LINKING NATURAL SUPERSYMMETRY TO FLAVOUR PHYSICS

HEP Chile 2013

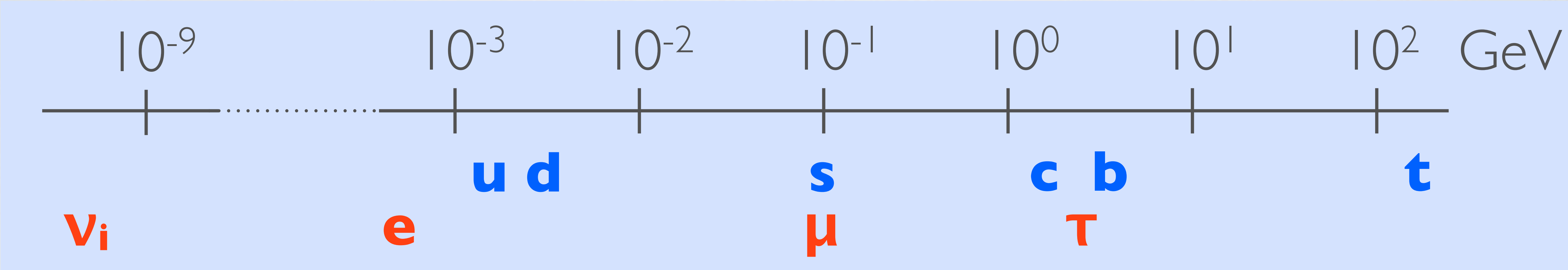
Gero von Gersdorff
December 2013

Based on 1308.1090 (with E. Dudas, S. Pokorski and R. Ziegler)

ICTP SAIFR
International Centre for Theoretical Physics
South American Institute for Fundamental Research

INTRODUCTION

- ✗ The flavor structure of the SM is very peculiar
- ✗ Strong hierarchy in the masses of the SM fermions:



- ✗ The quark mixing is pretty much the identity:

$$V_{\mathrm{CKM}} = \begin{pmatrix} 0.97427 \pm 0.00015 & 0.22534 \pm 0.00065 & 0.00351^{+0.00015}_{-0.00014} \\ 0.22520 \pm 0.00065 & 0.97344 \pm 0.00016 & 0.0412^{+0.0011}_{-0.0005} \\ 0.00867^{+0.00029}_{-0.00031} & 0.0404^{+0.0011}_{-0.0005} & 0.999146^{+0.000021}_{-0.000046} \end{pmatrix}$$

INTRODUCTION

✗ Low energy New Physics comes with a FCNC/CP problem:

✗ In the SM, only 3 angles an 1 phase are left in the renormalizable Lagrangian (the W vertex), accounts very well for all the flavor transitions and CP violation

INTRODUCTION

✗ Low energy New Physics comes with a FCNC/CP problem:

✗ In the SM, only 3 angles an 1 phase are left in the renormalizable Lagrangian (the W vertex), accounts very well for all the flavor transitions and CP violation

✗ Generic new physics can be captured within an effective field theory prescription

$$\mathcal{L}_{eff} = \mathcal{L}_{SM} + \frac{C_{\mathcal{O}}}{\Lambda^{d-4}}\,\mathcal{O}^d$$

INTRODUCTION

✗ Low energy New Physics comes with a FCNC/CP problem:

✗ In the SM, only 3 angles an 1 phase are left in the renormalizable Lagrangian (the W vertex), accounts very well for all the flavor transitions and CP violation

✗ Generic new physics can be captured within an effective field theory prescription

$$\mathcal{L}_{eff} = \mathcal{L}_{SM} + \frac{C_{\mathcal{O}}}{\Lambda^{d-4}}\,\mathcal{O}^d$$

✗ No reasons why the coefficients should be flavour diagonal,

✗ All the quark rotation angles (and phases) matter

✗ This typically requires $\Lambda > 10^5 - 10^6\,\mathrm{TeV}$

HORIZONTAL SYMMETRIES

✗ Horizontal symmetries put quarks in representations of some Abelian or non-Abelian symmetry which consequently is spontaneously broken at a scale somewhat lower than the UV scale.

✗ Generates small order parameter that controls the size of Yukawa couplings

HORIZONTAL SYMMETRIES

✗ Horizontal symmetries put quarks in representations of some Abelian or non-Abelian symmetry which consequently is spontaneously broken at a scale somewhat lower than the UV scale.

✗ Generates small order parameter that controls the size of Yukawa couplings

✗ Simplest model: A single U(1) symmetry

Froggatt + Nielsen '79

$$\mathcal{L}_f = H \left(\hat{Y}^u_{ij} \; \bar{q}_i u_j \left[\frac{\chi}{\Lambda} \right]^{X_{\bar{q}_i} + X_{u_j}} + \hat{Y}^d_{ij} \; \bar{q}_i d_j \left[\frac{\chi}{\Lambda} \right]^{X_{\bar{q}_i} + X_{d_j}} \right) \qquad X_\chi = -1$$

HORIZONTAL SYMMETRIES

✗ Horizontal symmetries put quarks in representations of some Abelian or non-Abelian symmetry which consequently is spontaneously broken at a scale somewhat lower than the UV scale.

✗ Generates small order parameter that controls the size of Yukawa couplings

✗ Simplest model: A single U(1) symmetry

Froggatt + Nielsen '79

$$\mathcal{L}_f = H \left(\hat{Y}^u_{ij} \bar{q}_i u_j \left[\frac{\chi}{\Lambda}\right]^{X_{\bar{q}_i}+X_{u_j}} + \hat{Y}^d_{ij} \bar{q}_i d_j \left[\frac{\chi}{\Lambda}\right]^{X_{\bar{q}_i}+X_{d_j}} \right) \qquad X_\chi = -1$$

O(1) bare Yukawas $\hat{Y}^u_{ij}$

HORIZONTAL SYMMETRIES

✗ Horizontal symmetries put quarks in representations of some Abelian or non-Abelian symmetry which consequently is spontaneously broken at a scale somewhat lower than the UV scale.

✗ Generates small order parameter that controls the size of Yukawa couplings

✗ Simplest model: A single U(1) symmetry

Froggatt + Nielsen '79

$$\mathcal{L}_f = H\left(\hat{Y}^u_{ij}\bar{q}_i u_j \left[\frac{\chi}{\Lambda}\right]^{X_{\bar{q}_i}+X_{u_j}} + \hat{Y}^d_{ij}\bar{q}_i d_j \left[\frac{\chi}{\Lambda}\right]^{X_{\bar{q}_i}+X_{d_j}}\right) \qquad X_\chi = -1$$

O(1) bare Yukawas $\hat{Y}^u_{ij}$

$$\epsilon = \frac{\langle\chi\rangle}{\Lambda}$$

$$Y^u_{ij} \sim \epsilon^{X_{\bar{q}_i}+X_{u_j}}$$

HORIZONTAL SYMMETRIES

✗ Horizontal symmetries put quarks in representations of some Abelian or non-Abelian symmetry which consequently is spontaneously broken at a scale somewhat lower than the UV scale.

✗ Generates small order parameter that controls the size of Yukawa couplings

✗ Simplest model: A single U(1) symmetry

Froggatt + Nielsen '79

$$\mathcal{L}_f = H \left(\hat{Y}^u_{ij} \bar{q}_i u_j \left[\frac{\chi}{\Lambda}\right]^{X_{\bar{q}_i}+X_{u_j}} + \hat{Y}^d_{ij} \bar{q}_i d_j \left[\frac{\chi}{\Lambda}\right]^{X_{\bar{q}_i}+X_{d_j}} \right) \qquad X_\chi = -1$$

O(1) bare Yukawas $\hat{Y}^u_{ij}$

$$\epsilon = \frac{\langle\chi\rangle}{\Lambda}$$

$$Y^u_{ij} \sim \epsilon^{X_{\bar{q}_i}+X_{u_j}} \longrightarrow$$

Hierarchical eigen-values and angles!

PROBLEMS OF U(1) MODELS

✗ What does this imply for supersymmetry ?

✗ A general problem of U(1) models is that the suppressions tend to cancel out in the soft terms:

$$K \supset |X|^2 \; c_{ij} \; \epsilon^{|X_i - X_j|} \; \bar{Q}_i Q_i$$

✗ Not so small off-diagonal terms

✗ Diagonal terms completely unsuppressed: uncontrolled splitting

✗ Bounds on first two generation squarks > 100 TeV
(Note: this is also inconsistent with light stops because RG evolution will typically drive those tachyonic)

Arkani Hamed et al '97

✗ Still many parameters

U(2) MODELS

✗ Unify the first two generations in doublets of a U(2) symmetry

✗ Make the third generation a total flavor singlet (top Yukawa!).

$$\frac{\langle \phi \rangle}{\Lambda} = \begin{pmatrix} 0 \\ \epsilon_\phi \end{pmatrix} \qquad \frac{\langle \chi \rangle}{\Lambda} = \epsilon_\chi$$

Barbieri, Dvali, Hall '95
Barbieri, Hall, Romanino, '96

U(2) MODELS

✗ Unify the first two generations in doublets of a U(2) symmetry

✗ Make the third generation a total flavor singlet (top Yukawa!).

$$\frac{\langle\phi\rangle}{\Lambda} = \begin{pmatrix} 0 \\ \epsilon_\phi \end{pmatrix} \qquad \frac{\langle\chi\rangle}{\Lambda} = \epsilon_\chi$$

Barbieri, Dvali, Hall '95
Barbieri, Hall, Romanino, '96

$$Y_{ij} = \begin{pmatrix} 0 & Y_{12} & 0 \\ -Y_{12} & Y_{22} & Y_{23} \\ 0 & Y_{32} & Y_{33} \end{pmatrix}$$

✗ up and down sectors work in the same way ($Y^u \sim Y^d$)

(suppression factors $\epsilon_\chi, \ \epsilon_\phi$ are not displayed here...)

