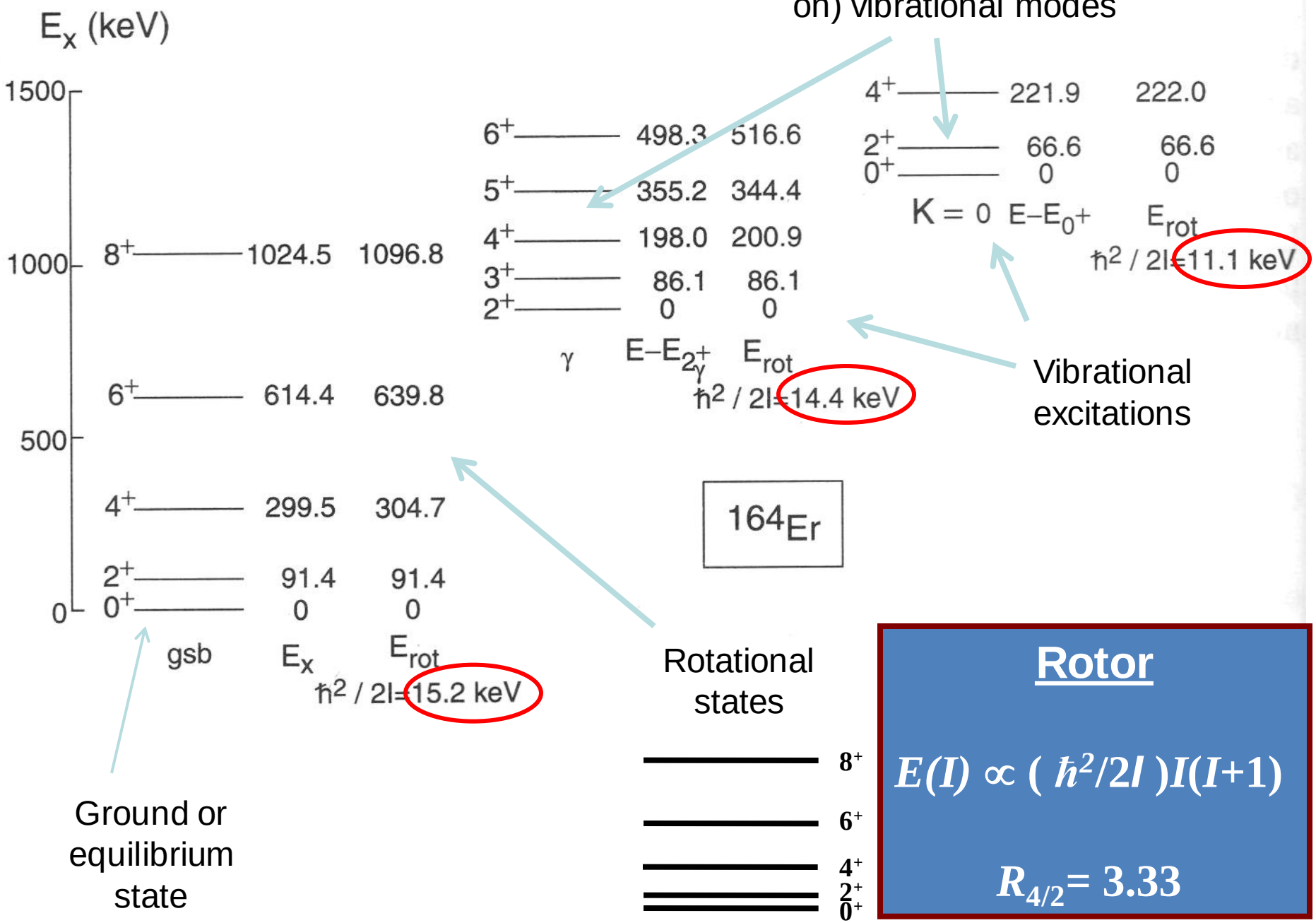
More on Collective models

Microscopic drivers: Valence p-n interactions

Simply estimating the properties of nuclei

Exotic nuclei

Typical deformed nucleus



Rotor

$$E(I) \propto (\hbar^2/2I)I(I+1)$$

$$R_{4/2} = 3.33$$

Guidelines about Collective states.

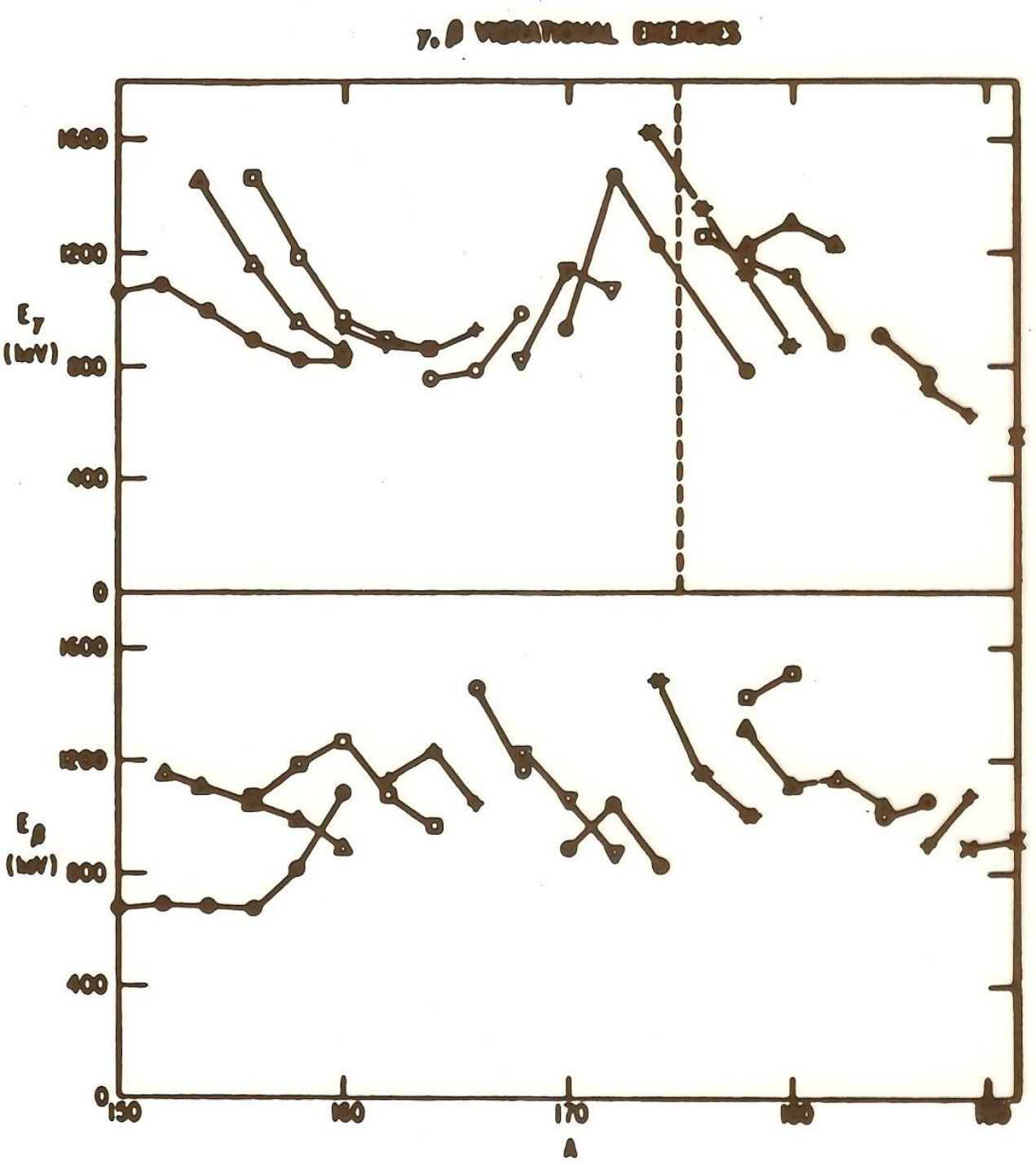
They arise from mixing and so should be relatively low lying in energy. The lower in energy, the more collective.

Collectivity inherently involves wave functions with many components corresponding to particles in many different orbit configurations (many states mix)

That implies that collective excitations do not change much from one nucleus to a neighboring (in Z and N) one.

Systematics and collectivity of the lowest vibrational modes in deformed nuclei

Notice that the the β mode is at higher energies (~ 1.5 times the γ vibration near mid-shell)* and fluctuates more. This points to lower collectivity of the β vibration.



Energies of collective states

Gamma ray transitions between collective states:

Intraband, interband, Alaga

Electromagnetic Transitions in Deformed Nuclei

$$\psi = \left[D_{IMK}\chi_K + (-1)^{J-K} D_{IM-K}\chi_{-K} \right]$$

E2 Matrix Elements $\langle \psi_f | E2 | \psi_i \rangle$

4 terms – simplify notations $D_{IMK} \rightarrow D_K$

$$\langle \psi_f \| E2 \| \psi_i \rangle = \langle D_{K_f}\chi_{K_f} + (-1)^{J-K_f} D_{-K_f}\chi_{-K_f} \| E2 \| D_{K_i}\chi_{K_i} + (-1)^{J-K_i} D_{-K_i}\chi_{-K_i} \rangle$$

Cross terms involve $\Delta K = K_i + K_f$

Usually not connected by E2 operator

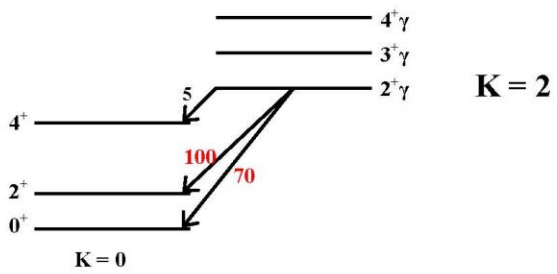
Two direct terms give equal results.

$$\therefore \quad \langle \psi_f \| E2 \| \psi_i \rangle = \langle D_{K_f}\chi_{K_f} \| E2 \| D_{K_i}\chi_{K_i} \rangle$$

Separability of rotational, intrinsic motion

$$\boxed{\langle \psi_f \| E2 \| \psi_i \rangle = \langle D_{K_f} \| E2 \| D_{K_i} \rangle \langle \chi_{K_f} \| E2 \| \chi_{K_i} \rangle}$$

Apply to transitions between two bands



Remember:

$$T(E2) + KE_\gamma^5 B(E2)$$

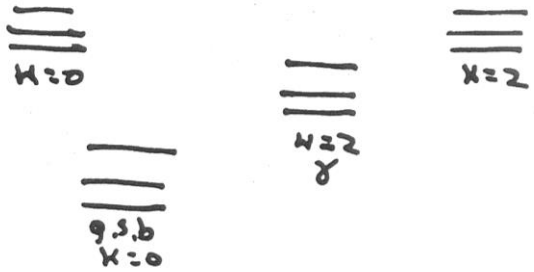
$$\frac{\langle J_i K_i \| E2 \| J_f K_f \rangle}{\langle J_i K_i \| E2 \| J_{f'} K_f \rangle} = \frac{\langle D_{J_i K_i} \| E2 \| D_{J_f K_f} \rangle}{\langle D_{J_i K_i} \| E2 \| D_{J_f' K_f} \rangle} \times \frac{\langle \chi_i \| E2 \| \chi_f \rangle}{\langle \chi_i \| E2 \| \chi_f \rangle}$$

BR is, again ratio of $(C - G)^2$

$$\frac{B(E2: J_i \to J_f)}{B(E2: J_i \to J_{f'})} = \frac{\langle J_i K_i 2 \ \Delta K | J_f K_f \rangle^2}{\langle J_i K_i 2 \ \Delta K | J_{f'} K_f \rangle^2}$$

$$\frac{B(E2: 2_\gamma^+ \to 0_g^+)}{B(E2: 2_\gamma^+ \to 2_g^+)} = 0.7$$

$$\frac{B(E2: 2_\gamma^+ \to 4_g^+)}{B(E2: 2_\gamma^+ \to 2_g^+)} = 0.05$$



Some useful Alaga rules for E2 transitions in deformed nuclei[a]

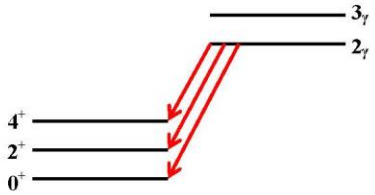
J_i	J_f	$\langle J_i K 2\Delta K \mid J_f K_f\rangle^2$ $K_i \to K_f$			
		$0 \to 0$	$2 \to 0$	$0 \to 2$	$2 \to 2$
0	2	1.0	—	1.0	—
2	0	0.200	0.200	—	—
	2	0.286	0.286	0.286	0.286
	3	—	—	0.500	0.500
	4	0.515	0.014	0.215	0.215
3	2	—	0.358	—	0.358
	3	—	—	—	0
	4	—	0.143	—	0.343
	5	—	—	—	0.300
4	2	0.286	0.120	0.008	0.120
	3	—	—	0.112	0.267
	4	0.260	0.351	0.351	0.042
	5	—	—	0.389	0.234
	6	0.455	0.031	0.142	0.340
5	3	—	—	—	0.191
	4	—	0.319	—	0.191
	5	—	—	—	0.093
	6	—	0.182	—	0.167
	7	—	—	—	0.360
6	4	0.315	0.098	0.021	0.235
	5	—	—	0.154	0.141
	6	0.255	0.364	0.364	0.130
	7	—	—	0.347	0.124
	8	0.431	0.039	0.116	0.371

70
100
180

70
100
5

[a]The entries are the squares of the Clebsch-Gordon coefficients for each indicated transition. Thus, relative B(E2) values connecting states J_i, J_f in bands with K_i, K_f are $B(E2; J_i K_i \to J_f K_f) = \langle J_i K 2\Delta K \mid J_f K_f\rangle^2$.

Relative B(E2 : $J_\gamma \rightarrow J_g$) in Deformed Nuclei



Compare Data to Alaga Rules

Table 6.8 *Comparison of some relative* B(E2: $\gamma \rightarrow g$) *values in deformed rare earth nuclei with the Alaga rules* *

J_i	J_f	Alaga	Relative B(E2 : $J_i \rightarrow J_f$) ^{154}Gd	^{168}Er	^{178}Hf
2	0	70	43	54	88
	2	100	100	100	100
	4	5	14	6.8	5.8
3	2	100	100	100	100
	4	40	105	65	52
4	2	34	16	20	18
	4	100	100	100	100
	6	9	—	14	—
5	4	100	—	100	100
	6	57	—	123	107
6	4	27	—	12	18
	6	100	—	100	100
	8	11	—	37	—

* One transition is normalized to 100 for each initial state. The Alaga rule entries are relative values from Table 6.7 for the $K = 2 \rightarrow K = 0$ case.

We have discussed spherical vibrators and deformed ellipsoidal rotors.

But how do nuclei evolve between these extreme limits?

Deformation can develop in several ways.

One of the most intriguing and challenging is very rapid onsets of deformation that have been described in terms of Quantum Phase Transitions and new "symmetries" for critical point nuclei called E(5) and X(5).

Spectroscopic observables

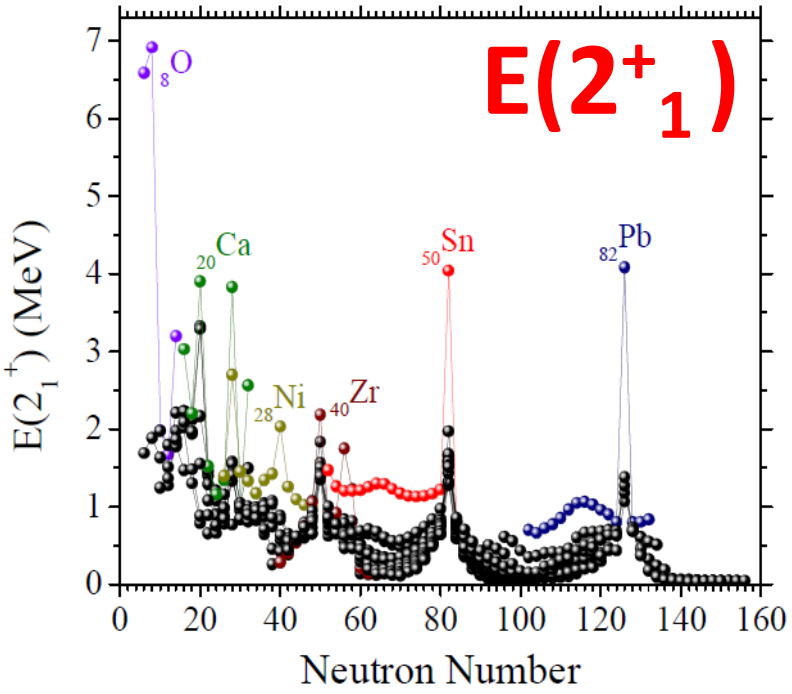
Two obvious features which capture much of the physics:

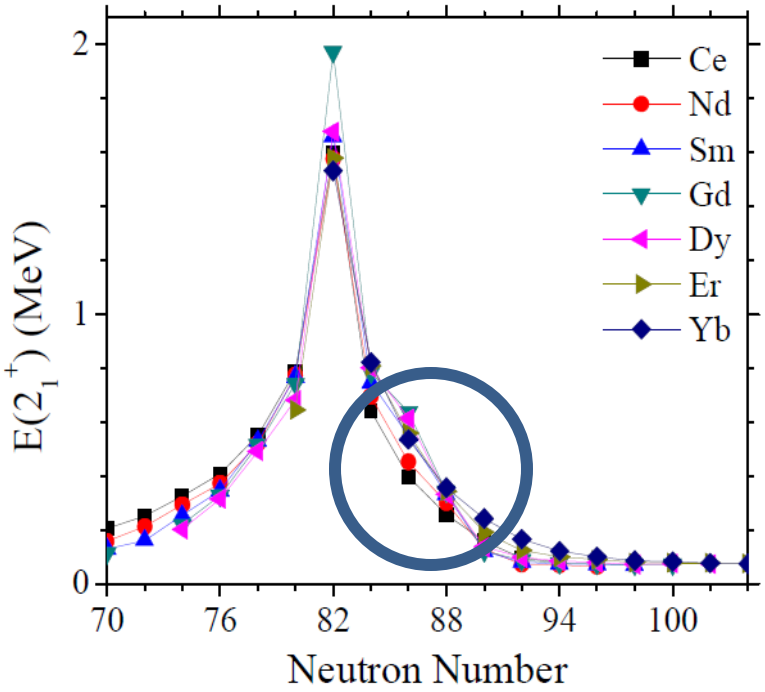
- **High values** at certain numbers, 2, 8, 20, 50, 82, 126…

