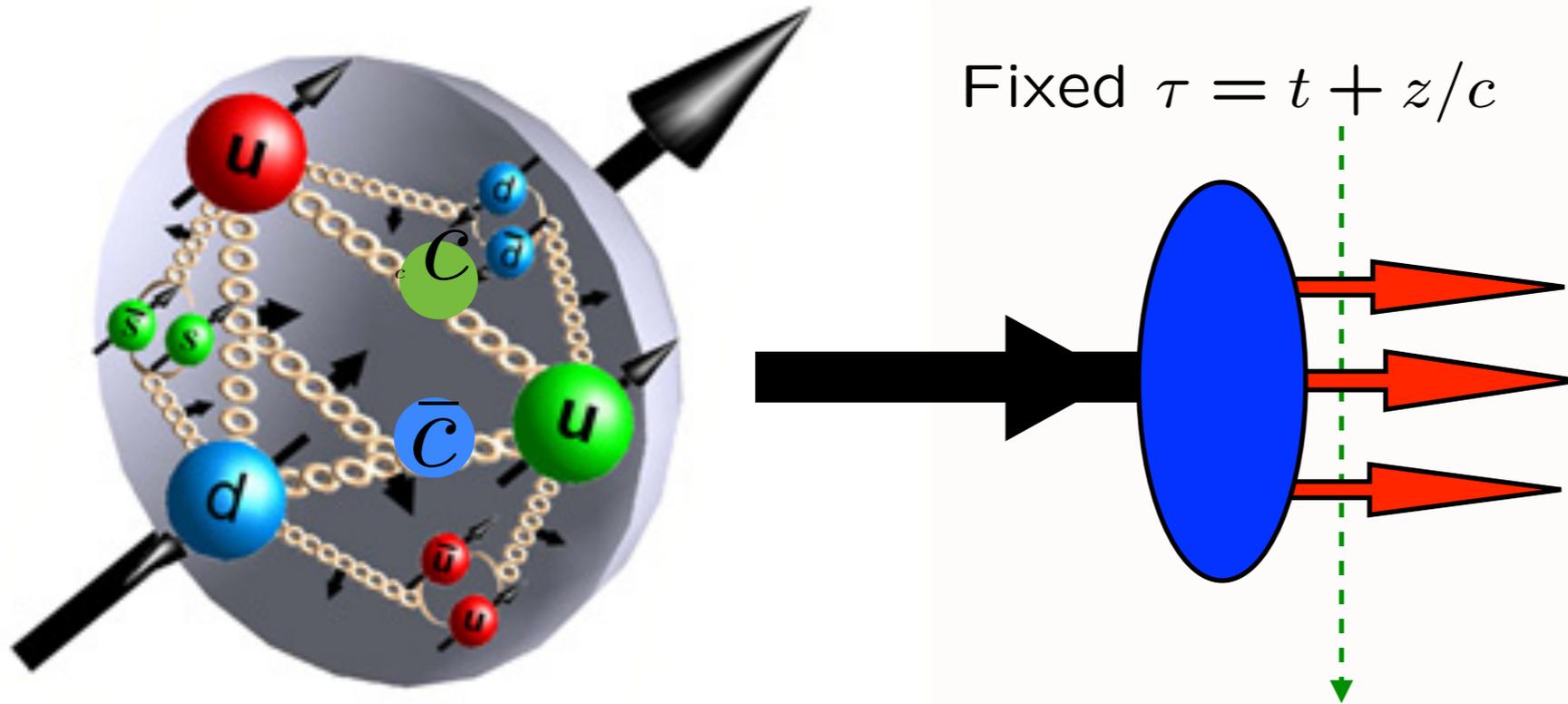


Novel QCD Phenomena and New Perspectives for Hadron Physics from Light-Front Holography



Stan Brodsky



High Energy Physics
in the LHC Era

5th International Workshop

Universidad Técnica
Federico Santa María

December 16-20, 2013

May 19-20, 2011

Goal: an analytic first approximation to QCD

- **As Simple as Schrödinger Theory in Atomic Physics**
- **Relativistic, Frame-Independent, Color-Confining**
- **Confinement in QCD -- What sets the QCD mass scale?**
- **QCD Coupling at all scales**
- **Hadron Spectroscopy**
- **Light-Front Wavefunctions**
- **Form Factors, Hadronic Observables, Constituent Counting Rules**
- **Hadronization at the Amplitude Level**
- **Insights into QCD Condensates**
- **Chiral Symmetry**
- **Systematically improvable**
- **Eliminate scale ambiguities**

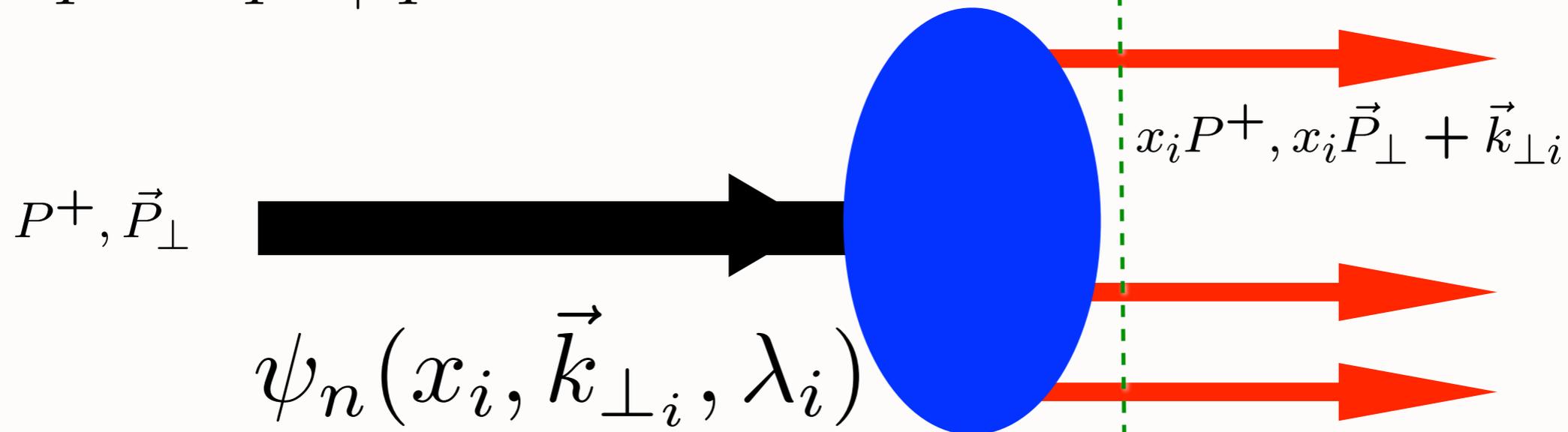


Light-Front Wavefunctions: **rigorous** representation of composite systems in quantum field theory

Eigenstate of LF Hamiltonian

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

Fixed $\tau = t + z/c$



$$|p, J_z \rangle = \sum_{n=3} \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; x_i, \vec{k}_{\perp i}, \lambda_i \rangle$$

$$\sum_i^n x_i = 1$$

$$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$$

Invariant under boosts! Independent of P^μ

Causal, Frame-independent. Creation Operators on Simple Vacuum, Current Matrix Elements are Overlaps of LFWFS

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

P^+, \vec{P}_\perp

$\psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$

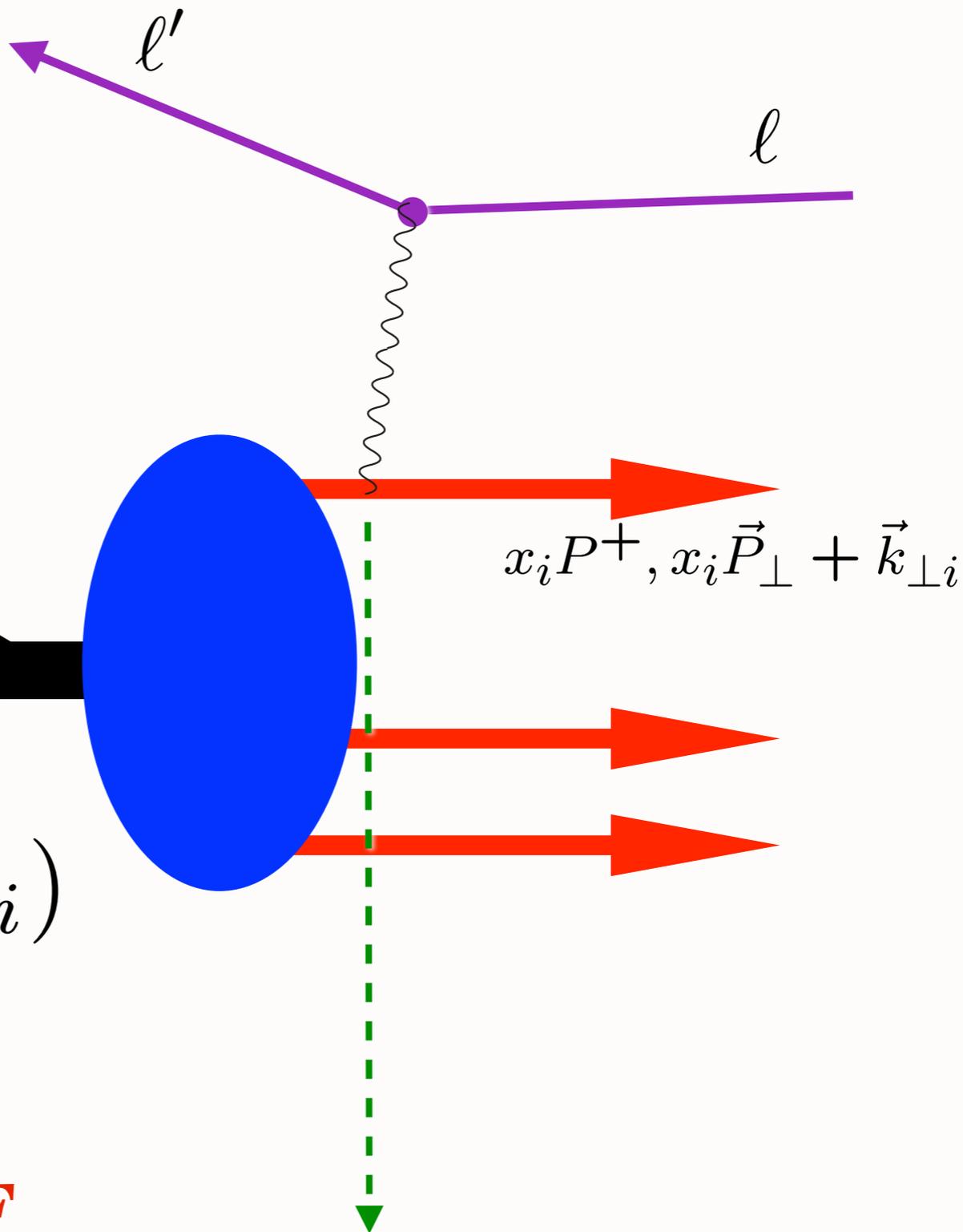
$x_i P^+, x_i \vec{P}_\perp + \vec{k}_{\perp i}$

Fixed $\tau = t + z/c$

$$x_{bj} = x = \frac{k^+}{P^+}$$

Measurements of hadron LF wavefunction are at fixed LF time

Like a flash photograph



$$\langle p + q | j^+(0) | p \rangle = 2p^+ F(q^2)$$

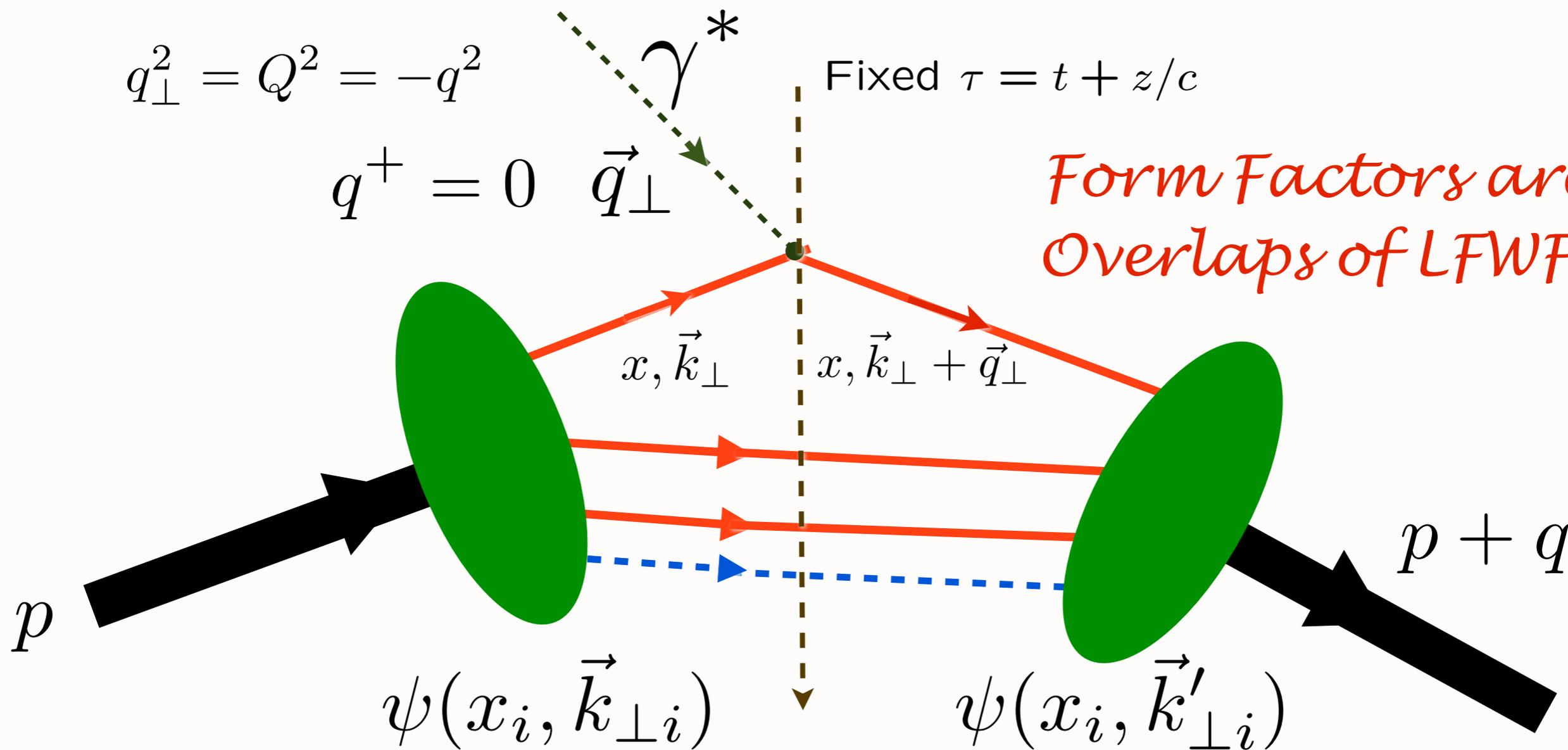
Interaction picture

$$q_{\perp}^2 = Q^2 = -q^2$$

$$q^+ = 0 \quad \vec{q}_{\perp}$$

Fixed $\tau = t + z/c$

Form Factors are Overlaps of LFWFs



$$\psi(x_i, \vec{k}_{\perp i})$$

$$\psi(x_i, \vec{k}'_{\perp i})$$

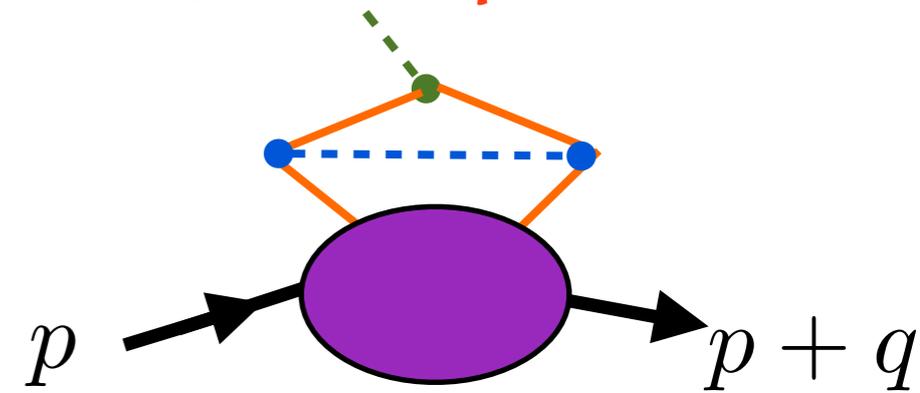
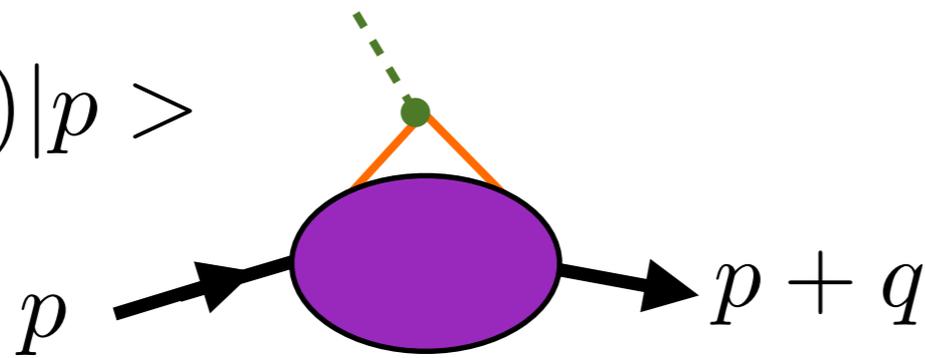
struck $\vec{k}'_{\perp i} = \vec{k}_{\perp i} + (1 - x_i)\vec{q}_{\perp}$

spectators $\vec{k}'_{\perp i} = \vec{k}_{\perp i} - x_i\vec{q}_{\perp}$

**Drell & Yan, West
Exact LF formula!**

Calculation of proton form factor in Instant Form

$$\langle p + q | J^\mu(0) | p \rangle$$



- **Need to boost proton wavefunction from p to $p + q$: Extremely complicated dynamical problem; even the particle number changes**
- **Need to couple to all currents arising from vacuum!! Remains even after normal-ordering**
- **Each time-ordered contribution is frame-dependent**
- **Divide by disconnected vacuum diagrams**
- **Instant form: acausal boundary conditions**

Exact LF Formula for Pauli Form Factor

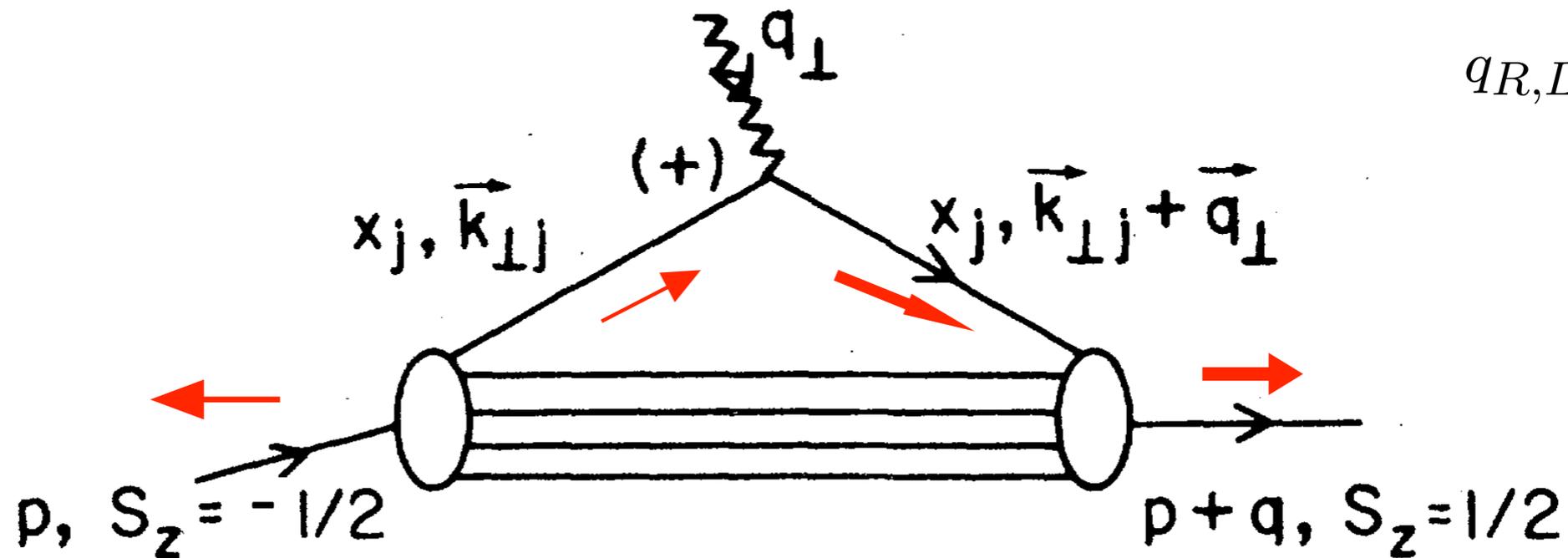
$$\frac{F_2(q^2)}{2M} = \sum_a \int [dx] [d^2\mathbf{k}_\perp] \sum_j e_j \frac{1}{2} \times$$

$$\left[-\frac{1}{q^L} \psi_a^{\uparrow*}(x_i, \mathbf{k}'_{\perp i}, \lambda_i) \psi_a^\downarrow(x_i, \mathbf{k}_{\perp i}, \lambda_i) + \frac{1}{q^R} \psi_a^{\downarrow*}(x_i, \mathbf{k}'_{\perp i}, \lambda_i) \psi_a^\uparrow(x_i, \mathbf{k}_{\perp i}, \lambda_i) \right]$$

$$\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i \mathbf{q}_\perp \qquad \mathbf{k}'_{\perp j} = \mathbf{k}_{\perp j} + (1 - x_j) \mathbf{q}_\perp$$

Drell, sjb

$$q_{R,L} = q^x \pm iq^y$$



Must have $\Delta l_z = \pm 1$ to have nonzero $F_2(q^2)$

Nonzero Proton Anomalous Moment -->
Nonzero orbital quark angular momentum

Angular Momentum on the Light-Front

$$J^z = \sum_{i=1}^n s_i^z + \sum_{j=1}^{n-1} l_j^z.$$

**Conserved
LF Fock-State by Fock-State
Every Vertex**

$$l_j^z = -i \left(k_j^1 \frac{\partial}{\partial k_j^2} - k_j^2 \frac{\partial}{\partial k_j^1} \right)$$

**n-1 orbital angular
momenta**

Nonzero Anomalous Moment <--> Nonzero orbital angular momentum

Drell, sjb, Schmidt

Parke-Taylor Amplitudes

Santiago-Cruz, Stasto

Single-spin asymmetries

Leading Twist Sivers Effect

Hwang, Schmidt, sjb

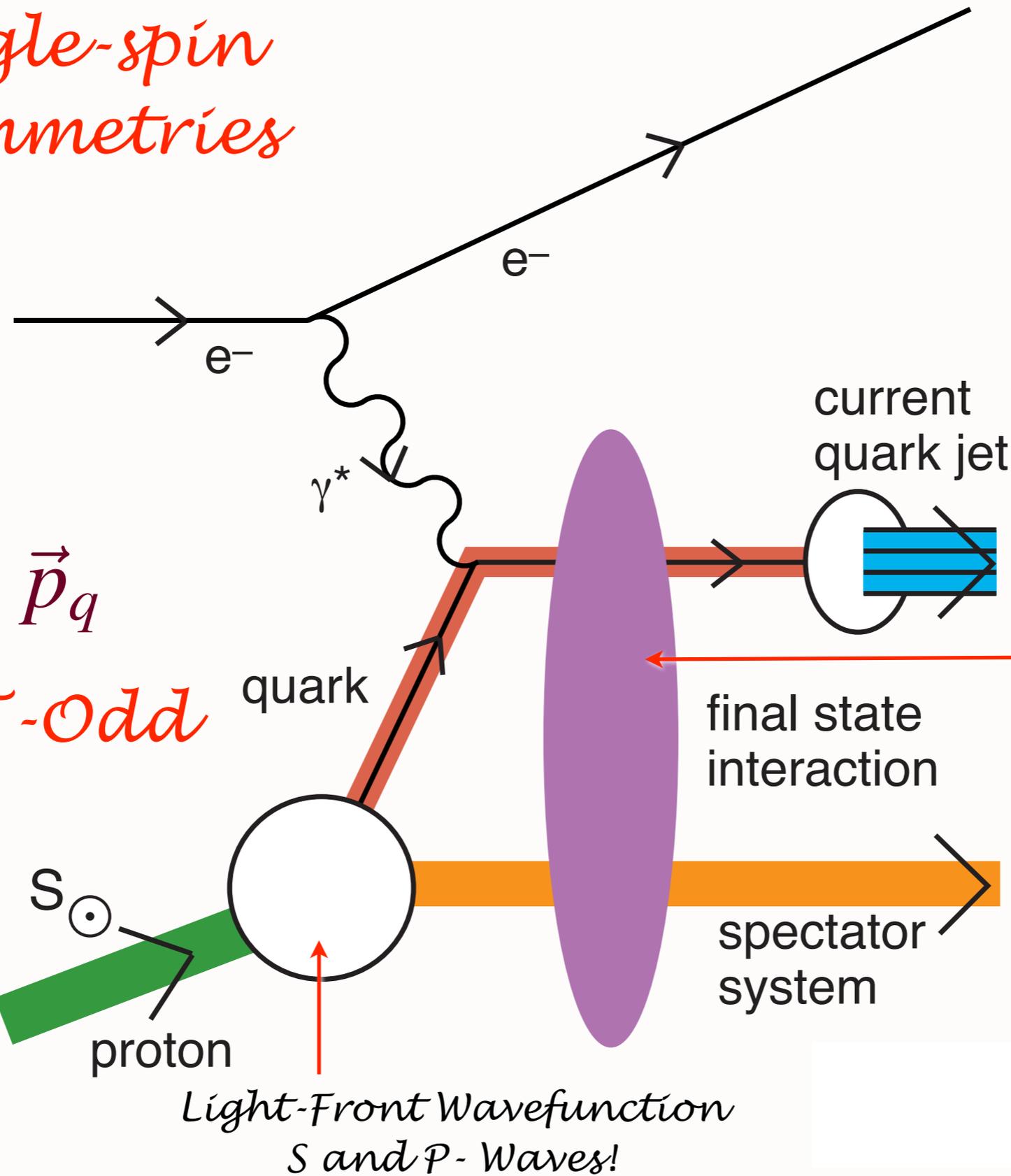
Collins, Burkardt, Ji, Yuan. Pasquini, ...

QCD S- and P-Coulomb Phases --Wilson Line

“Lensing Effect”

Leading-Twist Rescattering Violates pQCD Factorization!

Relation to confining interaction?



$$i \vec{S}_p \cdot \vec{q} \times \vec{p}_q$$

Pseudo-T-Odd

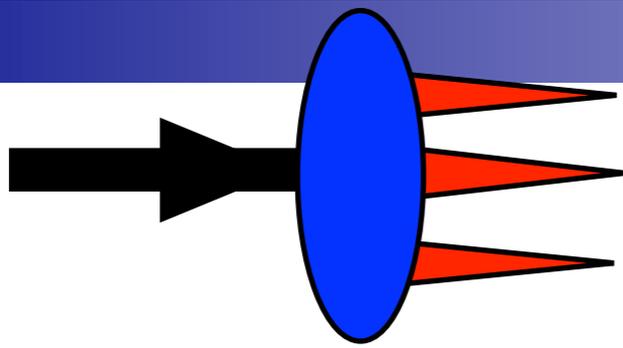
**QED:
Lensing
involves soft
scales**

S_{\odot}
proton

*Light-Front Wavefunction
S and P- Waves!*

Sign reversal in DY!

• *Light Front Wavefunctions:*



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

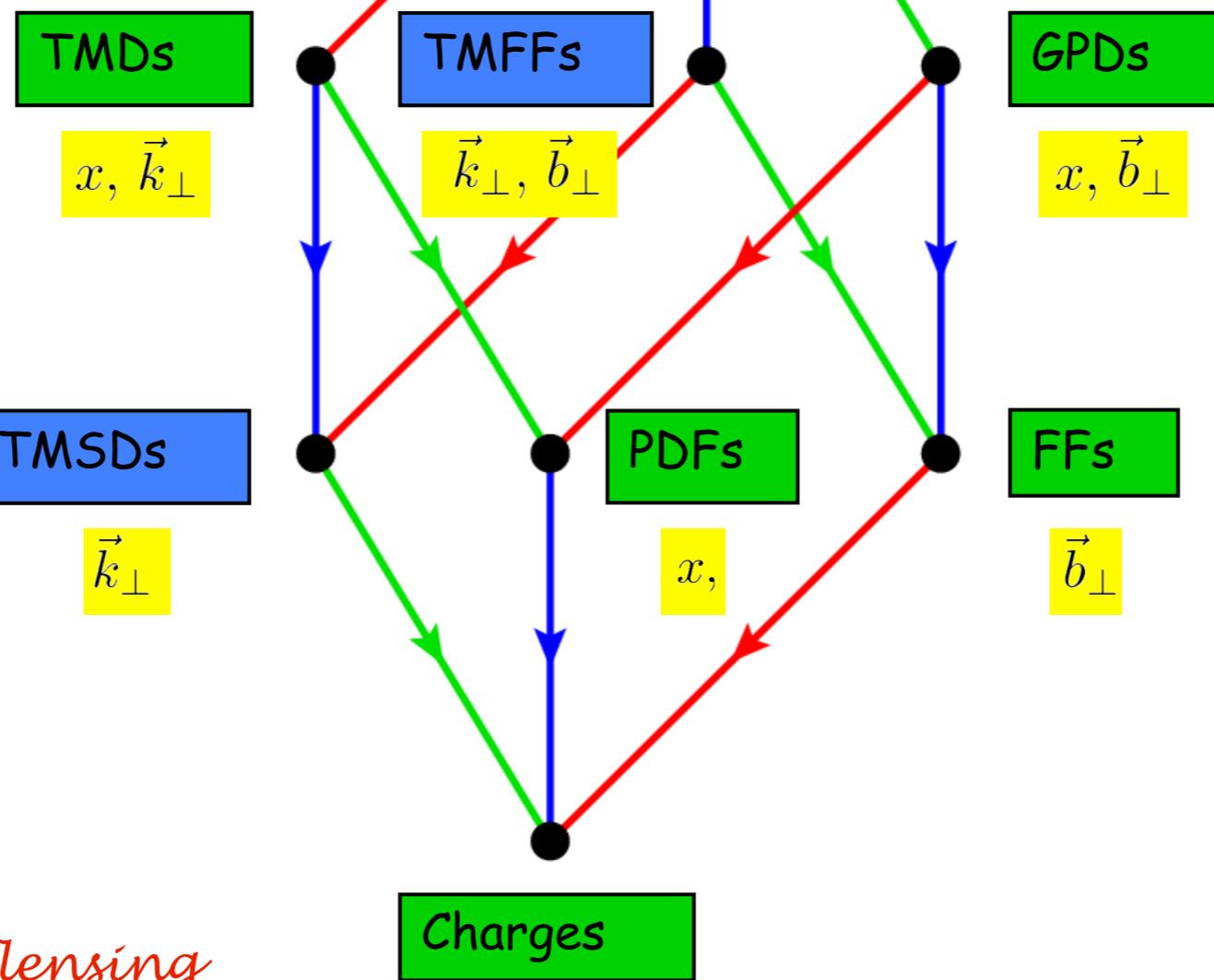
GTMDs

$$x, \vec{k}_{\perp}, \vec{b}_{\perp}$$

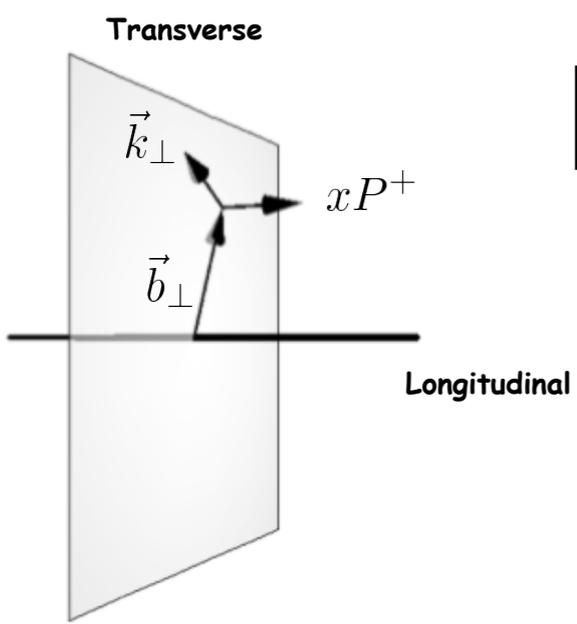
Momentum space $\vec{k}_{\perp} \leftrightarrow \vec{z}_{\perp}$ Position space
 $\vec{\Delta}_{\perp} \leftrightarrow \vec{b}_{\perp}$

Transverse density in momentum space

Transverse density in position space



*Lorce,
Pasquini*



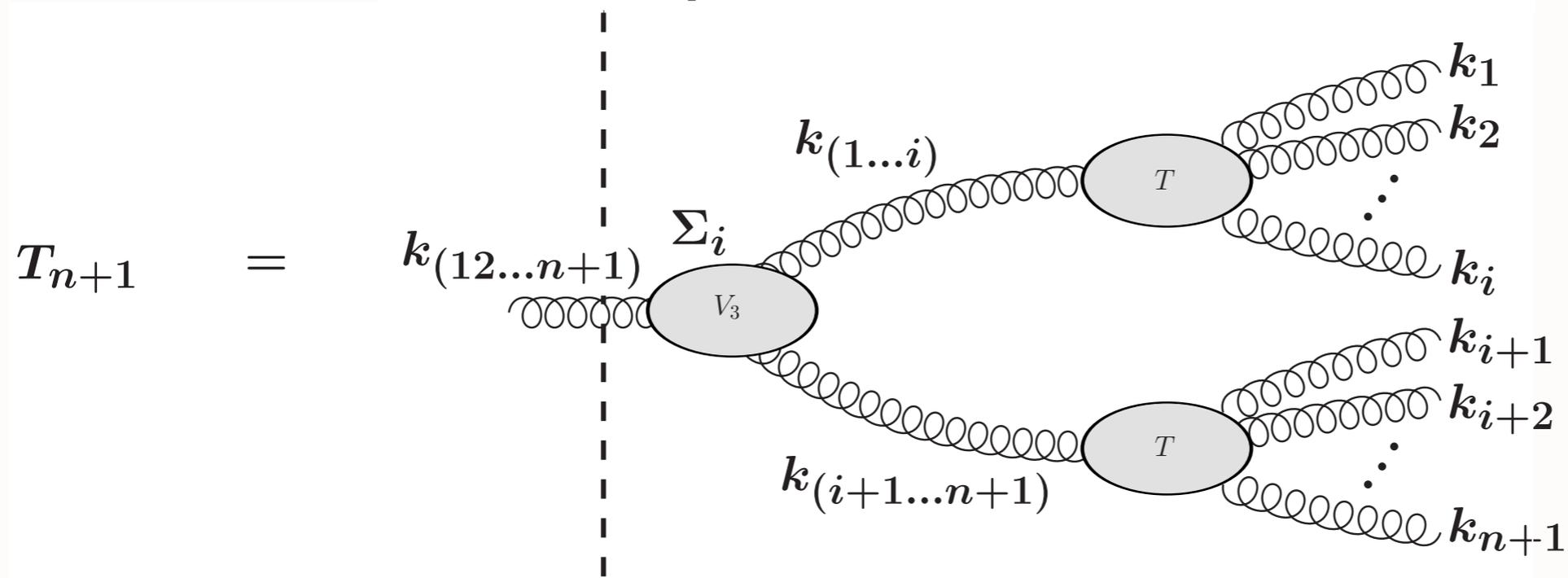
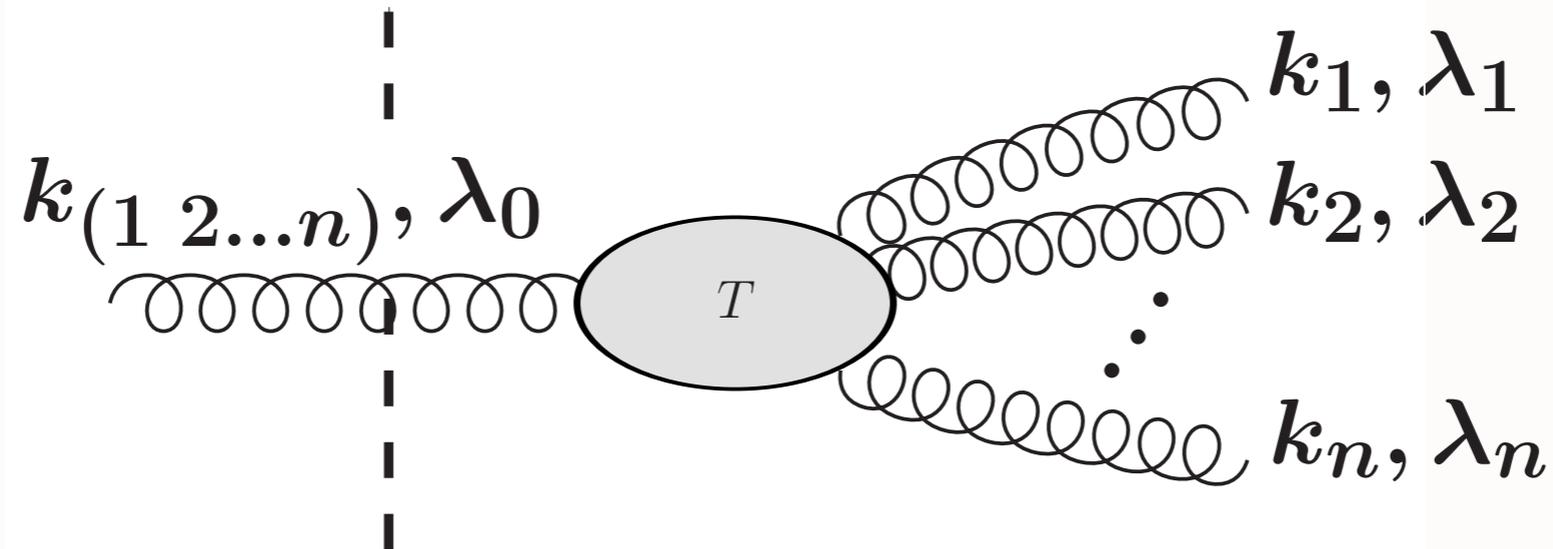
Sivers, T-odd from lensing

→ $\int d^2 b_{\perp}$
 → $\int dx$
 → $\int d^2 k_{\perp}$

Recursion Relations and Scattering Amplitudes in the Light-Front Formalism

Cruz-Santiago & Stasto

Cluster Decomposition Theorem for relativistic systems: C. Ji & sjb



Parke-Taylor amplitudes reflect LF angular momentum conservation

$$\langle ij \rangle = \sqrt{z_i z_j} \underline{\epsilon}^{(-)} \cdot \left(\frac{\underline{k}_i}{z_i} - \frac{\underline{k}_j}{z_j} \right) =$$

Advantages of the Dirac's Front Form for Hadron Physics



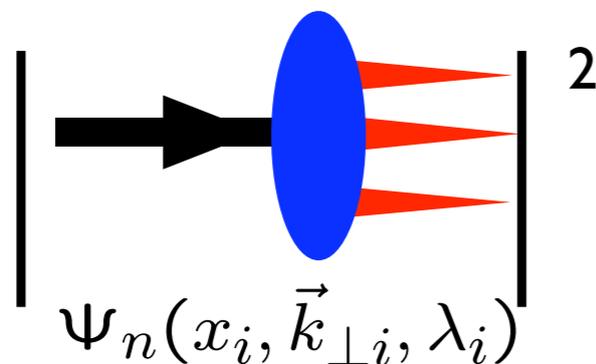
- **Measurements are made at fixed τ**
- **Causality is automatic**
- **Structure Functions are squares of LFWFs**
- **Form Factors are overlap of LFWFs**
- **LFWFs are frame-independent -- no boosts!**
- **No dependence on observer's frame**
- **LF Holography: Dual to AdS space**
- **LF Vacuum trivial -- no condensates!**
- **Profound implications for Cosmological**

Structure functions are not parton probabilities.

By Stanley J. Brodsky, Paul Hoyer,
Nils Marchal, Stephane Peigne, Francesco Sannino.
Phys.Rev. D65 (2002) 114025.

