Chiral magnetic effect in QED$_3$

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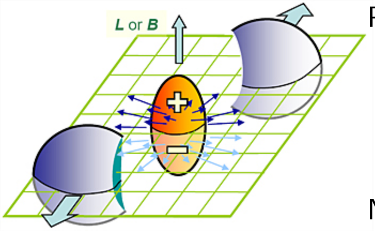
Chiral magnetic effect in relativistic HIC

Topological charge + magnetic field $\rightarrow P$ and CP violation

- induces chiral charges and an electric current in the $\boldsymbol{B}$ direction (helicities alignment)
- In Heavy-Ion collisions, $eB \sim 10^4\text{MeV}^2 \sim 10^{18}\text{Gauss}$
- topological charge fluctuations could be observed In in high T QCD
- It should be manifest in $\pi^+\ \pi^-$ imbalance



Problems

- short time life of $\boldsymbol{B}$
- fast screening
- statistical fluctuations

No significant evidence yet

QCD at high $T \longleftrightarrow$ QED in (2+1) dimensions

- QCD At high temperature is dimensionally reduced
- QCD at high T *abelianizes* for large N_f

$$\mathcal{L}_{\mathrm{QED}_3} = \bar{\psi}[\not\partial - \mathrm{e}\not\mathcal{A} - m_e + m_o\tau]\psi$$
$$-\frac{1}{2\xi}(\partial \cdot \mathcal{A})^2 - \frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} - \frac{\theta}{4}\epsilon^{\mu\nu\rho}\mathcal{A}_\mu\partial_\nu\mathcal{A}_\rho$$

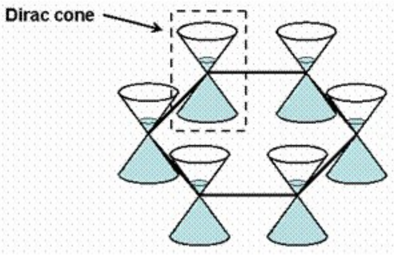
with indexes $\mu = (0, 1, 2)$.

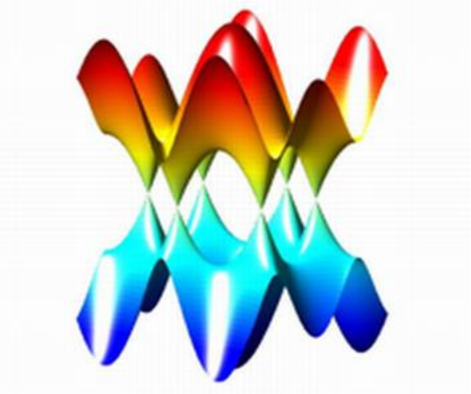
- Is invariant under $\psi \to \psi e^{i\alpha\tau}$, with $\tau = i\gamma_3\gamma_5$
- dynamical chiral symmetry breaking
- Confinement
- Chern-Simons term naturally arise

QED_3 and graphene physics

QED_3 is the best candidate for the description of fermion interactions in graphene

- quasi-electrons are relativistic (v_F is the speed of light)
- massless fermions propagates over the Fermi surface
- low temperature

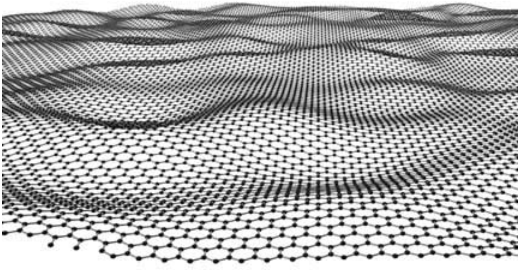


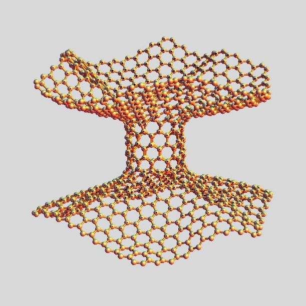


field theory and graphene

Interchange of effects in particle behavior
high T QCD $\leftrightarrow$ graphene

Other example: Gravitation techniques $\rightarrow$ metric



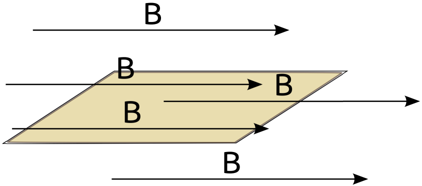


Chiral magnetic effect analog in QED$_3$

The most simple way to introduce CME in QED$_3$

$$\begin{aligned}\mathcal{L} &= \bar{\psi}\left[i\partial\!\!\!/ + \mu_A\gamma^0\gamma^5 - eA_3\gamma^3\right]\psi \\ &= \bar{\psi}_+\left[i\partial\!\!\!/ + \mu_A\gamma^0 - eA_3\gamma^3\right]\psi_+ + \bar{\psi}_-\left[i\partial\!\!\!/ - \mu_A\gamma^0 - eA_3\gamma^3\right]\psi_-\end{aligned}$$

- $A_3 = Bx_2$ an external field
- μ_A axial chemical potential
- $\psi_\pm = \frac{1}{2}(1 \pm \gamma^5)\psi$



Schwinger propagator

$$G(x,x') = \int_0^\infty ds \ \langle x| e^{-iHs}[\gamma^{\bar{\mu}}\Pi_{\bar{\mu}} + m]|x'\rangle$$

$$\begin{aligned} \Pi_\mu &= i\partial_\mu - eA_\mu \\ \Pi_3 &= -eA_3 \\ H &= (\gamma^{\bar{\mu}}\Pi_{\bar{\mu}})^2 - m^2 \end{aligned}$$

e^{-iHs} is the time evolution operator.