These show the rigidity to excitation of nuclei with these special numbers of nucleons

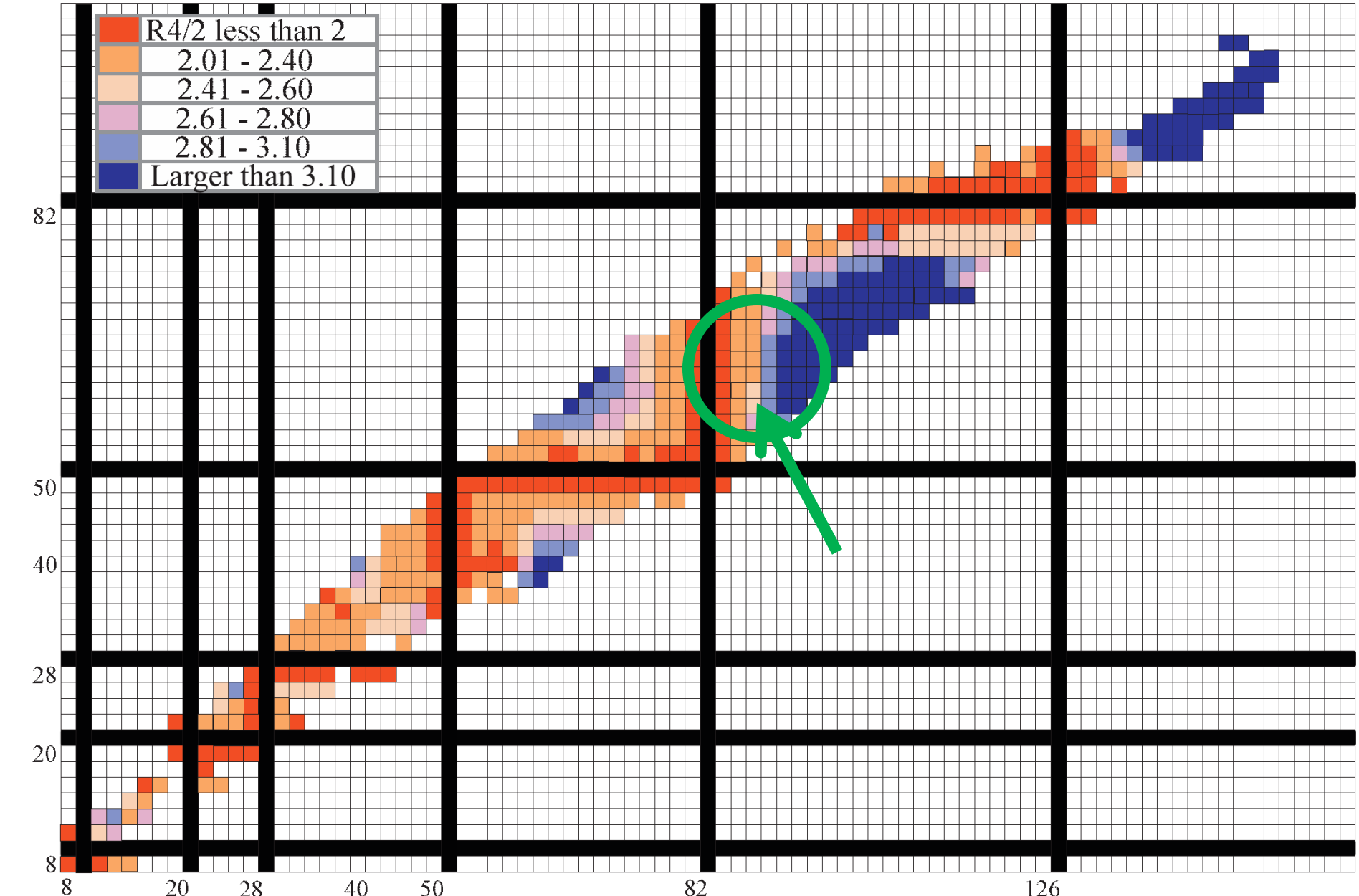
- **Sharp drops** thereafter.

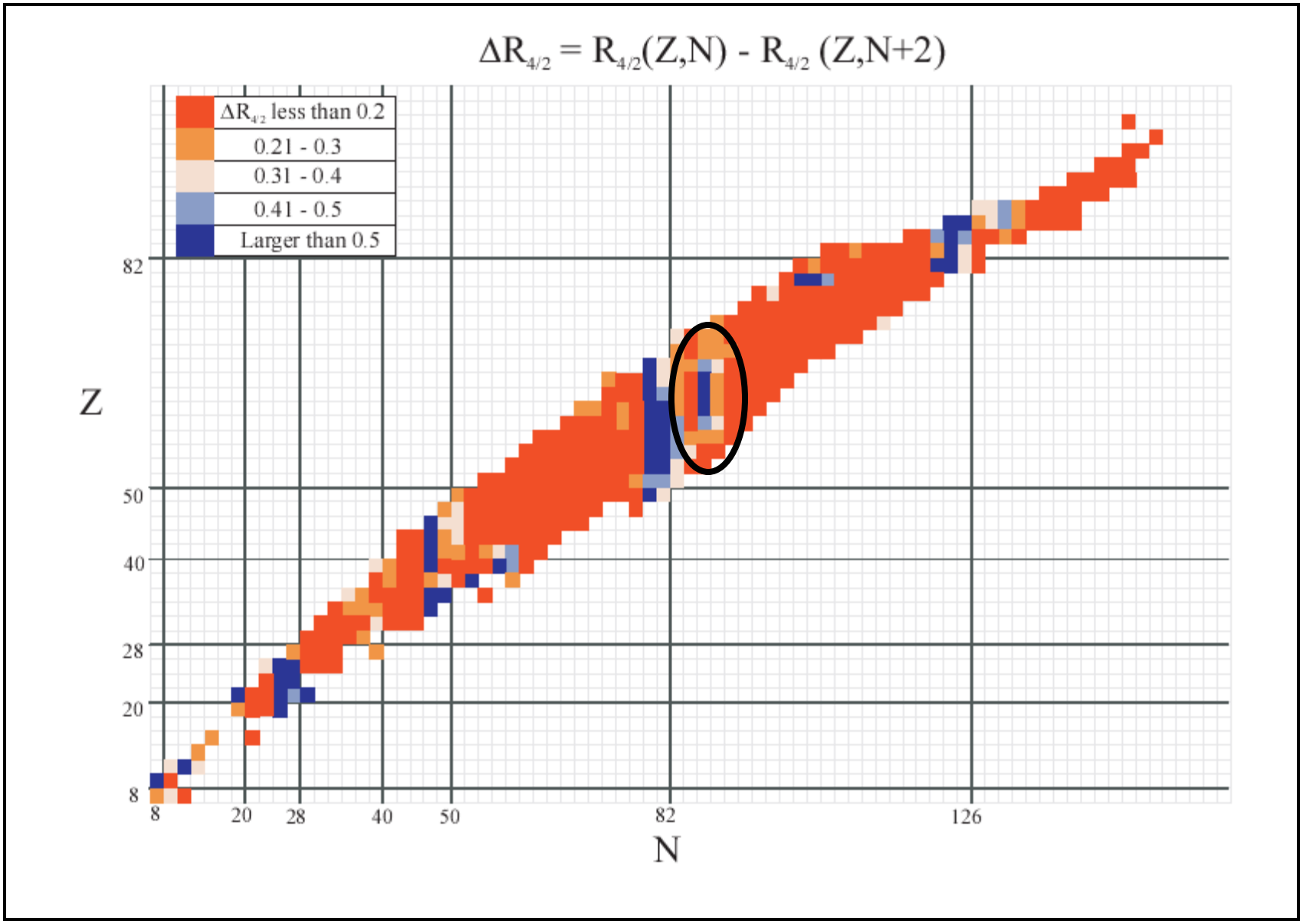
- Something must be special about these "magic" numbers



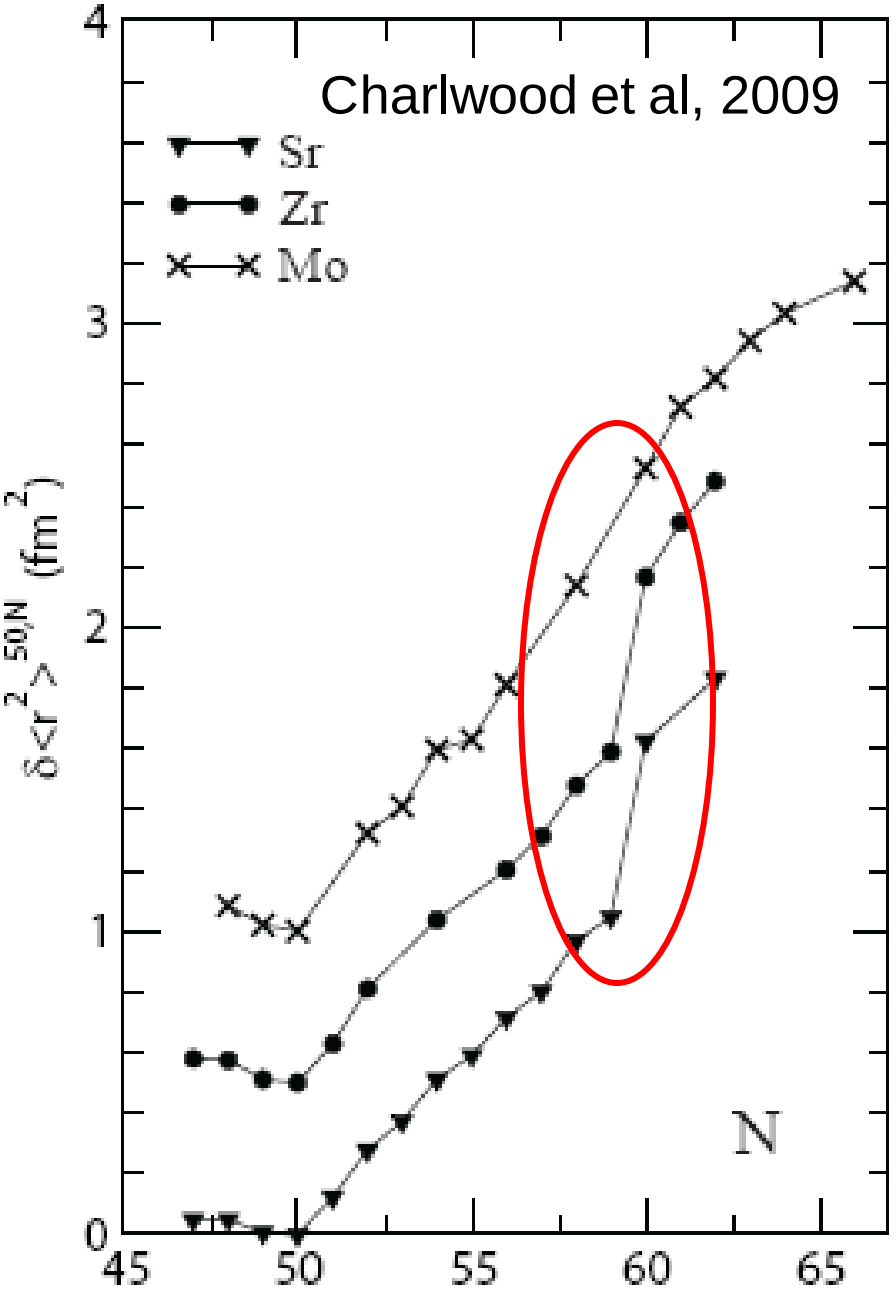


$R_{4/2}$


R4/2 less than 2
2.01 - 2.40
2.41 - 2.60
2.61 - 2.80
2.81 - 3.10
Larger than 3.10
82
50
40
28
20
8
8
20
28
40
50
82
126

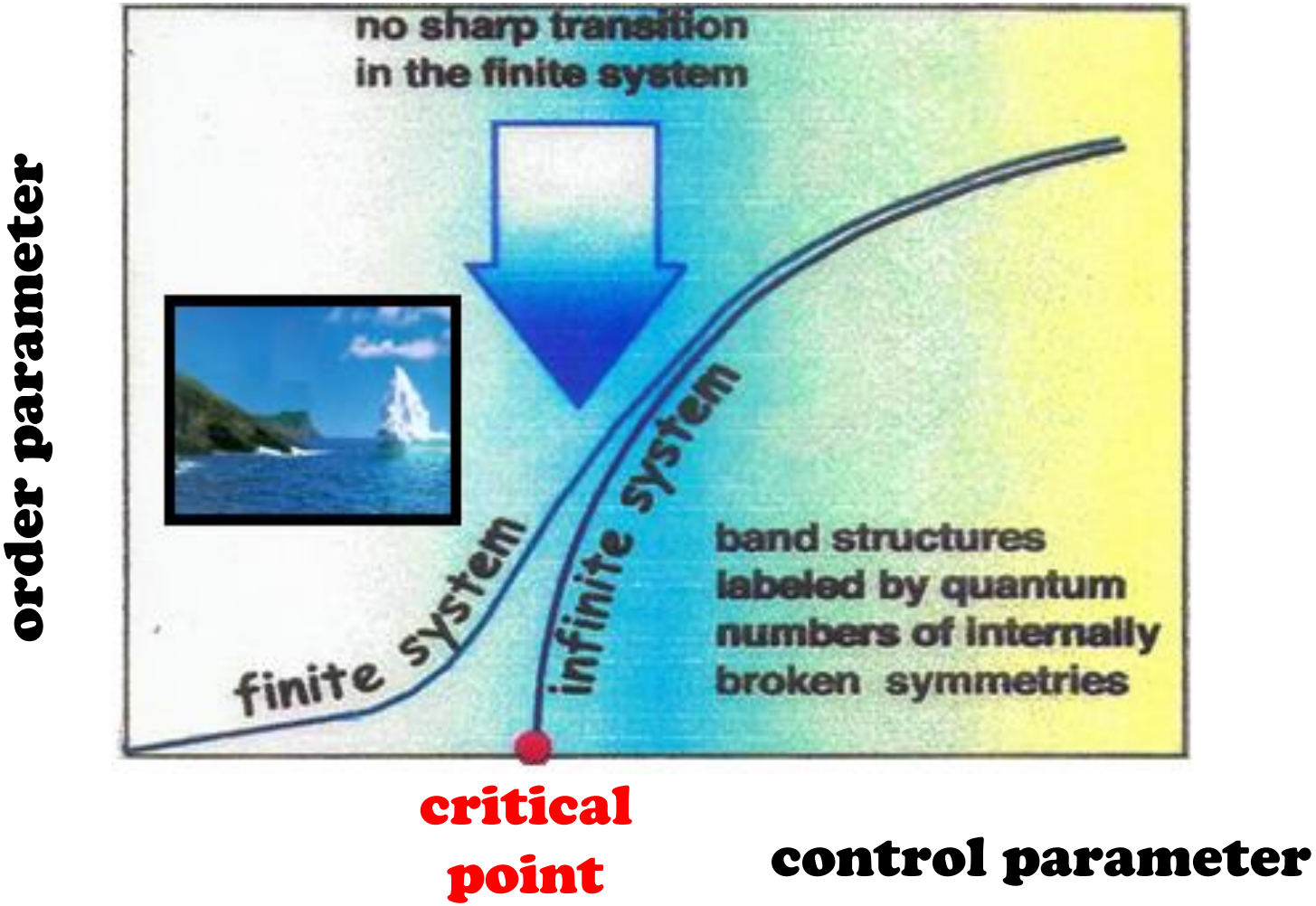

$\Delta R_{4/2} = R_{4/2}(Z,N) - R_{4/2}(Z,N+2)$
$\Delta R_{4/2}$ less than 0.2
0.21 - 0.3
0.31 - 0.4
0.41 - 0.5
Larger than 0.5
Z
N
8
20
28
40
50
82
126

Isotope shifts



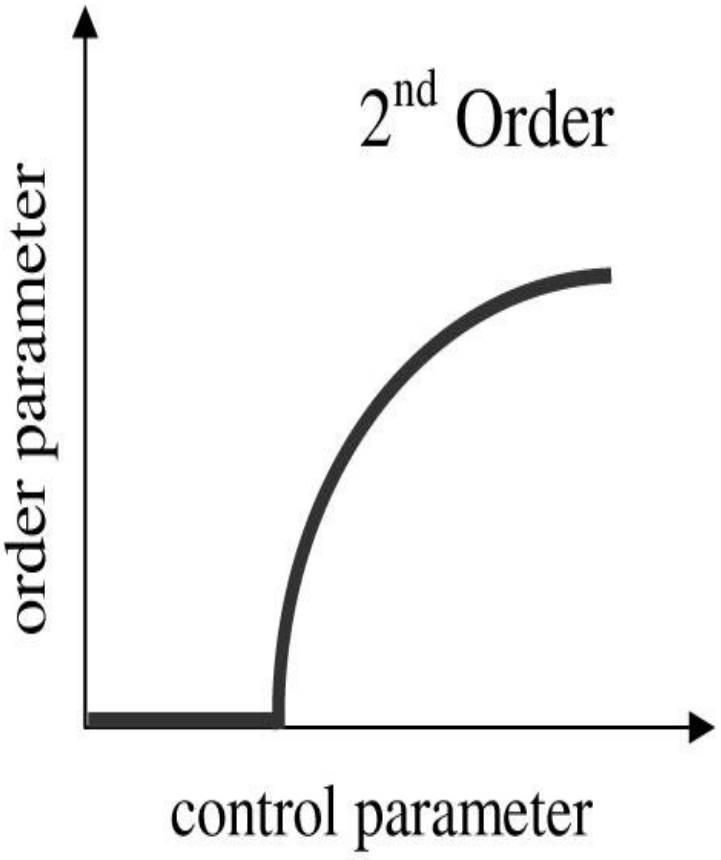
Li et al, 2009

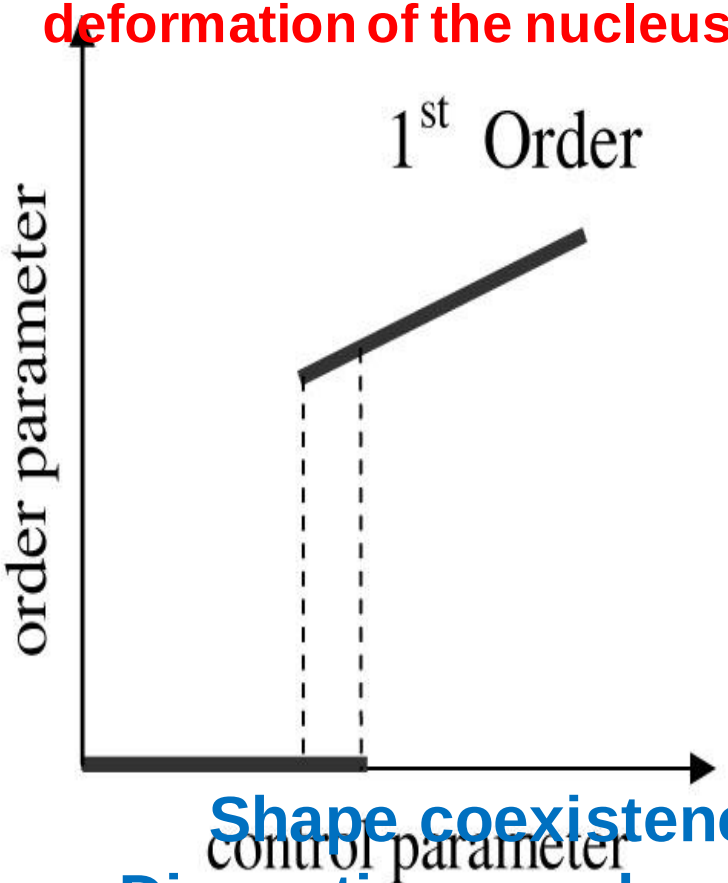
Quantum (equilibrium) phase transitions in the shapes of strongly interacting finite nuclei as a function of neutron and proton number