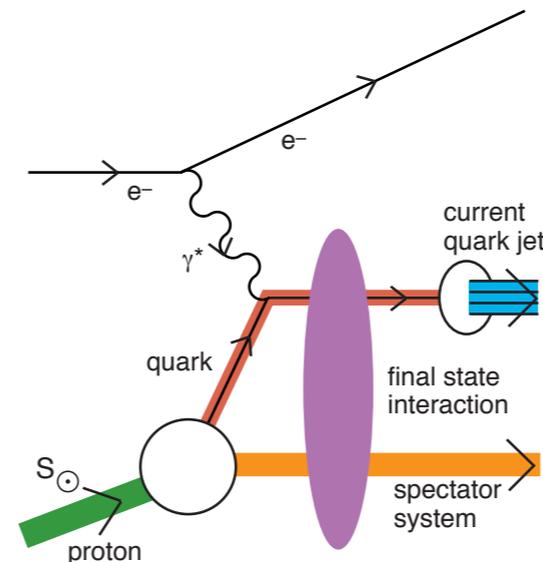
Static

- Square of Target LFWFs
- No Wilson Line
- Probability Distributions
- Process-Independent
- T-even Observables
- No Shadowing, Anti-Shadowing
- Sum Rules: Momentum and J^z
- DGLAP Evolution; mod. at large x
- No Diffractive DIS



Dynamic

- Modified by Rescattering: ISI & FSI
- Contains Wilson Line, Phases
- No Probabilistic Interpretation
- Process-Dependent - From Collision
- T-Odd (Sivers, Boer-Mulders, etc.)
- Shadowing, Anti-Shadowing, Saturation
- Sum Rules Not Proven
- DGLAP Evolution
- Hard Pomeron and Odderon Diffractive DIS



**Hwang,
Schmidt, sjb,**

Mulders, Boer

Qiu, Sterman

Collins, Qiu

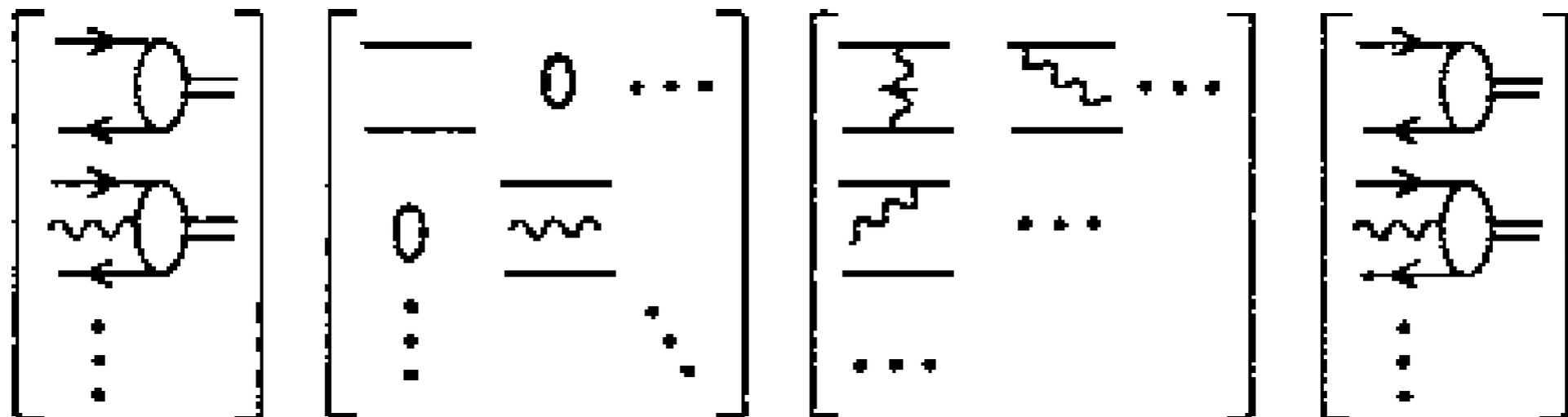
**Pasquini, Xiao,
Yuan, sjb**

LIGHT-FRONT MATRIX EQUATION

Rigorous Method for Solving Non-Perturbative QCD!

$$\left(M_\pi^2 - \sum_i \frac{\vec{k}_{\perp i}^2 + m_i^2}{x_i} \right) \begin{bmatrix} \psi_{q\bar{q}}/\pi \\ \psi_{q\bar{q}g}/\pi \\ \vdots \end{bmatrix} = \begin{bmatrix} \langle q\bar{q} | V | q\bar{q} \rangle & \langle q\bar{q} | V | q\bar{q}g \rangle & \cdots \\ \langle q\bar{q}g | V | q\bar{q} \rangle & \langle q\bar{q}g | V | q\bar{q}g \rangle & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} \psi_{q\bar{q}}/\pi \\ \psi_{q\bar{q}g}/\pi \\ \vdots \end{bmatrix}$$

$$A^+ = 0$$



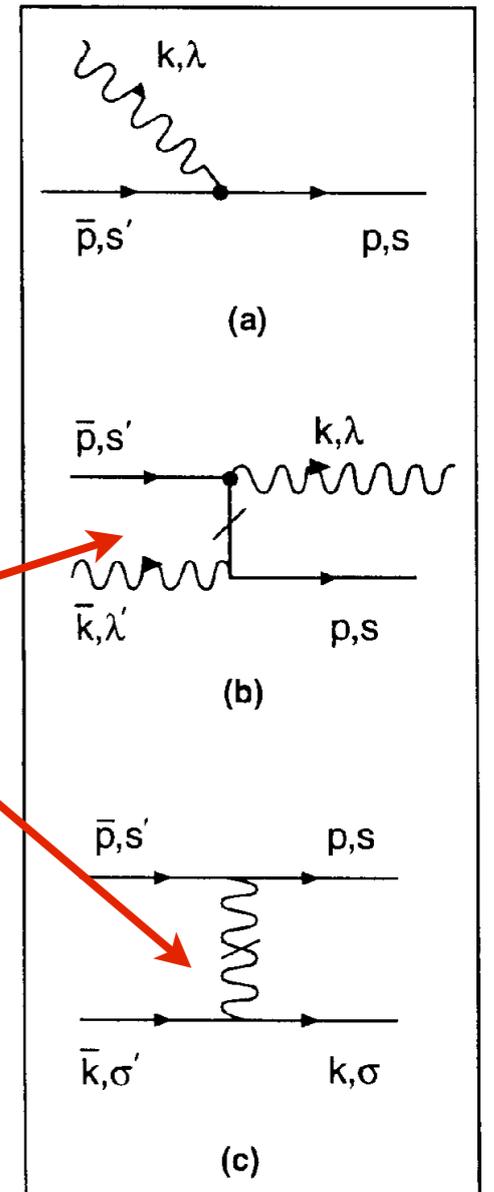
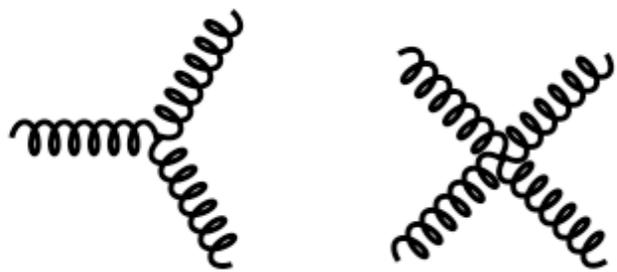
Minkowski space; frame-independent; no fermion doubling; no ghosts

- *Light-Front Vacuum = vacuum of free Hamiltonian!*

$$\mathcal{L}_{QCD} = -\frac{1}{4} \text{Tr}(G^{\mu\nu} G_{\mu\nu}) + \sum_{f=1}^{n_f} i\bar{\Psi}_f D_\mu \gamma^\mu \Psi_f + \sum_{f=1}^{n_f} m_f \bar{\Psi}_f \Psi_f$$

H_{QCD}^{LF}

$$\begin{aligned} &= \frac{1}{2} \int d^3x \bar{\psi} \gamma^+ \frac{(i\partial^\perp)^2 + m^2}{i\partial^+} \tilde{\psi} - A_a^i (i\partial^\perp)^2 A_{ia} \\ &- \frac{1}{2} g^2 \int d^3x \text{Tr} [\tilde{A}^\mu, \tilde{A}^\nu] [\tilde{A}_\mu, \tilde{A}_\nu] \\ &+ \frac{1}{2} g^2 \int d^3x \bar{\psi} \gamma^+ T^a \tilde{\psi} \frac{1}{(i\partial^+)^2} \bar{\psi} \gamma^+ T^a \tilde{\psi} \\ &- g^2 \int d^3x \bar{\psi} \gamma^+ \left(\frac{1}{(i\partial^+)^2} [i\partial^+ \tilde{A}^\kappa, \tilde{A}_\kappa] \right) \tilde{\psi} \\ &+ g^2 \int d^3x \text{Tr} \left([i\partial^+ \tilde{A}^\kappa, \tilde{A}_\kappa] \frac{1}{(i\partial^+)^2} [i\partial^+ \tilde{A}^\kappa, \tilde{A}_\kappa] \right) \\ &+ \frac{1}{2} g^2 \int d^3x \bar{\psi} \tilde{A} \frac{\gamma^+}{i\partial^+} \tilde{A} \tilde{\psi} \\ &+ g \int d^3x \bar{\psi} \tilde{A} \tilde{\psi} \\ &+ 2g \int d^3x \text{Tr} (i\partial^\mu \tilde{A}^\nu [\tilde{A}_\mu, \tilde{A}_\nu]) \end{aligned}$$



Rigorous First-Principle Formulation of Non-Perturbative QCD

Exact frame-independent formulation of nonperturbative QCD!

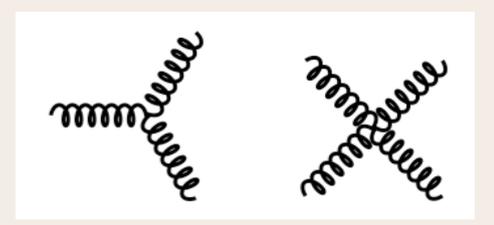
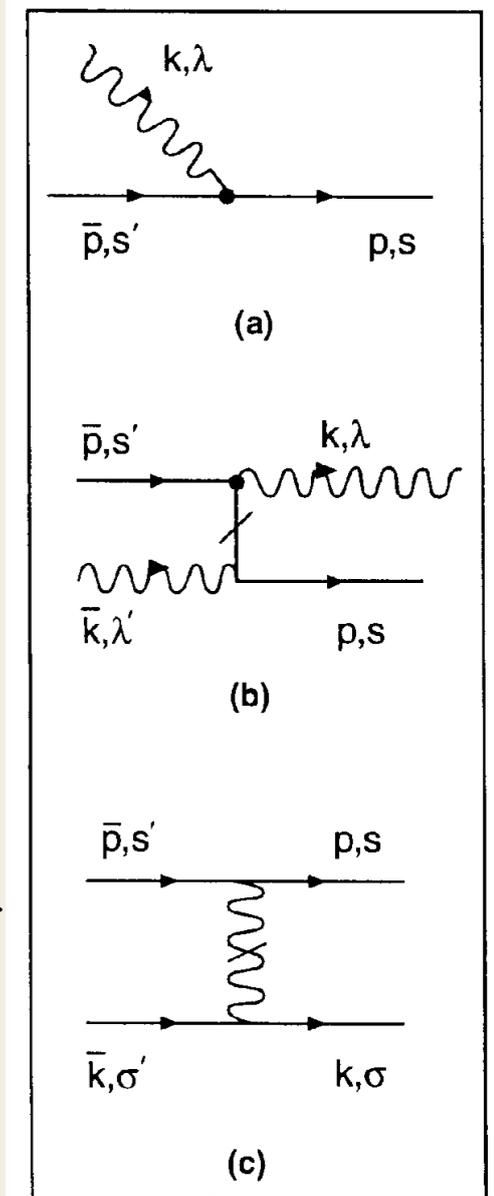
$$L^{QCD} \rightarrow H_{LF}^{QCD}$$

$$H_{LF}^{QCD} = \sum_i \left[\frac{m^2 + k_{\perp}^2}{x} \right]_i + H_{LF}^{int}$$

H_{LF}^{int} : Matrix in Fock Space

$$H_{LF}^{QCD} |\Psi_h\rangle = \mathcal{M}_h^2 |\Psi_h\rangle$$

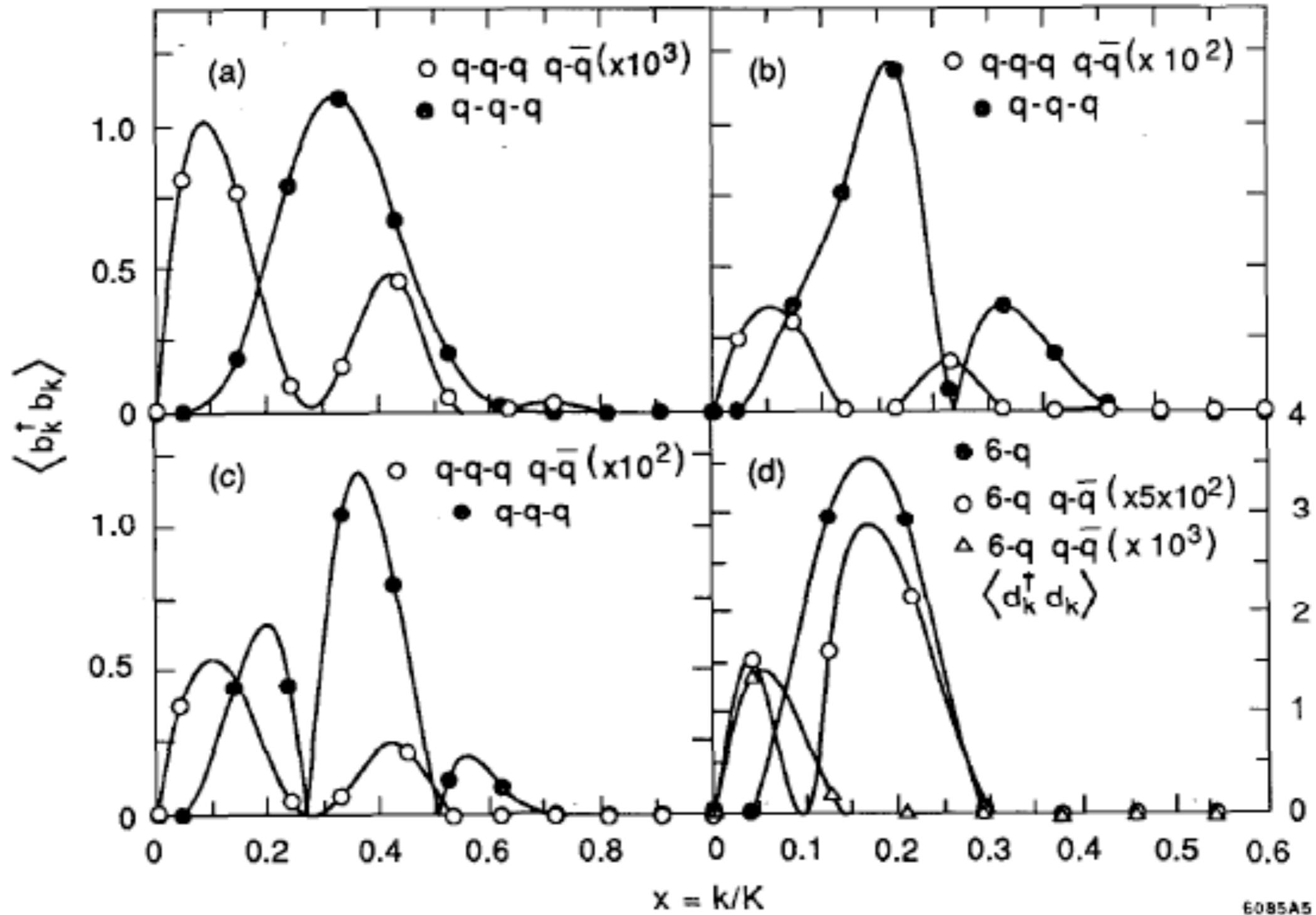
$$|p, J_z\rangle = \sum_{n=3} \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; x_i, \vec{k}_{\perp i}, \lambda_i\rangle$$



H_{LF}^{int}

Eigenvalues and Eigensolutions give Hadronic Spectrum and Light-Front wavefunctions

LFWFs: Off-shell in P- and invariant mass



a-c) First three states in $N = 3$ baryon spectrum, $2K=21$. d) First $B = 2$ state.

$$|p, S_z\rangle = \sum_{n=3} \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; \vec{k}_{\perp i}, \lambda_i\rangle$$

sum over states with $n=3, 4, \dots$ constituents

The Light Front Fock State Wavefunctions

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

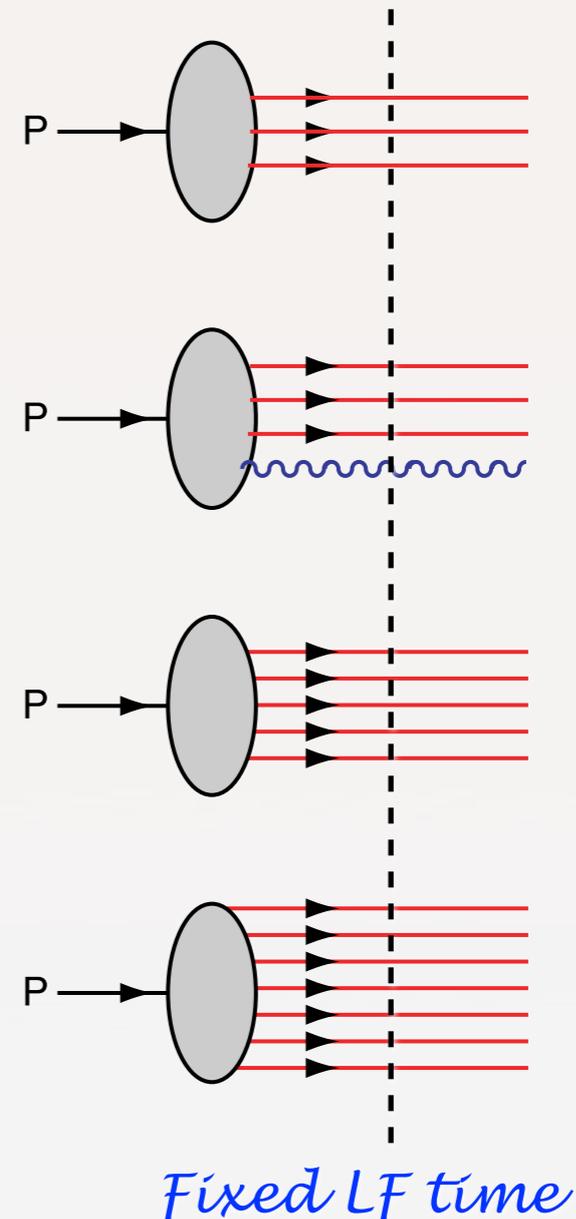
are boost invariant; they are independent of the hadron's energy and momentum P^μ .

The light-cone momentum fraction

$$x_i = \frac{k_i^+}{p^+} = \frac{k_i^0 + k_i^z}{P^0 + P^z}$$

are boost invariant.

$$\sum_i^n k_i^+ = P^+, \quad \sum_i^n x_i = 1, \quad \sum_i^n \vec{k}_i^\perp = \vec{0}^\perp.$$



Intrinsic heavy quarks
 $s(x), c(x), b(x)$ at high x !

$\bar{s}(x) \neq s(x)$
 $\bar{u}(x) \neq \bar{d}(x)$

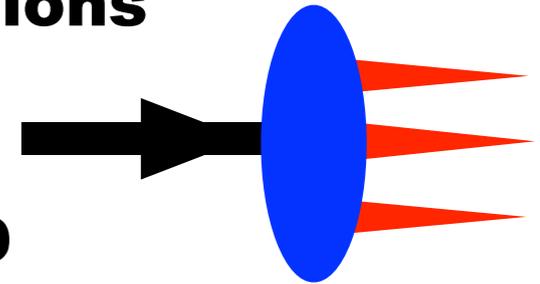
Mueller: gluon Fock states

BFKL Pomeron

Hidden Color

- **LF wavefunctions play the role of Schrödinger wavefunctions in Atomic Physics**

- **LFWFs=Hadron Eigensolutions: Direct Connection to QCD Lagrangian**



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

- **Relativistic, frame-independent: no boosts, no disc contraction, Melosh built into LF spinors**
- **Hadronic observables computed from LFWFs: Form factors, Structure Functions, Distribution Amplitudes, GPDs, TMDs, Weak Decays, modulo 'lensing' from ISIs, FSIs**
- **Cannot compute current matrix elements using instant form from eigensolutions alone -- need to include vacuum currents!**
- **Hadron Physics without LFWFs is like Biology without DNA!**

$$H_{QED}$$

QED atoms: positronium and muonium

$$(H_0 + H_{int}) |\Psi\rangle = E |\Psi\rangle$$

Coupled Fock states

*Eliminate higher Fock states
(retarded interactions)*

$$\left[-\frac{\Delta^2}{2m_{\text{red}}} + V_{\text{eff}}(\vec{S}, \vec{r})\right] \psi(\vec{r}) = E \psi(\vec{r})$$

Effective two-particle equation

Includes Lamb Shift, quantum corrections

$$\left[-\frac{1}{2m_{\text{red}}} \frac{d^2}{dr^2} + \frac{1}{2m_{\text{red}}} \frac{l(l+1)}{r^2} + V_{\text{eff}}(r, S, l)\right] \psi(r) = E \psi(r)$$

Spherical Basis r, θ, ϕ

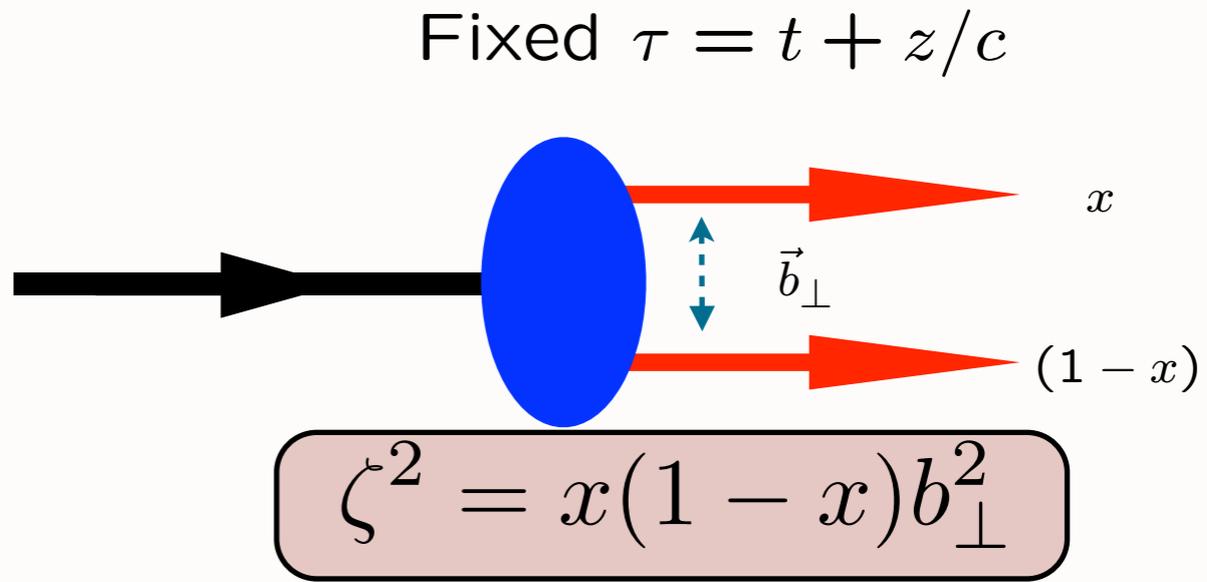
Coulomb potential

$$V_{\text{eff}} \rightarrow V_C(r) = -\frac{\alpha}{r}$$

*Semiclassical first approximation to QED --> **Bohr Spectrum***

Light-Front QCD

$$H_{QCD}^{LF}$$



$$(H_{LF}^0 + H_{LF}^I) |\Psi\rangle = M^2 |\Psi\rangle$$

Coupled Fock states

$$\left[\frac{\vec{k}_\perp^2 + m^2}{x(1-x)} + V_{\text{eff}}^{LF} \right] \psi_{LF}(x, \vec{k}_\perp) = M^2 \psi_{LF}(x, \vec{k}_\perp)$$

*Eliminate higher Fock states
(retarded interactions)*

Effective two-particle equation

$$\left[-\frac{d^2}{d\zeta^2} + \frac{m^2}{x(1-x)} + \frac{-1 + 4L^2}{4\zeta^2} + U(\zeta, S, L) \right] \psi_{LF}(\zeta) = M^2 \psi_{LF}(\zeta)$$

Azimuthal Basis

$$\zeta, \phi$$

AdS/QCD:

$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)$$

*Confining AdS/QCD
potential!*

Semiclassical first approximation to QCD

Sums an infinite # diagrams

Light-Front Schrödinger Equation

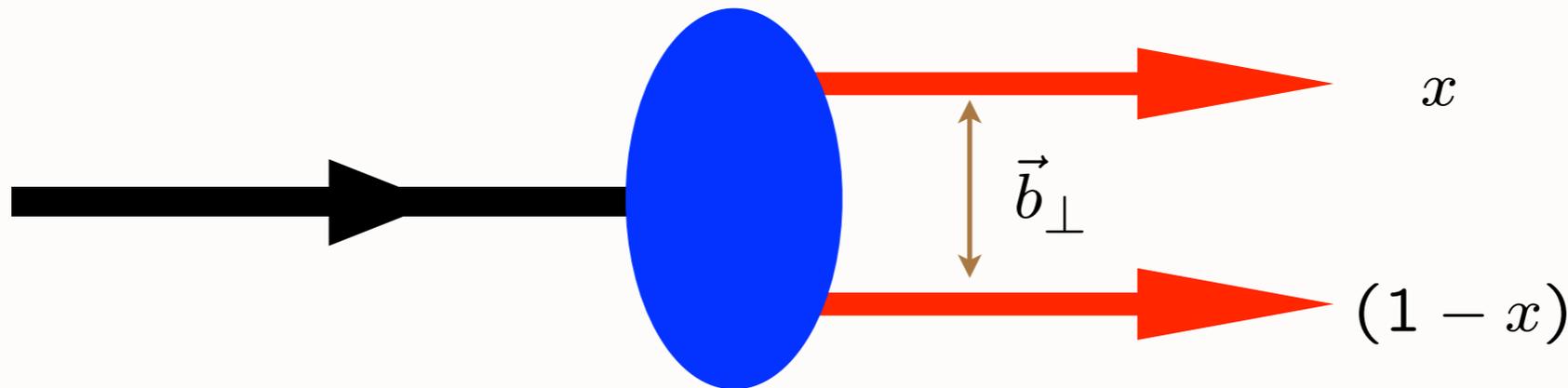
G. de Teramond, sjb

Relativistic LF single-variable radial equation for QCD & QED

Frame Independent!

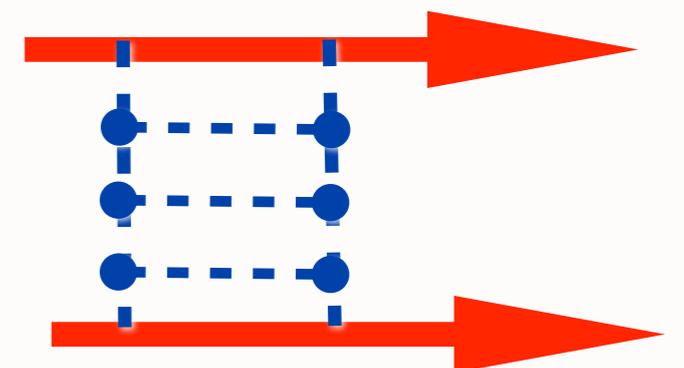
$$\left[-\frac{d^2}{d\zeta^2} + \frac{m^2}{x(1-x)} + \frac{-1 + 4L^2}{4\zeta^2} + U(\zeta, S, L) \right] \psi_{LF}(\zeta) = M^2 \psi_{LF}(\zeta)$$

$$\zeta^2 = x(1-x)\mathbf{b}_\perp^2.$$



U is the confining QCD potential
Conjecture: 'H'-diagrams generate

$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)$$

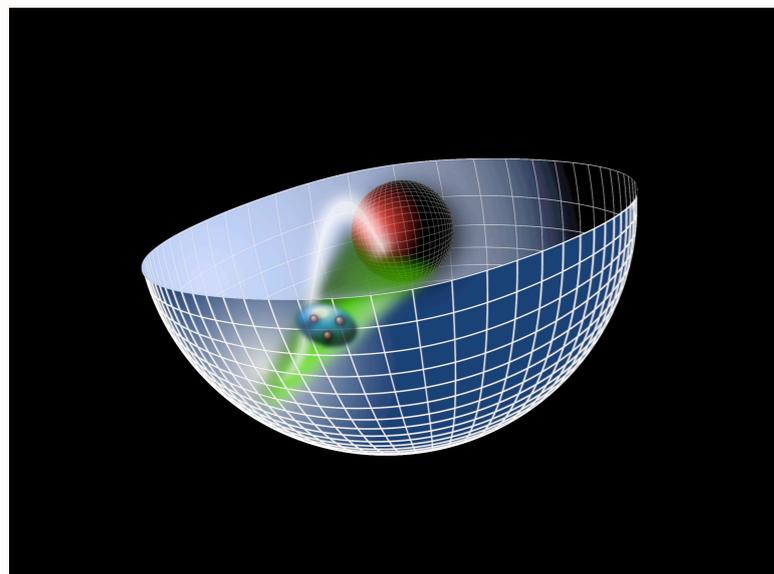


$$\phi(z)$$

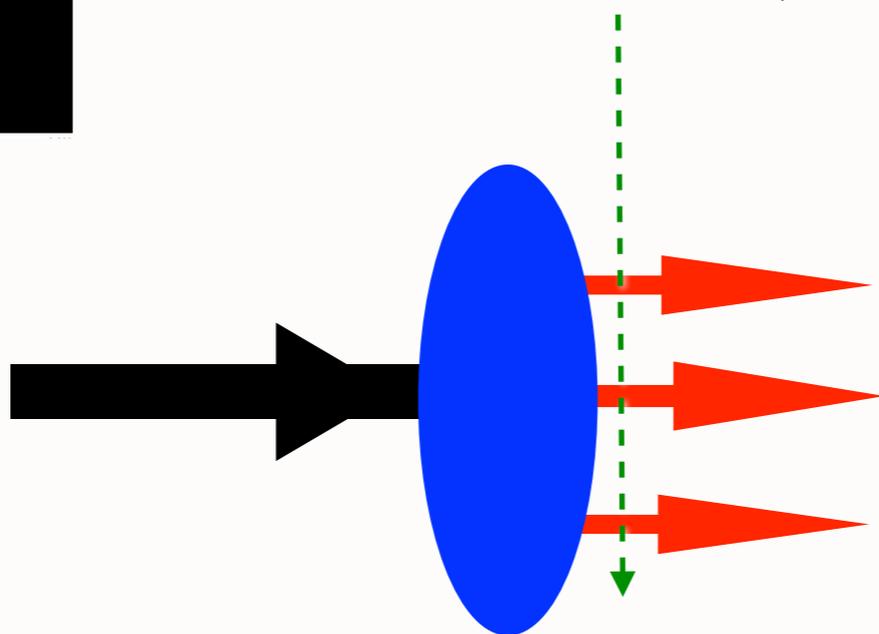
AdS₅: Conformal Template for QCD

• Light-Front Holography

Duality of AdS₅ with LF Hamiltonian Theory



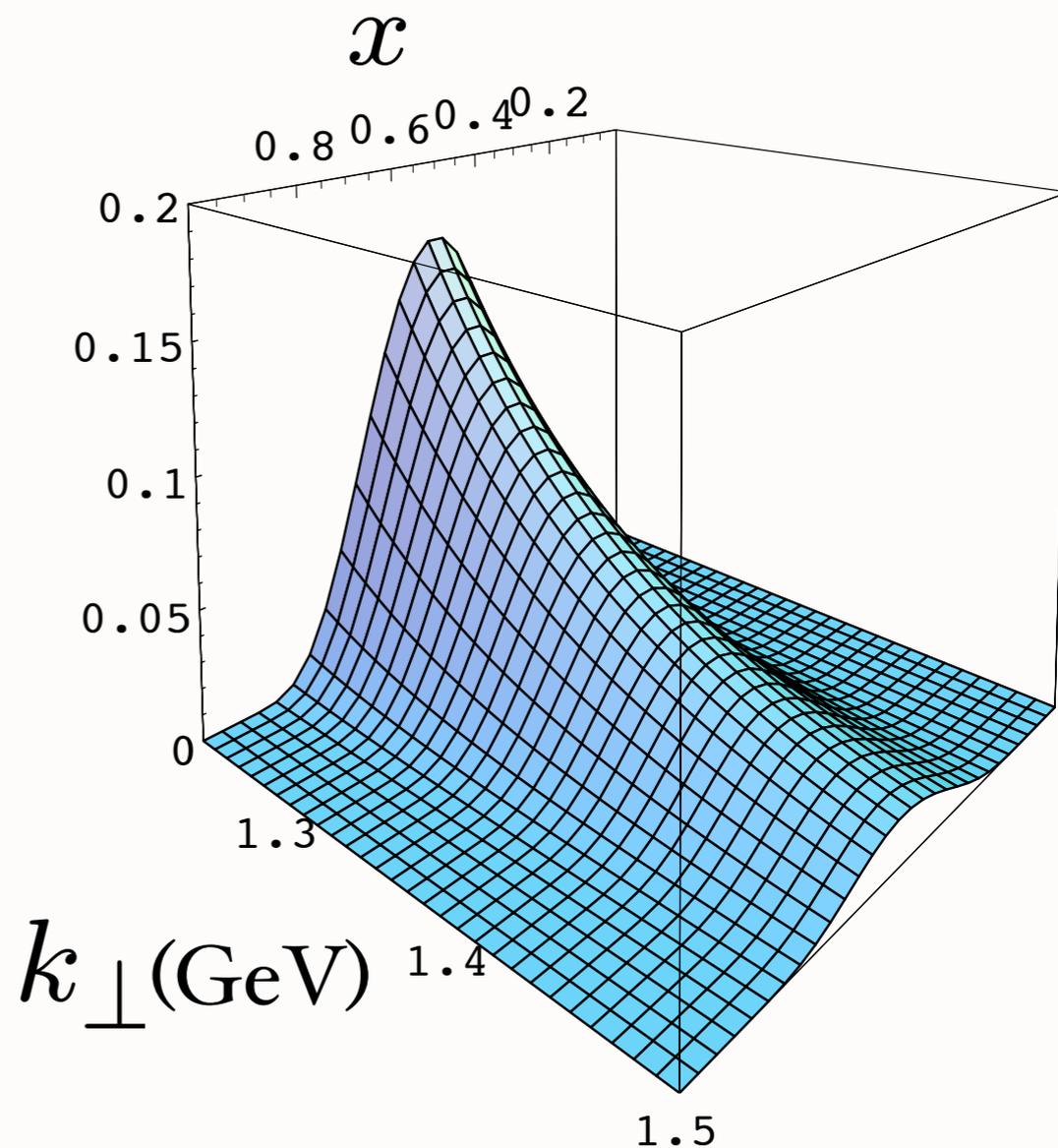
Fixed $\tau = t + z/c$



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

• Light Front Wavefunctions:

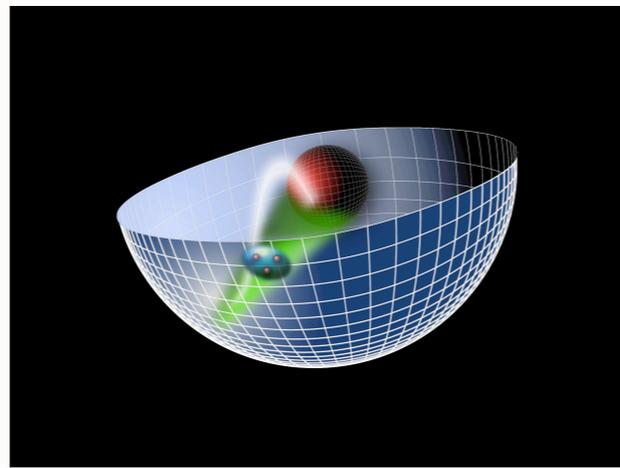
**Light-Front Schrödinger Equation
Spectroscopy and Dynamics**



AdS/QCD
Soft-Wall Model

Single scheme-independent
fundamental mass scale

κ



$$\zeta^2 = x(1-x)b_{\perp}^2.$$

Light-Front Holography

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta) \right] \psi(\zeta) = \mathcal{M}^2 \psi(\zeta)$$



Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2(L + S - 1)$$

$$\kappa \simeq 0.6 \text{ GeV}$$

$$1/\kappa \simeq 1/3 \text{ fm}$$

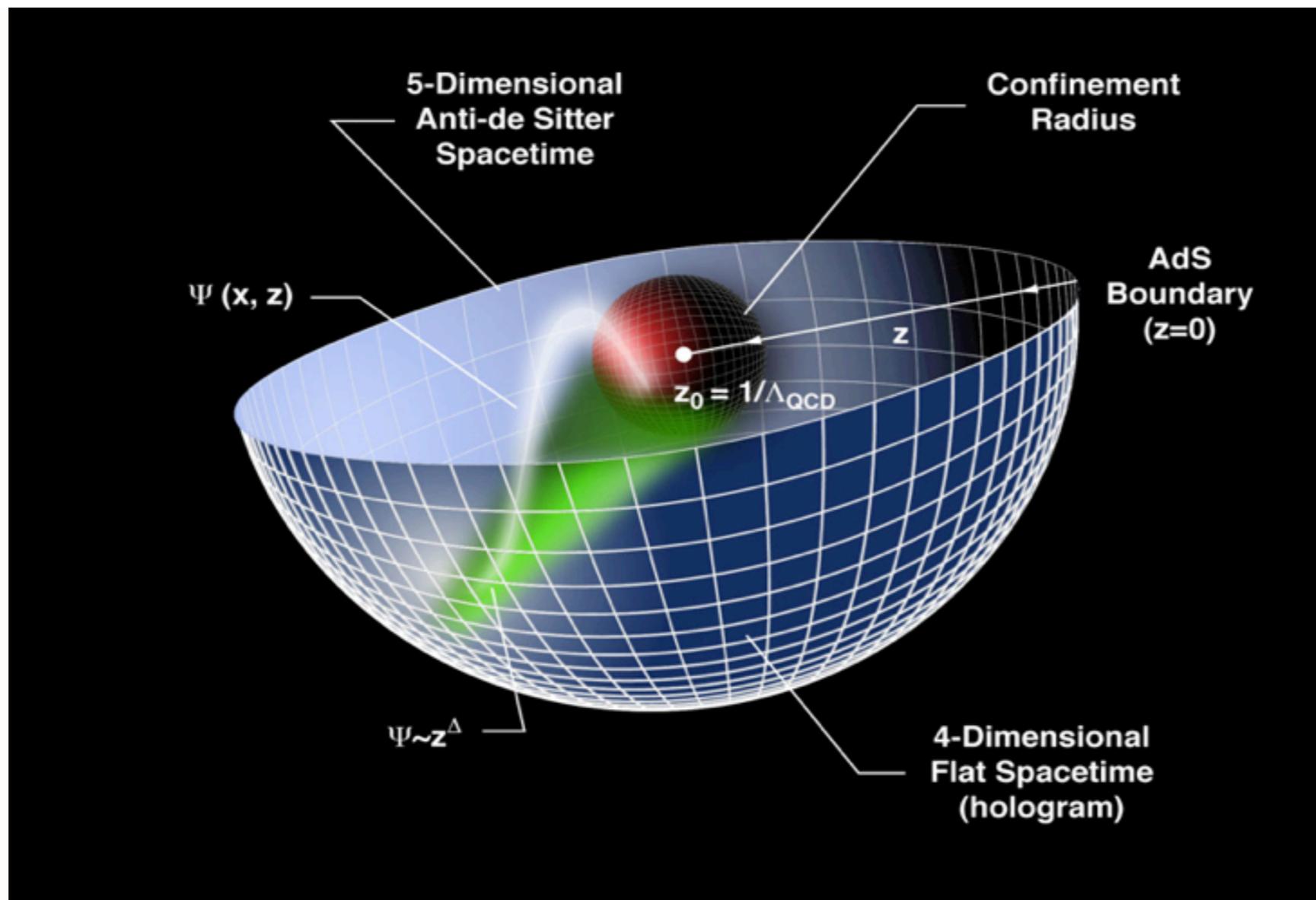
Confinement scale:

($\mathbf{m}_q=0$)

**Unique
Confinement Potential!**
Conformal Symmetry
of the action

● de Alfaro, Fubini, Furlan:

**Scale can appear in Hamiltonian and EQM
without affecting conformal invariance of action!**



Changes in physical length scale mapped to evolution in the 5th dimension z

- Truncated AdS/CFT (Hard-Wall) model: cut-off at $z_0 = 1/\Lambda_{\text{QCD}}$ breaks conformal invariance and allows the introduction of the QCD scale (Hard-Wall Model) **Polchinski and Strassler (2001)**.
- Smooth cutoff: introduction of a background dilaton field $\varphi(z)$ – usual linear Regge dependence can be obtained (Soft-Wall Model) **Karch, Katz, Son and Stephanov (2006)**.

AdS/CFT

- Isomorphism of $SO(4, 2)$ of conformal QCD with the group of isometries of AdS space

$$ds^2 = \frac{R^2}{z^2} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2),$$

invariant measure 

$x^\mu \rightarrow \lambda x^\mu, z \rightarrow \lambda z$, maps scale transformations into the holographic coordinate z .

- AdS mode in z is the extension of the hadron wf into the fifth dimension.
- Different values of z correspond to different scales at which the hadron is examined.

$$x^2 \rightarrow \lambda^2 x^2, \quad z \rightarrow \lambda z.$$

$x^2 = x_\mu x^\mu$: invariant separation between quarks

- The AdS boundary at $z \rightarrow 0$ correspond to the $Q \rightarrow \infty$, UV zero separation limit.

