Skyrme effective pseudopotential up to next-to-next-to leading order

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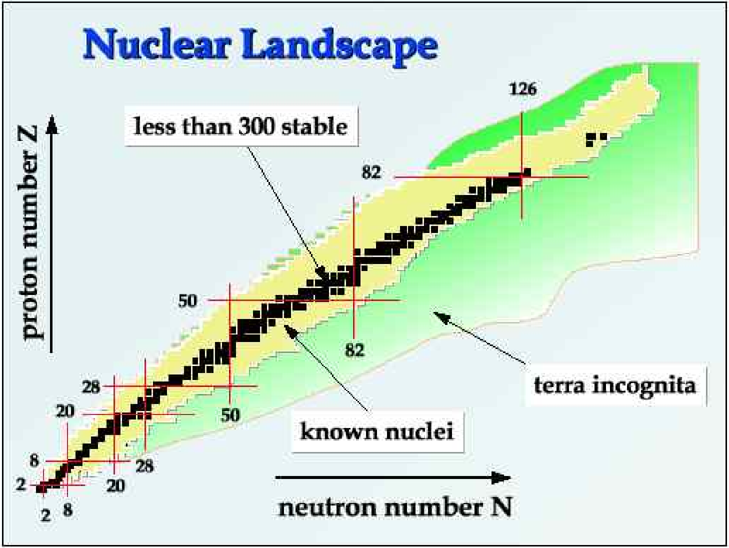

Nuclear Landscape
126
less than 300 stable
82
proton number Z
50
82
28
20
50
8
28
2
20
2 8
terra incognita
known nuclei
neutron number N

Skyrme interaction (Negele&Vautherin)

In 1959 Skyrme suggested a zero range interaction for mean field models.

[T.H.R.Skyrme Nuclear Phys. **9**, 615-634, 1959]

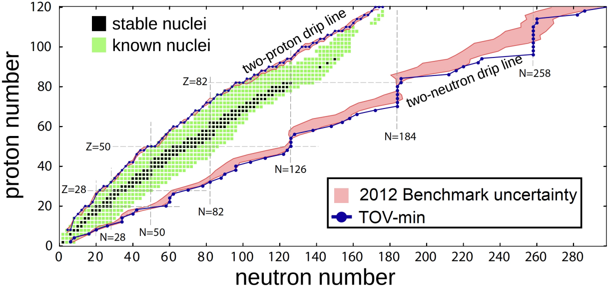
Two body

$$
\begin{aligned}
& t_0(1+x_0\hat{P}_\sigma)\delta_{\mathbf{r}} \\
& +\tfrac{1}{2}t_1(1+x_1\hat{P}_\sigma)\left[\mathbf{k}'^2\delta_{\mathbf{r}}+\delta_{\mathbf{r}}\mathbf{k}^2\right] \\
& +t_2(1+x_2\hat{P}_\sigma)\mathbf{k}'\cdot\delta_{\mathbf{r}}\mathbf{k} \\
& +\mathrm{i}W_0(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\left[\mathbf{k}'\times\delta_{\mathbf{r}}\mathbf{k}\right]
\end{aligned}
$$

Three body

$$t_3\delta(\mathbf{r}_1-\mathbf{r}_2)\delta(\mathbf{r}_3-\mathbf{r}_1)\rightarrow t_3\delta_{\mathbf{r}}\rho(\mathbf{R})^\gamma$$

A simple and successful model



[J. Erler et al., Nature **486** 509 (2012)]

Two important ingredients

- The fitting protocol to derive the coupling constants
- The form of the interaction

Extra terms:

T.H. Skyrme also suggested some extra terms usually neglected

Extra terms

- Tensor term

$$
\begin{aligned}
&+\frac{t_e}{2}\left\{3(\boldsymbol{\sigma}_1\cdot\mathbf{k}')\delta_{\mathbf{r}}(\boldsymbol{\sigma}_2\cdot\mathbf{k}')-(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)\mathbf{k}'^2\delta_{\mathbf{r}}+h.c.\right\}\\
&+\frac{t_o}{2}\left\{3(\boldsymbol{\sigma}_1\cdot\mathbf{k}')\delta_{\mathbf{r}}(\boldsymbol{\sigma}_2\cdot\mathbf{k})-(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)\mathbf{k}'\cdot\delta_{\mathbf{r}}\mathbf{k}+h.c\right\}
\end{aligned}
$$

[T. Lesinski *et al.* Phys. Rev C 76, 014312, 2007]