Quantum phase transitions in equilibrium shapes of nuclei with *N*, *Z*

Potential as function of the ellipsoidal deformation of the nucleus

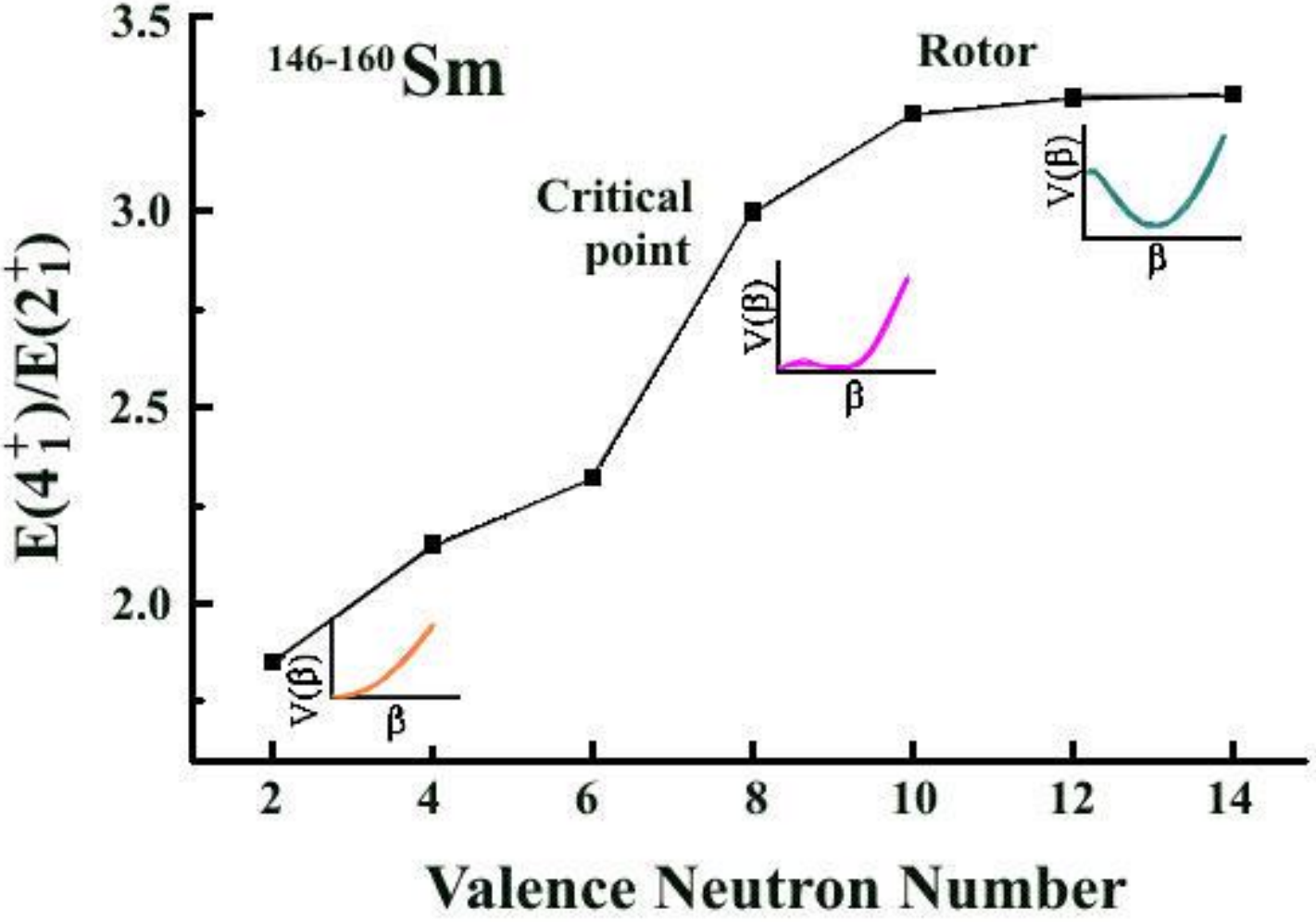




Shape coexistence

Discontinuous change in equilibrium deformation

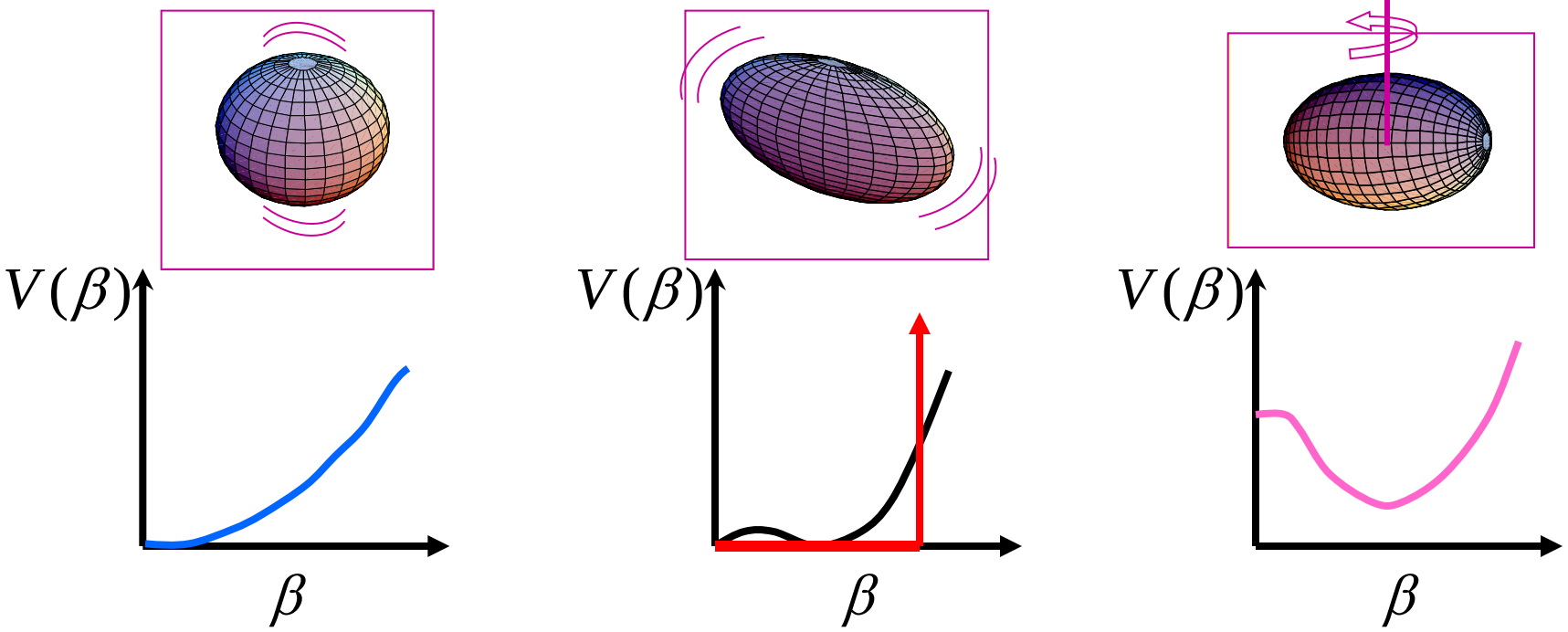
For nuclear shape phase transitions the control parameter is nucleon number and the order parameter is deformation


146-160 Sm
Rotor
Critical point
V(β)
β
V(β)
β
V(β)
β
E(4$_1^+$)/E(2$_1^+$)
3.5
3.0
2.5
2.0
2
4
6
8
10
12
14
Valence Neutron Number

Nuclear Shape Evolution

β - nuclear ellipsoidal deformation (β=0 is spherical)

Vibrational Region | Transitional Region | Rotational Region



$E_n = n\hbar\omega$

$R_{4/2}$= ~2.0

Critical Point

New analytical solutions, E(5) and X(5)

$E_J \sim J(J+1)$

$R_{4/2}$= 3.33

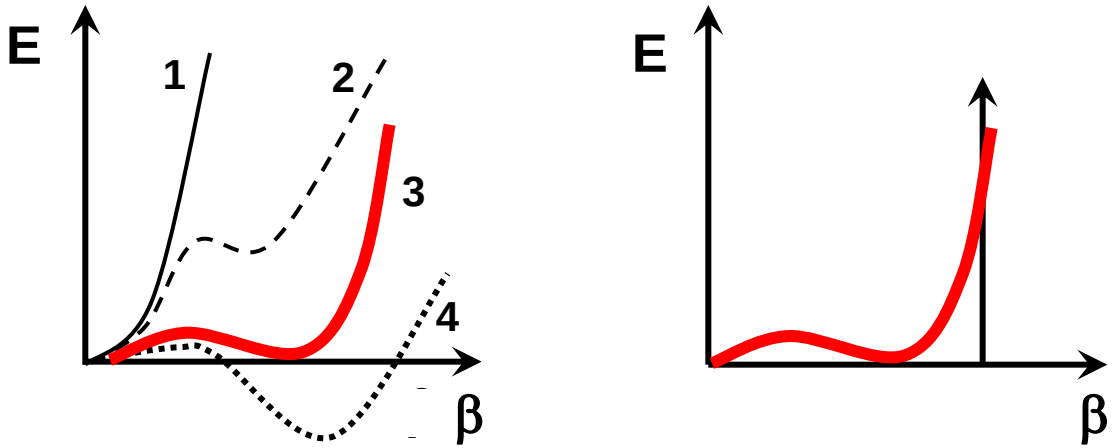
Few valence nucleons ⟶ **Many valence Nucleons**

Critical Point Symmetries

First Order Phase Transition – Phase Coexistence

X(5)

Energy surface changes with valence nucleon number

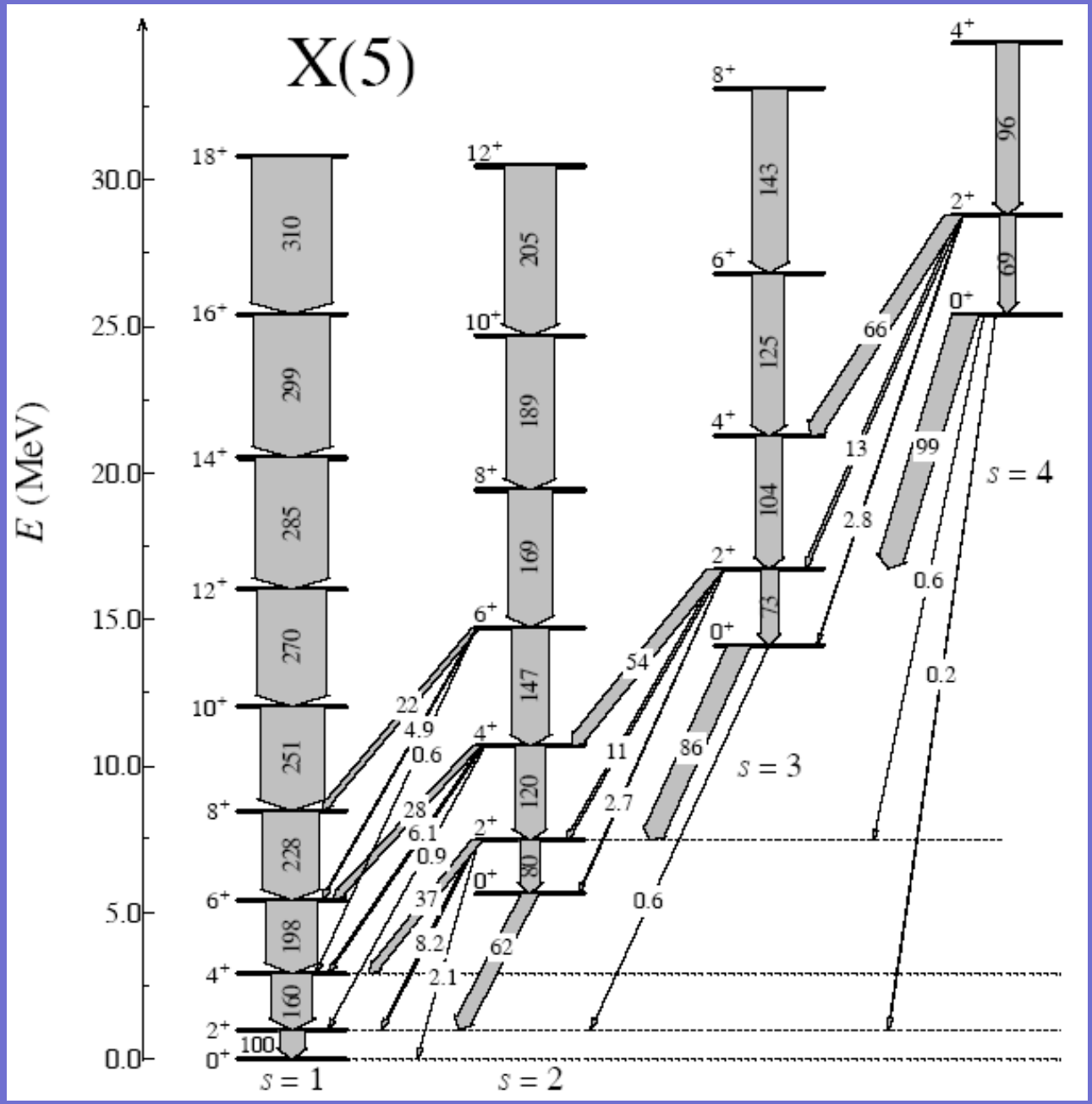


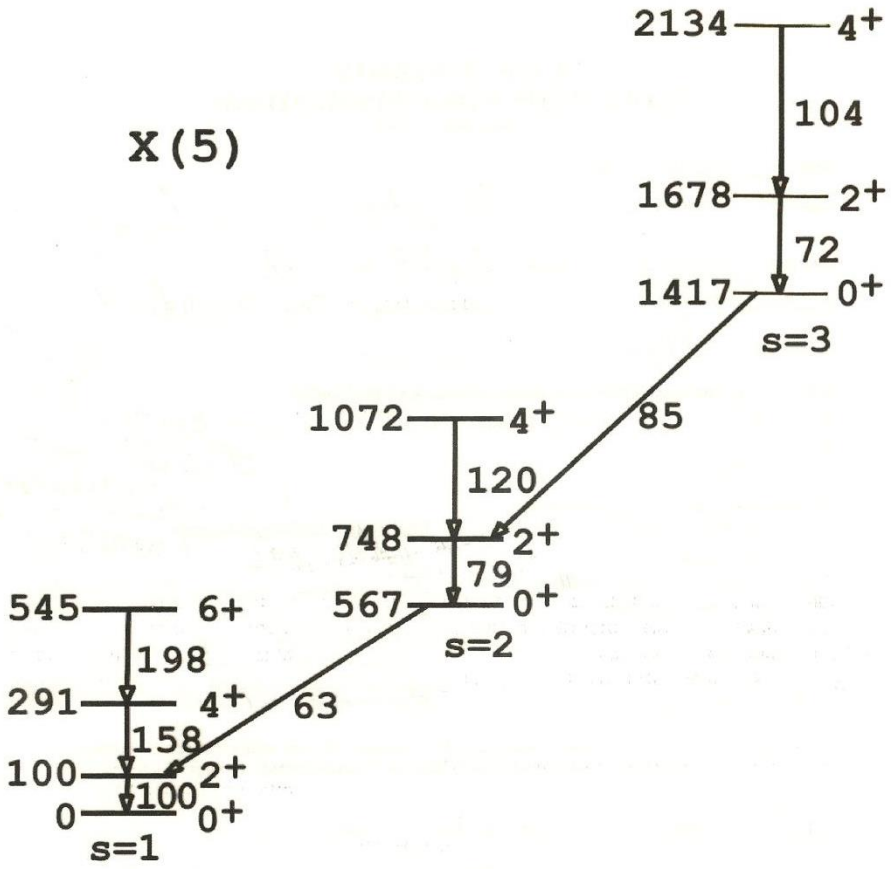
Bessel equation

$$\tilde{\xi}'' + \frac{\tilde{\xi}'}{z} + \left[1 - \frac{\nu^2}{z^2} \right] \tilde{\xi} = 0; \qquad \tilde{\xi}\left(\beta_w\right) = 0.$$

$$\nu = \left(\frac{L(L+1)}{3} + \frac{9}{4} \right)^{1/2}$$

Iachello


X(5)
E (MeV)
0.0
5.0
10.0
15.0
20.0
25.0
30.0
0+
2+
4+
6+
8+
10+
12+
14+
16+
18+
100
160
198
228
251
270
285
299
310
S = 1
0+
2+
4+
6+
8+
10+
12+
80
120
147
169
189
205
S = 2
0+
2+
4+
6+
8+
73
104
125
143
S = 3
0+
2+
4+
69
96
S = 4
22
4.9
0.6
28
6.1
0.9
37
8.2
2.1
62
54
11
2.7
0.6
86
66
13
2.8
99
0.6
0.2



