Thermal nonlocal Nambu-Jona-Lasinio model with complex poles.

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Outline

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The NJL model.

NJL model.

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi + \frac{G}{2}\left((\bar{\psi}\psi)^2 - (\bar{\psi}i\gamma^5\boldsymbol{\tau}\psi)^2\right) \quad (1)$$

- Proposed as a model of interacting nucleons.
- Reinterpreted as a model of interacting quarks after the discovery of QCD.

The nNJL model.

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$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi + \frac{G}{2} j_a(x) j_a(x) \tag{2}$$

- Nonlocal generalization of the NJL model.
- Nowadays, vastly used to study thermal properties of QCD.

$$j_a(x) = \int d^4y\, d^4z\, r(y-x) r(x-z) \bar{\psi}(x) \Gamma_a \psi(z) \tag{3}$$

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The regulator

There are many possible regulators to be used in the model. Among the most commonly used are Gaussian and Lorentzian regulators

$$r^2(p^2) = \mathrm{e}^{-\frac{q^2}{\Lambda^2}} \qquad r^2(p^2) = \frac{1}{1+\left(\frac{q^2}{\Lambda^2}\right)^n}. \tag{4}$$

A regulator was constructed in order to fit lattice data from the light quark propagator. (S. Noguera and N. N. Scoccola, Phys.Rev.D. 78 114002 (2008).)

$$r^2(q^2) = \frac{1}{1+\left(\frac{q^2}{\Lambda^2}\right)^{3/2}} \tag{5}$$

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The problem statement.

- Study the thermal nNJL model in the real time formalism.
- Study the behavior of the poles of the propagator.
- Particularly, study the possible effects of a complex cut in the regulator. (n=3/2 Lorentzian regulator)

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Some notation

Bosonic fields

We define scalar and pseudoscalar bosonic fields and work within the mean field approximation.

$$G\bar{\psi}\psi = \sigma = \bar{\sigma} + \delta\sigma \tag{6}$$

$$G\bar{\psi}i\gamma^5\boldsymbol{\tau}\psi = \boldsymbol{\pi} = \delta\pi \tag{7}$$

Some notation

The propagator

The propagator of the model

$$S_0(q) = i\frac{\not{q} + \Sigma(-q^2)}{q^2 - \Sigma^2(-q^2)}, \tag{8}$$

where

$$\Sigma(-q^2) = m + \bar{\sigma} r^2(-q^2) \tag{9}$$

Poles of the propagator at

$$q^2 = \mathcal{M}^2 = M^2 \pm iM\Gamma \tag{10}$$

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The plan

Choose a regulator (n=3/2 Lorentzian)$\Rightarrow$ Subtle definition in Minkowski space.

$\longrightarrow$ Compute the effective action

$\longrightarrow$ Compute the thermodynamical potential

$\longrightarrow$ Minimize the potential with respect to $\bar{\sigma}$ to obtain a gap equation

$\longrightarrow$ Solve the equation to get the behavior of $\bar{\sigma}$ as a function of T

$\longrightarrow$ Look for a phase transition to a chirally restored phase.

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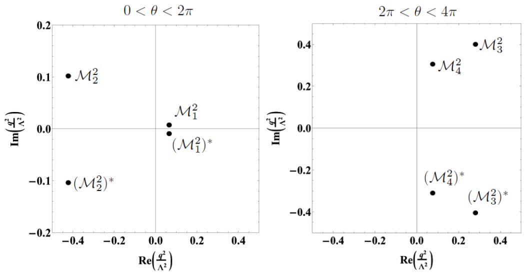
A look at the poles

We consider $r^2(q^2) = \frac{1}{1+\left(\frac{q^2}{\Lambda^2}\right)^{3/2}}$ in the chiral limit $(m = 0)$



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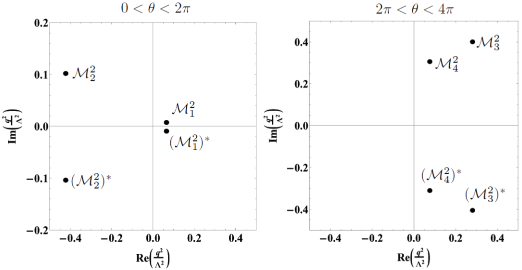

