Higgs plus 1 jet at NNLO

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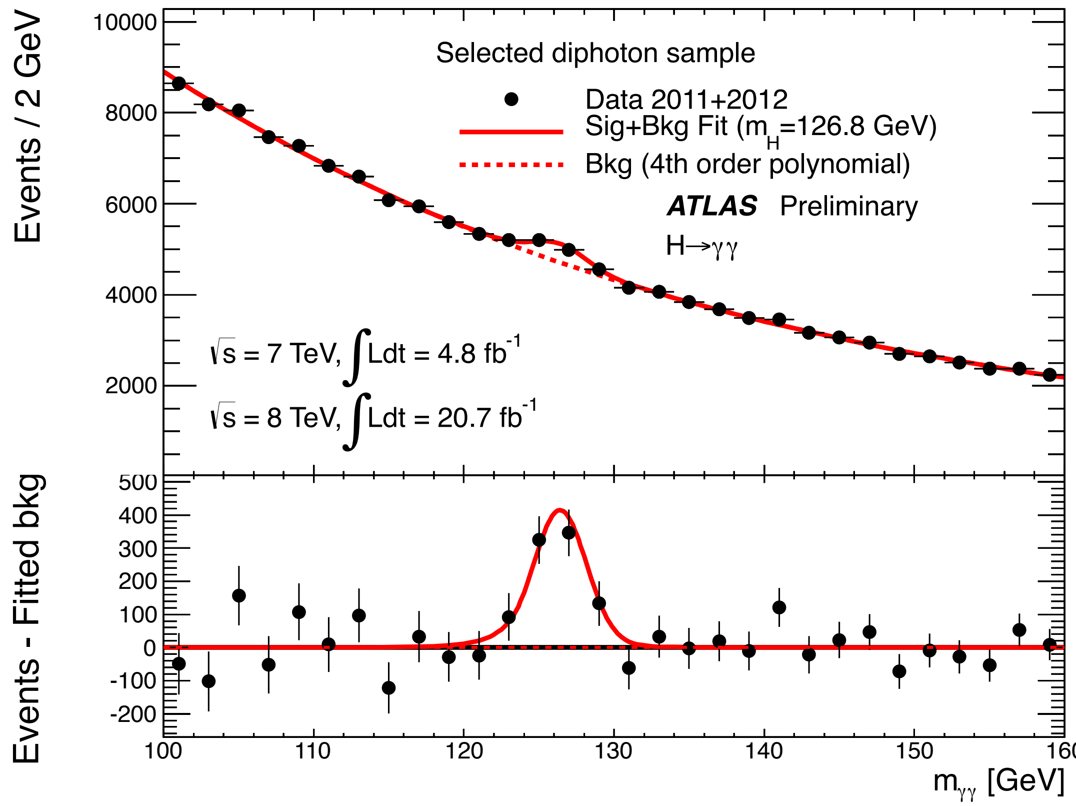
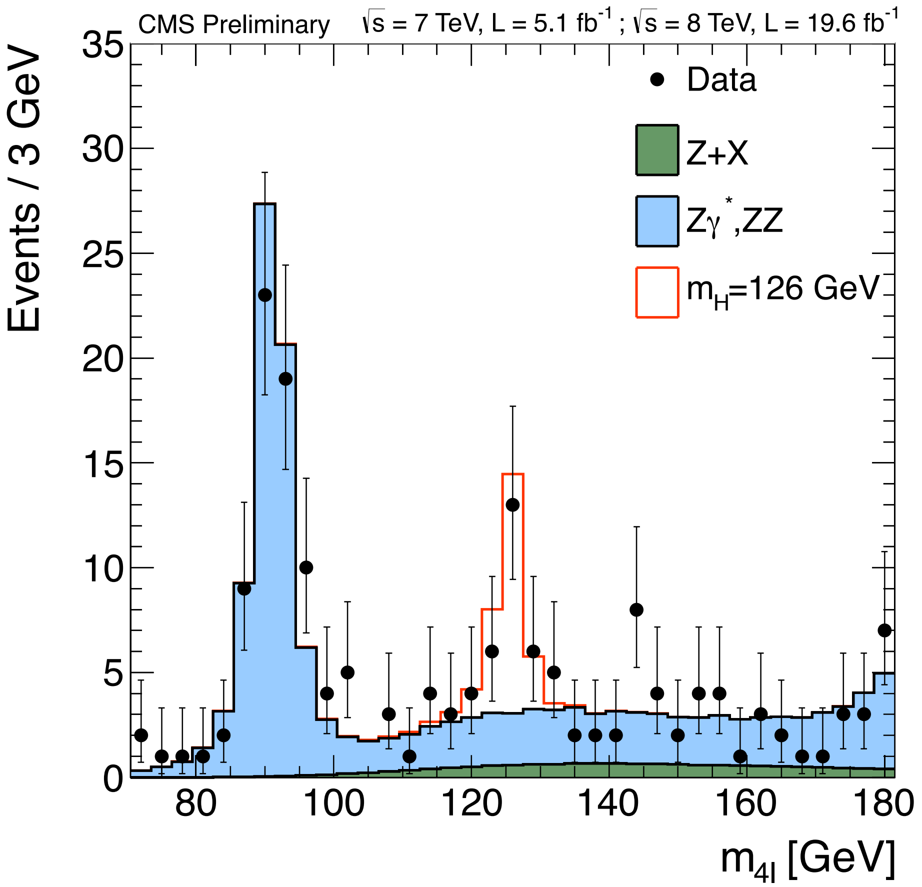
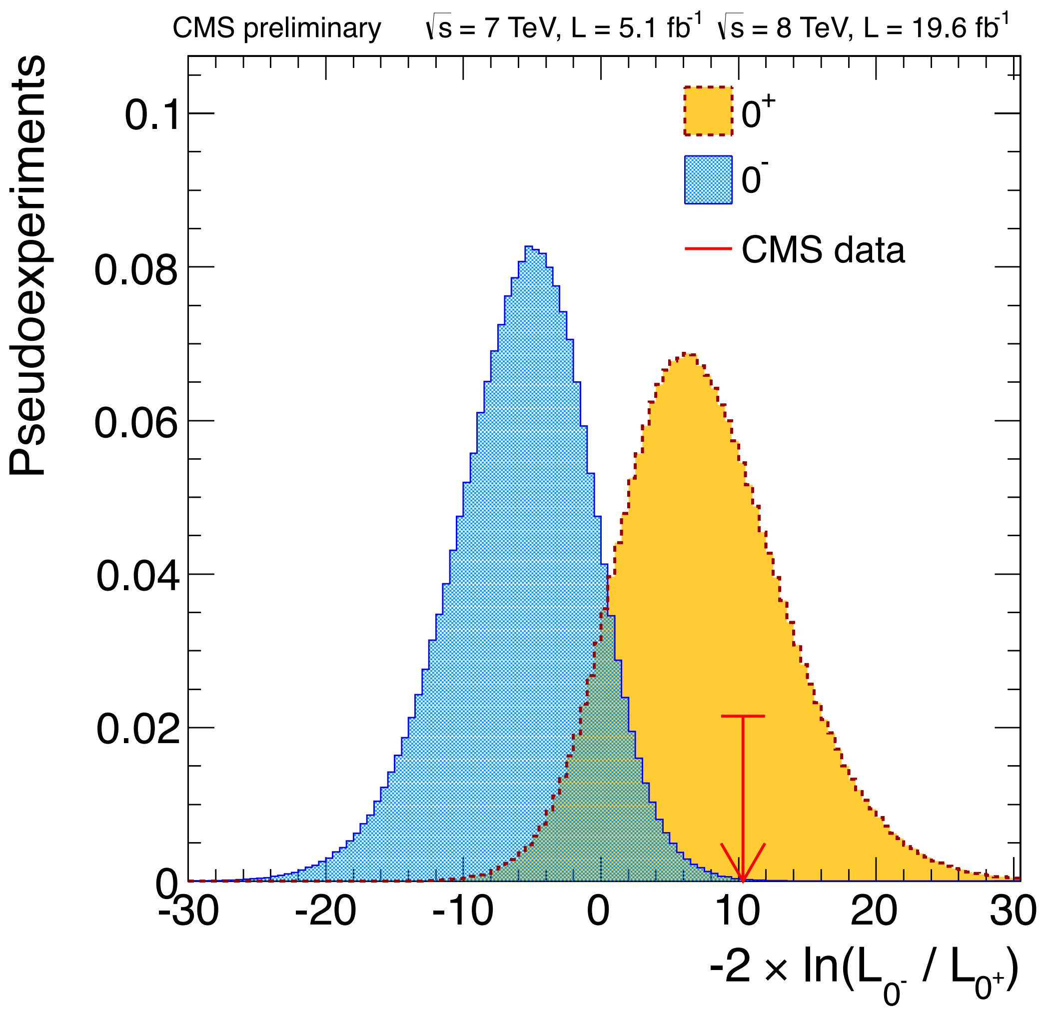
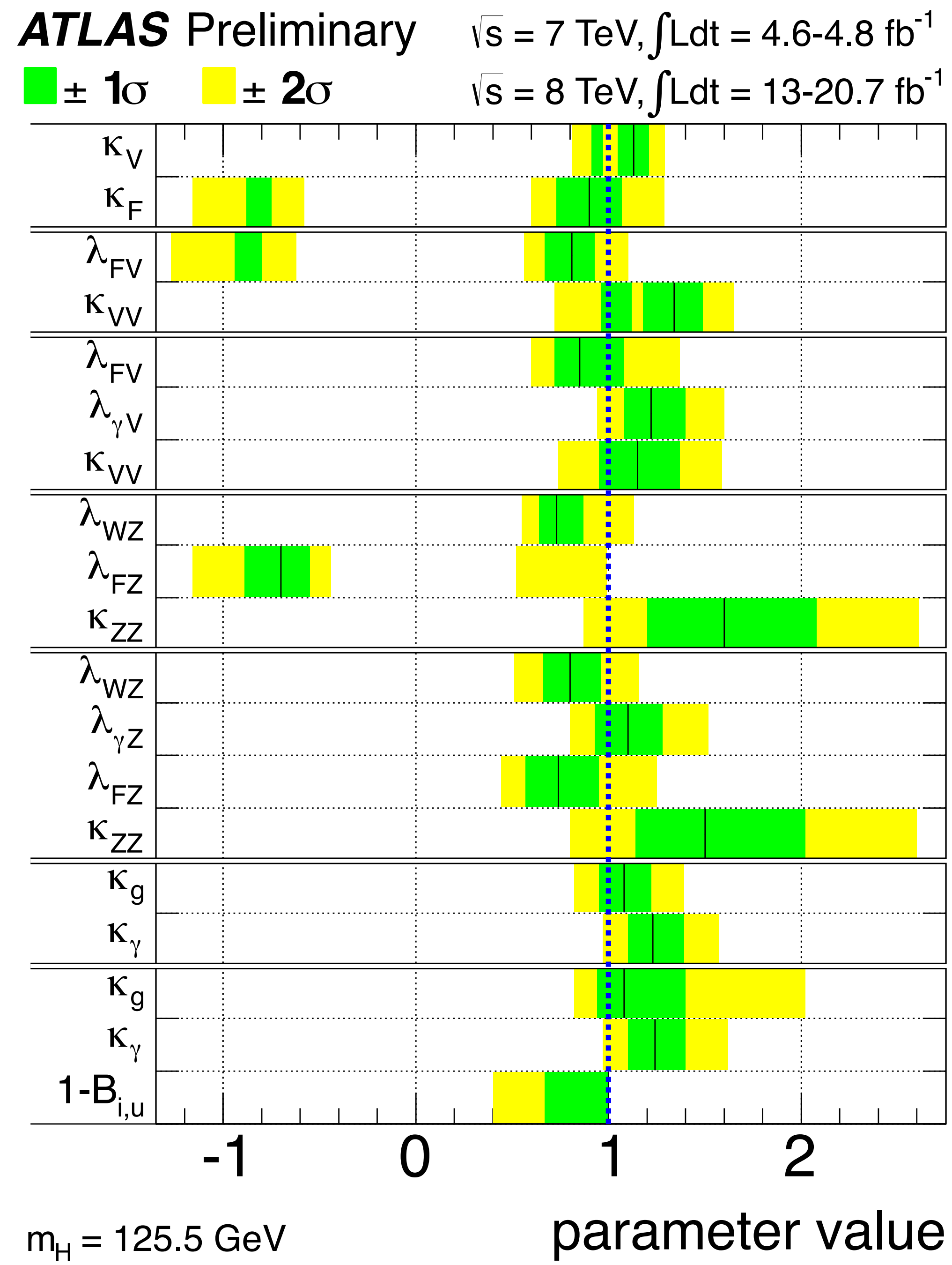
The Higgs Boson: from discovery...





