Bayesian Methods to Determine the Nucleon-Nucleon Interaction from Neutron Star Observations

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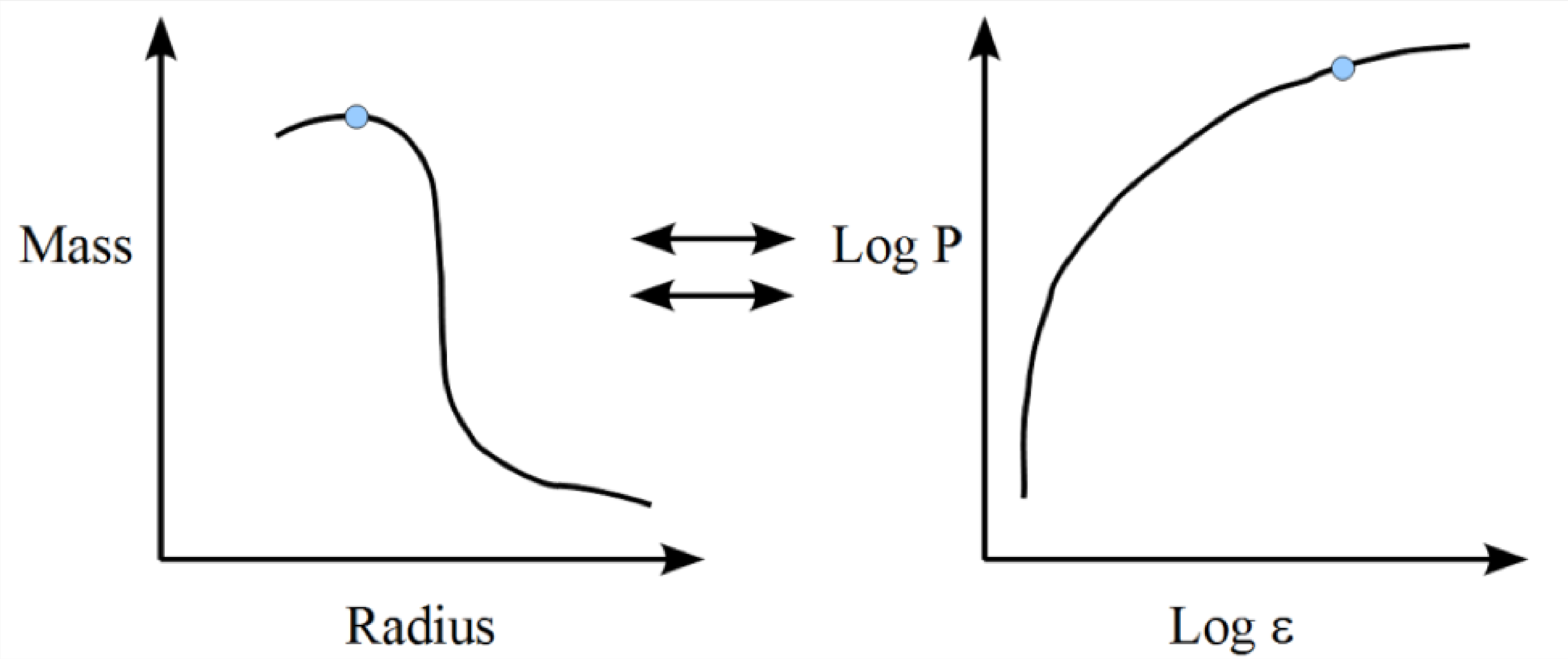
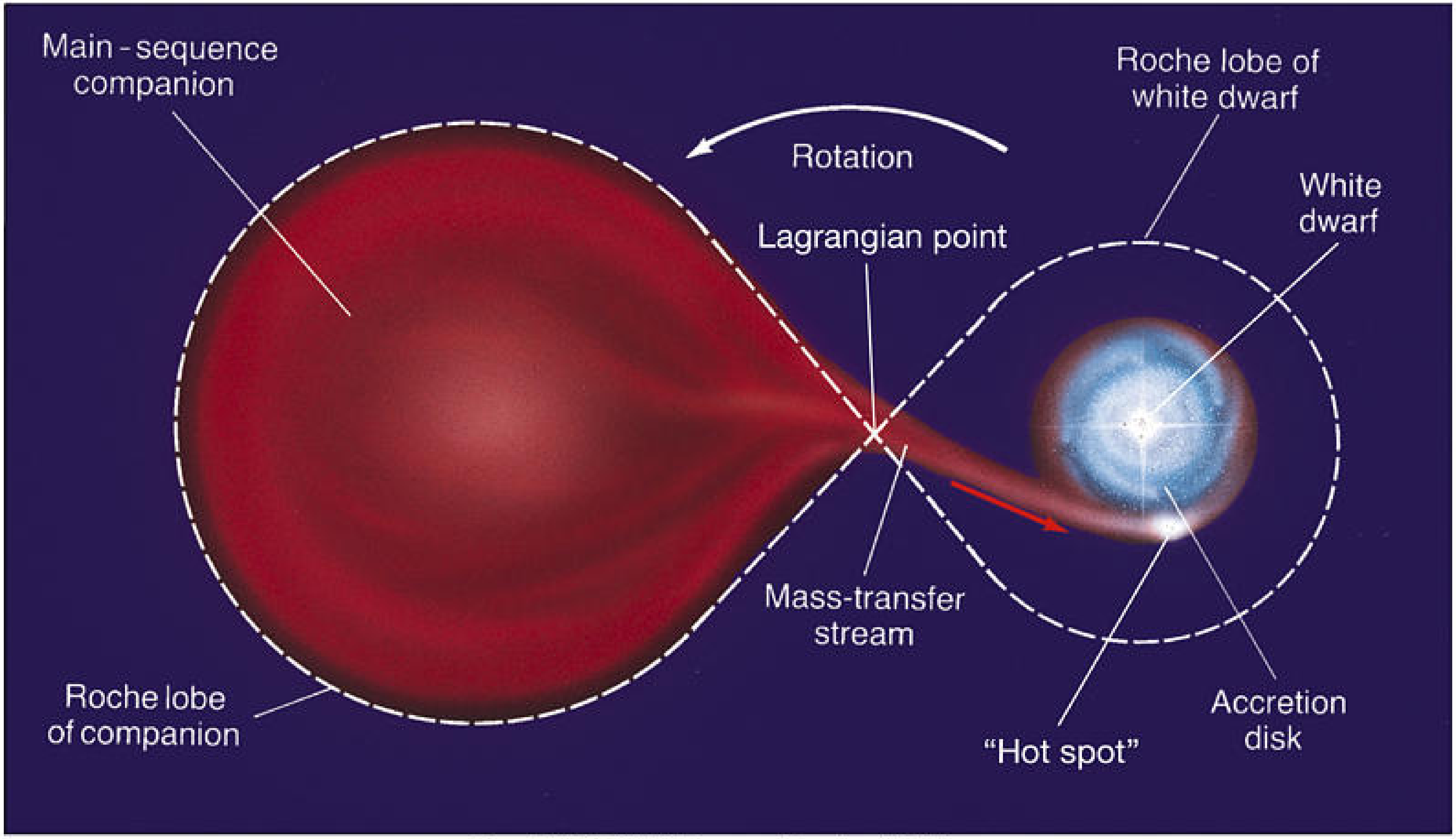
Neutron Star Masses and Radii and the EOS



