THE HIGH ENERGY JETS FRAMEWORK

Tuomas Hapola

Institute for Particle Physics Phenomenology
Durham University

LHCPhenoNet Annual Meeting, December 2, 2013

OUTLINE

- Motivation
- Multi-Regge-Kinematics
- Construction of HEJ matrix elements
- Master Formula
- Comparison with data
- Future developments
- Conclusions

MOTIVATION

- Capture the relation between Δy and the amount of

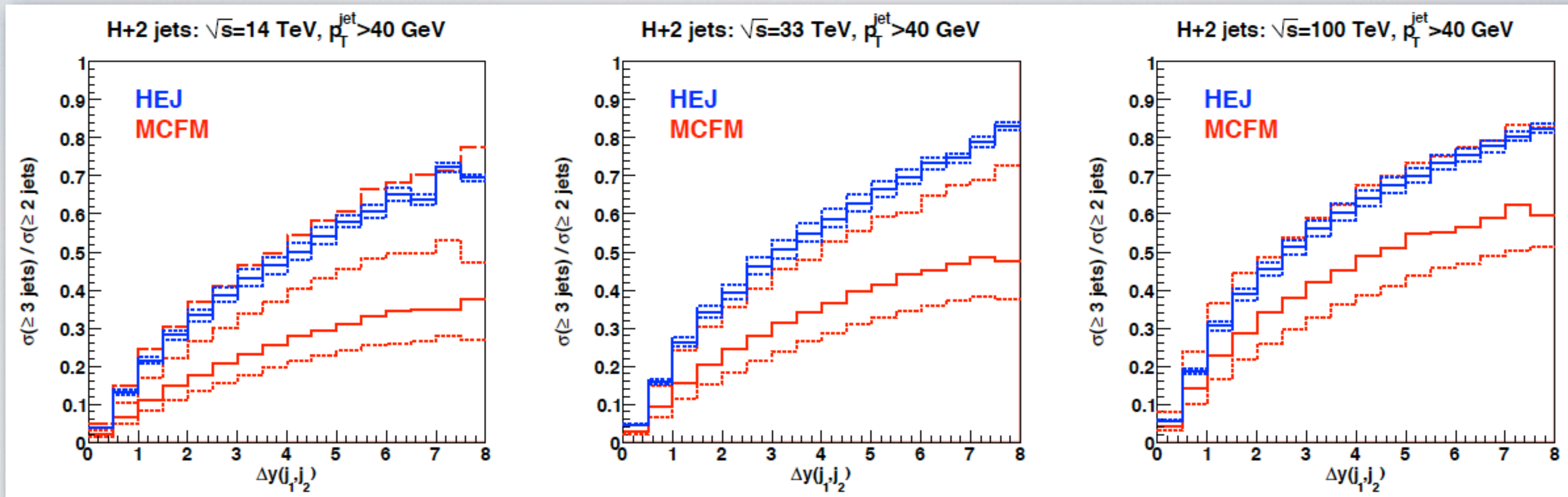
hard radiation

- Large rapidity separation between jets
    - ➡extra emissions
    - ➡Large logs

MOTIVATION

- Capture the relation between Δy and the amount of hard radiation
- Large rapidity separation between jets
  - ➡extra emissions
  - ➡Large logs $p_{h\perp}$ vs m_{jj}

MOTIVATION



arXiv:1310.5189

REGGE THEORY

- Regge limit: $s >> -t$, soft process
- S matrix properties
  - Unitarity
  - Analyticity
  - Crossing symmetry