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

Positive-sign dilaton

• Dosch, de Teramond, sjb

AdS Soft-Wall Schrodinger Equation for bound state of two scalar constituents:

$$\left[-\frac{d^2}{dz^2} - \frac{1 - 4L^2}{4z^2} + U(z) \right] \Phi(z) = \mathcal{M}^2 \Phi(z)$$

$$U(z) = \kappa^4 z^2 + 2\kappa^2 (L + S - 1)$$

Derived from variation of Action for Dilaton-Modified AdS₅

Identical to Light-Front Bound State Equation!

z



$$\zeta = \sqrt{x(1-x)} \vec{b}_\perp^2$$

Dilaton-Modified AdS/QCD

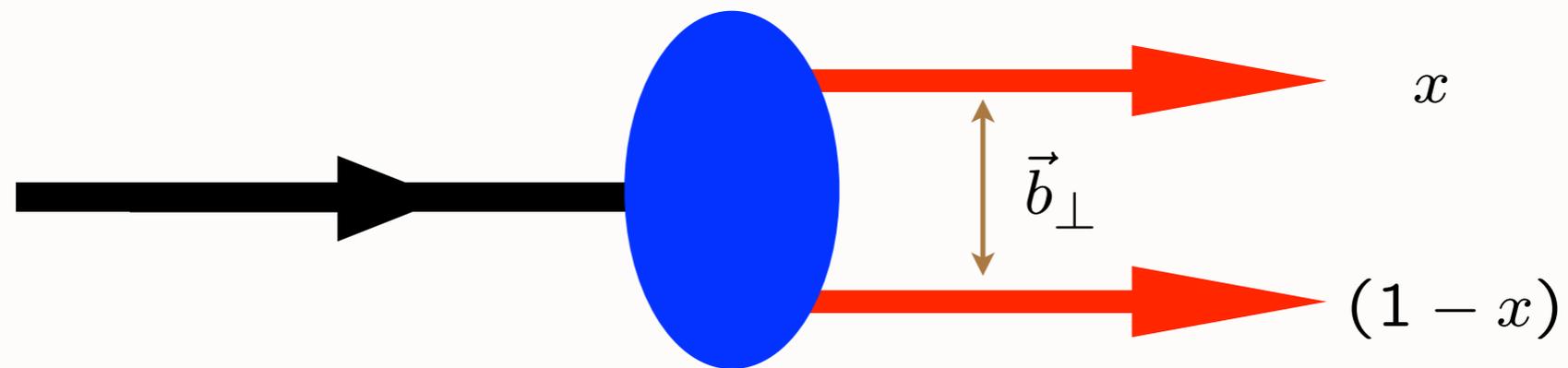
$$ds^2 = e^{\varphi(z)} \frac{R^2}{z^2} (\eta_{\mu\nu} x^\mu x^\nu - dz^2)$$

- **Soft-wall dilaton profile breaks conformal invariance** $e^{\varphi(z)} = e^{+\kappa^2 z^2}$
- **Color Confinement**
- **Introduces confinement scale κ**
- **Uses AdS₅ as template for conformal theory**

$LF(3+1) \longleftrightarrow AdS_5$

$\psi(x, \vec{b}_\perp) \longleftrightarrow \phi(z)$

$\zeta = \sqrt{x(1-x)} \vec{b}_\perp^2 \longleftrightarrow z$



$$\psi(x, \zeta) = \sqrt{x(1-x)} \zeta^{-1/2} \phi(\zeta)$$

$$(\mu R)^2 = L^2 - (J - 2)^2$$

Light-Front Holography: Unique mapping derived from equality of LF and AdS formula for EM and gravitational current matrix elements and identical equations of motion

$$ds^2 = \frac{R^2}{z^2} e^{\varphi(z)} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2)$$

$$z \Leftrightarrow \zeta, \quad \Phi_P(z) \Leftrightarrow |\psi(P)\rangle$$

General dilaton profile

- Upon substitution $z \rightarrow \zeta$ and $\phi_J(\zeta) \sim \zeta^{-3/2+J} e^{\varphi(z)/2} \Phi_J(\zeta)$ in AdS WE

$$\left[-\frac{z^{d-1-2J}}{e^{\varphi(z)}} \partial_z \left(\frac{e^{\varphi(z)}}{z^{d-1-2J}} \partial_z \right) + \left(\frac{\mu R}{z} \right)^2 \right] \Phi_J(z) = \mathcal{M}^2 \Phi_J(z)$$

find LFWE ($d = 4$)

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta) \right) \phi_J(\zeta) = M^2 \phi_J(\zeta)$$

with

$$U(\zeta) = \frac{1}{2} \phi''(\zeta) + \frac{1}{4} \phi'(\zeta)^2 + \frac{2J-3}{2\zeta} \phi'(\zeta)$$

and $(\mu R)^2 = -(2-J)^2 + L^2$

- AdS Breitenlohner-Freedman bound $(\mu R)^2 \geq -4$ equivalent to LF QM stability condition $L^2 \geq 0$
- Scaling dimension τ of AdS mode $\hat{\Phi}_J$ is $\tau = 2 + L$ in agreement with twist scaling dimension of a two parton bound state in QCD and determined by QM stability condition

Introduce "Dilaton" to simulate confinement analytically

- Nonconformal metric dual to a confining gauge theory

$$ds^2 = \frac{R^2}{z^2} e^{\varphi(z)} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2)$$

where $\varphi(z) \rightarrow 0$ at small z for geometries which are asymptotically AdS₅

- Gravitational potential energy for object of mass m

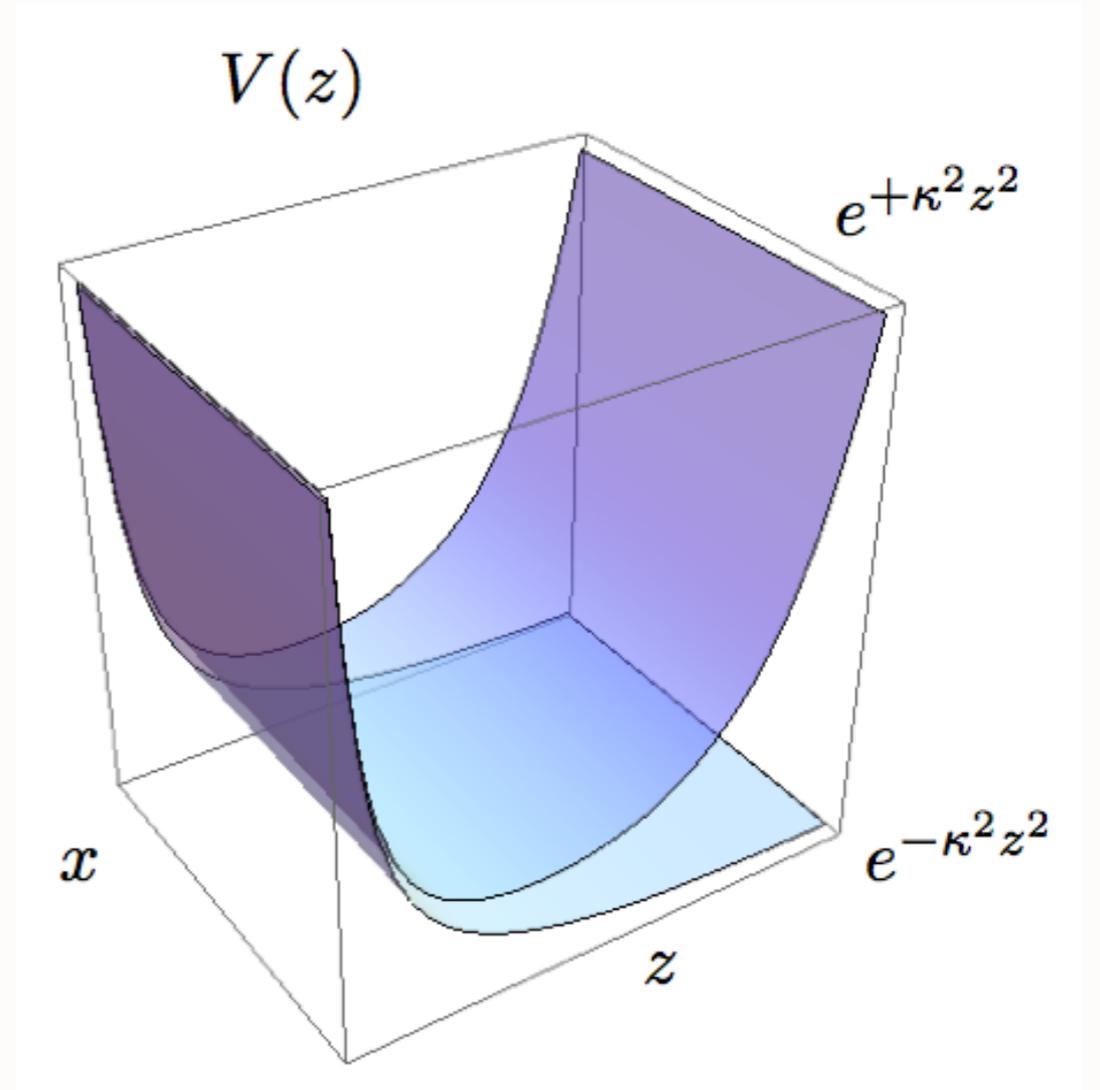
$$V = mc^2 \sqrt{g_{00}} = mc^2 R \frac{e^{\varphi(z)/2}}{z}$$

- Consider warp factor $\exp(\pm \kappa^2 z^2)$
- Plus solution: $V(z)$ increases exponentially confining any object in modified AdS metrics to distances $\langle z \rangle \sim 1/\kappa$

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

Positive-sign dilaton

- de Teramond, sjb



Klebanov and Maldacena

General-Spin Hadrons

- Obtain spin- J mode $\Phi_{\mu_1 \dots \mu_J}$ with all indices along 3+1 coordinates from Φ by shifting dimensions

$$\Phi_J(z) = \left(\frac{z}{R}\right)^{-J} \Phi(z)$$

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

- Substituting in the AdS scalar wave equation for Φ

$$\left[z^2 \partial_z^2 - (3 - 2J - 2\kappa^2 z^2) z \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2 \right] \Phi_J = 0$$

- Upon substitution $z \rightarrow \zeta$

$$\phi_J(\zeta) \sim \zeta^{-3/2+J} e^{\kappa^2 \zeta^2 / 2} \Phi_J(\zeta)$$

we find the LF wave equation

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1) \right) \phi_{\mu_1 \dots \mu_J} = \mathcal{M}^2 \phi_{\mu_1 \dots \mu_J}$$

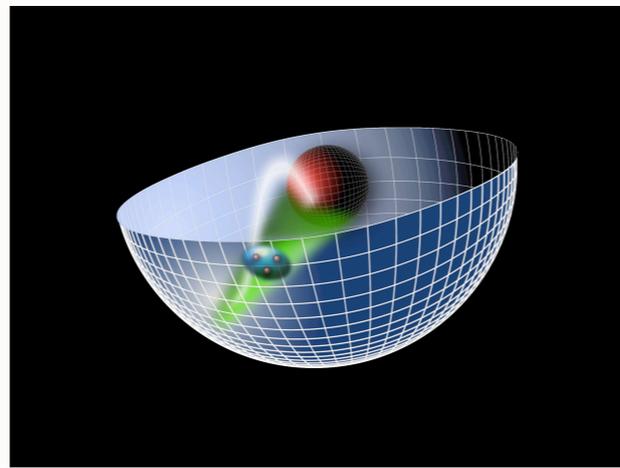


with $(\mu R)^2 = -(2 - J)^2 + L^2$

*AdS/QCD
Soft-Wall Model*

*Single scheme-independent
fundamental mass scale*

κ



$$\zeta^2 = x(1-x)b_{\perp}^2.$$

Light-Front Holography

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta) \right] \psi(\zeta) = \mathcal{M}^2 \psi(\zeta)$$



Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2(L + S - 1)$$

$$\kappa \simeq 0.6 \text{ GeV}$$

$$1/\kappa \simeq 1/3 \text{ fm}$$

Confinement scale:

($\mathbf{m}_q=0$)

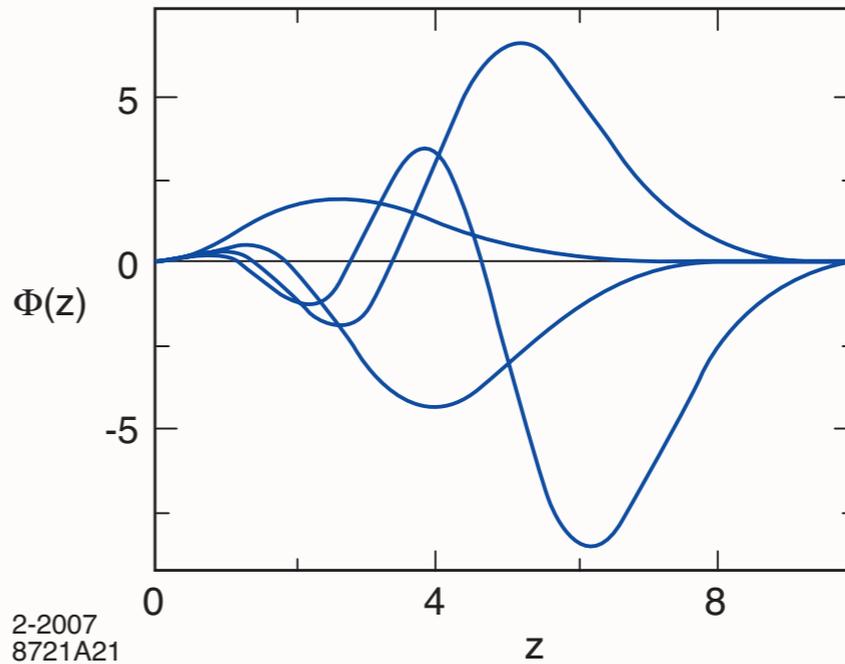
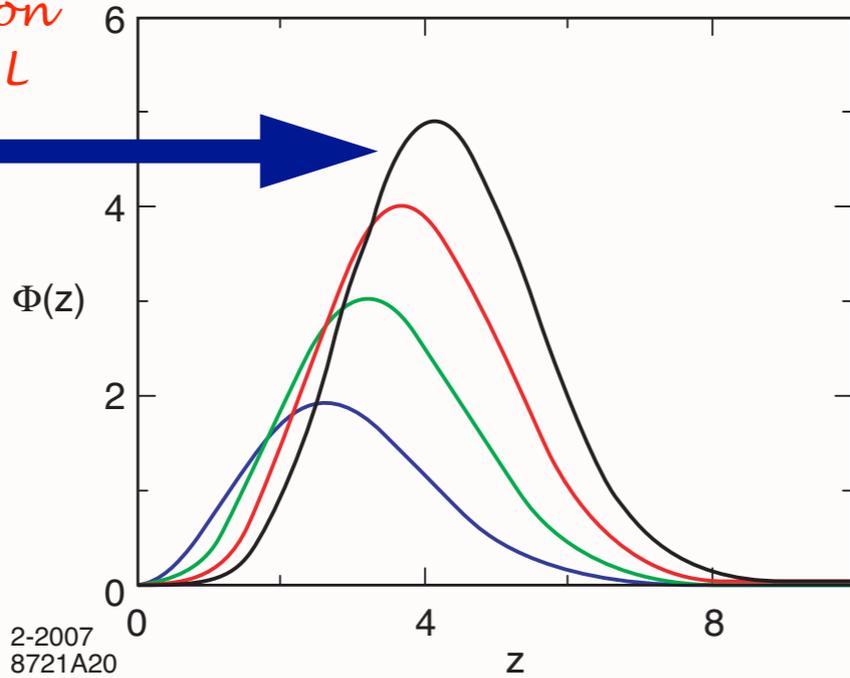
***Unique
Confinement Potential!***

*Conformal Symmetry
of the action*

● **de Alfaro, Fubini, Furlan:**

**Scale can appear in Hamiltonian and EQM
without affecting conformal invariance of action!**

Quark separation increases with L



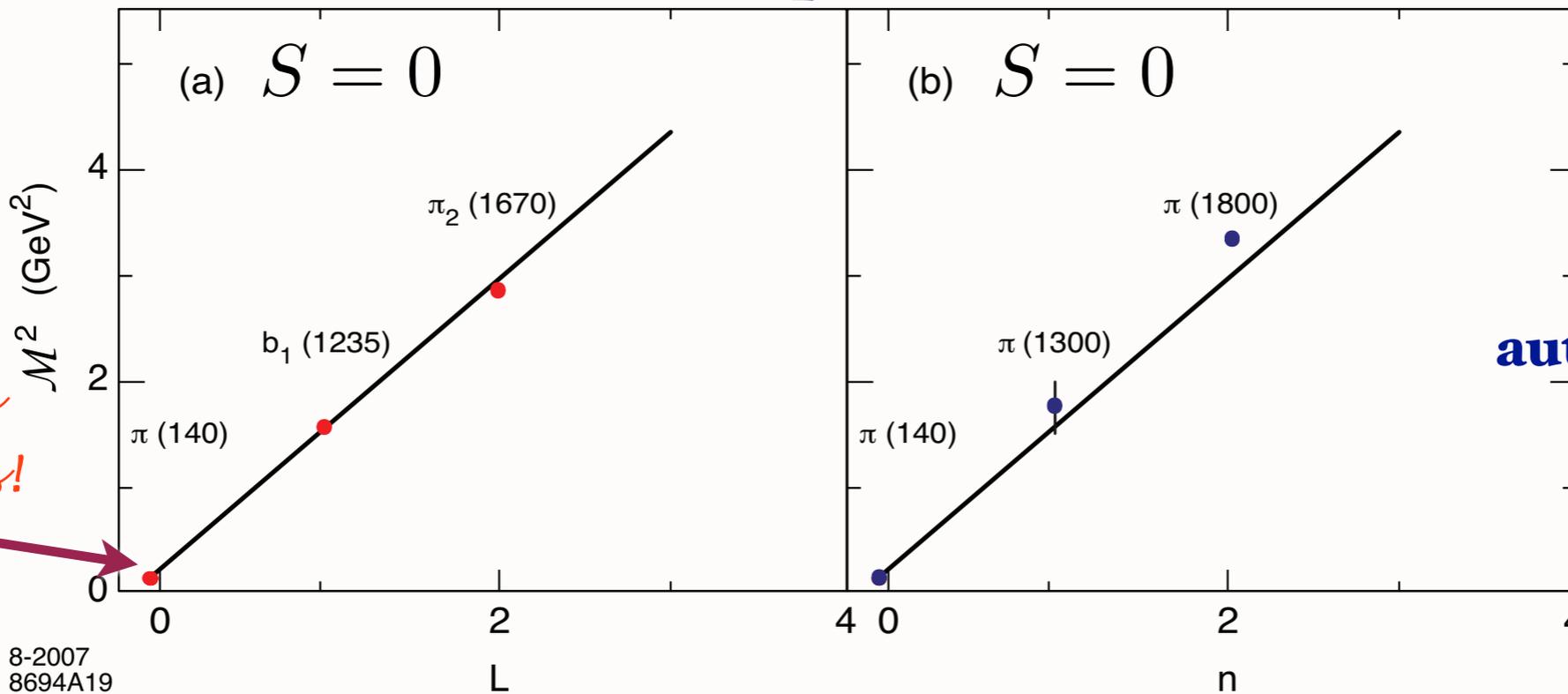
2-2007
8721A20

2-2007
8721A21

Fig: Orbital and radial AdS modes in the soft wall model for $\kappa = 0.6$ GeV .

Same slope in n and L !

Soft Wall Model



Pion has zero mass!



8-2007
8694A19

Pion mass automatically zero!

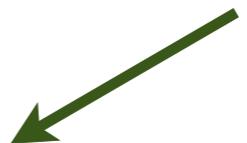
$$m_q = 0$$

Light meson orbital (a) and radial (b) spectrum for $\kappa = 0.6$ GeV.

New Perspectives for Hadron Physics

Meson Spectrum in Soft Wall Model

Pion: Negative term for $J=0$ cancels positive terms from LFKE and potential



- Effective potential: $U(\zeta^2) = \kappa^4 \zeta^2 + 2\kappa^2(J - 1)$

- LF WE

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + \kappa^4 \zeta^2 + 2\kappa^2(J - 1) \right) \phi_J(\zeta) = M^2 \phi_J(\zeta)$$

- Normalized eigenfunctions $\langle \phi | \phi \rangle = \int d\zeta \phi^2(z)^2 = 1$

$$\phi_{n,L}(\zeta) = \kappa^{1+L} \sqrt{\frac{2n!}{(n+L)!}} \zeta^{1/2+L} e^{-\kappa^2 \zeta^2 / 2} L_n^L(\kappa^2 \zeta^2)$$

- Eigenvalues

$$\mathcal{M}_{n,J,L}^2 = 4\kappa^2 \left(n + \frac{J+L}{2} \right)$$

- $J = L + S, I = 1$ meson families

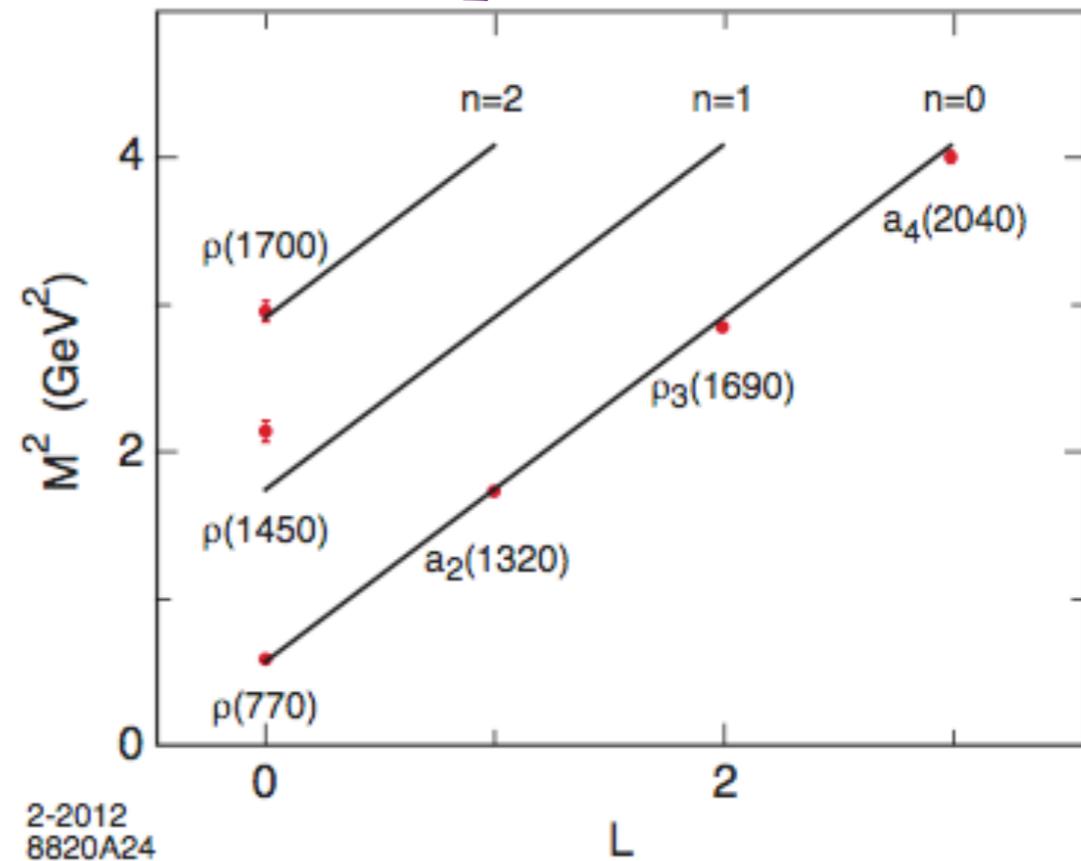
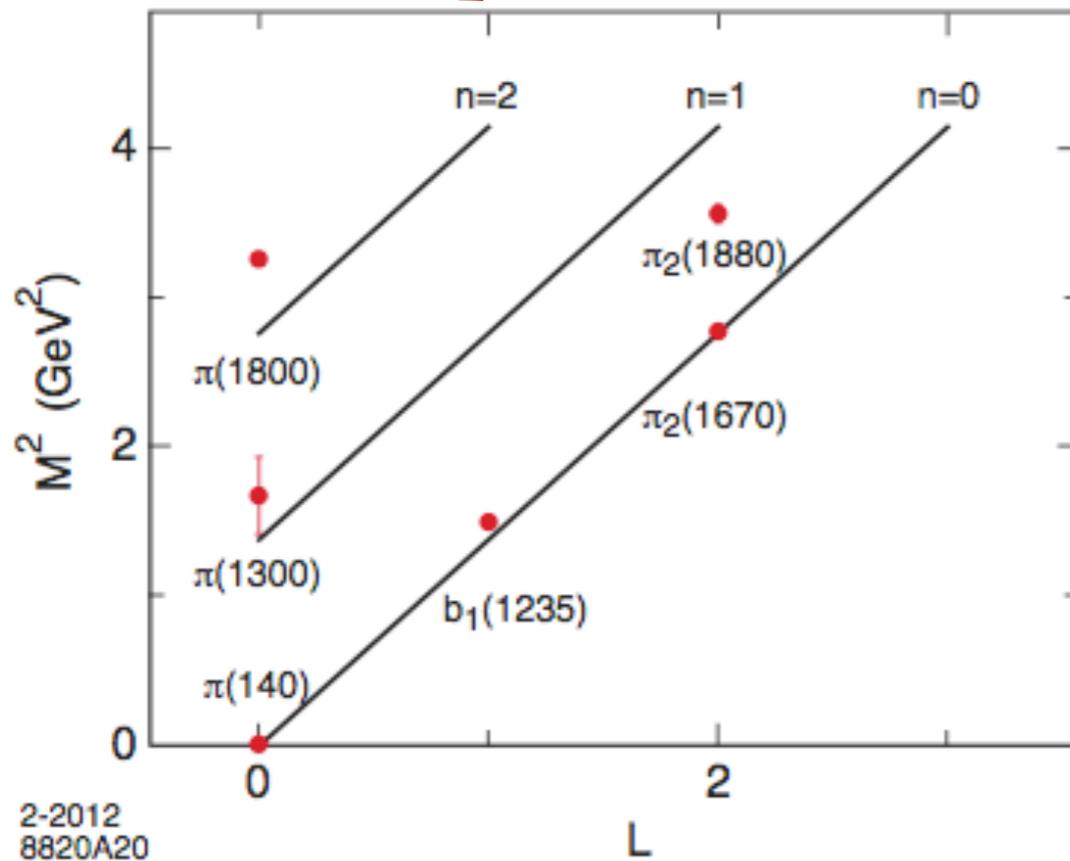
$$\mathcal{M}_{n,L,S}^2 = 4\kappa^2 (n + L + S/2)$$

$$\begin{aligned} 4\kappa^2 &\text{ for } \Delta n = 1 \\ 4\kappa^2 &\text{ for } \Delta L = 1 \\ 2\kappa^2 &\text{ for } \Delta S = 1 \end{aligned}$$

$$m_q = 0$$

Massless pion in Chiral Limit!

Same slope in n and L !



$I=1$ orbital and radial excitations for the π ($\kappa = 0.59$ GeV) and the ρ -meson families ($\kappa = 0.54$ GeV)

- Triplet splitting for the $I = 1, L = 1, J = 0, 1, 2$, vector meson a -states

$$\mathcal{M}_{a_2(1320)} > \mathcal{M}_{a_1(1260)} > \mathcal{M}_{a_0(980)}$$

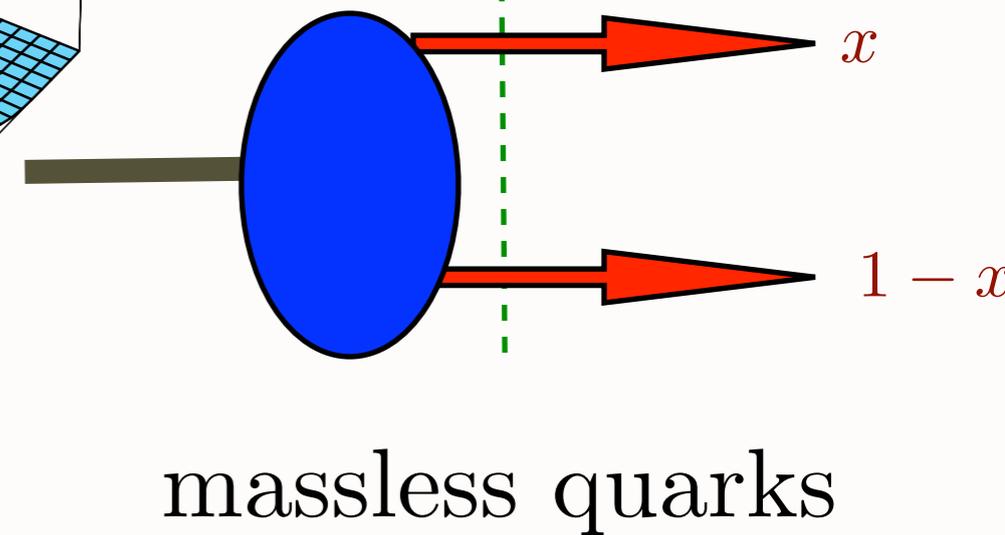
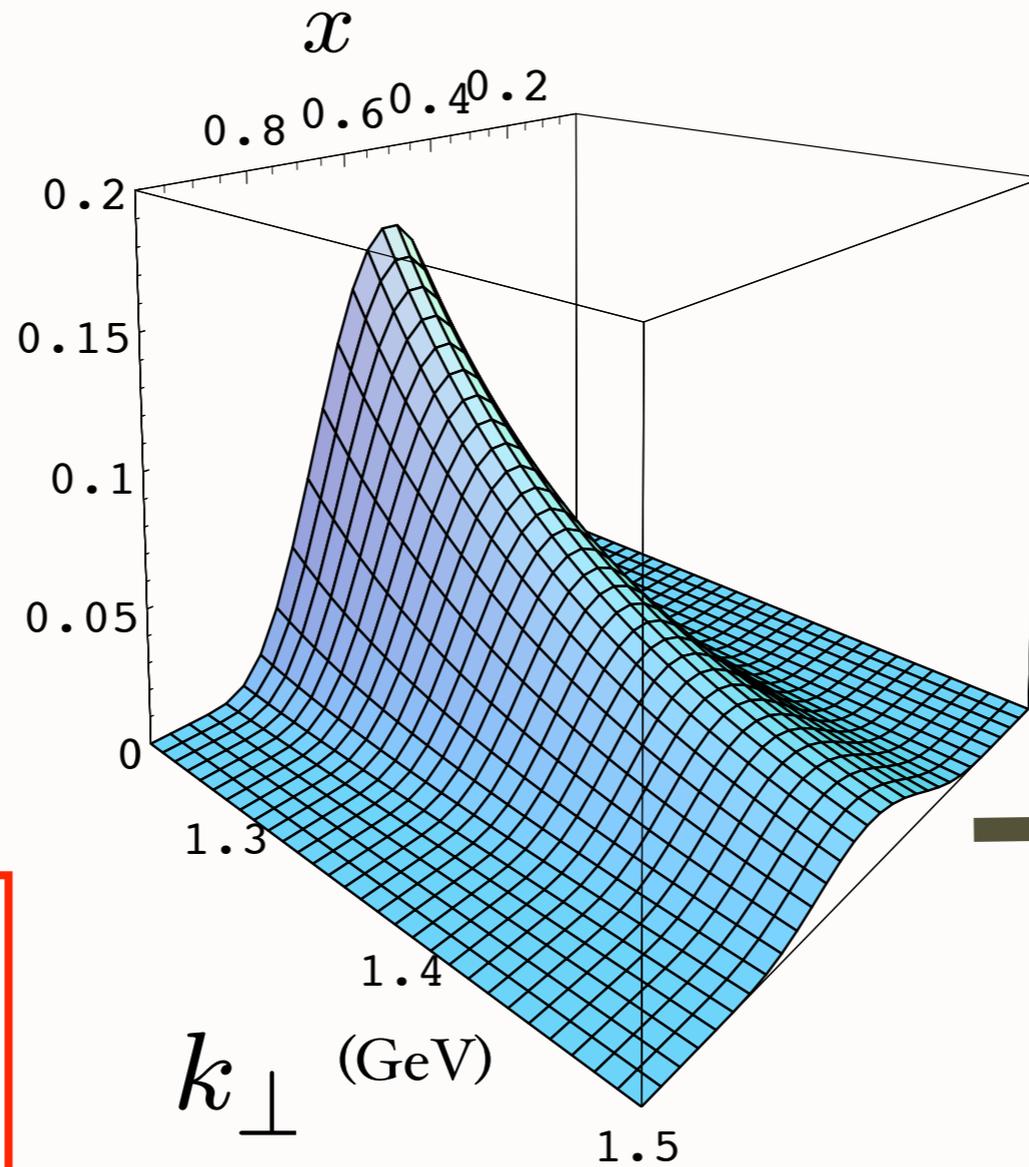
Mass ratio of the ρ and the a_1 mesons: coincides with Weinberg sum rules

Prediction from AdS/QCD: Meson LFWF

de Teramond,
Cao, sjb

“Soft Wall”
model

$$\psi_M(x, k_\perp^2)$$



Note coupling

$$k_\perp^2, x$$

$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa \sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

$$\phi_\pi(x) = \frac{4}{\sqrt{3}\pi} f_\pi \sqrt{x(1-x)}$$

$$f_\pi = \sqrt{P_{q\bar{q}}} \frac{\sqrt{3}}{8} \kappa = 92.4 \text{ MeV}$$

Provides Connection of Confinement to Hadron Structure

Hadron Form Factors from AdS/QCD

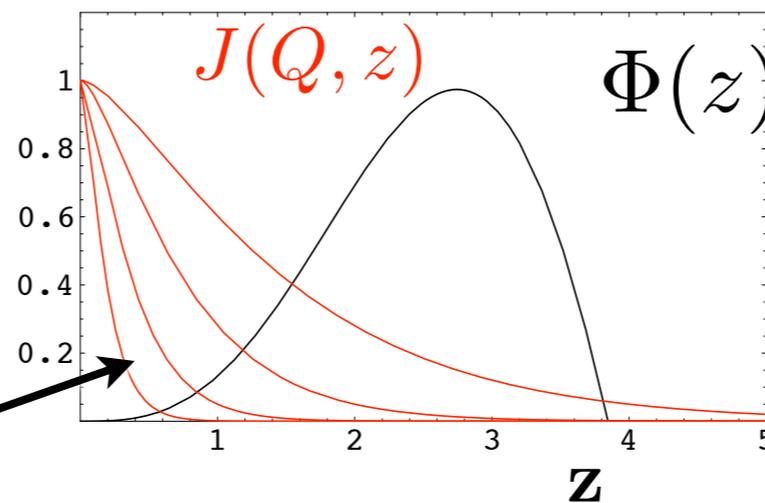
Propagation of external perturbation suppressed inside AdS.

$$J(Q, z) = zQ K_1(zQ)$$

$$F(Q^2)_{I \rightarrow F} = \int \frac{dz}{z^3} \Phi_F(z) J(Q, z) \Phi_I(z)$$

High Q^2
from
small $z \sim 1/Q$

high Q^2



Polchinski, Strassler
de Teramond, sjb

Consider a specific AdS mode $\Phi^{(n)}$ dual to an n partonic Fock state $|n\rangle$. At small z , $\Phi^{(n)}$ scales as $\Phi^{(n)} \sim z^{\Delta_n}$. Thus:

$$F(Q^2) \rightarrow \left[\frac{1}{Q^2} \right]^{\tau-1},$$

Dimensional Quark Counting Rules:
General result from
AdS/CFT and Conformal Invariance

where $\tau = \Delta_n - \sigma_n$, $\sigma_n = \sum_{i=1}^n \sigma_i$.

Twist $\tau = n + L$

Holographic Mapping of AdS Modes to QCD LFWFs

Drell-Yan-West: Form Factors are Convolution of LFWFs

- Integrate Soper formula over angles:

$$F(q^2) = 2\pi \int_0^1 dx \frac{(1-x)}{x} \int \zeta d\zeta J_0 \left(\zeta q \sqrt{\frac{1-x}{x}} \right) \tilde{\rho}(x, \zeta),$$

with $\tilde{\rho}(x, \zeta)$ QCD effective transverse charge density.

- Transversality variable

$$\zeta = \sqrt{x(1-x)} \vec{b}_\perp^2$$

- Compare AdS and QCD expressions of FFs for arbitrary Q using identity:

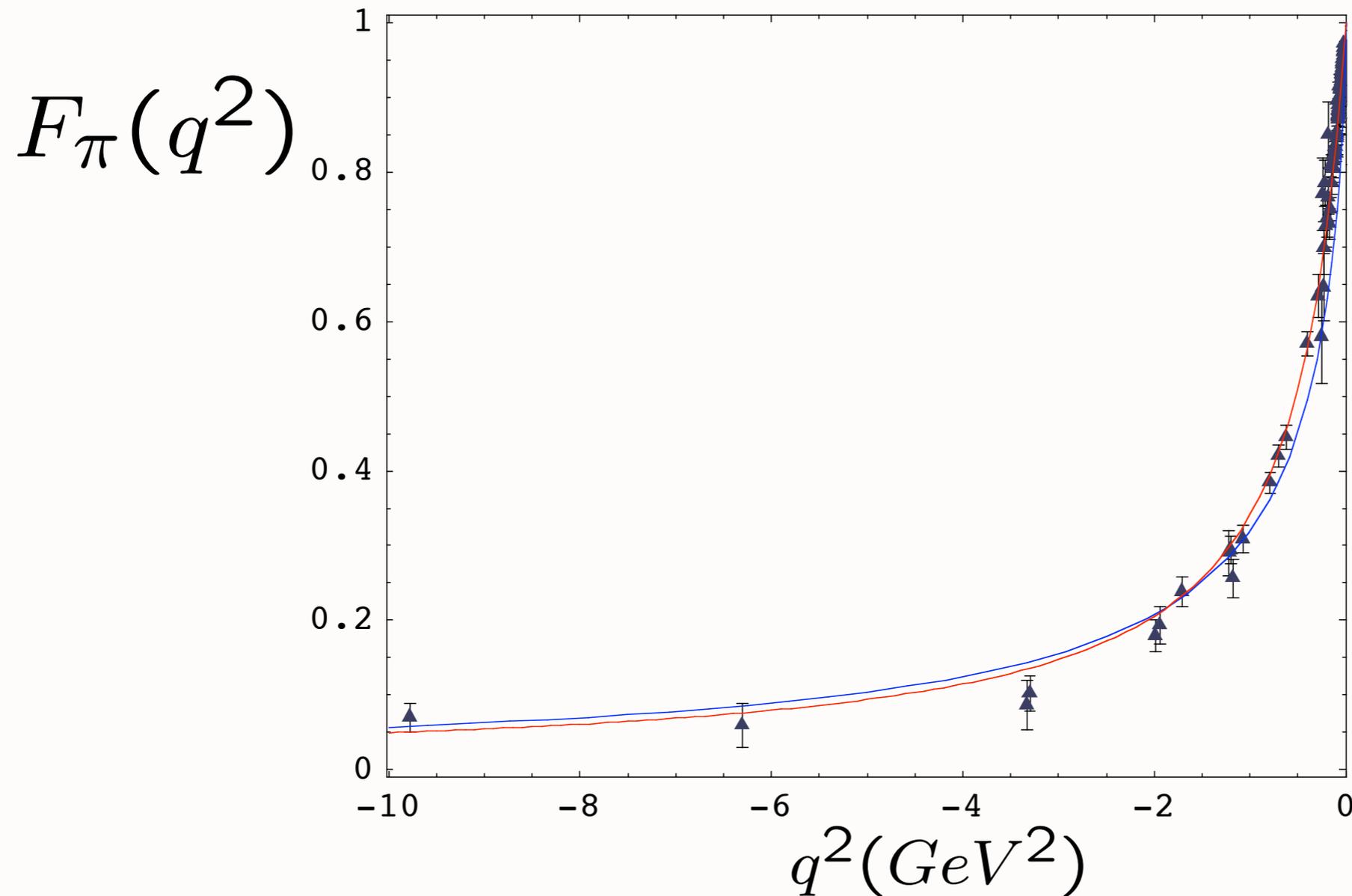
$$\int_0^1 dx J_0 \left(\zeta Q \sqrt{\frac{1-x}{x}} \right) = \zeta Q K_1(\zeta Q),$$

the solution for $J(Q, \zeta) = \zeta Q K_1(\zeta Q)$!

de Teramond, sjb

Identical to Polchinski-Strassler Convolution of AdS Amplitudes

Spacelike pion form factor from AdS/CFT



Data Compilation
Baldini, Kloe and Volmer

— Soft Wall: Harmonic Oscillator Confinement

— Hard Wall: Truncated Space Confinement

One parameter - set by pion decay constant

de Teramond, sjb
See also: Radyushkin

AdS/QCD Holographic Wave Function for the ρ Meson and Diffractive ρ Meson Electroproduction

J. R. Forshaw*

*Consortium for Fundamental Physics, School of Physics and Astronomy, University of Manchester,
Oxford Road, Manchester M13 9PL, United Kingdom*

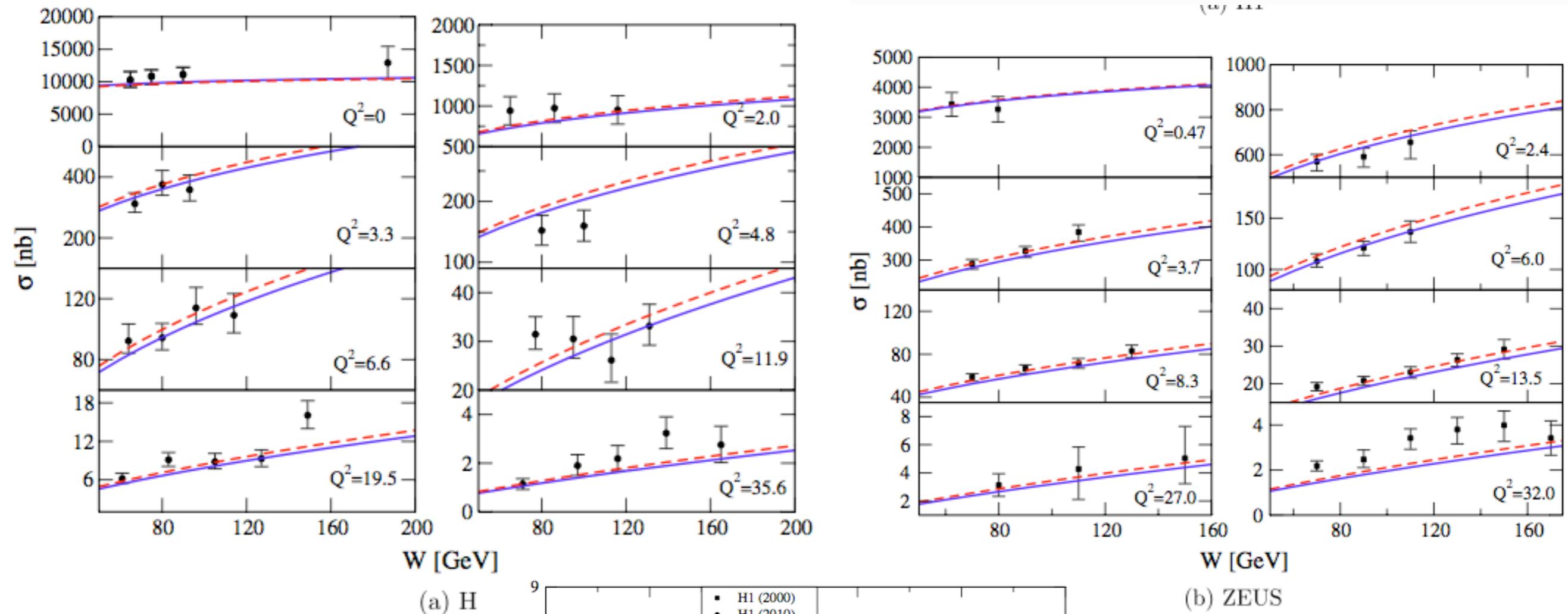
R. Sandapen†

Département de Physique et d'Astronomie, Université de Moncton, Moncton, New Brunswick E1A3E9, Canada
(Received 5 April 2012; published 20 August 2012)

We show that anti-de Sitter/quantum chromodynamics generates predictions for the rate of diffractive ρ -meson electroproduction that are in agreement with data collected at the Hadron Electron Ring Accelerator electron-proton collider.