- Unlike planets, neutron stars form a one-dimensional family
- Neutron stars (to better than 10%) all lie on one universal mass-radius curve
- Recent measurement of two $2\ M_\odot$ neutron stars
  Demorest et al. (2010), Antoniadis et al. (2013)
- Until recently, neutron star radii constrained to 8-15 km
  Lattimer and Prakash (2007)

Accreting Neutron Stars: LMXBs



- Most stars have companions: neutron stars can have main-sequence companions
- Accretion heats the crust and is episodic
- At high enough density, H and He are unstable to thermonuclear explosions

Radius Measurements in qLMXBs

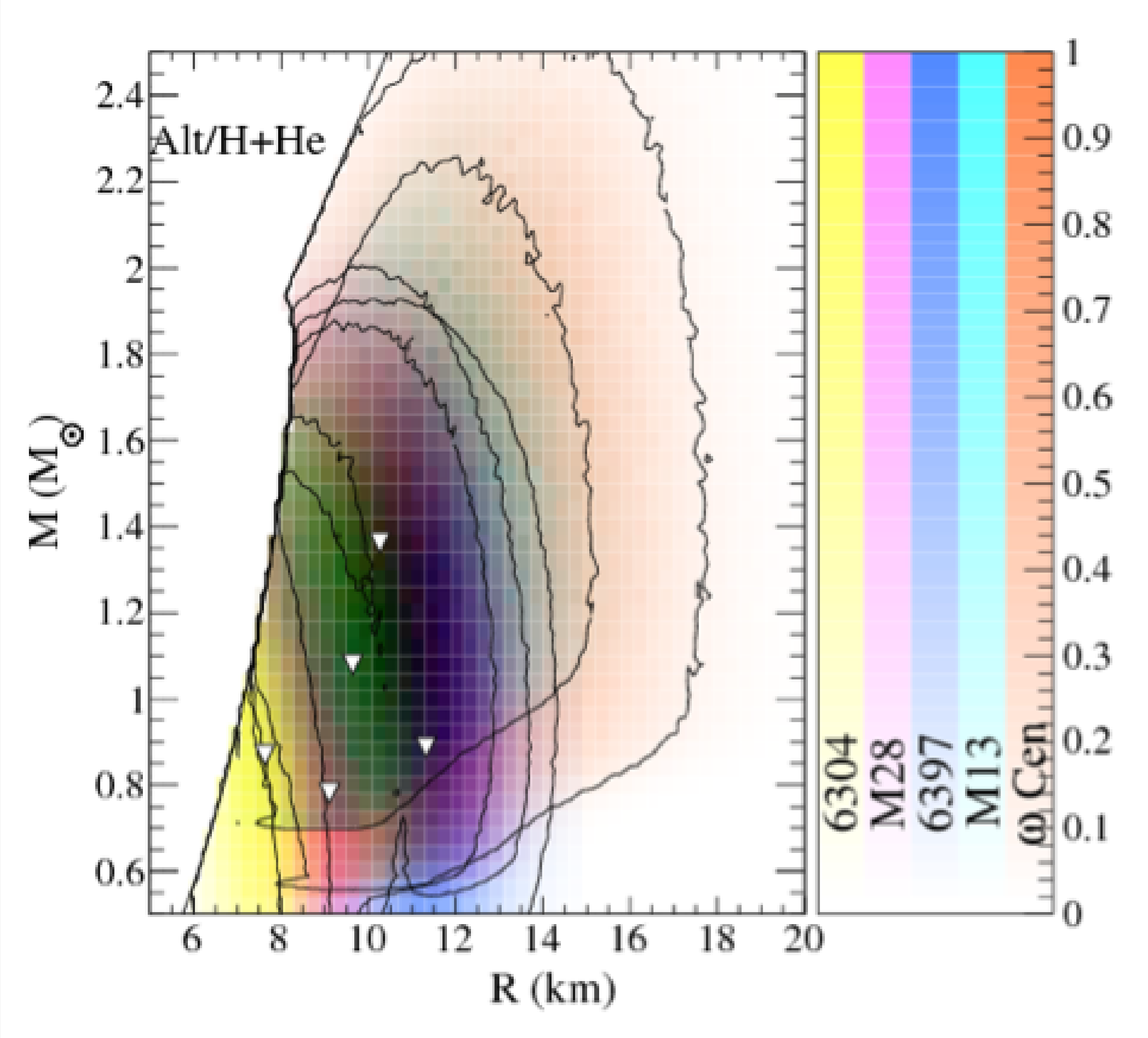
Quiescent LMXBs

- Measure flux of photons and their energy distribution
- Know distance if in a globular cluster
- Implies radius measurement

$$F \propto T_{\rm eff}^4 \left(\frac{R_\infty}{D} \right)^2$$

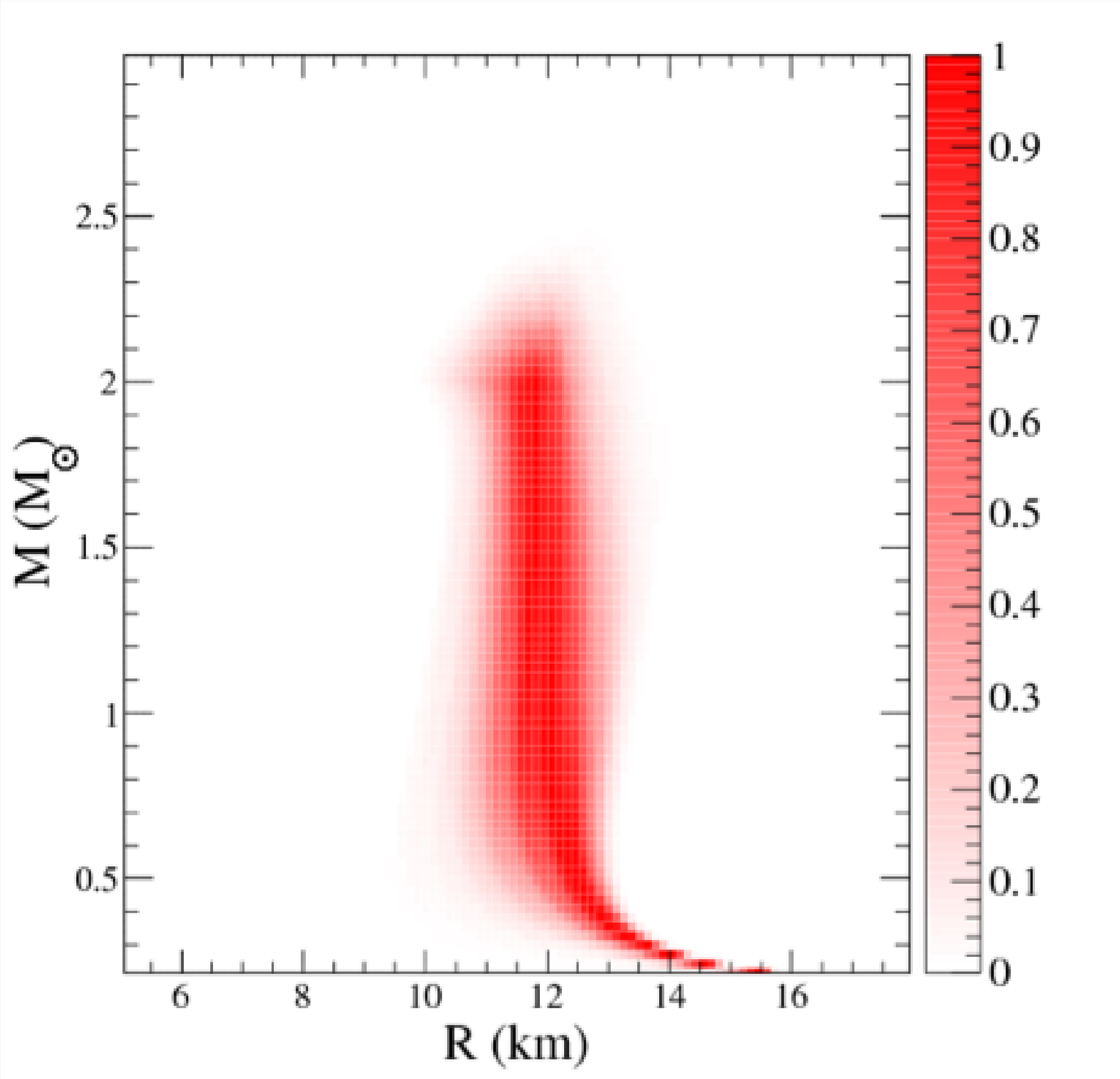
i.e. Rutledge et al. (1999)

Also information from PRE X-ray bursts, ~ 8-12 objects (more on the way)



Lattimer and Steiner (2013)

Why Bayes?



