Lectures on Nuclear Structure –

What nuclei do and why: An empirical overview from a simple perspective

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Outline

- **Introduction, empirical survey – what nuclei do**

- **Independent particle model and Residual interactions**
  - **Particles in orbits in the nucleus**
  - **Residual interactions: robust predictions and simple physical interpretation**

- **Collective models -- Geometrical, algebraic (The IBA)**

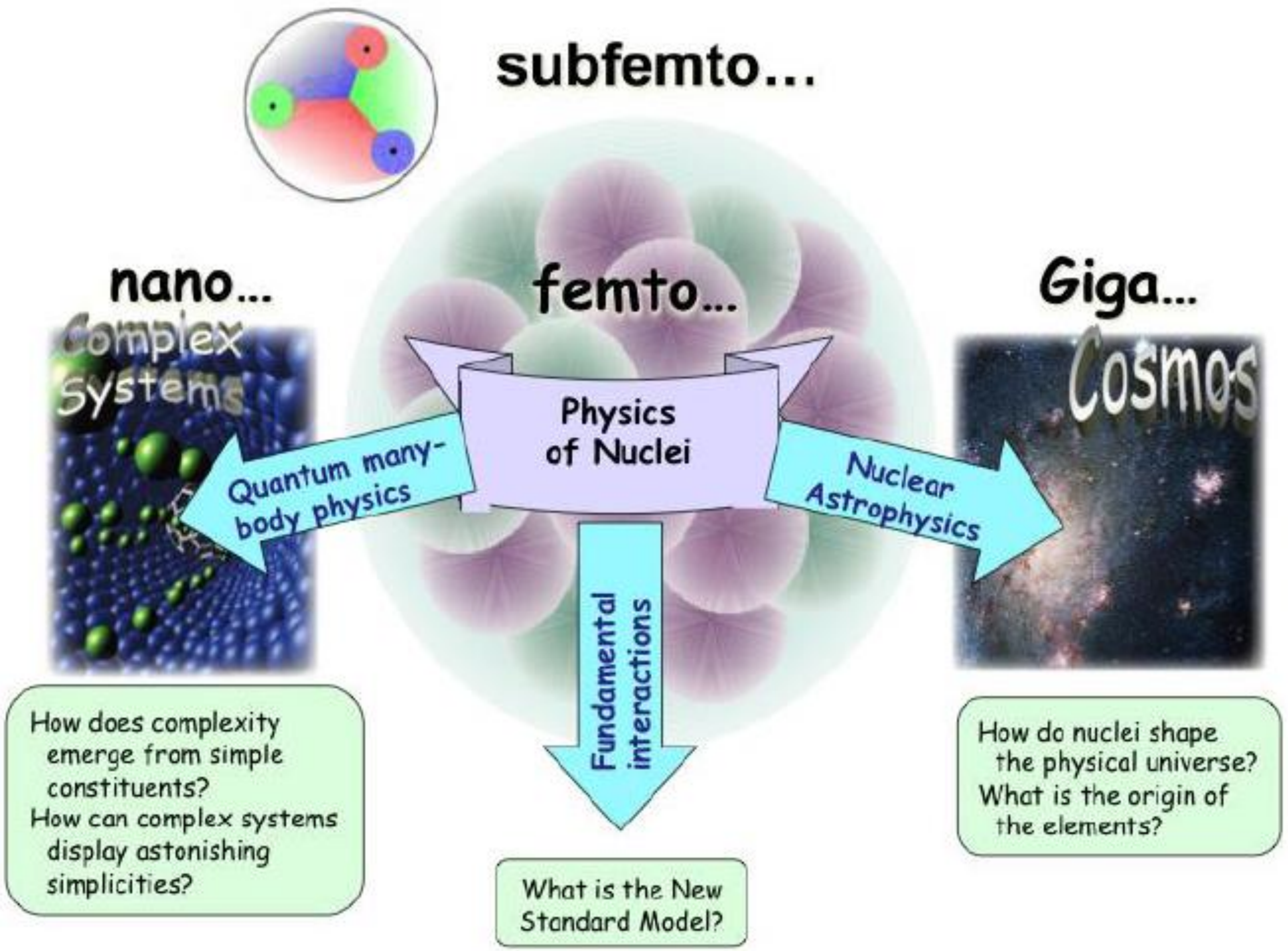
- **Linking microscopic and macroscopic – measuring the p-n interaction. Competition with pairing.**

- **Exotic Nuclei (maybe)**

TINSTAASQ

REALLY REALLY

Where do nuclei fit into the overall picture?



Constituents of nuclei

As I think you all know very well….

Protons and neutrons – nucleons - fermions, half integer spins: N + Z = A

A given element, say C or Fe or Sn or Pb, has a fixed number of protons (12,26,50,82) but can have a variety of neutrons – isotopes.

Odd – A nuclei: even Z - odd N or even N - odd Z Angular momenta ("spins") are half integer.

Even A nuclei: even Z – even N or odd Z – odd N with integer spins. (We focus most of our discussion on even-even (e –e) nuclei).

The nucleons interact with each other primarily with the strong and electromagnetic forces.

Strong, electromagnetic forces

In light nuclei like 4-He (Z = N = 2), strong force has strength of ~50 MeV. Coulomb repulsion of two protons is ~0.6 MeV. Strong force dominates.

However, strong force is short range (shorter than the sizes of most nuclei as we will see) while Coulomb force is long range. So (see later) each nucleon interacts by the strong force with maybe the 10 closest other nucleons while each of the protons interacts with all the others with the Coulomb force.

So the integrated strong force scales as the number of nucleons, while the integrated strength of the Coulomb repulsion scales as Z (Z-1).

For large enough Z, the Coulomb force will eventually dominate

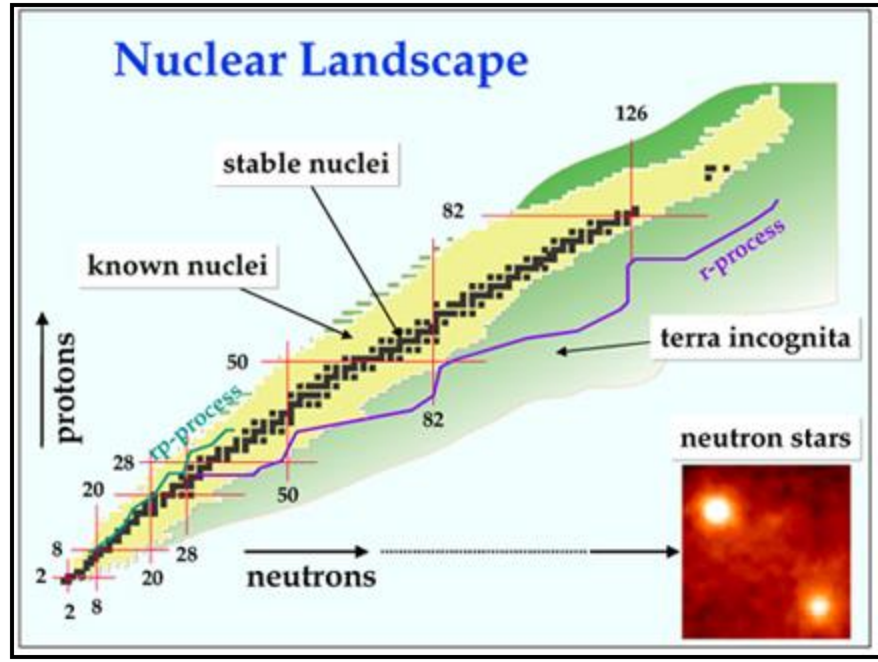
The heaviest nuclei (Z > 104) should not exist !!!

The scope of Nuclear Structure Physics

The Four Frontiers

1. Proton Rich Nuclei
2. Neutron Rich Nuclei
3. Heaviest Nuclei
4. Evolution of structure within these boundaries

Notice curvature to valley of stability. Light nuclei have N ~ Z. Heavy nuclei are neutron rich. Reason: rel. magnitudes of strong and electromagnetic forces we just discussed. Heavy nuclei need a neutron “buffer”



Terra incognita — huge gene pool of new nuclei

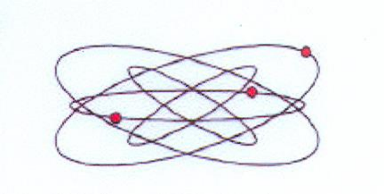
We can customize our system – fabricate “designer” nuclei to *isolate and amplify* specific physics or interactions

Themes and challenges of Modern Science

• **Complexity out of simplicity -- Microscopic**

How the world, with all its apparent complexity and diversity can be constructed out of a few elementary building blocks and their interactions

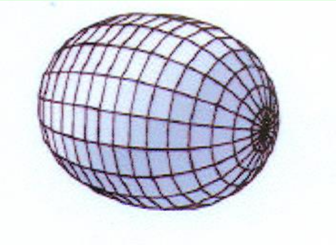
What is the force that binds nuclei?
Why do nuclei do what they do?



• **Simplicity out of complexity – Macroscopic**

How the world of complex systems can display such remarkable regularity and simplicity

What are the simple patterns that nuclei display and what is their origin ?



What is the structure of atomic nuclei

How can we determine it:

What observable quantities tell us what the nuclei look like and how they behave?

Properties of any quantum mechanical system:

A ground state and a set of discrete excited states, typically characterized by one or more quantum numbers such as total angular momentum and parity.

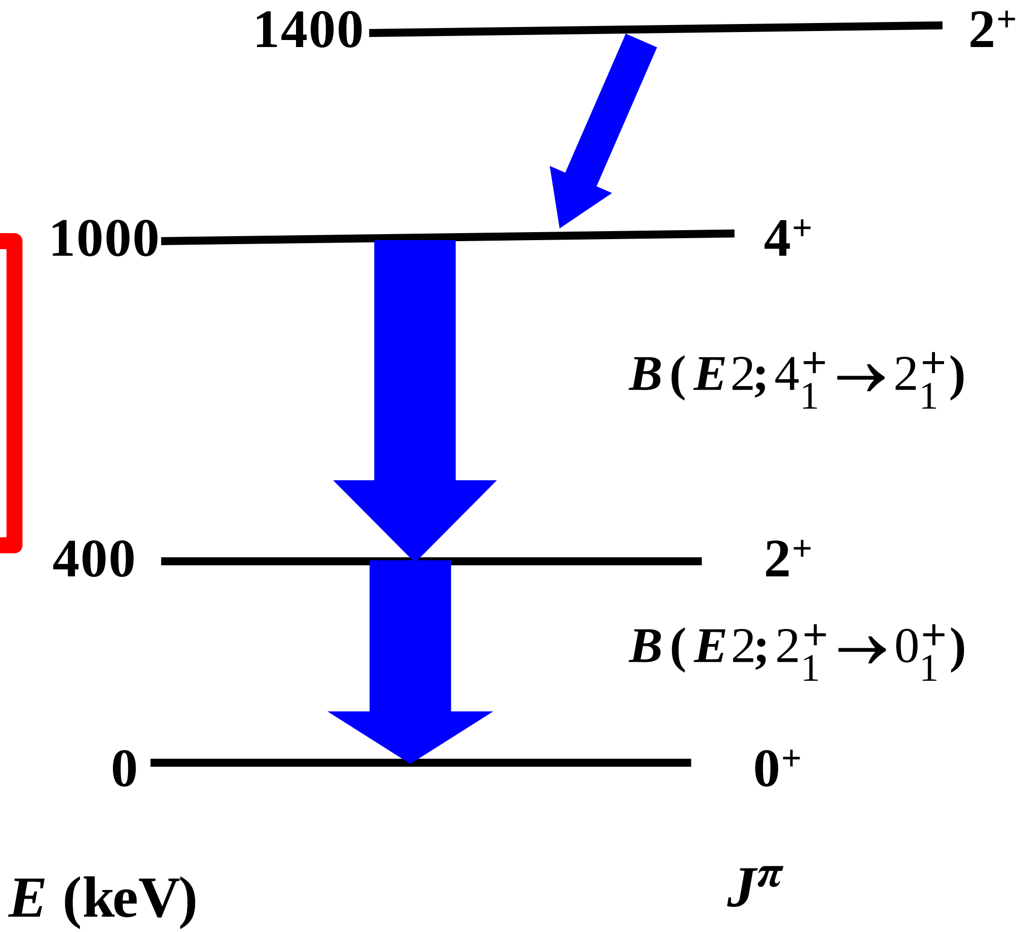
For odd A nuclei we will mostly just look at the ground state.