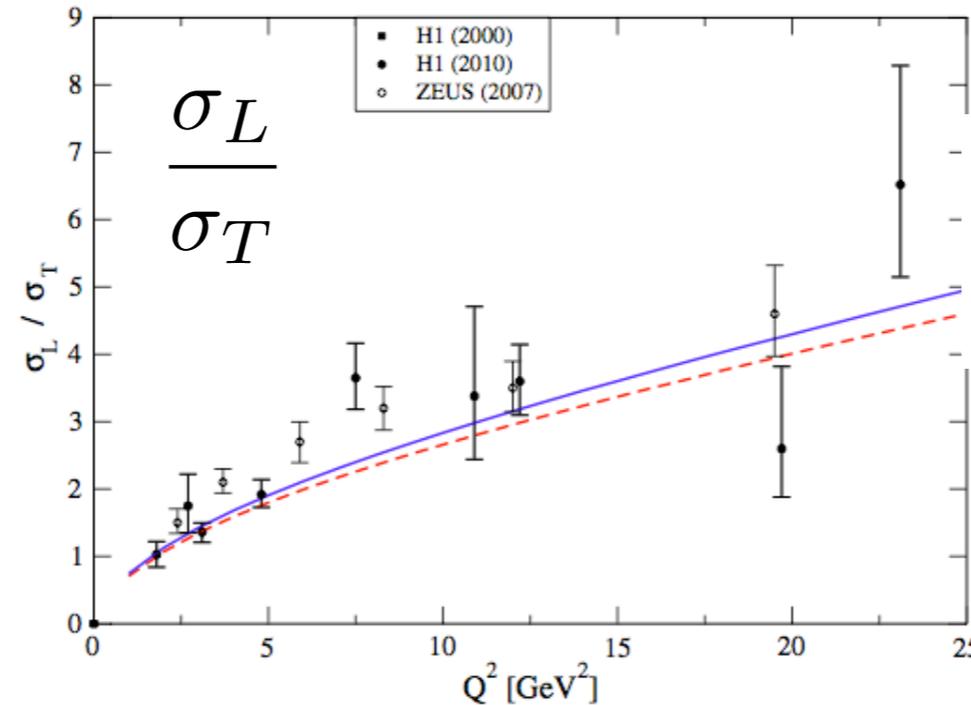
$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa \sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

AdS/QCD Holographic Wave Function for the ρ Meson and Diffractive ρ Meson Electroproduction



**J. R. Forshaw,
R. Sandapen**

$$\gamma^* p \rightarrow \rho^0 p'$$

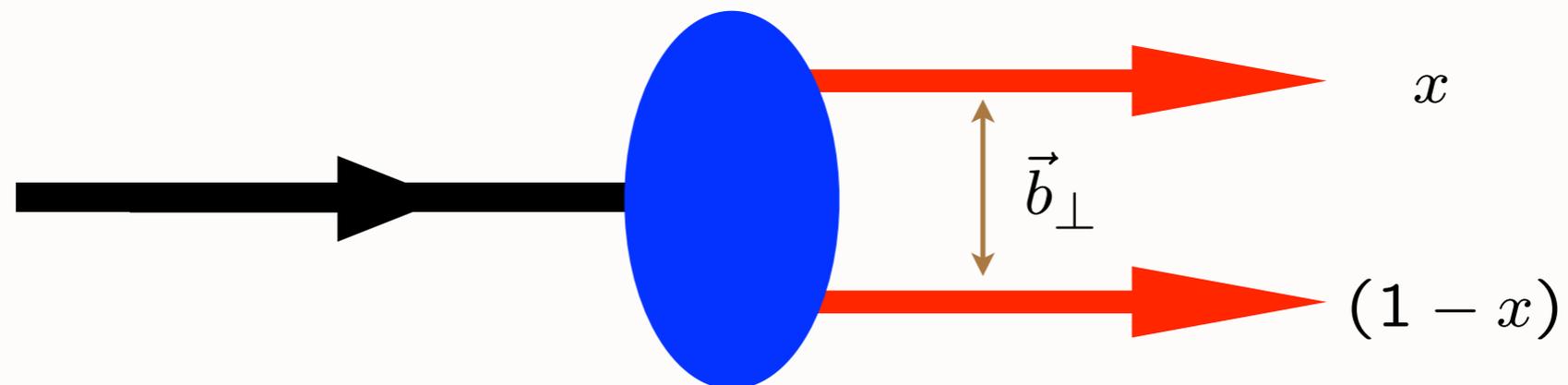


$$\tilde{\phi}(x, k) \propto \frac{1}{\sqrt{x(1-x)}} \exp\left(-\frac{M_{q\bar{q}}^2}{2\kappa^2}\right),$$

$LF(3+1) \longleftrightarrow AdS_5$

$\psi(x, \vec{b}_\perp) \longleftrightarrow \phi(z)$

$\zeta = \sqrt{x(1-x)} \vec{b}_\perp^2 \longleftrightarrow z$



$$\psi(x, \zeta) = \sqrt{x(1-x)} \zeta^{-1/2} \phi(\zeta)$$

Light-Front Holography: Unique mapping derived from equality of LF and AdS formula for EM and gravitational current matrix elements and identical equations of motion

An analytic first approximation to QCD

AdS/QCD + Light-Front Holography

- **As Simple as Schrödinger Theory in Atomic Physics**
- **LF radial variable ζ conjugate to invariant mass squared**
- **Relativistic, Frame-Independent, Color-Confining**
- **Unique confining potential!**
- **QCD Coupling at all scales: Essential for Gauge Link phenomena**
- **Hadron Spectroscopy and Dynamics from one parameter**
- **Wave Functions, Form Factors, Hadronic Observables, Constituent Counting Rules**
- **Insight into QCD Condensates: Zero cosmological constant!**
- **Systematically improvable with DLCQ-BLFQ Methods**

- Propagation of external current inside AdS space described by the AdS wave equation

$$\left[z^2 \partial_z^2 - z (1 + 2\kappa^2 z^2) \partial_z - Q^2 z^2 \right] J_\kappa(Q, z) = 0.$$

- Solution bulk-to-boundary propagator

$$J_\kappa(Q, z) = \Gamma\left(1 + \frac{Q^2}{4\kappa^2}\right) U\left(\frac{Q^2}{4\kappa^2}, 0, \kappa^2 z^2\right),$$

where $U(a, b, c)$ is the confluent hypergeometric function

$$\Gamma(a)U(a, b, z) = \int_0^\infty e^{-zt} t^{a-1} (1+t)^{b-a-1} dt.$$

- Form factor in presence of the dilaton background $\varphi = \kappa^2 z^2$

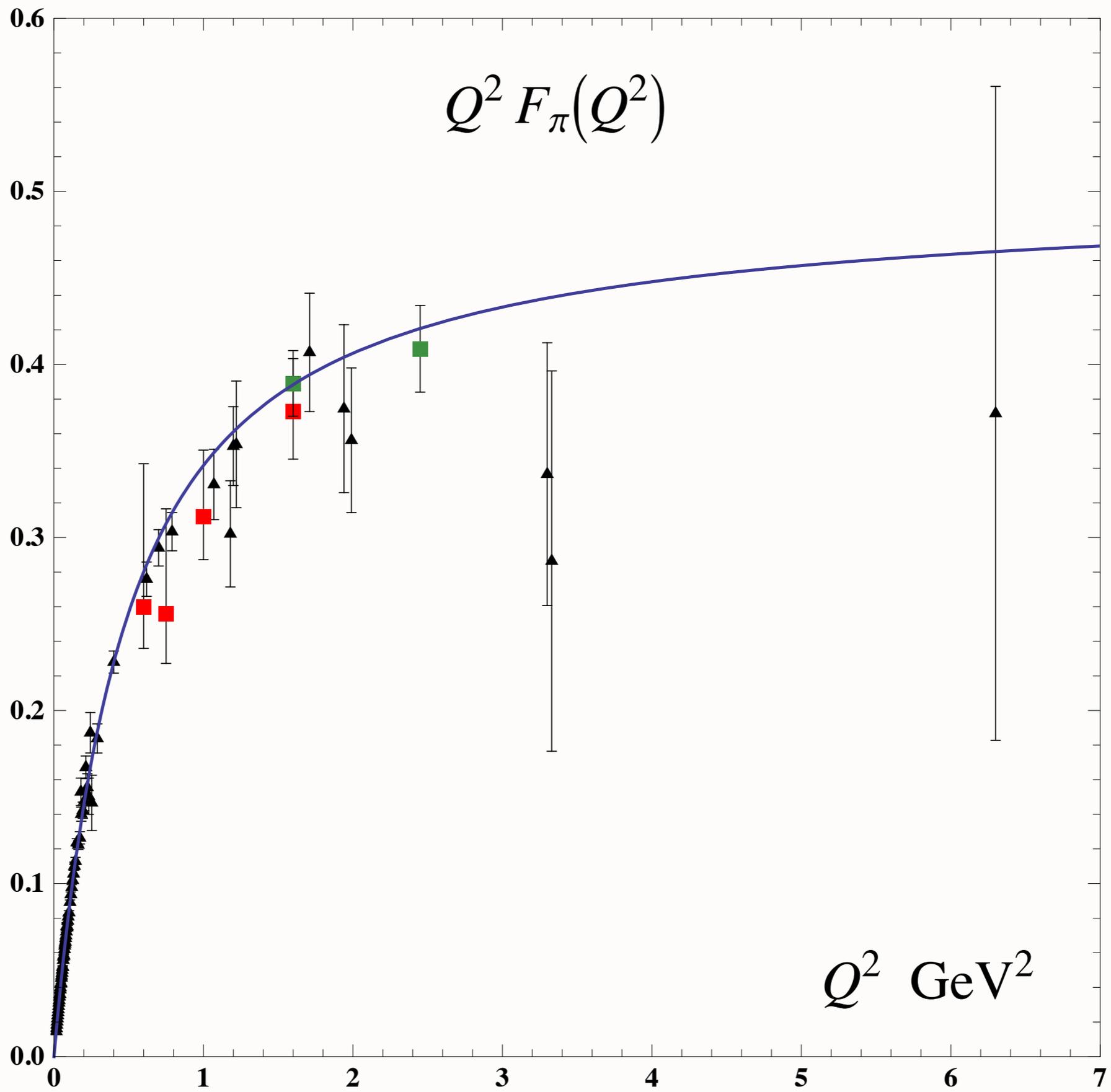
$$F(Q^2) = R^3 \int \frac{dz}{z^3} e^{-\kappa^2 z^2} \Phi(z) J_\kappa(Q, z) \Phi(z).$$

- For large $Q^2 \gg 4\kappa^2$

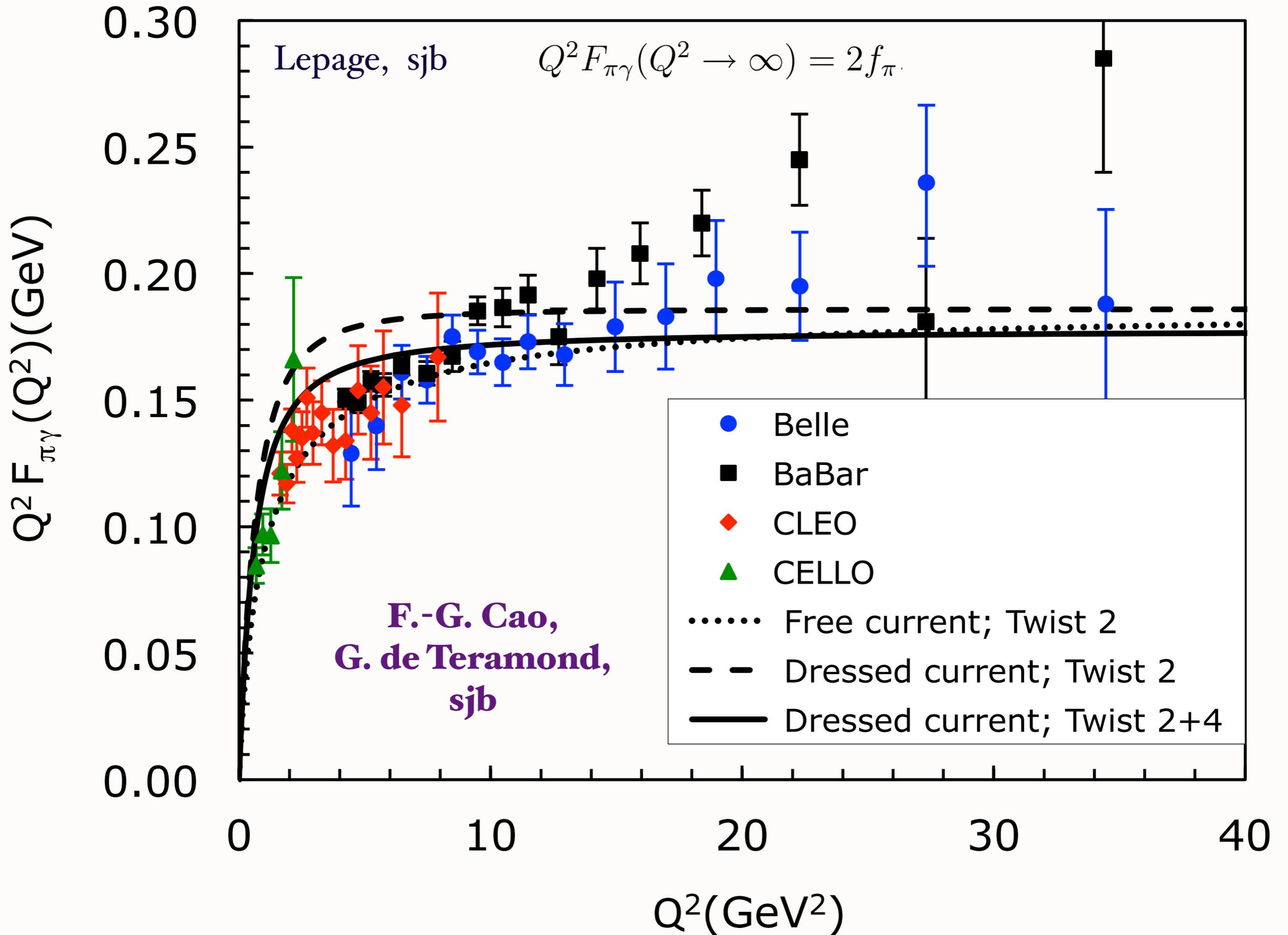
$$J_\kappa(Q, z) \rightarrow zQ K_1(zQ) = J(Q, z),$$

the external current decouples from the dilaton field.

*Dressed
Current
in Soft-Wall
Model*



Photon-to-pion transition form factor



Light-Front Holography

- **AdS₅/CFT₄ Duality between AdS₅ and Conformal Gauge Theory in 3+1 at fixed LF time** [G. de Téramond, H. G. Dosch, sjb](#)

Valery E. Lyubovitskij, Tanja Branz, Thomas Gutsche,
Ivan Schmidt, Alfredo Vega

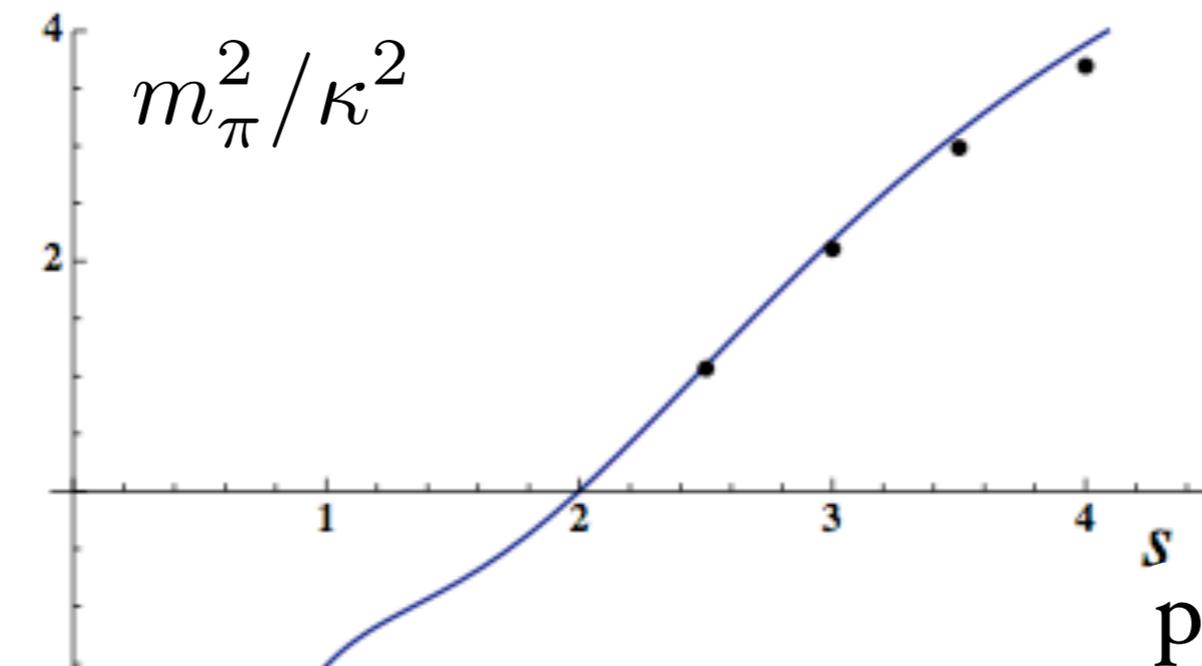
- “**AdS₄/CFT₃ Construction from Collective Fields**” [Robert de Mello Koch](#), [Antal Jevicki](#), [Kewang Jin](#), [João P. Rodrigues](#)

- “**Exact holographic mapping and emergent space-time geometry**” [Xiao-Liang Qi](#)

- **Ehrenfest arguments:** [Glazek and Trawinski](#)

Uniqueness of Dilaton

$$\varphi_p(z) = \kappa^p z^p$$



pion is massless in chiral limit iff
 $p=2!$

$$e^\varphi(z) = e^{+\kappa^2 z^2}$$

● **Dosch, de Teramond, sjb**

Uniqueness

de Teramond, Dosch, sjb

$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2(L + S - 1) \quad e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

- **ζ^2 confinement potential and dilaton profile unique!**
- **Linear Regge trajectories in n and L : same slope!**
- **Massless pion in chiral limit! No vacuum condensate!**
- **Conformally invariant action for massless quarks retained despite mass scale**
- **Same principle, equation of motion as de Alfaro, FurlanFubini, Conformal Invariance in Quantum Mechanics Nuovo Cim. A34 (1976) 569**

QCD Lagrangian

Fundamental Theory of Hadron and Nuclear Physics

gluon dynamics quark kinetic energy + quark-gluon dynamics quark mass term

$$\mathcal{L}_{QCD} = -\frac{1}{4} \text{Tr}(G^{\mu\nu} G_{\mu\nu}) + \sum_{f=1}^{n_f} i\bar{\Psi}_f D_\mu \gamma^\mu \Psi_f + \sum_{f=1}^{n_f} m_f \bar{\Psi}_f \Psi_f$$
$$iD^\mu = i\partial^\mu - gA^\mu \quad G^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu - g[A^\mu, A^\nu]$$

Classically Conformal if $m_q=0$

Yang Mills Gauge Principle: Color Rotation and Phase Invariance at Every Point of Space and Time

**Scale-Invariant Coupling
Renormalizable
Asymptotic Freedom
Color Confinement**

QCD Mass Scale from Confinement not Explicit

Conformal Invariance in Quantum Mechanics.

V. DE ALFARO

Istituto di Fisica Teorica dell'Università - Torino

Istituto Nazionale di Fisica Nucleare - Sezione di Torino

S. FUBINI and G. FURLAN (*)

CERN - Geneva

(ricevuto il 3 Maggio 1976)

Summary. — The properties of a field theory in one over-all time dimension, invariant under the full conformal group, are studied in detail. A compact operator, which is not the Hamiltonian, is diagonalized and used to solve the problem of motion, providing a discrete spectrum and normalizable eigenstates. The role of the physical parameters present in the model is discussed, mainly in connection with a semi-classical approximation.

$$G|\psi(\tau)\rangle = i\frac{\partial}{\partial\tau}|\psi(\tau)\rangle$$

$$G = uH + vD + wK$$

New term

$$G = H_\tau = \frac{1}{2}\left(-\frac{d^2}{dx^2} + \frac{g}{x^2} + \frac{4uw - v^2}{4}x^2\right)$$

Retains conformal invariance of action despite mass scale!

$$4uw - v^2 = \kappa^4 = [M]^4$$

Identical to LF Hamiltonian with unique potential and dilaton!

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1 - 4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L + S - 1)$$

dAFF: New Time Variable

$$\tau = \frac{2}{\sqrt{4uw - v^2}} \arctan \left(\frac{2tw + v}{\sqrt{4uw - v^2}} \right),$$

- **Identify with difference of LF time $\Delta x^+ / P^+$ between constituents**
- **Finite range**
- **Measure in Double Parton Processes**

Remarkable Features of Light-Front Schrödinger Equation

- **Relativistic, frame-independent**
- **QCD scale appears - unique LF potential**
- **Reproduces spectroscopy and dynamics of light-quark hadrons with one parameter**
- **Zero-mass pion for zero mass quarks!**
- **Regge slope same for n and L -- not usual HO**
- **Splitting in L persists to high mass -- contradicts conventional wisdom based on breakdown of chiral symmetry**
- **Phenomenology: LFWFs, Form factors, electroproduction**
- **Extension to heavy quarks**

$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)$$

Dirac Equation for Nucleons in Soft-Wall AdS/QCD

- We write the Dirac equation

$$(\alpha\Pi(\zeta) - \mathcal{M})\psi(\zeta) = 0,$$

in terms of the matrix-valued operator Π

$$\Pi_\nu(\zeta) = -i \left(\frac{d}{d\zeta} - \frac{\nu + \frac{1}{2}}{\zeta} \gamma_5 - \kappa^2 \zeta \gamma_5 \right),$$

and its adjoint Π^\dagger , with commutation relations

$$\left[\Pi_\nu(\zeta), \Pi_\nu^\dagger(\zeta) \right] = \left(\frac{2\nu + 1}{\zeta^2} - 2\kappa^2 \right) \gamma_5.$$

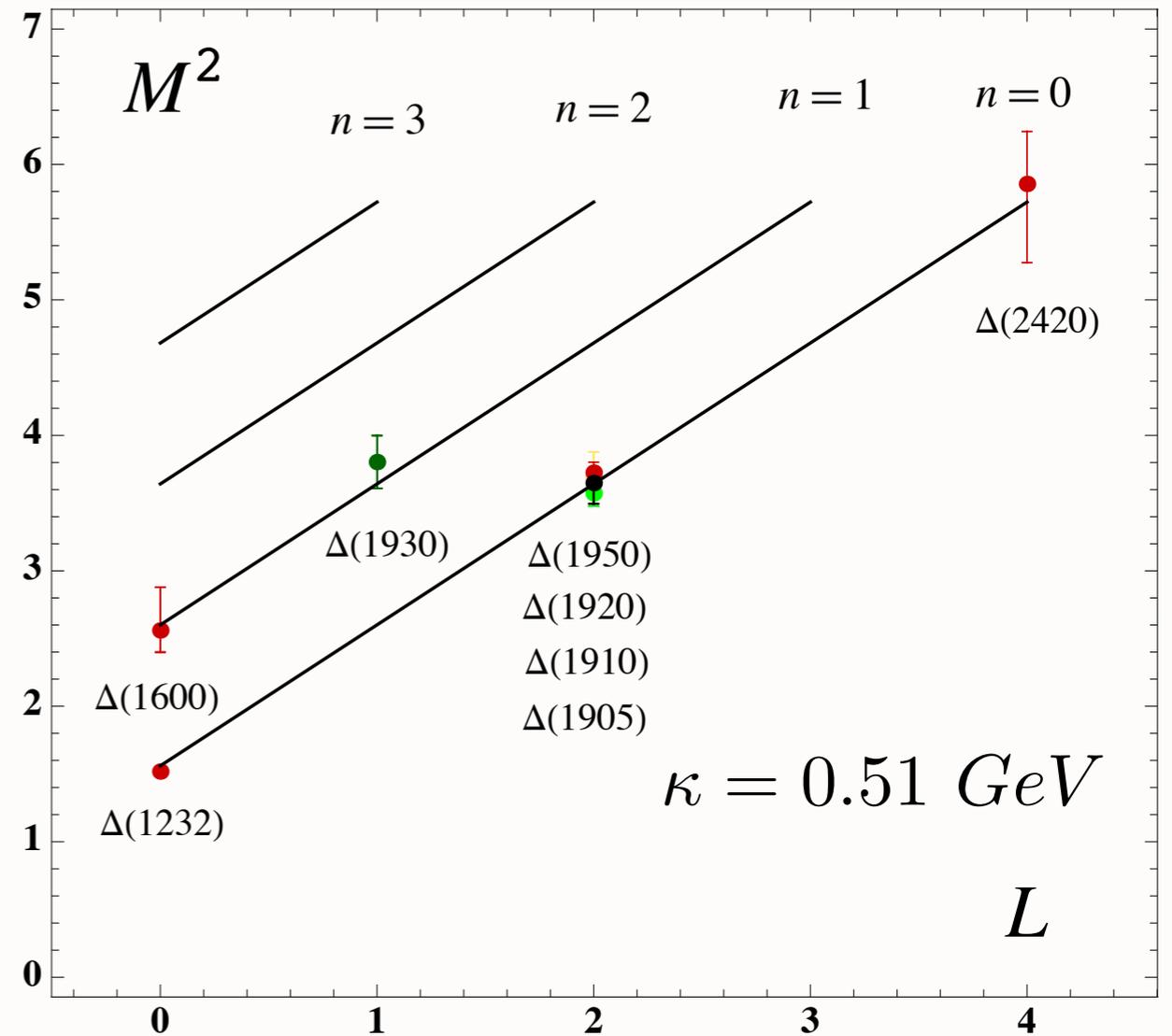
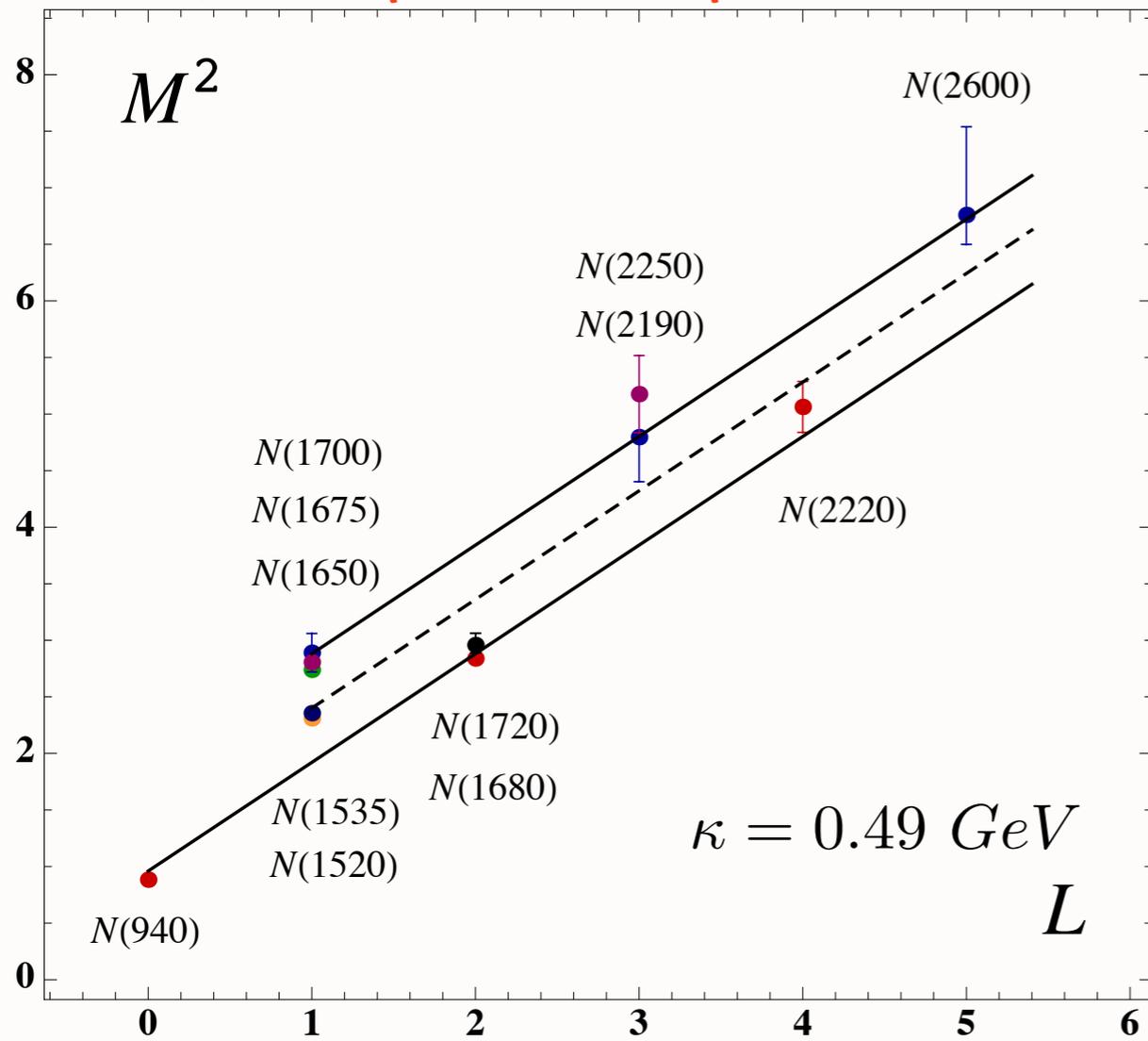
- Solutions to the Dirac equation

$$\begin{aligned} \psi_+(\zeta) &\sim z^{\frac{1}{2}+\nu} e^{-\kappa^2 \zeta^2 / 2} L_n^\nu(\kappa^2 \zeta^2), \\ \psi_-(\zeta) &\sim z^{\frac{3}{2}+\nu} e^{-\kappa^2 \zeta^2 / 2} L_n^{\nu+1}(\kappa^2 \zeta^2). \end{aligned} \quad \nu = L + 1$$

- Eigenvalues

$$\mathcal{M}^2 = 4\kappa^2(n + \nu + 1).$$

Baryon Spectroscopy from AdS/QCD and Light-Front Holography



de Teramond, sjb

$$\mathcal{M}_{n,L,S}^{2(+)} = 4\kappa^2 \left(n + L + \frac{S}{2} + \frac{3}{4} \right), \quad \text{positive parity}$$

$$\mathcal{M}_{n,L,S}^{2(-)} = 4\kappa^2 \left(n + L + \frac{S}{2} + \frac{5}{4} \right), \quad \text{negative parity}$$

**All confirmed
resonances
from PDG
2012**

See also Forkel, Beyer, Federico, Klempt

HEP-LHC 2013

New Perspectives for Hadron Physics

Stan Brodsky
SLAC
NATIONAL ACCELERATOR LABORATORY

Table 1: $SU(6)$ classification of confirmed baryons listed by the PDG. The labels S , L and n refer to the internal spin, orbital angular momentum and radial quantum number respectively. The $\Delta_{\frac{5}{2}}^{-}(1930)$ does not fit the $SU(6)$ classification since its mass is too low compared to other members **70**-multiplet for $n = 0$, $L = 3$.

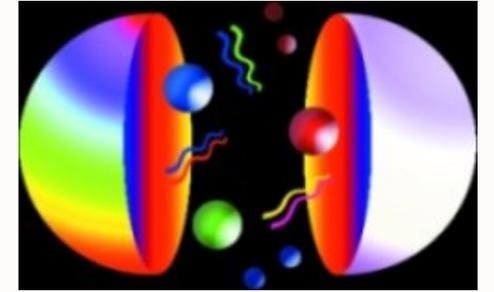
$SU(6)$	S	L	n	Baryon State				
56	$\frac{1}{2}$	0	0	$N_{\frac{1}{2}}^{1+}(940)$				
	$\frac{1}{2}$	0	1	$N_{\frac{1}{2}}^{1+}(1440)$				
	$\frac{1}{2}$	0	2	$N_{\frac{1}{2}}^{1+}(1710)$				
	$\frac{3}{2}$	0	0	$\Delta_{\frac{3}{2}}^{3+}(1232)$				
	$\frac{3}{2}$	0	1	$\Delta_{\frac{3}{2}}^{3+}(1600)$				
70	$\frac{1}{2}$	1	0	$N_{\frac{1}{2}}^{1-}(1535) \quad N_{\frac{3}{2}}^{3-}(1520)$				
	$\frac{3}{2}$	1	0	$N_{\frac{1}{2}}^{1-}(1650)$	$N_{\frac{3}{2}}^{3-}(1700)$	$N_{\frac{5}{2}}^{5-}(1675)$		
	$\frac{3}{2}$	1	1	$N_{\frac{1}{2}}^{1-}$	$N_{\frac{3}{2}}^{3-}(1875)$	$N_{\frac{5}{2}}^{5-}$		
	$\frac{1}{2}$	1	0	$\Delta_{\frac{1}{2}}^{1-}(1620) \quad \Delta_{\frac{3}{2}}^{3-}(1700)$				
56	$\frac{1}{2}$	2	0	$N_{\frac{3}{2}}^{3+}(1720) \quad N_{\frac{5}{2}}^{5+}(1680)$				
	$\frac{1}{2}$	2	1	$N_{\frac{3}{2}}^{3+}(1900) \quad N_{\frac{5}{2}}^{5+}$				
	$\frac{3}{2}$	2	0	$\Delta_{\frac{1}{2}}^{1+}(1910)$	$\Delta_{\frac{3}{2}}^{3+}(1920)$	$\Delta_{\frac{5}{2}}^{5+}(1905)$	$\Delta_{\frac{7}{2}}^{7+}(1950)$	
70	$\frac{1}{2}$	3	0	$N_{\frac{5}{2}}^{5-} \quad N_{\frac{7}{2}}^{7-}$				
	$\frac{3}{2}$	3	0	$N_{\frac{3}{2}}^{3-}$	$N_{\frac{5}{2}}^{5-}$	$N_{\frac{7}{2}}^{7-}(2190)$	$N_{\frac{9}{2}}^{9-}(2250)$	
	$\frac{1}{2}$	3	0	$\Delta_{\frac{5}{2}}^{5-} \quad \Delta_{\frac{7}{2}}^{7-}$				
56	$\frac{1}{2}$	4	0	$N_{\frac{7}{2}}^{7+} \quad N_{\frac{9}{2}}^{9+}(2220)$				
	$\frac{3}{2}$	4	0	$\Delta_{\frac{5}{2}}^{5+}$	$\Delta_{\frac{7}{2}}^{7+}$	$\Delta_{\frac{9}{2}}^{9+}$	$\Delta_{\frac{11}{2}}^{11+}(2420)$	
70	$\frac{1}{2}$	5	0	$N_{\frac{9}{2}}^{9-} \quad N_{\frac{11}{2}}^{11-}$				
	$\frac{3}{2}$	5	0	$N_{\frac{7}{2}}^{7-}$	$N_{\frac{9}{2}}^{9-}$	$N_{\frac{11}{2}}^{11-}(2600)$	$N_{\frac{13}{2}}^{13-}$	

PDG 2012

Fermionic Modes and Baryon Spectrum

[Hard wall model: GdT and S. J. Brodsky, PRL **94**, 201601 (2005)]

[Soft wall model: GdT and S. J. Brodsky, (2005), arXiv:1001.5193]



From Nick Evans

- Nucleon LF modes

$$\psi_+(\zeta)_{n,L} = \kappa^{2+L} \sqrt{\frac{2n!}{(n+L)!}} \zeta^{3/2+L} e^{-\kappa^2 \zeta^2 / 2} L_n^{L+1}(\kappa^2 \zeta^2)$$

$$\psi_-(\zeta)_{n,L} = \kappa^{3+L} \frac{1}{\sqrt{n+L+2}} \sqrt{\frac{2n!}{(n+L)!}} \zeta^{5/2+L} e^{-\kappa^2 \zeta^2 / 2} L_n^{L+2}(\kappa^2 \zeta^2)$$

- Normalization

$$\int d\zeta \psi_+^2(\zeta) = \int d\zeta \psi_-^2(\zeta) = 1$$

*Chiral Symmetry
of Eigenstate!*

- Eigenvalues

$$\mathcal{M}_{n,L,S=1/2}^2 = 4\kappa^2 (n+L+1)$$

- “Chiral partners”

$$\frac{\mathcal{M}_{N(1535)}}{\mathcal{M}_{N(940)}} = \sqrt{2}$$

Chiral Features of Soft-Wall AdS/QCD Model

- **Boost Invariant**
- **Trivial LF vacuum! No condensate, but consistent with GMOR**
- **Massless Pion**
- **Hadron Eigenstates (even the pion) have LF Fock components of different L^z**
- **Proton: equal probability $S^z = +1/2, L^z = 0; S^z = -1/2, L^z = +1$**
$$J^z = +1/2 : \langle L^z \rangle = 1/2, \langle S_q^z \rangle = 0$$
- **Self-Dual Massive Eigenstates: Proton is its own chiral partner.**
- **Label State by minimum L as in Atomic Physics**
- **Minimum L dominates at short distances**
- **AdS/QCD Dictionary: Match to Interpolating Operator Twist at $z=0$.**

No mass-degenerate parity partners!

Space-Like Dirac Proton Form Factor

- Consider the spin non-flip form factors

$$F_+(Q^2) = g_+ \int d\zeta J(Q, \zeta) |\psi_+(\zeta)|^2,$$

$$F_-(Q^2) = g_- \int d\zeta J(Q, \zeta) |\psi_-(\zeta)|^2,$$

where the effective charges g_+ and g_- are determined from the spin-flavor structure of the theory.

- Choose the struck quark to have $S^z = +1/2$. The two AdS solutions $\psi_+(\zeta)$ and $\psi_-(\zeta)$ correspond to nucleons with $J^z = +1/2$ and $-1/2$.
- For $SU(6)$ spin-flavor symmetry

$$F_1^p(Q^2) = \int d\zeta J(Q, \zeta) |\psi_+(\zeta)|^2,$$

$$F_1^n(Q^2) = -\frac{1}{3} \int d\zeta J(Q, \zeta) [|\psi_+(\zeta)|^2 - |\psi_-(\zeta)|^2],$$

where $F_1^p(0) = 1$, $F_1^n(0) = 0$.

