Light Front Wave Functions inspired by holography

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Outline

Introduction

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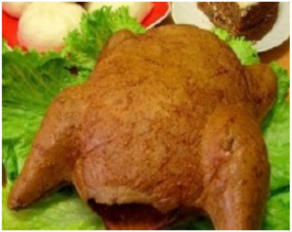
Applicability to QCD.

- One can take Gauge / Gravity as a motivating frame to build phenomenological models in extra dimensions that reproduce some features of QCD.



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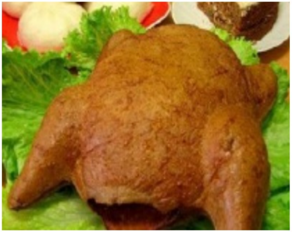

- Within the phenomenological models used recently in hadronic physics, some are based on the gauge/gravity duality.
- They suppose the existence of a gravity theory dual to QCD, and are divided into two classes, the top-down and the bottom-up approach.
- In turn, these last ones are divided into hard wall and soft wall models, depending on the way conformal invariance in the AdS side is broken.
- The bottom-up soft wall models have proven to be quite useful because of their simplicity and variety of successful applications.

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Examples of Gauge / Gravity applied to QCD problems.

- Hadron Spectroscopy. (Brodsky and de Teramond; Forkel, Beyer and Frederico; De Paula and Frederico; A.V, Schmidt, Gutsche and Lyubovitskij).
- Chiral symmetry breaking mechanism and light mesons. (Karch, Katz, Son and Stephanov; Da Rold and Pomarol).
- Wilson loop and Heavy quark potentials. (Maldacena; Sonnenschein; Jugeau).
- Barions. (Sakai and Sugimoto).
- Deep Inelastic Scattering. (Polchinski and Strassler; Braga and A.V).
- Hadronic wave functions. (Brodsky and de Teramond; A.V, Schmidt, Gutsche and Lyubovitskij; Chabysheva and Hiller; Forshaw and Sandapen).
- Form Factors and Generalized Parton Distribution Functions. (Abidin and Carlson; A.V, Schmidt, Gutsche and Lyubovitskij).

⋆ **Gauge / Gravity Dictionary.**

This tell us how are related elements involved in both sides of Gauge / Gravity duality.

Table: Summary of dictionary considered here.

QCD (4d)	Gravity (5d)
Operator ($\mathcal{O}$)	Normalizable Modes (Φ)
Hadron Mass (M)	Eigenvalues of Φ
Twist Dimension ([$\mathcal{O}$] - S)	Conformal Dimension (Δ)
Wave Function	Normalizable Modes (Φ) [1]

[1] Brodsky and de Teramond, Phys.Rev.Lett.96:201601,2006 (hep-ph/0602252), Phys.Rev.D77:056007,2008 (arXiv:0707.3859).

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Holographic Light Front Wave Functions

Comparison of Form Factors in light front and in AdS side, offer us a possibility to relate AdS modes that describe hadrons with LFWF.

- In Light Front

$$F(q^2) = 2\pi \int_0^1 dx \frac{(1-x)}{x} \int d\zeta \; \zeta J_0(\zeta q \sqrt{\tfrac{1-x}{x}}) \widetilde{\rho}(x,\zeta),$$

- In AdS

$$F(q^2) = \int_0^\infty dz \; \Phi(z) J(q^2, z) \Phi(z),$$

where $\Phi(z)$ correspond to AdS modes that represent hadrons, $J(q^2, z)$ it is dual to electromagnetic current, and usually an AdS metric is considered.

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Notice that if you consider $z = \zeta$, and if you writey si escribimos $J(q^2, z)$ as

$$J(q^2, \zeta) = \int_0^1 dx \; g(x) J_0(\zeta q \sqrt{\tfrac{1-x}{x}}),$$

(that it is possible in most popular soft wall model), then, comparing the previous form factors we get

$$\widetilde{\rho}(x, \zeta) = \frac{1}{2\pi} \frac{1}{\zeta} \frac{x}{(1-x)} g(x) |\Phi(\zeta)|^2.$$

Additionally if we consider the two parton case, where

$$\widetilde{\rho}(x, \zeta) = \frac{|\widetilde{\psi}(x,\zeta)|^2}{(1-x)^2},$$

where $\zeta^2 = x(1-x)\mathbf{b}_\perp^2$, it is possible to relate AdS model and mesonic LFWF $\widetilde{\psi}(x, \mathbf{b}_\perp)$.

So finally you can get

$$|\widetilde{\psi}(x,\zeta)|^2 = A\frac{1}{\zeta}x(1-x)g(x)|\Phi(\zeta)|^2,$$

where A is a normalization constant

Considering a soft wall model with cuadratic dilaton, Brodsky and de Teramond found

$$\psi(x,\mathbf{b}_\perp) = A\sqrt{x(1-x)}\; e^{-\frac{1}{2}\kappa^2 x(1-x)\mathbf{b}_\perp^2}.$$

Light Front Wave Functions Inspired by Holography

◇ **Summary of AdS / QCD wavefunction.**

The meson wavefunction can be written in the following factorized form

$$\psi(x,\zeta,\varphi) = \frac{\Phi(\zeta)}{\sqrt{2\pi\zeta}} f(x) e^{iL\varphi}$$

where L is the orbital quantum number, $\zeta^2 = x(1-x)b^2$, and the function $\Phi(\zeta)$ satisfies a Schrödinger like equation

$$(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta))\Phi(\zeta) = M^2\Phi(\zeta),$$

where for spin-J AdS mode in a soft wall model (with cuadratic dilaton and AdS metric)