No free parameters (except scale)

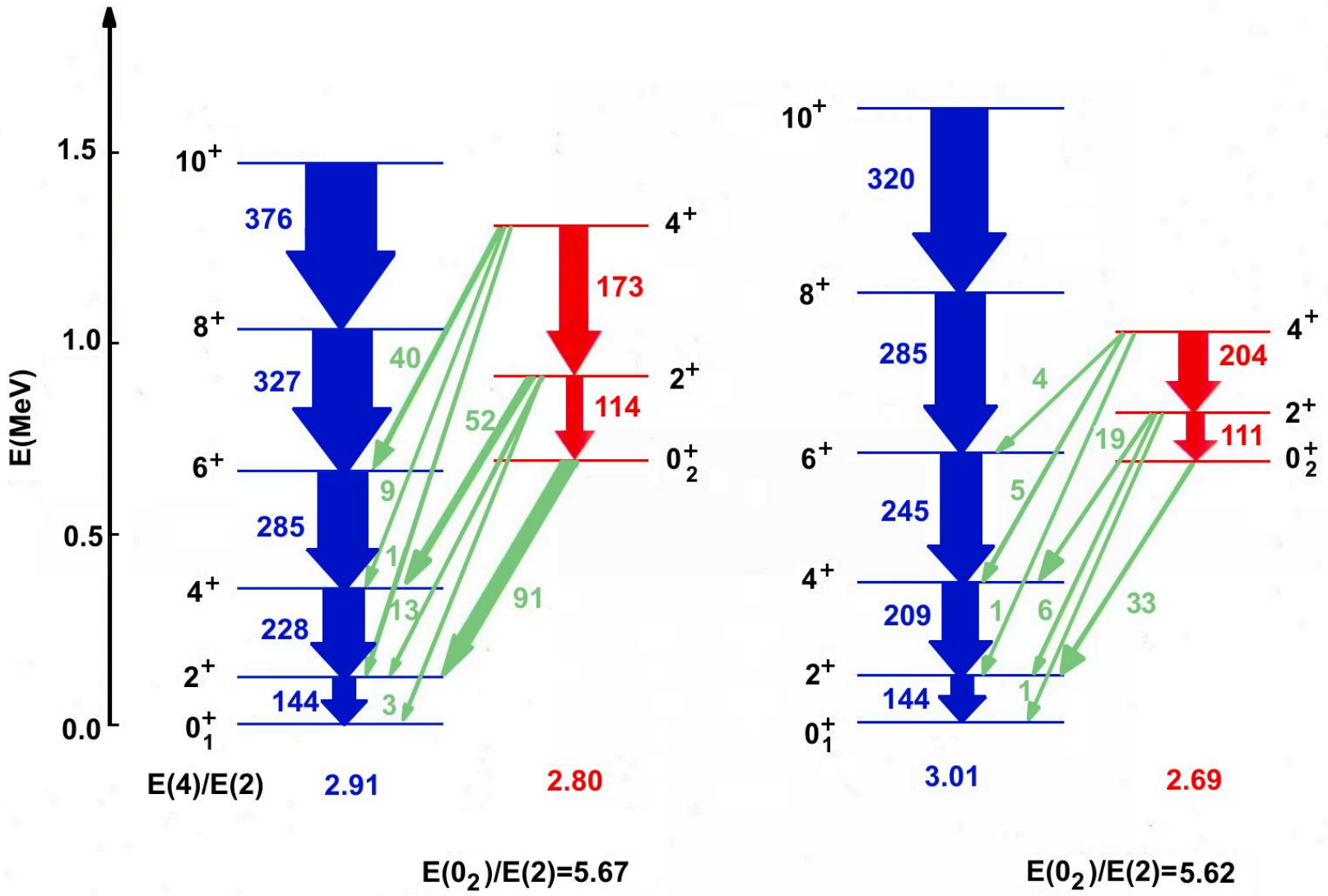
3 key signatures: $E(4_1^+)/E(2_1^+)=2.91$

$E(0_2^+)/E(2_1^+)=5.67$

$E(6_1^+) \cong E(0_2^+)$

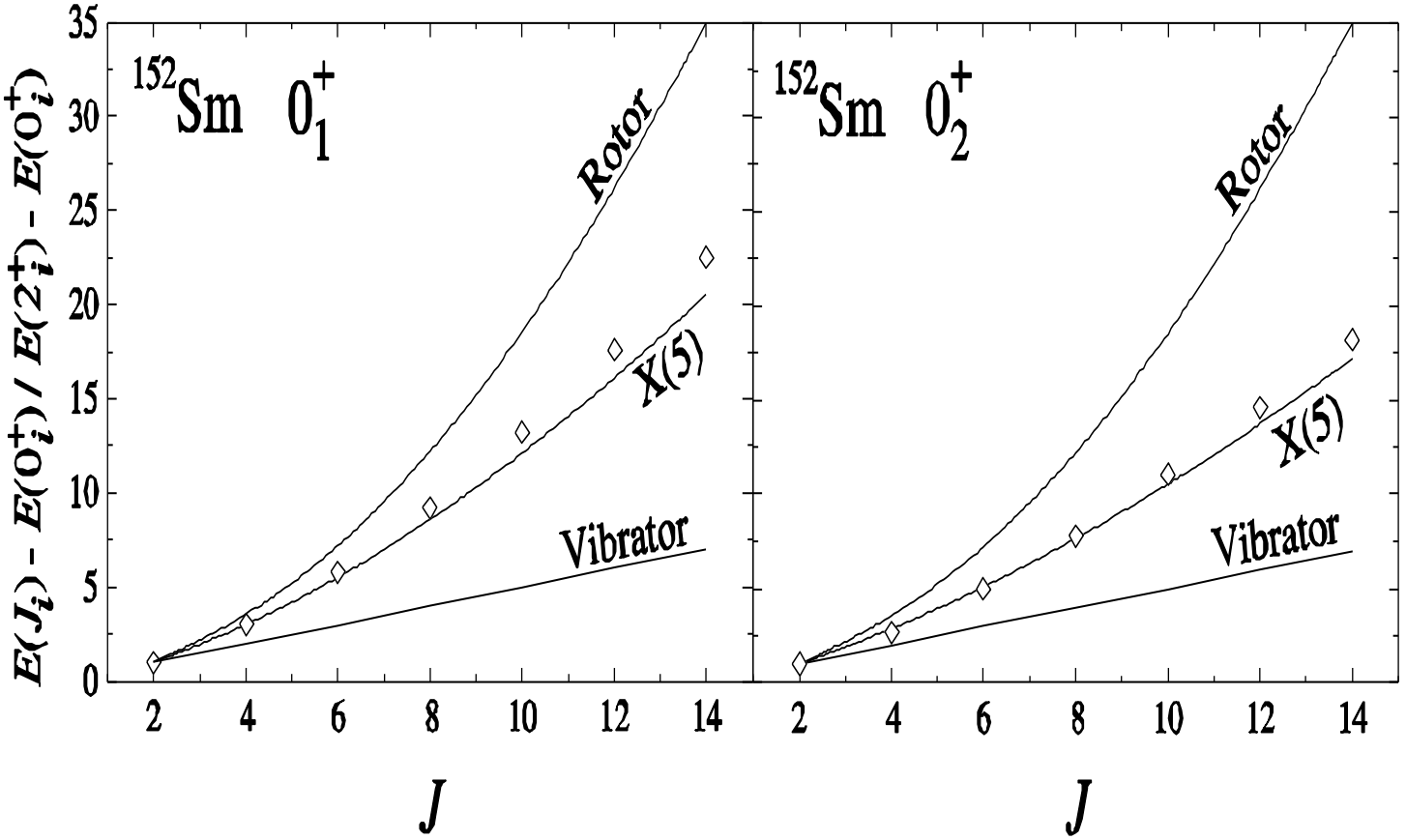
New Paradigm for Nuclear Structure

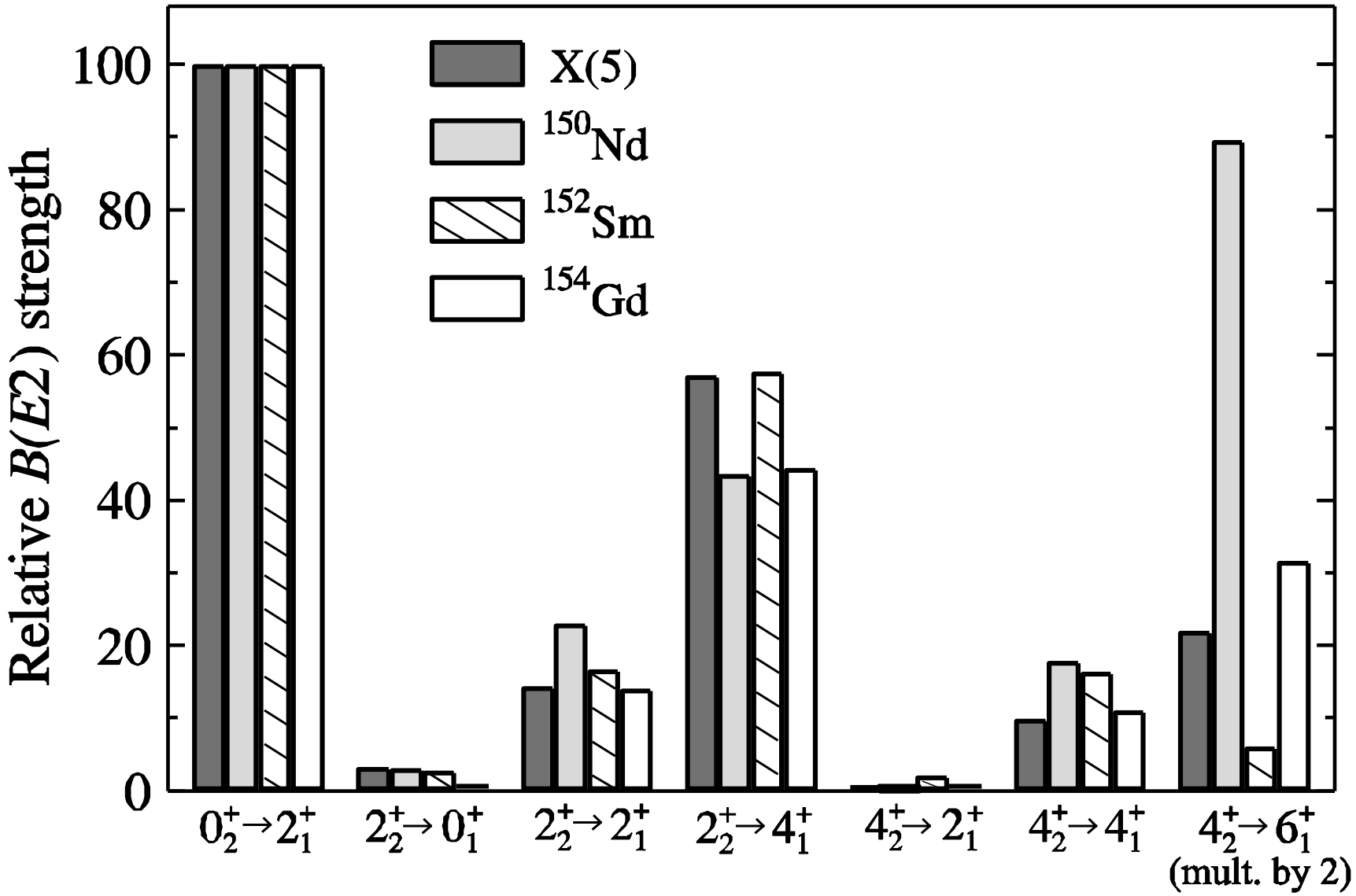
Critical Point Phase Transitional Nuclei

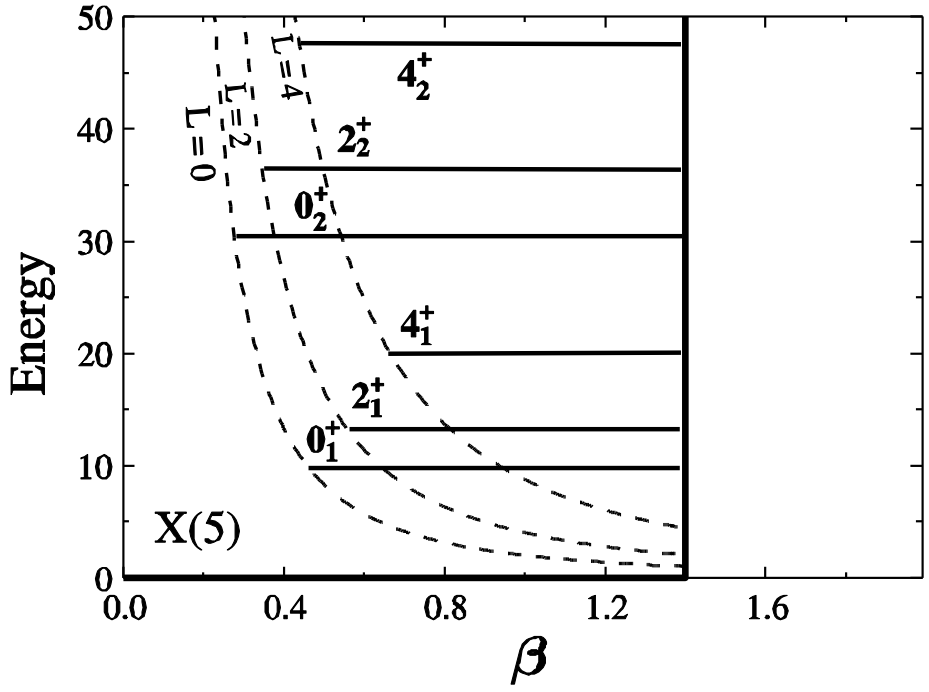


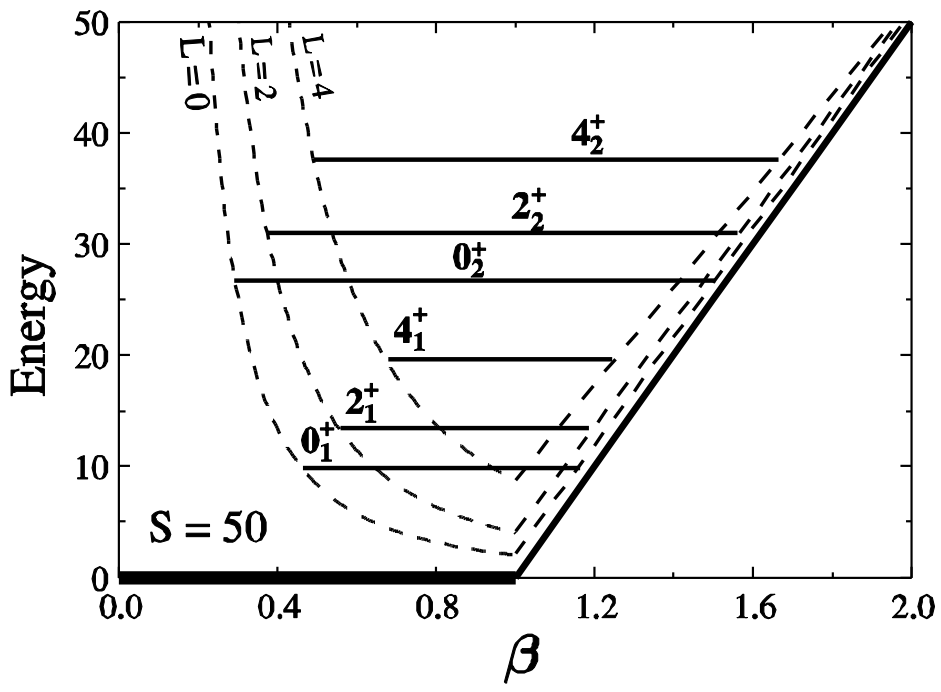
Casten and Zamfir

Comparison of relative energies with X(5)




Relative B(E2) strength
100
80
60
40
20
0
X(5)
150Nd
152Sm
154Gd
0+2→2+1
2+2→0+1
2+2→2+1
2+2→4+1
4+2→2+1
4+2→4+1
4+2→6+1
(mult. by 2)