U(2) MODELS

✗ Unify the first two generations in doublets of a U(2) symmetry

✗ Make the third generation a total flavor singlet (top Yukawa!).

$$\frac{\langle\phi\rangle}{\Lambda} = \begin{pmatrix} 0 \\ \epsilon_\phi \end{pmatrix} \qquad \frac{\langle\chi\rangle}{\Lambda} = \epsilon_\chi$$

Barbieri, Dvali, Hall '95
Barbieri, Hall, Romanino, '96

$$Y_{ij} = \begin{pmatrix} 0 & Y_{12} & 0 \\ -Y_{12} & Y_{22} & Y_{23} \\ 0 & Y_{32} & Y_{33} \end{pmatrix}$$

✗ up and down sectors work in the same way ($Y^u \sim Y^d$)

(suppression factors $\epsilon_\chi,\ \epsilon_\phi$ are not displayed here...)

✗ The quark rotations only depend on 4 angles (+4 phases)

✗ SUSY: Splitting of 1st and 2nd squark generation suppressed as ϵ_ϕ^2

PROBLEMS OF U(2) MODELS

✗ U(2) models make some striking predictions for quark data

$$|V_{us}| \approx \sqrt{m_d/m_s}\ , \qquad |V_{ub}/V_{cb}| \approx \sqrt{m_u/m_c}\ , \qquad |V_{td}/V_{ts}| \approx \sqrt{m_d/m_s}$$

✗ At leading order in the $\epsilon_\chi,\ \epsilon_\phi$ expansion, but no O(1) numbers!!

PROBLEMS OF U(2) MODELS

✗ U(2) models make some striking predictions for quark data

$$|V_{us}| \approx \sqrt{m_d/m_s}\ , \qquad |V_{ub}/V_{cb}| \approx \sqrt{m_u/m_c}\ , \qquad |V_{td}/V_{ts}| \approx \sqrt{m_d/m_s}$$

✗ At leading order in the $\epsilon_\chi,\ \epsilon_\phi$ expansion, but no O(1) numbers!!

$$\sqrt{m_d/m_s} = 0.22 \pm 0.02\ , \qquad \sqrt{m_u/m_c} = 0.046 \pm 0.008\ ,$$

$$|V_{us}| = 0.2253 \pm 0.0007\ , \quad |V_{ub}/V_{cb}| = 0.085 \pm 0.004\ , \quad |V_{td}/V_{ts}| = 0.22 \pm 0.01$$

PROBLEMS OF U(2) MODELS

✗ U(2) models make some striking predictions for quark data

$$|V_{us}| \approx \sqrt{m_d/m_s}\ , \qquad |V_{ub}/V_{cb}| \approx \sqrt{m_u/m_c}\ , \qquad |V_{td}/V_{ts}| \approx \sqrt{m_d/m_s}$$

✗ At leading order in the $\epsilon_\chi,\ \epsilon_\phi$ expansion, but no O(1) numbers!!

$$\sqrt{m_d/m_s} = 0.22 \pm 0.02\ , \qquad \sqrt{m_u/m_c} = 0.046 \pm 0.008\ ,$$

$$|V_{us}| = 0.2253 \pm 0.0007\ , \qquad |V_{ub}/V_{cb}| = 0.085 \pm 0.004\ , \qquad |V_{td}/V_{ts}| = 0.22 \pm 0.01$$

PROBLEMS OF U(2) MODELS

✗ U(2) models make some striking predictions for quark data

$$|V_{us}| \approx \sqrt{m_d/m_s}\ , \quad |V_{ub}/V_{cb}| \approx \sqrt{m_u/m_c}\ , \quad |V_{td}/V_{ts}| \approx \sqrt{m_d/m_s}$$

✓

✗ At leading order in the $\epsilon_\chi,\ \epsilon_\phi$ expansion, but no O(1) numbers!!

$$\sqrt{m_d/m_s} = 0.22 \pm 0.02\ , \qquad \sqrt{m_u/m_c} = 0.046 \pm 0.008\ ,$$

$$|V_{us}| = 0.2253 \pm 0.0007\ , \quad |V_{ub}/V_{cb}| = 0.085 \pm 0.004\ , \quad |V_{td}/V_{ts}| = 0.22 \pm 0.01$$

PROBLEMS OF U(2) MODELS

✗ U(2) models make some striking predictions for quark data

$$|V_{us}| \approx \sqrt{m_d/m_s}\ , \qquad |V_{ub}/V_{cb}| \approx \sqrt{m_u/m_c}\ , \qquad |V_{td}/V_{ts}| \approx \sqrt{m_d/m_s}$$

✓

✗ At leading order in the $\epsilon_\chi,\ \epsilon_\phi$ expansion, but no O(1) numbers!!

$$\sqrt{m_d/m_s} = 0.22 \pm 0.02\ , \qquad \sqrt{m_u/m_c} = 0.046 \pm 0.008\ ,$$

$$|V_{us}| = 0.2253 \pm 0.0007\ , \qquad |V_{ub}/V_{cb}| = 0.085 \pm 0.004\ , \qquad |V_{td}/V_{ts}| = 0.22 \pm 0.01$$

PROBLEMS OF U(2) MODELS

✗ U(2) models make some striking predictions for quark data

$$|V_{us}| \approx \sqrt{m_d/m_s}\ , \quad ✓ \qquad |V_{ub}/V_{cb}| \approx \sqrt{m_u/m_c}\ , \qquad |V_{td}/V_{ts}| \approx \sqrt{m_d/m_s} \quad ✓$$

✗ At leading order in the ϵ_χ, ϵ_ϕ expansion, but no O(1) numbers!!

$$\sqrt{m_d/m_s} = 0.22 \pm 0.02\ , \qquad \sqrt{m_u/m_c} = 0.046 \pm 0.008\ ,$$

$$|V_{us}| = 0.2253 \pm 0.0007\ , \quad |V_{ub}/V_{cb}| = 0.085 \pm 0.004\ , \quad |V_{td}/V_{ts}| = 0.22 \pm 0.01$$

PROBLEMS OF U(2) MODELS

✗ U(2) models make some striking predictions for quark data

$$|V_{us}| \approx \sqrt{m_d/m_s}\ , \quad |V_{ub}/V_{cb}| \approx \sqrt{m_u/m_c}\ , \quad |V_{td}/V_{ts}| \approx \sqrt{m_d/m_s}$$

✓ ✓

✗ At leading order in the $\epsilon_\chi,\ \epsilon_\phi$ expansion, but no O(1) numbers!!

$$\sqrt{m_d/m_s} = 0.22 \pm 0.02\ , \quad \sqrt{m_u/m_c} = 0.046 \pm 0.008\ ,$$

$$|V_{us}| = 0.2253 \pm 0.0007\ , \quad |V_{ub}/V_{cb}| = 0.085 \pm 0.004\ , \quad |V_{td}/V_{ts}| = 0.22 \pm 0.01$$

PROBLEMS OF U(2) MODELS

✗ U(2) models make some striking predictions for quark data

$$|V_{us}| \approx \sqrt{m_d/m_s} \ , \quad ✓ \qquad |V_{ub}/V_{cb}| \approx \sqrt{m_u/m_c} \ , \quad ✗ \qquad |V_{td}/V_{ts}| \approx \sqrt{m_d/m_s} \quad ✓$$

✗ At leading order in the ϵ_χ, ϵ_ϕ expansion, but no O(1) numbers!!

$$\sqrt{m_d/m_s} = 0.22 \pm 0.02 \ , \qquad \sqrt{m_u/m_c} = 0.046 \pm 0.008 \ ,$$

$$|V_{us}| = 0.2253 \pm 0.0007 \ , \quad |V_{ub}/V_{cb}| = 0.085 \pm 0.004 \ , \quad |V_{td}/V_{ts}| = 0.22 \pm 0.01$$

PROBLEMS OF U(2) MODELS

✗ U(2) models make some striking predictions for quark data

$$|V_{us}| \approx \sqrt{m_d/m_s}\ , \quad ✓ \qquad |V_{ub}/V_{cb}| \approx \sqrt{m_u/m_c}\ , \quad ✗ \qquad |V_{td}/V_{ts}| \approx \sqrt{m_d/m_s} \quad ✓$$

✗ At leading order in the $\epsilon_\chi,\ \epsilon_\phi$ expansion, but no O(1) numbers!!

$$\sqrt{m_d/m_s} = 0.22 \pm 0.02\ , \qquad \sqrt{m_u/m_c} = 0.046 \pm 0.008\ ,$$

$$|V_{us}| = 0.2253 \pm 0.0007\ , \quad |V_{ub}/V_{cb}| = 0.085 \pm 0.004\ , \quad |V_{td}/V_{ts}| = 0.22 \pm 0.01$$