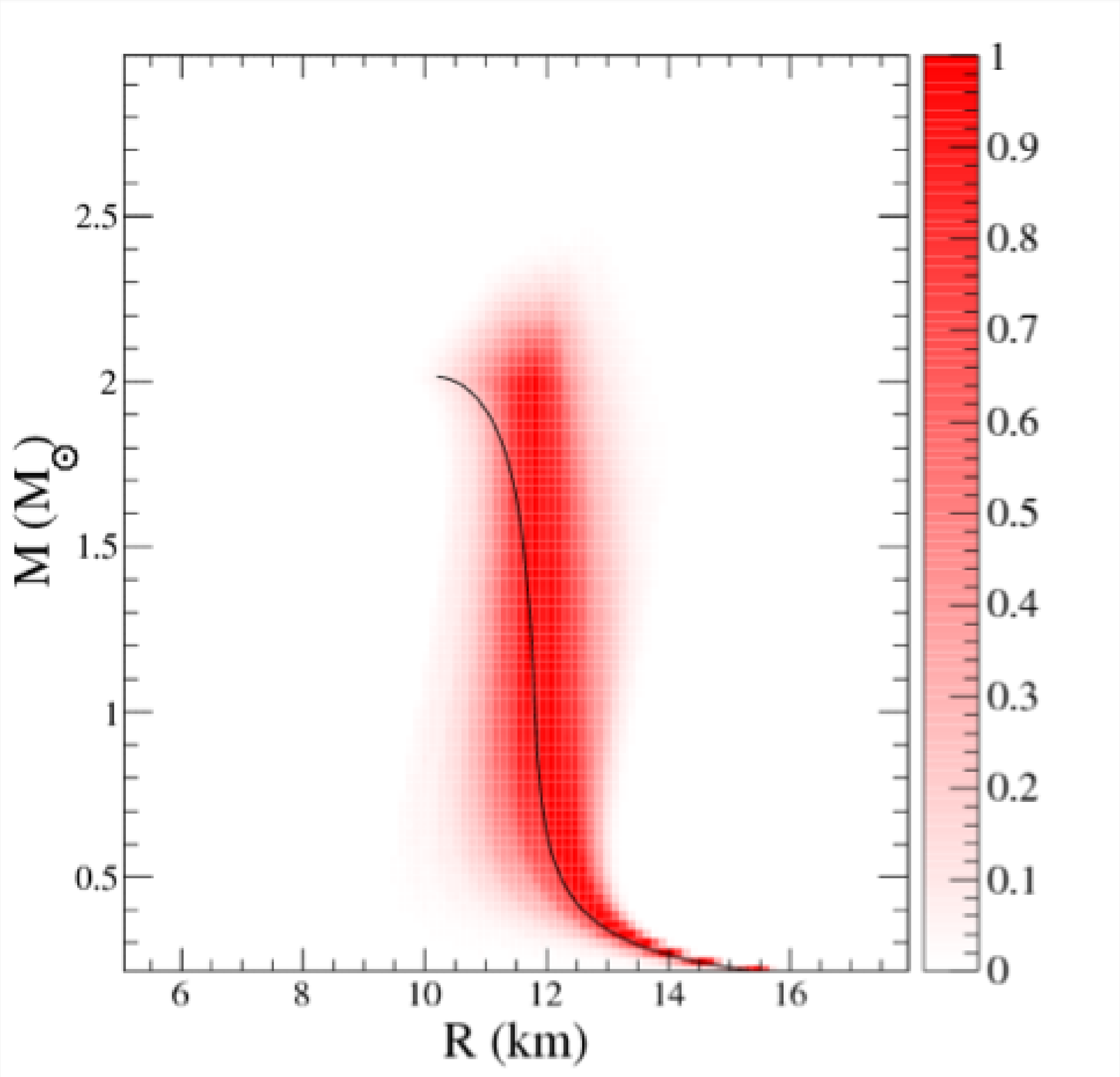
- Underconstrained problem
- Correlations handled easily
- Heterogeneous data set
- Explicit neutron star initial mass function
- Parameterizations based on known nuclear physics for low densities
- Bayes theorem:

$$P[\mathcal{M}_i|D] = \frac{P[D|\mathcal{M}_i]P[\mathcal{M}_i]}{\sum_j P[D|\mathcal{M}_j]P[\mathcal{M}_j]}$$

- Prior ⇔ EOS parameterization
- Determine parameters through marginalization, i.e.

$$P(\mathcal{M}_i^0) = \int \delta(\mathcal{M}_i - \mathcal{M}_i^0) P[\mathcal{M}_i|D]\, d\mathcal{M}$$