For e-e nuclei we consider several low lying states

Simple Observables - Even-Even Nuclei

$$R_{4/2} = \frac{E(4_1^+)}{E(2_1^+)}$$

Masses



$$B(E2; J_i \rightarrow J_f) \equiv \frac{1}{2J_i + 1} \left\langle \Psi_i \| E2 \| \Psi_f \right\rangle^2$$

Masses Reflect Nucleonic Interactions

Mass differences; interaction filters (double differences)

Total mass/binding energy: **Sum of all interactions**

Mass differences: **Separation energies, shell structure, phase transitions, collectivity**

Double differences of masses: **Interaction filters**

Macro

Micro

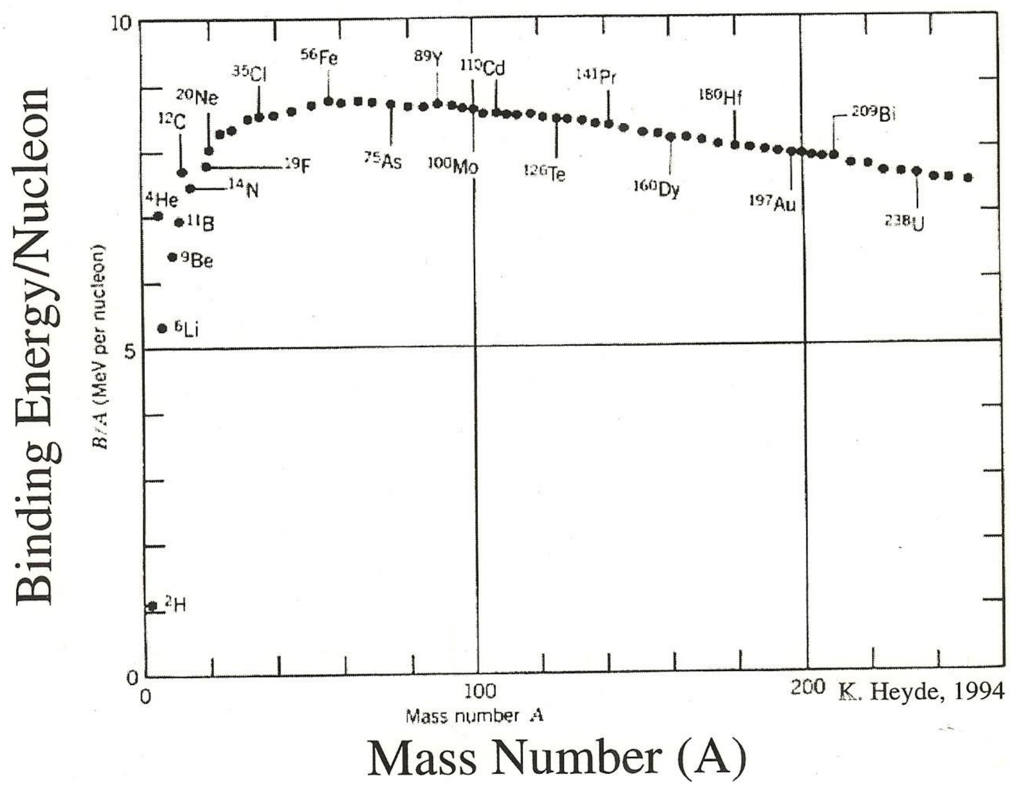
- Properties of the forces: ~ 100 MeV
- Shell structure : ~1 MeV
- **Quantum phase transitions: ~ 100s keV**
- **Collective effects: ~ 100s keV**
- **Interaction filters (e.g., p-n) ~ 10-15 keV**
- **Fundamental Interactions < 1 keV**

Simplest possible mass data. What does it tell us???

The plot gives B.E.s PER nucleon.

Note that they saturate. What does this tell us?

Binding Energies



Constant Nuclear Density

Nuclear Radius: $R = R_o A^{1/3}$ $R_o \sim 1.2$ fm

Consider the simplest possible model of nuclear binding.

Assume that each nucleon interacts with *n* others. Assume all such interactions are equal.

Look at the resulting binding as a function of *n* and A. Compare this with the B.E./A plot.

Each nucleon interacts with 10 or so others. Nuclear force is short range – shorter range than the size of heavy nuclei !!!

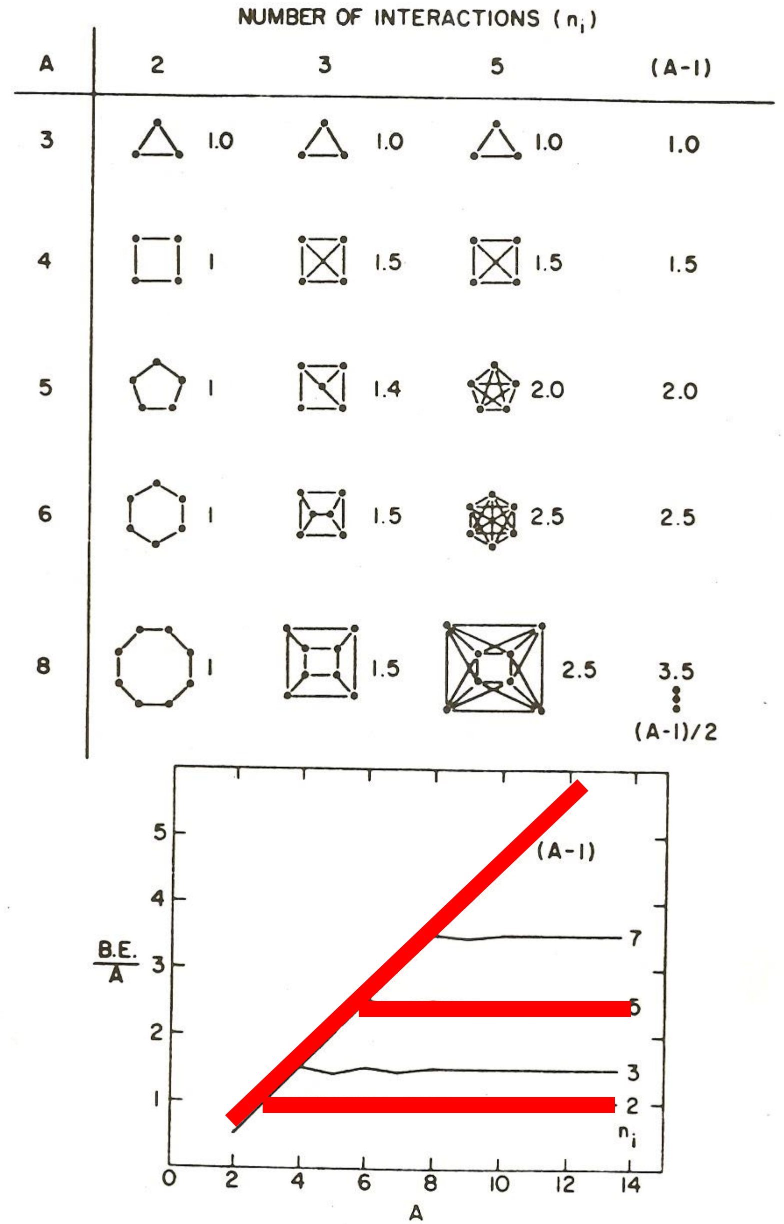
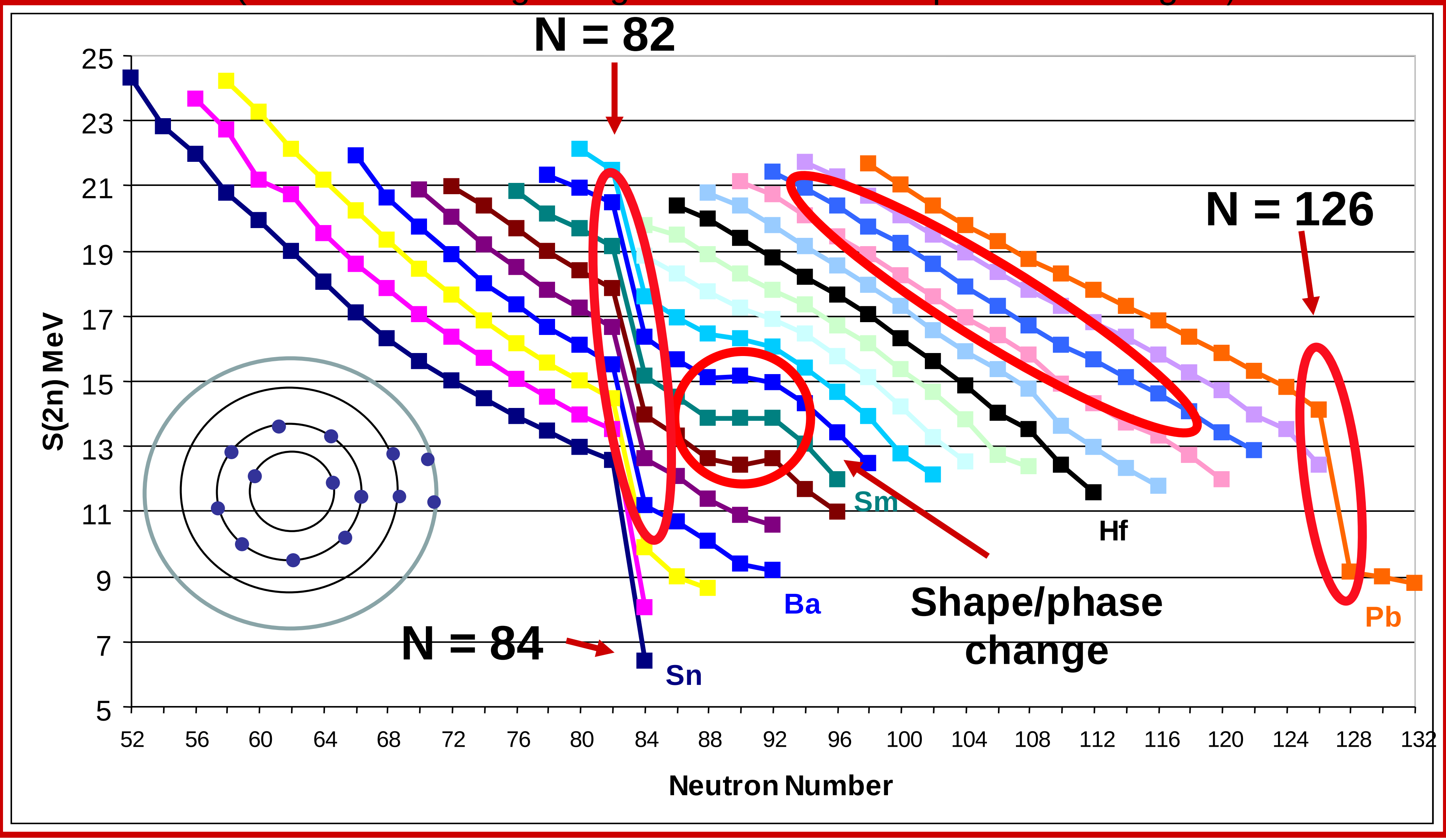


Fig.1.4. Highly schematic calculation of the binding energy per nucleon under different "saturation" assumptions concerning nuclear forces. The number of connections indicated, n_i, is the number of nucleons with which each other nucleon is assumed to interact. All such interactions are considered to be of equal strength. The lower part shows a plot of the resulting binding energies per nucleon.