The Higgs Boson: ... to precision measurements





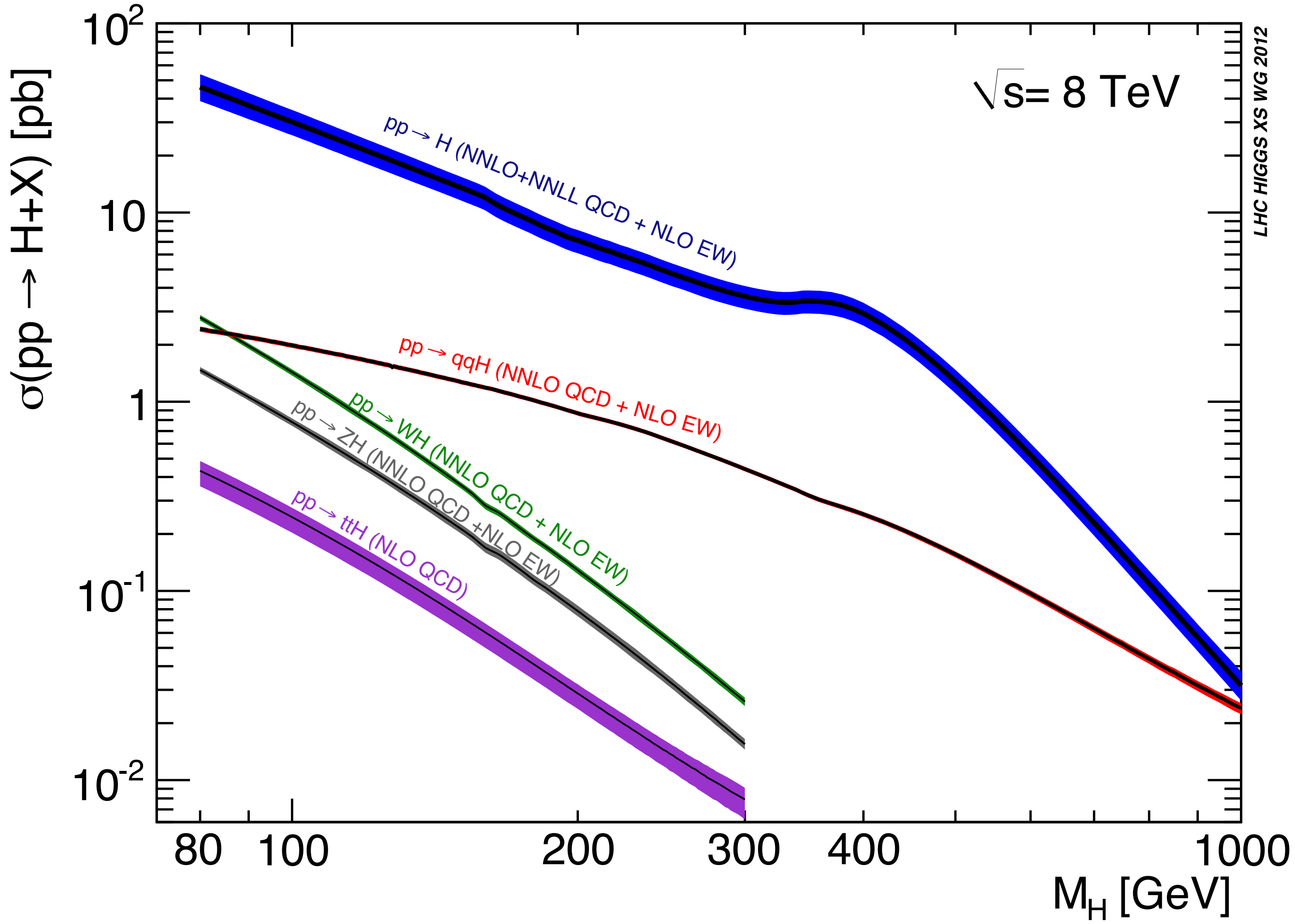
So far: very SM-like

Good control of theoretical predictions is required to search for small deviations

The Higgs Cross Section: what do we know

Gluon fusion: $\sim 10\%$

- NNLO QCD (inclusive and differential)
- NLO EW
- QCD resummations
- approximate NNNLO
- mixed QCD-EW
- 1/mt,mb corrections
- H+1j, H+2j @ NLO



VBF: $\sim 1\%$

- NNLO QCD (inclusive only)
- NLO EW
- VBF+1j @ NLO

Higgs-Strahlung: $\sim 1\%$

- NNLO QCD (differential)
- NLO EW
- VH+1j @ NLO

ttH: $\sim 10\%$

- NLO QCD, including PS matching

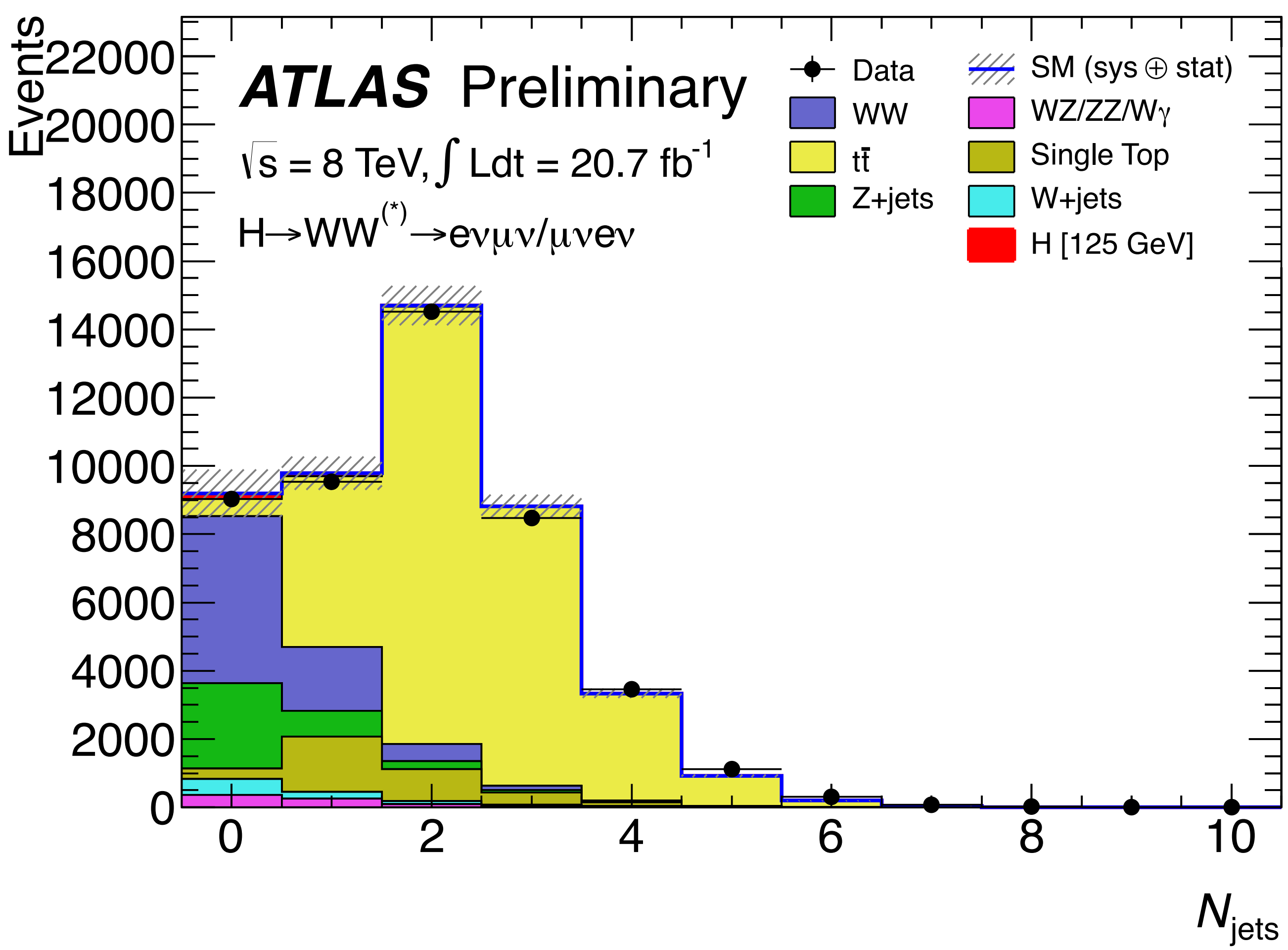
+ PDFs + MC tools + ...

Very good theoretical control

IS IT ENOUGH?