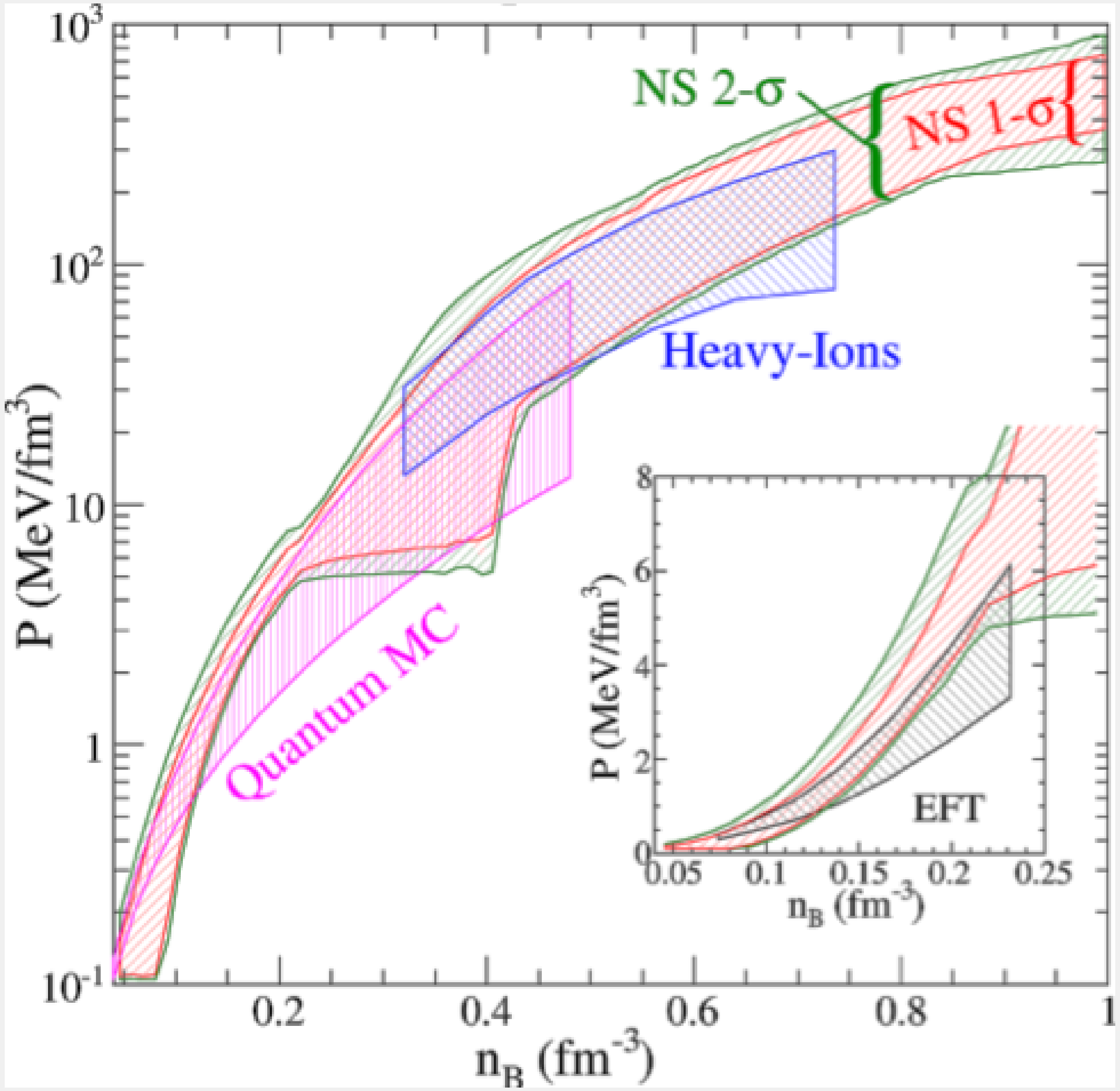
Statistical Complications



- Distributions are not Gaussian
- Odd-shaped constraints because of complicated relationship $(R, M) \leftrightarrow (\varepsilon, P)$

- Many standard methods assume something about the shape of the likelihood function near the maximum
- This fails in this case: the best fit not same as typical M-R curve
- Why? Causality + 2 solar mass NS + "geometry"
- Naive covariance analysis unrelated to typical M-R curve
- Over/under-constrained subspaces (Low vs. high densities)

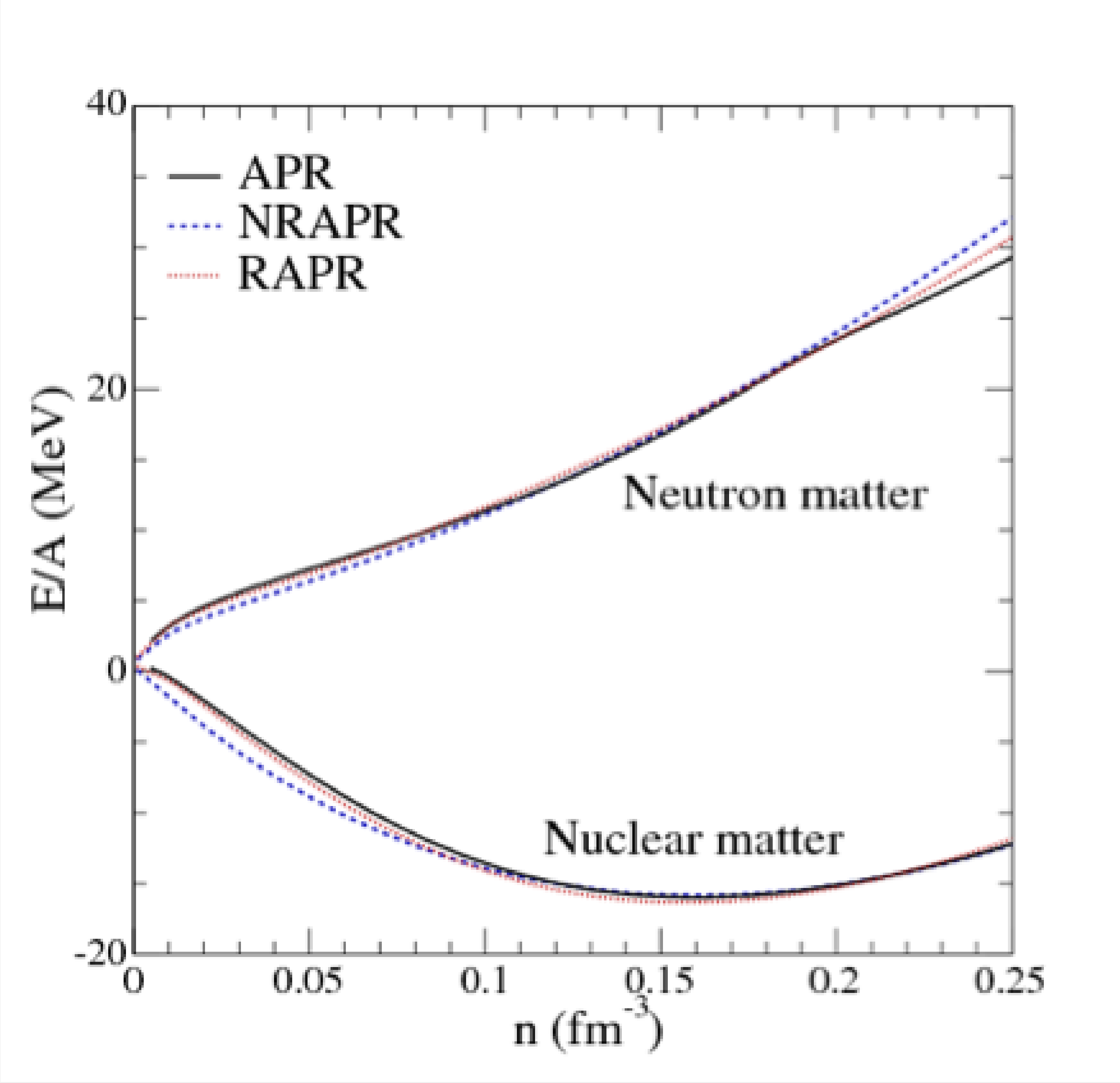
Constraining the EOS of dense matter



Steiner, Lattimer, and Brown (2013)