$$\mathcal{A} \sim s^{\alpha(t)}$$

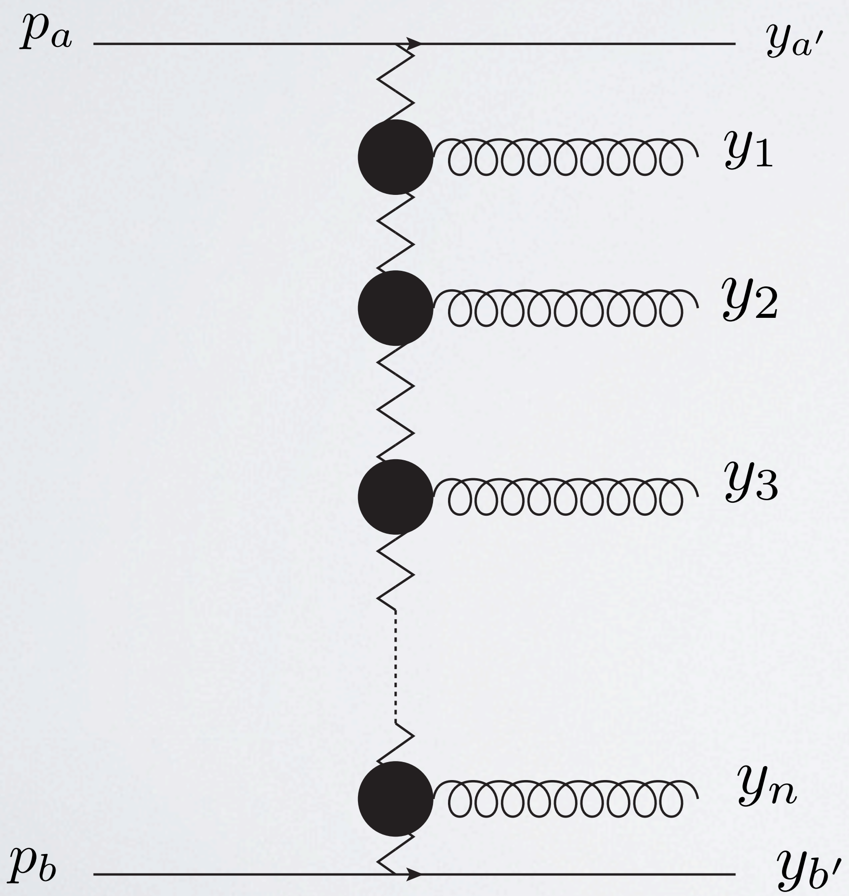
- Also gluons reggeizes

Sov.Phys.JETP 44 (1976) 443-450

MULTI REGGE KINEMATICS

- Leading Logarithmic approximation $\left[\alpha_s \ln\left(\frac{s}{|t|}\right)\right]^n$

$$y_{i-1} >> y_i >> y_{i+1}, \qquad |p_{\perp i}| \sim |p_{\perp j}|$$



$$\mathcal{A}_{2\to 2+n} \approx 2is(igf^{aa'c_1}g_{\mu_a\mu_{a'}})\frac{1}{t_1}\left(-\frac{s_{a',1}}{t_1}\right)^{\alpha(t_1)}$$

$$(igf^{c_1c_2a_1}V^{\mu_1}(q_1,q_2))\frac{1}{t_2}\left(-\frac{s_{1,2}}{t_2}\right)^{\alpha(t_2)}$$

$$\vdots$$

$$(igf^{c_nc_{n+1}a_n}V^{\mu_n}(q_n,q_{n+1}))\frac{1}{t_{n+1}}\left(-\frac{s_{n,n+1}}{t_{n+1}}\right)^{\alpha(t_{n+1})}$$

$$(igf^{a_ba_{b+1}c_{n+1}}g_{\mu_b\mu_b'})$$

HIGH ENERGY LIMIT

- Is this a good approximation?
- Squared matrix element:

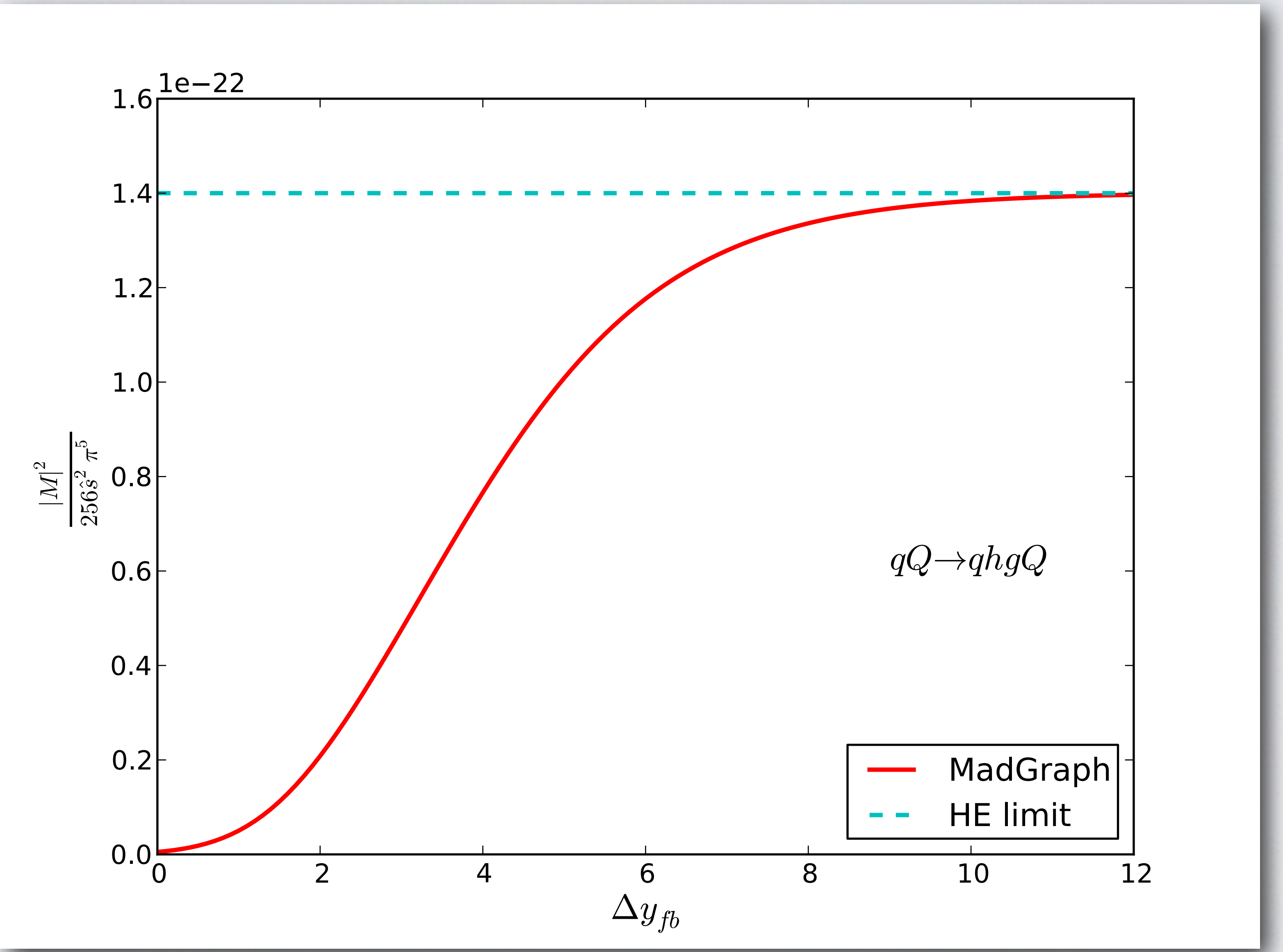
$$|\mathcal{M}^{qQ\to qhgQ}|^2 = \frac{4\hat{s}^2}{8}\frac{C_f g_s^2}{|p_{0\perp}|^4}|C^H(q_{a\perp}, q_{b\perp})|^2 \frac{4C_A g_s^2}{|p_{1\perp}|^4}\frac{C_f g_s^2}{|p_{2\perp}|^4}$$

$$C^H = i\frac{\alpha_s}{6\pi v}\left(|p_{h\perp}|^2 - |q_{a\perp}|^2 - |q_{b\perp}|^2\right)$$

arXiv:0808.3696

HIGH ENERGY LIMIT

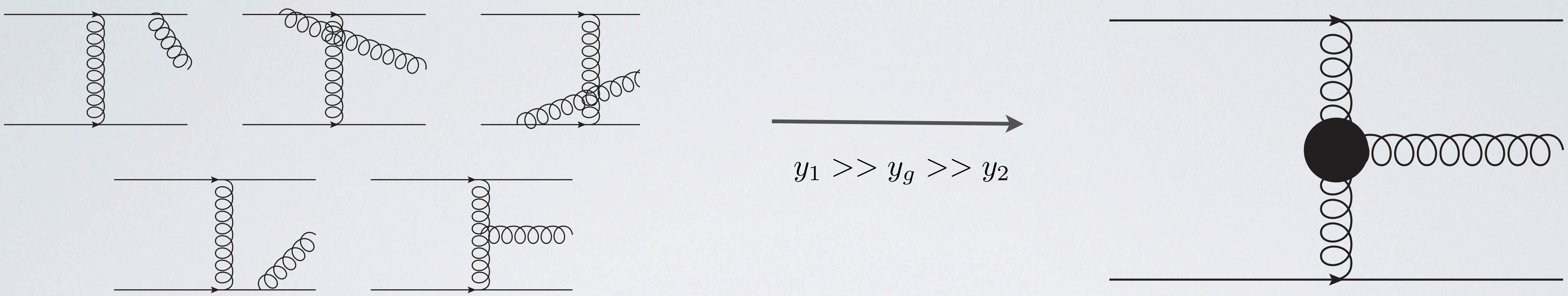
- Is this a good approximation?



