On regularizing the infrared singularities in QCD NLO splitting functions with the new Principal Value prescription

O. Gituliar, S. Jadach, A. Kusina, M. Skrzypek

IFJ PAN, Kraków, Poland

Partly supported by grants:
DEC-2011/03/B/ST2/02632, UMO-2012/04/M/ST2/00240,
PITN-GA-2010-264564 (LHCPhenoNet), DE-FG02-13ER41996

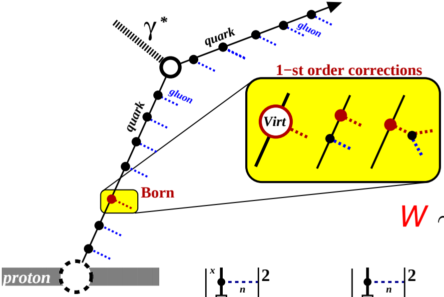
LHCPhenoNet Annual Meeting
CERN, 2-5 Dec. 2013

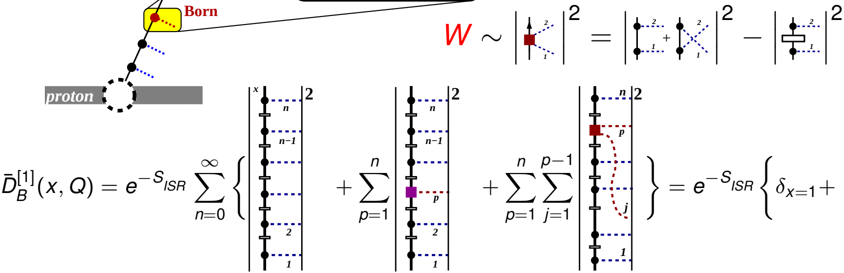
Plan

- Introduction
- New Principal Value prescription
- Results for NLO splitting functions
- Summary

NLO-corrected middle-of-the-ladder kernel, C_F^2

from S. Jadach talk

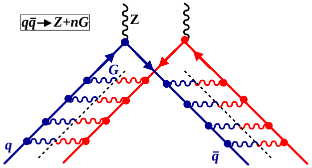




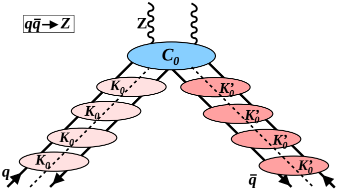
$$+\sum_{n=1}^{\infty}\left(\prod_{i=1}^{n}\int_{Q>a_i>a_{i-1}} d^3\eta_i\, \rho_{1B}^{(1)}(k_i)\right)\left[1+\sum_{p=1}^{n}\beta_0^{(1)}(z_p)+\sum_{p=1}^{n}\sum_{j=1}^{p-1} W(\tilde{k}_p,\tilde{k}_j)\right]\delta_{x=\prod_{j=1}^{n} x_j}\Big\}.$$

Construction of the evolution kernel
in collinear factorization

LO cascade and
construction of the ladder

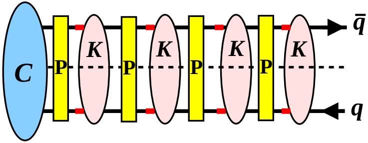


Include NLO and
group graphs in the ladder into
"two-particle-ireducible" sets K



Use "projection operators" P to
split the ladder and extract kernels

$$\Gamma_{qq} = \mathrm{Tr}\left[\frac{\hat{n}}{4nq} K \hat{p}\right]$$



PV prescription

☺ Axial gauge = physical interpretation as parton shower
☹ Axial gauge = spurious (unphysical) singularities

$$\text{gluon propagator:} \quad \frac{1}{l^2}\left(g^{\mu\nu} - \frac{l^\mu n^\nu + n^\mu l^\nu}{nl}\right)$$

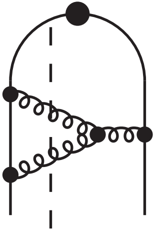
Spurious singularities cancel in full set of diags, but need regularization
Curci, Furmanski, Petronzio [80] Ellis, Vogelsang [96], Heinrich, Kunszt [97]:

$$\text{Principal Value:} \quad \left[\frac{1}{nl}\right]_{PV} = \frac{nl}{(nl)^2 + \delta^2 (pl)^2}$$

PV is more like "phenomenological rule"

Rigorous prescription: Mandelstam [83], Leibbrandt [84].
Difficult in calculations: Bassetto, Heinrich, Kunszt, Vogelsang [97].

Problem with real emission graph



Standard Heinrich, Kunszt [1998]:

$$N(\epsilon, Q^2)\Bigg[\frac{P_{qq}(x)}{\epsilon^3} - 2I_0\frac{P_{qq}(x)}{\epsilon^2} + \frac{p_{qq}(x)}{\epsilon}\Big(-2I_1 + 4I_0 + 2I_0\ln x - 2I_0\ln(1-x)\Big)\Bigg] + \mathcal{O}\Big(\frac{1}{\epsilon}\Big)$$

Not good for Parton Shower, needs Real-Virt. cancel.