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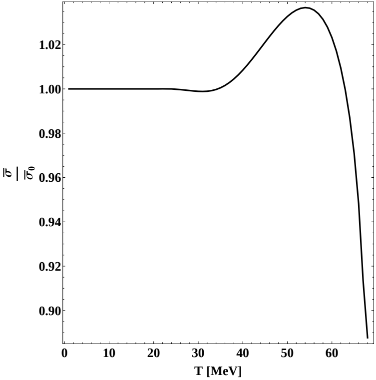
Solved gap equation



A whole new set of questions!

- What is that and why is it there?
- How do we get rid of that?
- Should we get rid of that?
- Does the same thing happens in the imaginary time formalism?

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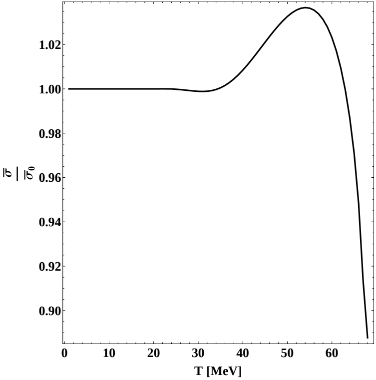

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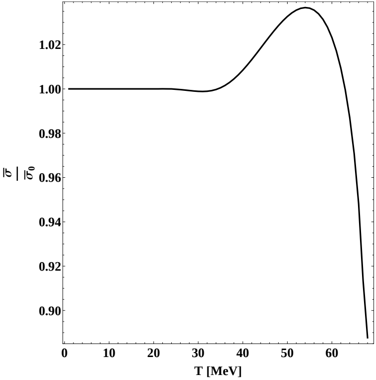

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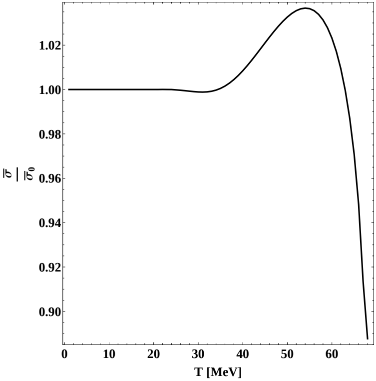

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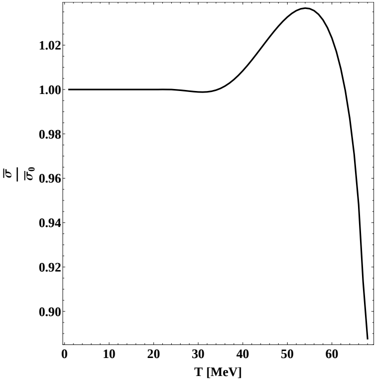

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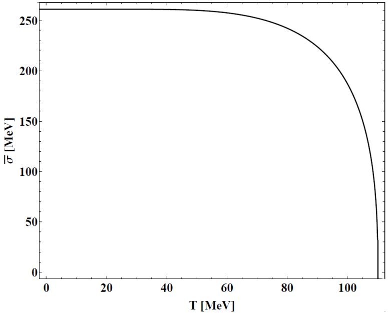
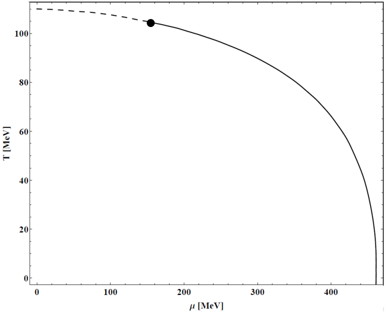
Getting rid of the instability





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Conclusions

- The instability seems to dissapear when we stop considering the oddly behaved pole.
- In that scenario we are able to reproduce the results of the imaginary time formalism.
- We find a chirally restored phase and first and second order phase transitions.
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