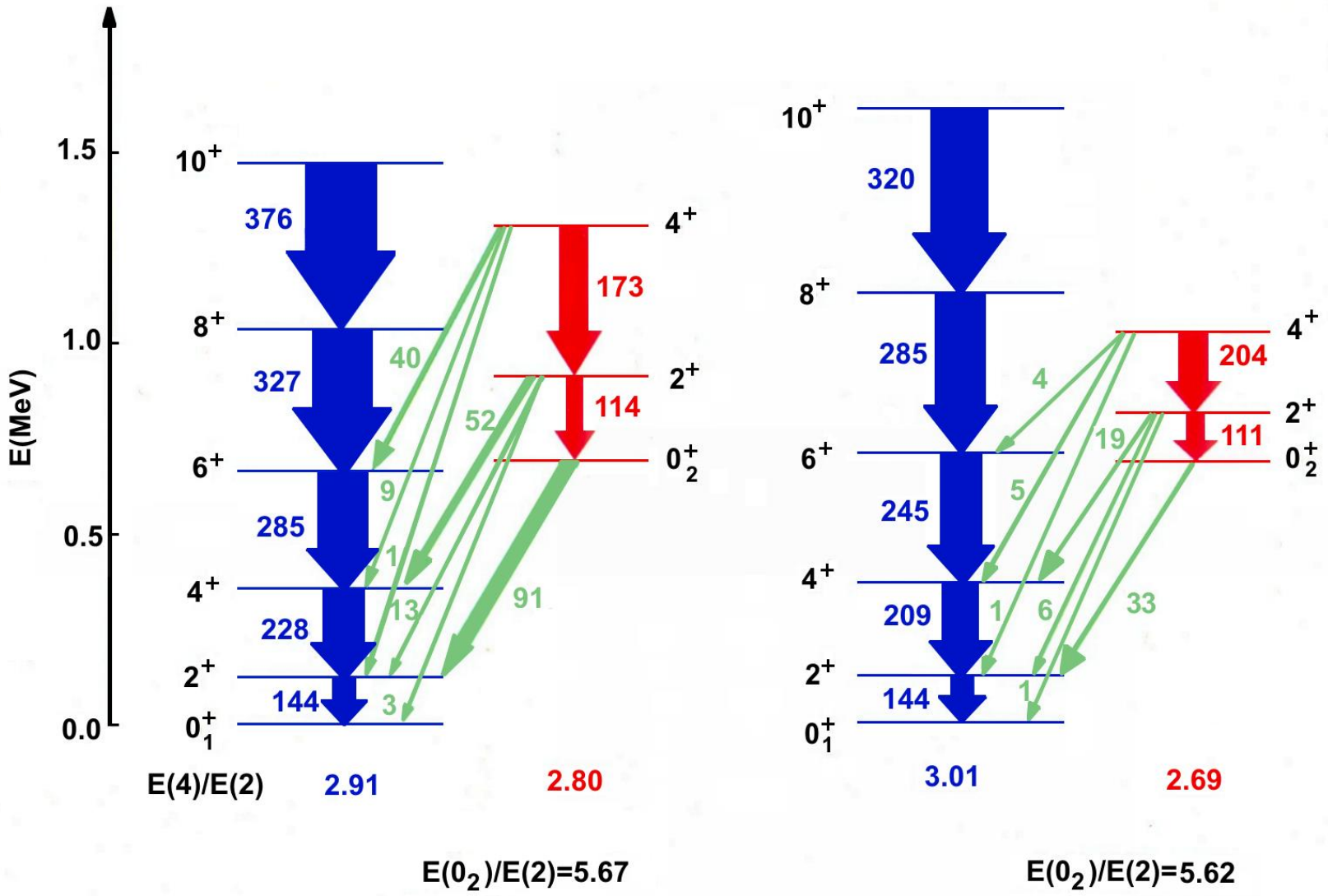




Based on idea of Mark Caprio

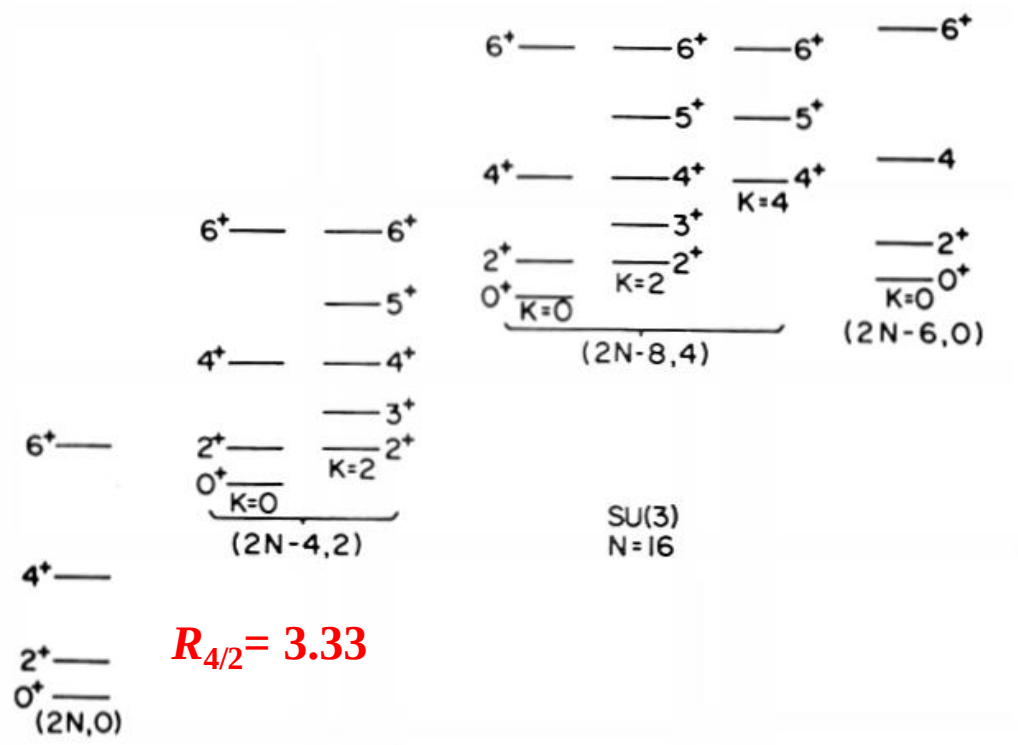
New Paradigm for Nuclear Structure

Critical Point Phase Transitional Nuclei



Casten and Zamfir

Borrowing from the later discussion --SU(3)



$R_{4/2}$= 3.33

Characteristic signatures:

- **Degenerate bands within a group**
- **Vanishing B(E2) values between groups**
- **Allowed transitions between bands within a group**

Hence BOTH the disagreements – for the energies in the excited band and for the B(E2) values are seen to be – not a whole series of unrelated discrepancies but, in both cases, a matter of scale.

This answers one question that was asked, namely how to react when a model doesn't work – sometimes you learn something !!

These collective models have been introduced ad hoc.

Is there a single comprehensive collective model that incorporates a variety of types of collectivity in a simple framework?

There are two: the Geometric Collective Model (GCM) and the Interacting Boson Approximation (IBA) model.

The latter is simpler, with fewer parameters, and has had continuing success. We will very briefly summarize the GCM and then discuss the IBA.

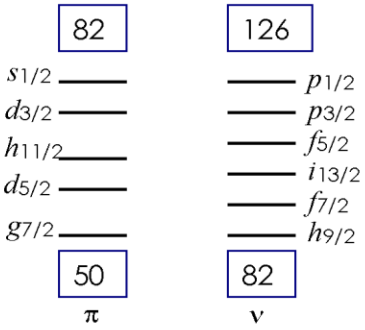
Recall the problem that faces descriptions of many-body systems: Too many "basis"states.

$^{154}_{62}Sm_{92}$

cl. sh. 50 82

$N_p = 12 \quad N_n = 10$

12 val. π in 50 – 82
10 val. ν in 82 – 126



How many 2⁺ states subject to Pauli Principle limits?

3 x 10^{14} ! ! !

^{154}Sm 2⁺ states within the valence shell space

Nuclear Shapes

- Need to specify the shape. Need two parameters, β and γ. The concept of "intrinsic frame".

  - β specifies the ellipsoidal deformation of the shape. (We consider quadrupole shapes only – American football or frisbee shapes.)

  - γ specifies the amount of axial asymmetry

- $H = T + V(\beta,\gamma)$ Models are primarily a question of choosing $V(\beta,\gamma)$

- Kinetic energy contains rotation if the nucleus is not spherical. So we must specify orientation of the nucleus in space (the lab frame). Introduces three more coordinates, Euler angles.

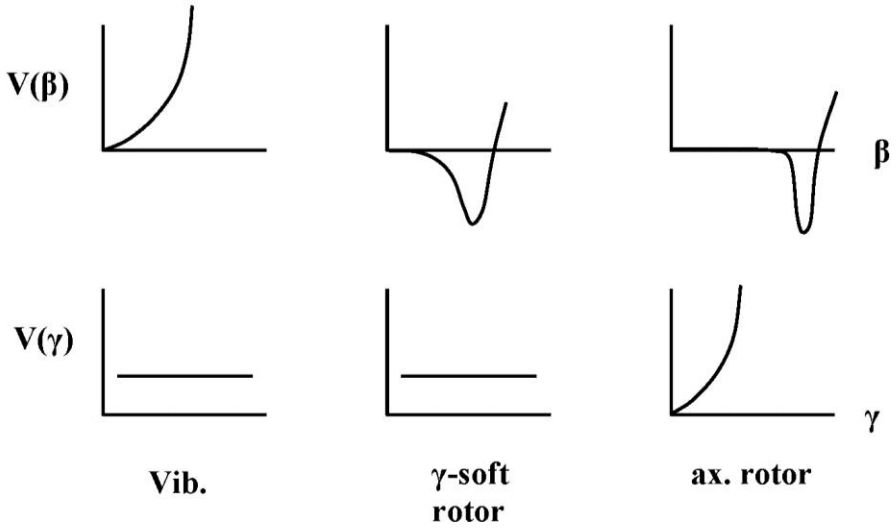
The Geometric Collective Model

$$H = T + T_{rot} + V(\beta,\gamma)$$

$$V \sim C_2\beta^2 + C_3\,\beta^3 \cos 3\gamma + C_4\beta^4 + \ldots.$$

Six terms in all for the potential. These three are normally the only ones used as they allow a rich variety of collective structures without an explosion of parameters. In addition, there is a kinetic energy term.

Geometric Collective Model



Vibrator:

$V = C_2' \beta^2$ $\qquad C_2' > 0$

γ-soft:

$V = C_2' \beta^2 + C_4' \beta^4$ $\qquad C_2' < 0,\ C_4' > 0$

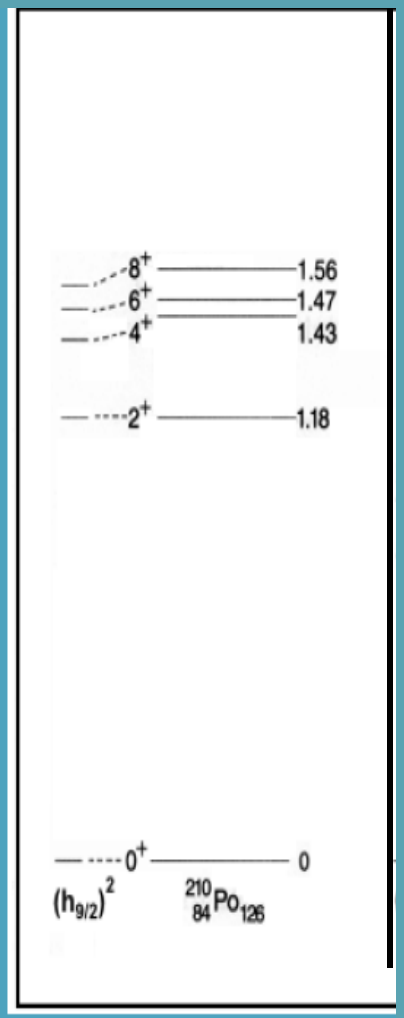
Rotor:

$V = C_2' \beta^2 + C_4' \beta^4 - C_3' \beta^3 \cos 3\gamma$ $\qquad C_2' < 0, C_3', C_4' > 0$

An algebraic approach
Collective behavior superposed on shell structure
IBA, a symmetry-based model (Iachello and Arima)

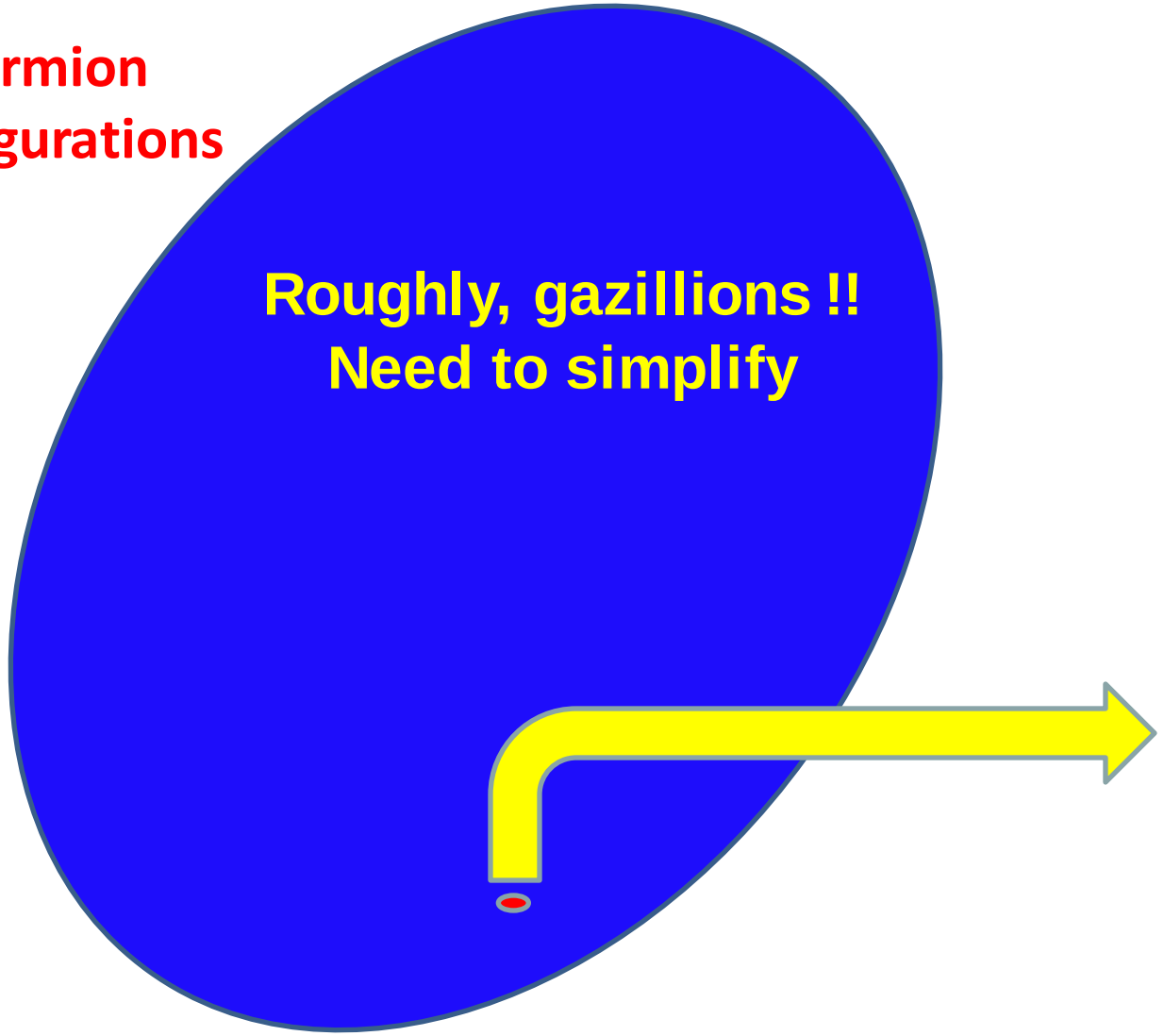
Drastic simplification of shell model

- Valence nucleons, in pairs as bosons
- Only certain configurations. Only pairs of nucleons coupled to angular momentum 0(**s**) and 2(**d**). Why?
- Simple Hamiltonian in terms of s an d boson creation, destruction operators – simple interactions
- **Group theoretical** underpinning
- **Why?** Because it works. And extremely parameter-efficient



Shell Model Configurations

Fermion configurations



The IBA

Boson configurations

(by considering only configurations of pairs of fermions with J = 0 or 2.)

Modeling a Nucleus

Why the IBA is the best thing since baseball, a jacket potato, aceto balsamico, Mt. Blanc, raclette, pfannekuchen, baklava, ….

^{154}Sm ➡ Shell model ➡ 3×10^{14} 2^+ states

Need to truncate

IBA assumptions

1. **Only valence nucleons**
2. **Fermions → bosons**

J = 0 (s bosons)

J = 2 (d bosons)

IBA: 26 2^+ states

Is it conceivable that these 26 basis states are correctly chosen to account for the properties of the low lying collective states?

IBA: Truncation of Shell Model with Group Theory structure

Brief, simple, trip into the Group Theory of the IBA

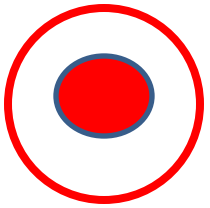
DON'T BE SCARED

You do not need to understand all the details but try to get the idea of the relation of groups to degeneracies of levels and quantum numbers

A more intuitive name for this application of Group Theory is

"Spectrum Generating Algebras"

IBA has a deep relation to Group theory



That relation is based on the operators that create, destroy *s* and *d* bosons

$s^\dagger$, s, $d^\dagger$, d operators $\qquad N_B = n_s + n_d = s^+s = d^+d$

Ang. Mom. 2 $\qquad d^\dagger_\mu , d_\mu \quad \mu = 2, 1, 0, -1, -2$

Hamiltonian is written in terms of *s, d* operators

$$\mathbf{H = H_s + H_d + H_{int}\,(s^\dagger s,\ s^\dagger d,\ d^\dagger s,\ d^\dagger d)}$$

Since boson number, N_B, is conserved for a given nucleus, *H* can only contain "bilinear" terms: 36 of them.

$s^\dagger s,\ s^\dagger d,\ d^\dagger s,\ d^\dagger d$ ⟹ Gr. Theor. classification of Hamiltonian

Group is called **U(6)**

U(6) has three subgroups corresponding to different shapes

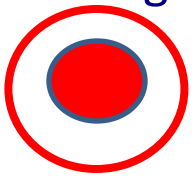
Concepts of group theory

First, some fancy words with simple meanings: Generators, Casimirs, Representations, conserved quantum numbers, degeneracy splitting

Generators of a group: Set of operators , O_i that close on commutation.

$[O_i , O_j] = O_i O_j - O_j O_i = O_k$ *i.e.*, their commutator gives back 0 or a member of the set

For IBA, the 36 operators $s^\dagger s,\ d^\dagger s,\ s^\dagger d,\ d^\dagger d$ are generators of the group U(6).

