Dimuon production from in-medium ρ decays from QCD Sum Rules

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This talk is based on the following articles:

Rho-meson resonance broadening in QCD at finite temperature: A. Ayala, C. A. Dominguez, M. Loewe and Y. Zhang. Hep-ph 1210.2588. Phys. Rev. D. 86 (2012) 114036

Dimuon production from in-medium rho decays form QCD Sum Rules: A. Ayala, C. A. Dominguez, L. Hernández, M. Loewe and Ana J. Mizher. hep-ph 1309.4135. To appear in Phys. Rev. D

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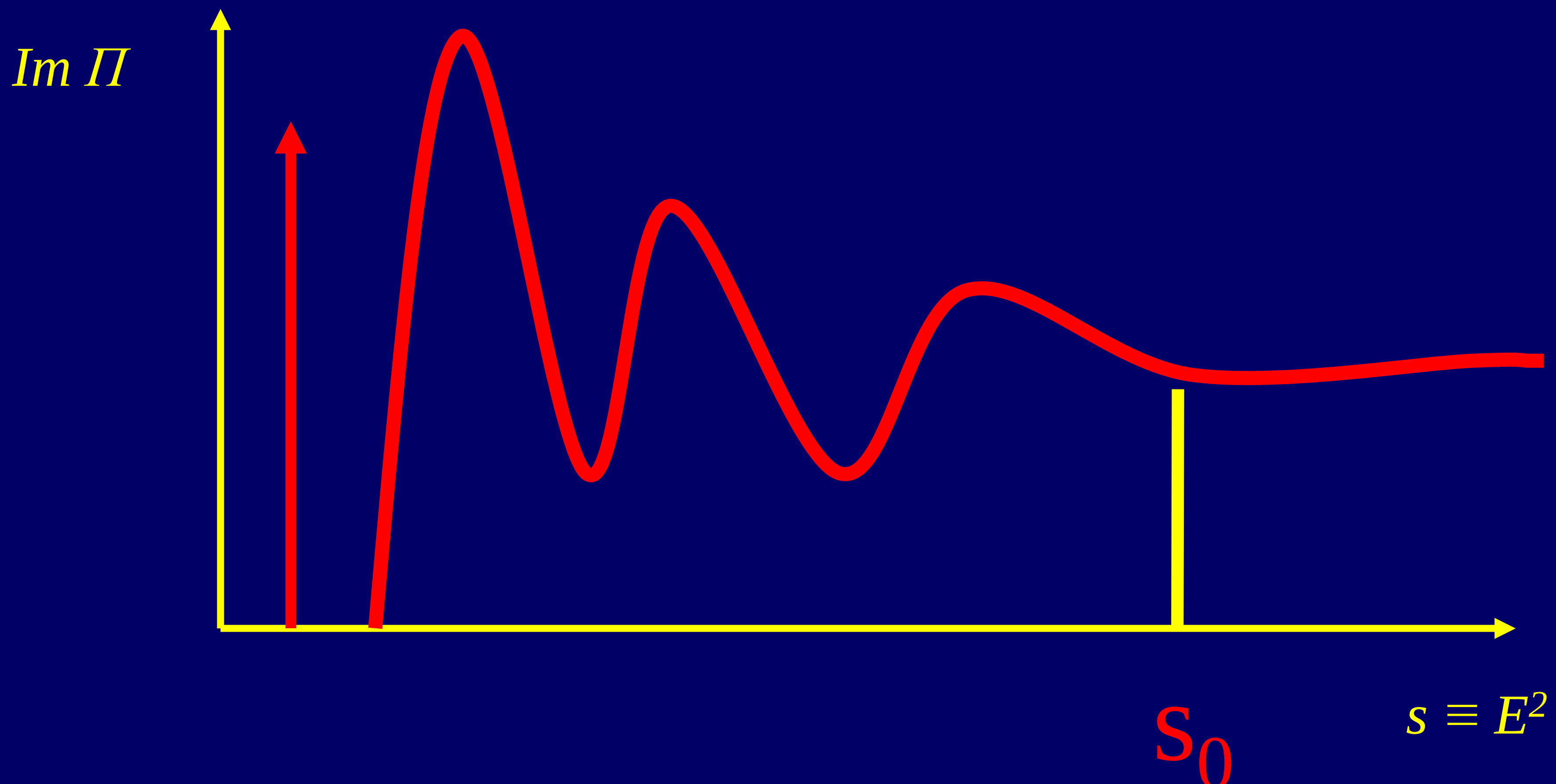
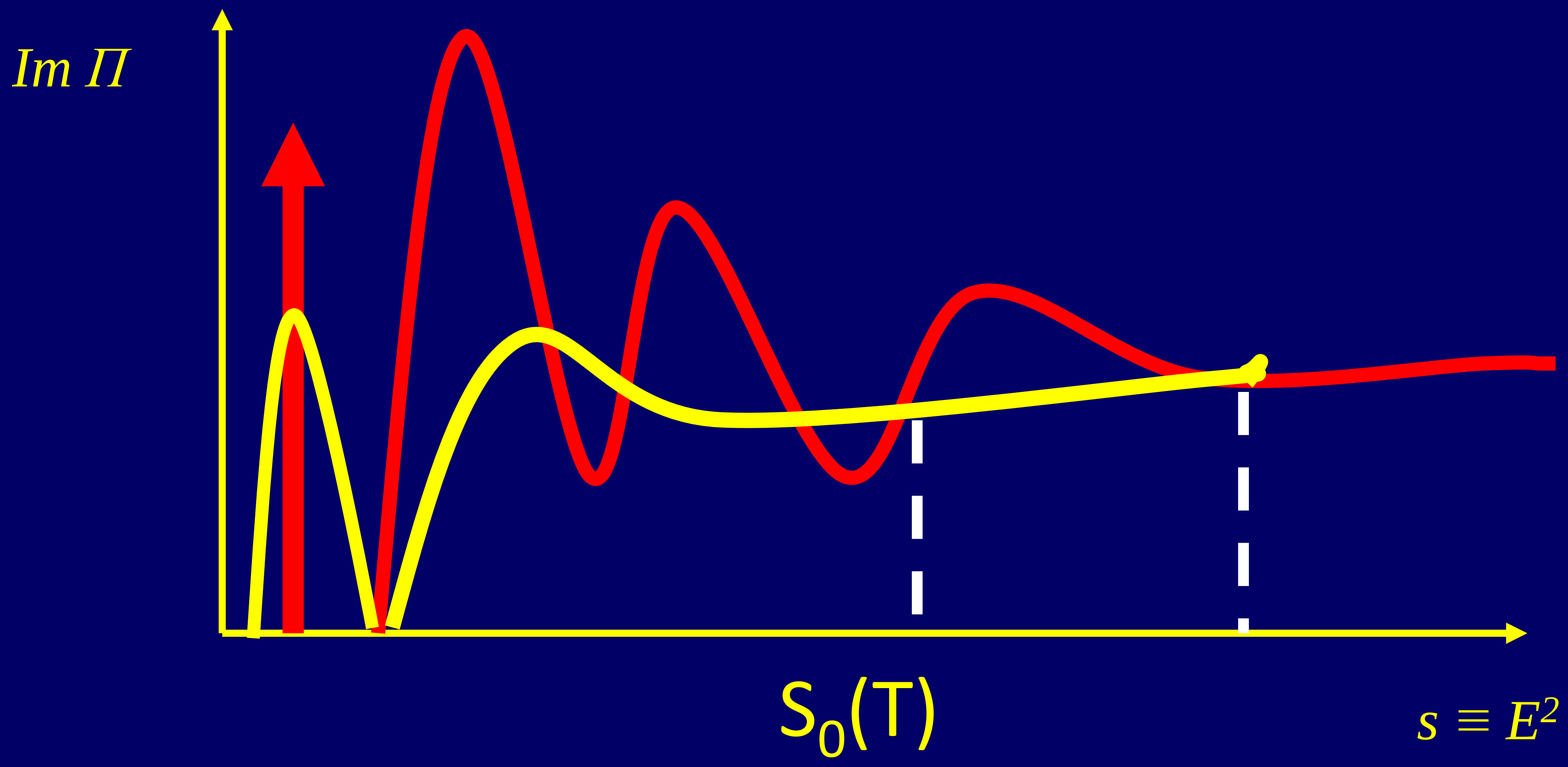
- The idea is to calculate the di-muon-excess invariant mass distribution at the ρ-meson peak in relativistic heavy ion collisions at the SPS.