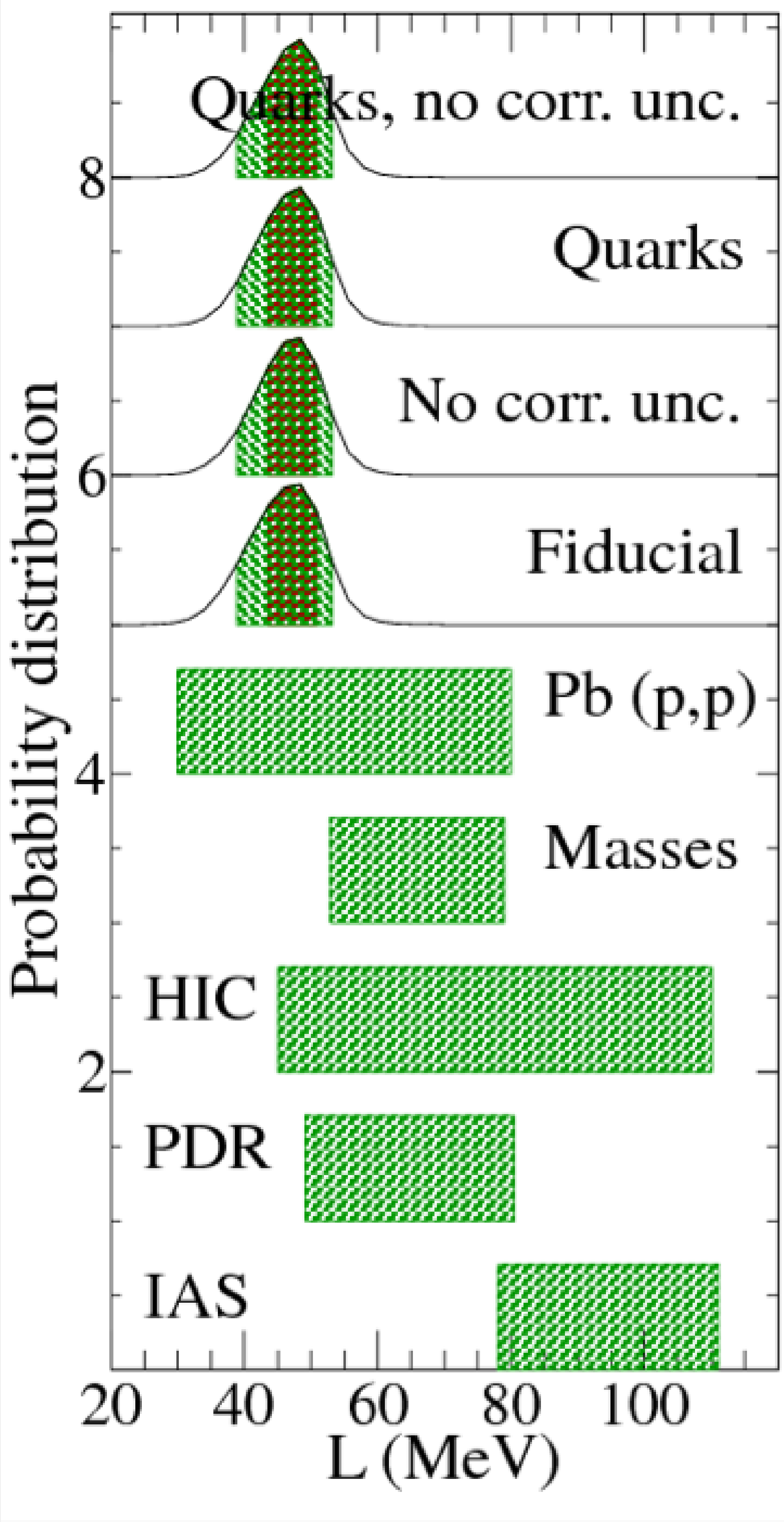
- $P(\varepsilon)$ determined to within about 60%
- We find concordance between nuclear physics data and astronomical observations
- Probe densities inaccessible to experiment and to perturbation theory in QCD

Constraints on the Nuclear Symmetry Energy



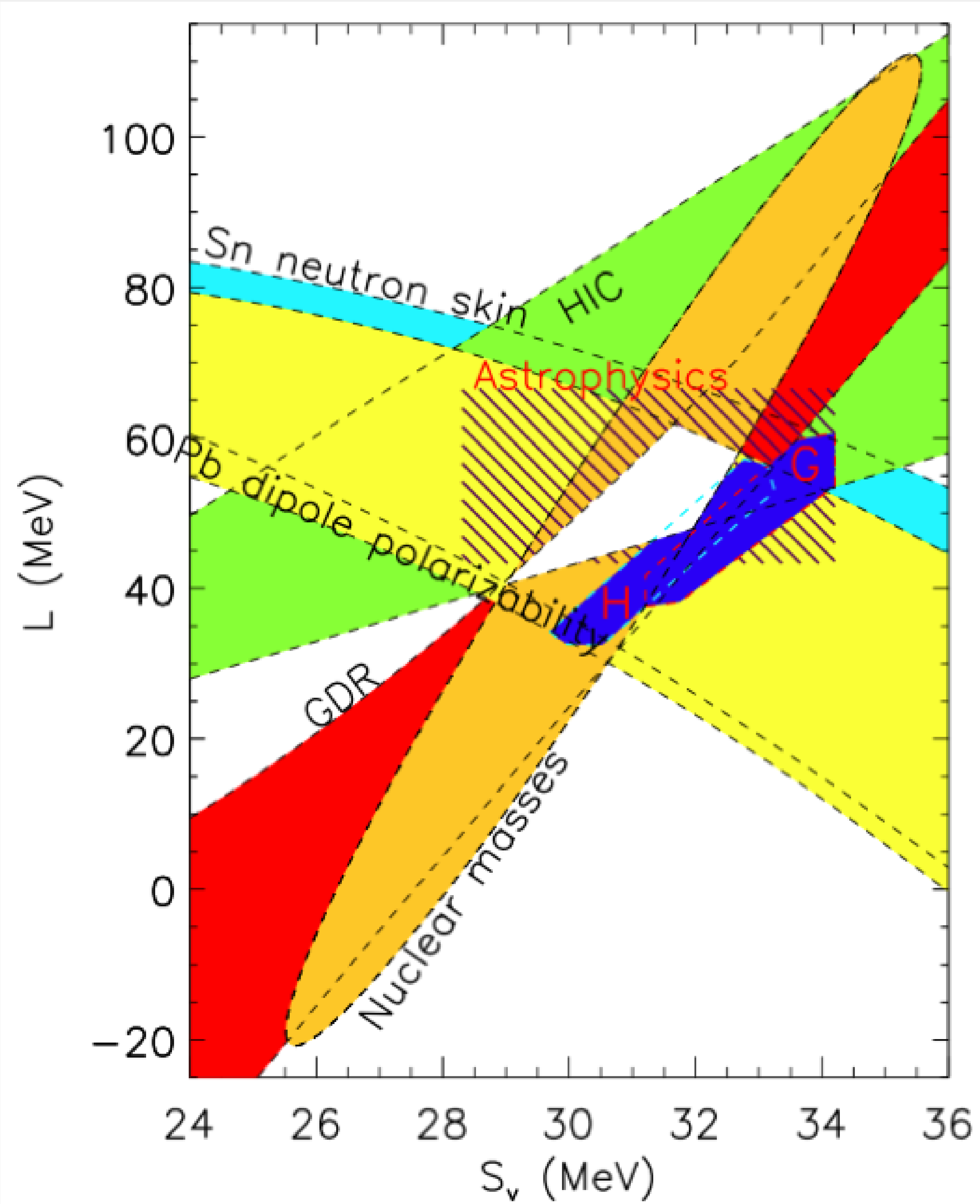
Steiner et al. (2005)

- S is the value at the nuclear saturation density $S = S(n_0)$
- L is the derivative, $L = 3n_0 S'(n_0)$
- Strong constraints on the derivative of the symmetry energy, L



Steiner and Gandolfi (2012)