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(J-1)$$

and

$$M^2 = 4\kappa^2(n + \frac{J}{2} + \frac{L}{2}).$$

◇ **Inclusion of massive quarks**[2].

$$\psi(x, \mathbf{b}_\perp) = A\sqrt{x(1-x)}\; e^{-\frac{1}{2}\kappa^2 x(1-x)\mathbf{b}_\perp^2}.$$

in momentum space

$$\psi_{q_1\bar{q}_2}(x, k) = \frac{4\pi A}{\kappa\sqrt{x(1-x)}} \exp\left(-\frac{k^2}{2\kappa_1^2 x(1-x)}\right).$$

Introduction of massive quarks:

$$\frac{k^2}{x(1-x)} \to \frac{k^2}{x(1-x)} + m_{12}^2 \quad , \quad m_{12}^2 = \frac{m_1^2}{x} + \frac{m_2^2}{1-x}.$$

Note that the change proposed is equivalent to the following change of the kinetic term in the equation of motion.

$$-\frac{d^2}{d\zeta^2} \to -\frac{d^2}{d\zeta^2} + m_{12}^2.$$

[2] S. J. Brodsky and G. F. de Teramond, arXiv:0802.0514 [hep-ph].

◇ **Chabysheva and Hiller extension**[3]**.**

$$M^2 \rightarrow M^2 - M_{||}^2$$

with a longitudinal equation

$$\left(\frac{m_1^2}{x} + \frac{m_2^2}{1-x} + U_{||}\right) f(x) = M_{||}^2 f(x).$$

considering a potential taken from t'Hooft model, $f(x)$ can be approximated by $x^{\beta_1}(1-x)^{\beta_2}$.

◇ **Forshaw and Sandapen extension**[4]**.**

Considering a parametrization that accommodates both the AdS / QCD and the boosted Gaussian wave function:

$$\psi(x, \zeta, \varphi) = N\,[x(1-x)]^{\beta} e^{-\frac{1}{2}\kappa^2\zeta^2} e^{-\frac{m^2}{2\kappa^2 x(1-x)}}$$

[3] S. Chabysheva and J. Hiller, Annals of Physics 337 (2013) 143 - 152.

[4] J. Forshaw and R. Sandapen, PRL 109, 081601 (2012).

◇ A different extension[5].

$$(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta) + \frac{m_1^2}{x} + \frac{m_2^2}{1-x})\Phi(\zeta) = M^2\Phi(\zeta),$$

$$\psi(x,\zeta,\varphi) = \frac{\Phi(\zeta)}{\sqrt{2\pi\zeta}}\sqrt{x(1-x)}f(x,m_1,m_2)e^{iL\varphi} \quad with \quad f(x,m_1,m_2) = N\, x^{\alpha_1}(1-x)^{\alpha_2}$$

then

$$M_{nJ}^2 = 4\kappa^2(n + \frac{L+J}{2}) + \int_0^1 (\frac{m_1^2}{x} + \frac{m_2^2}{1-x})\; f^2(x,m_1,m_2)\; dx$$

$$M_{nJ}^2 = 4\kappa^2(n + \frac{L+J}{2}) + (1 + 2\alpha_1 + 2\alpha_2)(\frac{m_1^2}{2\alpha_1} + \frac{m_2^2}{2\alpha_2})$$

In order to incorporate a mechanism for explicit breaking of chiral symmetry to reproduce GMOR relation, $\alpha_i \sim m_i/\sigma$.
For mesons with heavy quarks it is possible see that α_i depend on quark mass.

5 T. Gutsche, V. Lyubovitskij, I. Schmidt and A. V, PRD 87, 056001 (2013).

Conclusions

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- An interesting application of AdS / QCD models is the possibility to get a LFWF according to the Light Front Holography suggested by Brodsky and de Teramond .
- Here we review some alternatives to extend the original prescription, improving the longitudinal part in the LFWF and introducing massive quarks.
- Specially we discuss a way to introduce current quarks. In this case the procedure offer the possibility to incorporate chiral symmetry breaking, and symmetries related with heavy quarks.
- Although it is not discussed here, with the suggested extension for the LFWF, the AdS / QCD model considered allows us to obtain good mass spectra for mesons and leptonic decay constants.

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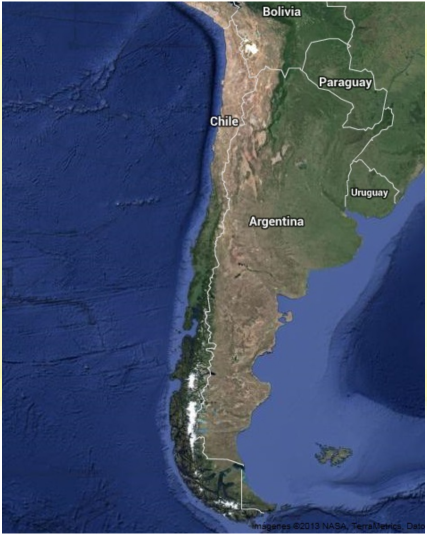

That's all Folks!