HEJ FORMALISM

- t-channel factorization as above, but also
  (1) full virtual 4-momenta
  (2) gauge invariance in all the phase space

EFFECTIVE VERTEX



$$
\begin{aligned}
\mathcal{A} =& (ig_s)^3\ T^g_{1i} T^d_{ia} T^d_{2b}\ \varepsilon_{1\nu} \frac{\langle 1|\nu|g\rangle\langle g|\mu|a\rangle + 2p_1^\nu\langle 1|\mu|a\rangle}{s_{1g} t_{b2}} \langle 2|\mu|b\rangle \\
&+ (ig_s)^3\ T^d_{1i} T^g_{ia} T^d_{2b}\ \varepsilon_{1\nu} \frac{2p_a^\nu\langle 1|\mu|a\rangle - \langle 1|\mu|g\rangle\langle g|\nu|a\rangle}{t_{ag} t_{b2}} \langle 2|\mu|b\rangle \\
&+ (ig_s)^3\ T^g_{2i} T^d_{ib} T^d_{1a}\ \varepsilon_{1\nu} \frac{\langle 2|\nu|g\rangle\langle g|\mu|b\rangle + 2p_2^\nu\langle 2|\mu|b\rangle}{s_{2g} t_{a1}} \langle 1|\mu|a\rangle \\
&+ (ig_s)^3\ T^d_{2i} T^g_{ib} T^d_{1a}\ \varepsilon_{1\nu} \frac{2p_b^\nu\langle 2|\mu|b\rangle - \langle 2|\mu|g\rangle\langle g|\nu|b\rangle}{t_{bg} t_{a1}} \langle 1|\mu|a\rangle \\
&- g_s^3\ f^{\alpha\beta\gamma} T^\alpha_{1a} T^\beta_{2b}\ \varepsilon_{1\nu} \frac{\langle 1|\rho|a\rangle\langle 2|\mu|b\rangle}{t_{a1} t_{b2}} \left(2p_g^\mu g^{\nu\rho} - 2p_g^\rho g^{\mu\nu} - (q_1+q_2)^\nu g^{\mu\rho}\right) .
\end{aligned}
$$

arXiv:0908.2786

EFFECTIVE VERTEX

$$\mathcal{A}^{\text{Eik}} = -\, ig_s^3\ \langle 1|\mu|a\rangle\langle 2|\mu|b\rangle\ \varepsilon_{1\nu} \\ \times\ \left(\frac{2p_1^\nu}{s_{1g}t_{b2}} T^g_{1i}T^d_{ia}T^d_{2b} + \frac{2p_a^\nu}{t_{ag}t_{b2}} T^d_{1i}T^g_{ia}T^d_{2b} + \frac{2p_2^\nu}{t_{a1}s_{2g}} T^g_{2i}T^d_{ib}T^d_{1a} + \frac{2p_b^\nu}{t_{bg}t_{a1}} T^d_{2i}T^g_{ib}T^d_{1a}\right).$$

Using approximations $\quad p_a \sim p_1 = p_+, \qquad p_b \sim p_2 = p_-$

$$\mathcal{A}^{\text{Eik}} \simeq -\, ig_s^3\ \langle 1|\mu|a\rangle\langle 2|\mu|b\rangle\ \varepsilon_{1\nu} \\ \times\ \left(\frac{p_+^\nu}{(p_+\cdot p_g)t_{b2}} T^d_{3b}\left(T^g_{1i}T^d_{ia} - T^d_{1i}T^g_{ia}\right) + \frac{p_-^\nu}{(p_-\cdot p_g)t_{a1}} T^d_{1a}\left(T^g_{2i}T^d_{ib} - T^d_{2i}T^g_{ib}\right)\right) \\ = g_s^3\ \langle 1|\mu|a\rangle\langle 2|\mu|b\rangle\ \varepsilon_{1\nu} f^{gde}\left(\frac{p_+^\nu}{(p_+\cdot p_g)t_{b2}} T^d_{2b}T^e_{1a} + \frac{p_-^\nu}{(p_-\cdot p_g)t_{a1}} T^d_{1a}T^e_{2b}\right) \\ = g_s^3\ \langle 1|\mu|a\rangle\langle 2|\mu|b\rangle\ \varepsilon_{1\nu} f^{gde} T^d_{2b}T^e_{1a}\ \frac{1}{t_{a1}t_{b2}}\left(\frac{p_+^\nu}{(p_+\cdot p_g)} t_{a1} - \frac{p_-^\nu}{(p_-\cdot p_g)} t_{b2}\right)$$

Restore the original momenta

$$\mathcal{A}^{\text{Eik}} \simeq g_s^3\ \langle 1|\mu|a\rangle\langle 2|\mu|b\rangle\ \varepsilon_{1\nu} f^{gde} T^d_{2b}T^e_{1a}\ \frac{1}{t_{a1}t_{b2}} \\ \times\ \frac{1}{2}\left(\frac{p_a^\nu}{(p_a\cdot p_g)} t_{a1} + \frac{p_1^\nu}{(p_1\cdot p_g)} t_{a1} - \frac{p_b^\nu}{(p_b\cdot p_g)} t_{b2} - \frac{p_2^\nu}{(p_2\cdot p_g)} t_{b2}\right).$$