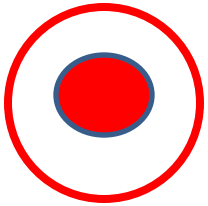


ex:
$$\left[d^\dagger s, s^\dagger s\right]\left|n_d n_s\right\rangle = \left(d^\dagger s s^\dagger s - s^\dagger s d^\dagger s\right)\left|n_d n_s\right\rangle$$
$$= d^\dagger s n_s \left|n_d n_s\right\rangle - s^\dagger s d^\dagger s \left|n_d n_s\right\rangle$$
$$= \left(n_s - s^\dagger s\right) d^\dagger s \left|n_d n_s\right\rangle$$
$$= \left(n_s - s^\dagger s\right)\sqrt{n_d+1}\sqrt{n_s}\left|n_d+1, n_s-1\right\rangle$$
$$= \sqrt{n_d+1}\sqrt{n_s}\left[n_s - \left(n_s-1\right)\right]\left|n_d+1, n_s-1\right\rangle$$
$$= \sqrt{n_d+1}\sqrt{n_s}\left|n_d+1, n_s-1\right\rangle$$
$$= d^\dagger s \left|n_d n_s\right\rangle$$

e.g:
$$\left[N, s^\dagger \tilde{d}\right]\Psi = \left[N\left(s^\dagger \tilde{d}\right) - s^\dagger \tilde{d} N\right]\Psi$$
$$= Ns^\dagger \tilde{d}\Psi - s^\dagger \tilde{d} N\Psi$$
$$= Ns^\dagger \tilde{d}\Psi - Ns^\dagger \tilde{d}\Psi = 0$$

or:
$$\left[d^\dagger s, s^\dagger s\right] = d^\dagger s$$

Sub-groups:



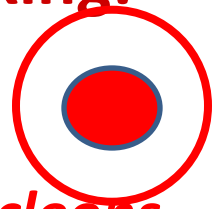
Subsets of generators that commute among themselves.

e.g: $d^{\dagger}d$ 25 generators—span U(5)

They conserve n_d (# *d* bosons)

Set of states with same n_d are the representations of the group [U(5)]

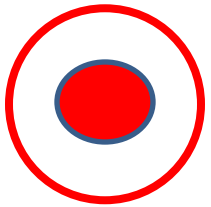
Let's illustrate group chains and degeneracy-breaking.



Consider a Hamiltonian that is a function ONLY of: $s^\dagger s + d^\dagger d$

Note that $s^\dagger s = n_s$ *and* $d^\dagger d = n_d$ *and that* $n_s + n_d = N = \frac{1}{2}$ *val nucleons*

That is: $H = a(s^\dagger s + d^\dagger d) = a\,(n_s + n_d) = aN$



$H' = H \qquad = aN$

Now, add a term to this Hamiltonian:

Now the energies depend not only on N but also on n_d

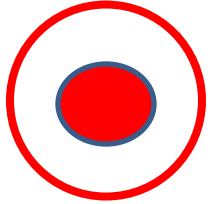
States of a given n_d are now degenerate. They are "representations" of the group U(5). States with different n_d are not degenerate

2a N + 2

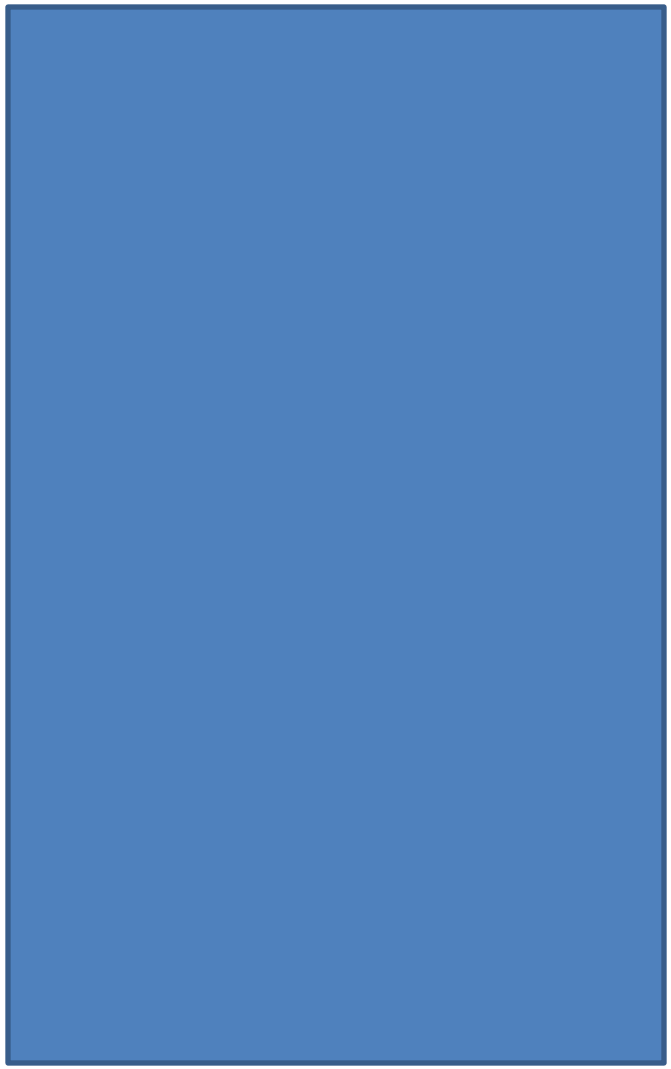
H = aN

U(6)

= a N



a N + 1



Etc. with further terms

0 N

E

U(6)

H = aN

Example of a nuclear dynamical symmetry -- O(6)

Spectrum generating algebra

N

U (6)

E= E_0

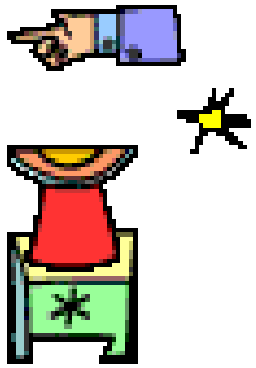
Each successive term:

- **Introduces a new sub-group**
- **A new quantum number to label the states described by that group**
- **Adds an eigenvalue term that is a function of the new quantum number, and therefore**
- **Breaks a previous degeneracy**

Group Structure of the IBA

6-Dim. problem

U(6) →



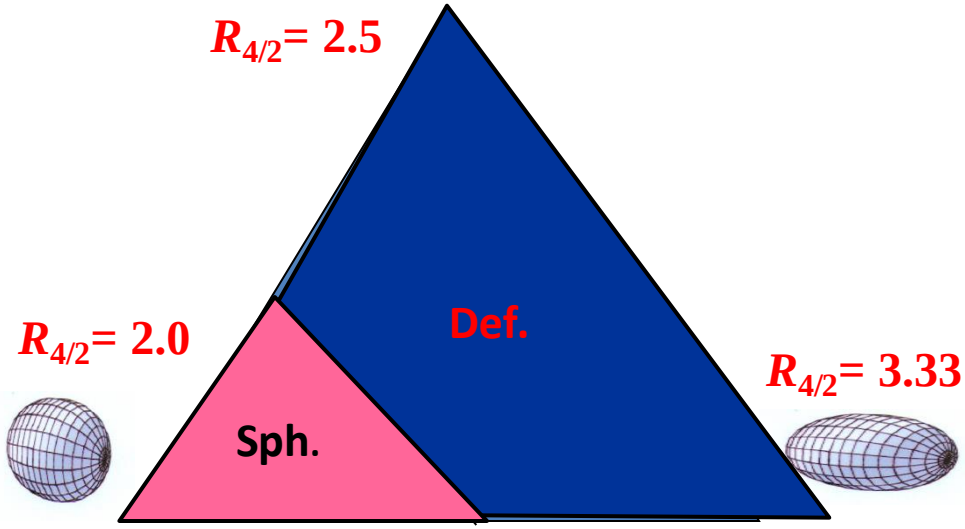
Magical group theory stuff happens here

U(5)
vibrator

SU(3)
rotor

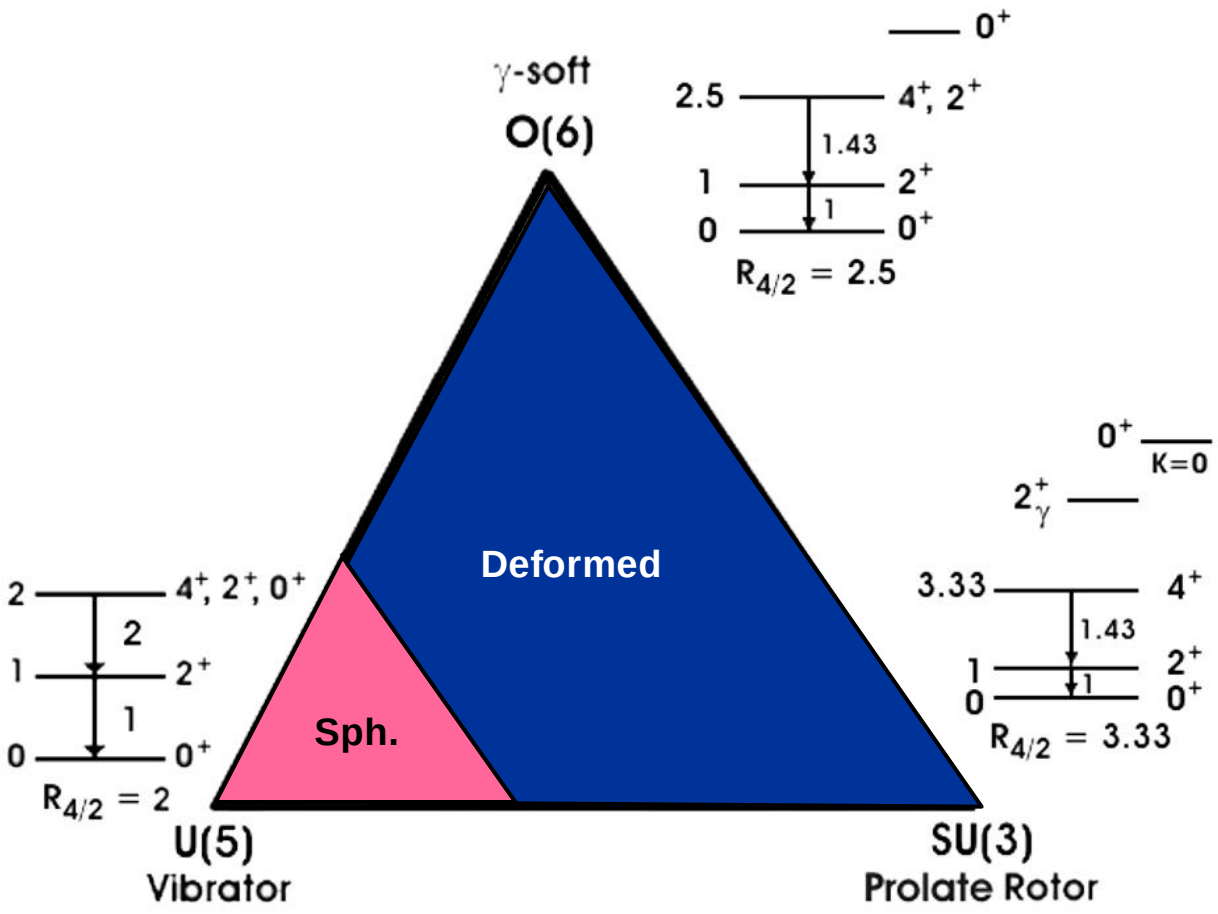
O(6)
γ-soft

Three Dynamic symmetries, nuclear shapes



Symmetry Triangle of the IBA

Classifying Structure -- The Symmetry Triangle

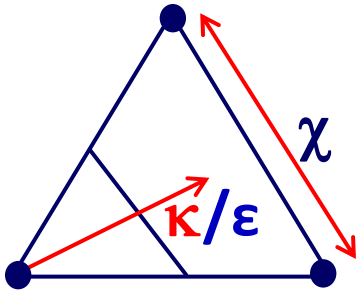


Most nuclei do not exhibit the idealized symmetries but rather lie in transitional regions. Mapping the triangle.

Calculations with the IBA Hamiltonian: Symmetries and symmetry-breaking

Truncated form of with just two parameters (+ scale):

$$H = \varepsilon n_d - \kappa Q \cdot Q$$



$$Q = e[s^{\dagger}\tilde{d} + d^{\dagger}s + \chi (d^{\dagger}\tilde{d})^{(2)}]$$

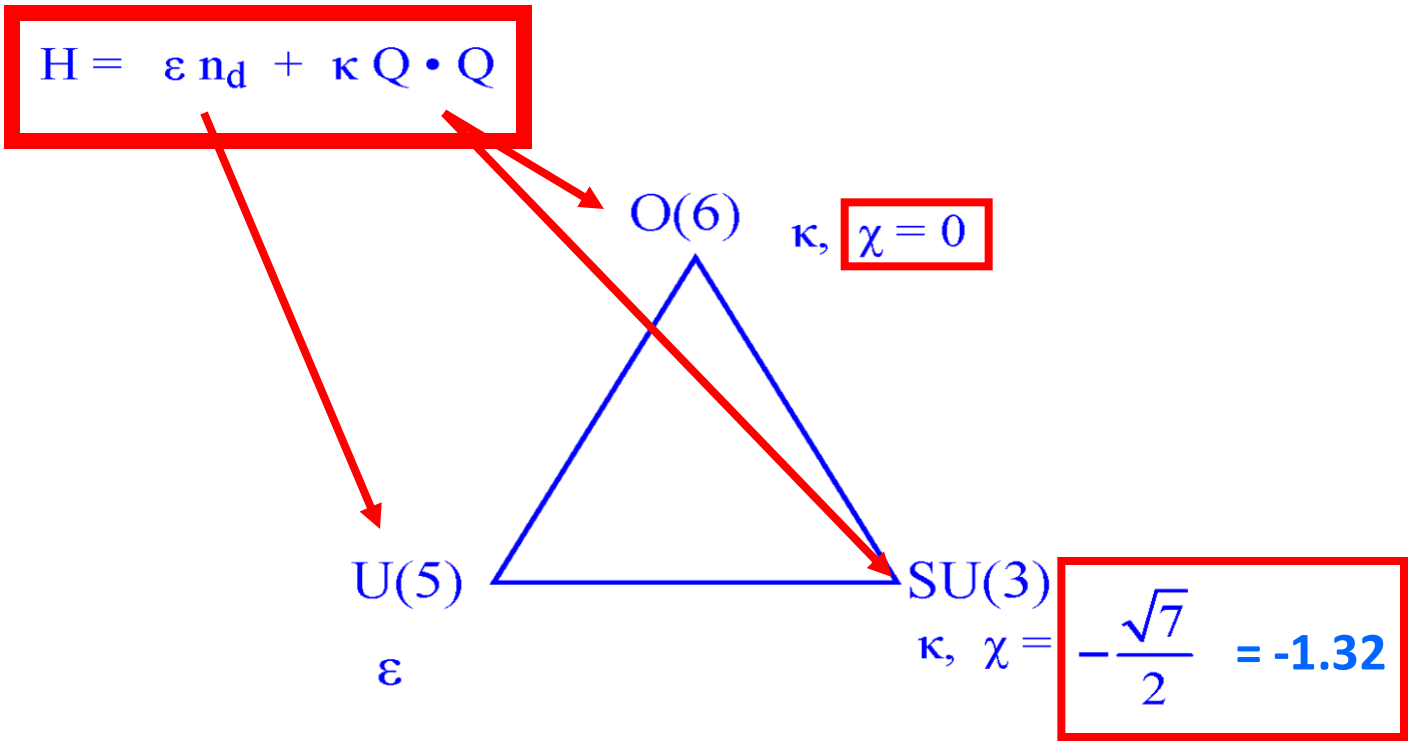
Competition:

εn_d	Counts quad bosons: vibrator.
$\kappa Q \cdot Q$	Gives deformed nuclei.
χ	Determines axial asymmetry

Hence structure is given by two parameters, ε/ κ and χ

More complicated forms exist but this is the form usually used. It works extremely well in most cases.

Relation of IBA Hamiltonian to Group Structure



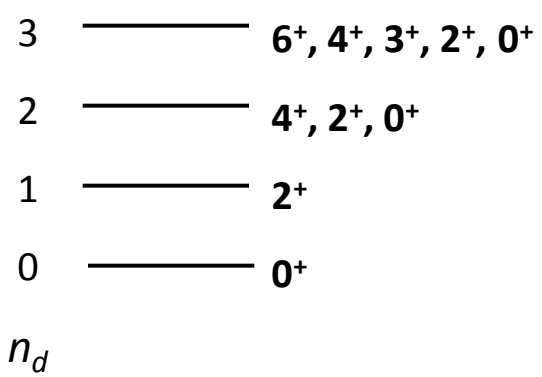
We will see later that this same Hamiltonian allows us to calculate the properties of a nucleus ANYWHERE in the triangle simply by choosing appropriate values of the parameters

Simplest Possible IBA Hamiltonian – given by energies of the bosons with NO interactions

$$H = \varepsilon n_d$$

U(5) **Harmonic form**

What is spectrum? Equally spaced levels defined by number of *d* bosons



What *J*'s? M-scheme

Look familiar? Same as quadrupole vibrator.

U(5) also includes anharmonic spectra

E2 Transition in the IBA

$$T(E2) = e_B Q = e_B \left[(s^\dagger \tilde{d} + d^\dagger s) + \chi (d^\dagger \tilde{d}) \right]$$

U(5) $\Delta n_d \quad \chi = 0$

$$\left\langle n_d n_s \left| s^\dagger \tilde{d} \right| n_d + 1, n_s - 1 \right\rangle = \sqrt{n_d + 1}\sqrt{n_s} \left\langle n_d n_s \middle| n_d n_s \right\rangle$$

$$= \sqrt{n_d + 1}\sqrt{N_B - n_d}$$

$\therefore$ $B(E2; J+2 \rightarrow J)_{yrast} = e_B^2 (n_d + 1)(N_B - n_d) \rightarrow N_B$ as $N_B \rightarrow \infty$

$$n_d = J/2$$

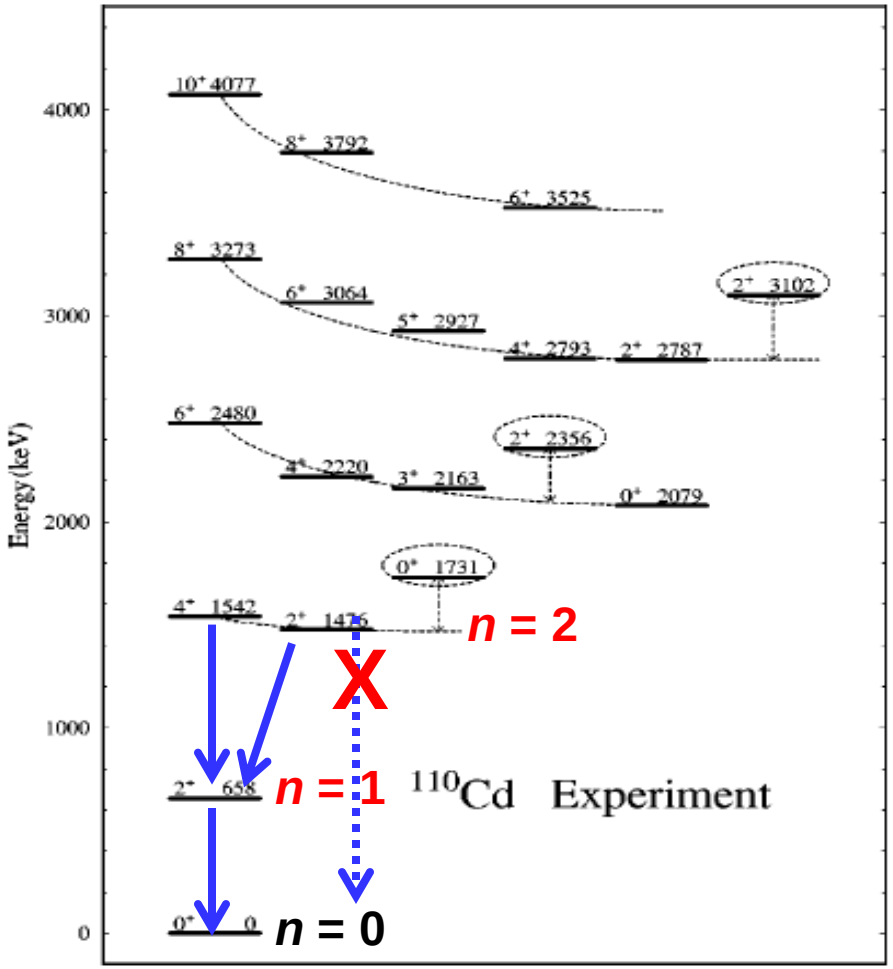
Spherical vibrator – U(5)

Vibrator (H.O.)