- The width, mass and leptonic decay constant of the ρ were determined by QCD Sum Rules taking into account the T and μ dependence

Realistic Spectral Function



Realistic Spectral Function (T)



- We consider the light quark vector channel which at T=0

$$\Pi_{\mu\nu}(q^2) = i \int d^4x\, e^{iqx} < 0|T(V_\mu(x)V_\nu^\dagger(0))|0 >$$

$$= (-g_{\mu\nu}\, q^2 + q_\mu q_\nu)\, \Pi(q^2)\,,$$

Where

$$V_\mu(x) = \frac{1}{2}[:\ \bar{u}(x)\gamma_\mu\, u(x) - \bar{d}(x)\gamma_\mu\, d(x)\ :]$$

Our normalization

$$\mathrm{Im}\,\Pi(q^2) = \frac{1}{8\pi}\left[1 + \mathcal{O}\left(\alpha_s(q^2)\right)\right]$$

It is not difficult to arrive to the following set of FESR

$$(-1)^{N-1} C_{2N} \langle O_{2N} \rangle = 8\pi^2 \left[\int_0^{s_0} ds\, s^{N-1} \frac{1}{\pi} \,\mathrm{Im}\Pi^{\mathrm{HAD}}(s) - \int_0^{s_0} ds\, s^{N-1} \frac{1}{\pi} \,\mathrm{Im}\Pi^{\mathrm{pQCD}}(s) \right]$$

These Sum Rules emerge from two basic facts:

OPERATOR PRODUCT EXPANSION OF CURRENT CORRELATORS AT SHORT DISTANCES

(BEYOND PERTURBATION THEORY)

CAUCHY'S THEOREM IN THE COMPLEX ENERGY (SQUARED) S-PLANE

Q C D SUM RULES (SVZ)

$$\Pi(q^2) = \int d^4x\, e^{iqx} < 0 | T\,(J(x)\, J^+(0)) | 0 >$$

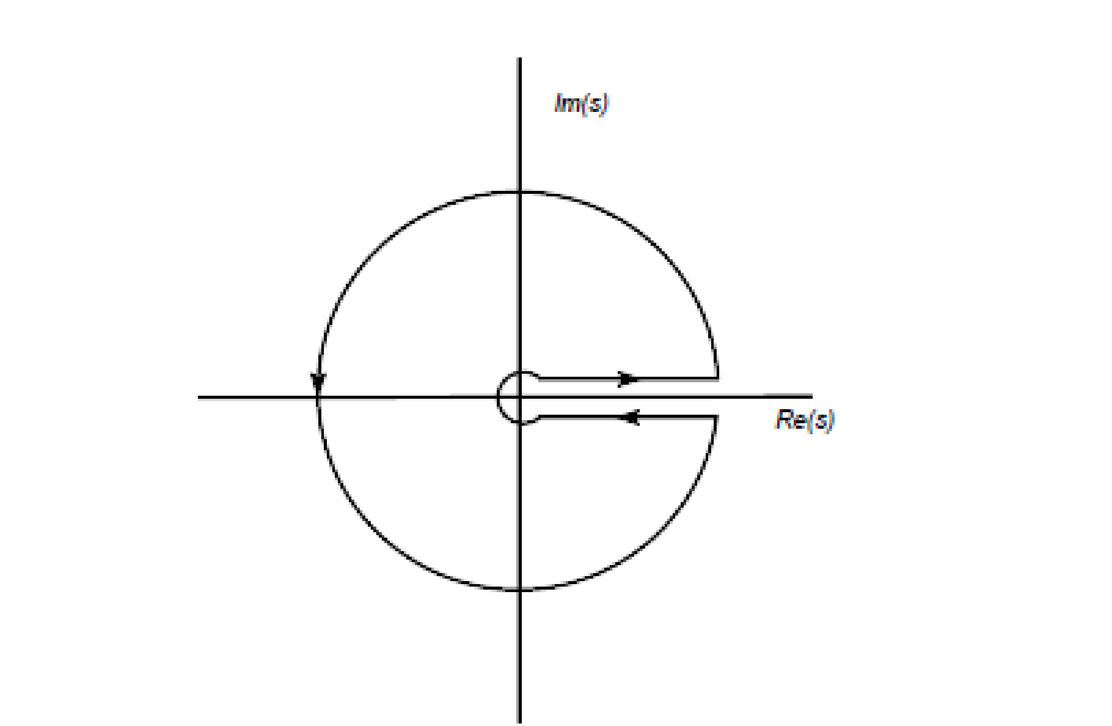
$$\Pi(q^2)\big|_{QCD} = I + \sum_{N=0} C_{2N+2}(q^2, \mu^2) < 0 | \hat{O}_{2N+2}(\mu^2) | 0 >$$