EFFECTIVE VERTEX

Three gluon vertex diagram

$$\mathcal{A}^{3\text{gv}} \simeq -\frac{g_s^3\ f^{\alpha\beta\gamma}T^\alpha_{1a}T^\beta_{2b}\ \varepsilon_{1\nu}}{t_{a1}t_{b2}} \left(8\left(p_g\cdot p_-\ p_+^\nu - p_g\cdot p_+\ p_-^\nu\right) - (q_1+q_2)^\nu\langle 1|\mu|a\rangle\langle 2|\mu|b\rangle\right)$$

$$= -\frac{g_s^3\ f^{\alpha\beta\gamma}T^\alpha_{1a}T^\beta_{2b}\ \varepsilon_{1\nu}}{t_{a1}t_{b2}}\ \langle 1|\mu|a\rangle\langle 2|\mu|b\rangle \left(4\left(\frac{p_g\cdot p_-}{\hat{s}}p_+^\nu - \frac{p_g\cdot p_+}{\hat{s}}p_-^\nu\right) - (q_1+q_2)^\nu\right)$$

$$= -\frac{g_s^3\ f^{\alpha\beta\gamma}T^\alpha_{1a}T^\beta_{2b}\ \varepsilon_{1\nu}}{t_{a1}t_{b2}}\ \langle 1|\mu|a\rangle\langle 2|\mu|b\rangle$$

$$\times\left(-(q_1+q_2)^\nu + \frac{1}{2}\left(\frac{p_g\cdot p_b}{p_a\cdot p_b}p_a^\nu + \frac{p_g\cdot p_2}{p_a\cdot p_2}p_a^\nu + \frac{p_g\cdot p_b}{p_1\cdot p_b}p_1^\nu + \frac{p_g\cdot p_2}{p_1\cdot p_2}p_1^\nu \right.\right.$$

$$\left.\left. - \frac{p_g\cdot p_a}{p_a\cdot p_b}p_b^\nu - \frac{p_g\cdot p_1}{p_b\cdot p_1}p_b^\nu - \frac{p_g\cdot p_a}{p_a\cdot p_2}p_2^\nu - \frac{p_g\cdot p_1}{p_1\cdot p_2}p_2^\nu\right)\right)$$

EFFECTIVE VERTEX

- Amplitude can be written as a contraction of two currents times an effective vertex

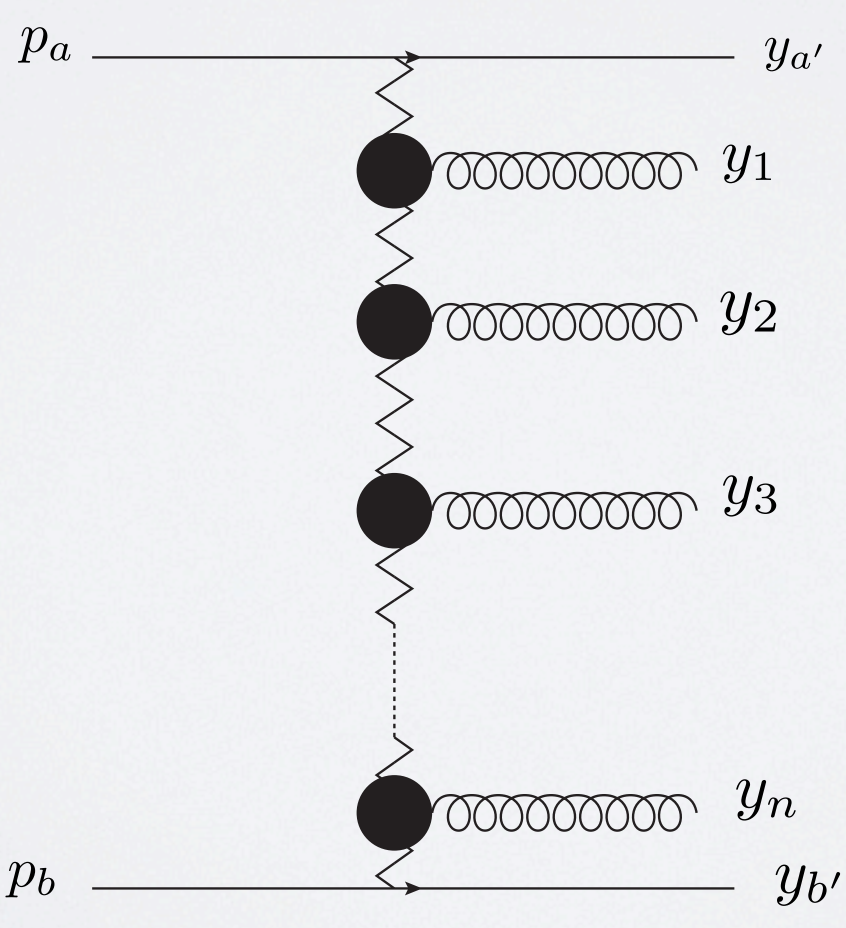
$$\mathcal{A} = g_s^3 \mathcal{C}_g \epsilon_\nu \frac{S_{qQ\to qQ}}{q_1^2 q_2^2} V^\nu$$