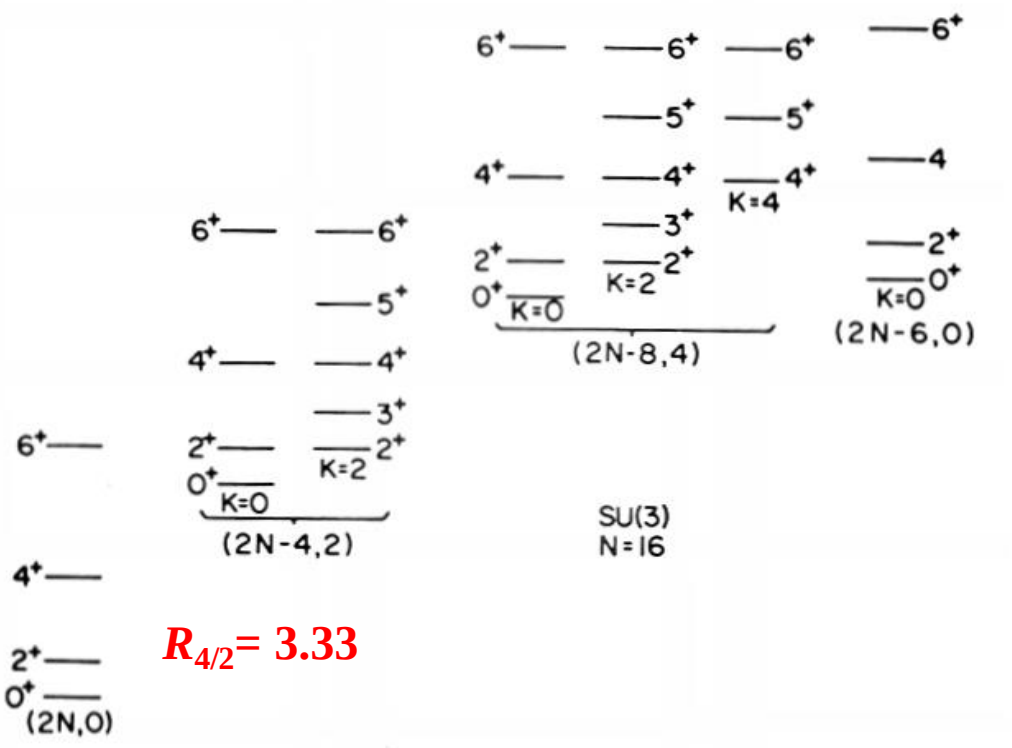
$$E(J) = n\,(\hbar\,\omega_0)$$

$$R_{4/2} = 2.0$$

Gamma-ray transitions:
Selection rule: Can destroy only one phonon



SU(3)



$R_{4/2}$= 3.33

Characteristic signatures:

- **Degenerate bands within a group**
- **Vanishing B(E2) values between groups**
- **Allowed transitions between bands within a group**

Where? N~ 104, Yb, Hf

$$E(\lambda,\mu,J) = A[\lambda^2 + \mu^2 + \lambda\mu + 3(\lambda+\mu)] + BJ(J+1)$$

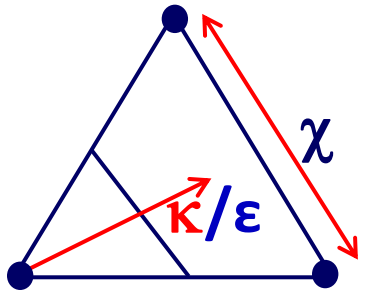
SU(3) ⊃ **O(3)**

K **bands in (λ,μ):** $K = 0, 2, 4, - - - - \mu$

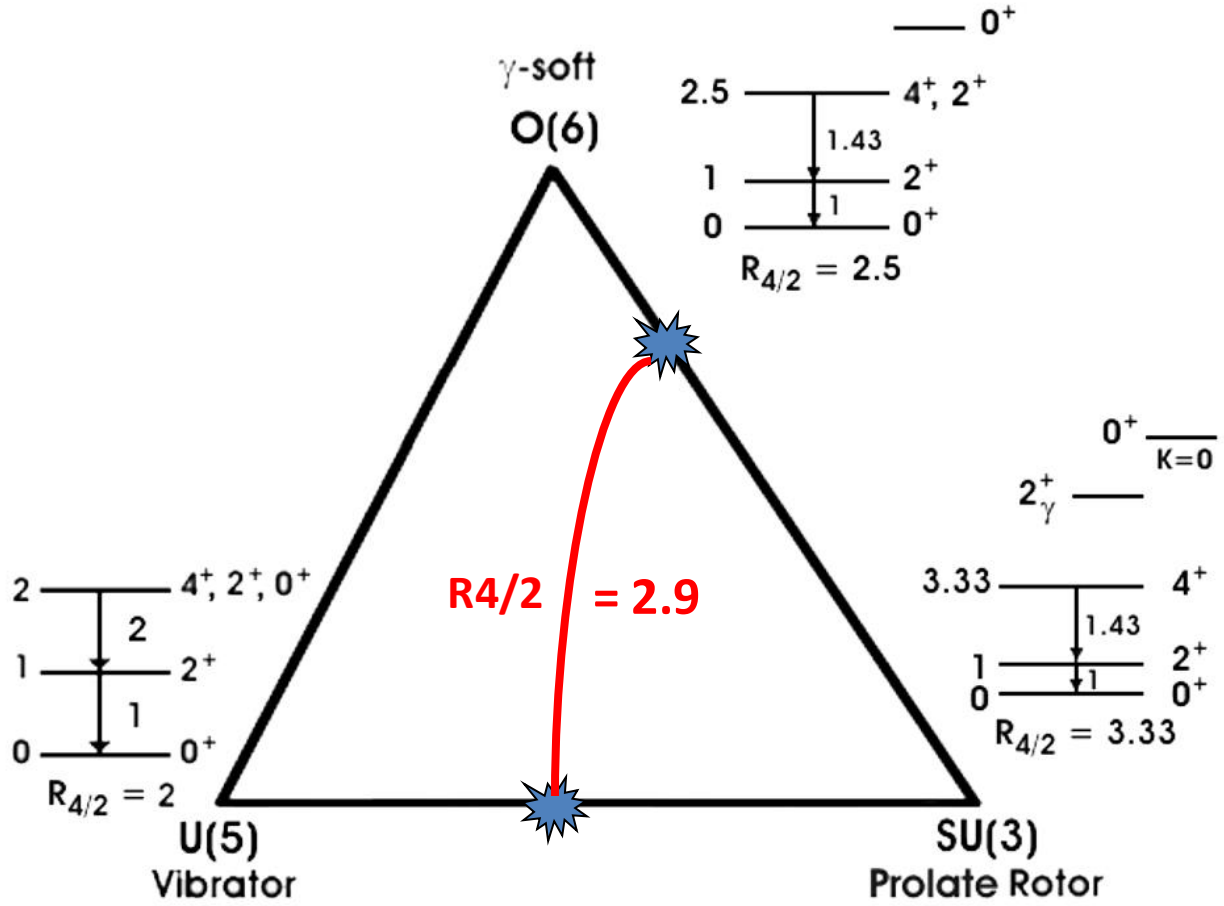
Now, what about all those nuclei that do not manifest a dynamical symmetry? How does the IBA deal with those

$$H = \varepsilon n_d - \kappa Q \cdot Q$$

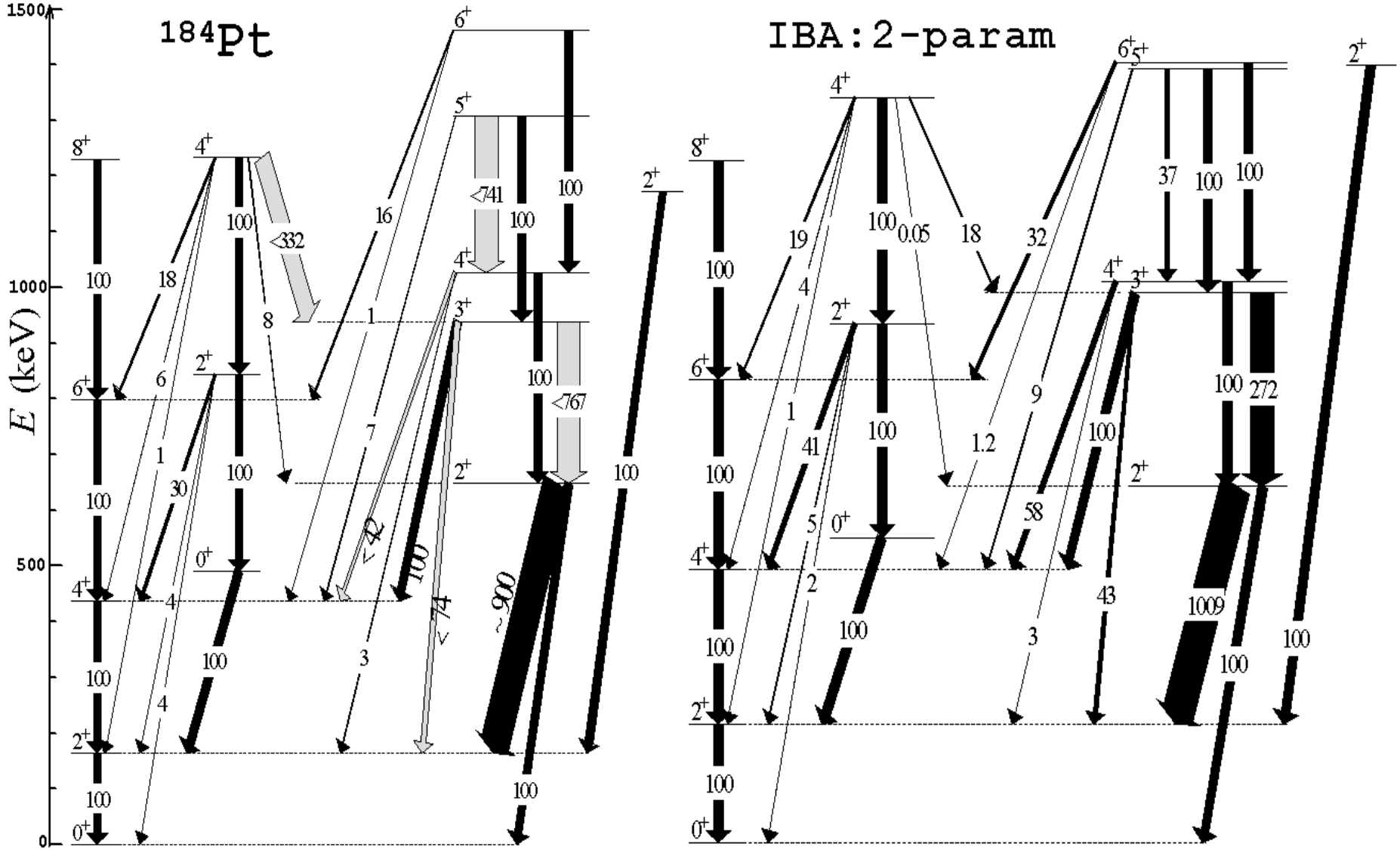
$$Q = e[s^\dagger d + d^\dagger s + \chi (d^\dagger d)^{(2)}]$$



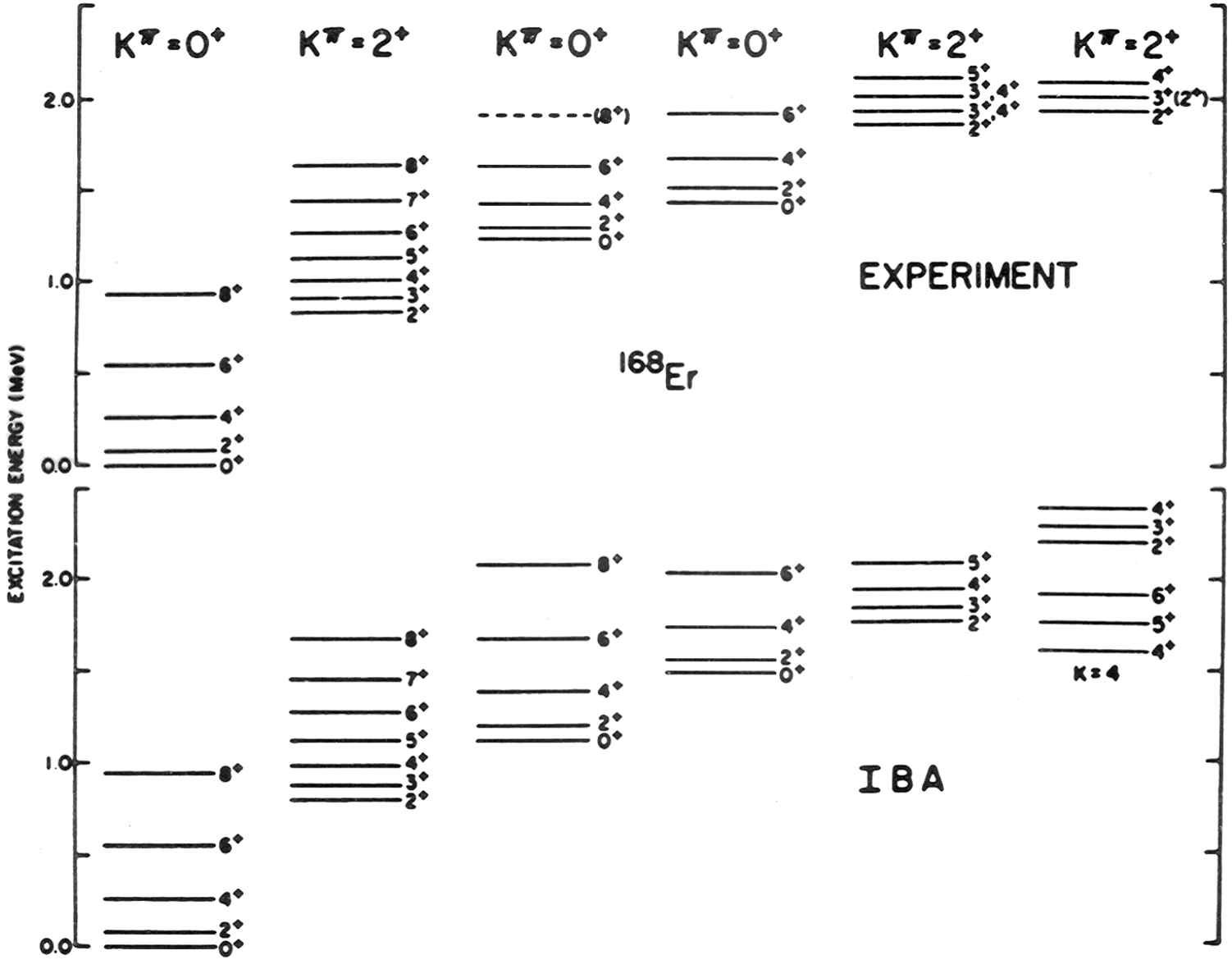
H has two parameters. A given observable can only specify one of them. That is, a given observable has a contour (locus) of constant values in the triangle



Structure varies considerably along this trajectory, so we need a second observable.