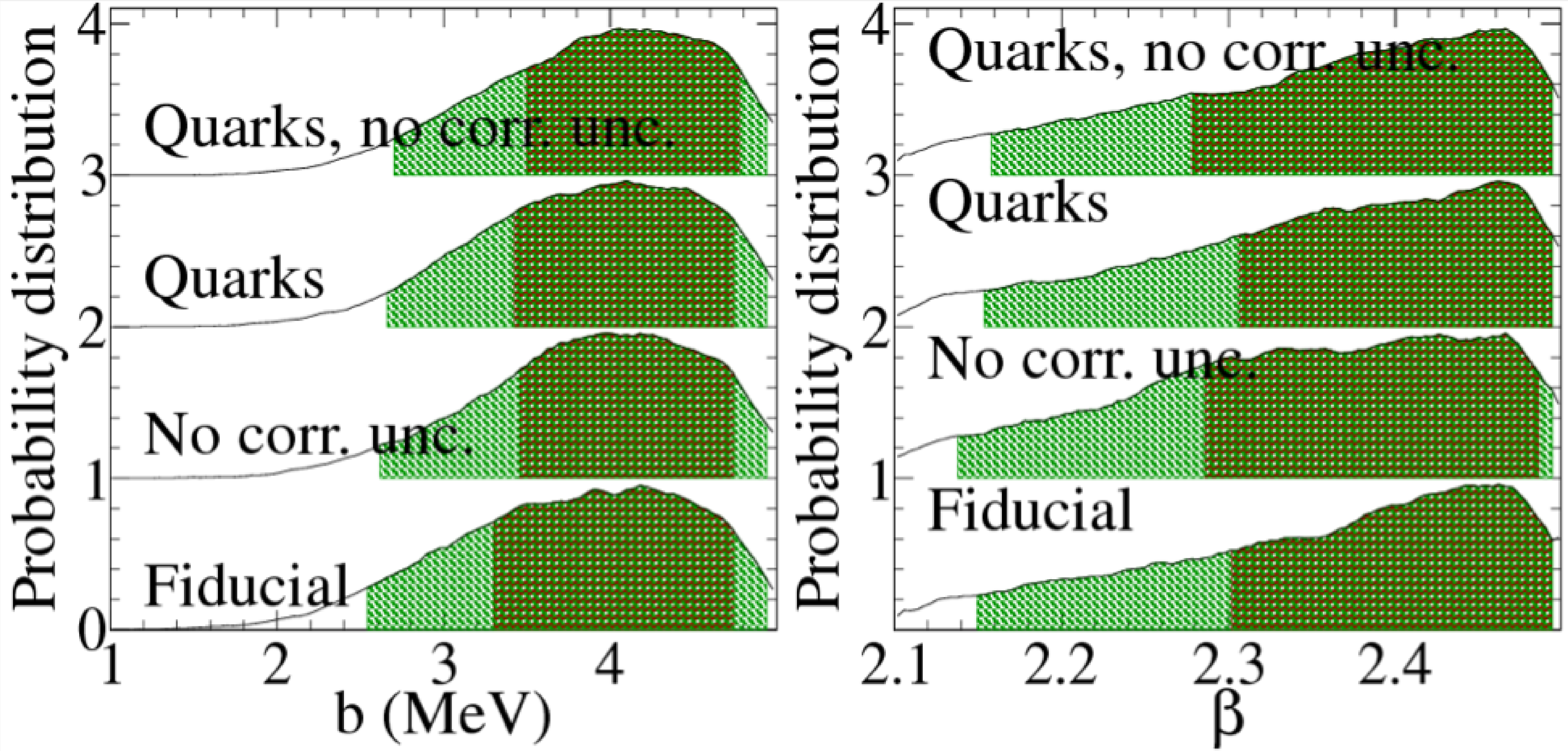
Constraints on the Nuclear Symmetry Energy



Lattimer and Lim (2012)

- Constraints on L from neutron star observations
- Correlations between S and L from masses, QMC and CET
- Other information from structure and reactions

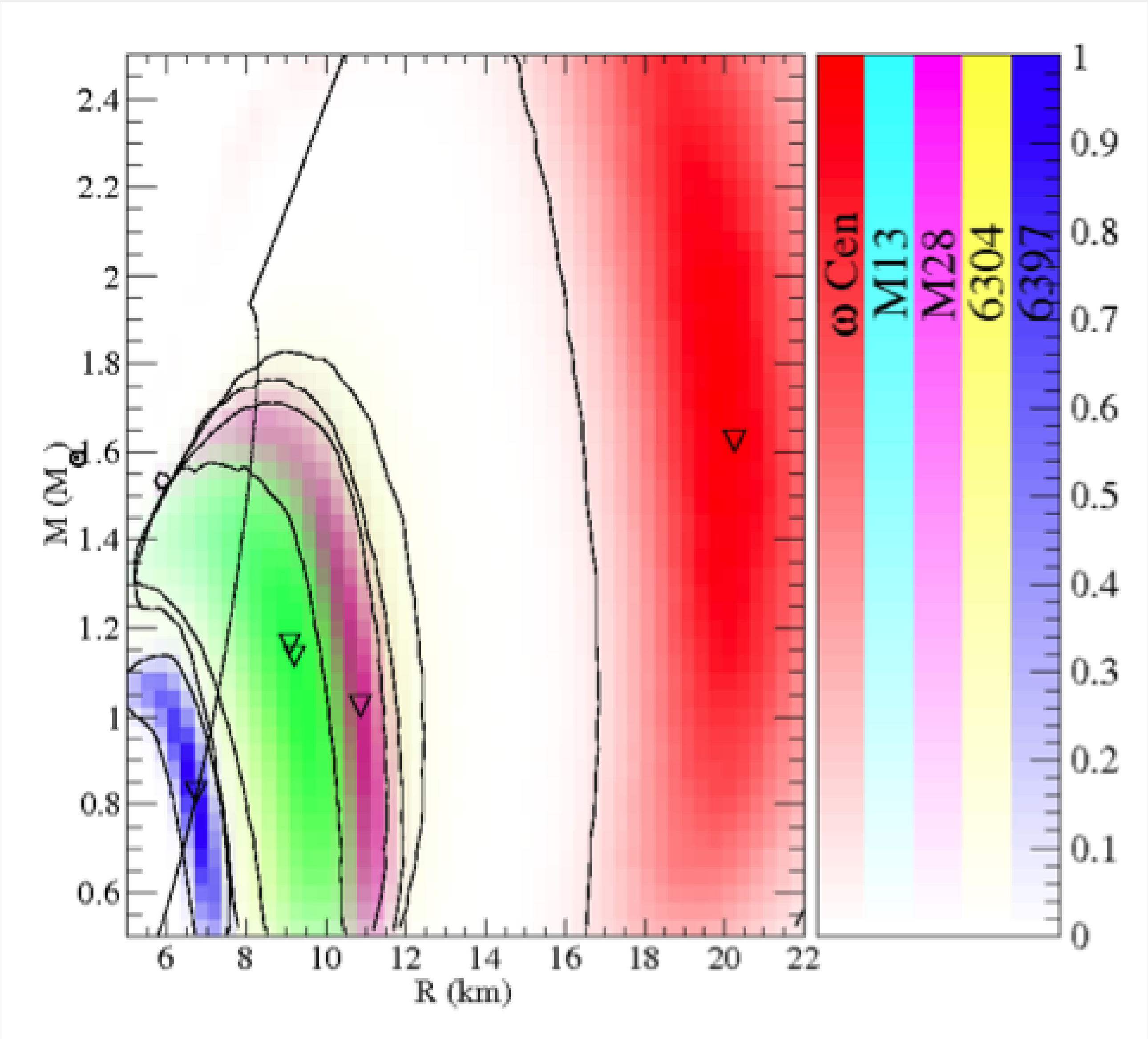
Constraints on Three-Body Force Parameters