✗ The reason of this discrepancy turns out to be the fact that b_R is a total flavor singlet ($X_{b_R} = 0$) which leads to a strong RH hierarchy in the down sector:

$$Y^d_{i1} \ll Y^d_{i2} \ll Y^d_{i3}$$

PROBLEMS OF U(2) MODELS

✗ U(2) models make some striking predictions for quark data

$$|V_{us}| \approx \sqrt{m_d/m_s}\ , \checkmark \qquad |V_{ub}/V_{cb}| \approx \sqrt{m_u/m_c}\ , ✗ \qquad |V_{td}/V_{ts}| \approx \sqrt{m_d/m_s} \checkmark$$

✗ At leading order in the $\epsilon_\chi,\ \epsilon_\phi$ expansion, but no O(1) numbers!!

$$\sqrt{m_d/m_s} = 0.22 \pm 0.02\ , \qquad \sqrt{m_u/m_c} = 0.046 \pm 0.008\ ,$$

$$|V_{us}| = 0.2253 \pm 0.0007\ , \quad |V_{ub}/V_{cb}| = 0.085 \pm 0.004\ , \quad |V_{td}/V_{ts}| = 0.22 \pm 0.01$$

✗ The reason of this discrepancy turns out to be the fact that b_R is a total flavor singlet ($X_{b_R} = 0$) which leads to a strong RH hierarchy in the down sector:

$$\cancel{Y^d_{i1} \ll Y^d_{i2} \ll Y^d_{i3}} \xrightarrow{\text{Fit prefers}} Y^d_{i1} \sim Y^d_{i2} \sim Y^d_{i3}$$

QUARK NUMEROLOGY

Typical model:

$$(Y^d)_{ij} \sim \epsilon^{X_{q^i} + X_{d^j}}$$

$$(Y^u)_{ij} \sim \epsilon^{X_{q^i} + X_{u^j}}$$

QUARK NUMEROLOGY

Typical model:

$$(Y^d)_{ij} \sim \epsilon^{X_{q^i} + X_{d^j}}$$

$$(Y^u)_{ij} \sim \epsilon^{X_{q^i} + X_{u^j}}$$

Eigenvalues:

$$y_i^d \sim \epsilon^{X_{q^i} + X_{d^i}}$$

$$y_i^u \sim \epsilon^{X_{q^i} + X_{u^i}}$$

QUARK NUMEROLOGY

Typical model:

$$(Y^d)_{ij} \sim \epsilon^{X_{q^i} + X_{d^j}}$$

$$(Y^u)_{ij} \sim \epsilon^{X_{q^i} + X_{u^j}}$$

Eigenvalues:

$$y_i^d \sim \epsilon^{X_{q^i} + X_{d^i}}$$

$$y_i^u \sim \epsilon^{X_{q^i} + X_{u^i}}$$

CKM angles:

$$|V_{ij}| \sim \epsilon^{|X_{q^i} - X_{q^j}|}$$

QUARK NUMEROLOGY

Typical model:

$$(Y^d)_{ij} \sim \epsilon^{X_{q^i}+X_{d^j}}$$

$$(Y^u)_{ij} \sim \epsilon^{X_{q^i}+X_{u^j}}$$

Eigenvalues:

$$y_i^d \sim \epsilon^{X_{q^i}+X_{d^i}}$$

$$y_i^u \sim \epsilon^{X_{q^i}+X_{u^i}}$$

CKM angles:

$$|V_{ij}| \sim \epsilon^{|X_{q^i}-X_{q^j}|}$$

$$\epsilon^{X_{q^1}} : \epsilon^{X_{q^2}} : \epsilon^{X_{q^3}} = 1 : 5 : 125$$

QUARK NUMEROLOGY

Typical model:

$$(Y^d)_{ij} \sim \epsilon^{X_{q^i} + X_{d^j}}$$

$$(Y^u)_{ij} \sim \epsilon^{X_{q^i} + X_{u^j}}$$

Eigenvalues:

$$y_i^d \sim \epsilon^{X_{q^i} + X_{d^i}}$$

$$y_i^u \sim \epsilon^{X_{q^i} + X_{u^i}}$$

CKM angles:

$$|V_{ij}| \sim \epsilon^{|X_{q^i} - X_{q^j}|}$$

$$\epsilon^{X_{q^1}} : \epsilon^{X_{q^2}} : \epsilon^{X_{q^3}} = 1 : 5 : 125$$

⇒ hierarchy

QUARK NUMEROLOGY

Typical model:

$$(Y^d)_{ij} \sim \epsilon^{X_{q^i}+X_{d^j}}$$

$$(Y^u)_{ij} \sim \epsilon^{X_{q^i}+X_{u^j}}$$

Eigenvalues:

$$y_i^d \sim \epsilon^{X_{q^i}+X_{d^i}}$$

$$y_i^u \sim \epsilon^{X_{q^i}+X_{u^i}}$$

CKM angles:

$$|V_{ij}| \sim \epsilon^{|X_{q^i}-X_{q^j}|}$$

$$\epsilon^{X_{q^1}} : \epsilon^{X_{q^2}} : \epsilon^{X_{q^3}} = 1 : 5 : 125$$

⇒ hierarchy

$$\epsilon^{X_{q^1}+X_{u^1}} : \epsilon^{X_{q^2}+X_{u^2}} : \epsilon^{X_{q^3}+X_{u^3}} = 1 : 560 : 75000$$

QUARK NUMEROLOGY

Typical model:

$$(Y^d)_{ij} \sim \epsilon^{X_{q^i} + X_{d^j}}$$

$$(Y^u)_{ij} \sim \epsilon^{X_{q^i} + X_{u^j}}$$

Eigenvalues:

$$y_i^d \sim \epsilon^{X_{q^i} + X_{d^i}}$$

$$y_i^u \sim \epsilon^{X_{q^i} + X_{u^i}}$$

CKM angles:

$$|V_{ij}| \sim \epsilon^{|X_{q^i} - X_{q^j}|}$$

$$\epsilon^{X_{q^1}} : \epsilon^{X_{q^2}} : \epsilon^{X_{q^3}} = 1 : 5 : 125$$

$$\epsilon^{X_{u^1}} : \epsilon^{X_{u^2}} : \epsilon^{X_{u^3}} = 1 : 110 : 600$$

$\Rightarrow$ hierarchy

QUARK NUMEROLOGY

Typical model:

$$(Y^d)_{ij} \sim \epsilon^{X_{q^i} + X_{d^j}}$$

$$(Y^u)_{ij} \sim \epsilon^{X_{q^i} + X_{u^j}}$$

Eigenvalues:

$$y_i^d \sim \epsilon^{X_{q^i} + X_{d^i}}$$

$$y_i^u \sim \epsilon^{X_{q^i} + X_{u^i}}$$

CKM angles:

$$|V_{ij}| \sim \epsilon^{|X_{q^i} - X_{q^j}|}$$

$$\epsilon^{X_{q^1}} : \epsilon^{X_{q^2}} : \epsilon^{X_{q^3}} = 1 : 5 : 125$$

⇒ hierarchy

$$\epsilon^{X_{u^1}} : \epsilon^{X_{u^2}} : \epsilon^{X_{u^3}} = 1 : 110 : 600$$

⇒ hierarchy

QUARK NUMEROLOGY

Typical model:

$$(Y^d)_{ij} \sim \epsilon^{X_{q^i}+X_{d^j}}$$

$$(Y^u)_{ij} \sim \epsilon^{X_{q^i}+X_{u^j}}$$

Eigenvalues:

$$y_i^d \sim \epsilon^{X_{q^i}+X_{d^i}}$$

$$y_i^u \sim \epsilon^{X_{q^i}+X_{u^i}}$$

CKM angles:

$$|V_{ij}| \sim \epsilon^{|X_{q^i}-X_{q^j}|}$$

$$\epsilon^{X_{q^1}} : \epsilon^{X_{q^2}} : \epsilon^{X_{q^3}} = 1 : 5 : 125$$

$\Rightarrow$ hierarchy

$$\epsilon^{X_{u^1}} : \epsilon^{X_{u^2}} : \epsilon^{X_{u^3}} = 1 : 110 : 600$$

$\Rightarrow$ hierarchy

$$\epsilon^{X_{q^1}+X_{d^1}} : \epsilon^{X_{q^2}+X_{d^2}} : \epsilon^{X_{q^3}+X_{d^3}} = 1 : 20 : 800$$