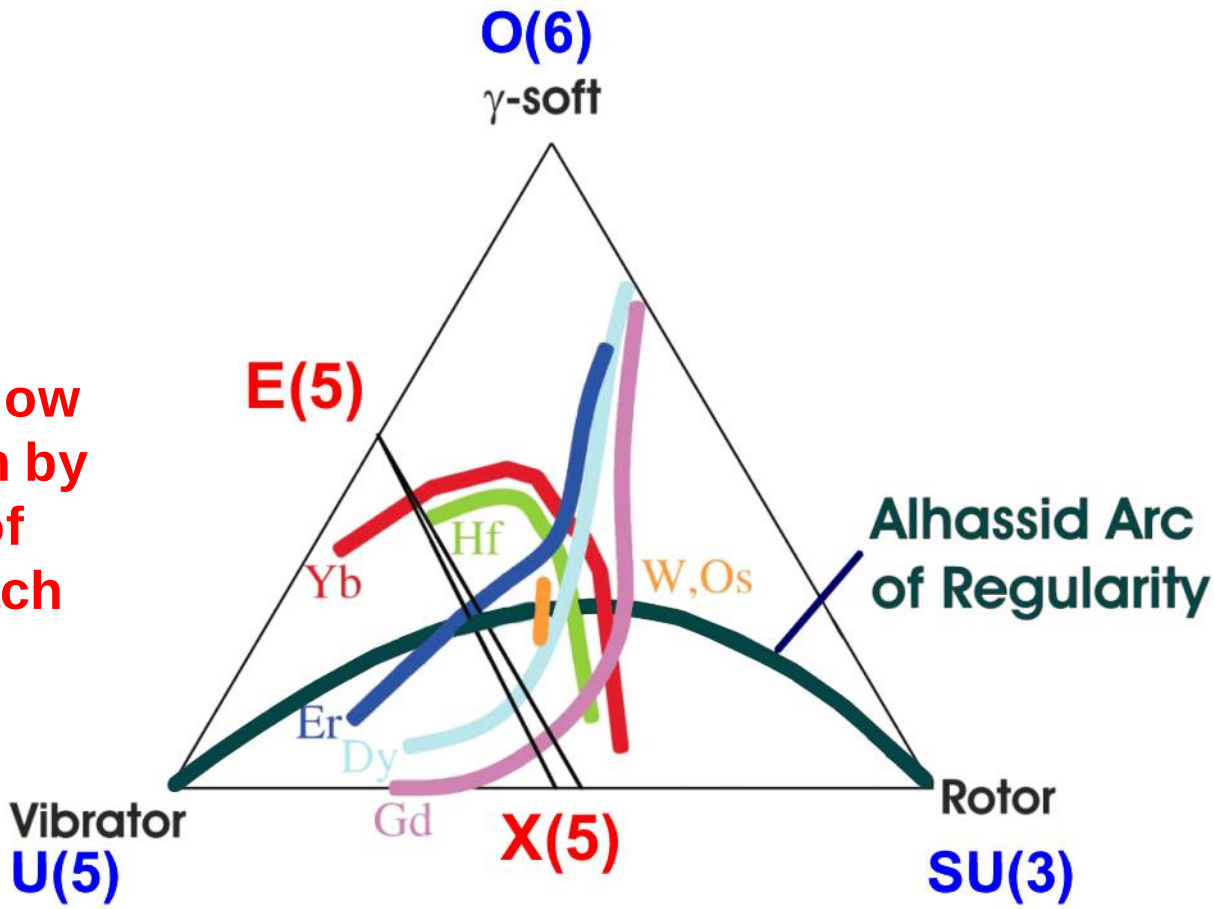
R. Burcu Cakirli



Warner, Borner, and Davidson

Evolution of Structure

Trajectories show the paths taken by sequences of isotopes of each element



McCutchan, Zamfir

Complementarity of macroscopic and microscopic approaches. Why do certain nuclei exhibit specific symmetries? Why these evolutionary trajectories?

What will happen far from stability in regions of proton-neutron asymmetry and/or weak binding?

Appendix --- doing IBA calculations

Nuclear Model Codes at Yale
Computer name: Titan

Connecting to SSH: Quick connect

Host name: titan.physics.yale.edu

User name: phy664

Port Number 22

Password: nuclear_codes

---

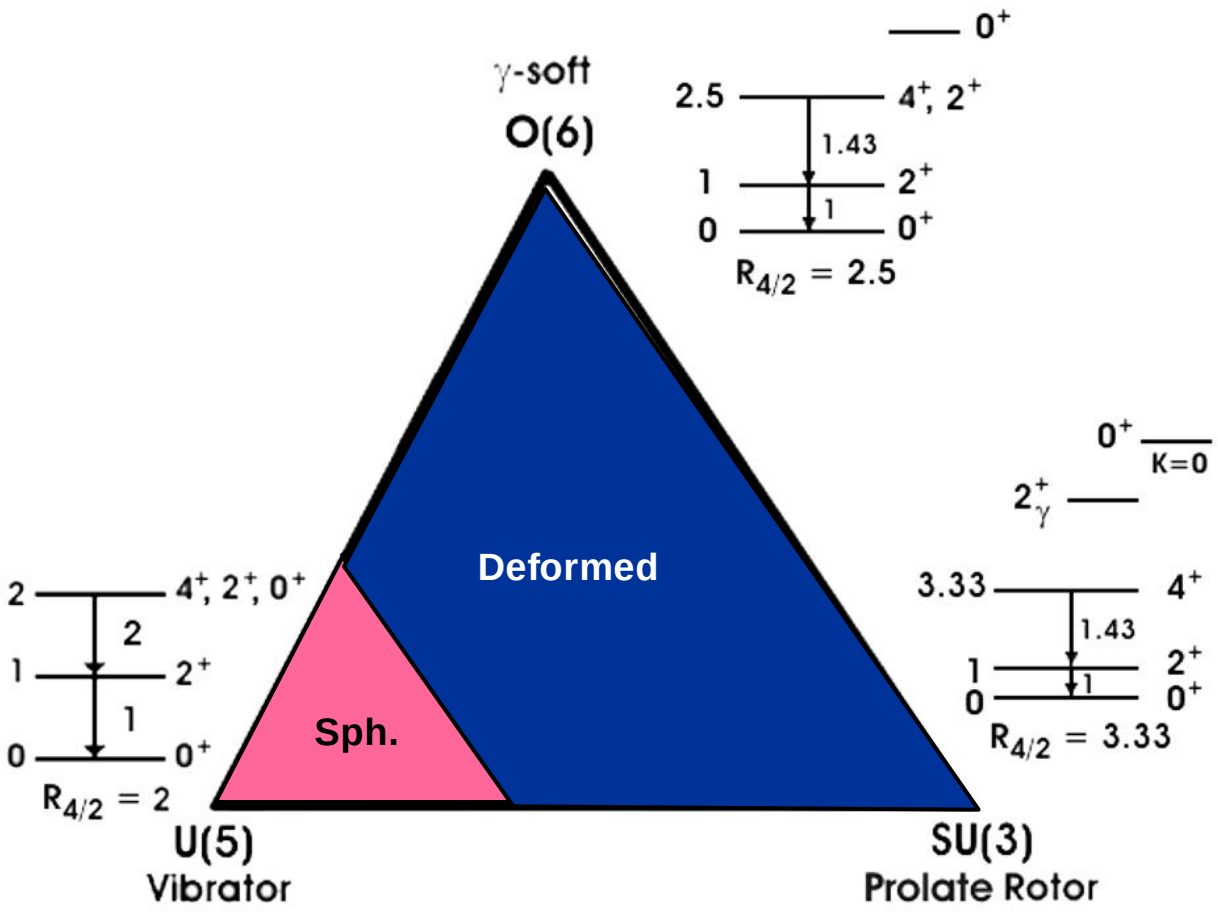
cd phintm

pico filename.in (ctrl x, yes, return)

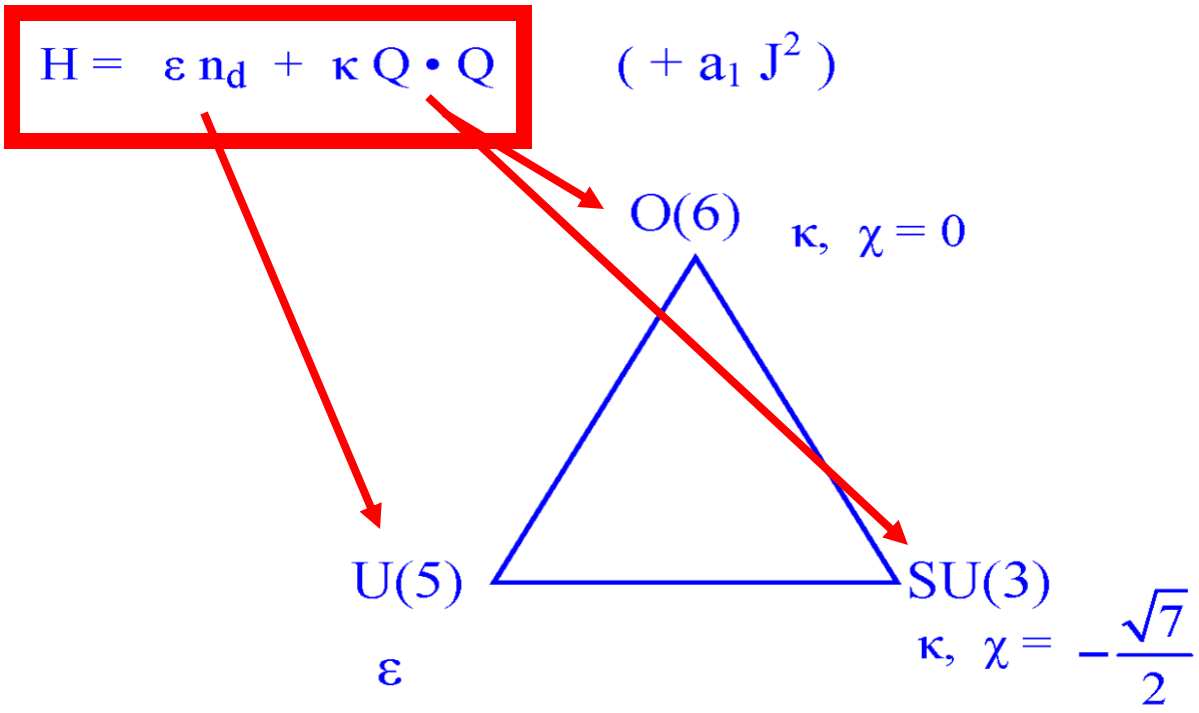
runphintm filename (w/o extension)

pico filename.out (ctrl x, return)

Lets first do the three symmetries. Okey, dokey?



Relation of IBA Hamiltonian to Group Structure



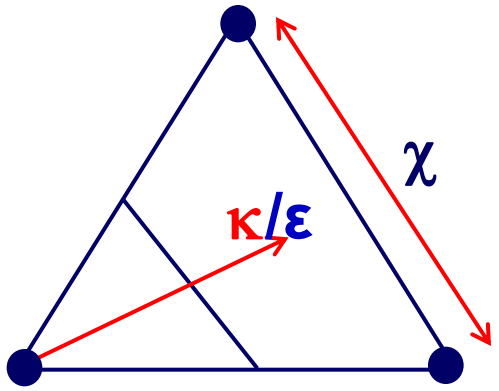
Mapping the Entire Triangle with a minimum of data

$H = \varepsilon n_d - \kappa Q \cdot Q$ **Parameters: κ/ε, χ (within Q)**

2 parameters

κ/ε varies from 0 to infinity

2-D surface



Note: The natural size of QQ is much larger than n_d so, in typical fits, κ is on the order of 10's of keV and ε is ~ hundreds of keV

Note: we usually keep κ fixed at 0.02 MeV and just vary ε. When we have a good fit to RELATIVE energies, we then scale BOTH κ and ε by the same factor to reproduce the experimental scale of energies

U(5)

Input

```
 $diag
 eps = 0.20,  kappa = 0.00,  chi =-0.00,
 nphmax = 6,
 iai = 0, iam = 6, neig = 3,
 mult=.t.,ell=0.0,pair=0.0,oct=0.0,ippm=1,print=.t.
$
$em
 E2SD=1.0,   E2DD=-0.00
$
 SLCT 2 2+  0+  2

99999
```

Output

```
-------------------------

L P =  0+

            Basis vectors
              |NR>  = |ND,NB,NC,LD,NF,L   P>
             -------------------------
             | 1> = | 0,  0,  0,  0,  0,  0+>
             | 2> = | 2,  1,  0,  0,  0,  0+>
             | 3> = | 3,  0,  1,  0,  0,  0+>
             | 4> = | 4,  2,  0,  0,  0,  0+>
             | 5> = | 5,  1,  1,  0,  0,  0+>
             | 6> = | 6,  0,  2,  0,  0,  0+>
             | 7> = | 6,  3,  0,  0,  0,  0+>

  Energies
    0.0000    0.4000    0.6000    0.8000    1.0000    1.2000    1.2000

  Eigenvectors

    1:   1.000   0.000   0.000
    2:   0.000   1.000   0.000
    3:   0.000   0.000   1.000
    4:   0.000   0.000   0.000
    5:   0.000   0.000   0.000
    6:   0.000   0.000   0.000
    7:   0.000   0.000   0.000
```

Basis

Energies

Pert. Wave Fcts.

```
-------------------------
L P =  1+
No states
-------------------------
L P =  2+

  Energies
    0.2000    0.4000    0.6000    0.8000    0.8000    1.0000    1.0000    1.2000    1.2000
-------------------------
L P =  3+

  Energies
    0.6000    1.0000    1.2000
-------------------------
L P =  4+

 Energies
    0.4000    0.6000    0.8000    0.8000    1.0000    1.0000    1.2000    1.2000    1.2000
-------------------------
L P =  5+

  Energies
    0.8000    1.0000    1.2000
-------------------------
L P =  6+

  Energies
    0.6000    0.8000    1.0000    1.0000    1.2000    1.2000    1.2000
-------------------------

Transitions:  2+  ->  0+  (BE2)
  2+,1 -> 0+,1:   6.00000   2+,1 -> 0+,2:   2.00000   2+,1  -> 0+,3:   0.00000
  2+,2 -> 0+,1:   0.00000   2+,2 -> 0+,2:   0.00000   2+,2  -> 0+,3:   2.40000
  2+,3 -> 0+,1:   0.00000   2+,3 -> 0+,2:   5.60000   2+,3  -> 0+,3:   0.00000
   and  0+ ->  2+  (BE2)
  0+,1 -> 2+,1:  30.00000   0+,2 -> 2+,1:  10.00000   0+,3 -> 2+,1:   0.00000
  0+,1 -> 2+,2:   0.00000   0+,2 -> 2+,2:   0.00000   0+,3 -> 2+,2:  12.00000
  0+,1 -> 2+,3:   0.00000   0+,2 -> 2+,3:  28.00000   0+,3 -> 2+,3:   0.00000

Transitions:  4+  ->  2+  (BE2)
  4+,1 -> 2+,1:  10.00000   4+,1 -> 2+,2:   0.00000   4+,1 -> 2+,3:   2.28571
  4+,2 -> 2+,1:   0.00000   4+,2 -> 2+,2:   6.28571   4+,2 -> 2+,3:   0.00000
  4+,3 -> 2+,1:   0.00000   4+,3 -> 2+,2:   0.00000   4+,3 -> 2+,3:   3.85714
```

O(6)

Input

```
 $diag
 eps = 0.0, kappa = 0.02, chi =-0.0,
 nphmax = 6,
 iai = 0, iam = 6, neig = 5,
 mult=.t.,ell=0.0,pair=0.0,oct=0.0,ippm=1,print=.t.
$
$em
 E2SD=1.0,  E2DD=-0.00
$
99999
```

Output

```
-------------------------

L P =  0+
          Basis vectors
            |NR>  = |ND,NB,NC,LD,NF,L   P>
            --------------------------
            | 1> = | 0,  0, 0, 0,  0, 0+>
            | 2> = | 2,  1, 0, 0,  0, 0+>
            | 3> = | 3,  0, 1, 0,  0, 0+>
            | 4> = | 4,  2, 0, 0,  0, 0+>
            | 5> = | 5,  1, 1, 0,  0, 0+>
            | 6> = | 6,  0, 2, 0,  0, 0+>
            | 7> = | 6,  3, 0, 0,  0, 0+>

  Energies
    0.0000    0.3600    0.5600    0.9200    0.9600    1.0800    1.2000

  Eigenvectors

    1:  -0.433   0.000   0.685   0.000   0.559
    2:  -0.750   0.000   0.079   0.000  -0.581
    3:   0.000  -0.886   0.000   0.463   0.000
    4:  -0.491   0.000  -0.673   0.000   0.296
    5:   0.000  -0.463   0.000  -0.886   0.000
    6:   0.000   0.000   0.000   0.000   0.000
    7:  -0.094   0.000  -0.269   0.000   0.512
-------------------------
```

Basis

Energies

Pert. Wave Fcts.

```
L P =  1+

No states
-------------------------
L P =  2+

  Energies
    0.0800    0.2000    0.5600    0.6400    0.7600    0.8000    1.0400    1.1200    1.1600
-------------------------

L P =  3+

  Energies
    0.3600    0.9200    1.0800
-------------------------
L P =  4+

  Energies
    0.2000    0.3600    0.5600    0.7600    0.8000    0.9200    1.0800    1.1200    1.1600
-------------------------

L P =  5+

  Energies
    0.5600    0.8000    1.1200
-------------------------

L P =  6+

  Energies
    0.3600    0.5600    0.8000    0.9200    1.0800    1.0800    1.1200

-------------------------

  Binding energy = -0.6000  , eps-eff = -0.1200
```

SU(3)

```
********** Input file contents **********

 $diag
 eps = 0.00,  kappa = 0.02,  chi =-1.3229,
 nphmax = 6,
 iai = 0, iam = 6, neig = 5,
 mult=.t.,ell=0.0,pair=0.0,oct=0.0,ippm=1,print=.t.
 $
 $em
 E2SD=1.0,   E2DD=-2.598

 $
 99999
**********************************************************
```

L P = 0+

Basis vectors

|NR> = |ND,NB,NC,LD,NF,L P>

| 1> = | 0, 0, 0, 0, 0, 0+>
| 2> = | 2, 1, 0, 0, 0, 0+>
| 3> = | 3, 0, 1, 0, 0, 0+>
| 4> = | 4, 2, 0, 0, 0, 0+>
| 5> = | 5, 1, 1, 0, 0, 0+>
| 6> = | 6, 0, 2, 0, 0, 0+>
| 7> = | 6, 3, 0, 0, 0, 0+>

Energies

0.0000 0.6600 1.0800 1.2600 1.2600 1.5600 1.8000

Eigenvectors

1:	0.134	0.385	-0.524	-0.235	0.398
2:	0.463	0.600	-0.181	0.041	-0.069
3:	-0.404	-0.204	-0.554	-0.557	-0.308
4:	0.606	-0.175	0.030	-0.375	-0.616
5:	-0.422	0.456	-0.114	0.255	-0.432
6:	-0.078	0.146	-0.068	0.245	-0.415
7:	0.233	-0.437	-0.606	0.606	0.057

Wave fcts. in U(5) basis

L P = 1+

No states

L P = 2+

Energies

0.0450 0.7050 0.7050 1.1250 1.1250 1.3050 1.3050 1.6050

L P = 3+

Energies

0.7500 1.1700 1.6500

L P = 4+

Energies

0.1500 0.8100 0.8100 1.2300 1.2300 1.2300 1.4100 1.4100

L P = 5+

Energies

0.8850 1.3050 1.3050

L P = 6+

Energies

0.3150 0.9750 0.9750 1.3950 1.3950 1.5750 1.5750

Binding energy = -1.2000 , eps-eff = -0.1550