Higgs plus jet: need for improvement

Experimental analyses for $pp \to H \to WW$: binned according to jet multiplicity (different systematics)



- Signal/background ratio for H+1, H+2 jets: $\sim 10\%$
- Significance in the H+1jet bin smaller, but not much smaller, than significance in the H+0 jet bin
- LARGE THEORY ERROR

Selection	N_{obs}	N_{bkg}	N_{sig}	N_{WW}	N_{VV}	$N_{t\bar{t}}$	N_t	N_{Z/γ^*}	$N_{W+\text{jets}}$
$N_{\text{jet}} = 1$	9527	9460 ± 40	97 ± 1	1660 ± 10	270 ± 10	4980 ± 30	1600 ± 20	760 ± 20	195 ± 5
$N_{b\text{-jet}} = 0$	4320	4240 ± 30	85 ± 1	1460 ± 10	220 ± 10	1270 ± 10	460 ± 10	670 ± 10	160 ± 4
$Z \to \tau\tau$ veto	4138	4020 ± 30	84 ± 1	1420 ± 10	220 ± 10	1220 ± 10	440 ± 10	580 ± 10	155 ± 4
$m_{\ell\ell} < 50$	886	830 ± 10	63 ± 1	270 ± 4	69 ± 5	216 ± 6	80 ± 4	149 ± 5	46 ± 2
$\lvert\Delta\phi_{\ell\ell}\rvert < 1.8$	728	650 ± 10	59 ± 1	250 ± 4	60 ± 4	204 ± 6	76 ± 4	28 ± 3	34 ± 2

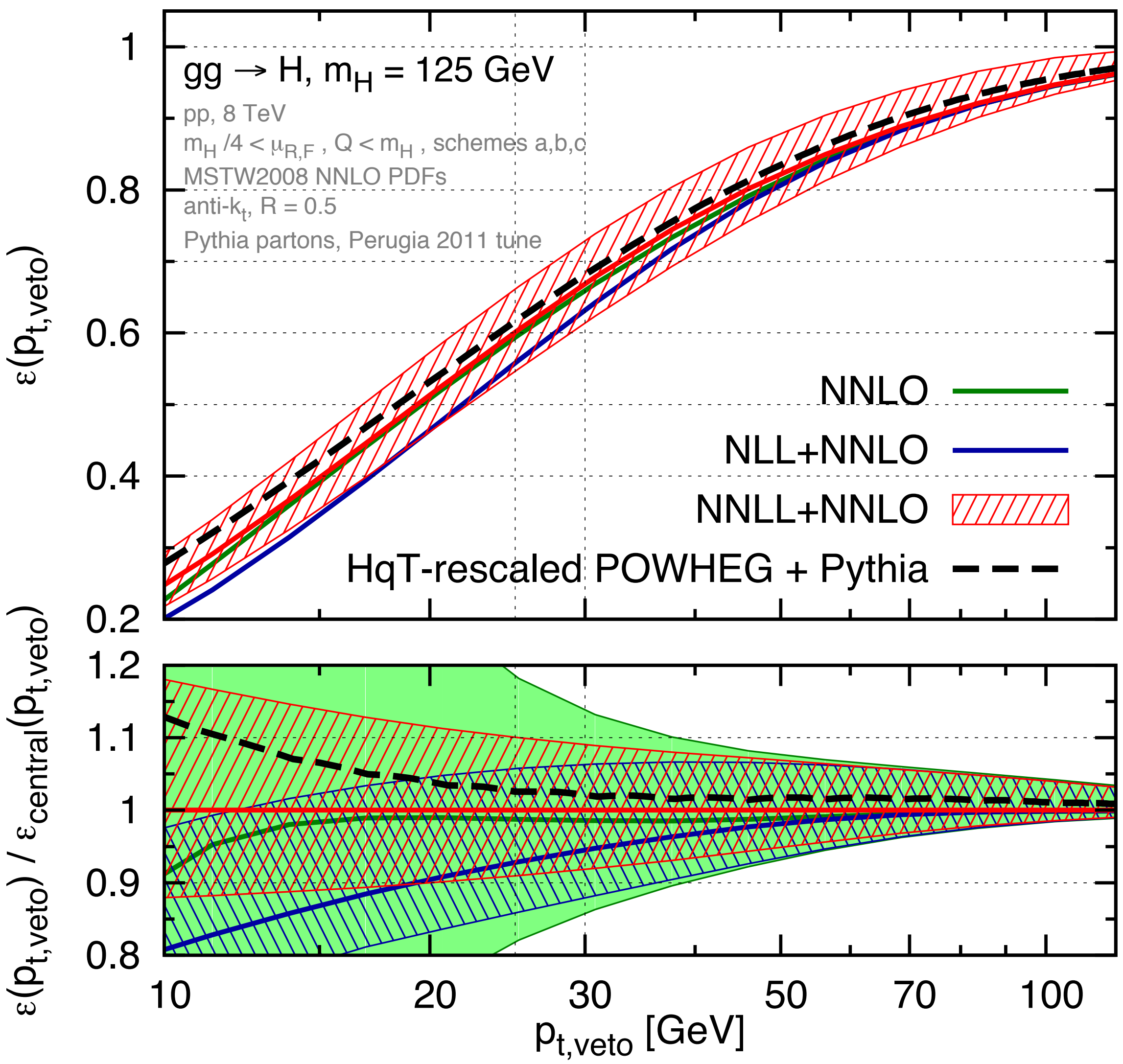
Higgs plus jet: need for improvement

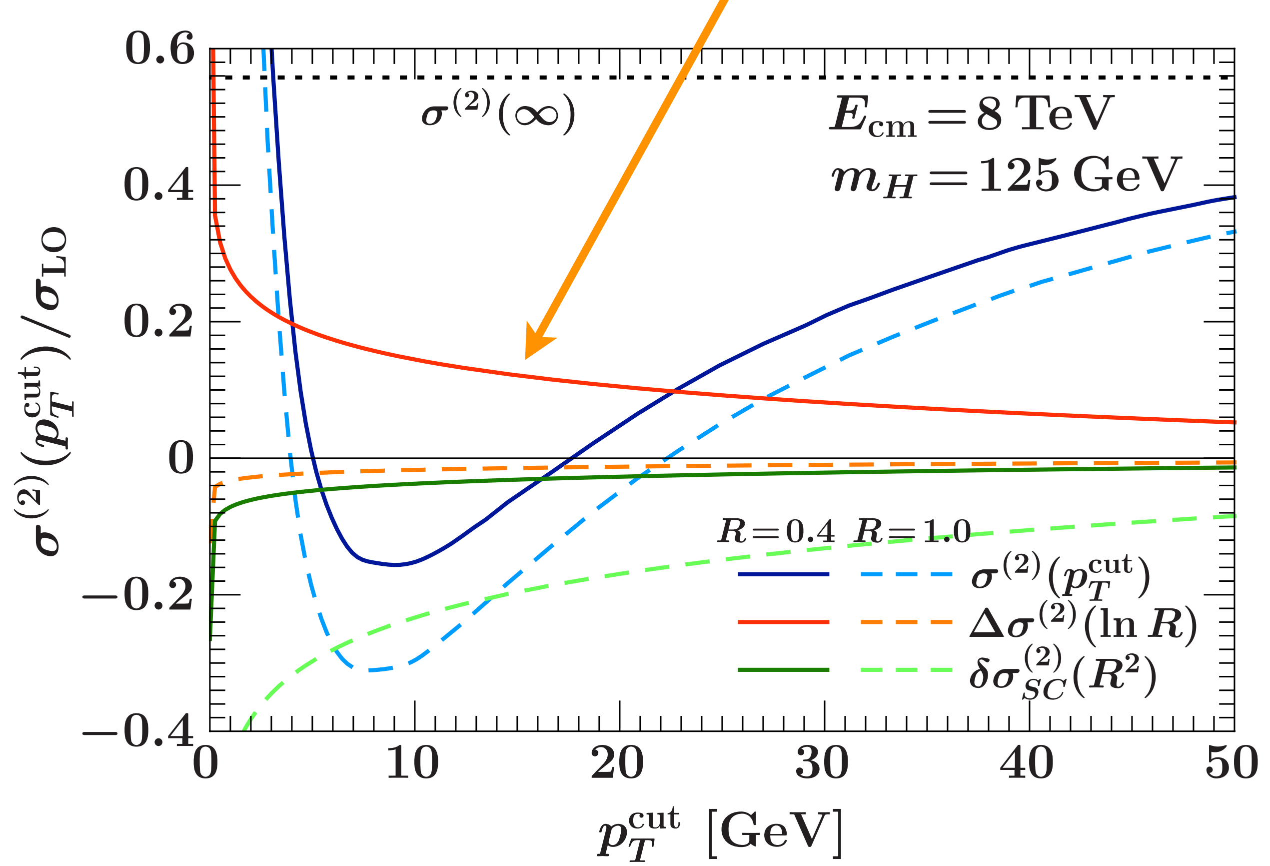
The 0-jet bin: jet-veto resummation

[Banfi et al. (2012), Tackmann et al. (2012)]
[1-jet bin: Liu and Petriello (2012, 2013)]

NNLL resummation for ln(pt/mh)

Challenging part: appearance of non-resummable (?) jet-algorithm dependence





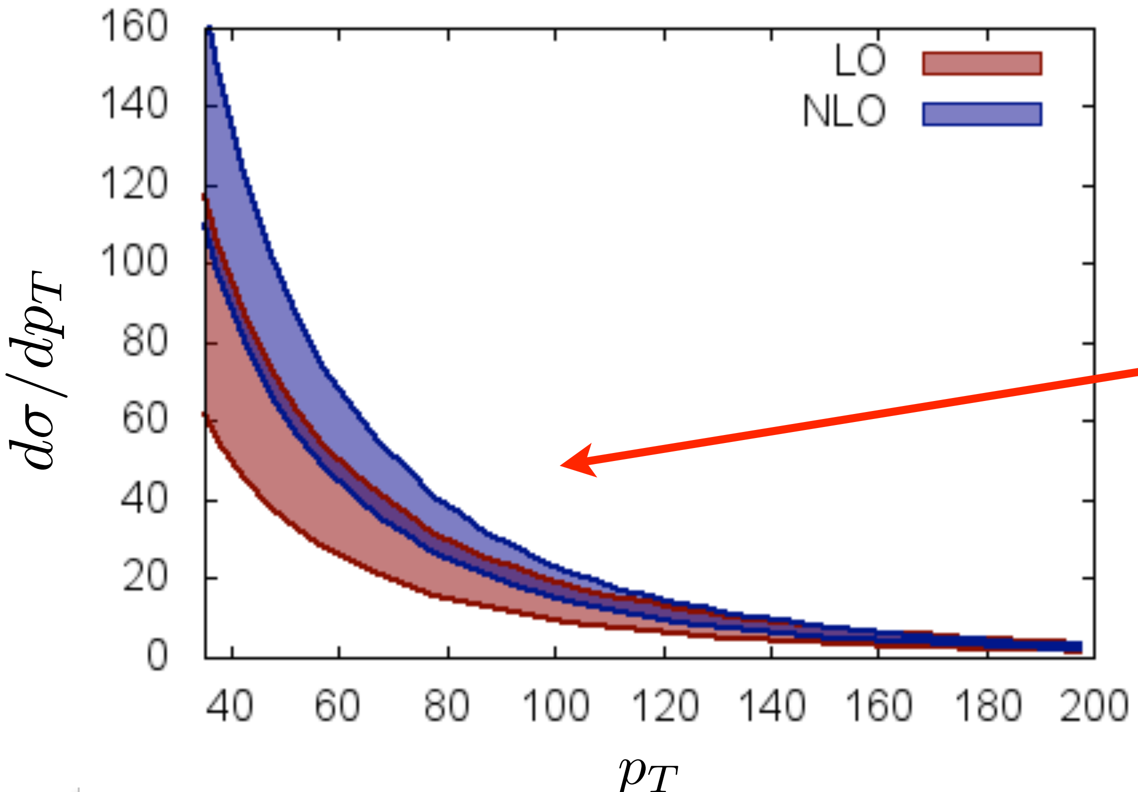
Uncertainty can be reduced by improving f.o. H+jets predictions

Higgs plus jet: need for improvement

The H+1 jet bin: large NLO K-factor and large theoretical uncertainty

Source (1-jet)	Signal (%)	Bkg. (%)
1-jet incl. ggF signal ren./fact. scale	27	0
2-jet incl. ggF signal ren./fact. scale	15	0
Missing transverse momentum	8	3
W+jets fake factor	0	7
b-tagging efficiency	0	7
Parton distribution functions	7	1

ATLAS



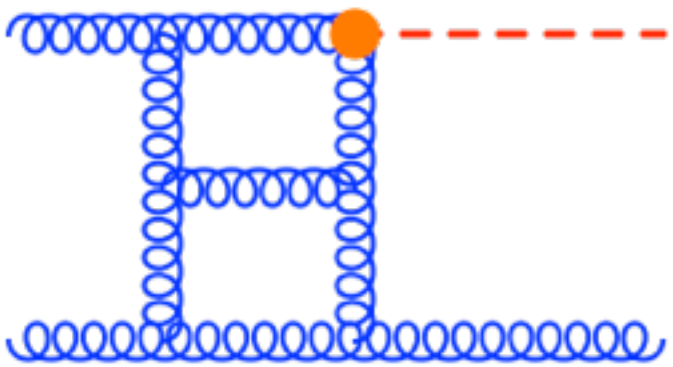
Need for higher orders!