- (Real) Three/Four body term

$$t_3\delta(\mathbf{r}_1-\mathbf{r}_2)\delta(\mathbf{r}_3-\mathbf{r}_1)+t_4\delta(\mathbf{r}_1-\mathbf{r}_2)\delta(\mathbf{r}_3-\mathbf{r}_1)\delta(\mathbf{r}_4-\mathbf{r}_1)$$

[J. Sadoudi *et al.* Phys.Scripta T154 (2013) 014013]

- Higher order (D wave)

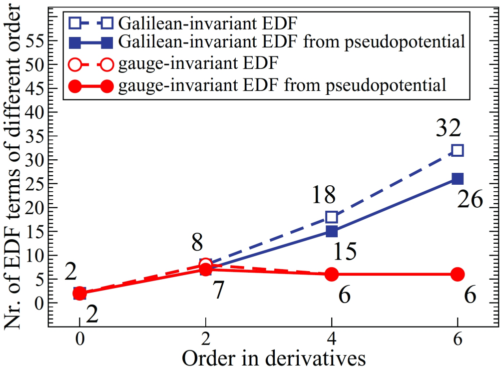
$$t_D[\mathbf{k}^2\mathbf{k}'^2-(\mathbf{k}\cdot\mathbf{k}')^2]\delta_{\mathbf{r}}$$

Systematic explorations

Carlsson suggested a systematic way to build high-order terms

$$\hat{V}^{\tilde{n}'\tilde{L}'}_{\tilde{n}\tilde{L},v_{12}S} = \frac{1}{2} i^{v_{12}} \left(\left[\left[K'_{\tilde{n}'\tilde{L}'} K_{\tilde{n}\tilde{L}} \right]_S \hat{S}_{v_{12}S} \right]_0 + (-1)^{v_{12}+S} \left[\left[K'_{\tilde{n}\tilde{L}} K_{\tilde{n}'\tilde{L}'} \right]_S \hat{S}_{v_{12}S} \right]_0 \right)$$
$$\times \left(1 - \hat{P}^M \hat{P}^\sigma \hat{P}^\tau \right) \delta(\mathbf{r}'_1 - \mathbf{r}_1)\delta(\mathbf{r}'_2 - \mathbf{r}_2)\delta(\mathbf{r}_1 - \mathbf{r}_2).$$

[B.G. Carlsson *et al.* Phys. Rev. C 78, 044326, 2008]



Why gauge invariance??

The gauge transformation reads

$$|\psi'\rangle = \text{exp}\left(i\sum_{j=1}^{A}\phi(r_j)\right)|\psi\rangle$$

Gauge invariance $\leftrightarrow$ Continuity equation

$$\frac{d}{dt}\rho(\mathbf{r},t) = -\frac{\hbar}{m}\nabla\mathbf{j}(\mathbf{r},t)$$

[F. Raimondi *et al.* Phys. Rev. C 83, 054311, 2011]

WARNING!!

We can neglect the continuity equation, but we need a new *physical* interpretation of a time-evolving quantal system

N2LO in Cartesian basis

The N2LO pseudo-potential with Galilean and gauge symmetry

$$
\begin{aligned}
\hat{V}^{(4)}_{\rm Sk} &= \frac{1}{4} t_1^{(4)} (1 + x_1^{(4)} P_\sigma) \left[(\mathbf{k}^2 + \mathbf{k}'^2)^2 + 4(\mathbf{k}' \cdot \mathbf{k})^2 \right] \\
&+ t_2^{(4)} (1 + x_2^{(4)} P_\sigma) (\mathbf{k} \cdot \mathbf{k}')(\mathbf{k}^2 + \mathbf{k}'^2) \\
&+ t_e^{(4)} \left[(\mathbf{k}^2 + \mathbf{k}'^2) T_e(\mathbf{k}', \mathbf{k}) + 2(\mathbf{k} \cdot \mathbf{k}') T_o(\mathbf{k}', \mathbf{k}) \right] \\
&+ t_o^{(4)} \left[5(\mathbf{k} \cdot \mathbf{k}') T_e(\mathbf{k}', \mathbf{k}) - \frac{1}{2} (\mathbf{k}^2 + \mathbf{k}'^2) T_o(\mathbf{k}', \mathbf{k}) \right],
\end{aligned}
$$

$$
\begin{aligned}
T_e(\mathbf{k}', \mathbf{k}) &= 3(\vec{\sigma}_1 \cdot \mathbf{k}')(\vec{\sigma}_2 \cdot \mathbf{k}') + 3(\vec{\sigma}_1 \cdot \mathbf{k})(\vec{\sigma}_2 \cdot \mathbf{k}) - (\mathbf{k}'^2 + \mathbf{k}^2)(\vec{\sigma}_1 \cdot \vec{\sigma}_2), \\
T_o(\mathbf{k}', \mathbf{k}) &= 3(\vec{\sigma}_1 \cdot \mathbf{k}')(\vec{\sigma}_2 \cdot \mathbf{k}) + 3(\vec{\sigma}_1 \cdot \mathbf{k})(\vec{\sigma}_2 \cdot \mathbf{k}') - 2(\mathbf{k}' \cdot \mathbf{k})(\vec{\sigma}_1 \cdot \vec{\sigma}_2),
\end{aligned}
$$