Masses: reflect all interactions. ~ 100GeV.

Separation energies to remove two neutrons ~ 16MeV

(2-neutron binding energies = 2-neutron “separation” energies)



$$S_{2n} = A + BN + S_{2n}\ (\mathrm{Coll.})$$

Spectroscopic observables

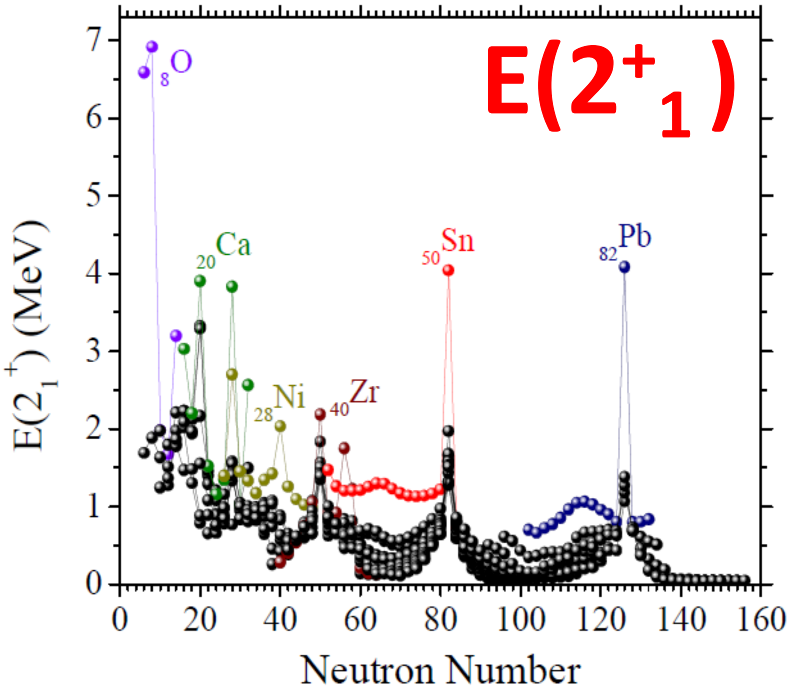
Two obvious features which capture much of the physics:

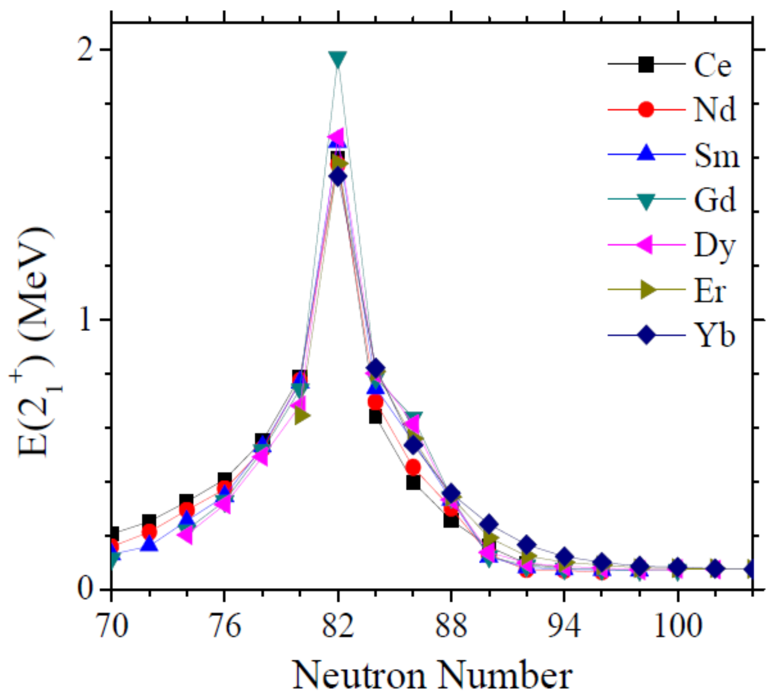
- High values at certain numbers, 2, 8, 20, 50, 82, 126…

These show the rigidity to excitation of nuclei with these special numbers of nucleons

- Sharp drops thereafter.

- Something must be special about these "magic" numbers





What do other observables tell us?

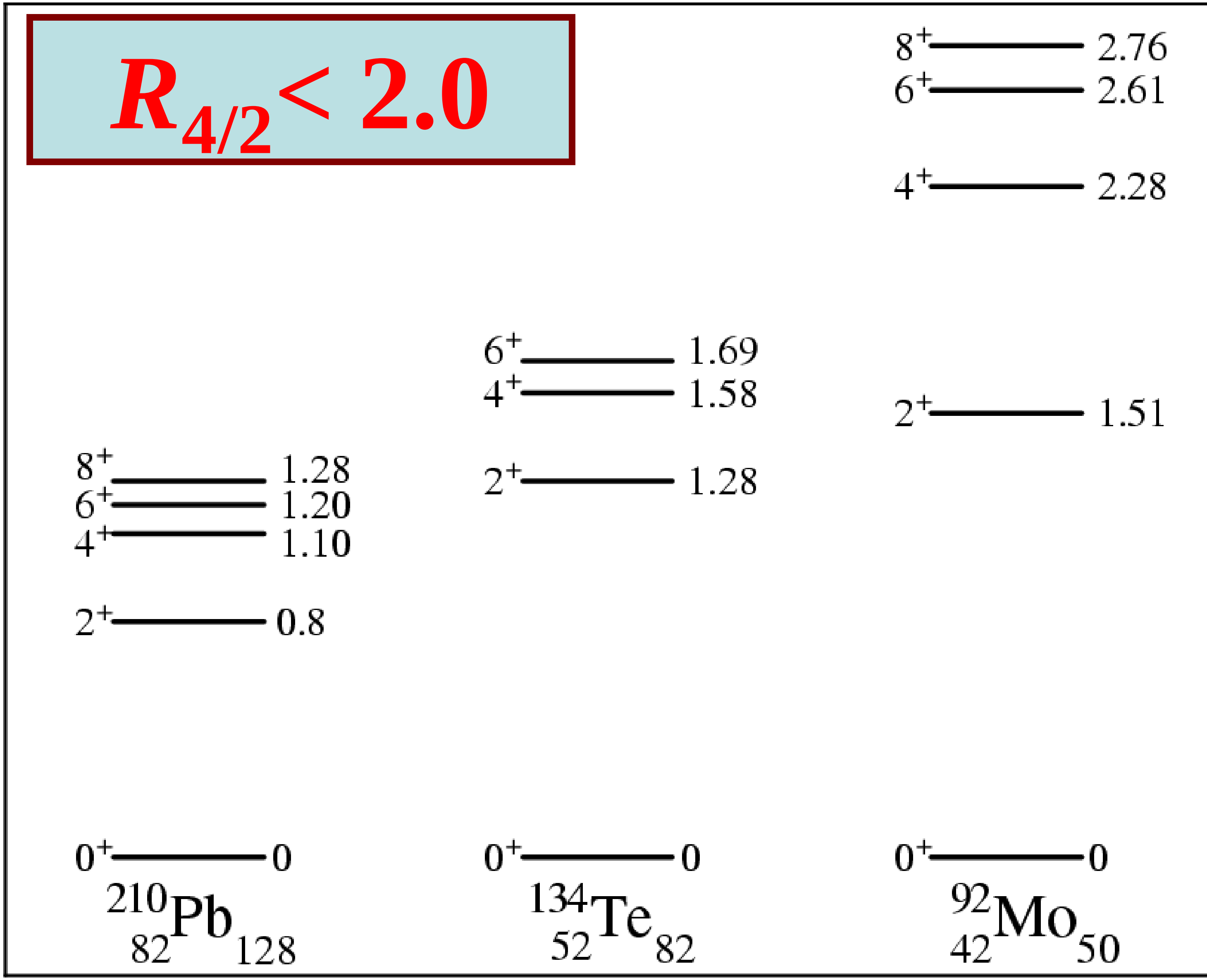
$R_{4/2}$

How does it vary, and why, and why do we care

- We care because it is almost the only observable that immediately tells us something about structure.
- We care because it is easy to measure.
- Why: It reflects the emergence of nuclear collectivity

- **3 special cases: <2, ~2, ~3.33**

Spectra of “2 valence nucleon” nuclei



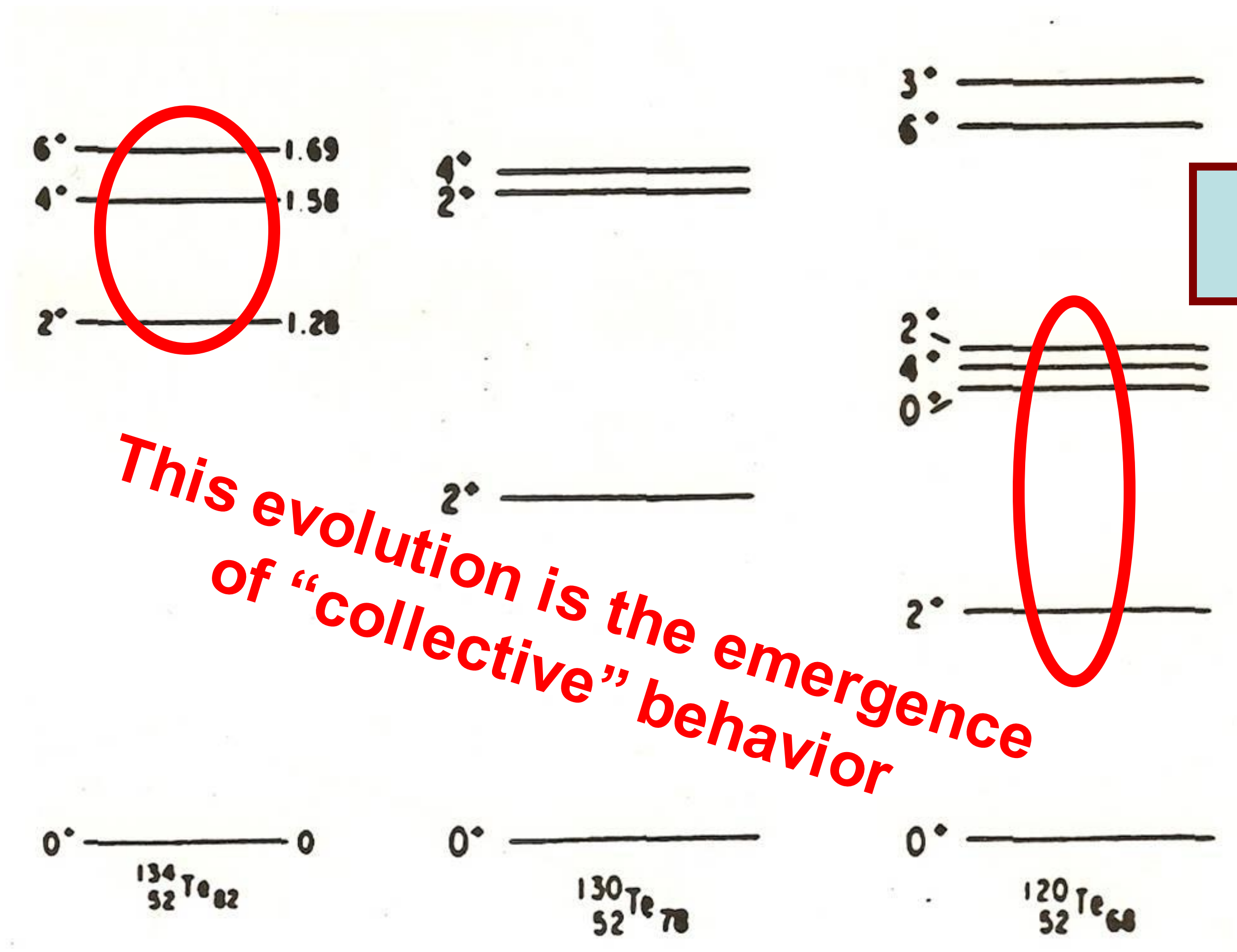
Starting from a doubly magic nucleus, what happens as the numbers of valence neutrons and protons **increase**?