- Compute Dirac proton form factor using SU(6) flavor symmetry

$$F_1^p(Q^2) = R^4 \int \frac{dz}{z^4} V(Q, z) \Psi_+^2(z)$$

- Nucleon AdS wave function

$$\Psi_+(z) = \frac{\kappa^{2+L}}{R^2} \sqrt{\frac{2n!}{(n+L)!}} z^{7/2+L} L_n^{L+1}(\kappa^2 z^2) e^{-\kappa^2 z^2/2}$$

- Normalization ($F_1^p(0) = 1$, $V(Q=0, z) = 1$)

$$R^4 \int \frac{dz}{z^4} \Psi_+^2(z) = 1$$

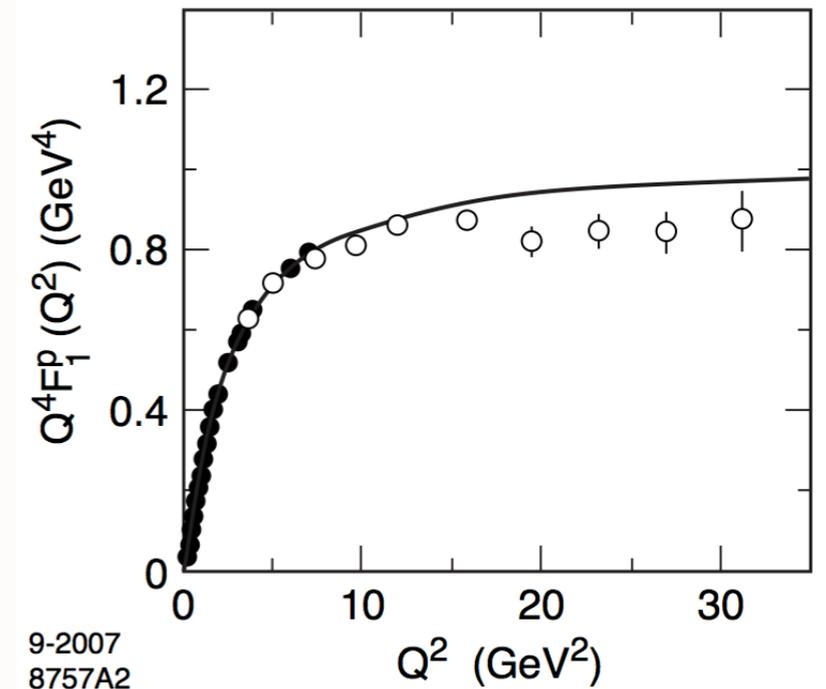
- Bulk-to-boundary propagator [Grigoryan and Radyushkin (2007)]

$$V(Q, z) = \kappa^2 z^2 \int_0^1 \frac{dx}{(1-x)^2} x^{\frac{Q^2}{4\kappa^2}} e^{-\kappa^2 z^2 x/(1-x)}$$

- Find

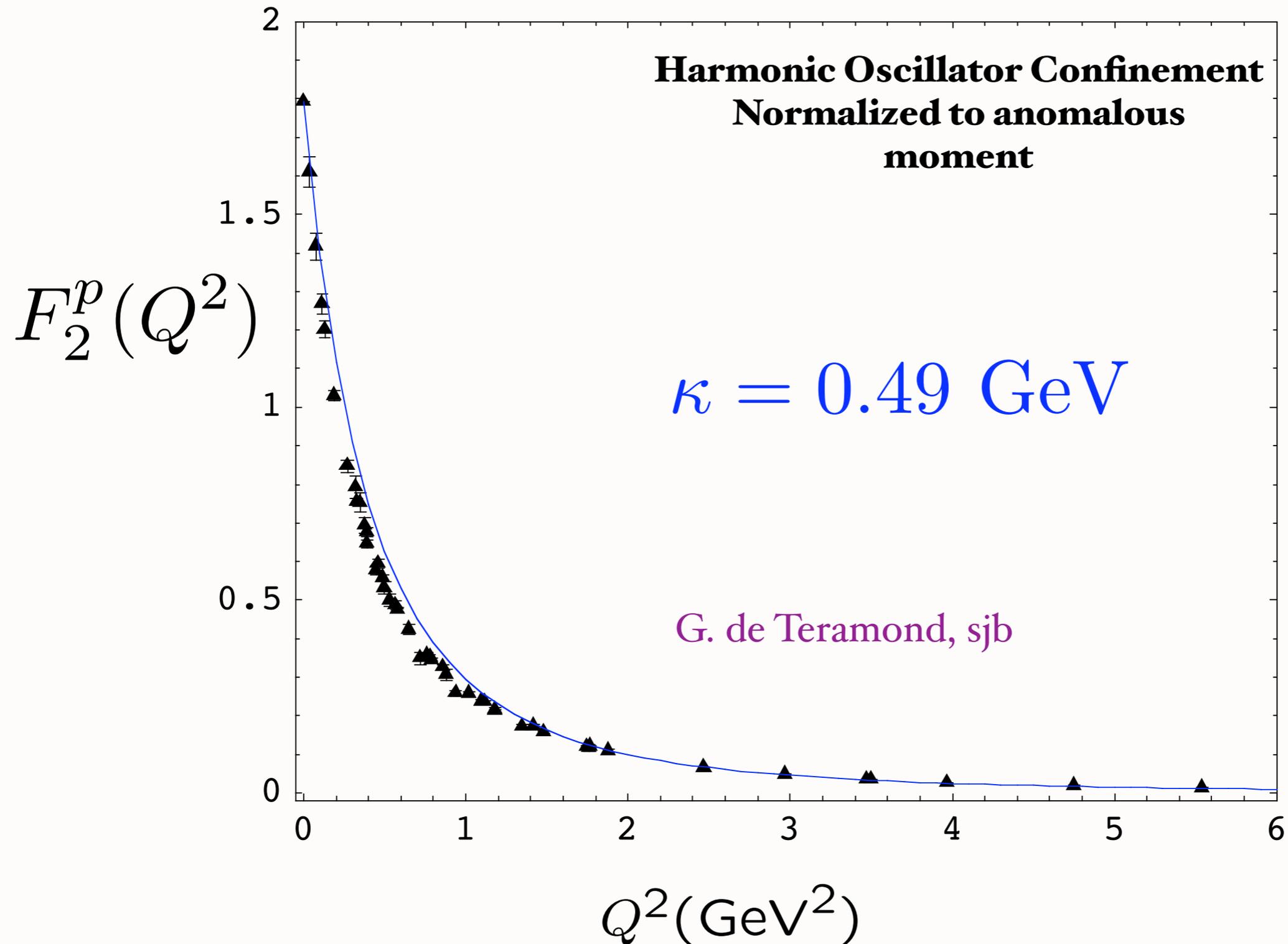
$$F_1^p(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{\mathcal{M}_{\rho}^2}\right) \left(1 + \frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)}$$

with $\mathcal{M}_{\rho_n}^2 \rightarrow 4\kappa^2(n + 1/2)$

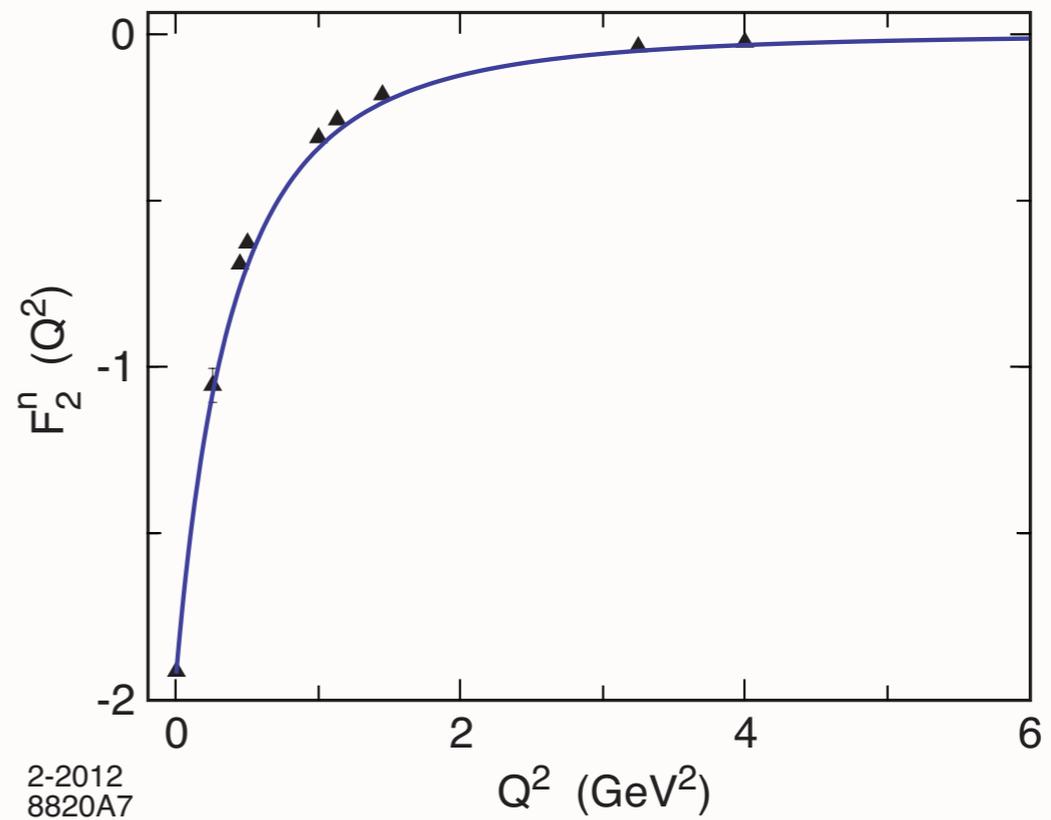
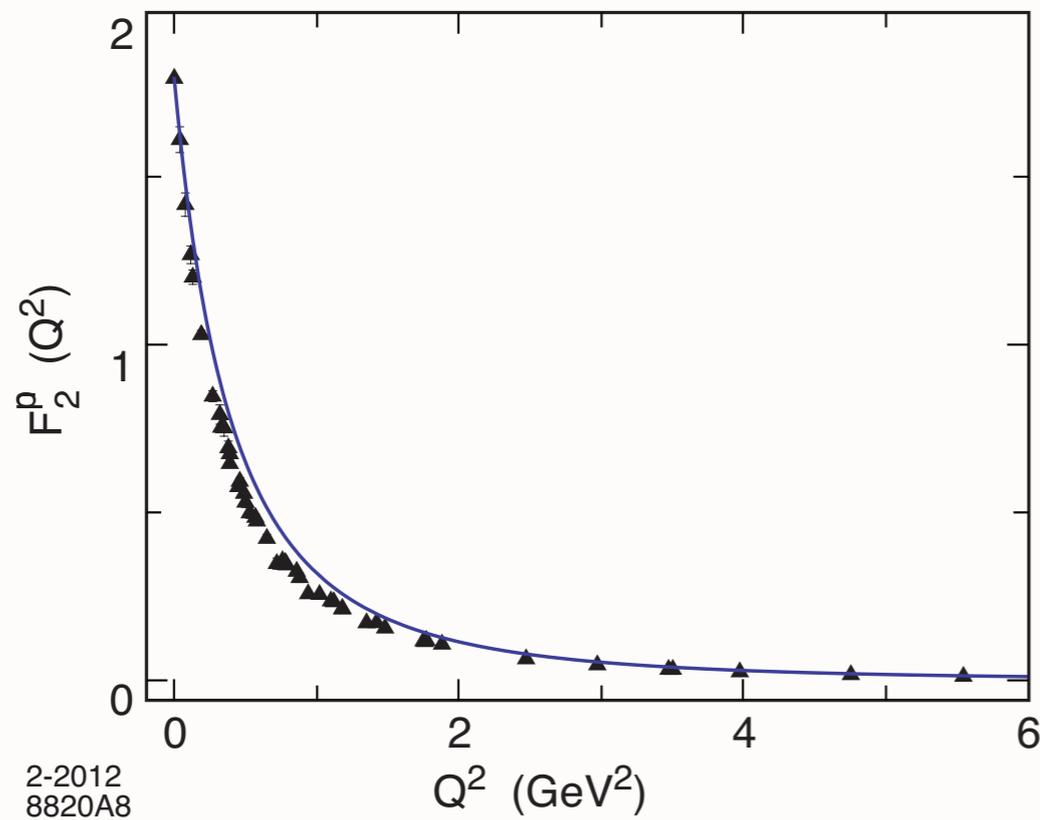
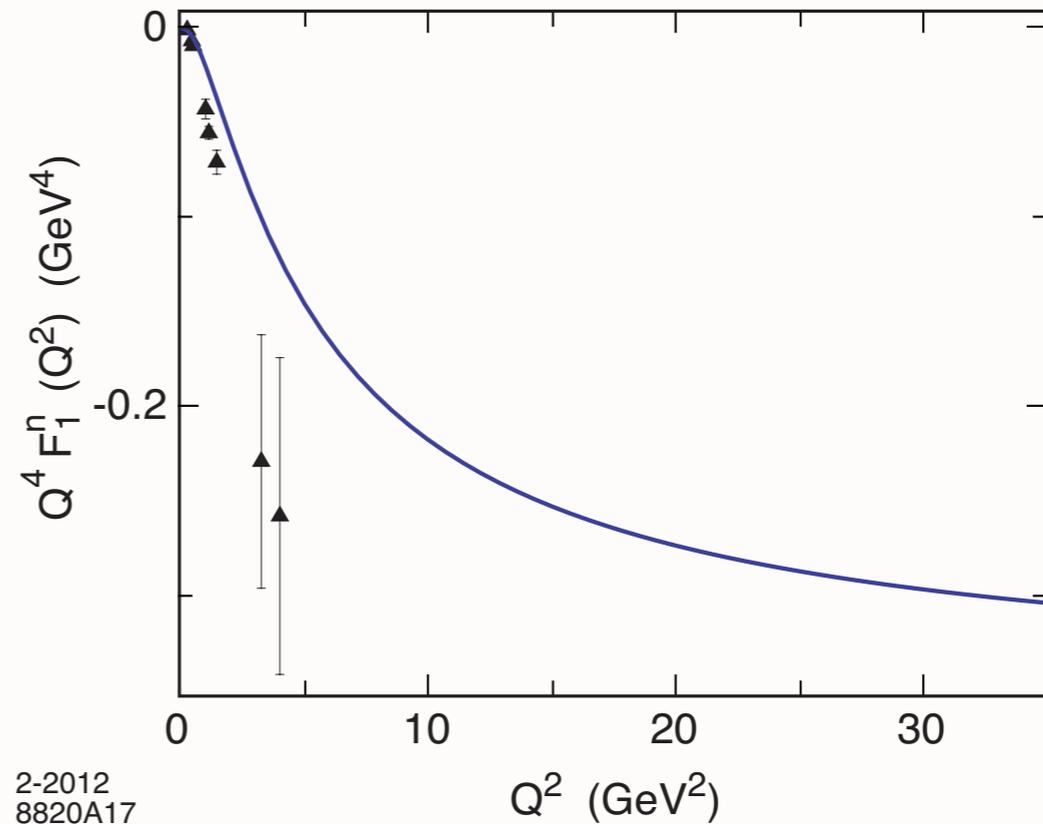
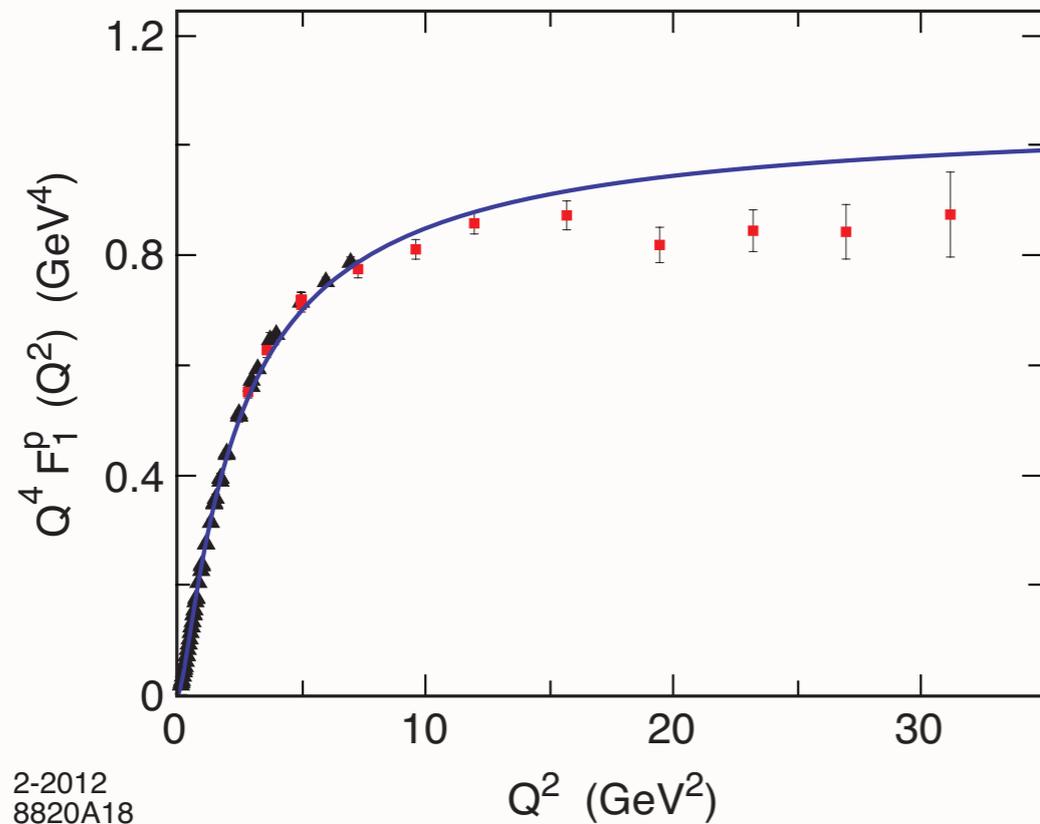


Spacelike Pauli Form Factor

From overlap of $L = 1$ and $L = 0$ LFWFs



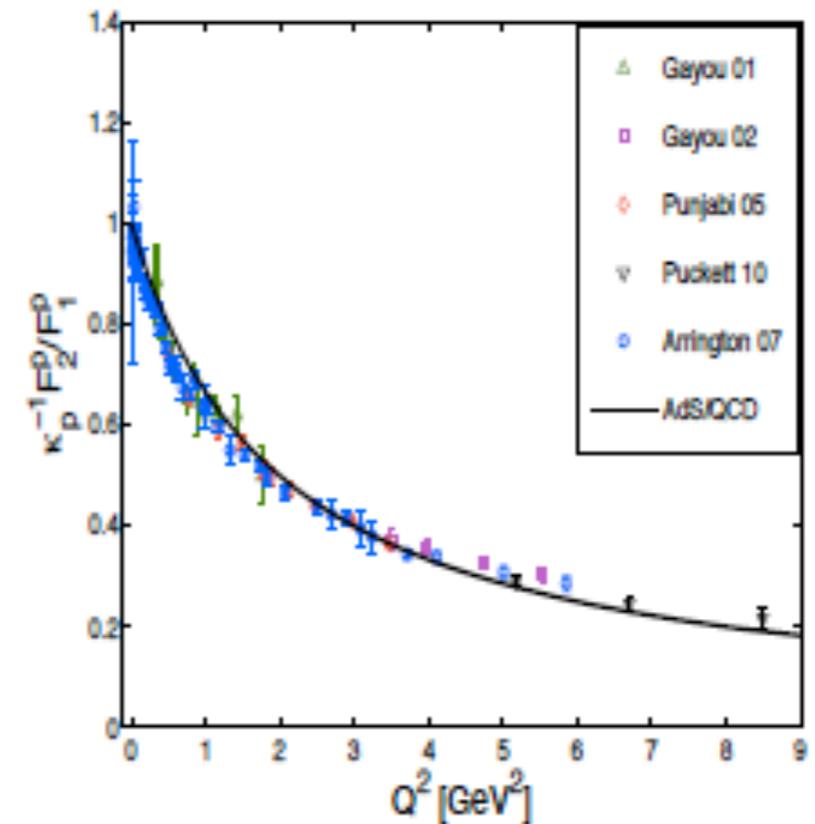
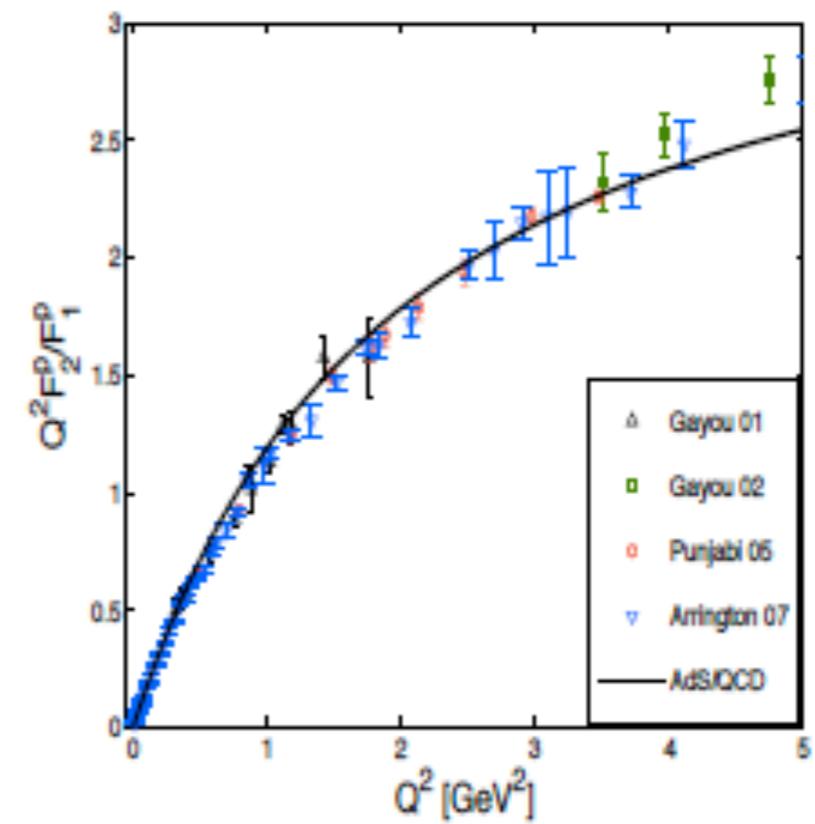
Using $SU(6)$ flavor symmetry and normalization to static quantities



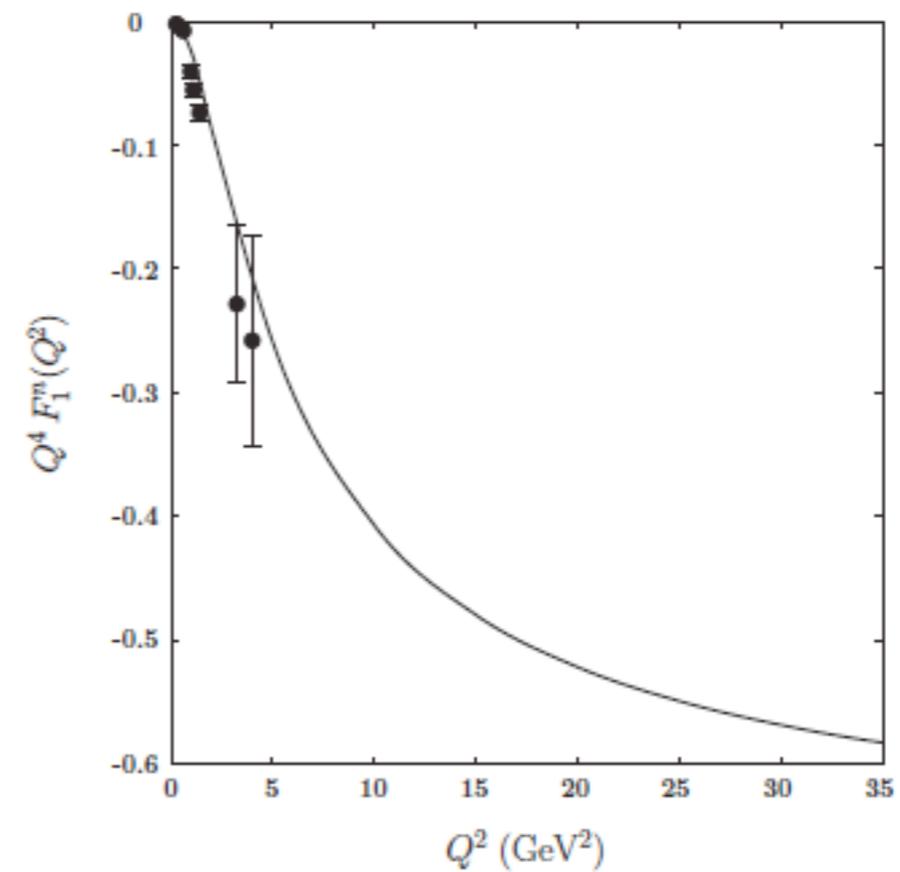
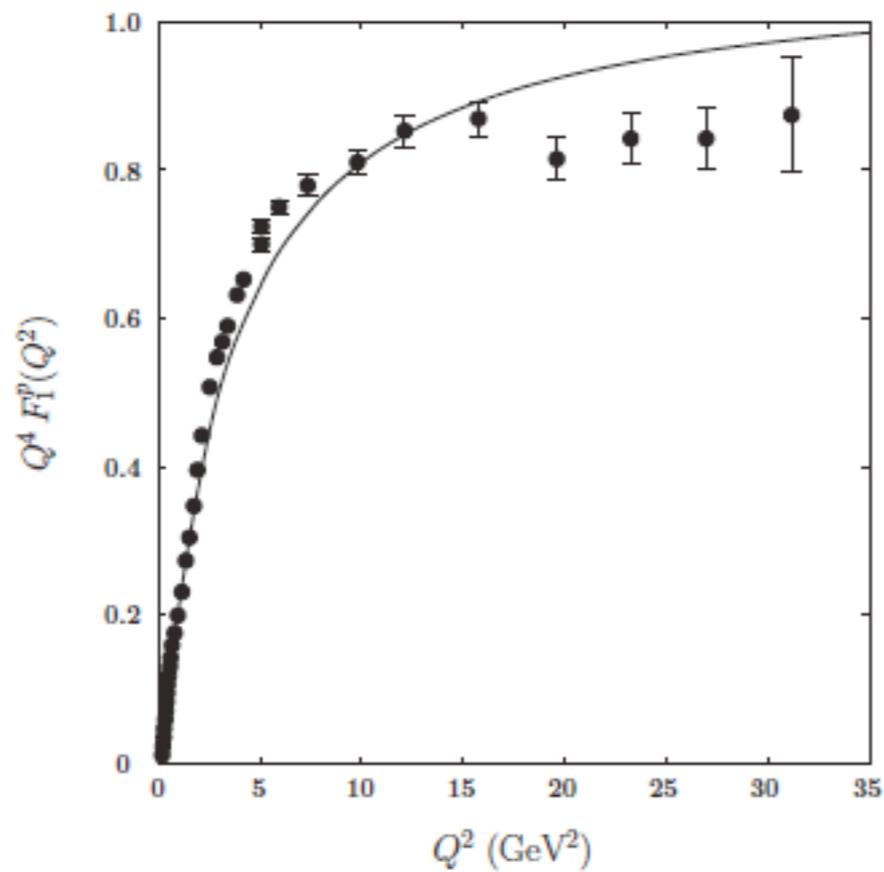
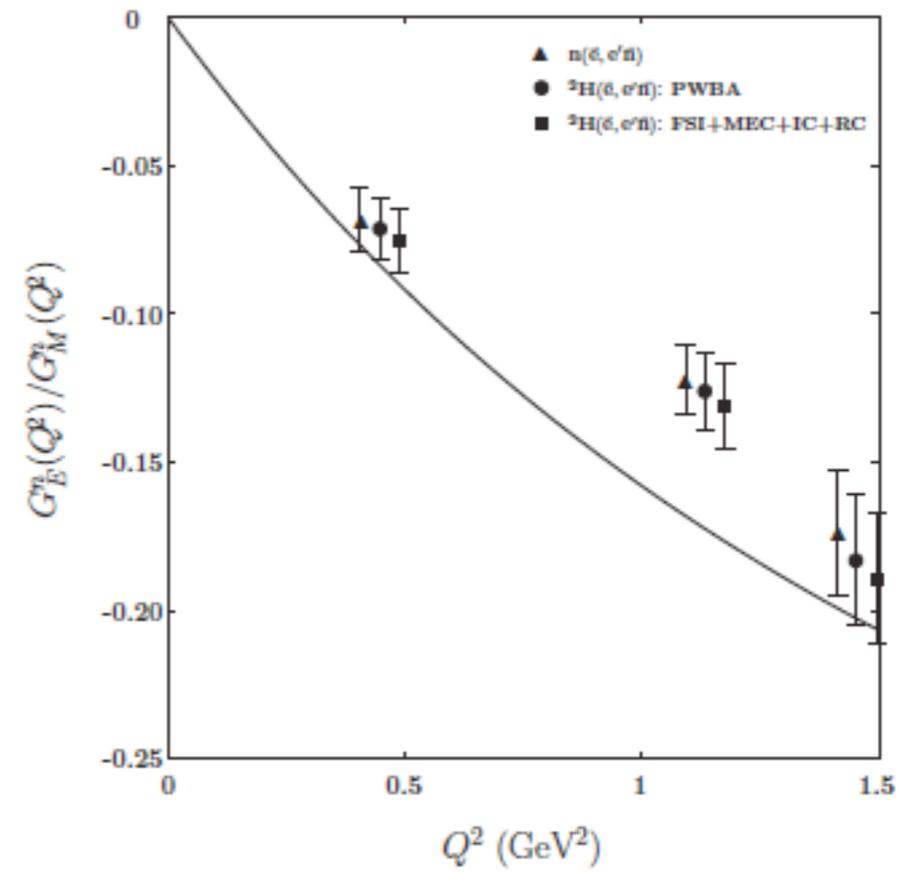
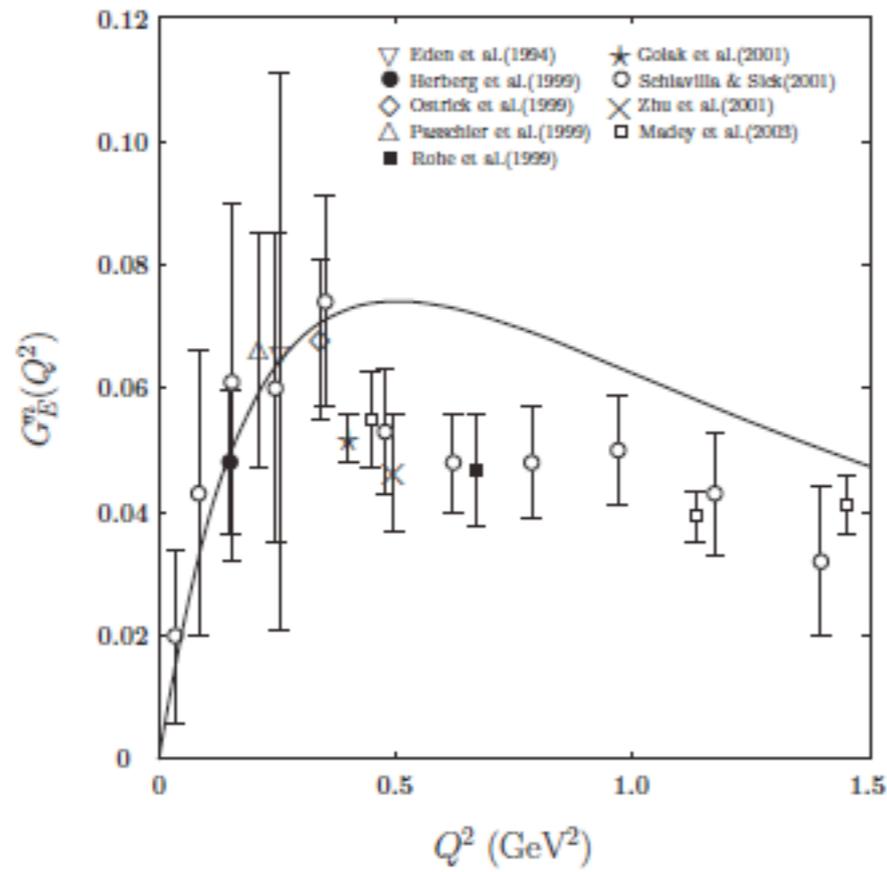
Nucleon and flavor form factors in a light front quark model in AdS/QCD

Dipankar Chakrabarti, Chandan Mondal

¹Department of Physics, Indian Institute of Technology Kanpur, Kanpur-208016, India.



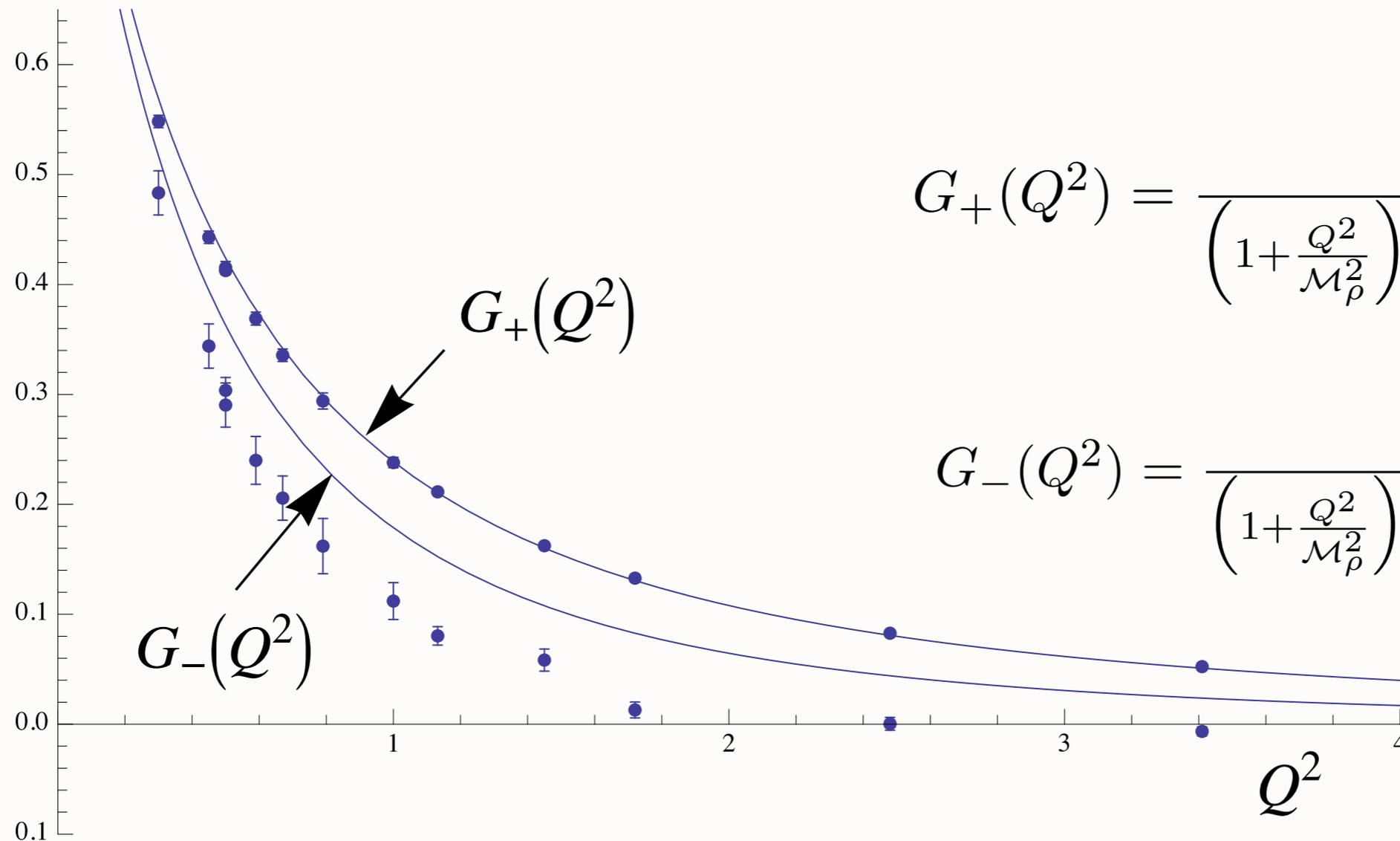
Baryon structure in AdS/QCD



Flavor Decomposition of Elastic Nucleon Form Factors

G. D. Cates *et al.* Phys. Rev. Lett. **106**, 252003 (2011)

- Proton SU(6) WF: $F_{u,1}^p = \frac{5}{3}G_+ + \frac{1}{3}G_-$, $F_{d,1}^p = \frac{1}{3}G_+ + \frac{2}{3}G_-$
- Neutron SU(6) WF: $F_{u,1}^n = \frac{1}{3}G_+ + \frac{2}{3}G_-$, $F_{d,1}^n = \frac{5}{3}G_+ + \frac{1}{3}G_-$

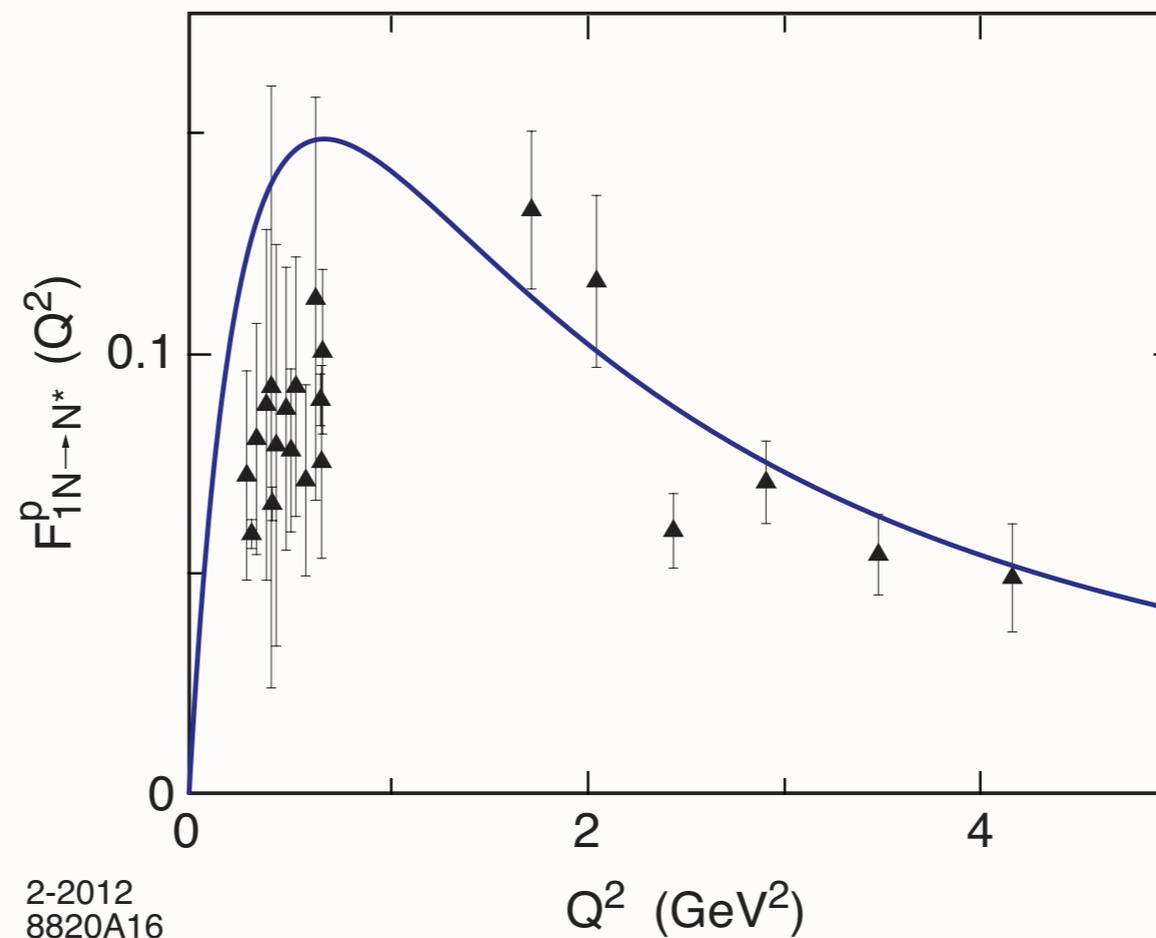


$$G_+(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{M_\rho^2}\right) \left(1 + \frac{Q^2}{M_{\rho'}^2}\right)}$$

$$G_-(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{M_\rho^2}\right) \left(1 + \frac{Q^2}{M_{\rho'}^2}\right) \left(1 + \frac{Q^2}{M_{\rho''}^2}\right)}$$

Nucleon Transition Form Factors

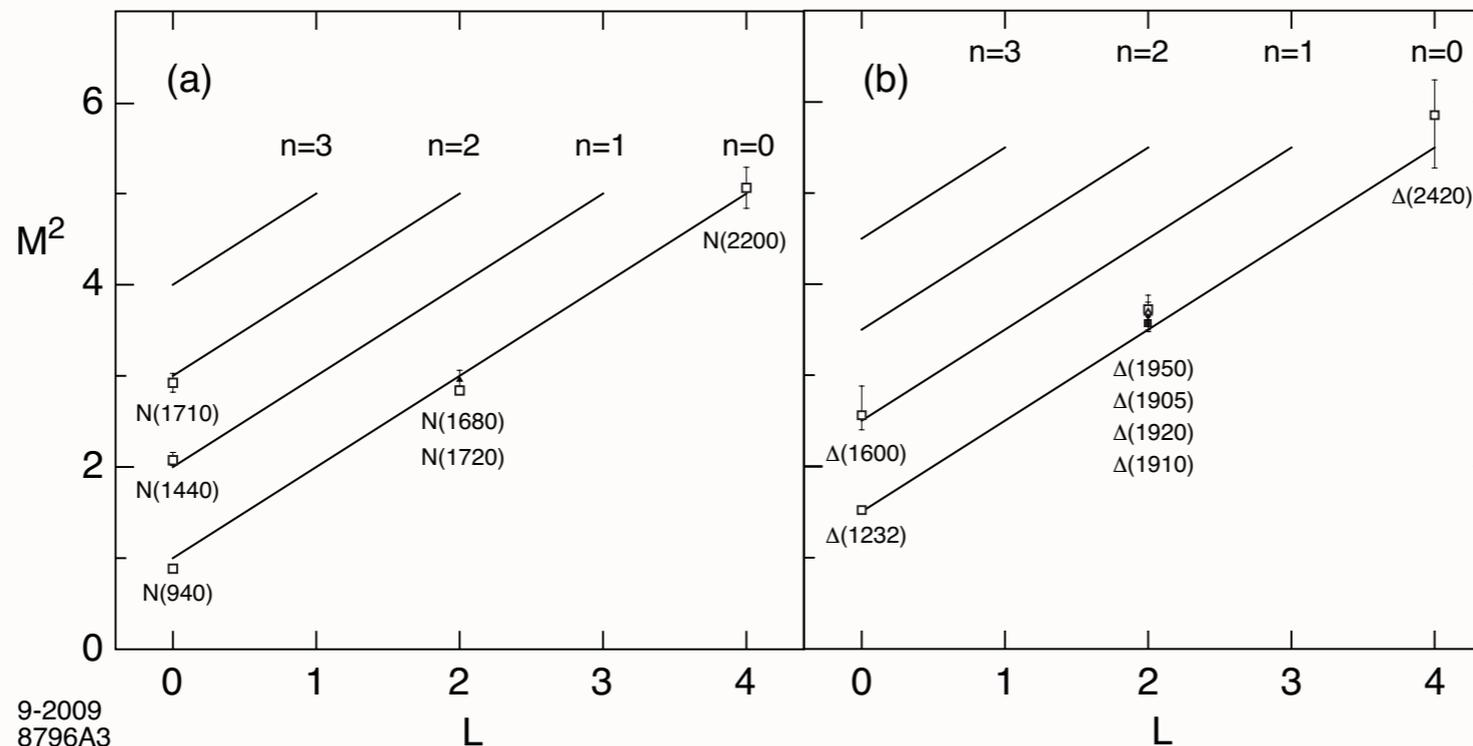
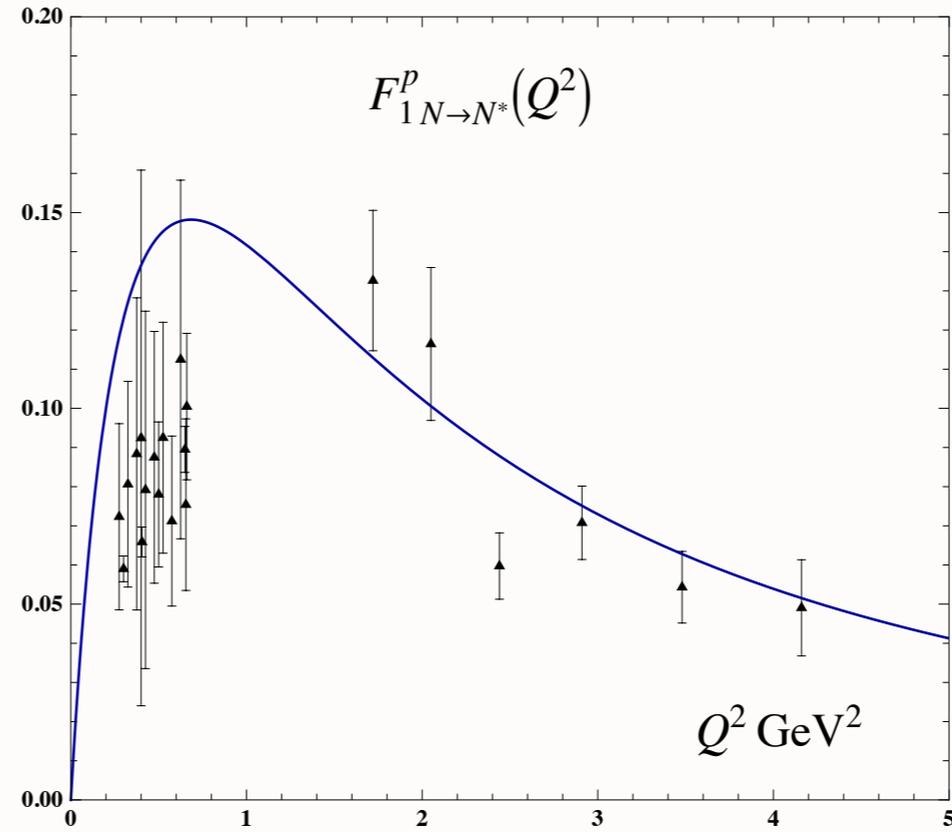
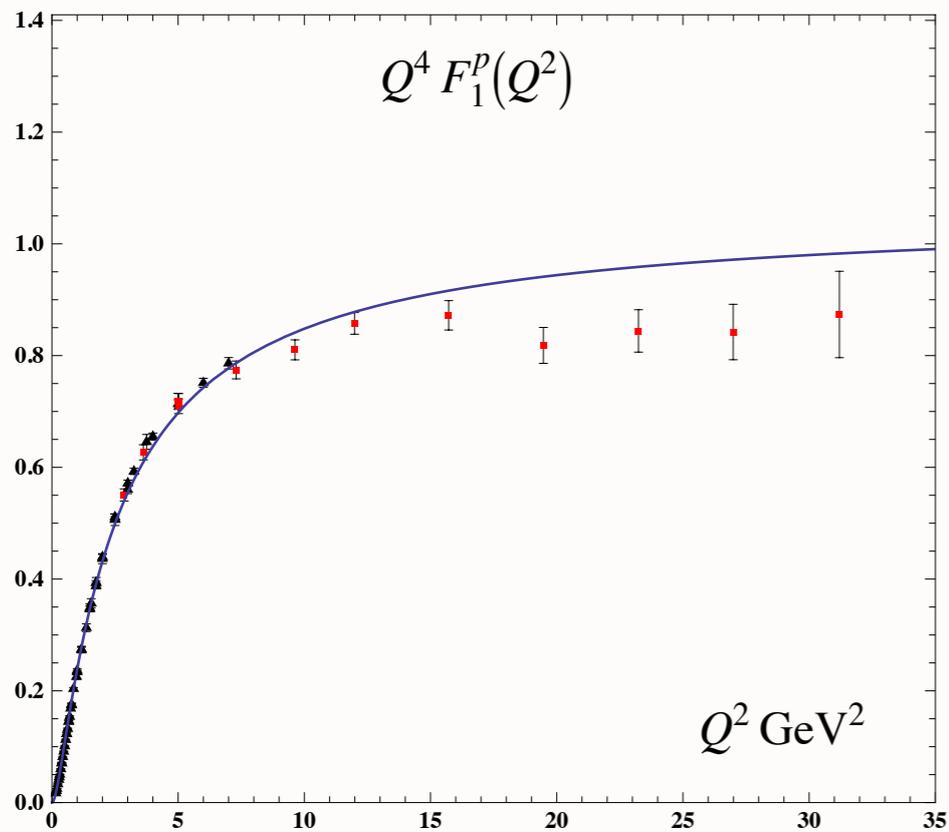
$$F_{1N \rightarrow N^*}^p(Q^2) = \frac{\sqrt{2}}{3} \frac{\frac{Q^2}{M_\rho^2}}{\left(1 + \frac{Q^2}{M_\rho^2}\right) \left(1 + \frac{Q^2}{M_{\rho'}^2}\right) \left(1 + \frac{Q^2}{M_{\rho''}^2}\right)}.$$