QUARK NUMEROLOGY

Typical model:

$$(Y^d)_{ij} \sim \epsilon^{X_{q^i} + X_{d^j}}$$

$$(Y^u)_{ij} \sim \epsilon^{X_{q^i} + X_{u^j}}$$

Eigenvalues:

$$y_i^d \sim \epsilon^{X_{q^i} + X_{d^i}}$$

$$y_i^u \sim \epsilon^{X_{q^i} + X_{u^i}}$$

CKM angles:

$$|V_{ij}| \sim \epsilon^{|X_{q^i} - X_{q^j}|}$$

$$\epsilon^{X_{q^1}} : \epsilon^{X_{q^2}} : \epsilon^{X_{q^3}} = 1 : 5 : 125$$

⇒ hierarchy

$$\epsilon^{X_{u^1}} : \epsilon^{X_{u^2}} : \epsilon^{X_{u^3}} = 1 : 110 : 600$$

⇒ hierarchy

$$\epsilon^{X_{d^1}} : \epsilon^{X_{d^2}} : \epsilon^{X_{d^3}} = 1 : 4 : 6.5$$

QUARK NUMEROLOGY

Typical model:

$$(Y^d)_{ij} \sim \epsilon^{X_{q^i}+X_{d^j}}$$

$$(Y^u)_{ij} \sim \epsilon^{X_{q^i}+X_{u^j}}$$

Eigenvalues:

$$y_i^d \sim \epsilon^{X_{q^i}+X_{d^i}}$$

$$y_i^u \sim \epsilon^{X_{q^i}+X_{u^i}}$$

CKM angles:

$$|V_{ij}| \sim \epsilon^{|X_{q^i}-X_{q^j}|}$$

$$\epsilon^{X_{q^1}} : \epsilon^{X_{q^2}} : \epsilon^{X_{q^3}} = 1 : 5 : 125$$

⇒ hierarchy

$$\epsilon^{X_{u^1}} : \epsilon^{X_{u^2}} : \epsilon^{X_{u^3}} = 1 : 110 : 600$$

⇒ hierarchy

$$\epsilon^{X_{d^1}} : \epsilon^{X_{d^2}} : \epsilon^{X_{d^3}} = 1 : 4 : 6.5$$

⇒ no hierarchy

QUARK NUMEROLOGY

Typical model:

$$(Y^d)_{ij} \sim \epsilon^{X_{q^i}+X_{d^j}}$$

$$(Y^u)_{ij} \sim \epsilon^{X_{q^i}+X_{u^j}}$$

Eigenvalues:

$$y_i^d \sim \epsilon^{X_{q^i}+X_{d^i}}$$

$$y_i^u \sim \epsilon^{X_{q^i}+X_{u^i}}$$

CKM angles:

$$|V_{ij}| \sim \epsilon^{|X_{q^i}-X_{q^j}|}$$

$$\epsilon^{X_{q^1}} : \epsilon^{X_{q^2}} : \epsilon^{X_{q^3}} = 1 : 5 : 125$$

$\Rightarrow$ hierarchy

$$\epsilon^{X_{u^1}} : \epsilon^{X_{u^2}} : \epsilon^{X_{u^3}} = 1 : 110 : 600$$

$\Rightarrow$ hierarchy

$$\epsilon^{X_{d^1}} : \epsilon^{X_{d^2}} : \epsilon^{X_{d^3}} = 1 : 4 : 6.5$$

$\Rightarrow$ no hierarchy

THE SU(2) X U(1) MODEL

✗ Giving up RH down hierarchy ⇒ correction to the exact relations

THE SU(2) X U(1) MODEL

✗ Giving up RH down hierarchy ⇒ correction to the exact relations

$$|V_{td}/V_{ts}| \approx |\sqrt{m_d/m_s} + e^{i\beta'}\Delta\, t_d|\sqrt{c_d}$$

$$|V_{ub}/V_{cb}| \approx |\sqrt{m_u/m_c} + e^{i\beta}\Delta\, t_d\sqrt{c_d}|$$

$$|V_{us}| \approx \sqrt{m_d/m_s}\sqrt{c_d}$$

$$\Delta = \frac{\sqrt{m_s m_d}}{|V_{cb}|m_b} \approx 0.09$$

$$t_d = \tan\theta_d = \frac{Y^d_{32}}{Y^d_{33}}$$

Roberts et al '01
Dudas et al '13

THE SU(2) X U(1) MODEL

✗ Giving up RH down hierarchy ⇒ correction to the exact relations

$$|V_{td}/V_{ts}| \approx |\sqrt{m_d/m_s} + e^{i\beta'}\Delta\, t_d|\sqrt{c_d}$$

$$|V_{ub}/V_{cb}| \approx |\sqrt{m_u/m_c} + e^{i\beta}\Delta\, t_d\sqrt{c_d}|$$

$$|V_{us}| \approx \sqrt{m_d/m_s}\sqrt{c_d}$$

$$\Delta = \frac{\sqrt{m_s m_d}}{|V_{cb}|m_b} \approx 0.09$$

$$t_d = \tan\theta_d = \frac{Y^d_{32}}{Y^d_{33}}$$

Roberts et al '01
Dudas et al '13

✗ The exact relations get corrected by the RH down 23 angle

✗ Fit requires $t_d \approx 0.5$

✗ SUSY: RH sbottom must be heavy!

$$V^d_R = \begin{pmatrix} 1 & \cdot & \cdot \\ \cdot & c_d & s_d \\ \cdot & -s_d & c_d \end{pmatrix}$$

THE SU(2) X U(1) MODEL

✗ Giving up RH down hierarchy ⇒ correction to the exact relations

$$|V_{td}/V_{ts}| \approx |\sqrt{m_d/m_s} + e^{i\beta'}\Delta\, t_d|\sqrt{c_d}$$

$$|V_{ub}/V_{cb}| \approx |\sqrt{m_u/m_c} + e^{i\beta}\Delta\, t_d\sqrt{c_d}|$$

$$|V_{us}| \approx \sqrt{m_d/m_s}\sqrt{c_d}$$

$$\Delta = \frac{\sqrt{m_s m_d}}{|V_{cb}|m_b} \approx 0.09$$

$$t_d = \tan\theta_d = \frac{Y^d_{32}}{Y^d_{33}}$$

Roberts et al '01
Dudas et al '13

✗ The exact relations get corrected by the RH down 23 angle

✗ Fit requires $t_d \approx 0.5$

✗ SUSY: RH sbottom must be heavy!

$$V_R^d = \begin{pmatrix} 1 & \cdot & \cdot \\ \cdot & c_d & s_d \\ \cdot & -s_d & c_d \end{pmatrix}$$

Only the two stops can remain truly light

⇒ Spectrum of Natural SUSY

FITTING THE QUARK DATA

✗ An explicit fit to quark masses exhibits the following possibilities:
(Also impose here that the $U(1)_X$ commutes with $SU(5)$)

Model	ϵ_ϕ	ϵ_χ	$\tan\beta$	X_ϕ	X_{Q_i,U_i}	X_{Q_3,U_3}	X_{D_i}	X_{D_3}
A	0.02	0.02	5	-1	1	0	1	1
B	0.1	0.2	5	-2	3	0	3	2
B′	0.1	0.2	20	-2	3	0	2	1
C	0.2	0.1	50	-1	2	0	1	0

Dudas et al '13

FITTING THE QUARK DATA

✗ An explicit fit to quark masses exhibits the following possibilities:
(Also impose here that the $U(1)_X$ commutes with $SU(5)$)

Model	ϵ_ϕ	ϵ_χ	$\tan\beta$	X_ϕ	X_{Q_i,U_i}	X_{Q_3,U_3}	X_{D_i}	X_{D_3}
A	0.02	0.02	5	-1	1	0	1	1
B	0.1	0.2	5	-2	3	0	3	2
B′	0.1	0.2	20	-2	3	0	2	1
C	0.2	0.1	50	-1	2	0	1	0

Dudas et al '13

✗ The smaller the SU(2) breaking, the less important the subleading effects from soft terms ⇒ FCNCs dominated by quark rotations

FITTING THE QUARK DATA

✗ An explicit fit to quark masses exhibits the following possibilities:
(Also impose here that the $U(1)_X$ commutes with $SU(5)$)

Model	ϵ_ϕ	ϵ_χ	$\tan\beta$	X_ϕ	X_{Q_i,U_i}	X_{Q_3,U_3}	X_{D_i}	X_{D_3}
A	0.02	0.02	5	-1	1	0	1	1
B	0.1	0.2	5	-2	3	0	3	2
B′	0.1	0.2	20	-2	3	0	2	1
C	0.2	0.1	50	-1	2	0	1	0

Dudas et al '13

✗ The smaller the SU(2) breaking, the less important the subleading effects from soft terms ⇒ FCNCs dominated by quark rotations

✗ This is the case in model A

✗ Also benefits from $X_{D_i} = X_{D_3}$ (additional FCNC suppression)

D-TERM DOMINANCE

Summary: would like to build a SUSY model with

✗ SU(2)xU(1) flavour symmetry

✗ Light stops (and light LH sbottom) => Naturalness

✗ All other squarks heavy

D-TERM DOMINANCE

Summary: would like to build a SUSY model with

✗ SU(2)xU(1) flavour symmetry

✗ Light stops (and light LH sbottom) => Naturalness

✗ All other squarks heavy

✗ A very natural way to realize this spectrum is assuming that the dominant contribution to the soft terms arises from D-terms of the flavour U(1)

$$m_I^2 = m_D^2 X_I$$

Binetruy+Dudas '96
Dvali+Pomarol '96

✗ Quark masses: Only vanishing charges are the (s)tops, i.e.

$$X_{Q^3} = X_{U^3} = 0$$

✗ Other soft terms are generated by subleading F terms

✗ A typical hierarchy is $m_D = 15\ \mathrm{TeV},\ m_F = 4\ \mathrm{TeV}$

D-TERM DOMINANCE

Summary: would like to build a SUSY model with

✗ SU(2)xU(1) flavour symmetry

✗ Light stops (and light LH sbottom) => Naturalness

✗ All other squarks heavy

✗ A very natural way to realize this spectrum is assuming that the dominant contribution to the soft terms arises from D-terms of the flavour U(1)

$$m_I^2 = m_D^2 X_I$$

Binetruy+Dudas '96
Dvali+Pomarol '96

✗ Quark masses: Only vanishing charges are the (s)tops, i.e.

$$X_{Q^3} = X_{U^3} = 0$$

✗ Other soft terms are generated by subleading F terms

✗ A typical hierarchy is $m_D = 15 \text{ TeV}, \; m_F = 4 \text{ TeV}$

@ high scale!