Need NNLO for H+Jet(s) to Fix These Issues

Higgs plus 1 jet at NNLO

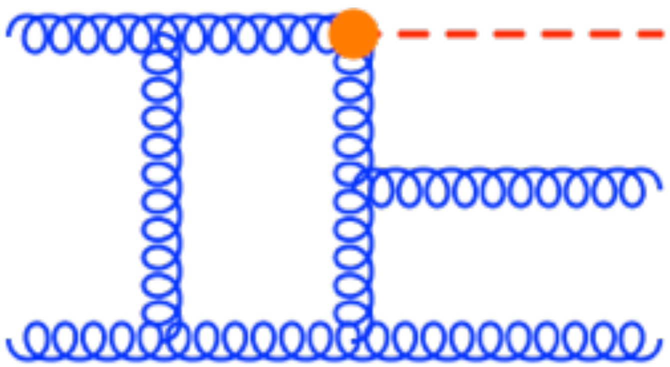
Anatomy of a NNLO computation

VV



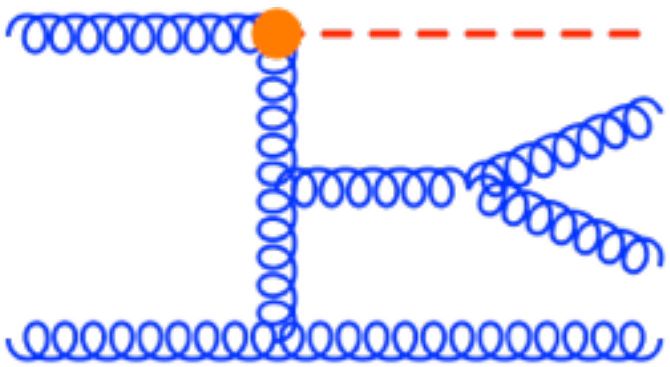
[Gehrmann et al. (2011)]

RV



[Badger et al. (2011)]

RR



[Del Duca et al., Dixon et al. (2004)]
[Badger]

Individual ingredients known for a while.

What prevented from doing the computation?

A (generic) procedure to extract IR poles from RV and RR was unknown until very recently

What about existing NNLO results?

Until very recently, all NNLO computations relied on
SPECIFIC PROPERTIES OF THE PROCESS UNDER CONSIDERATION

- Sector decomposition: simple enough phase space
  Higgs, Drell-Yan, dijets in e^+e^- [Anastasiou, Melnikov, Petriello; Melnikov, Petriello]
- e^+e^- antenna subtraction: no partons in the initial state
  dijets and trijets in e^+e^- [Gehrmann-De Ridder, Gehrmann, Glover et al.]
- qT resummation: no colored particles in the final state
  Higgs, Drell-Yan, dibosons and WH [Catani, Cieri, De Florian, Ferrera, Grazzini]

NONE OF THESE METHODS WOULD WORK
FOR HIGGS PLUS 1 JET

A successful strategy for simpler processes: Sector decomposition

[Binoth, Heinrich; Anastasiou, Melnikov, Petriello (2004)]

Basic idea: clever parametrization of the PS which makes IR SINGULARITIES MANIFEST:

$$\int |M|^2 d\Phi \rightarrow$$

$$\int [|M|^2 x]\{dy\} \frac{dx}{x^{1+\epsilon}} = -\frac{1}{\epsilon} F(0) + \int dx \frac{F(x) - F(0)}{x} + \ldots$$

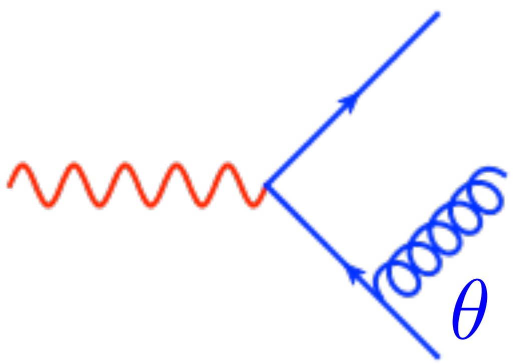
$$F(x) = \int [|M|^2 x]\{dy\}$$

Remap singular denominators on the hypercube

Singularities are extracted before integration

A toy example: simple parametrization

NLO: 1 sector

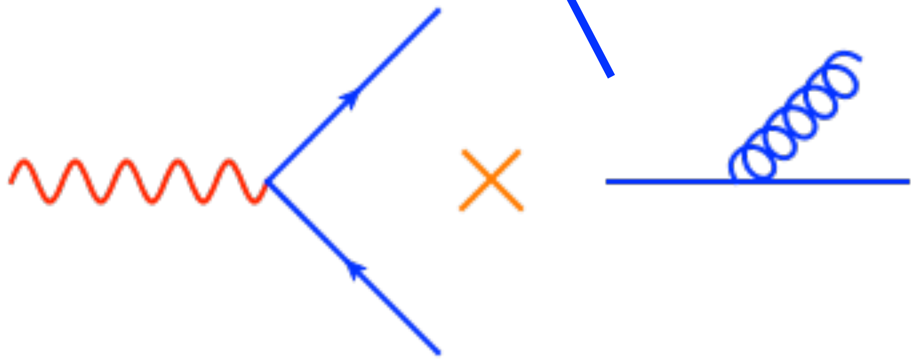


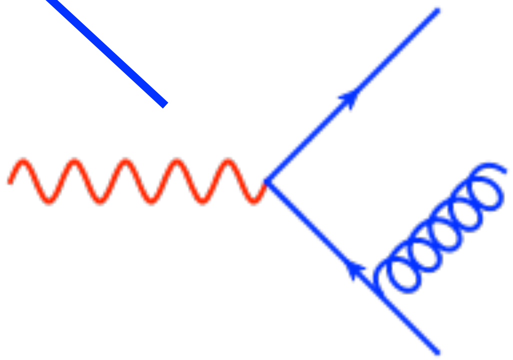
$$\frac{d^{d-1}g}{(2\pi)^{d-1}2E_g} \sim \left(1-\cos^2\theta\right)^{-\epsilon} d\cos\theta$$

$$|M|^2 \sim \frac{1}{1-\cos\theta} \longrightarrow \quad \cos\theta \to 1-2x$$

$$\int |M|^2 d\Phi \sim \int \frac{dx}{x^{1+\epsilon}} F(x,\{y\})\{dy\}$$

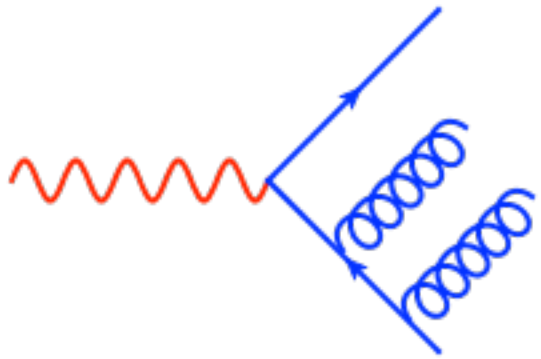
$$= -\frac{1}{\epsilon}\int F(0,\{y\})\{dy\} + \int \frac{F(x,\{y\})-F(0,\{y\})}{x} dx\{dy\} + \ldots$$





A toy example: sector decomposition

NNLO: overlapping divergences $\longrightarrow$ sector decomposition



$$|M|^2 \sim \frac{1}{s_{ijk}} = \frac{1}{s_{ij} + s_{ik} + s_{jk}}$$

$$\int |M|^2 d\Phi \sim \int \frac{dx_1 dx_2}{x_1^{1+\epsilon} x_2^{1+\epsilon} (x_1 + x_2)^\epsilon} F(\vec{x}; \{y\}) \{dy\}$$

- **Sector I:** $x_1 > x_2 \to x_2 = z x_1$

$$\int |M|^2 d\Phi \sim \int \frac{dx_1 dz}{x_1^{1+3\epsilon} z^{1+\epsilon} (1 + z)^\epsilon} F(\vec{x}; \{y\}) \{dy\}$$

- **Sector II:** $x_1 < x_2 \to x_1 = t x_2$

$$\int |M|^2 d\Phi \sim \int \frac{dt dx_2}{t^{1+\epsilon} x_2^{1+3\epsilon} (1 + t)^\epsilon} F(\vec{x}; \{y\}) \{dy\}$$

Sector decomposition: pro et contra

$$\int |M|^2 d\phi = \frac{F(0)}{\epsilon} + \int dx \frac{F(x) - F(0)}{x} + \ldots$$

Subtraction and integrated subtraction terms are for free

(no need for analytic PS integrations)

Powerful tool for fully differential NNLO computations:

- dijet production at LEP [Anastasiou, Melnikov, Petriello (2004)]
- Higgs production at hadron colliders [Anastasiou, Melnikov, Petriello (2005)]
- DY production at hadron colliders [Melnikov, Petriello (2006)]

BUT

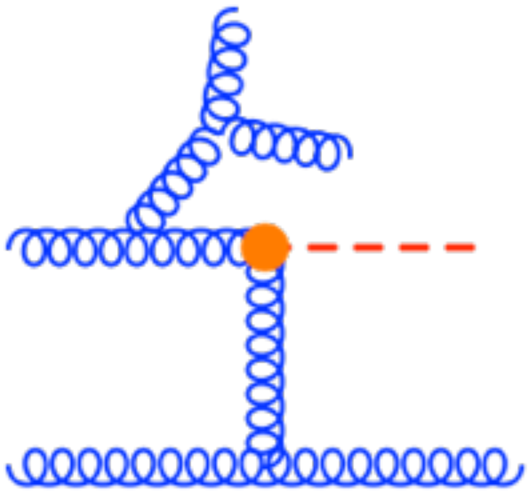
Parametrization become challenging for more complicated processes

Parametrization known only for ONE COLLINEAR DIRECTION

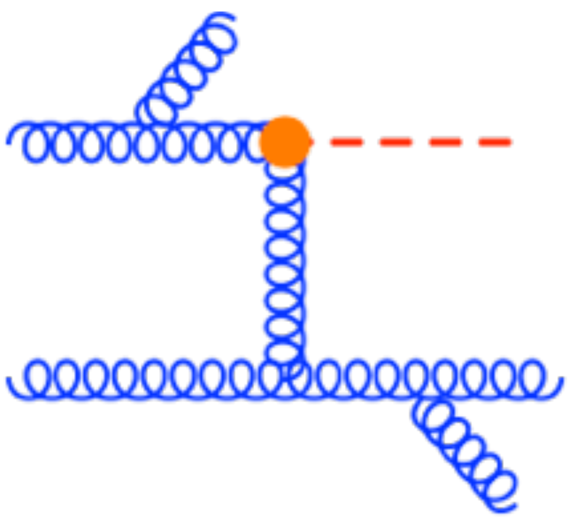
As it is, highly process-dependent framework