$$I \Rightarrow O(\alpha_s^4) \qquad\qquad C_{2N+2} \Rightarrow \frac{1}{(-q)^{2N+2}}$$

$$m_q < 0 | \overline{q}\; q | 0 >, \quad < 0 | \alpha_s\, G_{\mu\nu}\, G^{\mu\nu} | 0 >, \quad etc.$$

The correlator can also be expressed in the hadronic sector. Bound states correspond to poles on the real axis and resonances lie on the second Riemann sheet. (Complex s-plane)

In principle we have also cuts associated to non resonant multiple particle production.



$$-\frac{1}{2\pi i}\oint_{C(|s_0|)} ds\,\Pi_{QCD}(s) = \int_{s_{th}}^{s_0} ds\,\frac{1}{\pi}\,\mathrm{Im}\,\Pi(s)\,|_{HAD}$$

The leading order vacuum condensates in the chiral limit are the dimension d=4 gluon condensate

$$C_4\langle\hat{\mathcal{O}}_4\rangle = \frac{\pi}{3}\langle\alpha_s G^2\rangle$$

and the dimension d=6 condensate

$$C_6\langle\hat{\mathcal{O}}_6\rangle = -8\pi^3\alpha_s\left[\langle(\bar{q}\gamma_\mu\gamma_5\lambda^a q)^2\rangle + \frac{2}{9}\langle(\bar{q}\gamma_\mu\lambda^a q)^2\rangle\right]$$

- Notice that there is no convincing theoretical support for the validity of the vacuum saturation hipothesis that allows to relate both condensates.
- In the hadronic sector we use a Breit-Wigner as

an approximation for the Rho-resonance

$$\frac{1}{\pi}\mathrm{Im}\Pi|_{HAD}(s) = \frac{1}{\pi}\frac{1}{f_\rho^2}\frac{M_\rho^3\Gamma_\rho}{\left(s - M_\rho^2\right)^2 + M_\rho^2\Gamma_\rho^2}$$

where: fρ =5; Mρ = 0.776 GeV; Γ_ρ = 0.145 GeV

The normalization has been chosen such that the area is equal to the zero-width expression

$$\text{Im}\,\Pi|_{HAD}(s) = f_\rho^2\, M_\rho^2\, \delta(s - M_\rho^2)$$

A test of the FESR for N=1,2,3 can be performed by determining S_0, the gluon and the four quark condensate comparing them with data analyses. The results are completely reasonable.

$$s_0(0) = 1.7298\,\text{GeV}^2 \qquad C_4\langle O_4\rangle(0) = 0.412561\ \text{GeV}^4$$

$$C_6\langle O_6\rangle(0) = -0.951667\ \text{GeV}^6$$

This value of S_0 validates the ρ-dominance assumption.

Thermal Extension of the QCD Sum Rules

- There are important differences:

- 1) The vacuum is populated (a thermal vacuum)

- 2) A new analytic structure in the complex s-plane appears, due to scattering. This effect turns out to be very important

The current- current correlator in a thermal vacuum (a populated vacuum) corresponds to

$$G^R_{\mu,\nu}(\omega,\vec{p}) = i \int d^4x \; e^{ipx}\theta(x^0)\langle\langle[J_\mu(x), J_\nu(0)]\rangle\rangle$$

$$\langle\langle \cdots \rangle\rangle = \sum_n e^{-E_n/T}\langle n|\cdots|n\rangle$$

New effect in the pressence of a populated vacuum: Annihilation + Scattering contributions to the spectral function