CONSTRAINTS FROM KAONS

✗ Effective Lagrangian: Wilson coefficient of $\mathcal{O}_4 = (\bar{d}_R s_L)(\bar{d}_L s_R)$

$$\mathrm{Im}\,\Delta C_4 \;\approx 1.6\times 10^{-8}\left(\frac{|V_{23}^d|}{0.04}\right)^2\left(\frac{s_d^2}{0.2}\right)\left(\frac{\sin\alpha_{12}}{0.5}\right)\left(\tilde{m}_{d_R}^2-\tilde{m}_{b_R}^2\right)\,\frac{\log\left(\frac{\tilde{m}_{d_R}}{m_{\tilde{g}}}\right)+\frac{1}{4}}{(\tilde{m}_{d_R})^4}$$

CONSTRAINTS FROM KAONS

✗ Effective Lagrangian: Wilson coefficient of $\mathcal{O}_4 = (\bar{d}_R s_L)(\bar{d}_L s_R)$

$$\mathrm{Im}\,\Delta C_4 \approx 1.6 \times 10^{-8} \left(\frac{|V^d_{23}|}{0.04}\right)^2 \left(\frac{s^2_d}{0.2}\right) \left(\frac{\sin\alpha_{12}}{0.5}\right) (\tilde{m}^2_{d_R} - \tilde{m}^2_{b_R}) \,\frac{\log\left(\frac{\tilde{m}_{d_R}}{m_{\tilde{g}}}\right) + \frac{1}{4}}{(\tilde{m}_{d_R})^4}$$

angles

CONSTRAINTS FROM KAONS

✗ Effective Lagrangian: Wilson coefficient of $\mathcal{O}_4 = (\bar{d}_R s_L)(\bar{d}_L s_R)$

$$\mathrm{Im}\,\Delta C_4 \approx 1.6 \times 10^{-8} \left(\frac{|V^d_{23}|}{0.04}\right)^2 \left(\frac{s^2_d}{0.2}\right) \left(\frac{\sin\alpha_{12}}{0.5}\right) (\tilde{m}^2_{d_R} - \tilde{m}^2_{b_R}) \frac{\log\left(\frac{\tilde{m}_{d_R}}{m_{\tilde{g}}}\right) + \frac{1}{4}}{(\tilde{m}_{d_R})^4}$$

angles

phase

CONSTRAINTS FROM KAONS

✗ Effective Lagrangian: Wilson coefficient of $\mathcal{O}_4 = (\bar{d}_R s_L)(\bar{d}_L s_R)$

$$\mathrm{Im}\,\Delta C_4 \approx 1.6\times 10^{-8}\left(\frac{|V^d_{23}|}{0.04}\right)^2\left(\frac{s_d^2}{0.2}\right)\left(\frac{\sin\alpha_{12}}{0.5}\right)\left(\tilde{m}^2_{d_R}-\tilde{m}^2_{b_R}\right)\frac{\log\left(\frac{\tilde{m}_{d_R}}{m_{\tilde{g}}}\right)+\frac{1}{4}}{(\tilde{m}_{d_R})^4}$$

angles

phase

✗ dominant splitting
✗ GIM suppression!

CONSTRAINTS FROM KAONS

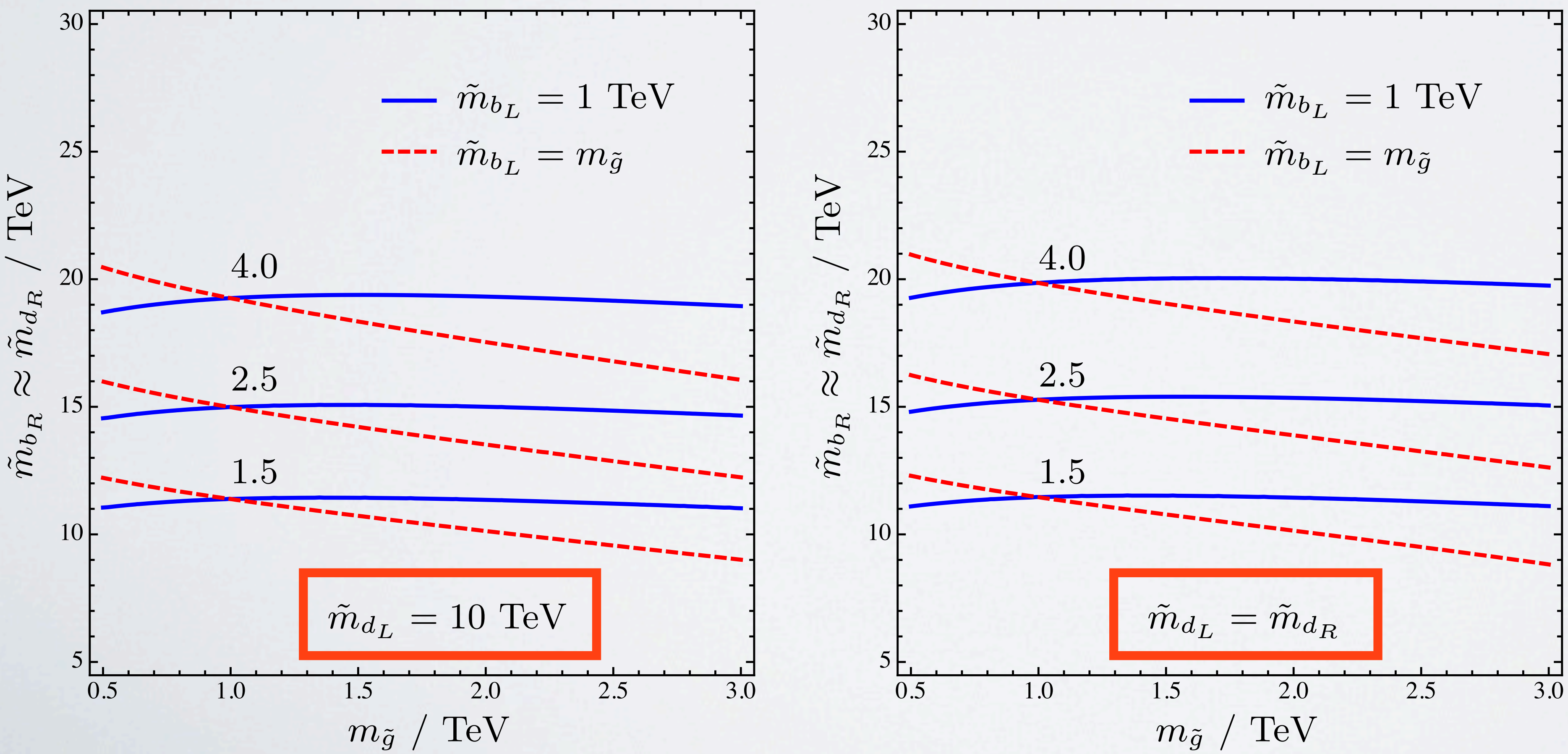
✗ Effective Lagrangian: Wilson coefficient of $\mathcal{O}_4 = (\bar{d}_R s_L)(\bar{d}_L s_R)$

$$\mathrm{Im}\,\Delta C_4 \approx 1.6 \times 10^{-8} \left(\frac{|V_{23}^d|}{0.04}\right)^2 \left(\frac{s_d^2}{0.2}\right) \left(\frac{\sin\alpha_{12}}{0.5}\right) (\tilde{m}_{d_R}^2 - \tilde{m}_{b_R}^2)\, \frac{\log\left(\frac{\tilde{m}_{d_R}}{m_{\tilde{g}}}\right) + \frac{1}{4}}{(\tilde{m}_{d_R})^4}$$

angles

phase

✗ dominant splitting
✗ GIM suppression!