Proton transition form factor to the first radial excited state. Data from JLab

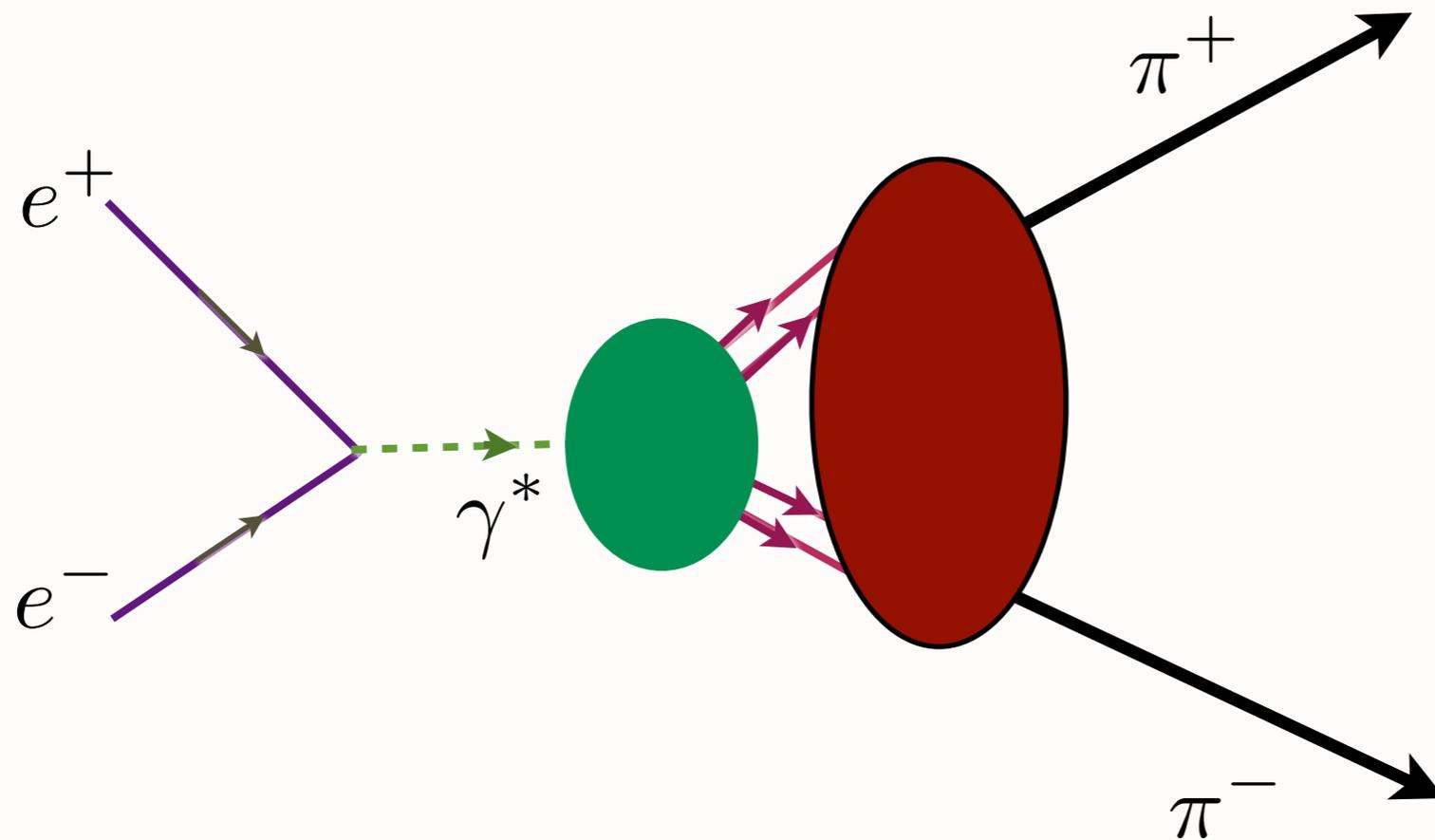
Excited Baryons in Holographic QCD

G. de Teramond & sjb



9-2009
8796A3

Dressed soft-wall current brings in higher Fock states and more vector meson poles

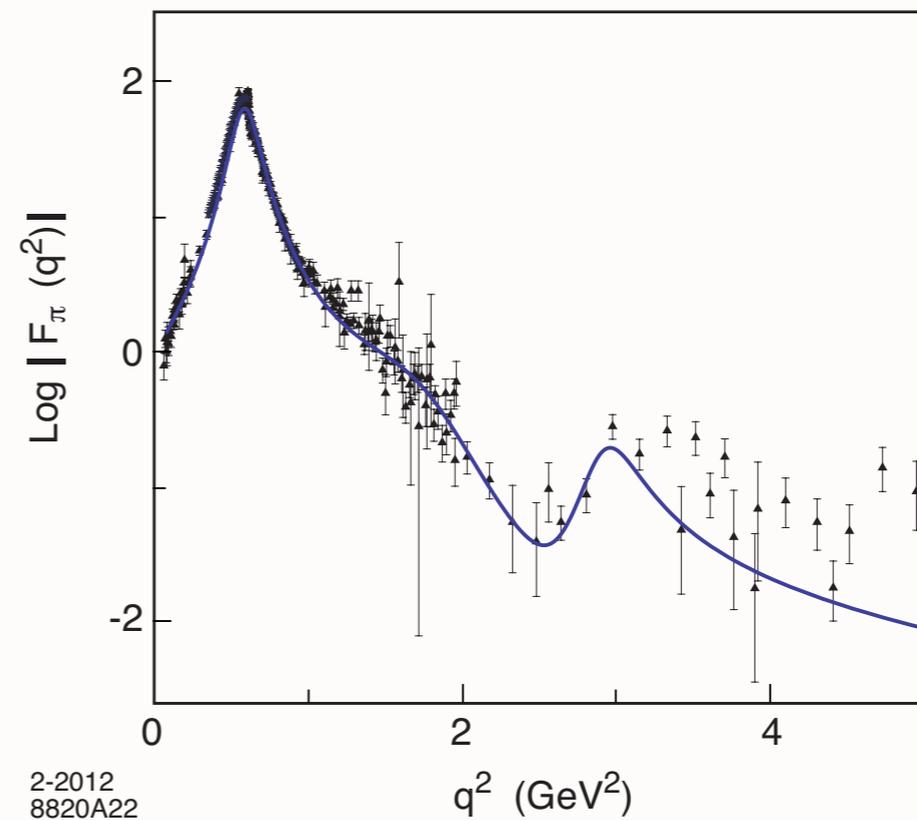
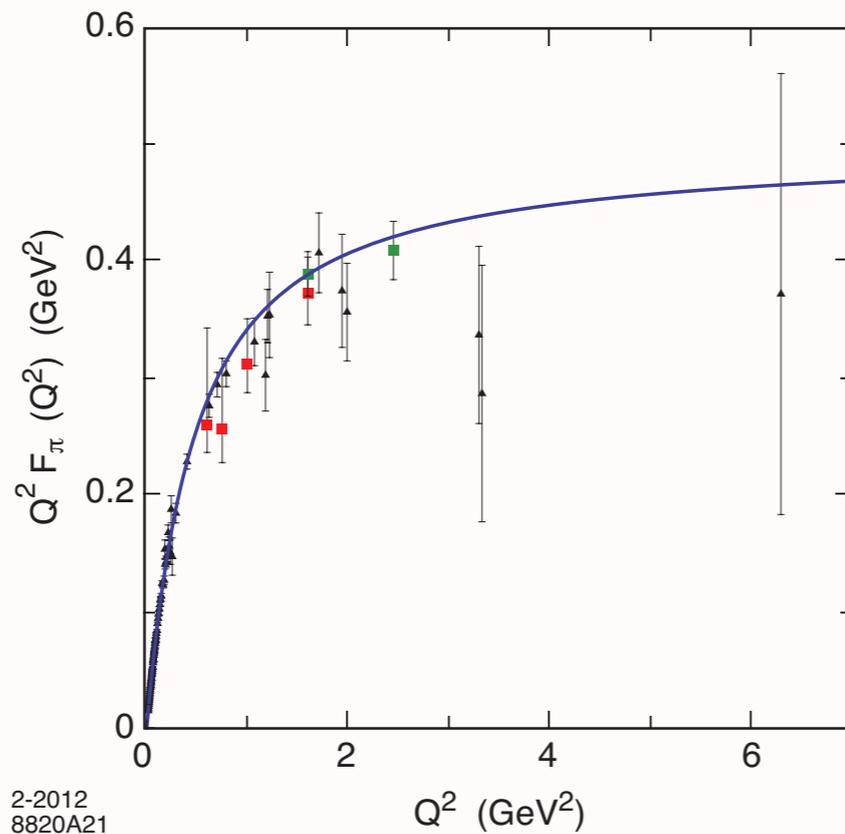


Higher Fock Components in LF Holographic QCD

- Effective interaction leads to $qq \rightarrow qq$, $q\bar{q} \rightarrow q\bar{q}$ but also to $q \rightarrow qq\bar{q}$ and $\bar{q} \rightarrow \bar{q}q\bar{q}$
- Higher Fock states can have any number of extra $q\bar{q}$ pairs, but surprisingly no dynamical gluons
- Example of relevance of higher Fock states and the absence of dynamical gluons at the hadronic scale

$$|\pi\rangle = \psi_{q\bar{q}/\pi} |q\bar{q}\rangle_{\tau=2} + \psi_{q\bar{q}q\bar{q}} |q\bar{q}q\bar{q}\rangle_{\tau=4} + \dots$$

- Modify form factor formula introducing finite width: $q^2 \rightarrow q^2 + \sqrt{2}i\mathcal{M}\Gamma$ ($P_{q\bar{q}q\bar{q}} = 13\%$)



Running Coupling from Modified AdS/QCD

Deur, de Teramond, sjb

- Consider five-dim gauge fields propagating in AdS₅ space in dilaton background $\varphi(z) = \kappa^2 z^2$

$$S = -\frac{1}{4} \int d^4x dz \sqrt{g} e^{\varphi(z)} \frac{1}{g_5^2} G^2$$

- Flow equation

$$\frac{1}{g_5^2(z)} = e^{\varphi(z)} \frac{1}{g_5^2(0)} \quad \text{or} \quad g_5^2(z) = e^{-\kappa^2 z^2} g_5^2(0)$$

where the coupling $g_5(z)$ incorporates the non-conformal dynamics of confinement

- YM coupling $\alpha_s(\zeta) = g_{YM}^2(\zeta)/4\pi$ is the five dim coupling up to a factor: $g_5(z) \rightarrow g_{YM}(\zeta)$
- Coupling measured at momentum scale Q

$$\alpha_s^{AdS}(Q) \sim \int_0^\infty \zeta d\zeta J_0(\zeta Q) \alpha_s^{AdS}(\zeta)$$

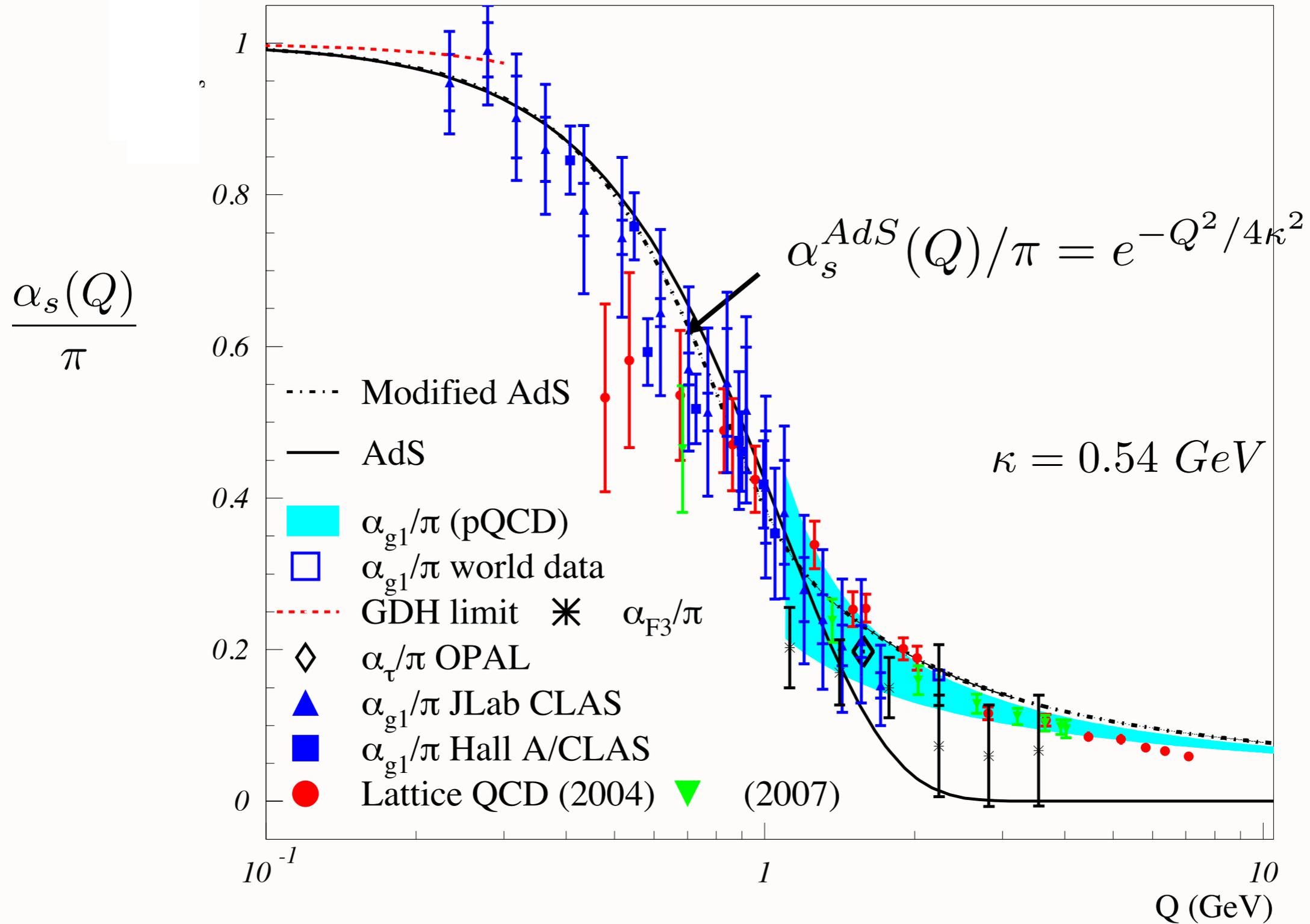
- Solution

$$\alpha_s^{AdS}(Q^2) = \alpha_s^{AdS}(0) e^{-Q^2/4\kappa^2}.$$

where the coupling α_s^{AdS} incorporates the non-conformal dynamics of confinement

Running Coupling from Light-Front Holography and AdS/QCD

Analytic, defined at all scales, IR Fixed Point



AdS/QCD dilaton captures the higher twist corrections to effective charges for $Q < 1 \text{ GeV}$

$$e^\varphi = e^{+\kappa^2 z^2}$$

Deur, de Teramond, sjb

Extensions of AdS/QCD LF Holography

- *Massive quarks*
- *Broken Chiral Symmetry*
- *Structure Functions*
- *Counting Rules at $x \sim 1$, Duality*
- *Nucleon GPDs*

Valery E. Lyubovitskij, Tanja Branz, Thomas Gutsche, Ivan Schmidt, Alfredo Vega
Ian Cloet, C. D. Roberts
Ruben Sandapen, Jeff Forshaw
Burkardt, Schmidt, Lepage, sjb

Light and heavy mesons in a soft-wall holographic model

Valery E. Lyubovitskij^{*1†}, Tanja Branz¹, Thomas Gutsche¹, Ivan Schmidt², Alfredo Vega²

¹ *Institut für Theoretische Physik, Universität Tübingen,
Kepler Center for Astro and Particle Physics,
Auf der Morgenstelle 14, D-72076 Tübingen, Germany*

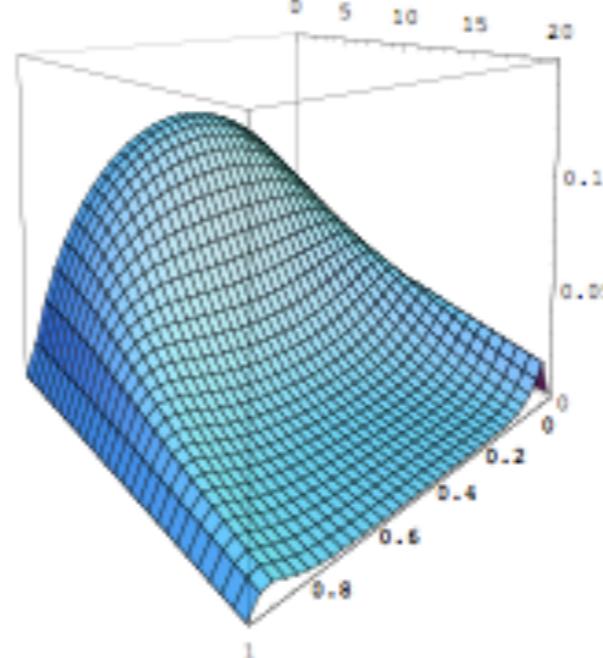
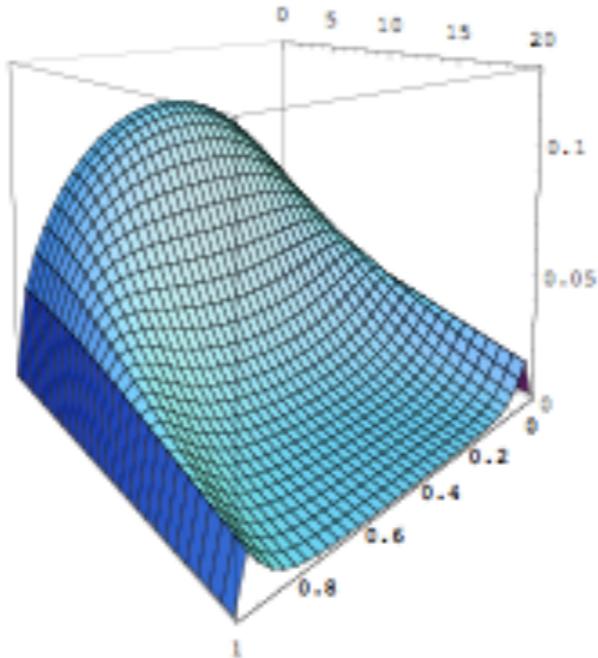
² *Departamento de Física y Centro Científico Tecnológico de Valparaíso (CCTVal),
Universidad Técnica Federico Santa María, Casilla 110-V, Valparaíso, Chile*

We study the spectrum and decay constants of light and heavy mesons in a soft-wall holographic approach, using the correspondence of string theory in Anti-de Sitter space and conformal field theory in physical space-time.

$$|\pi^+\rangle = |u\bar{d}\rangle$$

$$m_u = 2 \text{ MeV}$$

$$m_d = 5 \text{ MeV}$$

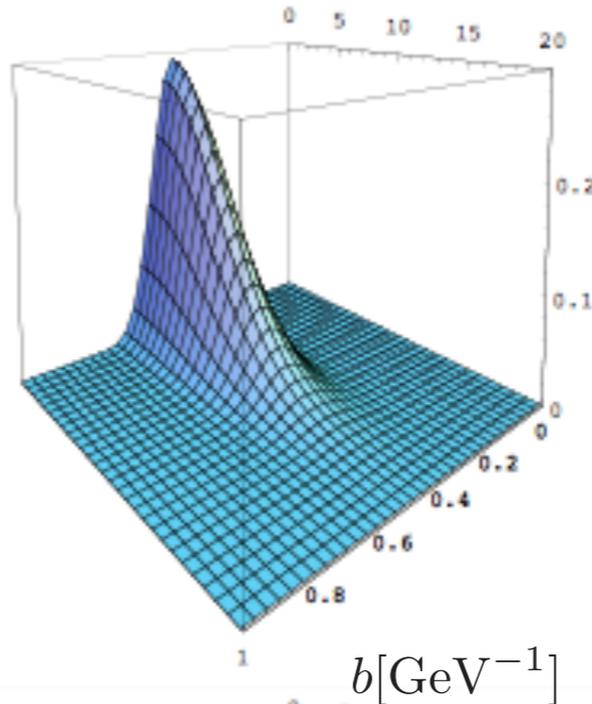
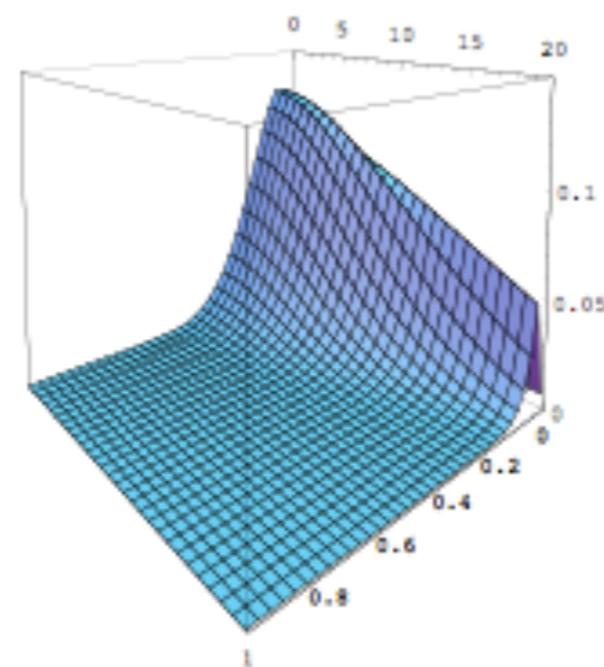


$$|K^+\rangle = |u\bar{s}\rangle$$

$$m_s = 95 \text{ MeV}$$

$$|D^+\rangle = |c\bar{d}\rangle$$

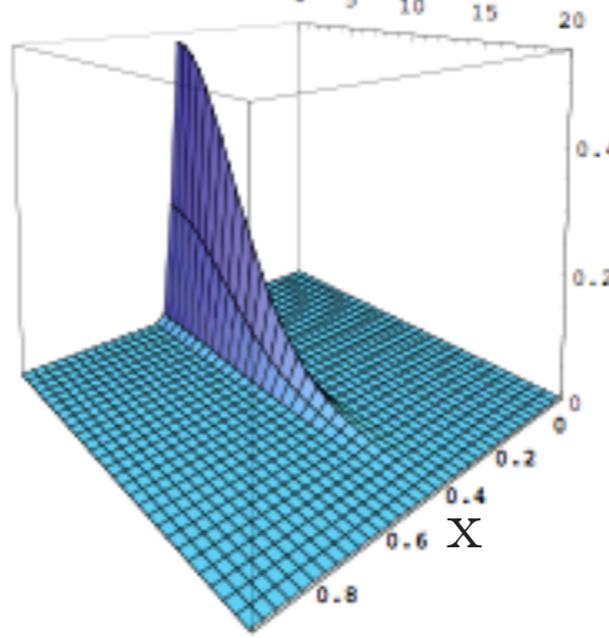
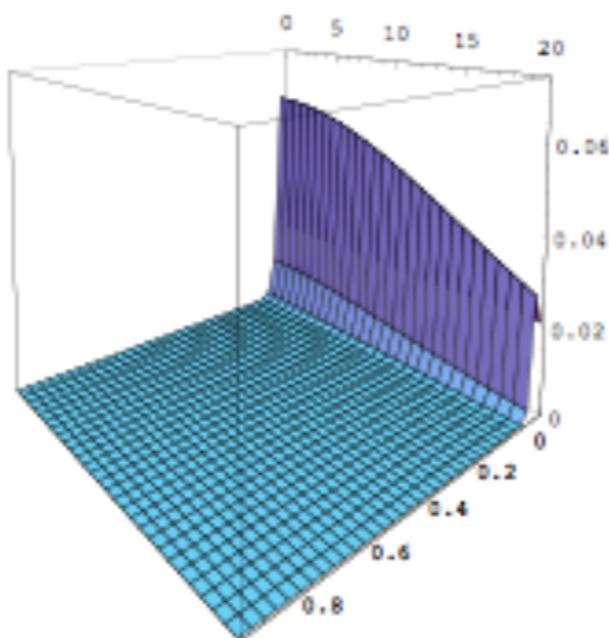
$$m_c = 1.25 \text{ GeV}$$



$$|\eta_c\rangle = |c\bar{c}\rangle$$

$$|B^+\rangle = |u\bar{b}\rangle$$

$$m_b = 4.2 \text{ GeV}$$



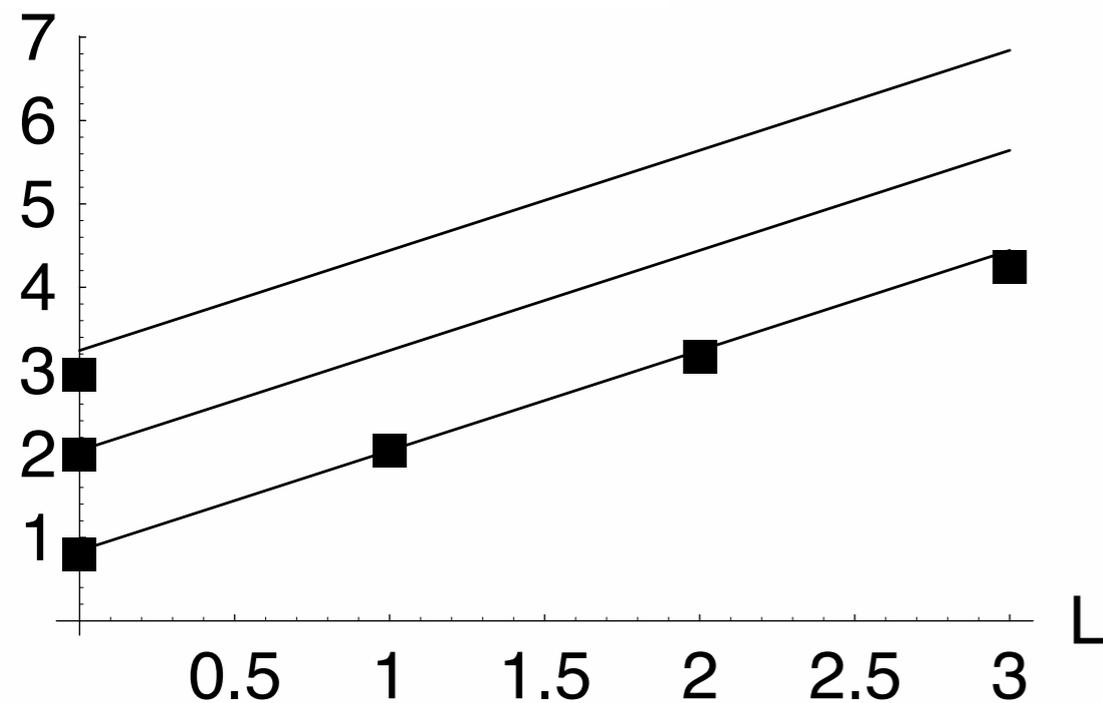
$$|\eta_b\rangle = |b\bar{b}\rangle$$

$$\kappa = 375 \text{ MeV}$$

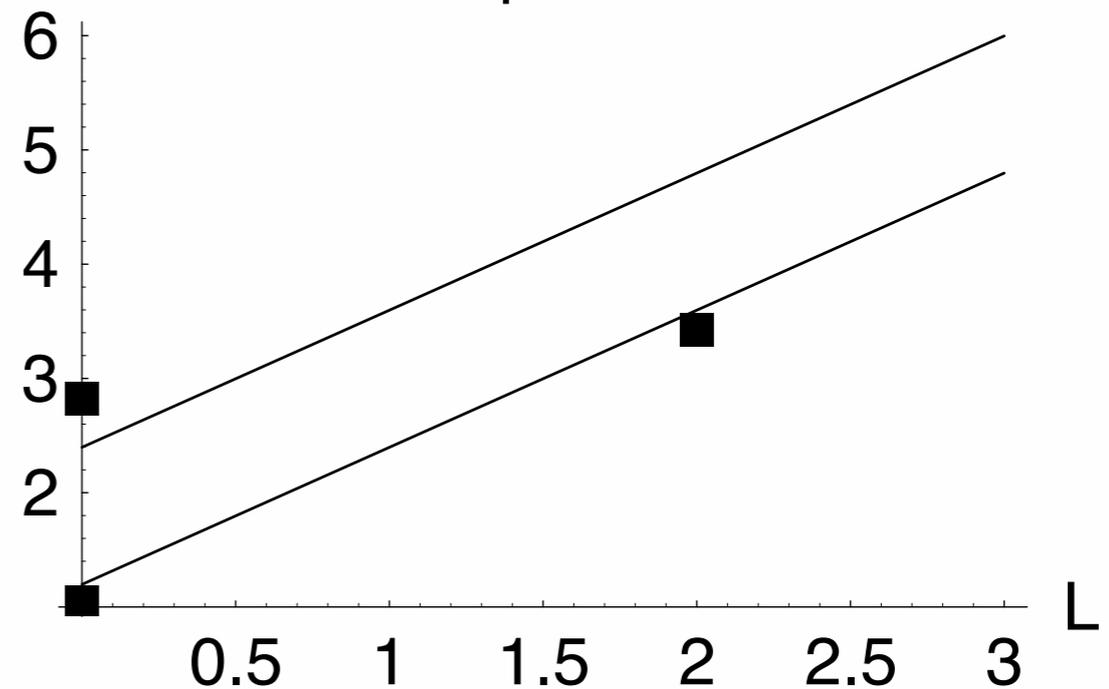
Application to Strange Hadrons

$$M^2 = M_0^2 + \left\langle X \left| \frac{m_q^2}{x} \right| X \right\rangle + \left\langle X \left| \frac{m_{\bar{q}}^2}{1-x} \right| X \right\rangle$$

$\mathcal{M}^2 (GeV^2)$



$\mathcal{M}^2 (GeV^2)$



$n = 0 : K^*(892), K^*(1410), K^*(1680), K^*(2045)$

$n = 1 : K^*(1430)$

$n = 2 : K^*(1789)$

$n = 0 : \phi(1012), \phi(1850)$

$n = 1 : \phi(1680)$

G. de Teramond, H. G. Dosch, sjb

Preliminary

Decay constants f_P in MeV of pseudoscalar mesons

Meson	Data [13]	Our
π^-	$130.4 \pm 0.03 \pm 0.2$	131
K^-	$156.1 \pm 0.2 \pm 0.8$	155
D^+	206.7 ± 8.9	167
D_s^+	257.5 ± 6.1	170
B^-	193 ± 11	139
B_s^0	$253 \pm 8 \pm 7$	144
B_c	$489 \pm 5 \pm 3$ [14]	159

Decay constants f_V in MeV of vector mesons

Meson	Data [13]	Our	Meson	Data [13]	Our
ρ^+	210.5 ± 0.6	170	ρ^0	154.7 ± 0.7	120
D^*	$245 \pm 20_{-2}^{+3}$ [15]	167	ω	45.8 ± 0.8	40
D_s^*	$272 \pm 16_{-20}^{+3}$ [16]	170	ϕ	76 ± 1.2	58
B^*	$196 \pm 24_{-2}^{+39}$ [15]	139	J/ψ	277.6 ± 4	116
B_s^*	$229 \pm 20_{-16}^{+41}$ [15]	144	$\Upsilon(1s)$	238.5 ± 5.5	56

Basis Light-Front Quantization Approach to Quantum Field Theory

BLFQ

Use AdS/QCD basis functions!

Xingbo Zhao

With Anton Ilderton,
Heli Honkanen, Pieter Maris,
James Vary, Stan Brodsky



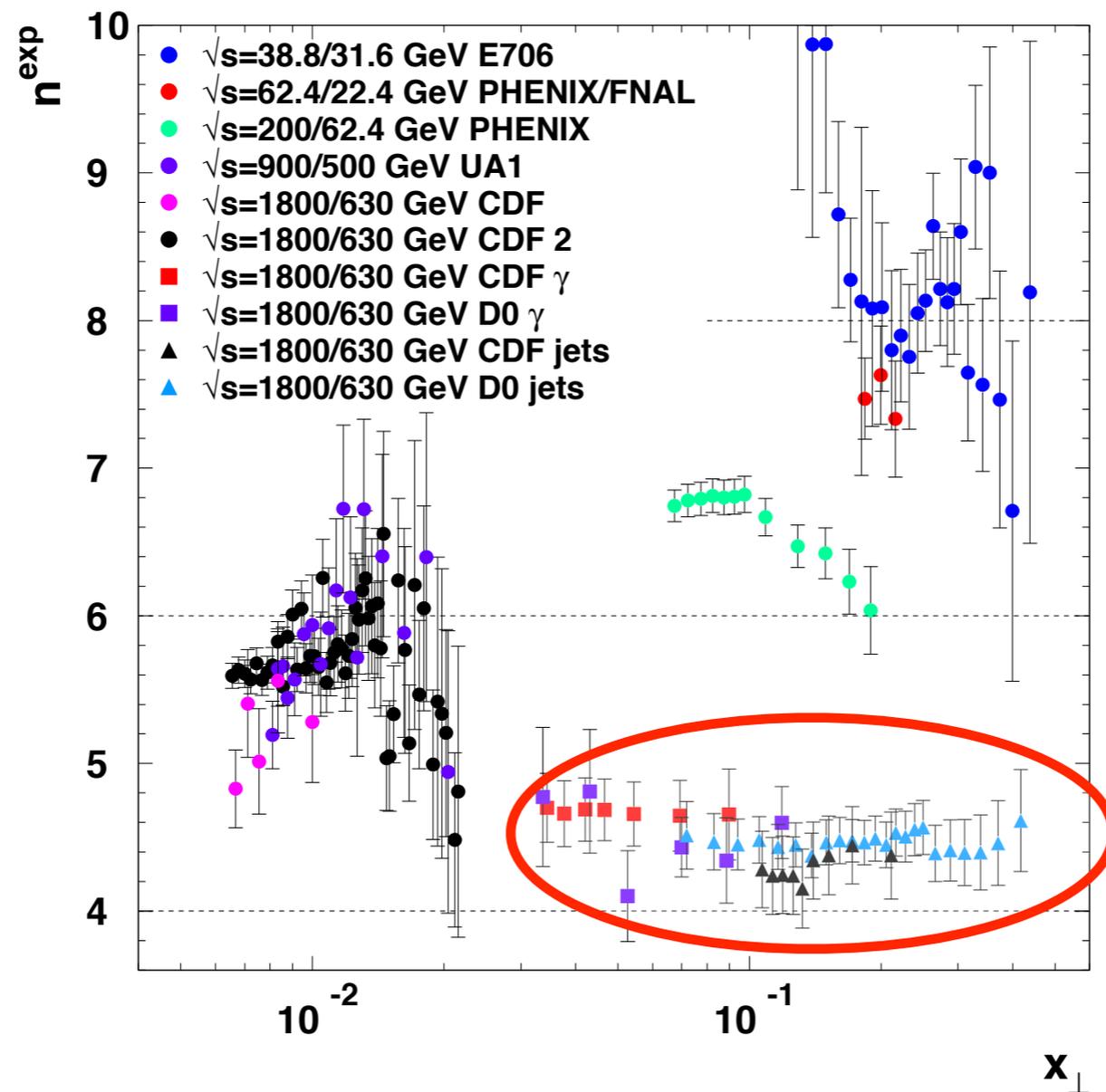
**Department of Physics and Astronomy
Iowa State University
Ames, USA**



Applications to Collider Physics

- **Non-Perturbative Structure Functions**
- **Fundamental understanding of angular momentum**
- **Higher Fock States: Intrinsic Heavy Quarks**
- **Higgs at High x_F**
- **Hadronization at the Amplitude Level**
- **Direct Higher-Twist Processes: Violation of leading twist scaling**
- **Collisions of Flux-Tubes: Ridge effect in p-p scattering**
- **Multiparton amplitudes: Cluster decomposition, J_z conservation, Parke-Taylor**
- **Multi-gluon initiated processes: Novel nuclear effects**
- **Non-Universal Anti-shadowing**
- **Hadronization from first principles -- at the Amplitude Level**
- **Principle of Maximum Conformality**
- **Connection to Pomeron (Shuryak)**

$$E \frac{d\sigma}{d^3p} (pp \rightarrow HX) = \frac{F(x_T, \theta_{cm} = \pi/2)}{p_T^n}$$



Photons and Jets
agree with PQCD
 x_T scaling
Hadrons do not!

Arleo, Hwang, Sickles, sjb

- Significant increase of the hadron n^{exp} with x_{\perp}
 - $n^{\text{exp}} \simeq 8$ at large x_{\perp}
- Huge contrast with photons and jets !
 - n^{exp} constant and slight above 4 at all x_{\perp}

Baryon can be made directly within hard subprocess

Coalescence within hard subprocess

Bjorken
 Blankenbecler, Gunion, sjb
 Berger, sjb
 Hoyer, et al: Semi-Exclusive

Sickles; sjb

*Small color-singlet
 Color Transparent
 Minimal same-side energy*

*Explains
 Baryon
 anomaly*

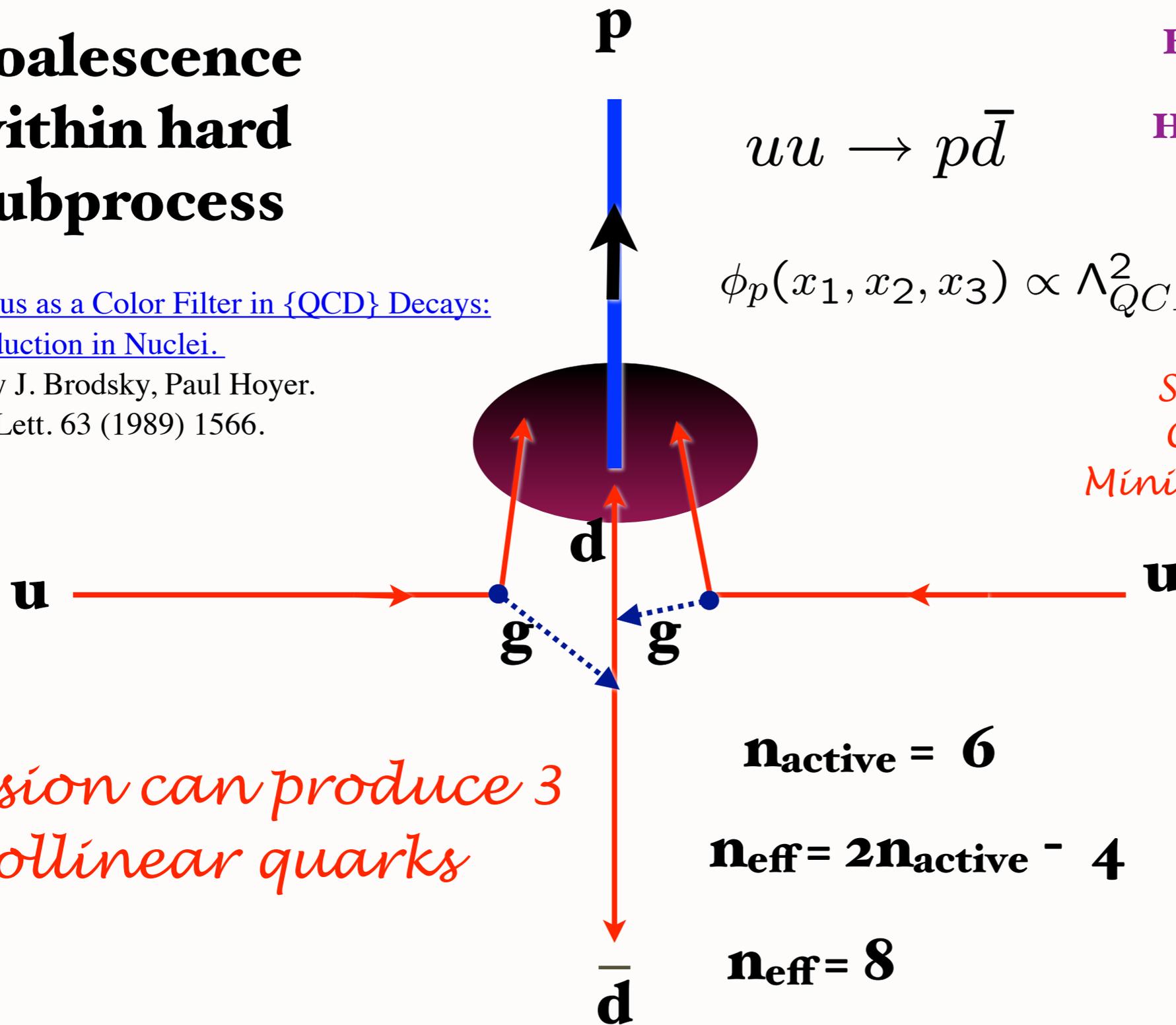
$qq \rightarrow B\bar{q}$

[The Nucleus as a Color Filter in {QCD} Decays:](#)

[Hadroproduction in Nuclei.](#)

By Stanley J. Brodsky, Paul Hoyer.