We are left with 6 additional free parameters (4 central+2 tensor)

$$
\begin{aligned}
\frac{1}{4} t_1^{(4)} &= \frac{3C^{22}_{22,00} + \sqrt{3} C^{22}_{22,20}}{12\sqrt{5}}, & \frac{1}{4} t_1^{(4)} x_1^{(4)} &= -\frac{C^{22}_{22,20}}{2\sqrt{15}}, \\
t_2^{(4)} &= \frac{3C^{31}_{11,00} + \sqrt{3} C^{31}_{11,20}}{18}, & t_2^{(4)} x_2^{(4)} &= -\frac{\sqrt{3} C^{31}_{11,20}}{9}, \\
t_e^{(4)} &= -\frac{C^{22}_{22,22}}{2\sqrt{105}}, & t_o^{(4)} &= -\frac{C^{33}_{11,22}}{30\sqrt{7}}.
\end{aligned}
$$

Schrödinger equation or not?

Two possibles D-wave terms

Skyrme original: $t_D[\mathbf{k}^2\mathbf{k}'^2 - (\mathbf{k}\cdot\mathbf{k}')^2]$

- We have a Schrödinger equation
- No contributions to IM properties (E/A, mass, Landau, ...)
- No gauge invariance

N2LO: $\frac{1}{4}t_1^{(4)}(1 + x_1^{(4)}P_\sigma)\left[(\mathbf{k}^2+\mathbf{k}'^2)^2 + 4(\mathbf{k}'\cdot\mathbf{k})^2\right]$

- No a Schrödinger equation (4th derivative....)
- Explicit contributions to IM properties (E/A, mass, Landau, ...)
- Gauge invariance

Without a Schrödinger equation no more r-space codes!!! Numerov method does not work....

New fitting protocol

The χ^2 takes contributions from IM and finite nuclei.

Contributions form IM

- Naturaleness analysis of high order terms [M. Kortelainen *et al.* Phys. Rev. C 82, 011304, 2010]
- 18 Landau inequalities for SNM and PNM at N2LO (not all independents!!!)
- EoS in the 4 (S,T) channels
- Compressibility, effective mass
- LR theory to avoid finite-size instabilities (not ready yet...)

Definition of an interval of variation of parameters *before* fitting finite nuclei. IM properties can also be check on *real time* since there is no CPU cost.

Properties of Landau parameters

- We can fit directly the Landau parameters of *any* interaction
- We can separate easily the different contributions

l	Contribution
f_0	$\frac{1}{4}L_0[f] + \frac{1}{8}k_F^2 L_2[f] + \frac{1}{6}k_F^4 L_4[f]$
f_1	$-\frac{1}{8}k_F^2 L_2[f] - \frac{1}{4}k_F^4 L_4[f]$
f_2	$\frac{1}{12}k_F^4 L_4[f]$
h_0	$\frac{1}{4}k_F^2 L_2[h] + \frac{1}{2}k_F^4 L_4[h]$
h_1	$-\frac{1}{2}k_F^4 L_4[h]$

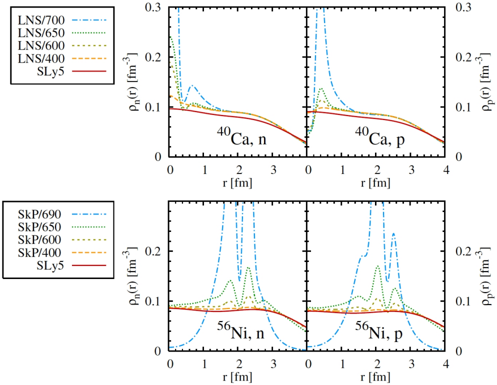
- Inequalities to avoid instabilities (F and G only)

$$F_l/(2l+1) > -1$$

- More complicated inequalities if tensor terms are present [J. Navarro *et al.* Phys. Rev. C 87, 044329, 2013]

Instabilities I

... a not well constrained coupling constant can lead to surprises....