CONSTRAINTS FROM KAONS

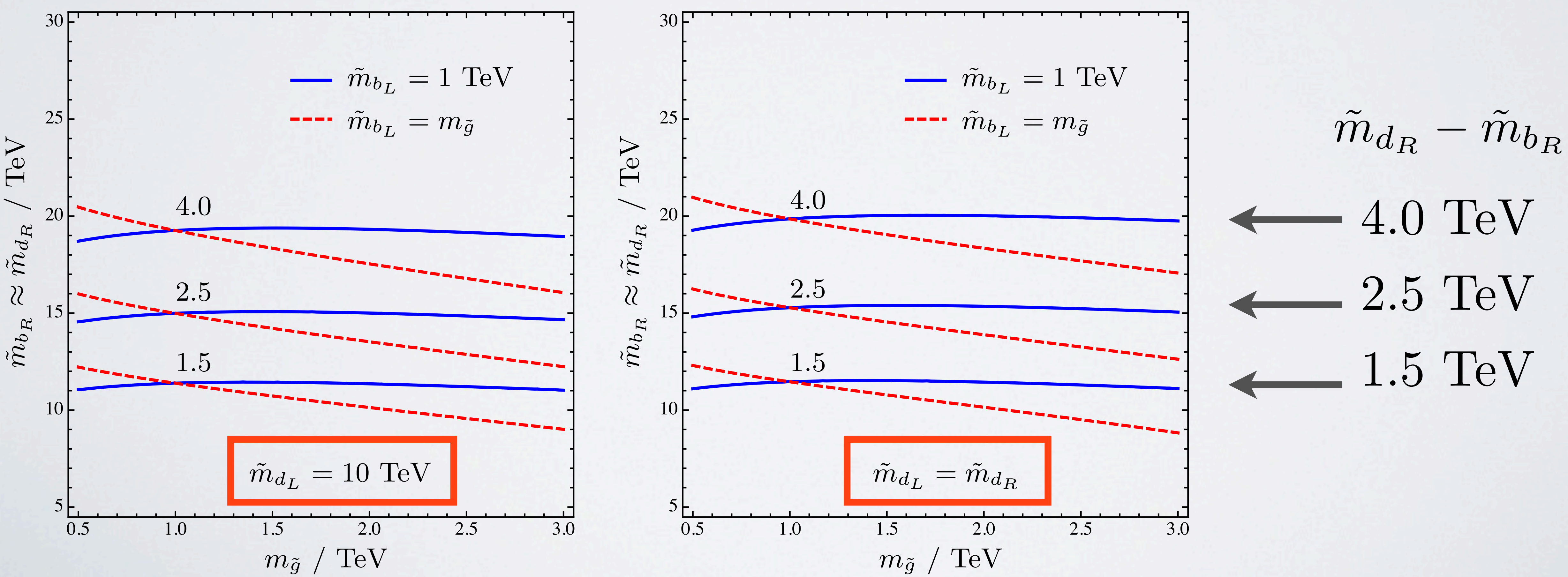
✗ Effective Lagrangian: Wilson coefficient of $\mathcal{O}_4 = (\bar{d}_R s_L)(\bar{d}_L s_R)$

$$\mathrm{Im}\,\Delta C_4 \approx 1.6 \times 10^{-8} \left(\frac{|V^d_{23}|}{0.04}\right)^2 \left(\frac{s^2_d}{0.2}\right) \left(\frac{\sin\alpha_{12}}{0.5}\right) \left(\tilde{m}^2_{d_R} - \tilde{m}^2_{b_R}\right) \frac{\log\left(\frac{\tilde{m}_{d_R}}{m_{\tilde{g}}}\right) + \frac{1}{4}}{(\tilde{m}_{d_R})^4}$$

angles

phase

✗ dominant splitting
✗ GIM suppression!



CONSTRAINTS FROM KAONS

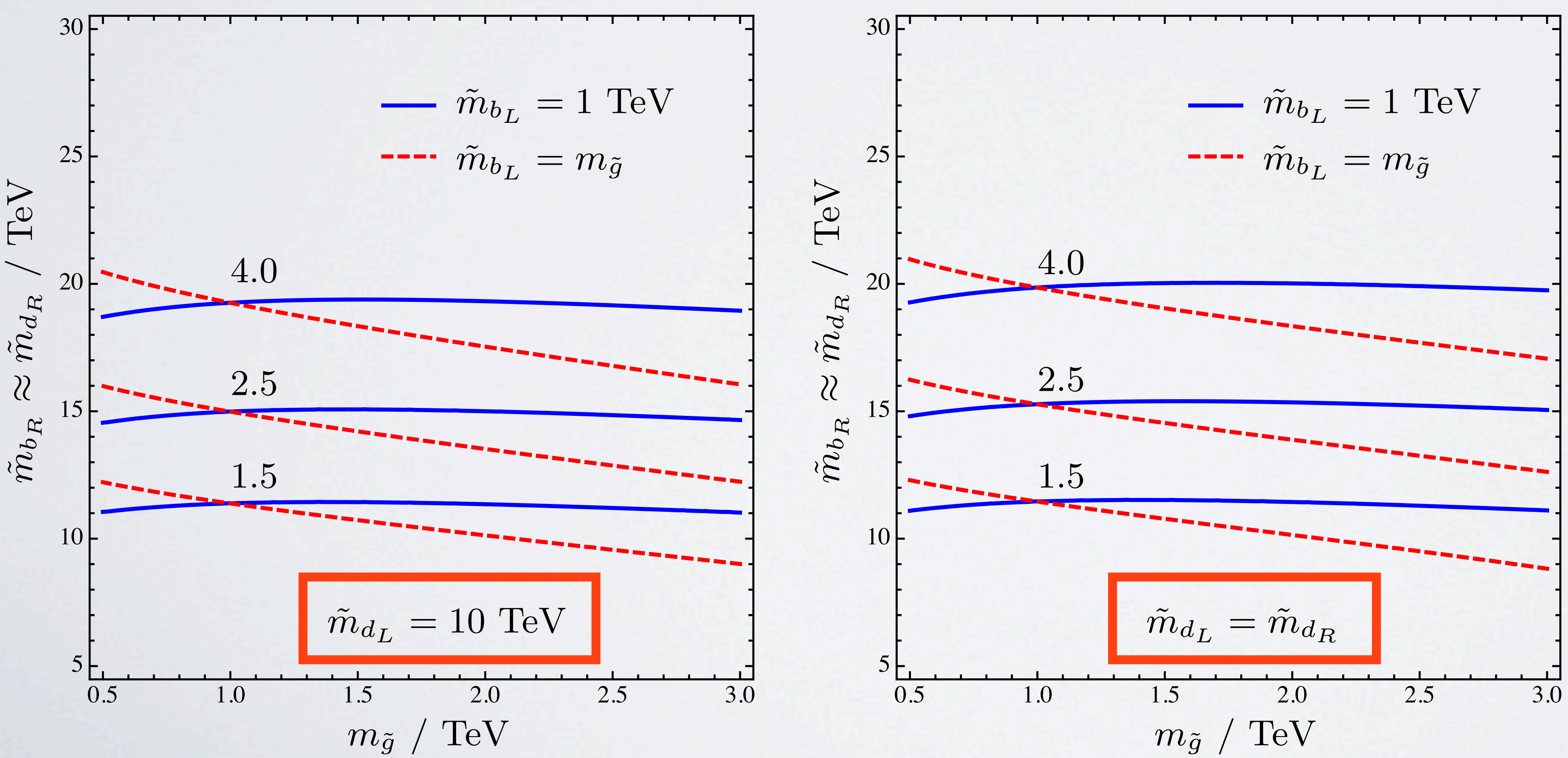
✗ Effective Lagrangian: Wilson coefficient of $\mathcal{O}_4 = (\bar{d}_R s_L)(\bar{d}_L s_R)$

$$\mathrm{Im}\,\Delta C_4 \approx 1.6\times 10^{-8} \left(\frac{|V^d_{23}|}{0.04}\right)^2 \left(\frac{s_d^2}{0.2}\right)\left(\frac{\sin\alpha_{12}}{0.5}\right)(\tilde{m}^2_{d_R}-\tilde{m}^2_{b_R})\,\frac{\log\left(\frac{\tilde{m}_{d_R}}{m_{\tilde{g}}}\right)+\frac{1}{4}}{(\tilde{m}_{d_R})^4}$$

angles

phase

✗ dominant splitting
✗ GIM suppression!



$\tilde{m}_{d_R} - \tilde{m}_{b_R}$

4.0 TeV

2.5 TeV

1.5 TeV

@TeV scale!