Annihilation term (survives when T → 0)

$$\rho^a_{\mu\nu}(\omega,\vec{p}) = \sum_q \int LIPS(\omega,\vec{p}\,;\ E_1,\vec{k}_1,E_2,\vec{k}_2)\langle 0|J_\mu|q\bar{q}\rangle\langle q\bar{q}|J_\nu|0\rangle$$
$$\times\{(1-n_F(E_1)(1-n_F(E_2)-n_F(E_1 n_F(E_2)\}$$

$$LIPS(\omega,\vec{p}\,;\ E_1,\vec{k}_1,E_2,\vec{k}_2) = \frac{d^3k_1}{2E_1(2\pi)^3}\frac{d^3k_2}{2E_2(2\pi)^3}\delta(\omega-E_1-E_2)\delta^{(3)}(\vec{P}-\vec{k}_1-\vec{k}_2)$$

New Effect:The current may scatter off particles in the populated vacuum. Notice that this term vanishes when $T \rightarrow 0$ (Bochkarev and Schaposnikov, Nucl. Phys. B268 (1986) 220)

$$\rho^{s}_{\mu\nu}(\omega,\vec{p}) = \sum_{q,\bar{q}} \int LIPS(\omega,\vec{p}\,;\; E_1,\vec{k}_1,-E_2,-\vec{k}_2)\langle q|J_\mu|\bar{q}\rangle\langle\bar{q}|J_\nu|q\rangle$$

$$\times\{(n_F(E_1)(1-n_F(E_2)-n_F(E_2(1-n_F(E_1)\}$$

Finite Temperature Effects

1) Time-like region: $\omega^2 - |\mathbf{q}|^2 > 0$

$$\mathrm{Im}\,\Pi^{+}(\omega, T) = \frac{1}{4\,\pi}\left[1 - 2\,n_F\left(\frac{\omega}{2T}\right)\right]$$

2) Space-like region: $\omega 2 - |\mathbf{q}| 2 < 0$

$$\begin{aligned}\mathrm{Im}\,\Pi^{-}(\omega, T) &= \frac{4}{\pi}\,\delta(\omega^2)\int_0^{\infty} y\,n_F\left(\frac{y}{T}\right)\,dy \\ &= \frac{\pi}{3}\,T^2\,\delta(\omega^2)\,,\end{aligned}$$

- where

$$n_F(z) = 1/(1+e^z)$$

In the hadronic sector f_ρ, M_ρ and Γ_ρ become temperature dependent.

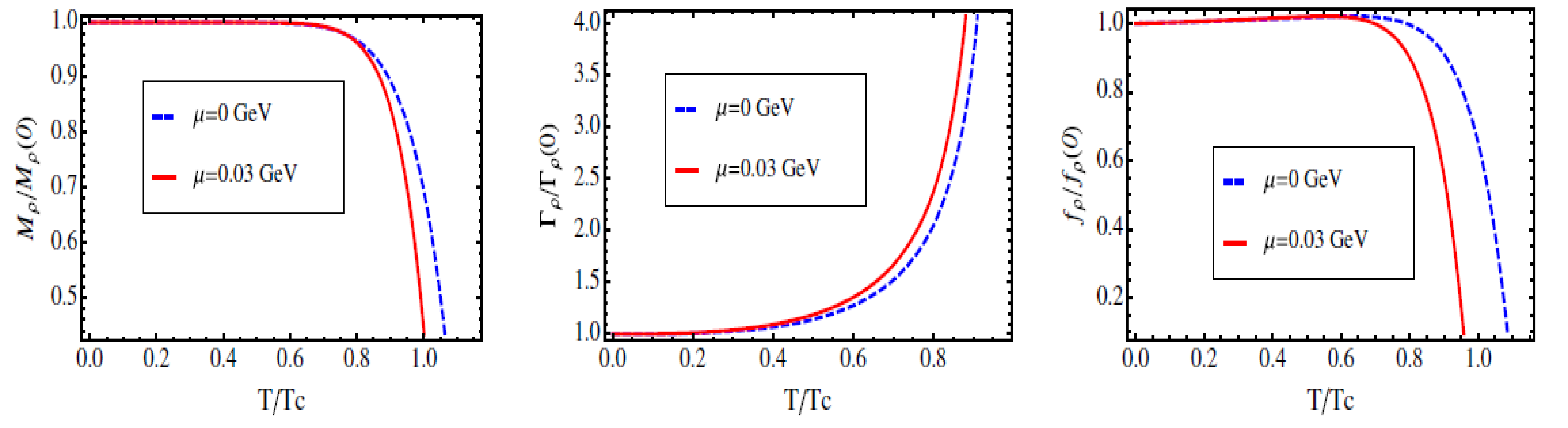
Due to the coupling of the current to two pions we have a scattering contribution in the hadronic sector