- cannot use the Schwinger method correctly
- We need to add a fourth spatial state, confine the system to a plane and normalize propperly

extended Schwinger propagator

We add a new state ζ

$$\Pi_3 \to \alpha \frac{\partial}{\partial \zeta} - eA_3$$

$$G(x, x') \to \int_0^\infty ds\, \mathcal{N}\, \langle x, \zeta | e^{-iHs} [\gamma^{\bar{\mu}} \Pi_{\bar{\mu}} + m] | x', \zeta \rangle$$

- α is a constant that will be set to cero at the end
- the normalization $\mathcal{N}$ avoids divergences $\langle \zeta | \zeta \rangle = \delta(0)$ through the condition

$$\lim_{s \to 0} \langle x(s), \zeta(s) | x', \zeta \rangle = \delta^3(x - x'), \qquad \Rightarrow \qquad \mathcal{N} \sim \sqrt{s}$$

Chemical potential and temperature

If $\mu > m$, the Shwinger propagator must be regularized differently.
If electric fields are absent $\Rightarrow$ Fourier transform of 0-component

$$\int_0^\infty ds \to \int_{-\infty}^\infty ds \left\{\theta(s)\theta[k_0(k_0+\mu)]e^{-s\epsilon} - \theta(-s)\theta[-k_0(k_0+\mu)]e^{s\epsilon}\right\}$$

At finite temperature

$$k_0 \to i\omega_n = i(2n+1)\pi T, \qquad \int \frac{dk_0}{2\pi} \to iT\sum_n$$

$$\int_0^\infty ds \to \int_{-\infty}^\infty ds \left\{\theta(s)\theta(\omega_n\mu) - \theta(-s)\theta(-\omega_n\mu)\right\}$$

Final propagator

In momentum space with $B^i = B\delta^{i1}$

$$S(p) = \frac{1}{2}(1+\gamma_5)S_+(p) + \frac{1}{2}(1-\gamma_5)S_-(p)$$

$$\begin{aligned} S_\pm(k) &= \int_{-\infty}^{\infty} ds \left\{\theta(s)\theta[k_0(k_0+\mu)]e^{-s\epsilon} - \theta(-s)\theta[-k_0(k_0+\mu)]e^{s\epsilon}\right\} \\ & \exp\left\{is\left[(k_0 \pm \mu_A)^2 - k_1^2 - k_2^2 \tan(eBs)/eBs\right]\right\} \\ & \{(k_0 \pm \mu_A)\gamma_0 - k_1\gamma_1 - k_2[\gamma_2 + \gamma_3 \tan(eBs)]\} \\ & [1 - \gamma_2\gamma_3 \tan(eBs)] \left[eBs/\tan(eBs)\right]^{1/2} \end{aligned}$$

Induced topological term

Polarization tensor $\Pi^{\mu\nu} = \Pi_A^{\mu\nu} + \Pi_S^{\mu\nu}$

$$\Pi_A^{\mu\nu}(p) = \epsilon^{\mu\nu\alpha} \int_{k,s,s'} \Theta_\alpha(s,k;s',p+k;\mu_A) F(s,k) F(s',p+k)$$

Expanded in external momentum

$$\mathcal{L}_\theta \sim \theta_0(\mu_A, B)\ \epsilon^{0\mu\nu} \mathcal{A}_\mu \partial_2 \mathcal{A}_\nu + \theta_2(\mu_A, B)\ \epsilon^{2\mu\nu} \mathcal{A}_\mu \partial_0 \mathcal{A}_\nu$$

Induced current

The chiral number density $n_A = n_+ - n_- = \langle \psi^\dagger \gamma^5 \psi \rangle$ and the induced electric current $j = \langle \bar{\psi} \gamma^1 \psi \rangle$ are

$$\begin{aligned} n_A &= j + \frac{eB}{\pi} \sum_{l=1}^{\infty} \left[n_F(m_l - \mu_A) - n_F(m_l + \mu_A) \right], \\ j &= \frac{eB}{2\pi} \tanh(\beta \mu_A / 2). \end{aligned}$$

with $m_l = \sqrt{2eBl}$. At zero temperature

$$\begin{aligned} n_A &= j + \mathrm{sign}(\mu_A) \frac{eB}{\pi} \sum_{l=1}^{\infty} \theta(|\mu_A| - m_l). \\ j &= \mathrm{sign}(\mu_A) \frac{eB}{2\pi}. \end{aligned}$$

In QCD: $j = (N_c \sum_f q_f^2)\ \mu_A \frac{eB}{2\pi^2}$

Conclusions and outlook

- The chiral magnetic effect analog is possible in QED_3
- Interesting link between Heavy ion physics and graphene
- Start with a Chern-Simons term to see the induced currents
- Improve the system $\rightarrow$ electric chemical potential
- look forward for reciprocal results to be applied in Heavy Ion physics
- An experiment to measure the induced current?

THANKS!