MAPPING TO THE HIGH SCALE

✗ The bounds apply to the physical masses at the TeV scale

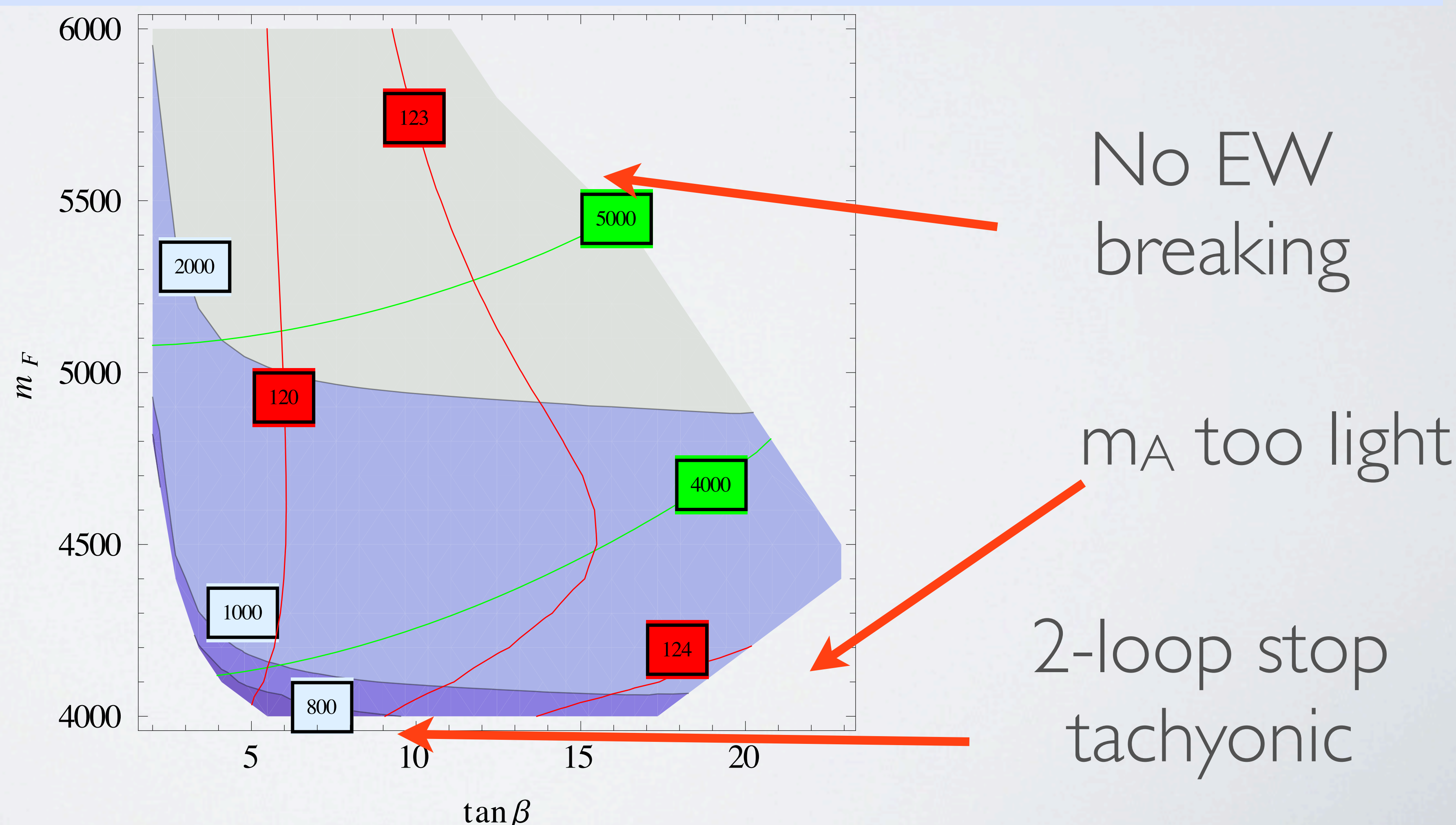
✗ Need to map this to the high scale, to determine $m_F,\ m_D$

✗ The splitting at the high scale between the 3rd and 1st two generations cannot be too large, otherwise the 3rd generation is driven to tachyonic values by 2-loop RG

$$m_D = 15\ \text{TeV}$$
$$m_{\tilde{g}}|_{M_{GUT}} = 0.6\ \text{TeV}$$

↓

$$m_{\tilde{q}^{1,2}_{L,R}} \approx m_{\tilde{b}_R} \approx 15\ \text{TeV}$$
$$m_{\tilde{g}} = 1.5\ \text{TeV}$$



MAPPING TO THE HIGH SCALE

✗ The bounds apply to the physical masses at the TeV scale

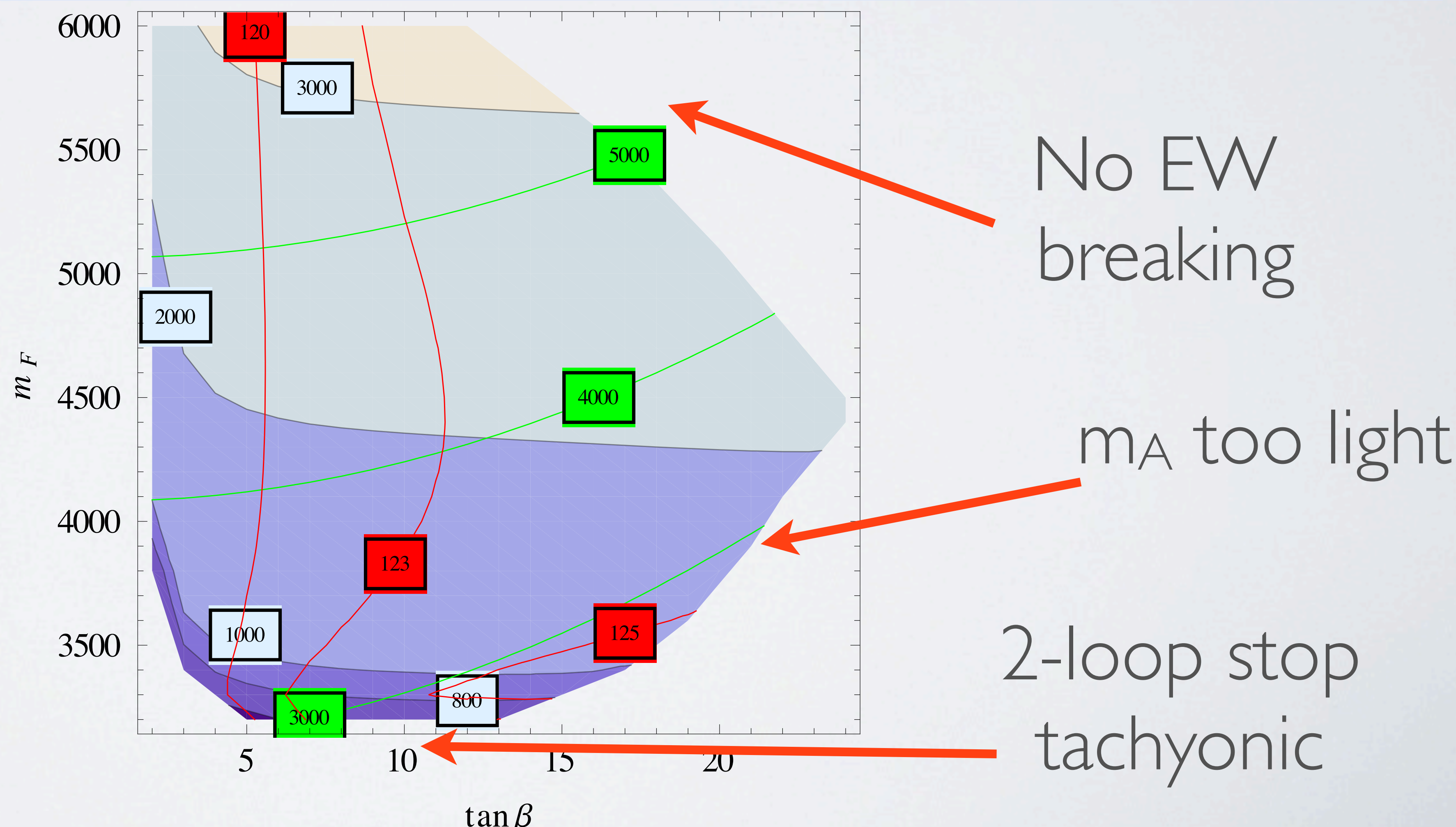
✗ Need to map this to the high scale, to determine $m_F,\ m_D$

✗ The splitting at the high scale between the 3rd and 1st two generations cannot be too large, otherwise the 3rd generation is driven to tachyonic values by 2-loop RG

$$m_D = 15\ \mathrm{TeV}$$
$$m_{\tilde{g}}|_{M_{GUT}} = 1.0\ \mathrm{TeV}$$

↓

$$m_{\tilde{q}^{1,2}_{L,R}} \approx m_{\tilde{b}_R} \approx 15\ \mathrm{TeV}$$
$$m_{\tilde{g}} = 2.5\ \mathrm{TeV}$$



MODEL BUILDING REQUIREMENTS

✗ Continuous SU(2) symmetry is problematic (Goldstone modes)

✗ Discrete subgroups may be a way out

✗ The simplest groups that do not spoil the Yukawa texture are the dicyclic groups $\tilde{D}_n, \; n \geq 3$ $\quad (\; t_1 = e^{\frac{\pi i}{n} \sigma_3} \, , \; t_2 = i\sigma_2 \;)$

Two possible ways:

✗ Nonrenormalizable breaking $SU(2) \to \tilde{D}_n$ (start with an SU(2) model + higher dim Kahler operators that break the Goldstone degeneracy and align the vev)

✗ Renormalizable breaking $SU(2) \to \tilde{D}_n$ (add fields in nontrivial representations of $\tilde{D}_n$), have constructed examples, but involved...

CONCLUSIONS

CONCLUSIONS

✗ SU(2)xU(1) provides an economical way of generating quark masses with relatively few parameters

✗ Quark rotations in the RH down sector are large!

CONCLUSIONS

✗ SU(2)xU(1) provides an economical way of generating quark masses with relatively few parameters

✗ Quark rotations in the RH down sector are large!

✗ LHC bounds on superpartners revive interest in "minimal" or "natural" susy, with just the stops and gluino light.

✗ Fits very well with the SU(2)xU(1) quark model

CONCLUSIONS

✗ SU(2)xU(1) provides an economical way of generating quark masses with relatively few parameters

✗ Quark rotations in the RH down sector are large!

✗ LHC bounds on superpartners revive interest in "minimal" or "natural" susy, with just the stops and gluino light.