$$\frac{1}{\pi}\mathrm{Im}\,\Pi^-|_{HAD}(\omega,T) = \frac{2}{3\pi^2}\delta(\omega^2)\int_0^\infty y\, n_B\left(\frac{y}{T}\right) dy$$

where

$$n_B(z) = 1/(e^z - 1)$$

- Strategy: We will use our previous results for the thermal evolution of the ρ to compute the dimuon thermal rate taking also finite density effects into account.



- A previous analysis by the mexican group

E. Gutierrez, A. Ahmad, A. Ayala, A. Bashir, and A. Raya, arXiv:1304.8065.

based on a Schwinger-Dyson discussion provided a parameterization for the cross-over transition line between chiral broken and chiral restored phases

$$T_c(\mu) = T_c(\mu = 0) - 0.218\mu - 0.139\mu^2$$

μ is also inserted at the level of the quark propagators inside the quark loops

- In this way for μ = 0.03 GeV we get

$$M_\rho(T,\mu) = M_\rho(0)[1 - 0.5597(T/T_c)^{12.18}]$$

$$\Gamma_\rho(T,\mu) = \Gamma_\rho(0)[1 - 1.0717(T/T_c)^{2.763}]^{-1}$$

$$f_\rho(T,\mu) = f_\rho(0)[1 - 0.3901(T/T_c(\mu))^{10.75} + 0.04155)(T/T_c(\mu)^{1.27}]$$

Notice that the baryonic chemical potentila satisfies

$\mu_B = 3\,\mu = 0.09$ GeV

Let us go into the di-muon production….

- The leading effect will come from pion-annhilation into ρ which in turn decay into dimuons by menas of vector meson dominance. The rate is given by

$$\frac{dN}{d^4x d^4K} = \frac{\alpha^2}{48\pi^4}\left(1+\frac{2m^2}{M^2}\right)\left(1-\frac{4m_\pi^2}{M^2}\right)\sqrt{1-\frac{4m^2}{M^2}}E \times e^{-K_0/T}\mathcal{R}(K,T)\,\mathrm{Im}\Pi_0^{\mathrm{res}}(M^2),$$

Where: m_π is the pion mas, m is the muon mass, M is the dimuon invariant mass, K is the momentum of the muons,

$K^\mu = (K_0, \mathbf{K})$ $(K = |\mathbf{K}|)$

- where

$$\mathcal{R} = \frac{T/K}{1 - e^{-K_0/T}} \times \ln\left[\left(\frac{e^{-E_{\max}/T} - 1}{e^{-E_{\min}/T} - 1}\right)\left(\frac{e^{E_{\min}/T} - e^{-K_0/T}}{e^{E_{\max}/T} - e^{-K_0/T}}\right)\right]$$

with

$$E_{\max} = \frac{1}{2}\left[K_0 + K\sqrt{1 - 4m_\pi^2/M^2}\right]$$

$$E_{\min} = \frac{1}{2}\left[K_0 - K\sqrt{1 - 4m_\pi^2/M^2}\right]$$

- In order to integrate we use

$$d^4K = \frac{1}{2} dM^2 d^2K_\perp dy, \qquad d^4x = \tau d\tau d\eta d^2x_\perp$$

y and η are the space and momentum rapidities and

$$\tau = \sqrt{t^2 - z^2}$$

To relate the temperature change to the time evolution we assume that the expansion is enterily longitudinal