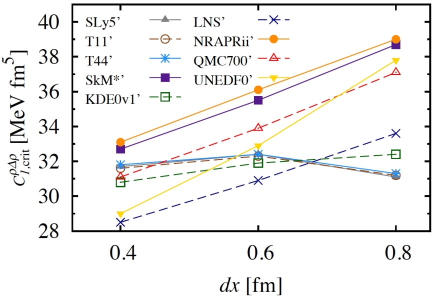


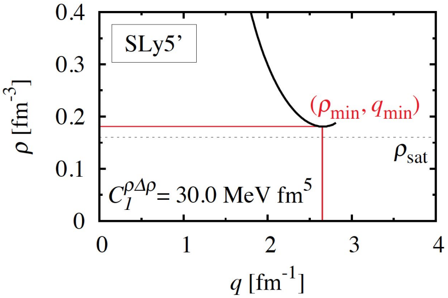
[T. Lesinski et al. Phys. Rev. C 76, 014312 (2007)]

Finite-size instability

Formation of polarized domains $\lambda \approx \frac{2\pi}{q}$

Instabilities II

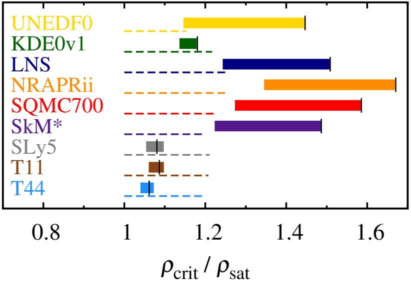




Numerical test

Calculations done with Ev8 spherical, deformed and superfluid nuclei

Instabilities III



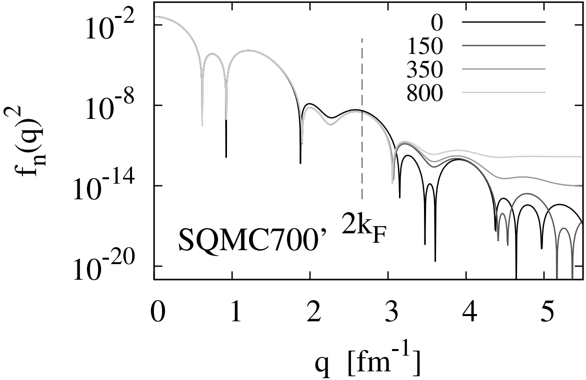
WARNING

A pole at ρ_{sat} leads to an instability. Which is the upper limit?

Conclusions

- Explicit derivation of N2LO terms in Cartesian basis
- Discussion of D-wave and spin-orbit terms
- Landau inequalities
- Definition of a new fitting protocol

THANK YOU!!!



$$f(\mathbf{q}) = \frac{1}{(2\pi)^{3/2}} \int e^{-i\mathbf{q}\cdot\mathbf{s}} \rho(0, \mathbf{s}) d^3\mathbf{s}$$

Spin-orbit and gauge invariance

Skyrme and Bell predicted a spin-orbit of the form

$$i(\sigma_1 + \sigma_2)(\mathbf{k}' \wedge \mathbf{k}) F\left(\mathbf{k}'^2, \mathbf{k}^2, (\mathbf{k} \cdot \mathbf{k}')\right)$$

Imposing gauge invariance we have that $F = 1$ is the only possibility!!!

Spin-orbit from high order terms

The spin-orbit can receive gauge-invariant contributions only via higher order tensor terms!

$$(\sigma_1 \cdot (\mathbf{k}' \wedge \mathbf{k}))(\sigma_2 \cdot (\mathbf{k}' \wedge \mathbf{k}))$$

How to constraint the new terms?

We can use different strategies

- Using DME starting from a finite range interaction as Gogny [B.G. Carlsson *et al.* Phys. Rev. Lett. 105, 122501, 2010]
- Using some properties of IM as E/A, effective mass, compressibility

$$(E/A)^{(4)} = \frac{9}{280}\left[3t_1^{(4)} + (5+4x_2^{(4)})t_2^{(4)}\right]\rho k_F^4,$$

- Landau parameters

$$\sum_l \left\{ f_l + f_l'(\tau_1\cdot\tau_2) + \left[g_l + g_l'(\tau_1\cdot\tau_2)\right](\sigma_1\cdot\sigma_2) \quad + \left[h_l + h_l'(\tau_1\cdot\tau_2)\right]\frac{k_{12}^2}{k_F^2}\, S_{12}\right\} P_l(\cos\theta)$$

Warning!!

The original Skyrme D-wave is not gauge invariant and it does not contribute to E/A or Landau!

$$t_D[\mathbf{k}^2\mathbf{k}'^2 - (\mathbf{k}\cdot\mathbf{k}')^2]\delta_{\mathbf{r}}$$