$$\begin{aligned} V^\nu = &-(q_1+q_2)^\nu \\ &+\frac{p_a^\nu}{2}\left(\frac{q_1^2}{(p_a\cdot p_g)} + \frac{p_g\cdot p_b}{p_a\cdot p_b} + \frac{p_g\cdot p_2}{p_a\cdot p_2}\right) + p_a \leftrightarrow p_1 \\ &-\frac{p_b^\nu}{2}\left(\frac{q_2^2}{(p_b\cdot p_g)} + \frac{p_g\cdot p_a}{p_a\cdot p_b} - \frac{p_g\cdot p_1}{p_b\cdot p_1}\right) - p_b \leftrightarrow p_2 \end{aligned}$$

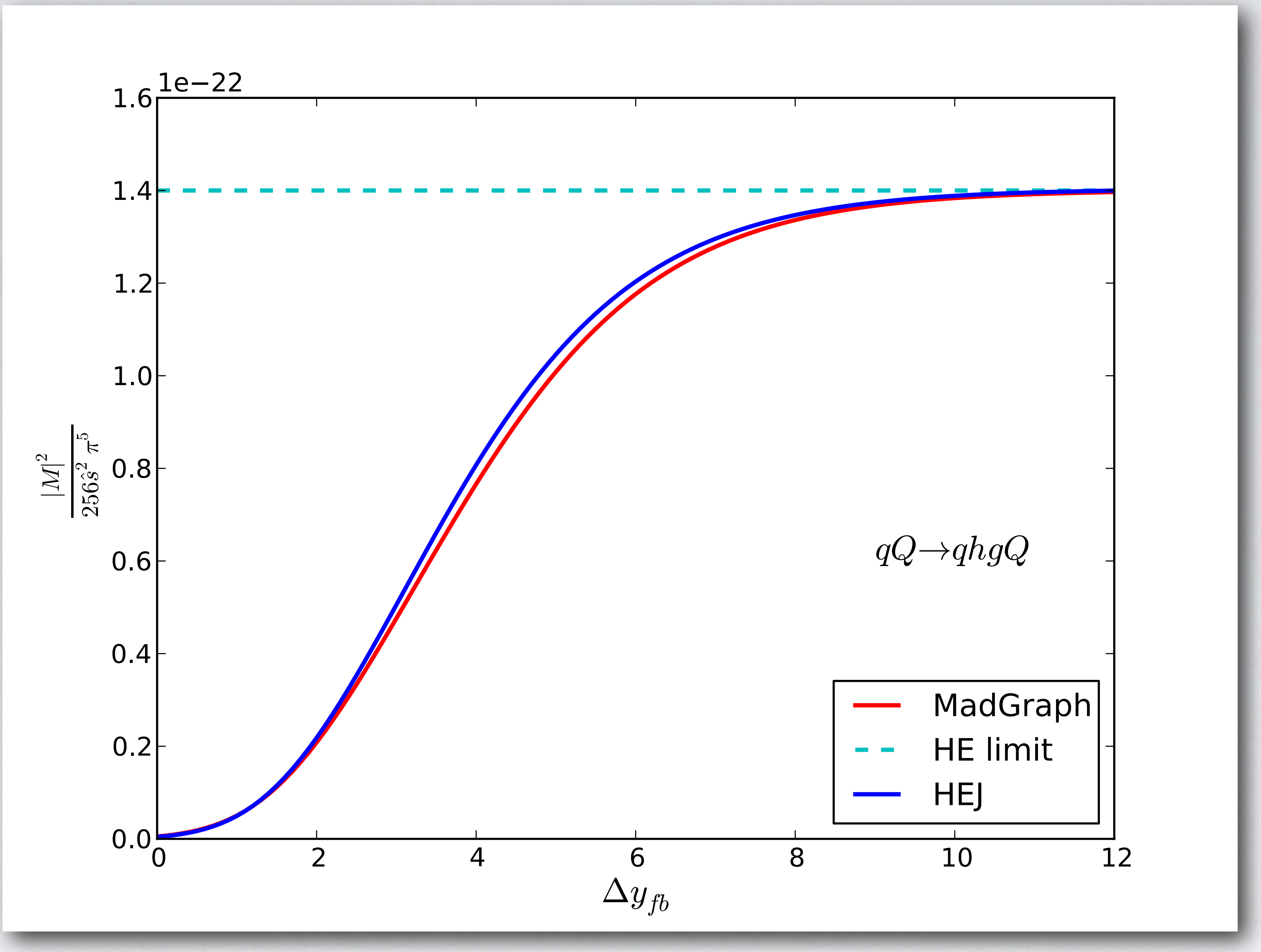
- Gauge invariant $\quad p_g \cdot V = 0$