$$T = T_0 \left(\frac{\tau_0}{\tau}\right)^{v_s^2} \qquad v_s^2 = 1/3$$

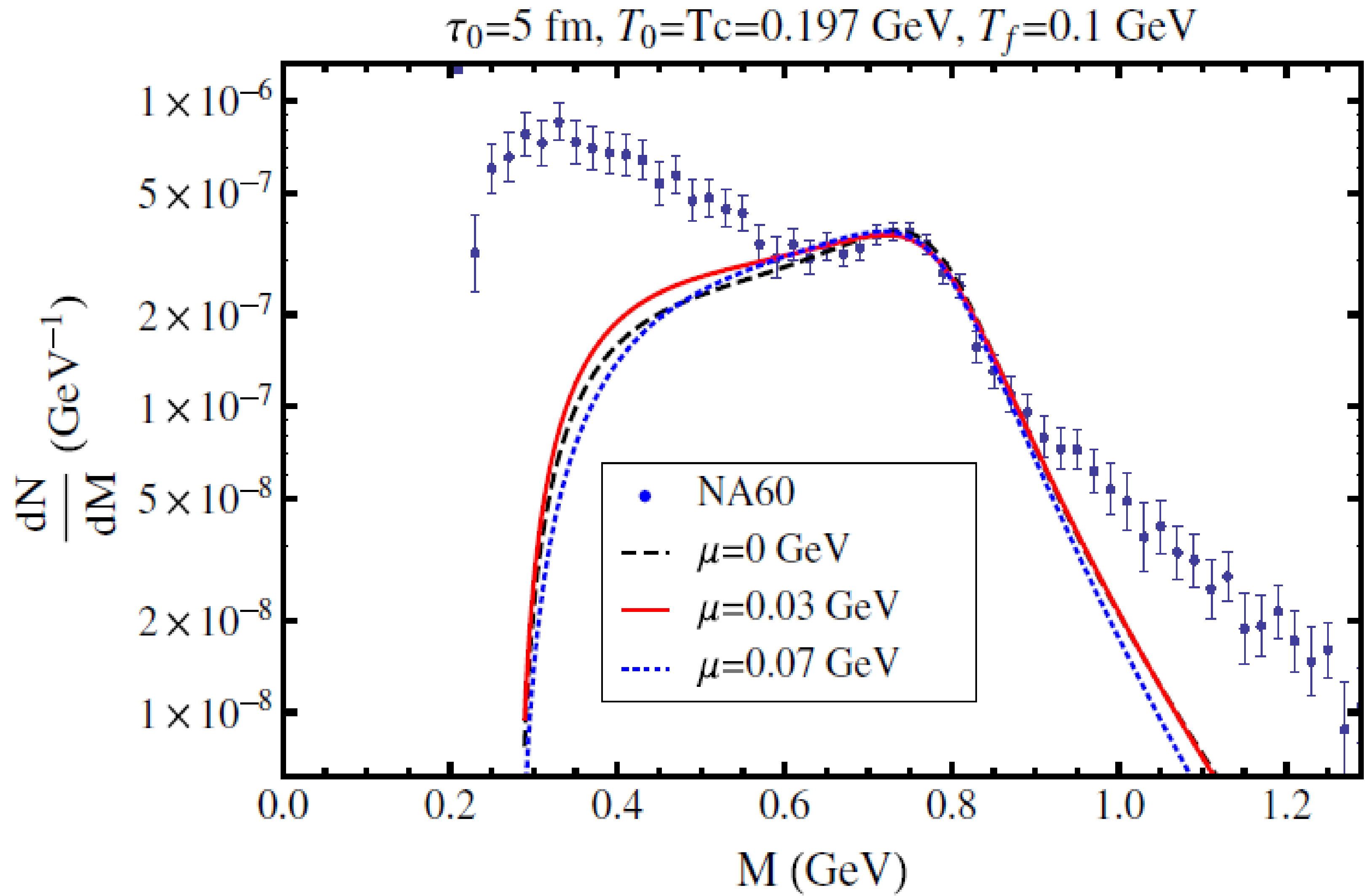
Bjorken's cooling law

- The object of interest is

$$\frac{dN}{dMdy} = \Delta y M \int_{\tau_0}^{\tau_f} \tau d\tau \int d^2 K_\perp \int d^2 x_\perp \frac{dN}{d^4 x d^4 K}$$

And in this way we get

Invariant dimuon mass compared to data from NA60



- CONCLUSIONS

QCD FESR are a powerful tool to compute the ρ meson parameters at finite temperature and chemical potential

We have use these parameters to compute the excess of di-muons in the vicinity of the ρ. The results are in a very good agreement with the NA60 data