Case of **few** valence nucleons:

Lowering of energies.

Monotonic sequences evolve into multiplets.

Two nucleons of one type

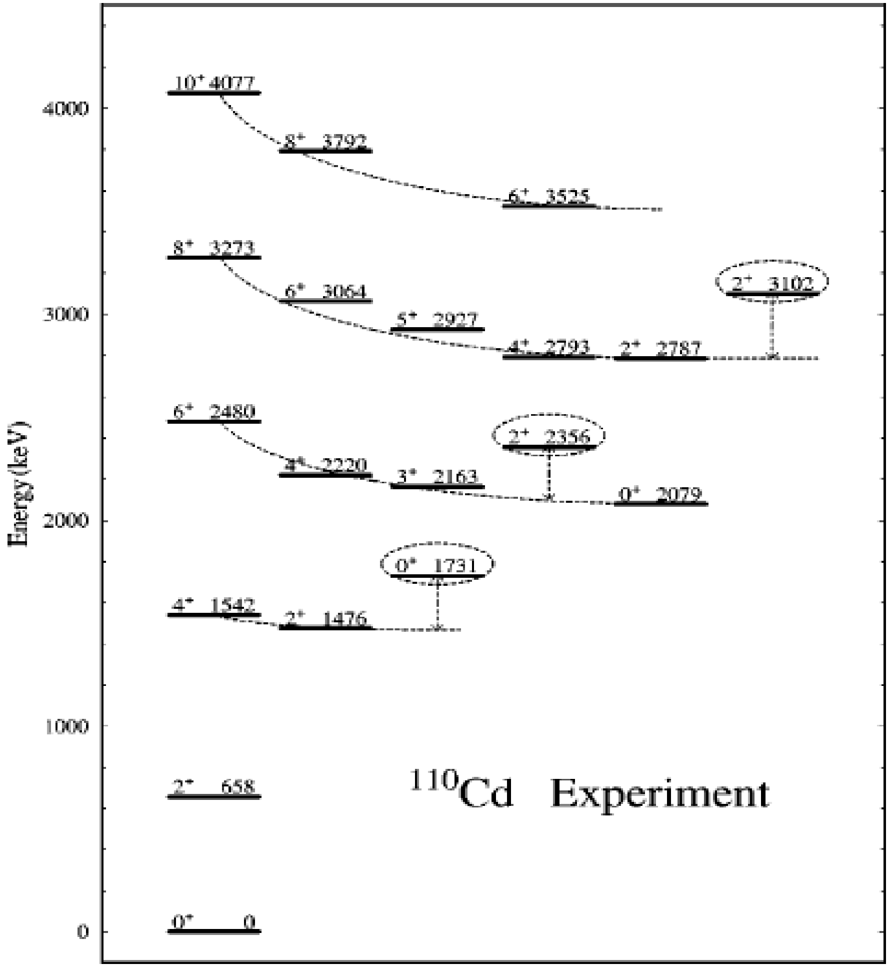


$R_{4/2}$~2.0

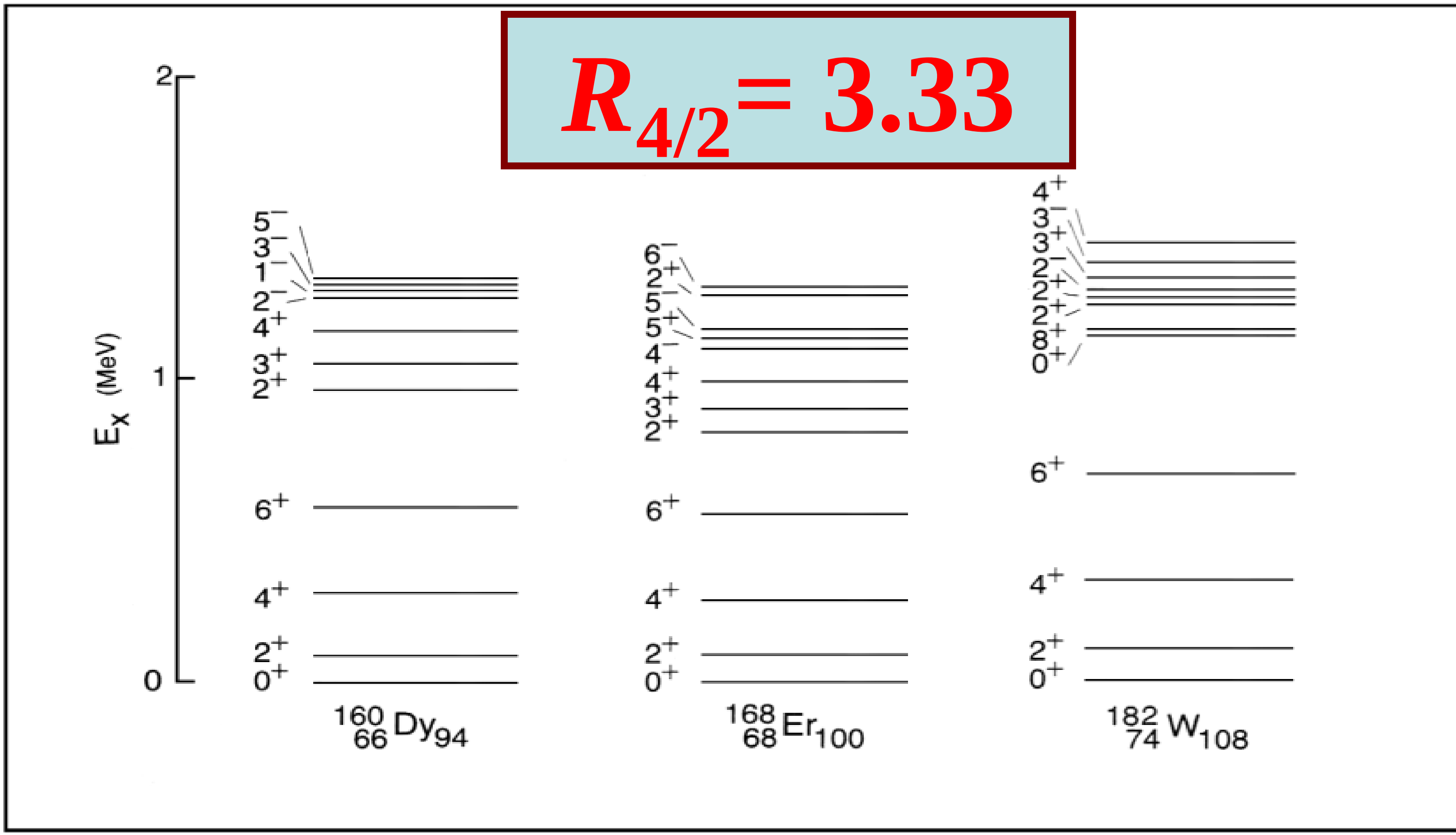
Few nucleons of both types

Few valence nucleons of each type:

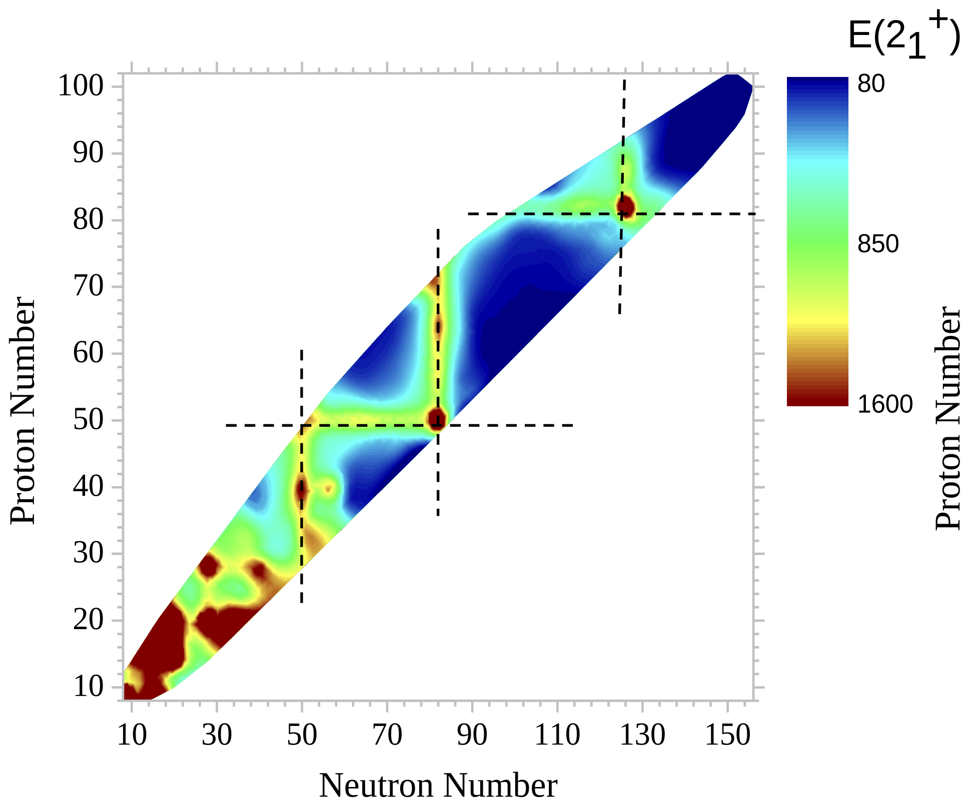
$$R_{4/2} = 2.0$$

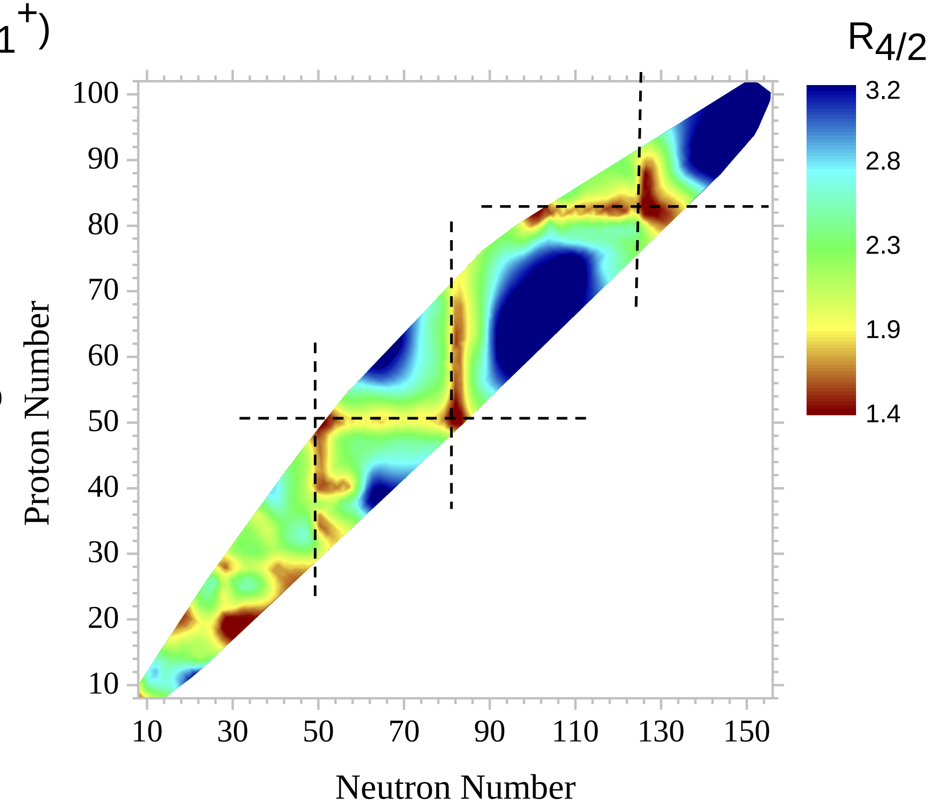


Lots of valence nucleons of both types:



Broad perspective on structural evolution





The remarkable regularity of these patterns is one of the beauties of nuclear systematics and one of the challenges to nuclear theory.
Whether they persist far off stability is one of the fascinating questions for the future

Cakirli

Key guides to structure

- Magic numbers:

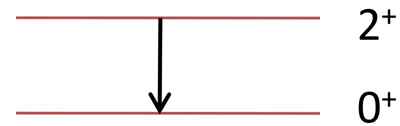
  2, 8, 20, 28, (40), 50, (64), 82, 126

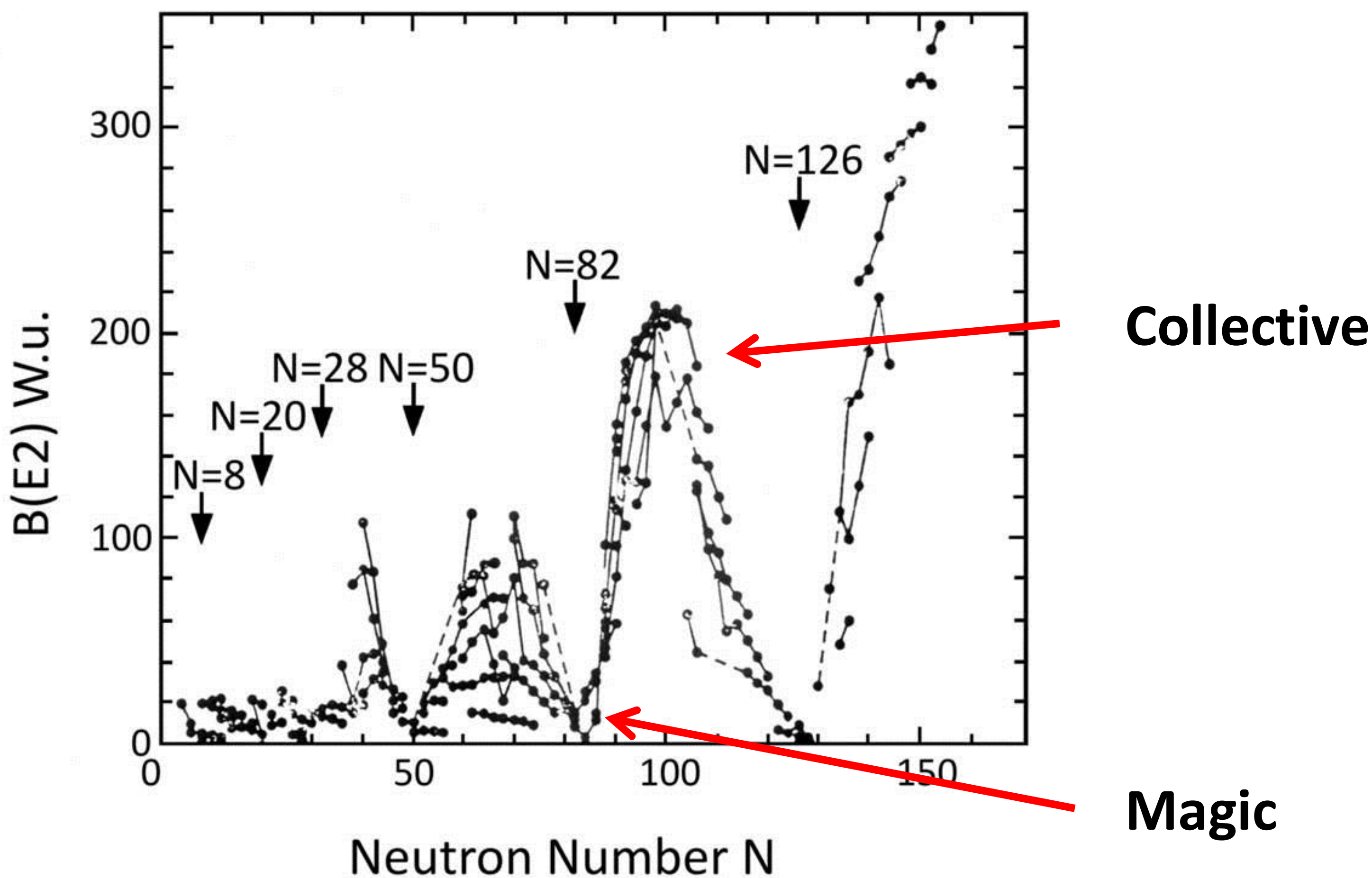
- Benchmark $R_{4/2}$ values

These are the only things you need to memorize.

Transition rates (half lives of excited levels) also tell us a lot about structure

$$B(E2:\ 2^+_1 \rightarrow 0^+_1) \propto \langle\, 2^+_1 \,||\, E2 \,||\, 0^+_1 \rangle^2$$

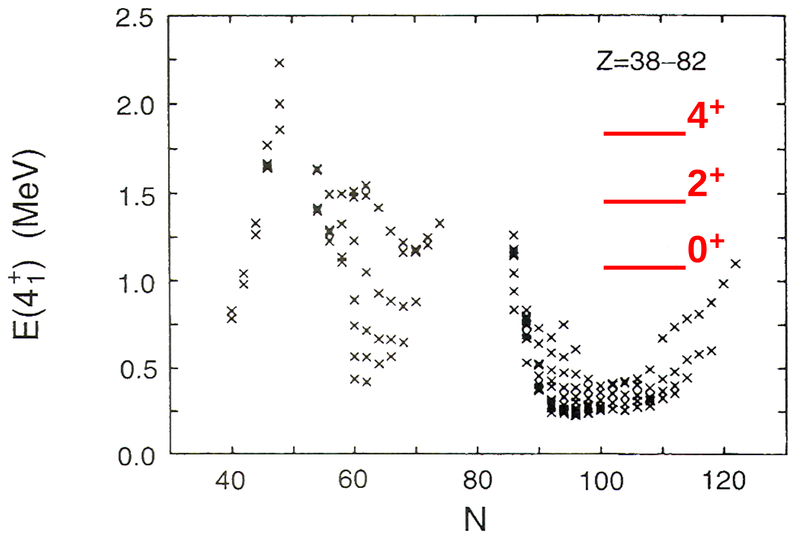




So far, everything we have plotted has been an individual observable against N or Z (or A)

Now we introduce the idea of correlations of different observables with each other.

Correlations of Observables



There is only one appropriate reaction to this result ….

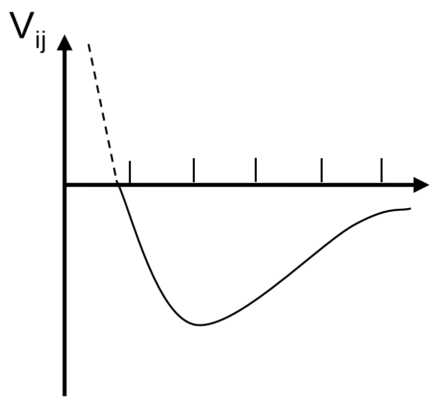
Wow !!!!!!!!

There is only one worry, however …. accidental or false correlations. Beware of lobsters !!!

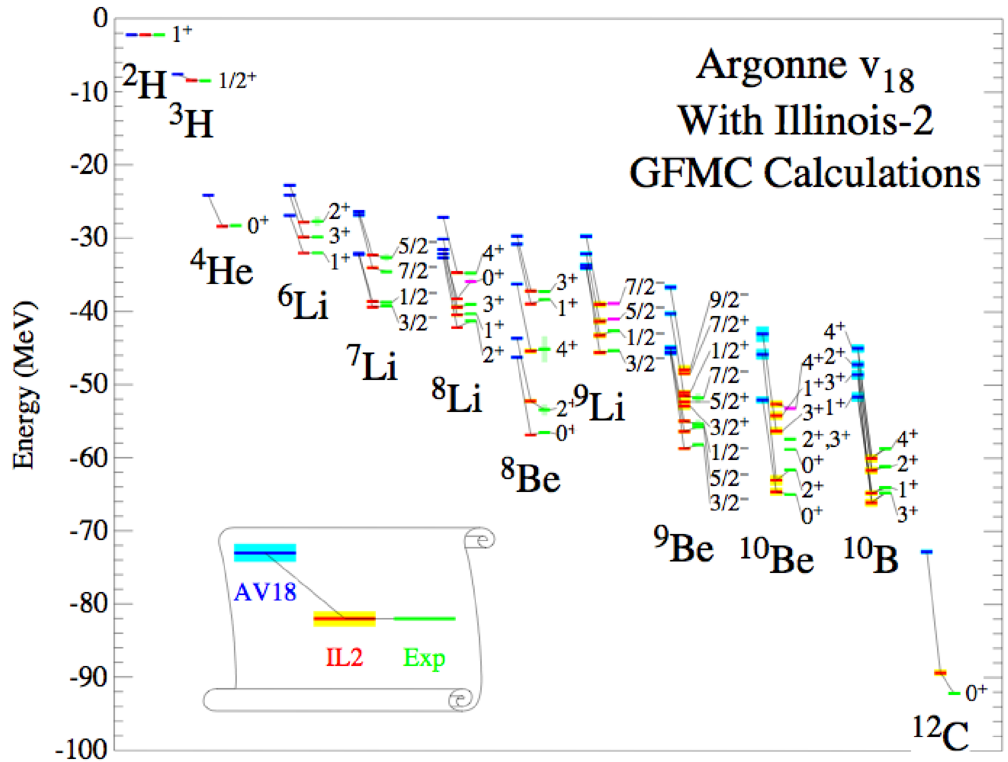

BEWARE OF FALSE CORRELATIONS!
LOBSTERS
FOR RENT
924-0616

So, how can we understand nuclear behavior?

Calculating the structure of nuclei -- start with the interaction of each nucleon with each other



Nucleon-nucleon force – very complex



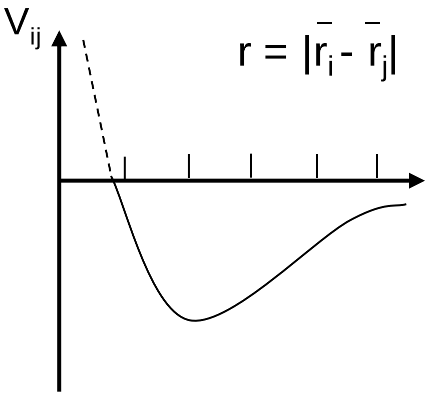
Ab initio calculations: An on-going success story. But thousands of hours of super-computer time

We will take a much simpler approach and try to reproduce the magic numbers and understand what they tell us and then introduce some simple collective models

Start with Independent particle model: magic numbers, shell gaps, valence nucleons.