Phys.Rev.Lett. 63 (1989) 1566.



$$uu \rightarrow p\bar{d}$$

$$\phi_p(x_1, x_2, x_3) \propto \Lambda_{QCD}^2$$

$$n_{\text{active}} = 6$$

$$n_{\text{eff}} = 2n_{\text{active}} - 4$$

$$n_{\text{eff}} = 8$$

*Collision can produce 3
 collinear quarks*

New Perspectives for QCD

Hoyer Fest

November 21, 2013

CP³ - Origins

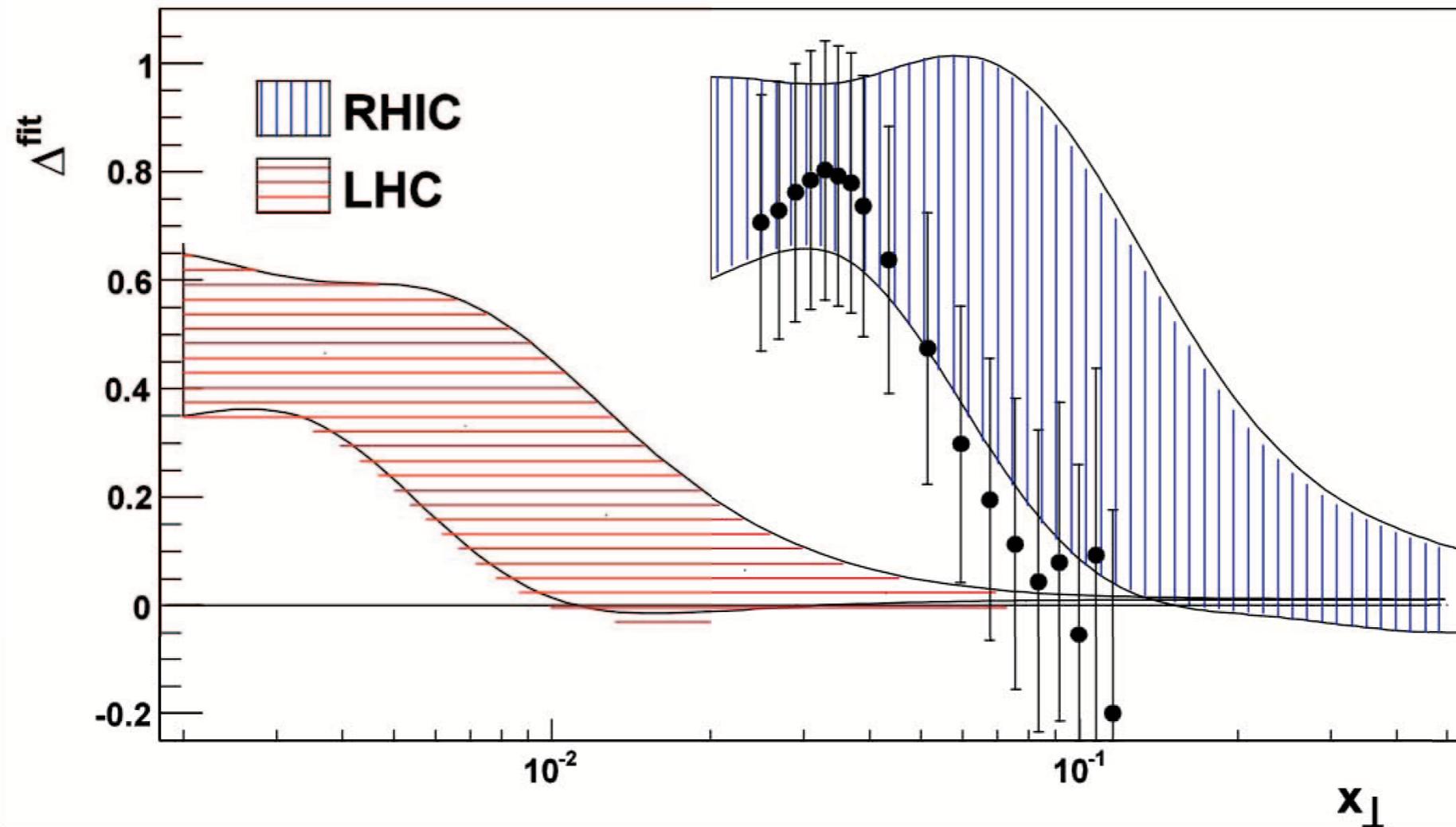
Stan Brodsky

SLAC
 NATIONAL ACCELERATOR LABORATORY

PHENIX results

Scaling exponents from $\sqrt{s} = 500$ GeV preliminary data

[A. Bezilevsky, APS Meeting



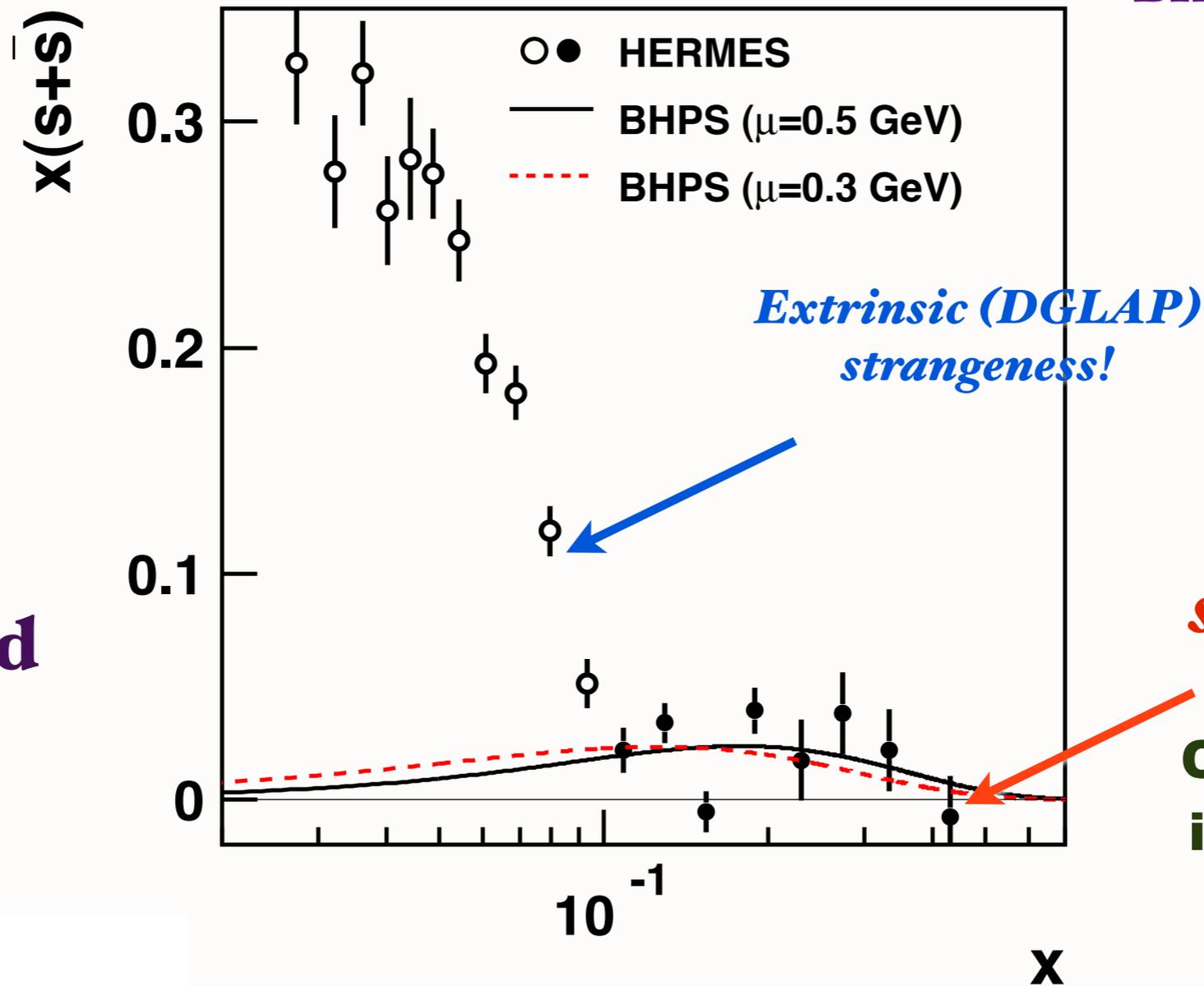
- Magnitude of Δ and its x_{\perp} -dependence consistent with predictions

$$\Delta = n_{\text{expt}} - n_{PQCD}$$

Arleo, Hwang, Sickles, sjb

HERMES: Two components to $s(x, Q^2)$!

BHPS: Hoyer, Sakai,
Peterson, sjb



*Intrinsic
strangeness!*

**Consistent with
intrinsic charm
data**

QCD: $\frac{1}{M_Q^2}$ scaling

Comparison of the HERMES $x(s(x) + \bar{s}(x))$ data with the calculations based on the BHPS model. The solid and dashed curves are obtained by evolving the BHPS result to $Q^2 = 2.5 \text{ GeV}^2$ using $\mu = 0.5 \text{ GeV}$ and $\mu = 0.3 \text{ GeV}$, respectively. The normalizations of the calculations are adjusted to fit the data at $x > 0.1$ with statistical errors only, denoted by solid circles.

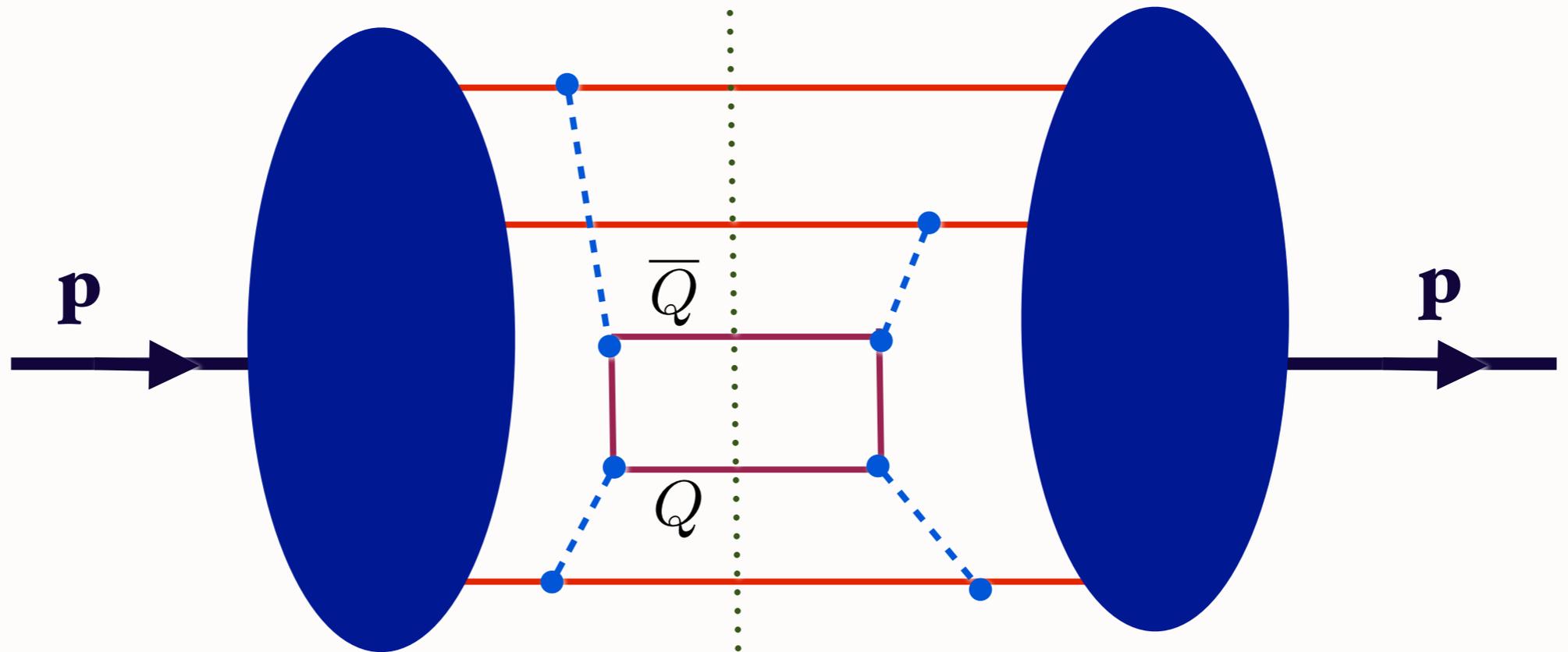
$$s(x, Q^2) = s(x, Q^2)_{\text{extrinsic}} + s(x, Q^2)_{\text{intrinsic}}$$

W. C. Chang and
J.-C. Peng
arXiv:1105.2381

Fixed LF time

$$x_Q \propto (m_Q^2 + k_{\perp}^2)^{1/2}$$

*Proton Self Energy
Intrinsic Heavy Quarks*

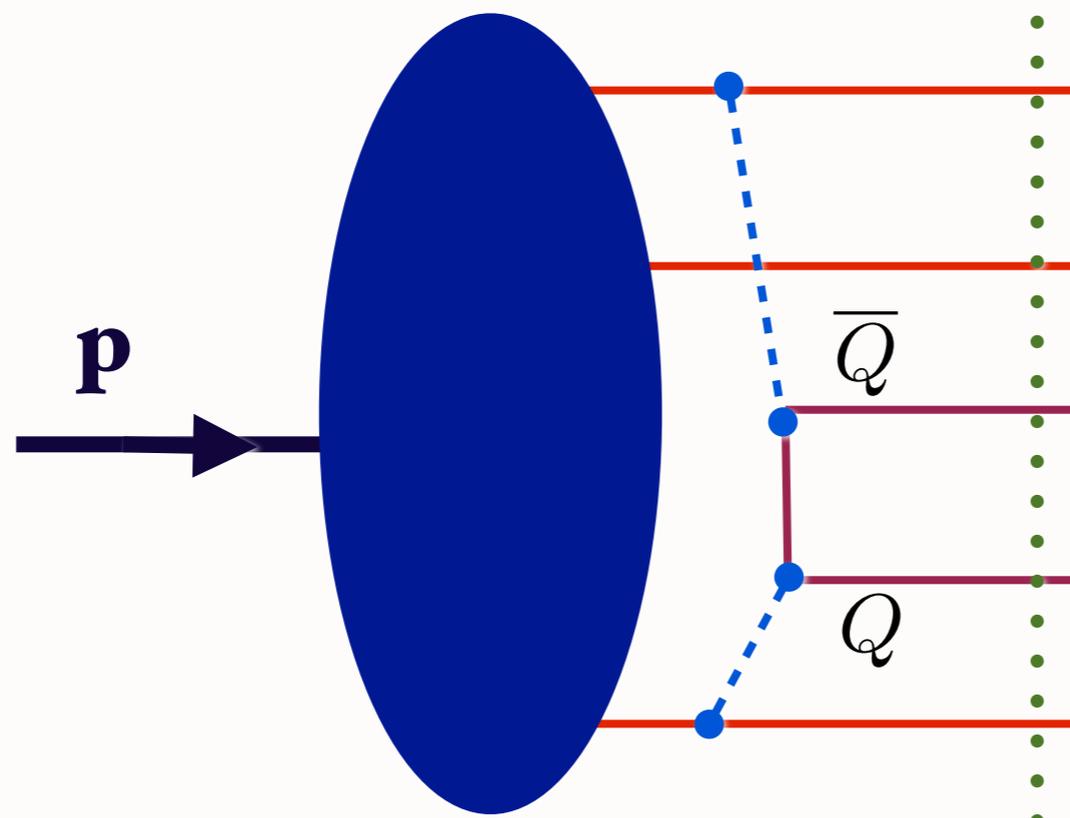


$$\text{Probability (QED)} \propto \frac{1}{M_{\ell}^4}$$

$$\text{Probability (QCD)} \propto \frac{1}{M_Q^2}$$

**Collins, Ellis, Gunion, Mueller, sjb
M. Polyakov, et al.**

*Proton 5-quark Fock State:
Intrinsic Heavy Quarks*



*QCD predicts
Intrinsic Heavy
Quarks at high x !*

Minimal off-shellness

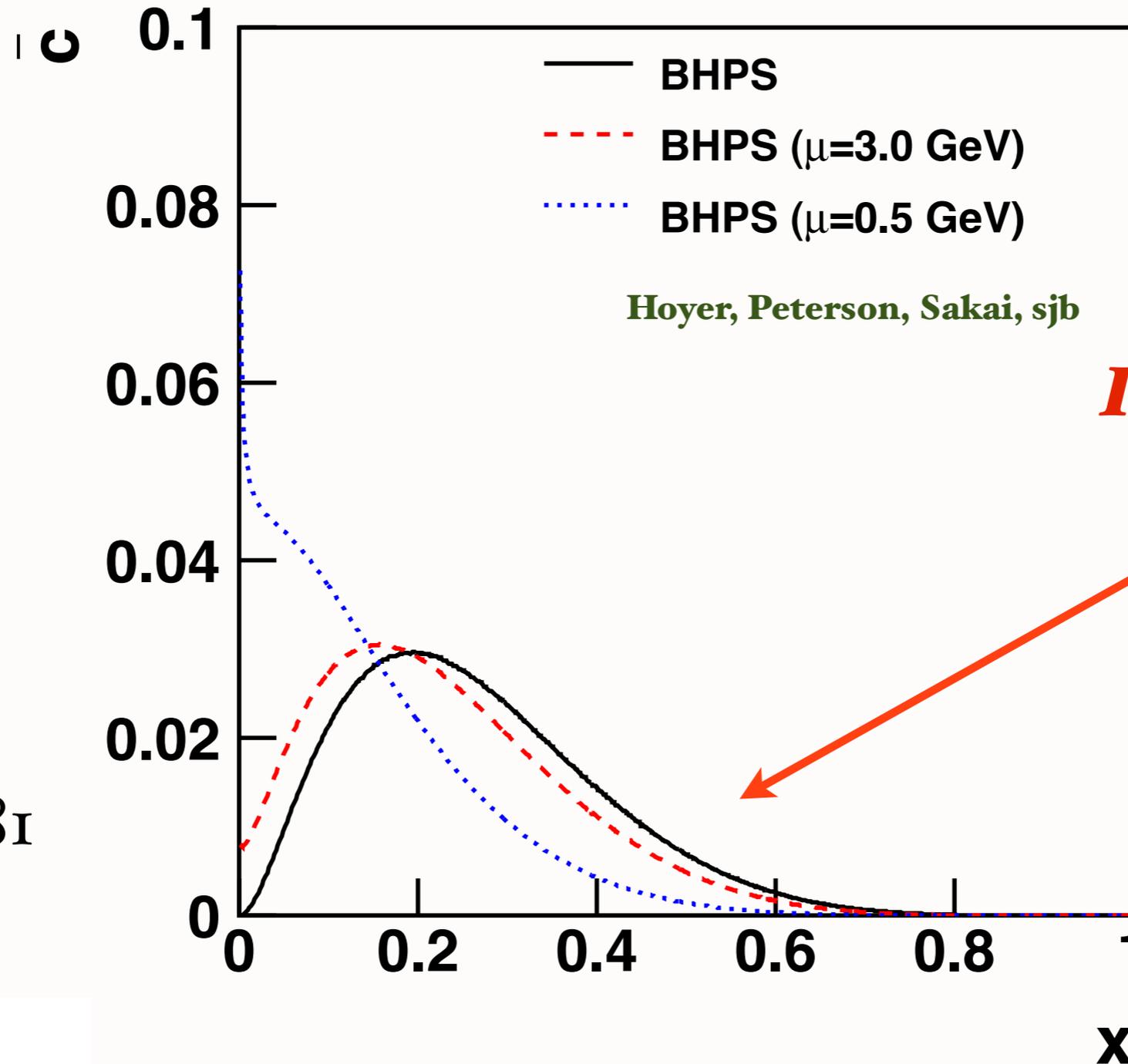
$$x_Q \propto (m_Q^2 + k_{\perp}^2)^{1/2}$$

Probability (QED) $\propto \frac{1}{M_{\ell}^4}$

Probability (QCD) $\propto \frac{1}{M_Q^2}$

**Collins, Ellis, Gunion, Mueller, sjb
Polyakov, et al.**

QCD ($1/m_Q^2$) scaling: predict IC !



W. C. Chang and
J.-C. Peng

arXiv:1105.2381

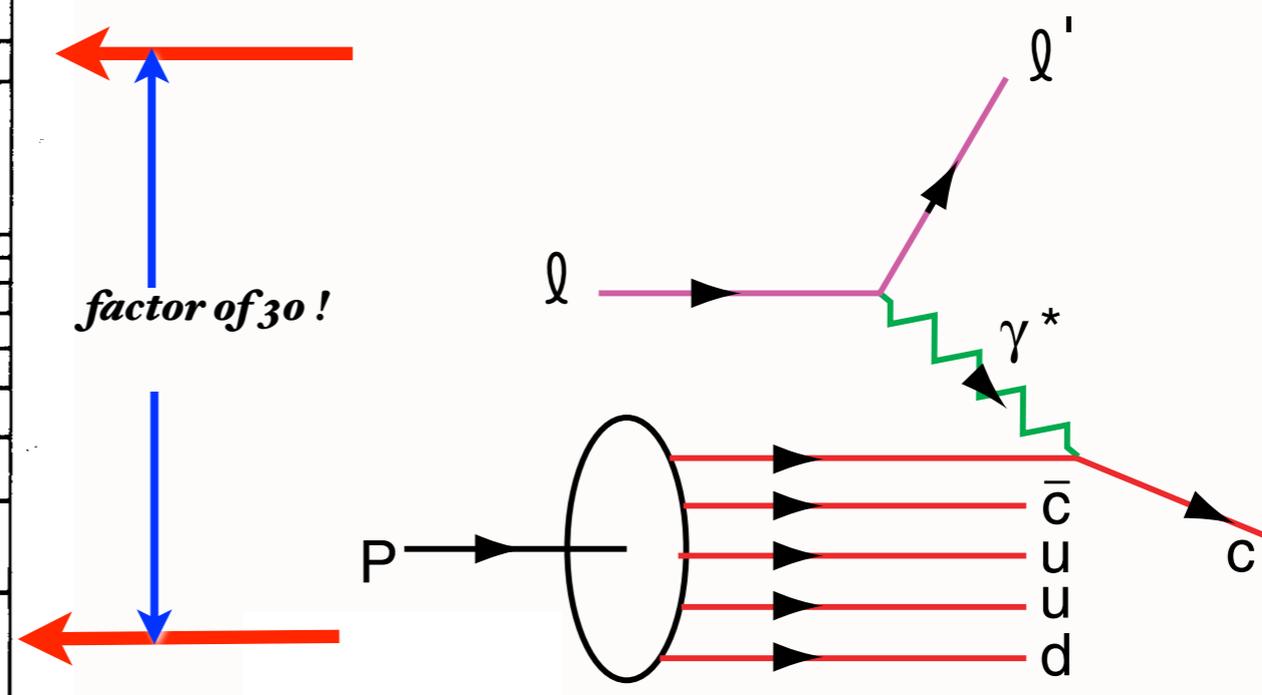
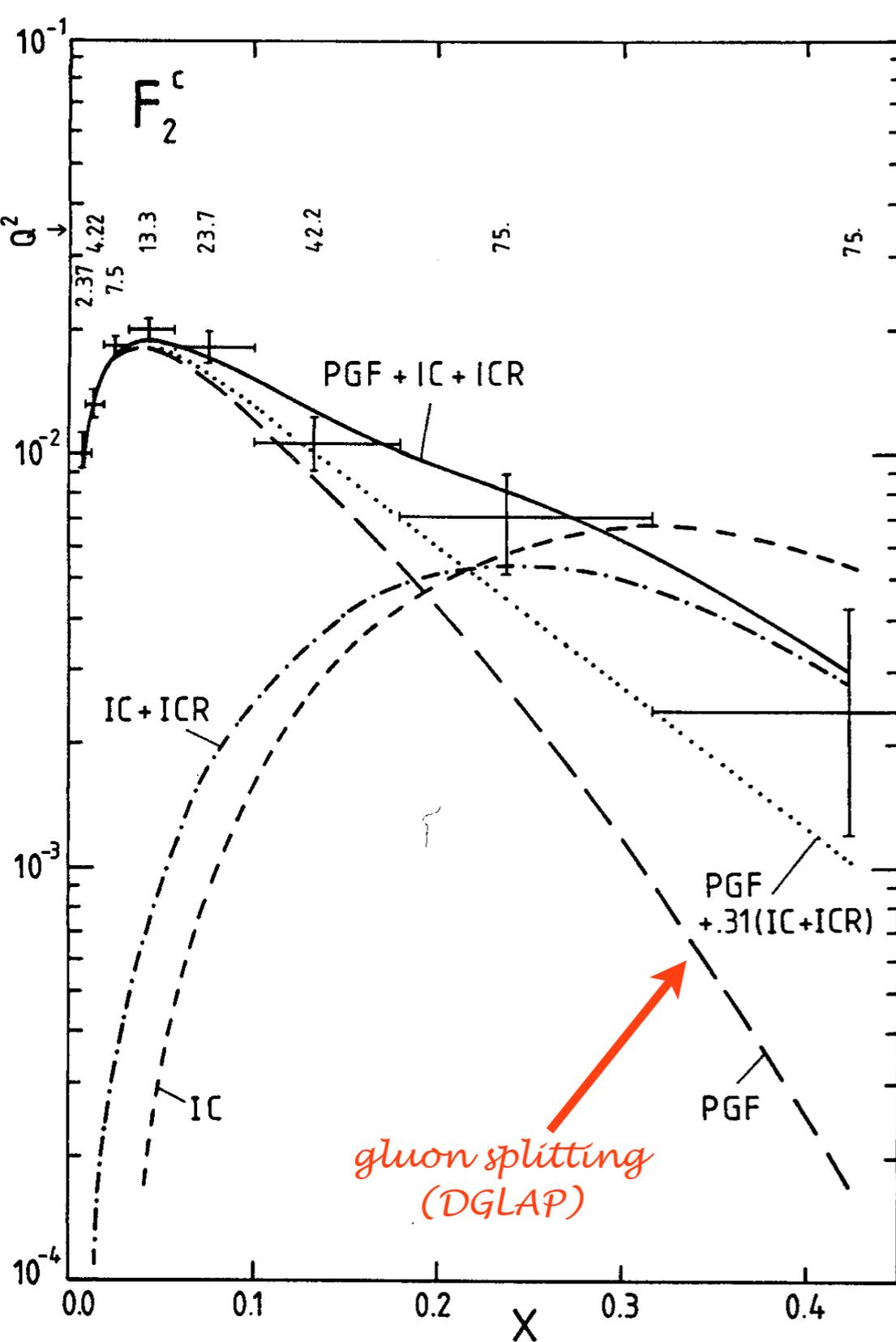
Calculations of the $\bar{c}(x)$ distributions based on the BHPS model. The solid curve corresponds to the calculation using Eq. 1 and the dashed and dotted curves are obtained by evolving the BHPS result to $Q^2 = 75 \text{ GeV}^2$ using $\mu = 3.0 \text{ GeV}$, and $\mu = 0.5 \text{ GeV}$, respectively. The normalization is set at $\mathcal{P}_5^{c\bar{c}} = 0.01$.

Consistent with EMC

Measurement of Charm Structure Function

J. J. Aubert et al. [European Muon Collaboration], "Production Of Charmed Particles In 250-GeV Mu+ - Iron Interactions," Nucl. Phys. B 213, 31 (1983).

First Evidence for Intrinsic Charm Hoyer, Peterson, Sakai, sjb



factor of 30!

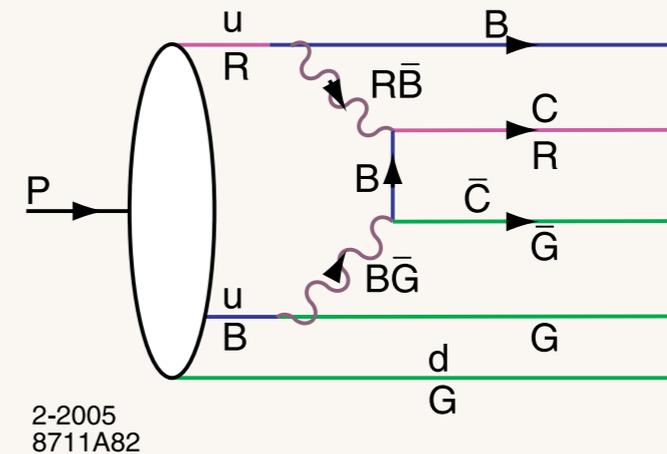
DGLAP / Photon-Gluon Fusion: factor of 30 too small

Two Components (separate evolution):

$$c(x, Q^2) = c(x, Q^2)_{\text{extrinsic}} + c(x, Q^2)_{\text{intrinsic}}$$

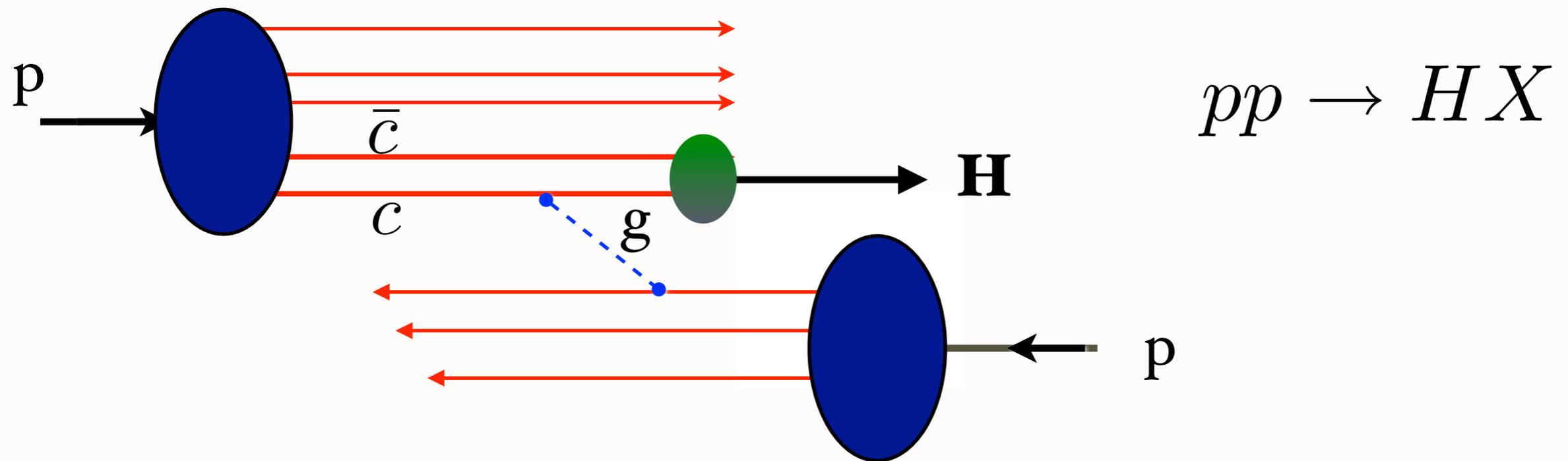
Intrinsic Heavy-Quark Fock States

- Rigorous prediction of QCD, OPE
- Color-Octet Color-Octet Fock State!



- Probability $P_{Q\bar{Q}} \propto \frac{1}{M_Q^2}$ $P_{Q\bar{Q}Q\bar{Q}} \sim \alpha_s^2 P_{Q\bar{Q}}$ $P_{c\bar{c}/p} \simeq 1\%$
- Large Effect at high x
- Greatly increases kinematics of colliders such as Higgs production (Kopeliovich, Schmidt, Soffer, sjb)
- Severely underestimated in conventional parameterizations of heavy quark distributions (Pumplin, Tung)
- Cannot use $c(x, Q^2)$ to determine $g(x, Q^2)$

*Intrinsic Charm Mechanism for Inclusive
High- x_F Higgs Production*



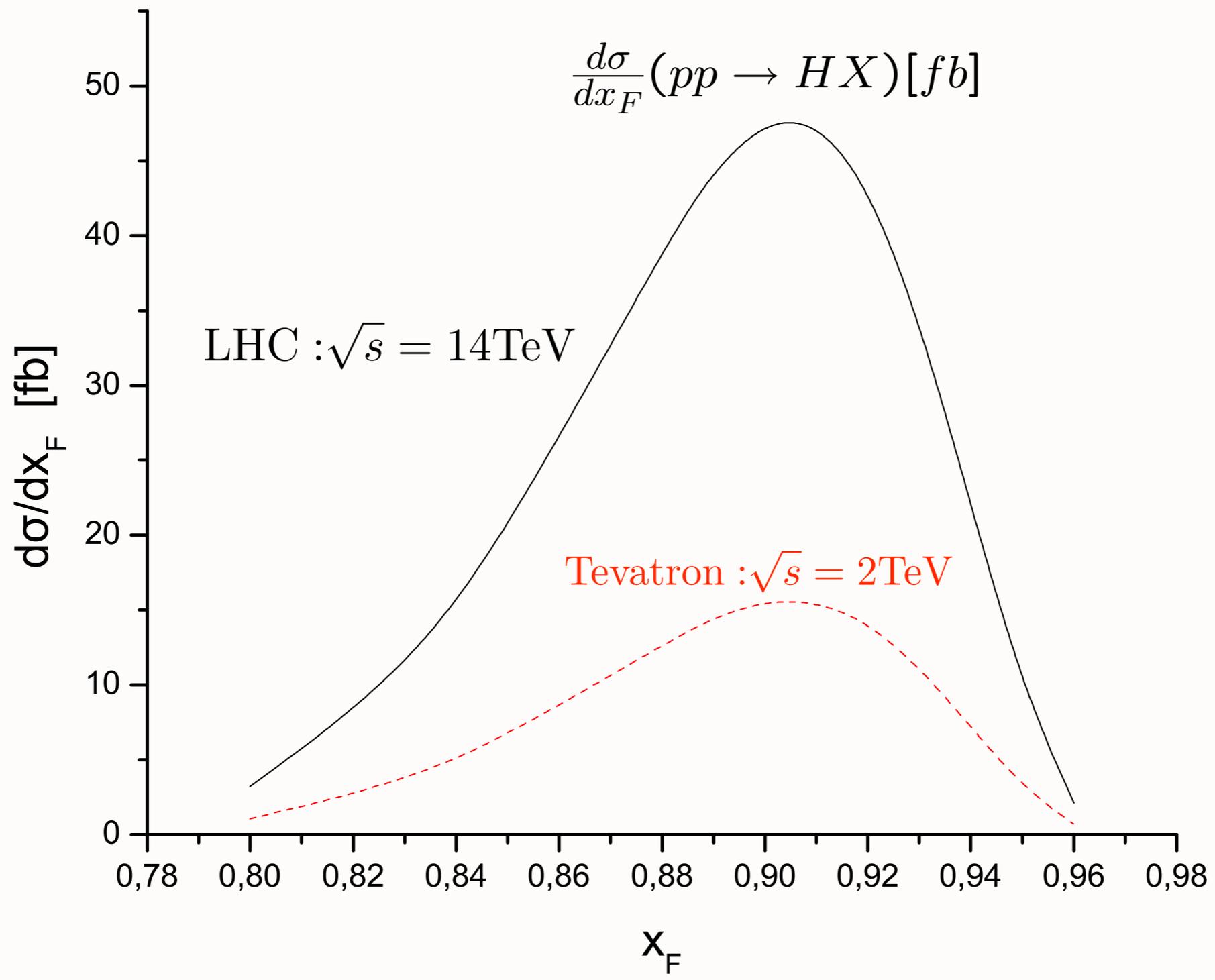
Also: intrinsic strangeness, bottom, top

Higgs can have > 80% of Proton Momentum!

New production mechanism for Higgs

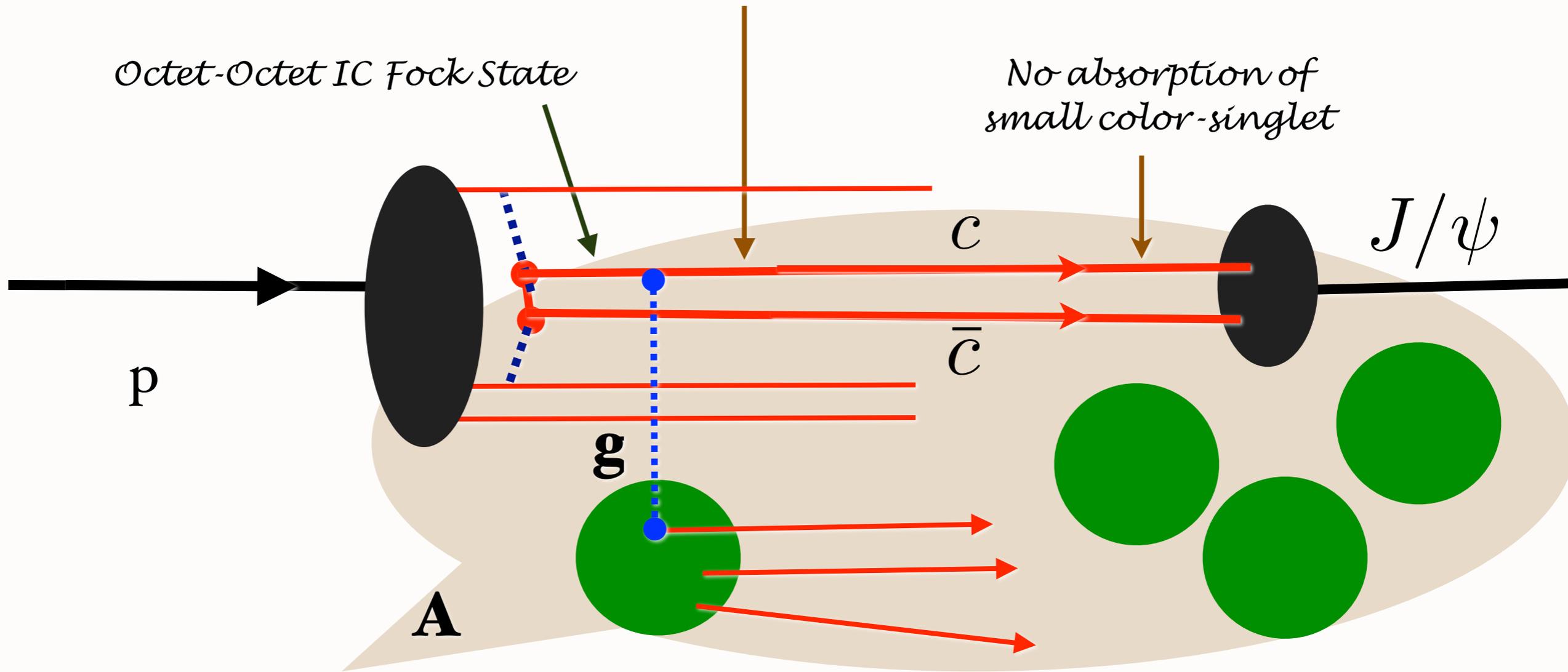
AFTER: Higgs production at threshold!