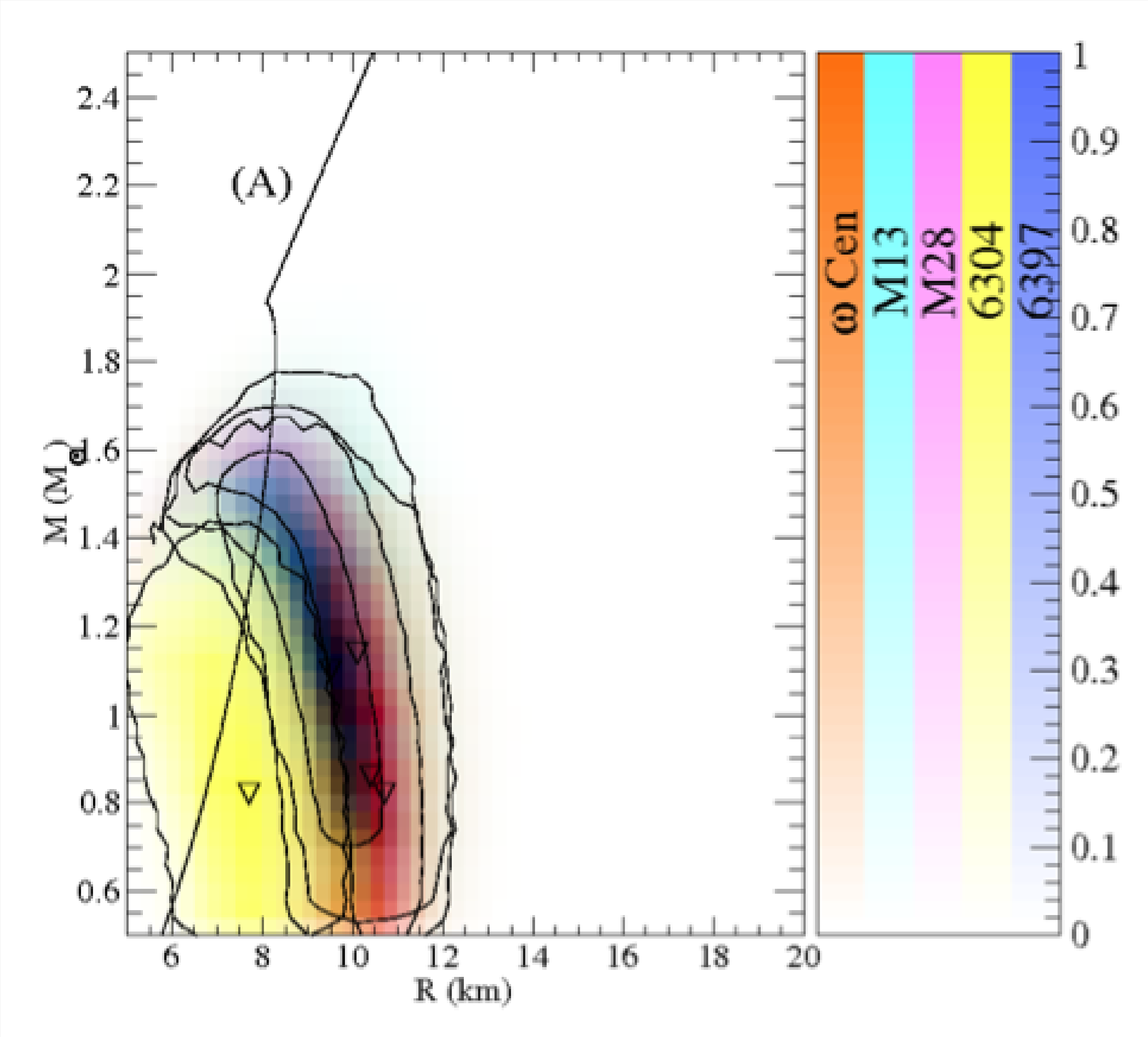


Steiner and Gandolfi (2012)

- $E_{\text{neut}} = a\left(\frac{n}{n_0}\right)^{\alpha} + b\left(\frac{n}{n_0}\right)^{\beta}$
- Values of a and α are unconstrained, but constraints on b and β
- Neutron star radii are constraining nuclear three-body forces

Nuclear Guidance on Neutron Star Observations





- Some observations, at first blush, appear inconsistent with expectations from nuclear physics
- Resolution: uncertainties in X-ray absorption and distance

- Alternative N_H
- The right panel is more consistent with nuclear physics
- Bayes factor ~ 10^6!

Lattimer and Steiner (2013) and adapted from Guillot et al. (2013)

Summary and Future

Current neutron star mass and radius observations provide novel constraints for nuclear physics

- Caveat emptor! Important systematic uncertainties in the astronomical measurements are problematic: distances, modeling, etc.

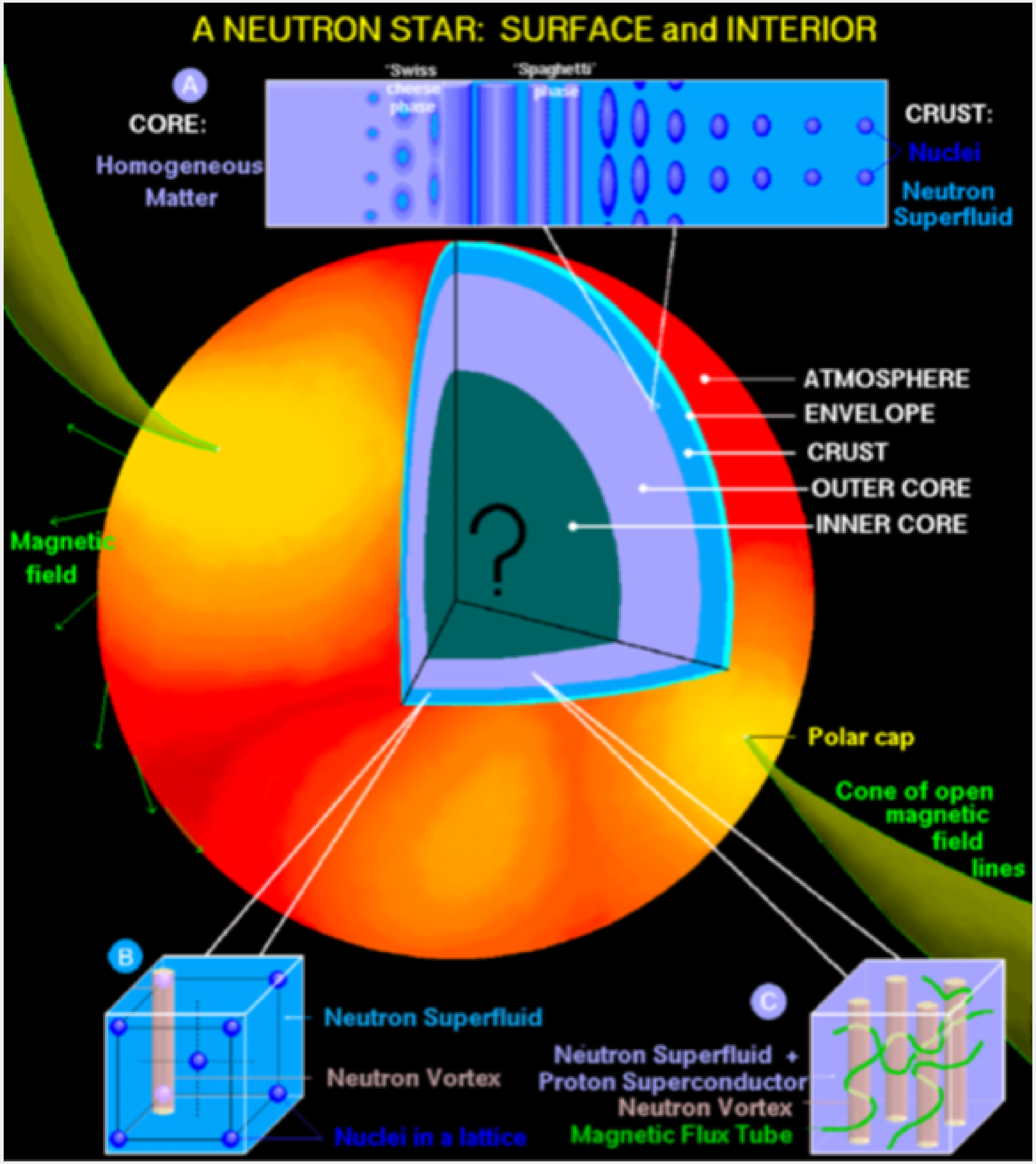
From NS radius observations:

- All neutron star radii are between 10.4 and 12.9 km
- The neutron skin thickness of $^{208}\mathrm{Pb} < 0.2$ fm
- $41 < L < 83$ MeV
- Ruling out some three-nucleon forces

Future:

- We have forgotten old-school data analysis because it was hard
- Computational advances $\Rightarrow$ Easier now
- Heteroskedasticity - Not all data points are created equal
- Vary your priors (Hamiltonians, many-body techniques, data set choices, constraints, etc.)

Neutron Star Composition



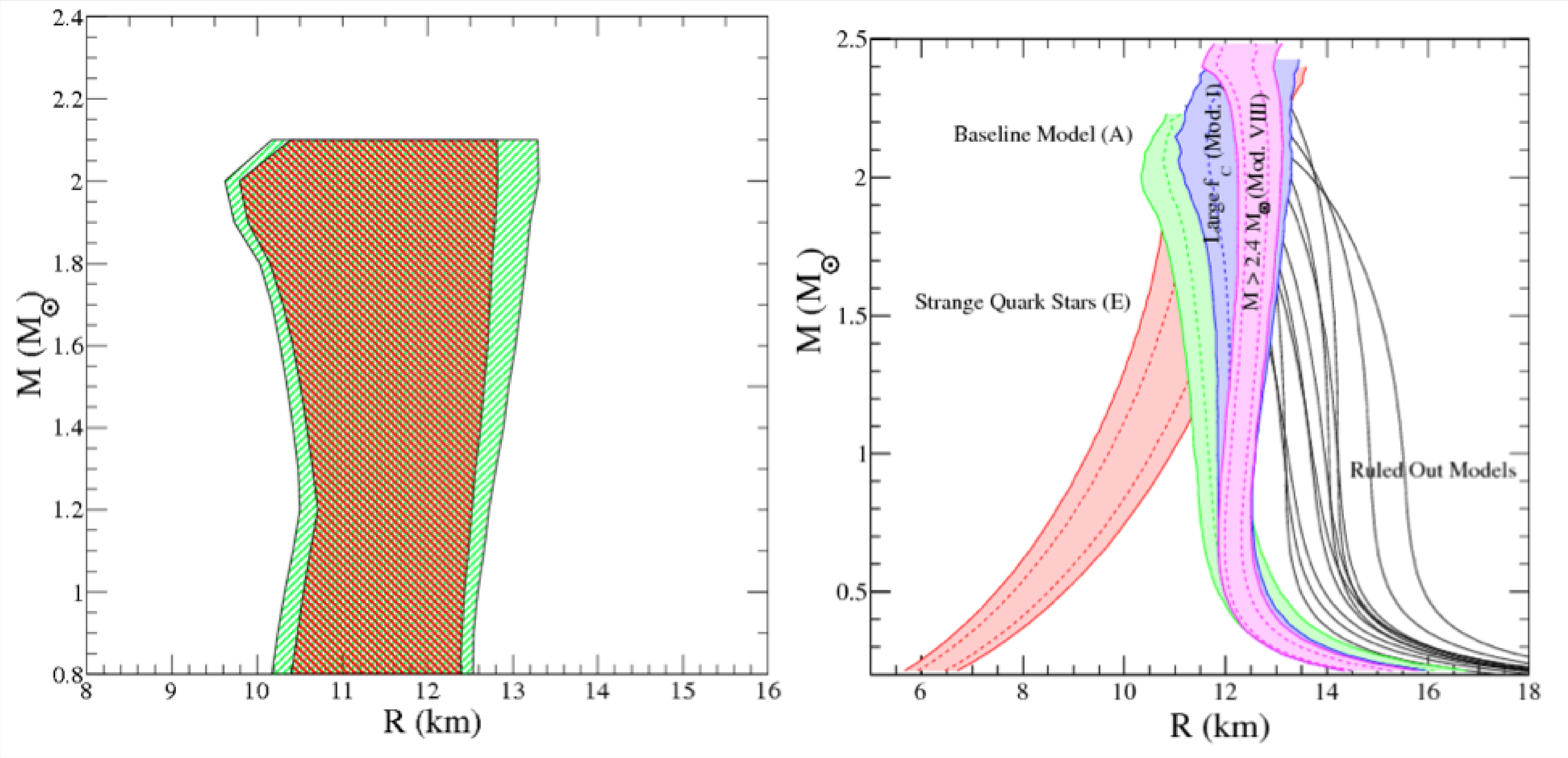
- Stable nuclei ⇒ neutron-rich nuclei ⇒ neutron-rich nuclei with quasi-free neutrons ⇒ nucleonic matter ⇒ exotica(?)
- Fundamental neutron star questions:
  - What are neutron star radii?
  - What is their composition?
- Connected to fundamental questions in QCD

Figure by Dany Page

Connections Between Neutron Stars and Nuclear Physics

- Mass and radius: EOS, n-n interaction, symmetry energy, composition
  Lattimer et al. 2001, Ozel et al. 2010, Steiner et al. 2013
- Moment of inertia and tidal deformation: EOS, composition
  Hinderer et al. 2010, Damour et al. 2012, Yagi et al. 2013
- Pulsar glitches: superfluidity
  Link 2012, Ho et al. 2012, Andersson et al. 2012
- r-mode oscillations: composition, superfluidity, shear and bulk viscosities
  Jaikumar et al. 2008, Ho et al. 2011, Schwenzer 2012, Mahmoodifar et al. 2013
- Thermal evolution: neutrino and photon emissivities, superfluidity, composition
  Page et al. 2009, Page et al. 2011, Shternin et al. 2011
- Crust cooling: composition, superfluidity
  Brown et al. 2009, Page et al. 2012
- Magnetar flares: EOS, composition of crust
  Gabler et al. 2012, Sotani et al. 2012, Passamonti et al. 2012, Deibel et al. 2013
- kHz QPOs: EOS, composition
  van der Klis 2006
- Mergers and gravitational waves: EOS, composition, r-process nucleosynthesis, transport properties
- X-ray bursts: EOS, symmetry energy, rp-process nucleosynthesis, nuclear structure
  Suleimanov et al. 2011, and many others
- r-Process Nucleosynthesis: symmetry energy, nuclear structure
  Roberts et al. 2012, and many others
- Birth of neutron stars and core-collapse supernovae: EOS, nucleon-nucleon interaction, symmetry energy, neutrino opacities

Mass and Radius Results



Steiner, Lattimer, and Brown (2013)

- Vary priors through different EOS parameterizations, choose smallest region enclosing all results
- Range of radii for a 1.4 solar mass star: 10.4 and 12.9 km (95% conf.)
- All neutron stars have nearly the same radius
- Several models are ruled out