Three key ingredients

First:

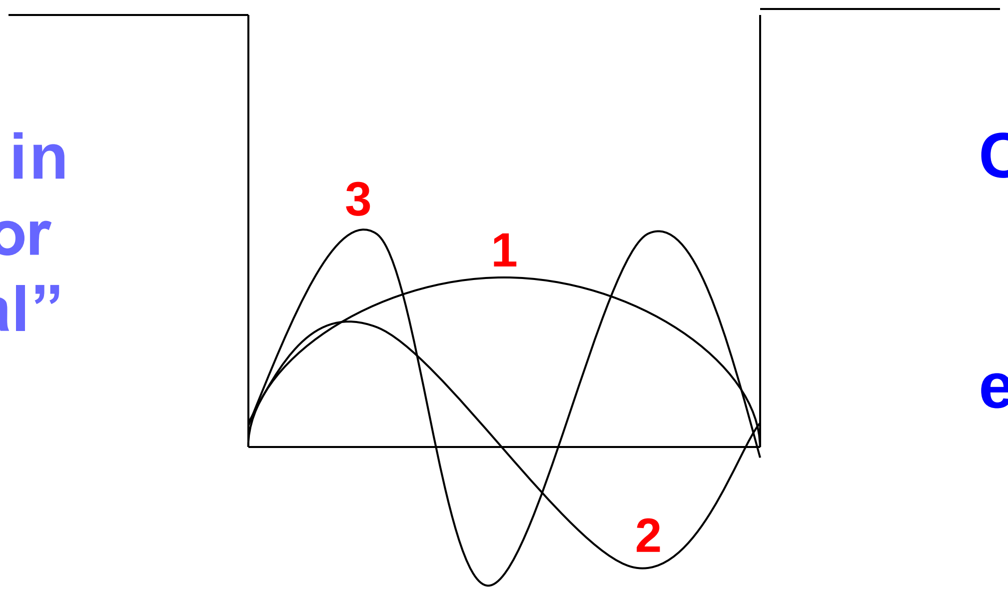


Nucleon-nucleon force – very complex

This extreme approximation cannot be the full story. Will need "residual" interactions. But it works surprisingly well in special cases.

Second key ingredient: Quantum mechanics

Particles in a “box” or “potential” well



Confinement is origin of quantized energies levels

Energy ~ 1 / wave length

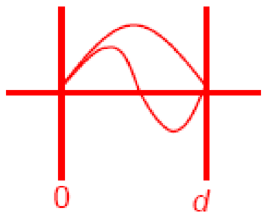
***n* = 1,2,3 is principal quantum number**

E ↑ up with *n* because wave length is shorter

Energies in an Infinite Square Well

(box)

Simple Derivation



$\Psi(0) = \Psi(d) = 0$

for containment

$$\therefore \frac{n\lambda}{2} = d \qquad n = 1, 2, \cdots$$

Now, use de Broglie relation

$$P = \frac{h}{\lambda} \qquad \text{and} \qquad E = \tfrac{1}{2} mv^2 = p^2/2m$$

$$\text{or} \quad p = \sqrt{2mE}$$

$$\therefore \frac{nh}{2p} = \frac{nh}{2\sqrt{2mE}} = d$$

$$\therefore \frac{n^2 h^2}{8mE} = d^2$$

or $$\boxed{E = \frac{n^2 h^2}{8md^2}} \qquad n = 1, 2, \cdots$$

Zero point motion

a) confinement
b) wave/particle relation

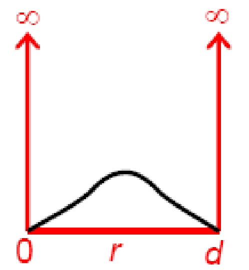
} → quantization

Potential Wells (l – d)

a) Infinite

$V = 0 \qquad 0 < r < d$

$V = \infty \qquad r \leq 0,\ r \geq d$



Schrödinger Eq.

$$\frac{-\hbar^2}{2m}\frac{d^2\Psi}{dr^2} + V\,(=0) = E\,\Psi$$

OR
$$\frac{d^2\Psi}{dr^2} = -\frac{1}{\hbar^2}\;2mE\;\;\Psi \equiv -k^2\,\Psi \qquad k = \frac{1}{\hbar}\sqrt{2mE} = \frac{2\pi}{\lambda}$$

Double difference $\Psi \rightarrow \Psi \Rightarrow \Psi$ is “exponential”

Double difference $\Psi \rightarrow$ neg # $\Rightarrow$ exponent contains i

$\Psi = A \sin kr + B \cos kr = A \sin kr \qquad \Psi\,(\,0\,) = 0$

$\Psi\,(\,d\,) = 0 \;\Rightarrow\; k\,d = n\,\pi \qquad$ or $\qquad k = n\,\pi\,/d$

$$\Psi = A \sin \frac{n\,\pi\,r}{d} \qquad \boxed{E} = \frac{k^2\hbar^2}{2m} = \frac{n^2\pi^2 h^2}{2md^2\left(2\pi\right)^2} = \boxed{\frac{n^2 h^2}{8md^2}}$$

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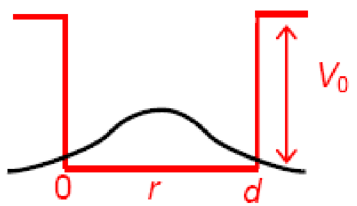
b) Finite

$V = V_0 \qquad r \leq 0\;\; r \geq d$



Inside: as above

Outside: $$\frac{d^2\Psi}{dr^2} = -\frac{2m}{\hbar^2}\left(E - V\right)\Psi = k^2\,\Psi \qquad k = \frac{1}{\hbar}\sqrt{2m\left(V_0 - E\right)}$$

Double difference $\Psi \rightarrow +\Psi \Rightarrow$ exponent without i

$\Psi = Ae^{-kr}$ (+ $k\,r$ grows without bound)

$\boxed{E \text{ lower}}$

Uncertainty Principle and Zero Point Motion

Uncertainty Principle $\Rightarrow$ particle cannot fall to bottom of well

If $\Delta p = 0$ (*i.e.*, stopped in bottom of pot)
then $\Delta x \to \infty$ (particle is unbound)

OR:

If $\Delta x = 0$ (particle exactly localized)
then $\Delta p \to \infty$ (particle jumps out of potential)

Example: Particle in well (3 - d)

$$\Delta p_x = \frac{\hbar}{2\Delta x} = \frac{\hbar}{2r} \qquad r = \text{radius of well}$$

$$\therefore \quad \boxed{\Delta E_x} = \frac{\Delta P_2^2}{2m} = \boxed{\frac{\hbar^2}{8mr^2}}$$

Compare e^- in atom, p in nucleus. Take $m_e = 1$, $m_p = 10^3\ m_e$

Atom $\quad \Delta E_{min} \sim \dfrac{1}{\left(10^{-8}\right)^2} = \dfrac{1}{10^{-16}\ \text{cm}^2}$

Nucleus $\quad \Delta E_{min} \sim \dfrac{1}{10^3\left(10^{-12}\right)^2} = \dfrac{1}{10^{-21}\ \text{cm}^2}$

Nuclear scale is ~ 10^5 times larger

$(10\ \text{eV})_{\text{atom}} \to (1\ \text{MeV})_{\text{nucleus}}$

Nuclei are 3-dimensional

- What is new in 3 dimensions?
  - Angular momentum
  - Centrifugal effects

Central Potential $U(r)$ $U(r) \cdot$ 0 at $r = 0$

Schrödinger eq.:

$$H\,\Psi = \left(\frac{P^2}{2M} + U(r) \right) \Psi_{nlm}(r) = E_{nlm}\,\Psi_{nlm}(r)$$

Separable into radial and angular parts:

$$\Psi_{nlm}(r) = \Psi_{nlm}(r\,\theta\,\Phi) = \frac{1}{r}\,R_{nl}(r)\,\Psi_{nl}(\theta\,\Phi)$$

n = radial q. # = # nodes
l = orbital angular momentum
m = substate (direction in space)

Nomenclature:

$l = 0, 1, 2, 3, 4, 5\ldots$ s, p, d, f, g, h…

$m = l, l-1, \ldots\ldots\ldots\ldots\ldots(-l)$. $2l + 1$ m substates

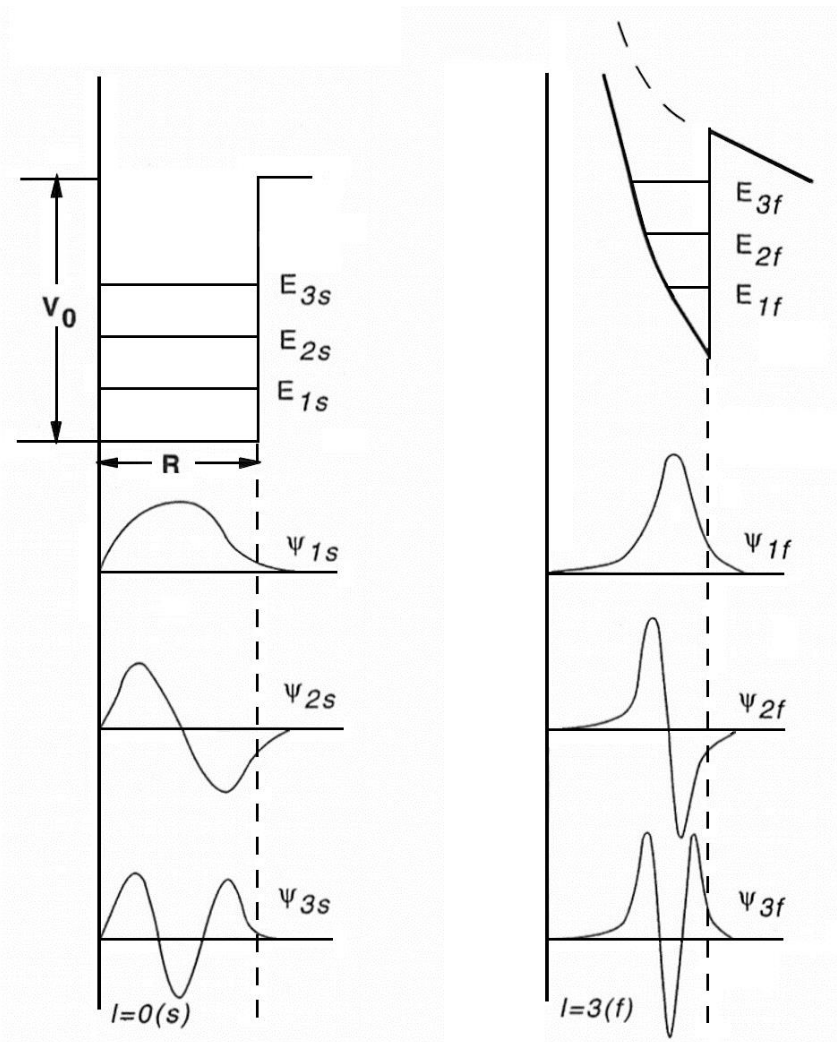
$$E(l\,m_i) = E(l\,m_j)$$

Radial Schrödinger eq.:

$$\frac{\hbar^2}{2M}\frac{d^2R_{nl}(r)}{d\,r^2} + \left[E_{nl} - U(r) - \frac{\hbar^2}{2M}\frac{l\,(l+1)}{r^2} \right] R_{nl}(r) = 0$$

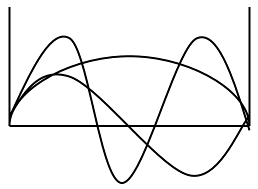
Radial Schroedinger wave function

$$\frac{h^2}{2m}\frac{d^2R_{nl}(r)}{dr^2} + \left[E_{nl} - U(r) - \frac{h^2}{2m}\frac{l(l+1)}{r^2}\right]R_{nl}(r) = 0$$



Higher Ang Mom: potential well is raised and squeezed. Wave functions have smaller wave lengths. Energies rise

Energies also rise with principal quantum number, *n*.



Raising one, lowering the other can give similar energies – "level clustering":

H.O: $E = \hbar\omega\,(2n+l)$

$E\,(n,l) = E\,(n-1,\, l+2)$

e.g., $E\,(2s) = E\,(1d)$

Properties of the Solutions

1) Higher n ⇒ higher E (more KE)

2) Higher l ⇒ higher E (larger radius, less bound)

⇒ lowest state is $1s_{1/2}$ $n = 1$, $l = 0$

(Explains gr. st. deuteron: $L = l_1 + l_2 = 0$)

1) and 2) ⇒

3) Can have similar energies for 2 orbits if one has larger n and smaller l (or vice versa)

Third key ingredient

Pauli Principle

- Two fermions, like protons or neutrons, can NOT be in the same place at the same time: can NOT occupy the same orbit.
- Orbit with total Ang Mom, j, has 2j + 1 substates, hence can only contain 2j + 1 neutrons or protons.