Intrinsic Heavy Quark Contribution to Inclusive Higgs Production



*Color-Opaque IC Fock state
interacts on nuclear front surface*

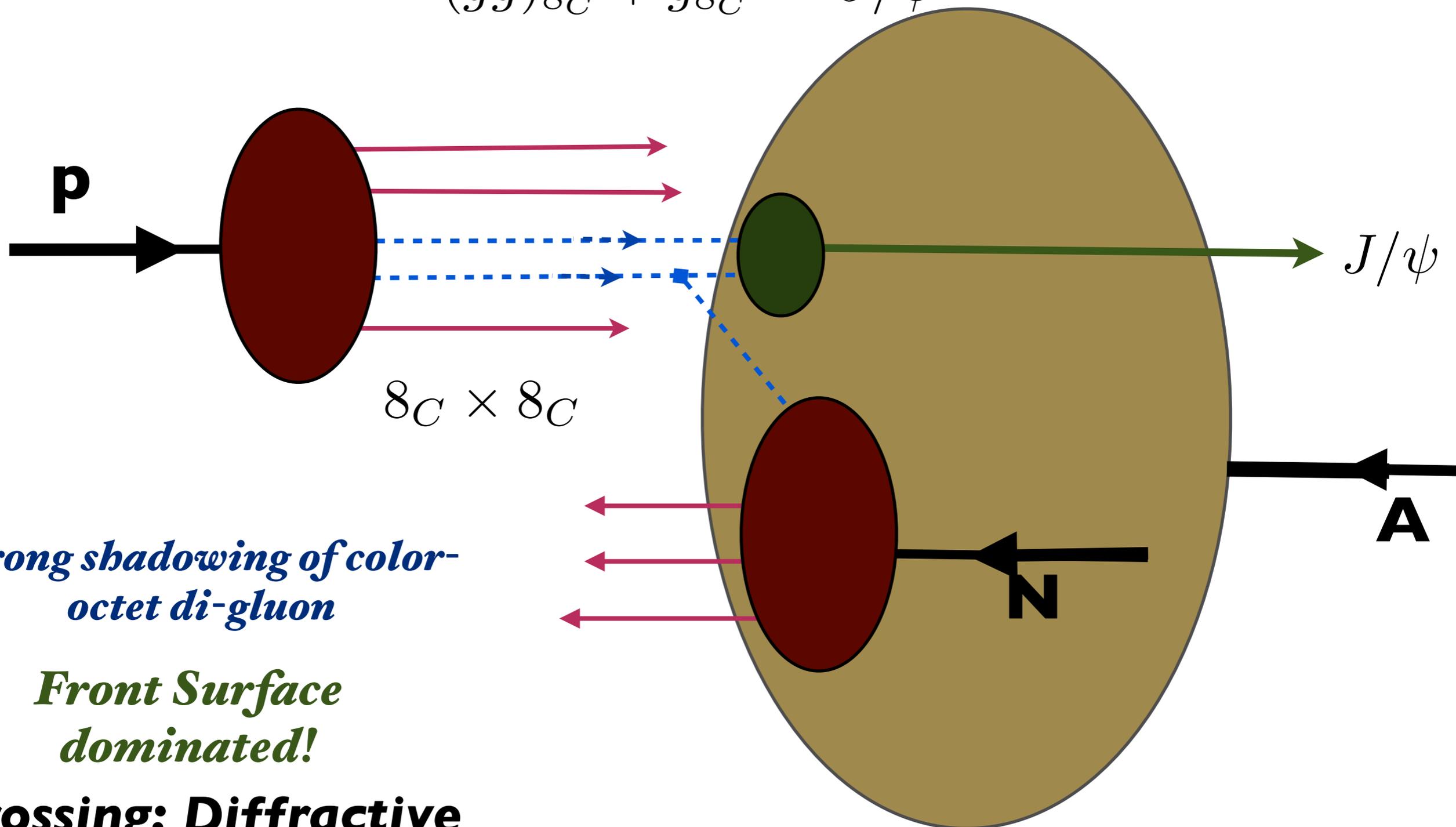
Scattering on front-face nucleon produces color-singlet $c\bar{c}$ pair



$$\frac{d\sigma}{dx_F}(pA \rightarrow J/\psi X) = A^{2/3} \times \frac{d\sigma}{dx_F}(pN \rightarrow J/\psi X)$$

$$pA \rightarrow J/\psi X$$

$$(gg)_{8_C} + g_{8_C} \rightarrow J/\psi$$



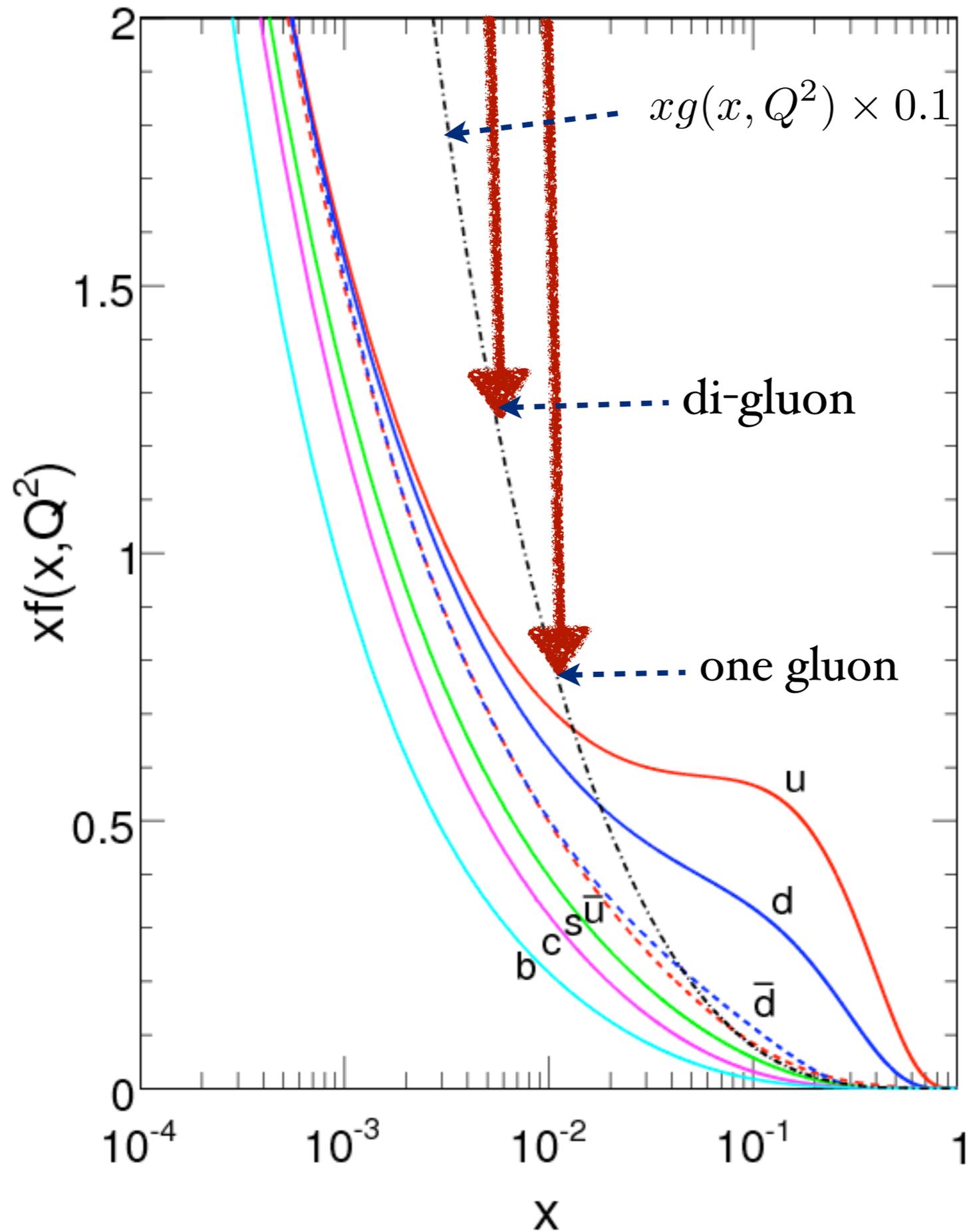
Strong shadowing of color-octet di-gluon

Front Surface dominated!

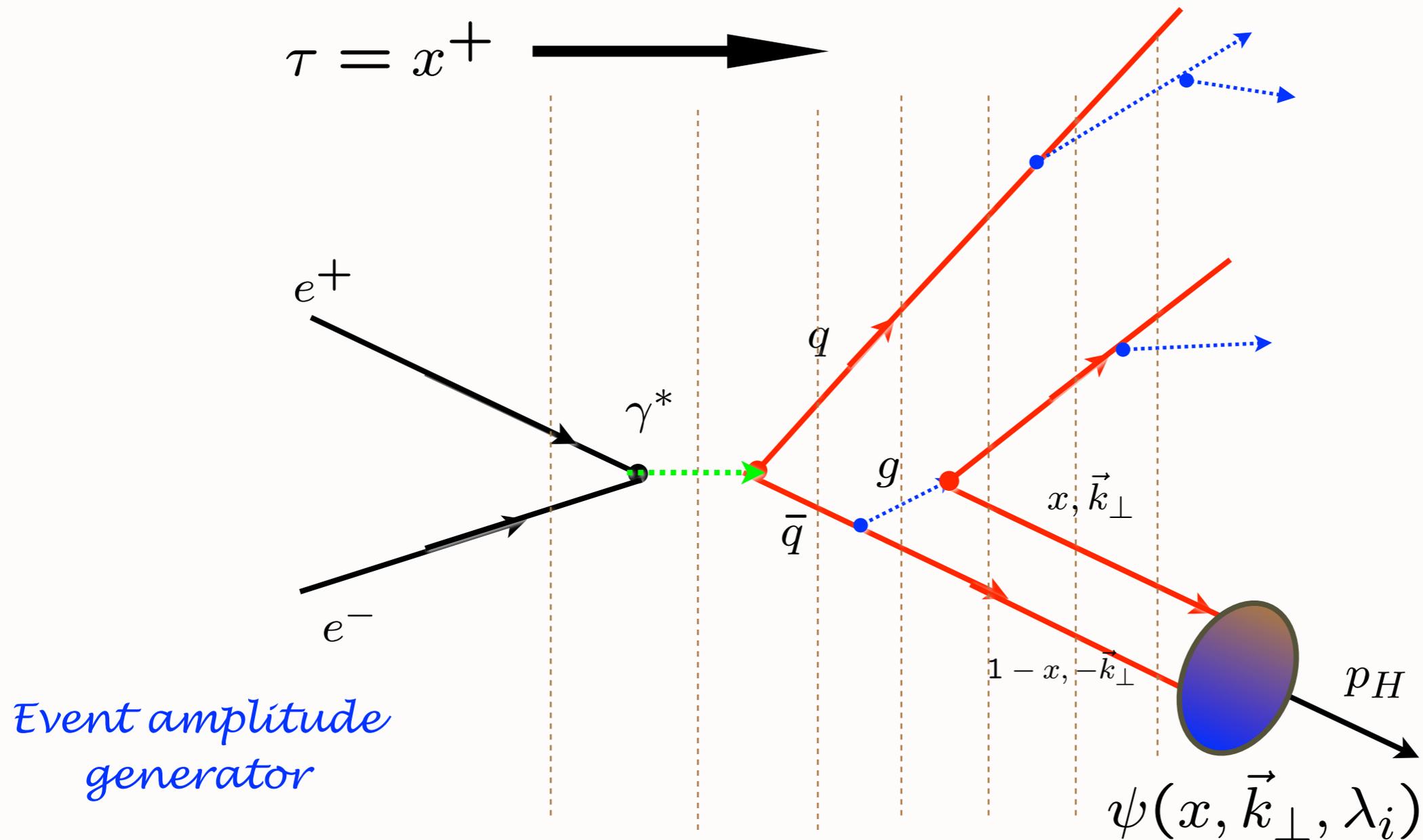
Crossing: Diffractive & pomeron exchange

Double-gluon subprocess

Two gluons at $g(0.005) \sim \frac{13}{0.005} = 2600$ vs. one gluon at $g(0.01) \sim \frac{8}{0.01} = 800$



Hadronization at the Amplitude Level



Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs

QCD Myths

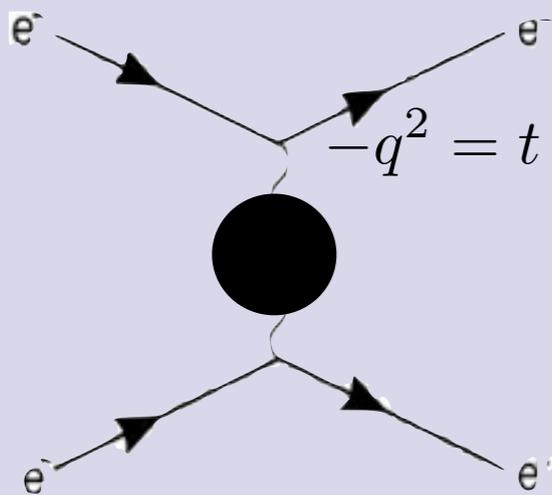
- **Anti-Shadowing is Universal**
- **ISI and FSI are higher twist effects and universal**
- **High transverse momentum hadrons arise only from jet fragmentation -- baryon anomaly!**
- **heavy quarks only from gluon splitting**
- **renormalization scale cannot be fixed**
- **QCD condensates are vacuum effects**
- **Infrared Slavery**
- **Nuclei are composites of nucleons only**
- **Real part of DVCS arbitrary**

Lessons from QED

In the (physical) Gell Mann-Low scheme, the momentum scale of the running coupling is the virtuality of the exchanged photon; independent of initial scale.

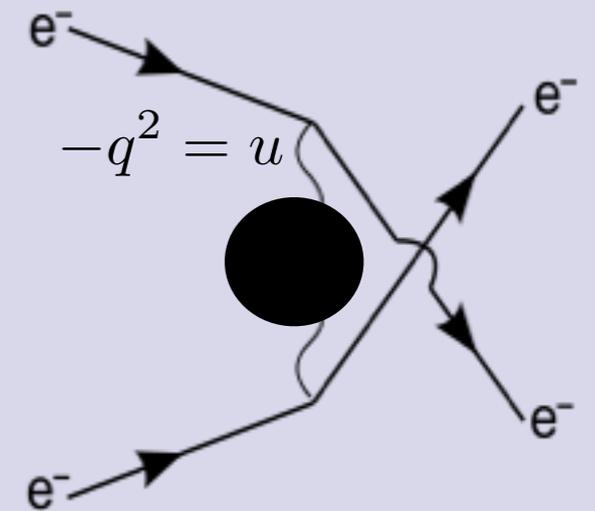
$$\alpha(t) = \frac{\alpha(t_0)}{1 - \Pi(t, t_0)} \quad \Pi(t, t_0) = \frac{\Pi(t) - \Pi(t_0)}{1 - \Pi(t_0)}$$

Example: ee-scattering



$$\mathcal{M}_{ee \rightarrow ee} = \frac{8\pi s}{t} \alpha(t) + \frac{8\pi s}{u} \alpha(u)$$

Two separate scales;
one for each skeleton graph.



For any other scale choice an infinite set of diagrams must be taken into account to obtain the correct result!

In any other scheme, the correct scale displacement must be used

$$\log \frac{\mu_{MS}^2}{m_\ell^2} = 6 \int_0^1 dx x(1-x) \log \frac{m_\ell^2 + Q^2 x(1-x)}{m_\ell^2}, \quad Q^2 \gg m_\ell^2 \rightarrow \log \frac{Q^2}{m_\ell^2} - \frac{5}{3}$$

$$\alpha_{MS}(e^{-5/3} q^2) = \alpha_{GM-L}(q^2).$$

Principle of Maximum Conformality (PMC)

QCD Observables

$$\mathcal{O} = C(\alpha_s(\mu_0^2)) + B(\beta \log \frac{Q^2}{\mu_0^2}) + D(\frac{m_q^2}{Q^2}) + E(\frac{\Lambda_{QCD}^2}{Q^2}) + F(\frac{\Lambda_{QCD}^2}{m_Q^2}) + G(\frac{m_q^2}{m_Q^2})$$

↑
**Scale-Free
Conformal Series**

↖
**Running Coupling
Effects**

↖
**Higher Twist from
Hadron Dynamics**

↖
**Intrinsic Heavy
Quarks**

↑
**Light by Light
Loops**

BLM/PMC: Absorb β -terms into running coupling

$$\mathcal{O} = C(\alpha_s(Q^{*2})) + D(\frac{m_q^2}{Q^2}) + E(\frac{\Lambda_{QCD}^2}{Q^2}) + F(\frac{\Lambda_{QCD}^2}{m_Q^2}) + G(\frac{m_q^2}{m_Q^2})$$

Principle of Maximum Conformality (PMC)

- **Sets pQCD renormalization scale correctly at every finite order**
- **Predictions are scheme-independent**
- **Satisfies all principles of the renormalization group**
- **Agrees with Gell Mann-Low procedure for pQED in Abelian limit**
- **Shifts all β terms into α_s , leaving conformal series**
- **Automatic procedure: R_δ scheme**
- **Number of flavors n_f set**
- **Eliminates $n!$ renormalon growth**
- **Choice of initial scale irrelevant**
- **Eliminates unnecessary systematic error -- conventional guess is scheme-dependent, disagrees with QED**
- **Reduces disagreement with pQCD for top/anti-top asymmetry at Tevatron from 3σ to 1σ**

**Xing-Gang Wu, Martin Mojaza
Leonardo di Giustino, SJB**



Systematic All-Orders Method to Eliminate Renormalization-Scale and Scheme Ambiguities in Perturbative QCD

Matin Mojaza*

*CP3-Origins, Danish Institute for Advanced Studies, University of Southern Denmark, DK-5230 Odense, Denmark
and SLAC National Accelerator Laboratory, Stanford University, Stanford, California 94039, USA*

Stanley J. Brodsky†

SLAC National Accelerator Laboratory, Stanford University, Stanford, California 94039, USA

Xing-Gang Wu‡

*Department of Physics, Chongqing University, Chongqing 401331, People's Republic of China
(Received 13 January 2013; published 10 May 2013)*

We introduce a generalization of the conventional renormalization schemes used in dimensional regularization, which illuminates the renormalization scheme and scale ambiguities of perturbative QCD predictions, exposes the general pattern of nonconformal $\{\beta_i\}$ terms, and reveals a special degeneracy of the terms in the perturbative coefficients. It allows us to systematically determine the argument of the running coupling order by order in perturbative QCD in a form which can be readily automatized. The new method satisfies all of the principles of the renormalization group and eliminates an unnecessary source of systematic error.

δ -Renormalization Scheme (\mathcal{R}_δ scheme)

In dim. reg. $1/\epsilon$ poles come in powers of [Bollini & Gambiagi, 't Hooft & Veltman, '72]

$$\ln \frac{\mu^2}{\Lambda^2} + \frac{1}{\epsilon} + c$$

In the **modified minimal subtraction** scheme ($\overline{\text{MS}}$ -bar) one subtracts together with the pole a constant [Bardeen, Buras, Duke, Muta (1978) on DIS results]:

$$\ln(4\pi) - \gamma_E$$

This corresponds to a shift in the scale:

$$\mu_{\overline{\text{MS}}}^2 = \mu^2 \exp(\ln 4\pi - \gamma_E)$$

A finite subtraction from infinity is arbitrary. *Let's make use of this!*

Subtract an arbitrary constant and keep it in your calculation: \mathcal{R}_δ -scheme

$$\ln(4\pi) - \gamma_E - \delta,$$

$$\mu_\delta^2 = \mu_{\overline{\text{MS}}}^2 \exp(-\delta) = \mu^2 \exp(\ln 4\pi - \gamma_E - \delta)$$

Exposing the Renormalization Scheme Dependence

Observable in the \mathcal{R}_δ -scheme:

$$\rho_\delta(Q^2) = r_0 + r_1 a(\mu) + [r_2 + \beta_0 r_1 \delta] a(\mu)^2 + [r_3 + \beta_1 r_1 \delta + 2\beta_0 r_2 \delta + \beta_0^2 r_1 \delta^2] a(\mu)^3 + \dots$$

$$\mathcal{R}_0 = \overline{\text{MS}}, \quad \mathcal{R}_{\ln 4\pi - \gamma_E} = \text{MS} \quad \mu^2 = \mu_{\overline{\text{MS}}}^2 \exp(\ln 4\pi - \gamma_E), \quad \mu_{\delta_2}^2 = \mu_{\delta_1}^2 \exp(\delta_2 - \delta_1)$$

Note the divergent 'renormalon series' $n! \beta^n \alpha_s^n$

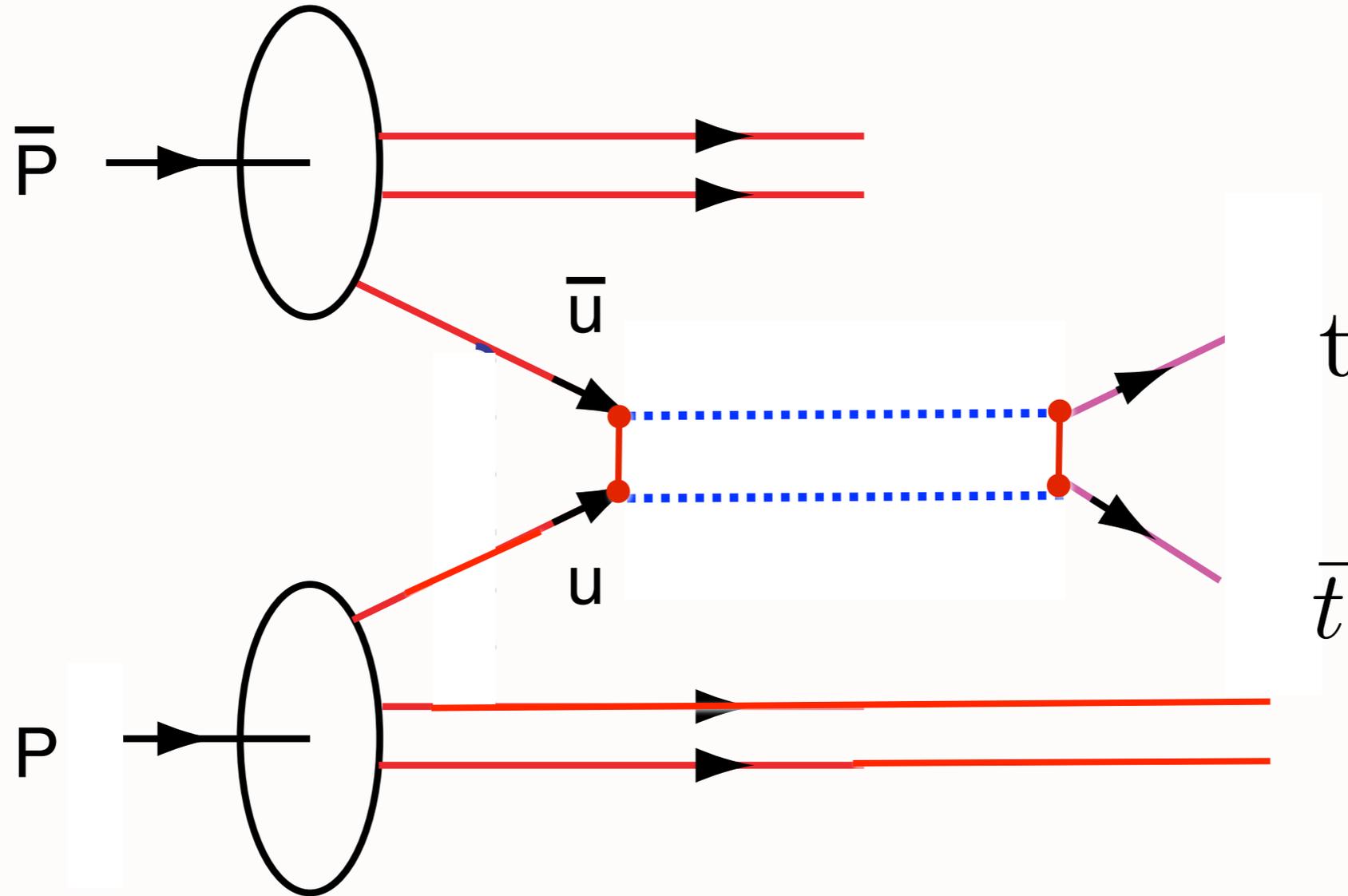
Renormalization Scheme Equation

$$\frac{d\rho}{d\delta} = -\beta(a) \frac{d\rho}{da} \stackrel{!}{=} 0 \quad \longrightarrow \text{PMC}$$

$$\rho_\delta(Q^2) = r_0 + r_1 a_1(\mu_1) + (r_2 + \beta_0 r_1 \delta_1) a_2(\mu_2)^2 + [r_3 + \beta_1 r_1 \delta_1 + 2\beta_0 r_2 \delta_2 + \beta_0^2 r_1 \delta_1^2] a_3(\mu_3)^3$$

The $\delta_k^p a^n$ -term indicates the term associated to a diagram with $1/\epsilon^{n-k}$ divergence for any p . Grouping the different δ_k -terms, one recovers in the $N_c \rightarrow 0$ Abelian limit the dressed skeleton expansion.

Contributes to the $\bar{p}p \rightarrow \bar{t}tX$ asymmetry at the Tevatron

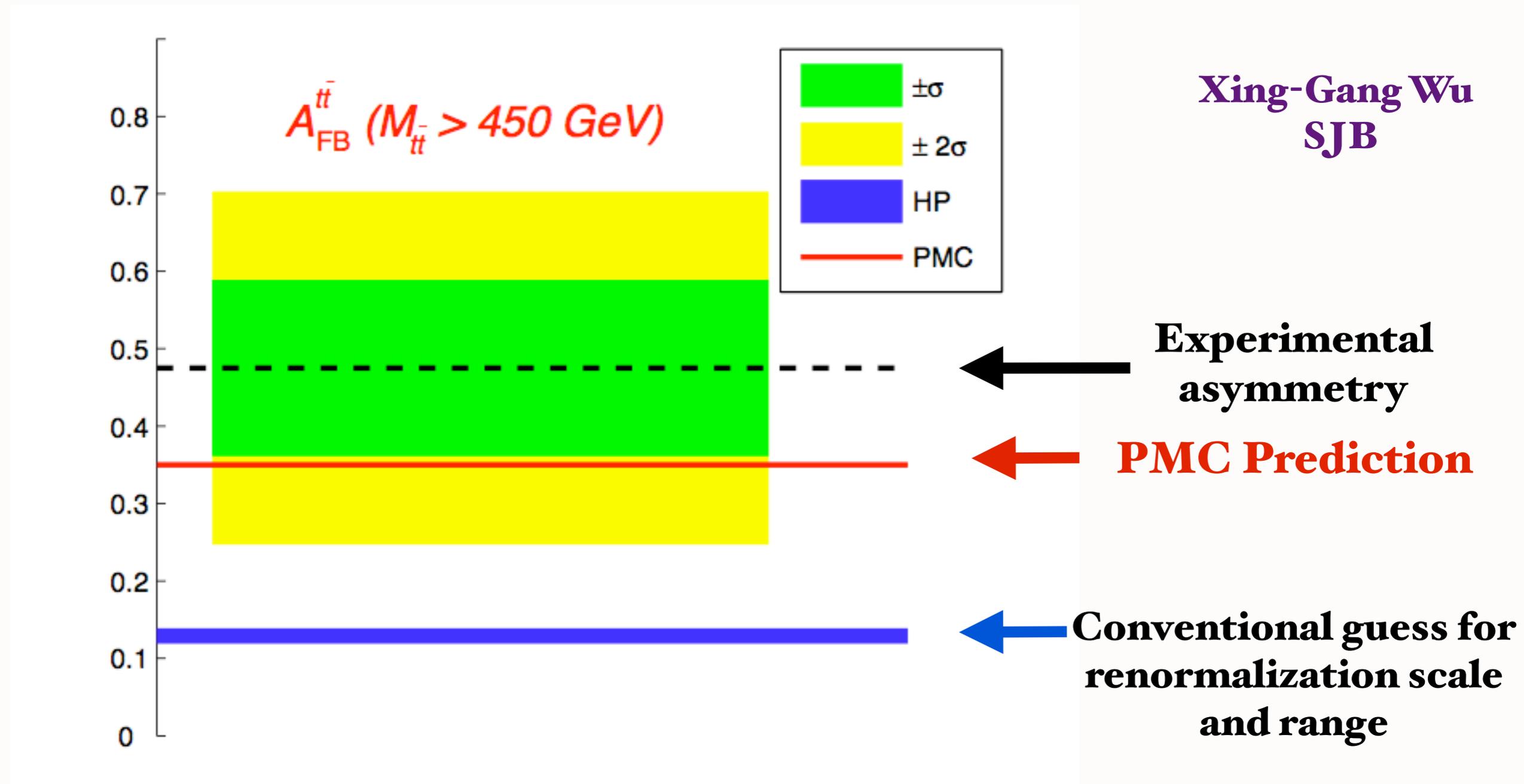


Interferes with Born term.

Small value of renormalization scale increases asymmetry

Xing-Gang Wu, sjb

The Renormalization Scale Ambiguity for Top-Pair Production Eliminated Using the 'Principle of Maximum Conformality' (PMC)



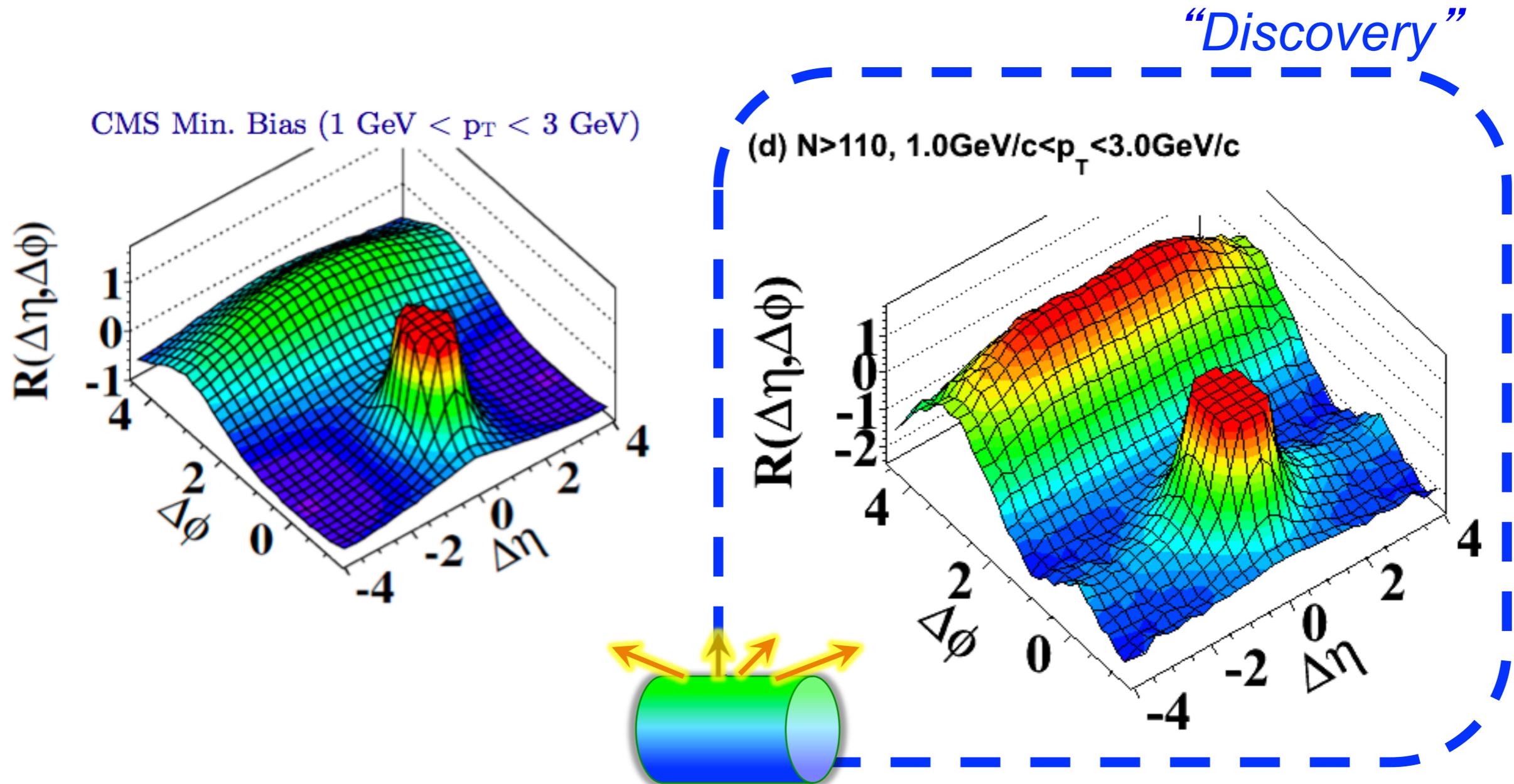
Top quark forward-backward asymmetry predicted by pQCD NNLO within 1σ of CDF/D0 measurements using PMC/BLM scale setting

Conformal Template

- **Self-Consistent breaking of scale invariance--Unique Confining Potential and Dilaton**
- **Non-Perturbative QCD Running Coupling**
- **Principle of Maximum Conformality -- sets renormalization scale in PQCD -- result is scheme independent!**
- **ERBL evolution and eigensolutions**

Frishman, Sachrajda, Lepage, sjb; Braun

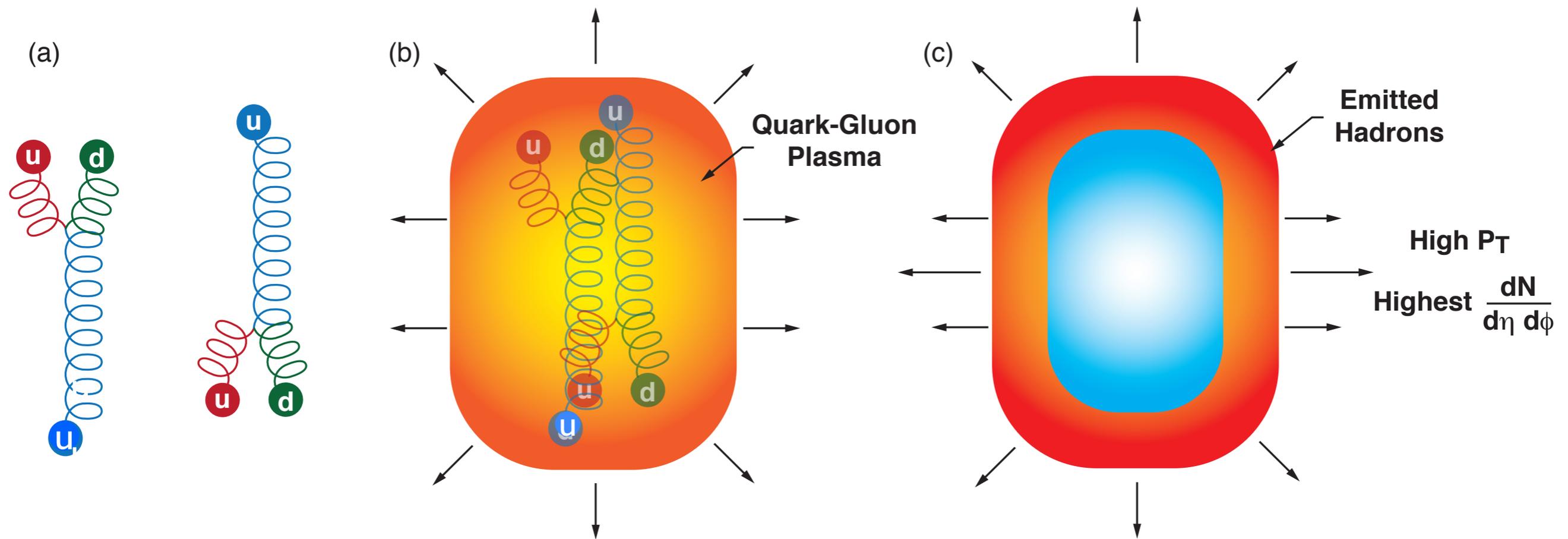
Two particle correlations: CMS results



- ◆ Ridge: Distinct long range correlation in η collimated around $\Delta\Phi \approx 0$ for two hadrons in the intermediate $1 < p_T, q_T < 3 \text{ GeV}$

Possible origin of same-side CMS ridge in p p Collisions

Bjorken, Goldhaber, sjb



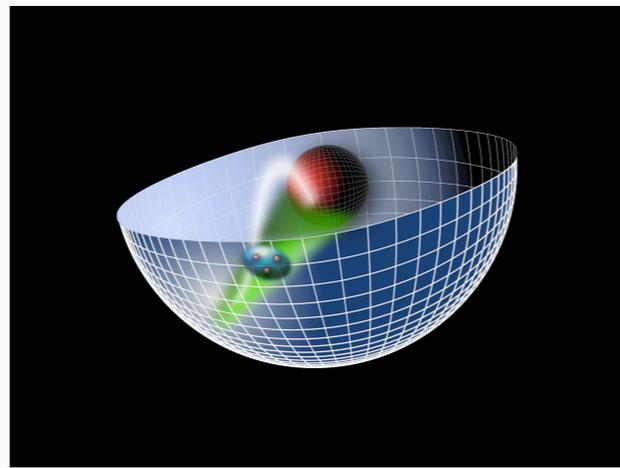
$$\vec{V} = \sum_{i=1}^N [\cos 2\phi_i \hat{x} + \sin 2\phi_i \hat{y}]$$

Possible multiparticle ridge-like correlations in very high multiplicity proton-proton collisions

Bjorken, Goldhaber, sjb

We suggest that this “ridge”-like correlation may be a reflection of the rare events generated by the collision of aligned flux tubes connecting the valence quarks in the wave functions of the colliding protons.

The “spray” of particles resulting from the approximate line source produced in such inelastic collisions then gives rise to events with a strong correlation between particles produced over a large range of both positive and negative rapidity.



*AdS/QCD
Soft-Wall Model*

Light-Front Holography

$$\zeta^2 = x(1-x)b_{\perp}^2.$$

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta) \right] \psi(\zeta) = \mathcal{M}^2 \psi(\zeta)$$



Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2(L + S - 1)$$

$$\kappa \simeq 0.6 \text{ GeV}$$

Confinement scale:

$$1/\kappa \simeq 1/3 \text{ fm}$$

***Unique
Confinement Potential!
Conformal Symmetry
of the action***

● **de Alfaro, Fubini, Furlan:**

**Scale can appear in Hamiltonian and EQM
without affecting conformal invariance of action!**

AdS/QCD and Light-Front Holography

- **Light-Front Holography**
- **LF Schrödinger Equation**
- **Color Confinement -- Unique Potential, Unique dilaton**
- **Single scheme-independent mass scale $\kappa \sim 0.6 \text{ GeV}$**
- **Retains conformal invariance of chiral QCD action**
- **Condensates -- A new view**
- **QCD: Zero contribution to the Cosmological Constant**

AdS/QCD and Light-Front Holography

$$\mathcal{M}_{n,J,L}^2 = 4\kappa^2 \left(n + \frac{J+L}{2} \right)$$

- **Zero mass pion for $m_q = 0$ ($n=J=L=0$)**
- **Regge trajectories: equal slope in n and L**
- **Form Factors at high Q^2 : Dimensional counting** $[Q^2]^{n-1} F(Q^2) \rightarrow \text{const}$
- **Space-like and Time-like Meson and Baryon Form Factors**
- **Running Coupling for NPQCD** $\alpha_s(Q^2) \propto e^{-\frac{Q^2}{4\kappa^2}}$
- **Meson Distribution Amplitude** $\phi_\pi(x) \propto f_\pi \sqrt{x(1-x)}$

An analytic first approximation to QCD

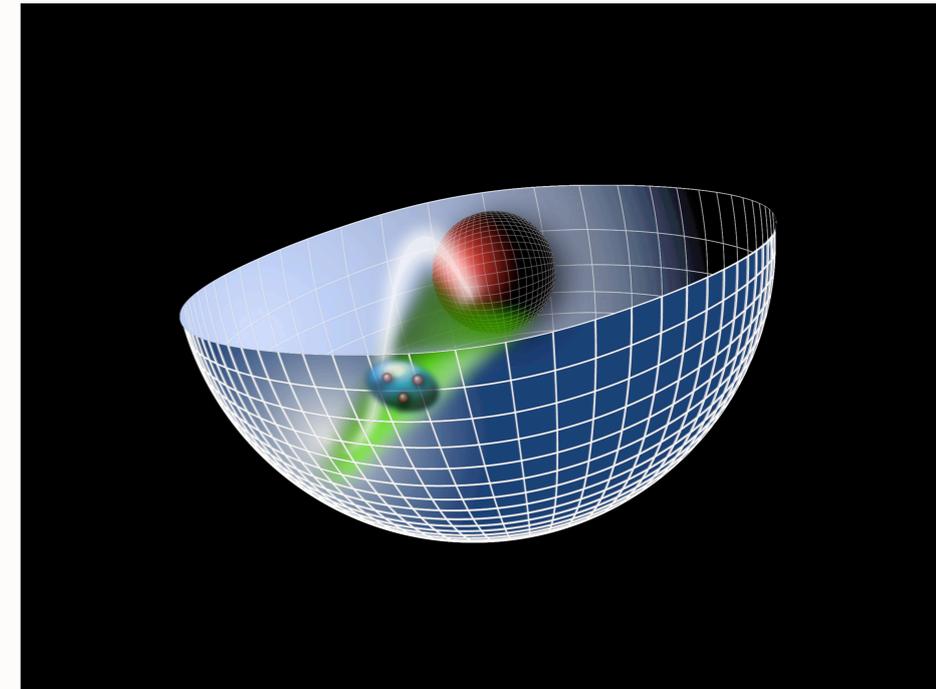
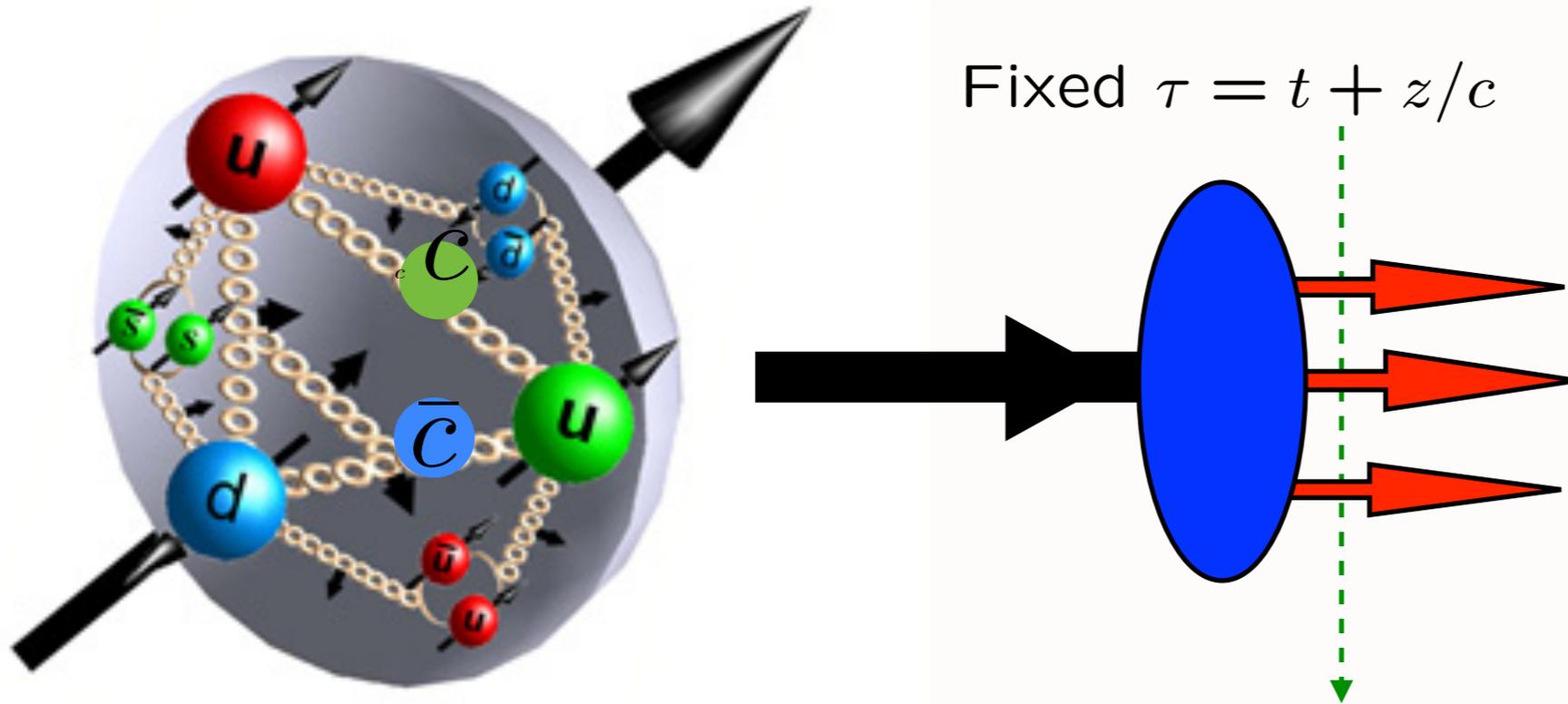
AdS/QCD + Light-Front Holography

- **As Simple as Schrödinger Theory in Atomic Physics**
- **LF radial variable ζ conjugate to invariant mass squared**
- **Relativistic, Frame-Independent, Color-Confining**
- **Unique confining potential!**
- **QCD Coupling at all scales: Essential for Gauge Link phenomena**
- **Hadron Spectroscopy and Dynamics from one parameter**
- **Wave Functions, Form Factors, Hadronic Observables, Constituent Counting Rules**
- **Insight into QCD Condensates: Zero cosmological constant!**
- **Systematically improvable with DLCQ-BLFQ Methods**

New Perspectives for QCD

- **Light-Front QCD and Holography**
- **Unique Color Confinement Potential**
- **Principle of Maximal Conformality**
- **Non-Universal Anti-Shadowing and other Novel Nuclear Effects**
- **Lensing effects and Factorization Breaking**
- **Direct and multiparton Processes**
- **Heavy Quark Distributions and Novel Higgs Production Mechanisms**
- **Ridge Correlations at the LHC**
- **The QCD Vacuum and the Cosmological Constant**

Novel QCD Phenomena and New Perspectives for Hadron Physics from Light-Front Holography



Stan Brodsky



High Energy Physics
in the LHC Era

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December 16-20, 2013

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