Higgs plus jet: singularity structure

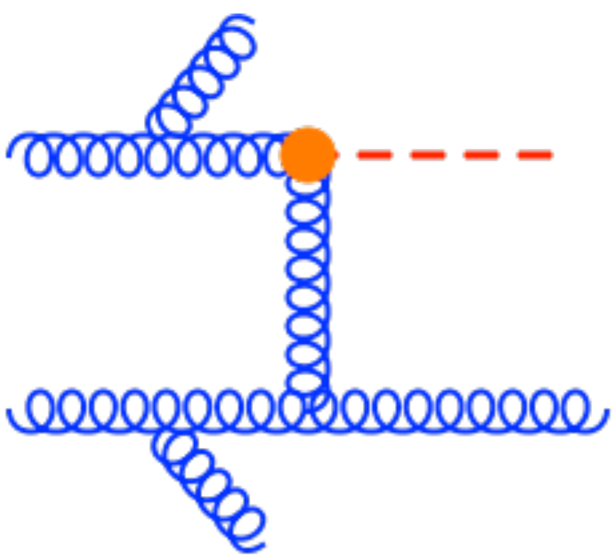
Much more complicated singularity structure. Collinear:



$$\sim \frac{P_{ggg} \otimes |M_j|^2}{s_{igg}}, \quad \frac{P_{gg} \otimes |M_{jj}|^2}{s_{gg}}$$ x3



x2,



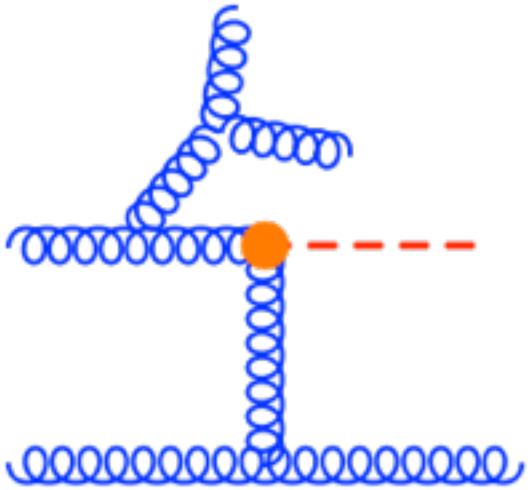
$$\sim \frac{P_{gg} P_{gg} \otimes |M_j|^2}{s_{ig} s_{jg}}$$

Potential troubles: $s_{1g}, s_{2g}, s_{3g}, s_{gg}, s_{1gg}, s_{2gg}, s_{3gg}$ and combinations

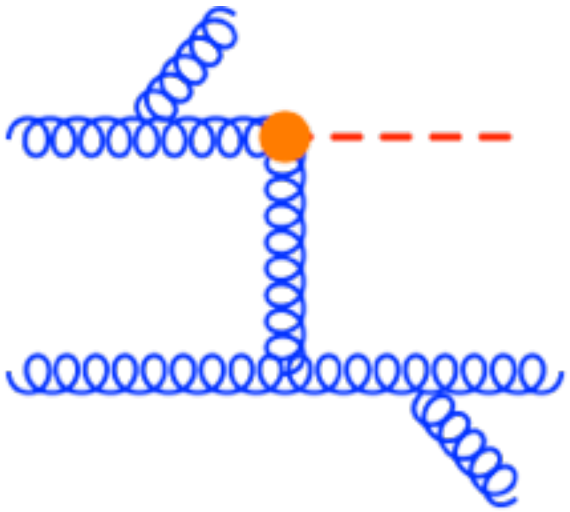
Finding a 'good' global parametrization is (very) hard

Sector-improved subtraction scheme

HOWEVER: collinear sing. cannot occur all together [Czakon (2010)]



Troubles:
s_{igg}, s_{gg} only



Troubles:
s_{ig}, s_{jg} only

Can we make use of it, i.e.
can we single out different collinear directions?

YES, just use the Frixione-Kunszt-Signer (FKS) partitioning
[Czakon (2010)]

$$1 = \sum \Delta^{g_1||i,g_2||j}$$

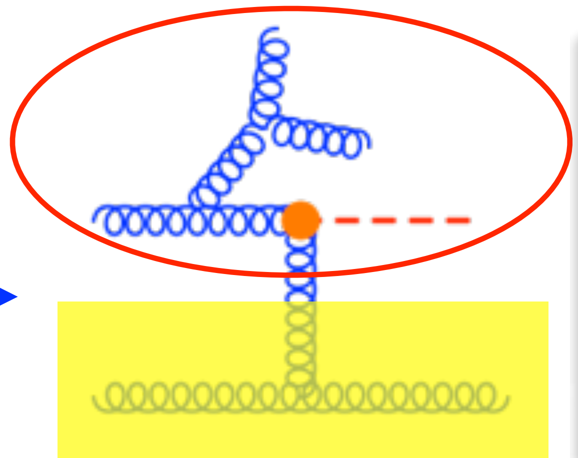
$$\Delta_s^{g_1||i,g_2||j} \to 0 \text{ when } g_1||p_l,\ g_2||p_m,\ l \neq i,\ m \neq j$$

Sector-improved subtraction scheme

Sector decomposition + FKS [Czakon (2010)]

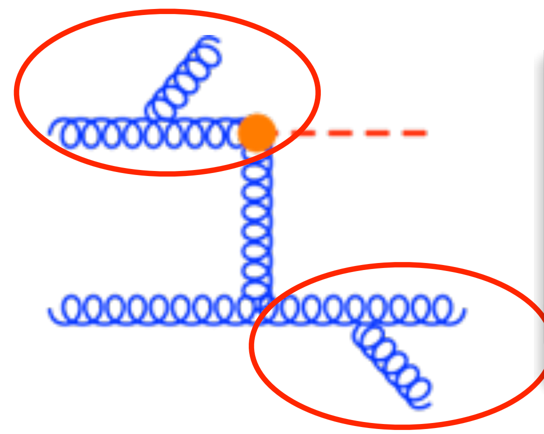
$$\int |M|^2 d\phi = \sum_s \int |M|^2 d\phi \Delta_s^{g_1||i, g_2||j}$$

$$\int |M|^2 d\phi \Delta^{g_1||1, g_2||1} \longrightarrow$$



Single collinear direction
~ parametrization of
ggH, DY, e^+e^- → dijets

$$\int |M|^2 d\phi \Delta^{g_1||1, g_2||3} \longrightarrow$$



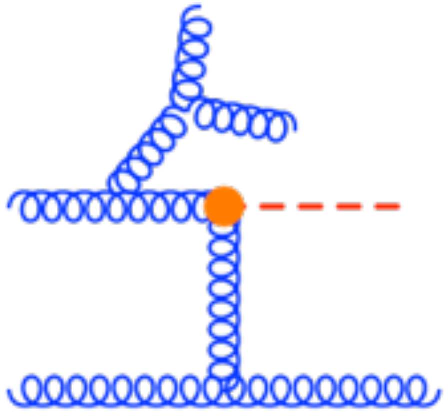
Two (~uncorrelated) dir.
~ NLO^2

<u>No matter how complicated the process is,</u>
it can be reduced to the sum of individual contributions. For each of them, we know a sector decomposition-friendly PS parametrization

Sector-improved subtraction and H+j

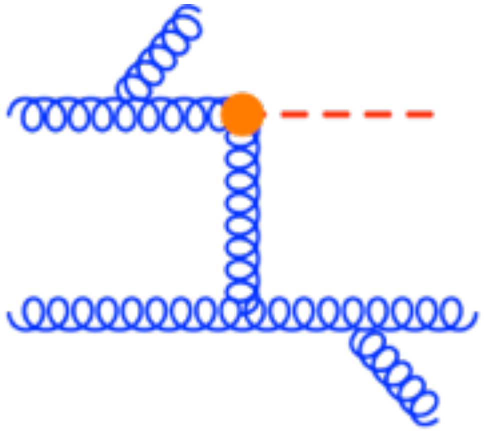
Worked-out details for RR: [Czakon (2010)]

(Although we use a slightly different parametrization and sector definition)



Three triple-collinear partitions

Each: 5 sectors



Six double-collinear (energy ordering)

No sector decomposition required

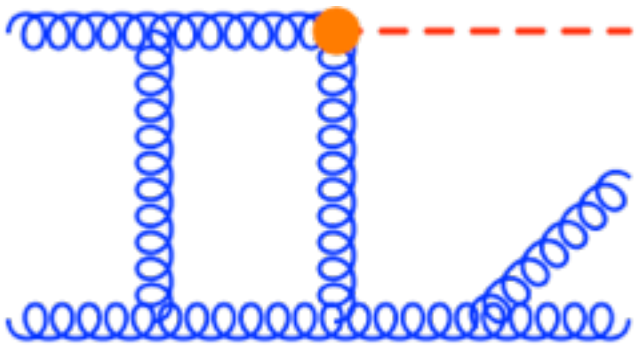
$$\mathrm{RR}_i = \int F_i(x_1, x_2, x_3, x_4, \{y\}) \prod \frac{dx_i}{x_i^{1+a_i\epsilon}} \{dy\} =$$

$$\int \{dy\} \left\{ \frac{F_i(\vec{0}, \{y\})}{a\epsilon^4} + \frac{1}{\epsilon^3} \left[\left(\frac{F_i(x_1, 0, 0, 0, \{y\}) - F_i(\vec{0}, \{y\})}{b x_1} \right) dx_1 + \ldots \right] + \ldots \right\}$$

Sector-improved subtraction and H+j

Worked-out details for RV: [Boughezal, Melnikov, Petriello (2011)]

(Although we need a slight generalization)



Three collinear partitions
(same of NLO)

Phase-space is simple (same of NLO), but amplitudes have non trivial branch-cuts

$$\mathrm{RV}_i = \int \{dy\} \frac{dx_1}{x_1^{1+2\epsilon}} \frac{dx_2}{x_2^{1+\epsilon}} \left(F_{i,1} + (x_1^2 x_2)^{-\epsilon} F_{i,2} + x_1^{-2\epsilon} F_{i,3}\right) =$$

$$= \int \{dy\} \left[\frac{A}{\epsilon^4} + \frac{B}{\epsilon^3} + \frac{C}{\epsilon^2} + \frac{D}{\epsilon} + E\right]$$

Sector-improved subtraction and H+j: building blocks

Recall the general structure: $F(x) = \int [|M|^2 x]\{dy\}$

$$\int |M|^2 d\phi = \frac{F(0)}{\epsilon} + \int dx \frac{F(x) - F(0)}{x} + \ldots$$

We need to provide

- $F(\vec{x};\{y\})$: fully-resolved matrix element (RR and RV)
- $\lim_{x_i \to 0} F(\vec{x};\{y\})$: matrix element in a singular configuration

↓

$\lim_{x_i \to 0} F(\vec{x};\{y\})$: reduced (=lower multiplicity) matrix element times universal eikonals / splitting functions

[Catani, Grazzini (1998, 2000); Kosower, Uwer (1999)]

At the end: ~ 170 different limits contribute

H+j: building blocks

Because of gluon spin correlations, we are forced to work in full CDR

Apart from eikonals/splitting functions, we require

- tree-level H+3j [Del Duca et al., Dixon et al. (2004), Badger]
- tree-level H+2j [Badger et al. (2011)] up to $\mathcal{O}(\epsilon^2)$
- tree-level H+1j up to $\mathcal{O}(\epsilon)$
- one-loop H+2j [Badger et al. (2011)]
- one-loop H+1j up to $\mathcal{O}(\epsilon^2)$ (although see [Weinzierl (2011)])
- two-loop H+1j [Gehrmann et al. (2011)]
- renormalization, collinear subtractions

Amplitudes are evaluated near to singular configurations:
have to be very stable (and possibly fast) →
ANALYTIC RESULTS, SPINOR-HELICITY FORMALISM

EXTREMELY GRATEFUL TO MCFM FOR PROVIDING EXCELLENT AMPLITUDES ALREADY AS A FORTRAN CODE!