This, plus the clustering of levels in simple potentials, gives nuclear SHELL STRUCTURE

Consider SHO Levels

nlj: Pauli Prin. 2j + 1 nucleons

Levels	# nucleons (either *p* or *n*)	Total up to *E*
$3s_{1/2}$, $2d_{3/2}$, $2d_{5/2}$, $1g_{7/2}$, $1g_{9/2}$	2 + 4 + 6 + 8 + 10 = 30	70
$2p_{1/2}$, $2p_{3/2}$, $1f_{5/2}$, $1f_{7/2}$	2 + 4 + 6 + 8 = 20	40
$2s_{1/2}$, $1d_{3/2}$, $1d_{5/2}$	2 + 4 + 6 = 12	20
$1p_{1/2}$, $1p_{3/2}$	2 + 4 = 6	8
$1s_{1/2}$	2	2

(Next higher is 112)

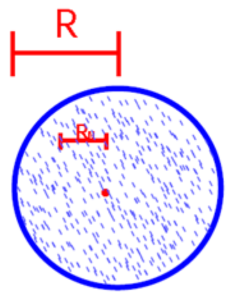
First few are known magic numbers

Higher ones are not. And known magic numbers

50, 82, 126 do not appear

Form of Central Potential

Consider nucleus of radius $R >> R_{nucl.\ force}$ and nucleon in "interior"



Surrounded more or less uniformly by nucleons on all sides within range of force

$\therefore$ **No net force,** $V_{int} \sim const$

Perhaps square well might be better approximation

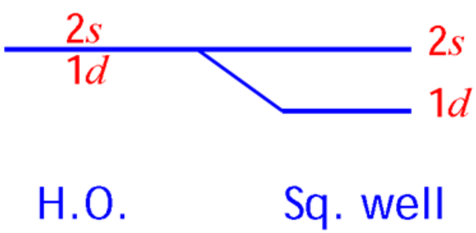
Compared to SHO, will mostly affect orbits at large radii – higher angular momentum states

Comparison of energy levels in H.O. and square well

Sq. well:

Potential deeper (more attractive) at large radii.

Lowers E of higher l states whose probability density is, on average, at larger radius.



So, square off the potential, add in a spin-orbit force that lowers states with angular momentum

$J = l + ½$

compared to those with

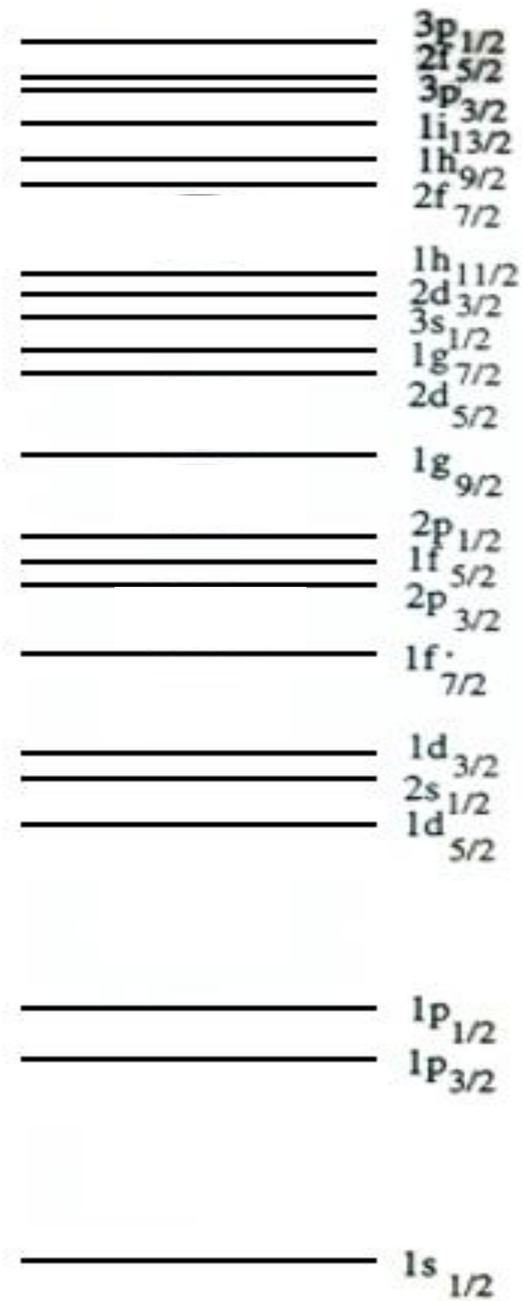
$J = l – ½$

→ Clustering of levels.

Add in Pauli Principle → magic numbers, inert cores

Concept of valence nucleons – key to structure. Many-body → few-body: each body counts.

Addition of 2 neutrons in a nucleus with 150 can drastically alter structure



Independent Particle Model

- Put nucleons (protons and neutrons separately) into orbits.
- Key question – how do we figure out the total angular momentum of a nucleus with more than one particle? Need to do vector combinations of angular momenta subject to the Pauli Principal. More on that later. However, there is one trivial yet critical case.
- Put **2j + 1** identical nucleons (fermions) in an orbit with angular momentum **j**. Each one MUST go into a different magnetic substate. Remember, angular momenta add vectorially but projections (m values) add algebraically.
- So, total M is sum of m's

$$M = j + (j - 1) + (j - 2) + \ldots + 1/2 + (-1/2) + \ldots + [-(j - 2)] + [-(j - 1)] + (-j) = 0$$

M = 0. **So, if the only possible M is 0, then J= 0**

Thus, a full shell of nucleons always has total angular momentum 0. This simplifies things enormously !!!

INDEPENDENT PARTICLE MODEL

Consider $^{41}_{20}Ca_{21}$

^{40}Ca is doubly magic

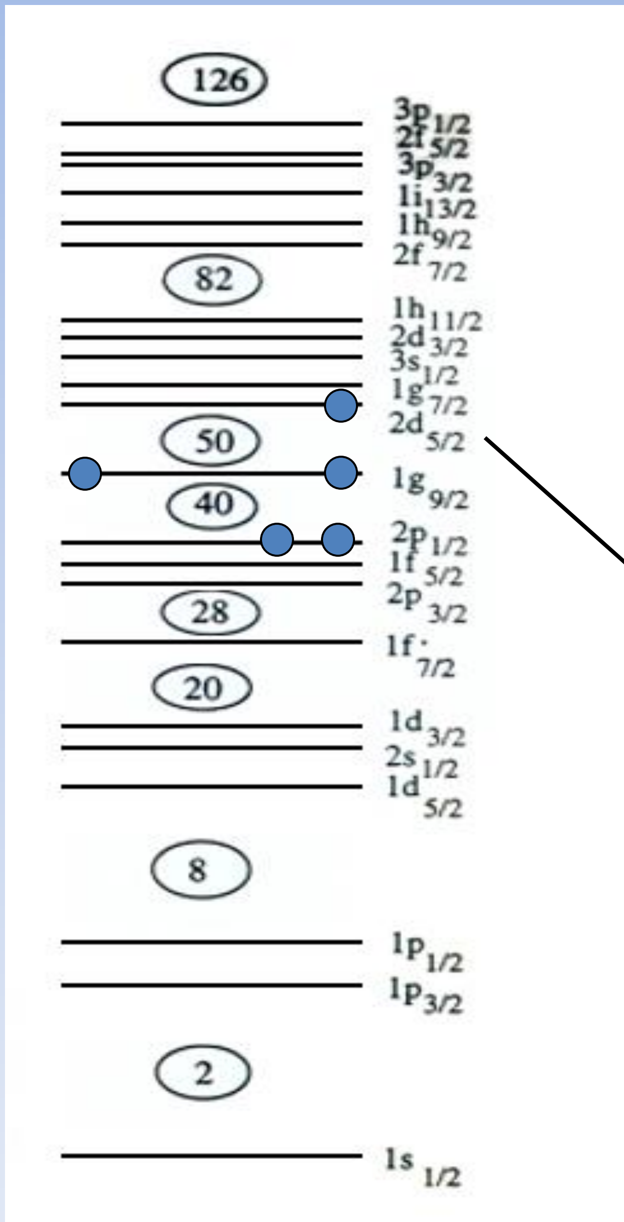
Proton, neutron shells filled: All magnetic substates filled,
hence $\sum_i m_i = M = 0$ **Hence J = 0**

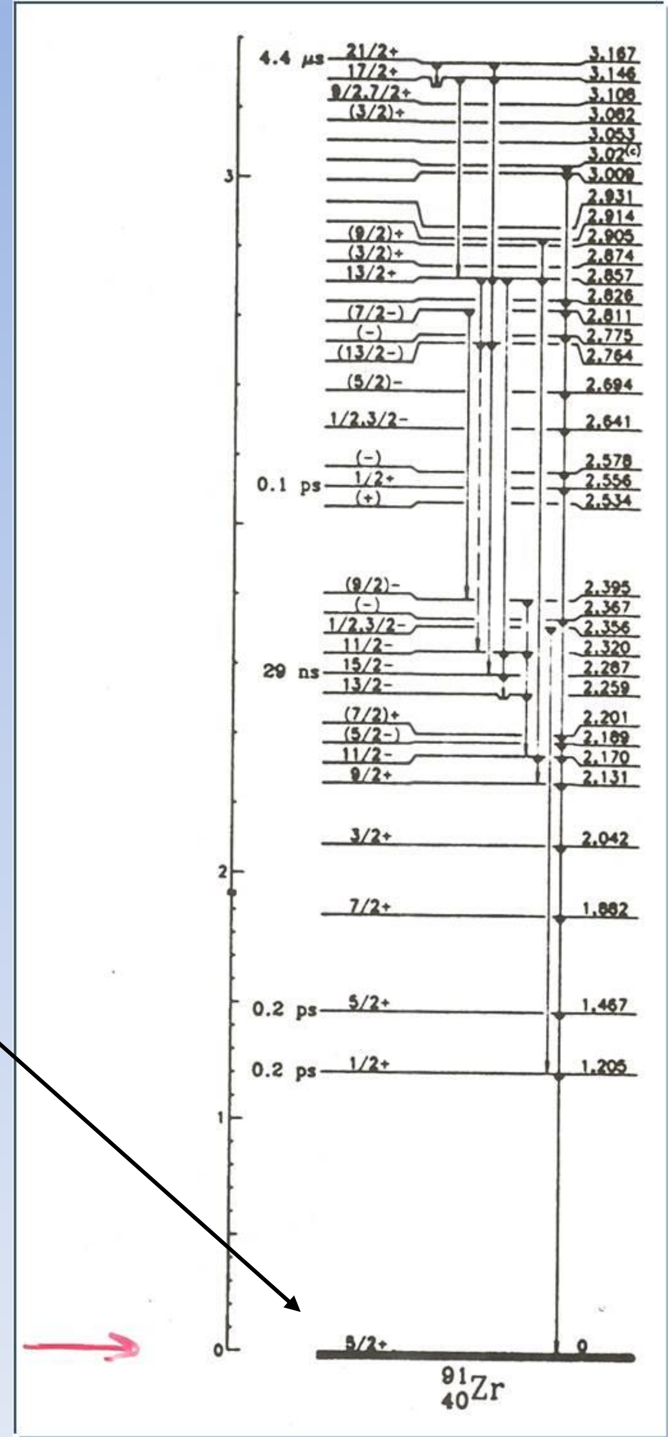
Ground state of ^{41}Ca determined by last odd neutron in

$1f_{7/2}$ orbit

$\therefore$ g.s. ^{41}Ca is $\frac{7}{2}^-$ **Agrees with experiment**

Let's do $^{91}{}_{40}Zr_{51}$





Paradigm Shifts in Physics/ Nuclear Physics

("Impossible" $\xrightarrow{PS}$ "Possible")

Changes of co-ordinate systems

Lab to center-of-mass

Masses to separation energies

Masses to excitation energies

Complex systems to simpler approximations

Most nuclear models represent some sort of paradigm shift

(up to 50 orders simpler!)