TREE LEVEL MATRIX ELEMENT

$$\left|\overline{\mathcal{M}}^t_{qQ\to qg..gQ}\right|^2 = \frac{1}{4(N_c^2-1)}||S_{qQ\to qQ}||^2 \left(g^2 C_F \frac{1}{t_1}\right)\left(g^2 C_F \frac{1}{t_{n-1}}\right)$$
$$\prod_{i=1}^{n-2}\left(\frac{-g^2 C_A}{t_i t_{i+1}} V^\mu(q_i,q_{i+1}) V_\mu(q_i,q_{i+1})\right)$$



DOES IT WORK?



1e−22
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
$\frac{|M|^2}{256\hat{s}^2\pi^5}$
0
2
4
6
8
10
12
Δy_{fb}
$qQ \to qhgQ$
MadGraph
HE limit
HEJ

VIRTUAL CORRECTIONS

- Lipatov ansatz:

  Replace gluon propagator with reggeized propagator

$$\frac{1}{t_i} \to \frac{1}{t_i} \exp[\alpha(q_i)(y_{y-1} - y_1)]$$

$$\alpha(q_i) = -g^2 C_a \frac{\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}} \frac{2}{\epsilon} (\mathbf{q}^2/\mu^2)^\epsilon$$

CANCELLATION OF POLES

- In the soft limit $|p_{g,i}| \to 0$

$$\frac{-1}{t_i t_{i+1}} V^{\mu}(q_i, q_{i+1}) V_{\mu}(q_i, q_{i+1}) \to \frac{4}{\mathbf{p}_i^2}$$

- Integrating over the soft region yields

$$\int_0^{\lambda} \frac{d^{2+2\epsilon}\mathbf{p_i} dy_i}{(2\pi)^{2+2\epsilon}4\pi} \frac{4}{\mathbf{p_i}^2} \mu^{-2\epsilon} = \frac{4\Delta_{y_{i-1,i+1}}}{(2\pi)^{2+2\epsilon}4\pi} \frac{\pi^{1+\epsilon}}{\Gamma(1+\epsilon)\epsilon} (\lambda^2/\mu^2)^{\epsilon}$$

- The poles cancel and we are left with

$$\Delta_{y_{i-1,i+1}} \frac{\alpha_s N_c}{\pi} \ln\left(\frac{\lambda^2}{\mathbf{q}^2}\right)$$

MATRIX ELEMENT

$$\overline{|\mathcal{M}^{\mathrm{reg}}|}^2 = \frac{1}{4(N_c^2-1)}||S_{qQ\to qQ}||^2 \left(g^2 C_F \frac{1}{t_1}\right)\left(g^2 C_F \frac{1}{t_{n-1}}\right)$$

$$\prod_{i=1}^{n-2}\left(\frac{-g^2 C_A}{t_i t_{i+1}} V^{\mu}(q_i,q_{i+1}) V_{\mu}(q_i,q_{i+1}) - \frac{g^2 C_A 4}{\mathbf{p}_i^2}\theta(\mathbf{p}_i^2 < \lambda^2)\right)$$

$$\prod_{j=1}^{n-1} \exp[\omega^0(q_j,\lambda)(y_{j-1}-y_j)],$$

$$\omega^0(q_j,\lambda) = -\frac{\alpha_s N_c}{\pi}\ln\frac{\mathbf{q}_j^2}{\lambda^2}$$

MASTER FORMULA

$$\sigma_{2j}^{\mathrm{resum,match}} = \sum_{f_1,f_2} \sum_{n=2}^{\infty} \prod_{i=1}^{n} \left(\int_{p_{i\perp}=\lambda}^{p_{i\perp}=\infty} \frac{\mathrm{d}^2\mathbf{p}_{i\perp}}{(2\pi)^3} \int \frac{\mathrm{d}y_i}{2} \right) \frac{\overline{\left|\mathcal{M}_{\mathrm{HEJ}}^{f_1 f_2 \to f_1 g \cdots g f_2}(\{p_i\})\right|}^2}{\hat{s}^2}$$
$$\times \sum_m \mathcal{O}_{mj}^e(\{p_i\})\ w_{m-\mathrm{jet}}$$
$$\times\ \ x_a f_{A,f_1}(x_a, Q_a)\ x_2 f_{B,f_2}(x_b, Q_b)\ (2\pi)^4\ \delta^2\!\left(\sum_{i=1}^{n} \mathbf{p}_{i\perp}\right)\ \mathcal{O}_{2j}(\{p_i\})$$