H+j: spinor-helicity in higher dimension

Because of gluon spin correlations, we are forced to work in full CDR

To get $\mathcal{O}(\epsilon^2)$ tree- and loop-level amplitudes:
Dimensional reconstruction: $\mathcal{O}(\epsilon)$ and $\mathcal{O}(\epsilon^2)$ from spinor-helicity in higher dimensions

Scalar-like gluons with polarization vectors pointing in the D=5,6 subspaces

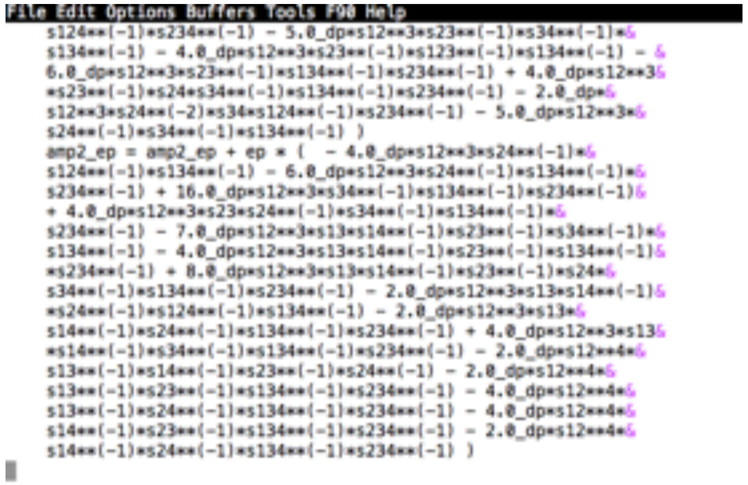
Similar to what is done for 1-loop in D-dimensional unitarity

- although slightly more tricky if quarks are around [$\bar{u}\gamma^\mu\hat{p}_1...\hat{p}_n\gamma^\mu v$ (1-loop) vs $\bar{u}\gamma^\mu\hat{p}_1...\hat{p}_k v$ (here)]
- and analytic-friendly

WE GET COMPACT AND STABLE RESULTS ALSO FOR FULL AMPLITUDES IN D-DIMENSIONS

$H + 4g: \sum_{pol} |M|^2$ from traditional methods:

...



Lots of $\sim 1/s_{ij}^2$ spurious singularities

$\mathcal{O}(\epsilon)$: 5890 lines

-UU-:----F1 out_gggg_me2colsum.f90 39% L5890 (F90)

Typical amplitudes from spinor-helicity in higher dimension

$$A(1^s, 2^+, 3^s, 4^+) = -\frac{\langle 1|p_h|4]\langle 3|p_h|4]}{s_{123}\langle 12\rangle\langle 23\rangle} + \frac{\langle 1|p_h|2]\langle 3|p_h|2]}{s_{134}\langle 14\rangle\langle 34\rangle} + \frac{m_h^2\langle 13\rangle^2}{\langle 12\rangle\langle 14\rangle\langle 23\rangle\langle 34\rangle},$$

$$A(1_{\bar{q}}^-, 2_q^+, 3^s, 4^{s'}) = \frac{\langle 23\rangle}{2\langle 13\rangle}\left(1 - \frac{m_h^2}{s_{123}}\right) - \frac{[41]}{2[42]}\left(1 - \frac{m_h^2}{s_{124}}\right)$$

Higgs plus 1 jet at NNLO: results (gg only)

Checks: generic

Two entirely independent computations (JHU/ANL-Northwestern)

Phase space parametrization and partitioning

- correct D-dimensional PS volume in each partition
- rotational invariance in D-dimensions (spin-correlations)

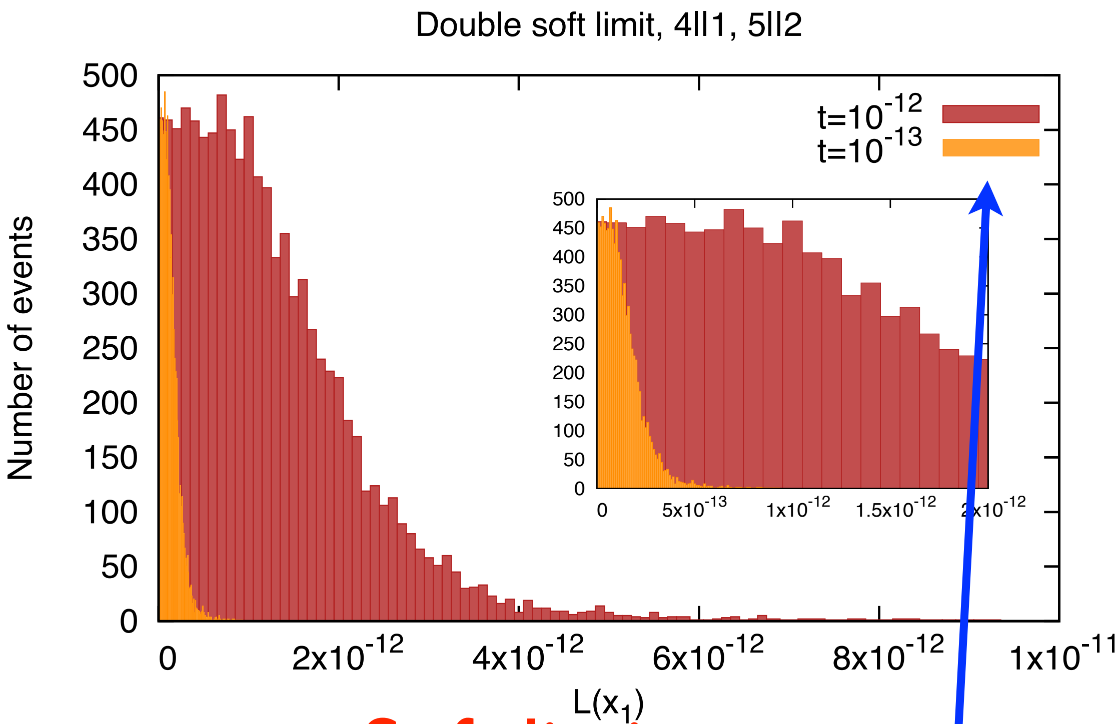
Amplitudes

- tree-level amplitudes tested against MadGraph
- loop-amplitudes implementation checked against original MCFM
- singular limits (see below)
- D-dimensional helicity amplitudes checked against brute-force computation for $\sum_{pol} |M|^2$

Checks: limits and scaling

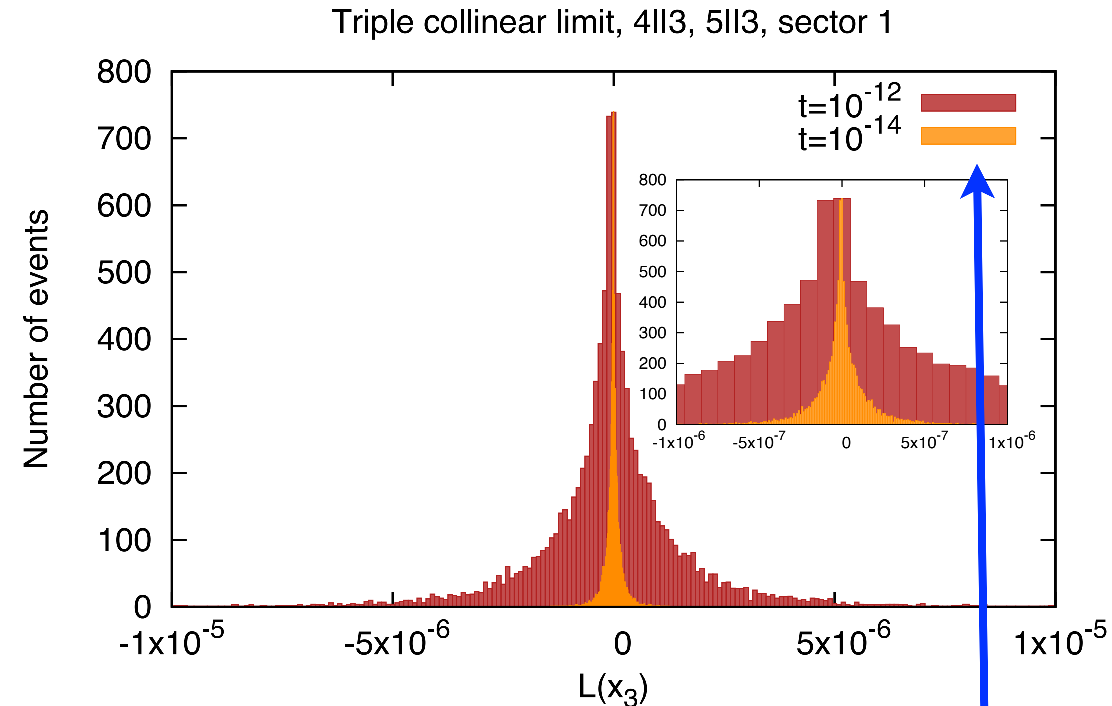
Subtraction terms should match the full amplitude in singular limits

Non-trivial since subtraction terms computed from
reduced matrix element and eikonals/splitting functions



Soft limits:

$$\lim_{x_1 \to 0} 1 - F(x_1)/F(0) \sim x_1$$



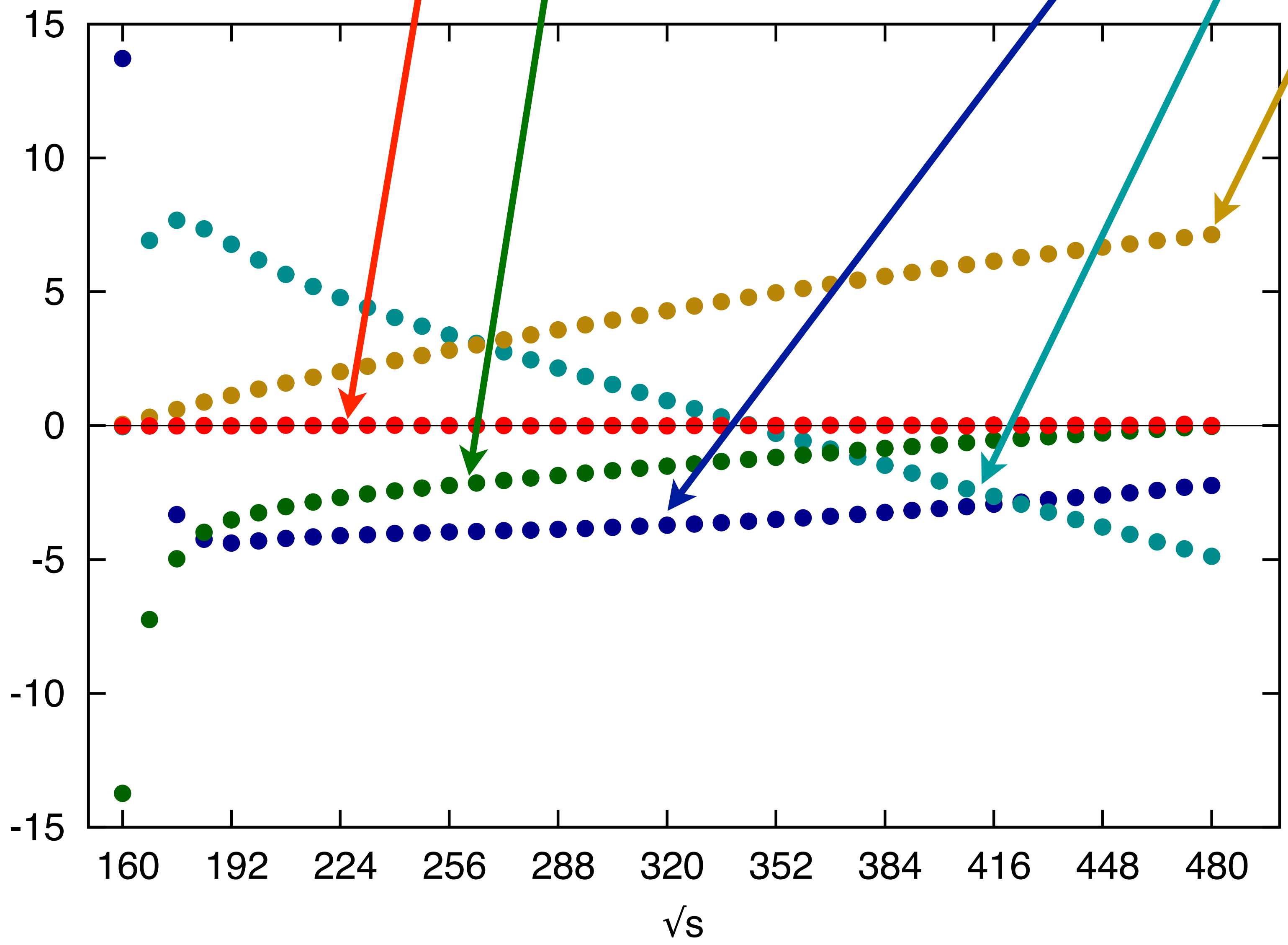
Collinear limits:

$$\lim_{x_2 \to 0} 1 - F(x_2)/F(0) \sim \sqrt{x_2}$$

Correct scaling is the ultimate test for limits

Checks: poles cancellation

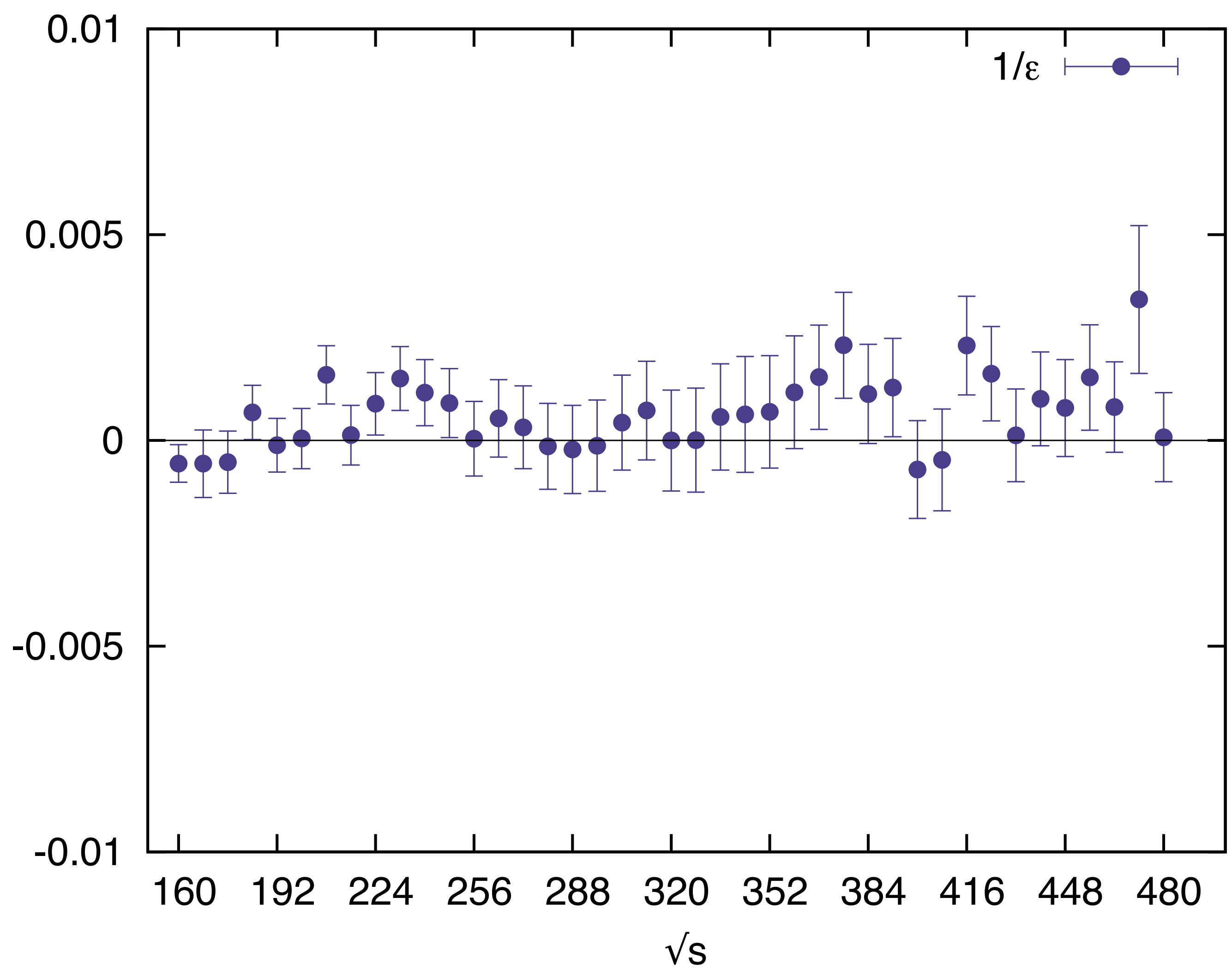
NUMERICAL CANCELLATION between renormalization and coll. couterterms, RR, RV, VV



$1/\epsilon$ poles, summing individual contributions

Checks: poles cancellation

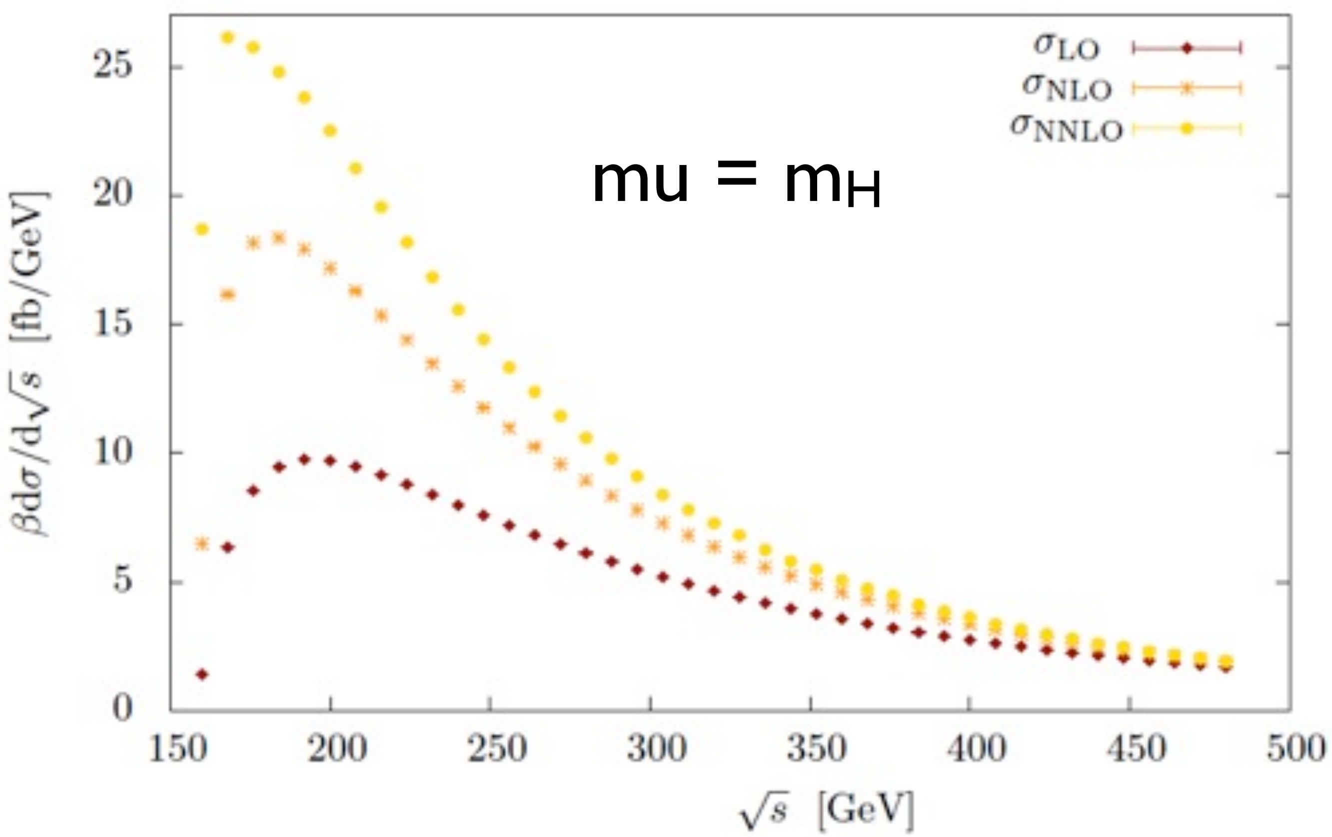
NUMERICAL CANCELLATION between
renormalization and coll. couterterms, RR, RV, VV