✗ Fits very well with the SU(2)xU(1) quark model

✗ Natural realization: D-term dominance (stops are light because their U(1) charges are vanishing)

✗ One model (A) particular nice (small SU(2) breaking, $X_{D_i} = X_{D_3}$)

CONCLUSIONS

✗ SU(2)xU(1) provides an economical way of generating quark masses with relatively few parameters

✗ Quark rotations in the RH down sector are large!

✗ LHC bounds on superpartners revive interest in "minimal" or "natural" susy, with just the stops and gluino light.

✗ Fits very well with the SU(2)xU(1) quark model

✗ Natural realization: D-term dominance (stops are light because their U(1) charges are vanishing)

✗ One model (A) particular nice (small SU(2) breaking, $X_{D_i} = X_{D_3}$)

✗ Dominant FCNC bounds from ϵ_K, leading to $m_D > 10 - 20\ \mathrm{TeV}$

CONCLUSIONS

✗ SU(2)xU(1) provides an economical way of generating quark masses with relatively few parameters

✗ Quark rotations in the RH down sector are large!

✗ LHC bounds on superpartners revive interest in "minimal" or "natural" susy, with just the stops and gluino light.

✗ Fits very well with the SU(2)xU(1) quark model

✗ Natural realization: D-term dominance (stops are light because their U(1) charges are vanishing)

✗ One model (A) particular nice (small SU(2) breaking, $X_{D_i} = X_{D_3}$)

✗ Dominant FCNC bounds from ϵ_K, leading to $m_D > 10 - 20\ \mathrm{TeV}$

✗ Realistic model would be based on discrete subgroup of SU(2) (More model building needed)

BACKUP

QUARK ROTATIONS

$$V_L^u = \begin{pmatrix} e^{-i\alpha_{12}} & & \\ & 1 & \\ & & e^{i\alpha_{23}} \end{pmatrix} \begin{pmatrix} 1 & |V_{12}^u| & 0 \\ -|V_{12}^u| & 1 & |V_{23}^u| \\ |V_{12}^u V_{23}^u| & -|V_{23}^u| & 1 \end{pmatrix} \begin{pmatrix} e^{i(\tilde{\alpha}_{12}+\alpha_{12})} & & \\ & 1 & \\ & & e^{-i(\tilde{\alpha}_{23}+\alpha_{23})} \end{pmatrix}$$

exact rotations:

$$V_L^d = \begin{pmatrix} 1 & |V_{12}^d| & |V_{13}^d|\, e^{i\alpha_d} \\ -|V_{12}^d| & 1 & |V_{23}^d| \\ |V_{12}^d V_{23}^d| - |V_{13}^d|\, e^{-i\alpha_d} & -|V_{23}^d| & 1 \end{pmatrix} \begin{pmatrix} e^{i\tilde{\alpha}_{12}} & & \\ & 1 & \\ & & e^{-i\tilde{\alpha}_{23}} \end{pmatrix}$$

$$V_R^d = \begin{pmatrix} 1 & -|V_{12}^d|/c_d & 0 \\ |V_{12}^d| & c_d & |V_{32}^d| \\ -|V_{12}^d V_{32}^d|/c_d & -|V_{32}^d| & c_d \end{pmatrix} \begin{pmatrix} e^{-i\tilde{\alpha}_{12}} & & \\ & 1 & \\ & & e^{i(\tilde{\alpha}_{23}-\alpha_d)} \end{pmatrix}$$

$$V_R^u = \begin{pmatrix} 1 & -|V_{12}^u| & 0 \\ |V_{12}^u| & 1 & |V_{32}^u| \\ -|V_{12}^u V_{32}^u| & -|V_{32}^u| & 1 \end{pmatrix} \begin{pmatrix} e^{-i(\tilde{\alpha}_{12}+\alpha_{12})} & & \\ & 1 & \\ & & e^{i(\tilde{\alpha}_{23}+\alpha_{23}-\alpha_u)} \end{pmatrix}$$

with:

$$|V_{12}^u| = \sqrt{m_u/m_d} \qquad |V_{13}^d| = \sqrt{m_d m_s/m_b^2}\; s_d/\sqrt{c_d}$$

$$|V_{12}^d| = \sqrt{m_d/m_s}\sqrt{c_d} \qquad |V_{32}^d| = s_d$$

The remaining 4 angles ($V_{23}^d,\ V_{32}^d,\ V_{23}^u,\ V_{32}^u$) are free parameters

SQUARK MASS MATRICES

The full D-term squark mass matrices are

Q and U sectors

$$
\begin{aligned}
(\tilde{m}^2_{q,D})_{11} &= \left(X_{10} + z^q_{11}\, X_\phi\, \epsilon^2_\phi\right) \tilde{m}^2_D \\
(\tilde{m}^2_{q,D})_{22} &= \left(X_{10} + z^q_{22}\, X_\phi\, \epsilon^2_\phi\right) \tilde{m}^2_D \\
(\tilde{m}^2_{q,D})_{33} &= \left(z^q_{33} X_\phi\, \epsilon^2_\phi - z'^q_{33}\, \epsilon^2_\chi\right) \tilde{m}^2_D \\
(\tilde{m}^2_{q,D})_{12} &= 2\, z^q_{12}\, X_\phi\, \epsilon^2_\phi \epsilon_\chi^{2|X_\phi|}\, \tilde{m}^2_D \\
(\tilde{m}^2_{q,D})_{13} &= z^q_{13}\, (X_\phi - \tfrac{1}{2} X_{10})\, \epsilon_\phi \epsilon_\chi^{X_{10}-X_\phi}\, \tilde{m}^2_D \\
(\tilde{m}^2_{q,D})_{23} &= \left[z^q_{23}\, (X_\phi - \tfrac{1}{2} X_{10})\, \epsilon^2_\phi - z'^q_{23}\, (1 + \tfrac{1}{2} X_{10})\, \epsilon^2_\chi\right] \epsilon_\phi \epsilon_\chi^{X_{10}+X_\phi}\, \tilde{m}^2_D
\end{aligned}
$$

D sector

$$
\begin{aligned}
(\tilde{m}^2_{d,D})_{11} &= \left(X_{\bar{5}} + z^d_{11}\, X_\phi\, \epsilon^2_\phi\right) \tilde{m}^2_D\,, \\
(\tilde{m}^2_{d,D})_{22} &= \left(X_{\bar{5}} + z^d_{22}\, X_\phi\, \epsilon^2_\phi\right) \tilde{m}^2_D\,, \\
(\tilde{m}^2_{d,D})_{33} &= \left(X_3 + \left(z^d_{33}\, X_\phi \epsilon^2_\phi - z'^d_{33}\, \epsilon^2_\chi\right)\right) \tilde{m}^2_D\,, \\
(\tilde{m}^2_{d,D})_{12} &= 2\, z^d_{12}\, X_\phi\, \epsilon^2_\phi \epsilon_\chi^{2|X_\phi|}\, \tilde{m}^2_D\,, \\
(\tilde{m}^2_{d,D})_{13} &= z^d_{13}\, (X_\phi - \tfrac{1}{2} X_{53})\, \epsilon_\phi \epsilon_\chi^{X_{53}-X_\phi}\, \tilde{m}^2_D\,, \\
(\tilde{m}^2_{d,D})_{23} &= z^d_{23}\, (X_\phi + \tfrac{1}{2} X_{53})\, \epsilon_\phi \epsilon_\chi^{-X_{53}-X_\phi}\, \tilde{m}^2_D
\end{aligned}
$$

CONSTRAINTS FROM KAONS

Using unitarity of the rotations, one can cast the C4 coefficient as

$$\Delta C_4 \; = \; \frac{\alpha_s^2}{m_{\tilde g}^2}\left(\hat\delta_{12}^{d,RR}\Delta_{31}^{R} + \tilde\delta_{12}^{d,RR}\Delta_{21}^{R}\right)\left\{-\frac{1}{3}\left[\hat\delta_{12}^{d,LL}\, x_1^R\,\partial_R\left(\tilde f_4(x_3^L, x_1^R) - \tilde f_4(x_1^L, x_1^R)\right)\right.\right.$$
$$\left.\left. + \tilde\delta_{12}^{d,LL}\Delta_{21}^{L}\; x_1^L x_1^R\,\partial_L\partial_R\,\tilde f_4(x_1^L, x_1^R)\right] + \frac{7}{3}\left[\tilde f_4 \to f_4\right]\right\} \; ,$$

Keeping only the dominant splitting

$$\tilde\delta_{12}^{d,RR} \equiv (V_R^d)_{21}(V_R^d)^*_{22}\,, \qquad \hat\delta_{12}^{d,RR} \equiv (V_R^d)_{31}(V_R^d)^*_{32}$$
$$\tilde\delta_{12}^{d,LL} \equiv (V_L^d)^*_{21}(V_L^d)_{22}\,, \qquad \hat\delta_{12}^{d,LL} \equiv (V_L^d)^*_{31}(V_L^d)_{32}$$

$$\mathrm{Im}\,\Delta C_4 \;\approx 1.6\times 10^{-8}\left(\frac{|V_{23}^d|}{0.04}\right)^2\left(\frac{s_d^2}{0.2}\right)\left(\frac{\sin\alpha_{12}}{0.5}\right)(\tilde m_{d_R}^2 - \tilde m_{b_R}^2)\,\frac{\log\left(\frac{\tilde m_{d_R}}{m_{\tilde g}}\right) + \frac{1}{4}}{(\tilde m_{d_R})^4}$$