Most simplifications of complex systematics of nuclear properties involve paradigm shifts.

Look for, exploit, paradigm shifts

Shell Model:	P.S. 1	2-body → 1-body Pot.
	P.S. 2	All nucleons → valence

^{91}Zr:

From incredibly complex situation of 91 particles interacting with strong and Coulomb forces ⟶ 90 + 1 particles and then ⟶ 1 !!!

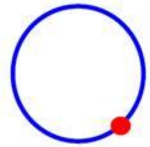
GROUND STATE J^{π} VALUES OF SOME ODD MASS NUCLEI

Z = 20	$3/2^+$ $^{37}\mathrm{Ca}$	$3/2^+$ $^{39}\mathrm{Ca}$	$7/2^-$ $^{41}\mathrm{Ca}$	$7/2^-$ $^{43}\mathrm{Ca}$	$7/2^-$ $^{45}\mathrm{Ca}$	$7/2^-$ $^{47}\mathrm{Ca}$	$3/2^-$ $^{49}\mathrm{Ca}$
Z = 40		$9/2^+$ $^{87}\mathrm{Zr}$	$9/2^+$ $^{89}\mathrm{Zr}$	$5/2^+$ $^{91}\mathrm{Zr}$	$5/2^+$ $^{93}\mathrm{Zr}$	$5/2^+$ $^{95}\mathrm{Zr}$	
Z = 39	$1/2^-$ $^{85}\mathrm{Y}$	$1/2^-$ $^{87}\mathrm{Y}$	$1/2^-$ $^{89}\mathrm{Y}$	$1/2^-$ $^{91}\mathrm{Y}$	$1/2^-$ $^{93}\mathrm{Y}$	$1/2^-$ $^{95}\mathrm{Y}$	$1/2^-$ $^{97}\mathrm{Y}$
Z = 41		$9/2^+$ $^{91}\mathrm{Nb}$	$9/2^+$ $^{93}\mathrm{Nb}$	$9/2^+$ $^{95}\mathrm{Nb}$	$9/2^+$ $^{97}\mathrm{Nb}$	$9/2^+$ $^{99}\mathrm{Nb}$	
N = 50	$3/2^-$ $^{85}_{35}\mathrm{Br}$	$3/2^-$ $^{87}_{37}\mathrm{Rb}$	$1/2^-$ $^{89}_{39}\mathrm{Y}$	$9/2^+$ $^{91}_{41}\mathrm{Nb}$	$9/2^+$ $^{93}_{43}\mathrm{Tc}$	$9/2^+$ $^{95}_{45}\mathrm{Rh}$	

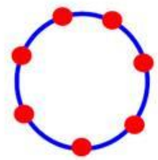
Particle-hole symmetry
(basic ideas—it can get more subtle)

Consider 1 particle in $f_{7/2}^{-}$ orbit

Then $J^{\pi} = j^{\pi} = 7/2^{-}$



Consider 7 particles in $f_{7/2}$ orbit

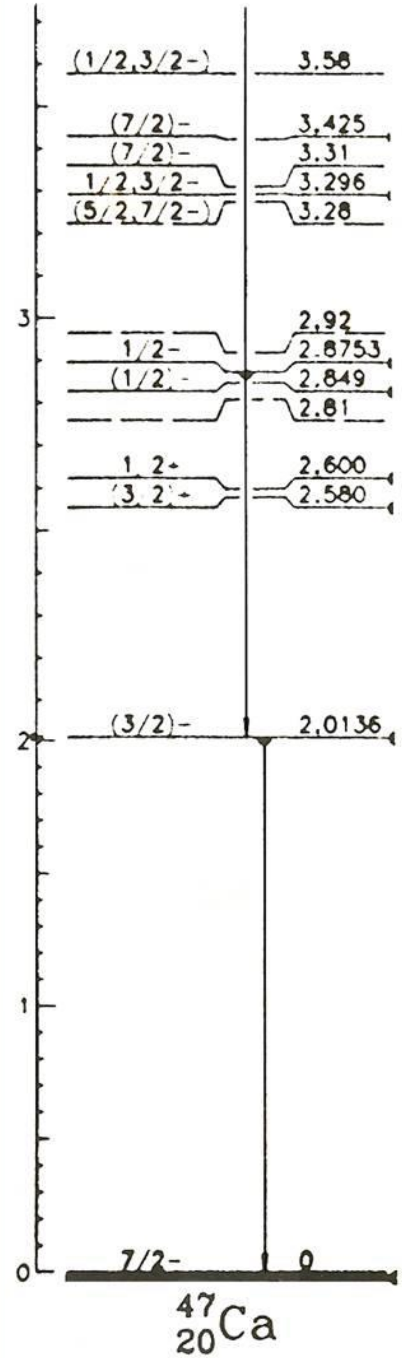


When shell is full (8 particles) $J = 0$

$$\therefore \quad \bar{J}_{\text{full shell}} = \bar{J}_{\text{full shell -1}} + \bar{7/2} = 0$$

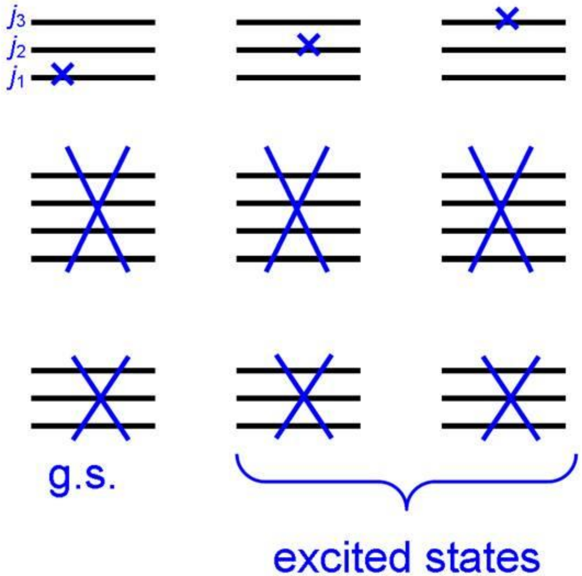
$$\Rightarrow \quad J_{\text{full shell -1}} = 7/2 \qquad \text{e.g.: } {}^{47}_{20}\mathrm{Ca}_{27}$$

- 1 hole has same J as one particle



GROUND STATE J^π VALUES OF SOME ODD MASS NUCLEI

Z = 20	$3/2^+$ $^{37}\mathrm{Ca}$	$3/2^+$ $^{39}\mathrm{Ca}$	$7/2^-$ $^{41}\mathrm{Ca}$	$7/2^-$ $^{43}\mathrm{Ca}$	$7/2^-$ $^{45}\mathrm{Ca}$	$7/2^-$ $^{47}\mathrm{Ca}$	$3/2^-$ $^{49}\mathrm{Ca}$
Z = 40		$9/2^+$ $^{87}\mathrm{Zr}$	$9/2^+$ $^{89}\mathrm{Zr}$	$5/2^+$ $^{91}\mathrm{Zr}$	$5/2^+$ $^{93}\mathrm{Zr}$	$5/2^+$ $^{95}\mathrm{Zr}$	
Z = 39	$1/2^-$ $^{85}\mathrm{Y}$	$1/2^-$ $^{87}\mathrm{Y}$	$1/2^-$ $^{89}\mathrm{Y}$	$1/2^-$ $^{91}\mathrm{Y}$	$1/2^-$ $^{93}\mathrm{Y}$	$1/2^-$ $^{95}\mathrm{Y}$	$1/2^-$ $^{97}\mathrm{Y}$
Z = 41		$9/2^+$ $^{91}\mathrm{Nb}$	$9/2^+$ $^{93}\mathrm{Nb}$	$9/2^+$ $^{95}\mathrm{Nb}$	$9/2^+$ $^{97}\mathrm{Nb}$	$9/2^+$ $^{99}\mathrm{Nb}$	
N = 50	$3/2^-$ $^{85}_{35}\mathrm{Br}$	$3/2^-$ $^{87}_{37}\mathrm{Rb}$	$1/2^-$ $^{89}_{39}\mathrm{Y}$	$9/2^+$ $^{91}_{41}\mathrm{Nb}$	$9/2^+$ $^{93}_{43}\mathrm{Tc}$	$9/2^+$ $^{95}_{45}\mathrm{Rh}$	



$E_{ex} = E_{sm}$ (all filled orbits)

$- E_{sm}$ (g.s.)

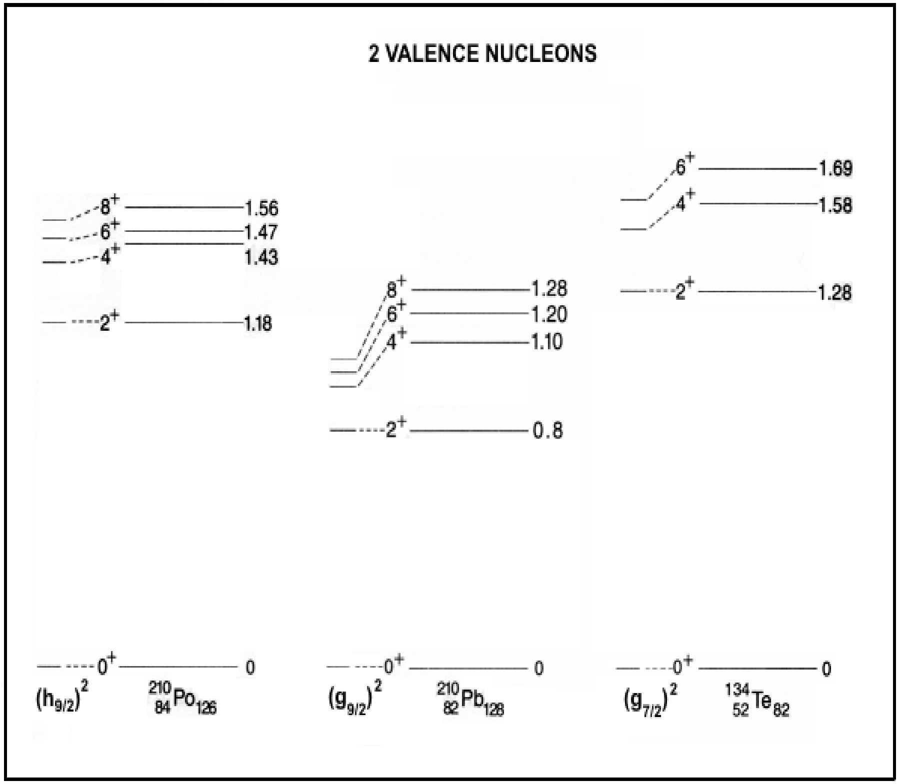
$$E_{ex} = E_{j_i} - E_{j_0}$$

So, what have we understood?

A lot, but not so much. We now understand something about magic numbers, and we can predict the lowest levels of nuclei with one particle (or one "hole") away from a doubly magic nucleus.

NICE !!! BUT, that is about 40 nuclei out of the thousands that we have data on. What about the others, with more than one valence nucleon?

We will immediately see that there are problems we need to deal with even in going to nuclei with just two valence nucleons!



Why these spins, these patterns?

Is there a way, starting with the IPM, to deal with multi-valence nucleon systems?

How do angular momenta combine? What accounts for the specific order of the states? Why is the ground state 0^+? Why are the excited energies so high and monotonic in spin?

Backups