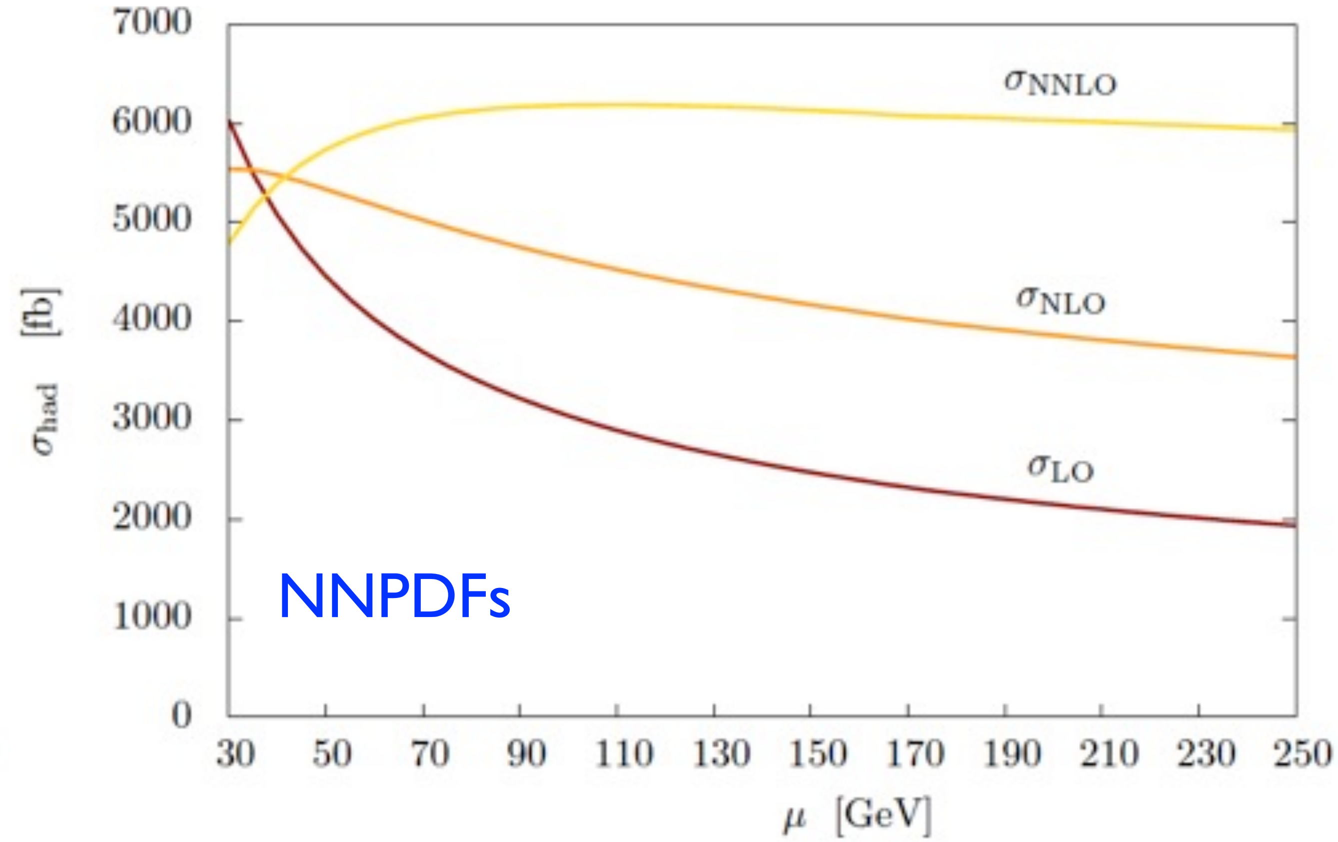


$1/\epsilon$ poles, degree of cancellation $\sim 10^{-3}$

$(1/\epsilon^2 : \ \sim 10^{-4})$

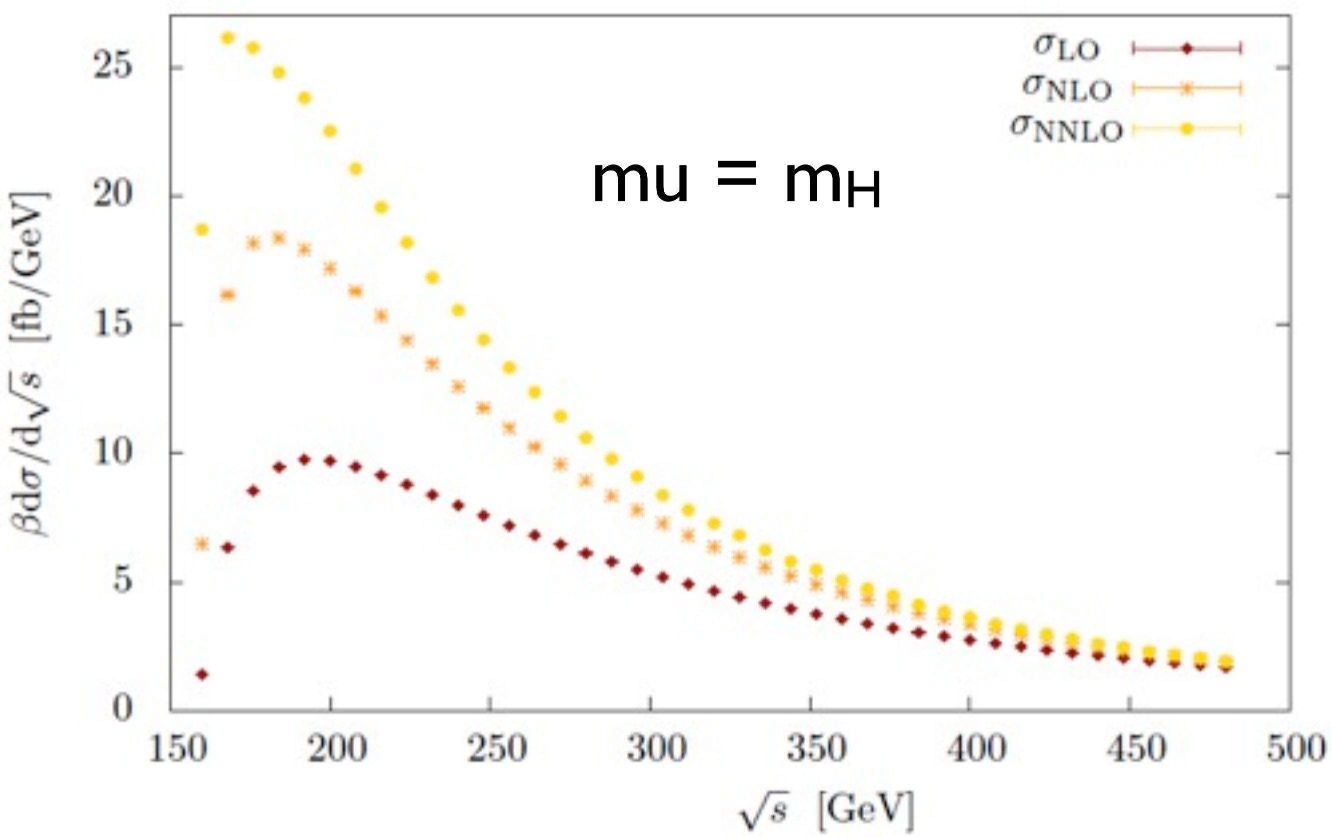
H+j @ NNLO (gg only)

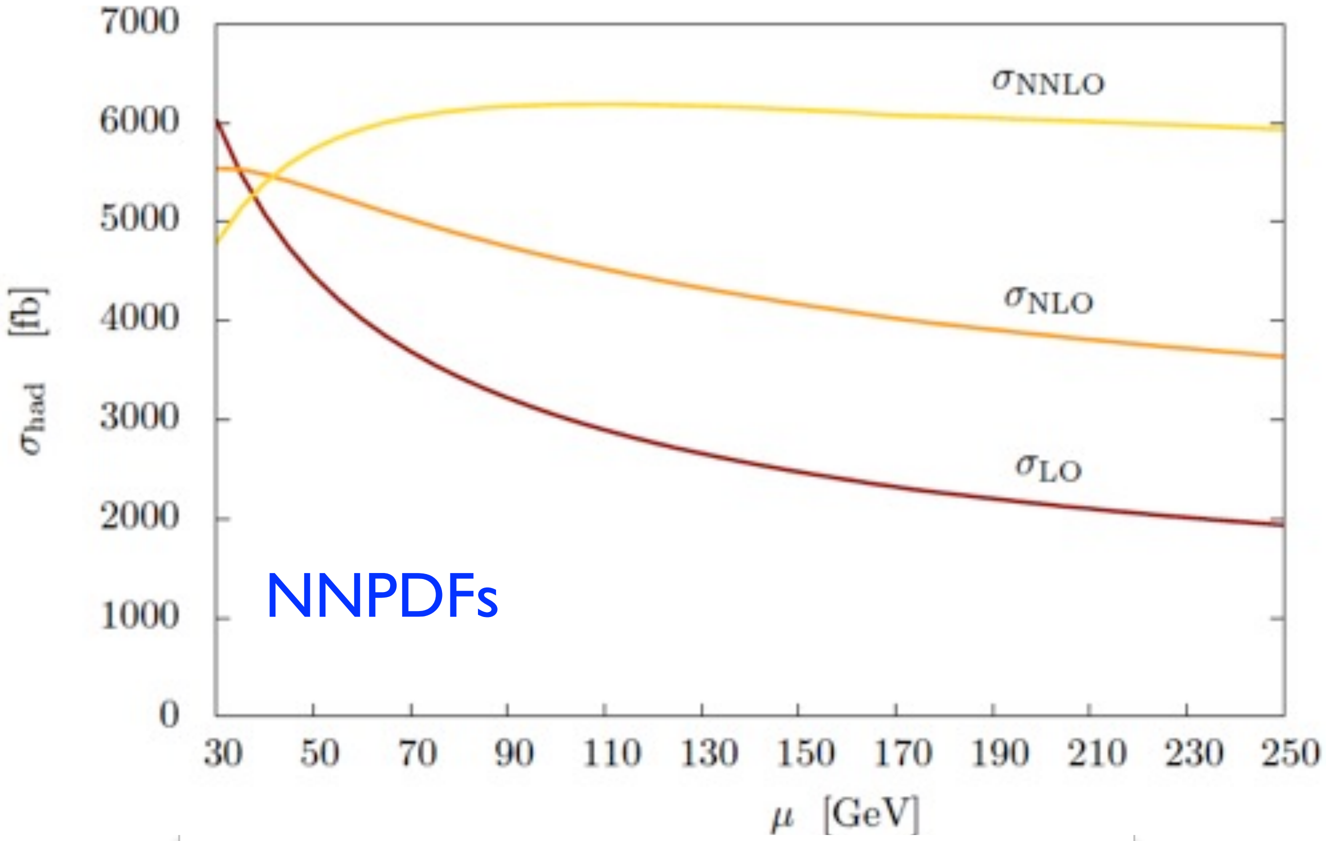




- Partonic cross section for gg → Hj @ LO, NLO, NNLO
- Realistic jet algorithm, k_T with R=0.5, p_T > 30 GeV
- Hadronic cross-section pp → Hj using latest NNPDF sets
- Scale variation in the range $m_H/2 < \mu < 2\, m_H$, m_H = 125 GeV

H+j @ NNLO (gg only)





$$\sigma_{\rm LO}(pp \to Hj) = 2713^{+1216}_{-776}\ {\rm fb},$$
$$\sigma_{\rm NLO}(pp \to Hj) = 4377^{+760}_{-738}\ {\rm fb},$$
$$\sigma_{\rm NNLO}(pp \to Hj) = 6177^{-204}_{+242}\ {\rm fb}.$$

Large K-factors

$\sigma_{NLO}/\sigma_{LO} = 1.6$

$\sigma_{NNLO}/\sigma_{NLO} = 1.3$

Significantly reduced O(4%) scale dependence

Conclusions

- We presented results for H+1j @ NNLO (gg only)
- Result urgently needed to reduce theoretical uncertainties in jet-bin based analyses
- gg channel: ~ 70% of the full result (NLO), and corrections to other channels expected to be smaller (color charges)
- Result already useful for preliminary phenomenological studies
- Large O(30%) NNLO/NLO K-factor
- Improved scale variation: 30% (NLO) → 4% (NNLO)
- PDFs uncertainty: 1-2%

Conclusions

- One of the first NNLO QCD results for $2 \to 2$ processes whose existence depends on a jet algorithm [dijet: Gehrmann et al.]
- Prototype of a generic NNLO computation
  - most generic singularity structure (initial, final and mixed collinear singularities)
  - large number of diagrams, but compact results with spinor-helicity techniques
  - maximal presence of spin correlations
- Robust test of the theoretical framework
  - [Czakon (2010)], [Boughezal, Melnikov, Petriello (2011)]
  - Very similar to the framework used for the computation of the NNLO ttbar cross section [Czakon, Mitov et al.]

Outlook

- Include quark-gluon channel, for reliable phenomenology
  - reliable results in the 1-jet bin
  - more precise description of jet-vetoed cross section
  - genuine NNLO analysis of the Higgs p_T spectrum
- Include Higgs decays (trivial)
- Compute differential distributions
  - already done within this framework for top decay and charmless b-decay [Brucherseifer, FC, Melnikov (2013)]
- Run with ATLAS/CMS setup
- Technical improvements
  - implement α-parameters
  - develop a D=4 framework

Thank you for your attention!