Parton Shower oriented Jadach, et.al. [2011]:

$$\frac{p_{qq}(x)}{\epsilon}\Big(+2I_1 + 4I_0 + 2I_0\ln x - 2I_0\ln(1-x)\Big) + \mathcal{O}\Big(\frac{1}{\epsilon}\Big)$$

Good for PS, δ is a cut-off in 4-dimensions, easy to generate in MC:

$$I_0 = \int_0^1 \frac{dx}{[x]_{PV}} \sim \int_\delta^1 \frac{dx}{x} = -\ln\delta, \qquad I_1 = \int_0^1 \frac{dx\ln x}{[x]_{PV}} \sim -\frac{1}{2}\ln^2\delta,$$

$$P_{qq}(x) = p_{qq} + \epsilon(1-x), \qquad p_{qq} = \frac{1+x^2}{1-x}$$

New use of PV prescription

Standard: regularize with PV only the gluon propagator
leave other singularities in (+)-component of integration momenta

$$\frac{d^m l}{l_+^{1-\epsilon}}, \qquad l_+ = \frac{nl}{np}$$

New proposal: regularize with PV all singularities of the integrand
in (+)-component of integration momenta, real & virtual

$$\frac{d^m l}{l_+^{1-\epsilon}} \to d^m l \left[\frac{1}{l_+}\right]_{PV} \Big(1 + \epsilon \ln l_+ + \epsilon^2 \frac{1}{2} \ln^2 l_+ + \dots\Big)$$

All (+)-singularities cancel in the final expression (kernel), so extension of "phenomenological PV rule" of Curci-Furmanski-Petronzio possible

Example: virtual three point integral

Must perform (+)-integral as the last one, Ellis, Vogelsang [1996]:

$$\int \frac{d^m l}{(2\pi)^m} \; \frac{f(l_+)}{l^2(l-q)^2(l-p)^2} =$$

$$= \frac{-i}{16\pi^2 q^2}\left(\frac{4\pi}{-q^2}\right)^{-\epsilon} \frac{\Gamma(1-\epsilon)}{-\epsilon}\left[\int_0^x dy f(l_+) \frac{z^\epsilon(1-z)^\epsilon}{1-y}\left(1+2\epsilon \ln\frac{1-y}{1-z}\right)\right.$$

$$\left. + 2\frac{\Gamma^2(1+\epsilon)}{\Gamma(1+2\epsilon)}(1-x)^{-\epsilon}\int_x^1 dy f(l_+)(1-y)^{-1+2\epsilon}\right],$$

$$x = q_+/p_+, \quad y = l_+/p_+, \quad z = y/x, \quad p^2 = (p-q)^2 = 0, \quad m = 4+2\epsilon,$$

Singularities at $y = 0$ and $y = x$: only from gluon propagator.

Singularity at $y = 1$: not from gluon propagator!

Proposal: treat all (+)-singularities on equal footing

Note: (+)-singularities lead to $1/\epsilon^3$ poles in kernel

Example: scalar non-axial integral

kinematics: $p^2 = (p-q)^2 = 0$

$$J_3^{\mathrm{F}} = \int \frac{d^m l}{(2\pi)^m} \frac{1}{l^2 (q-l)^2 (p-l)^2}$$

The PV prescription:

$$J_3^{\mathrm{F}} = C\left(-\frac{1}{\epsilon^2} + \frac{\pi^2}{6}\right), \qquad C = i\frac{\Gamma(1-\epsilon)}{(4\pi)^2 |q^2|}\left(\frac{4\pi}{|q^2|}\right)^{-\epsilon}$$

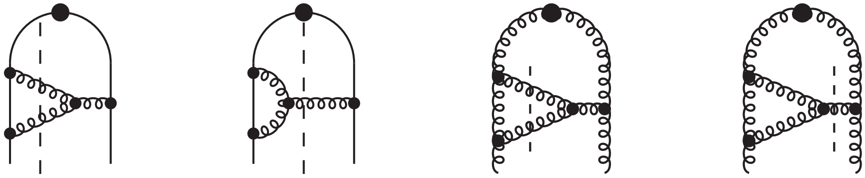
New PV prescription:

$$J_3^{\mathrm{F}} = C\left(-\frac{2I_0 + \ln(1-x)}{\epsilon} - 4I_1 + 2I_0 \ln(1-x) + \frac{\ln^2(1-x)}{2}\right),$$

$\frac{1}{\epsilon^2}$ replaced by I_1 and $\frac{1}{\epsilon} I_0$

NLO kernels P_{qq} and P_{gg} in New PV scheme

There are only four graphs with $1/\epsilon^3$ singularity:

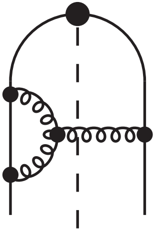


$\tilde{\Gamma}^{(d_R)}_{qq}(x,\epsilon)$ $\quad$ $\tilde{\Gamma}^{(d_V)}_{qq}(x,\epsilon)$ $\quad$ $\tilde{\Gamma}^{(d_R)}_{gg}(x,\epsilon)$ $\quad$ $\tilde{\Gamma}^{(d_V)}_{gg}(x,\epsilon)$

related to P_{qq} and P_{gg} splitting functions in a standard way:

$$\tilde{\Gamma}_{qq(gg)}(x,\epsilon) = \delta_{1-x} + \frac{1}{\epsilon}\left(\frac{\alpha_S}{2\pi}P^{LO}_{qq(gg)}(x) + \frac{1}{2}\left(\frac{\alpha_S}{2\pi}\right)^2 P^{NLO}_{qq(gg)}(x) + \dots\right) + \mathcal{O}\left(\frac{1}{\epsilon^2}\right)$$

Virtual contribution to NLO P_{qq} kernel in NPV



The virtual graph contributes:

$$\tilde{\Gamma}_{qq}^{(d_V)}(x,\epsilon) = -\frac{1}{\epsilon^2} P_{qq}\big(1+\epsilon\ln(1-x)\big)\tilde{Z}_{d_V}$$

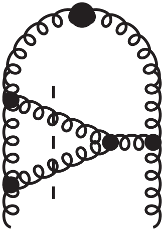
$$-\frac{1}{\epsilon} p_{qq}\Big[l_0\big(2\ln x + 2\ln(1-x)\big) - 6 l_1 - \mathrm{Li}_2(1-x)$$

$$+\ln^2 x - 3 + \frac{8}{12}\pi^2\Big] + \frac{1}{\epsilon}\frac{1}{2}\frac{1+x}{1-x},$$

$$\tilde{Z}_{d_V} = 4 l_0 + 2\ln(1-x) + \ln x - \frac{3}{2},$$

Inclusive sum of Real and Virtual graphs $\tilde{\Gamma}_{qq}^{(d)}$ identical as in standard PV scheme

Real contribution to NLO P_{gg} kernel in NPV



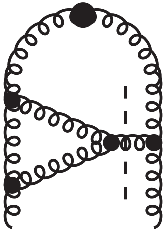
Only ϵ^{-1} poles
→ the calculation can be done in 4-dimensions,
→ much simpler than in standard PV

The real graph in New PV prescription:

$$\tilde{\Gamma}_{gg}^{(d_R)}(x,\epsilon) = C_S^{(d_R)} \frac{1}{\epsilon} \Big[P_{gg} \Big(-4 I_1 + 4 I_0 \big(\ln(1-x) - \ln(x) - 2\big) + 2\ln^2(1-x) + 2\ln^2(x) - 4\ln(x)\ln(1-x) - 8\ln(1-x) + \frac{11}{3}\ln(x) + 2\frac{\pi^2}{6} + 4 \Big) + \ln(x)\Big(\frac{11}{3}x^2 + \frac{23}{6}x + \frac{23}{6} + \frac{11}{3x}\Big) - \frac{22}{3}x^2 + \frac{24}{3}x - \frac{25}{3} + \frac{22}{3x} \Big].$$

Virtual contribution to NLO P_{gg} kernel in NPV

The virtual graph in New PV prescription:



$$\Gamma^{(d_V)}_{gg,NPV}(x,\epsilon) = C_S^{(d_V)} P_{gg} \Bigg[\frac{1}{\epsilon^2}\big(1+\epsilon\ln(1-x)\big)\tilde{Z}^V_{GS} + \frac{1}{\epsilon}\Bigg(4 I_0 \ln(1-x) + 8 I_0 \ln(x) - 16 I_1 + 4\ln^2(x) + 12\frac{\pi^2}{6} - \frac{134}{9}\Bigg)\Bigg] - C_S^{(d_V)}\frac{1}{3\epsilon}x$$

$$\tilde{Z}^V_{GS} = 12 I_0 + 4\ln(1-x) + 4\ln(x) - \frac{22}{3},$$

Inclusive sum of Real and Virtual graphs $\tilde{\Gamma}^{(d)}_{gg}$
identical as in standard PV scheme

Both schemes, PV and New PV,
give the same P_{qq} and P_{gg} kernels

Summary

- We modified the way of using the PV prescription in the light-cone gauge by applying it to all the singularities in the plus component of the integration momentum.
- In the New PV prescription the NLO kernels P_{qq} and P_{gg} are reproduced correctly.
- Partial results of four graphs contributing to the kernels, differ in PV and New PV: the $1/\epsilon^3$ poles, present in PV, are replaced by $(1/\epsilon)\ln^2\delta$ etc.
- Real graphs, free of $1/\epsilon^3$ and $1/\epsilon^2$ poles, can be calculated in four dimensions and are usable for the stochastic simmulations.
- The higher order poles cancel separately for real and for virtual components.