$$w_{n-\mathrm{jet}} \equiv \frac{\overline{\left|\mathcal{M}^{f_1 f_2 \to f_1 g \cdots g f_2}\left(\left\{p_{\mathcal{J}_l}^{\mathrm{new}}(\{p_i\})\right\}\right)\right|}^2}{\overline{\left|\mathcal{M}^{t, f_1 f_2 \to f_1 g \cdots g f_2}\left(\left\{p_{\mathcal{J}_l}^{\mathrm{new}}(\{p_i\})\right\}\right)\right|}^2}$$

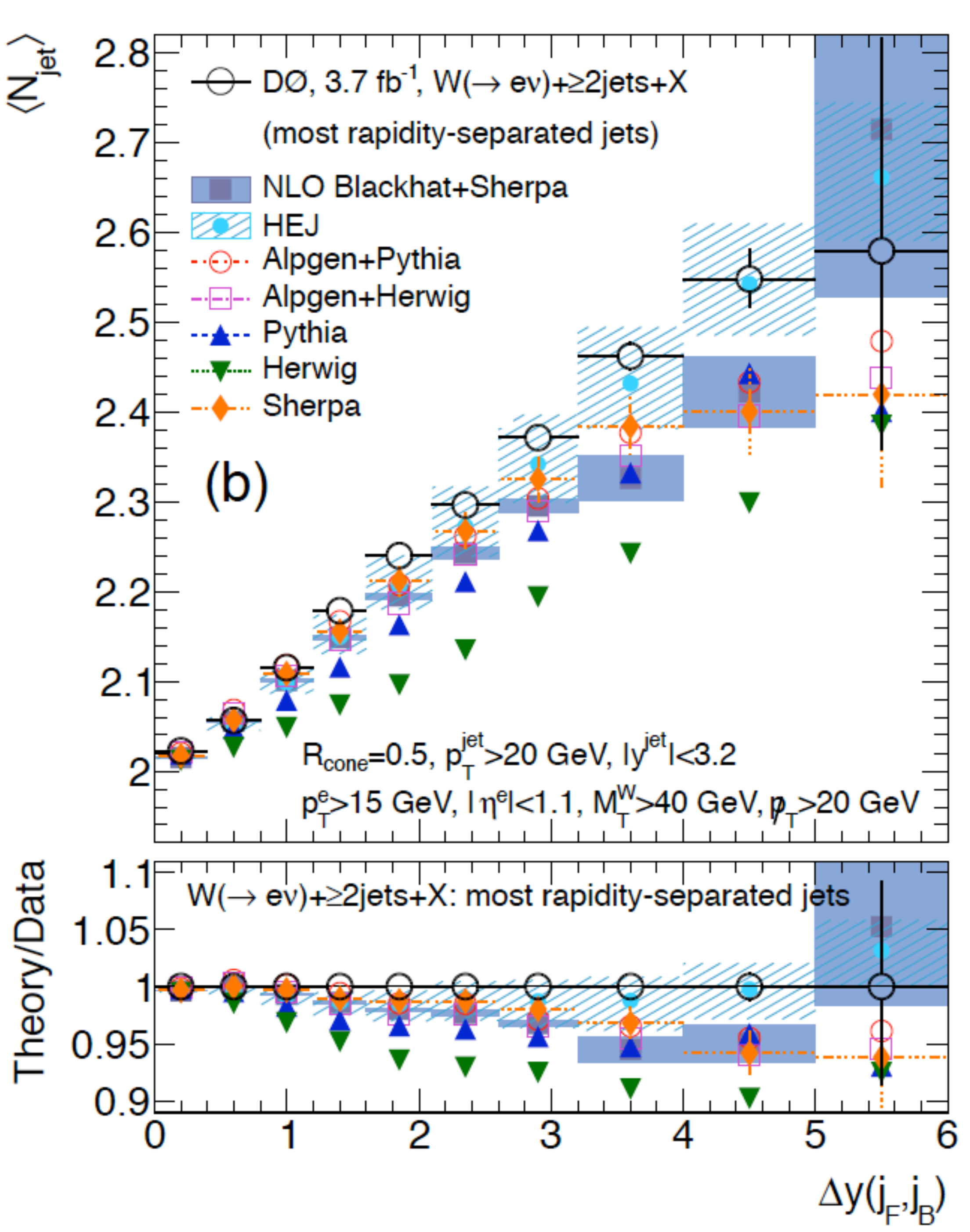
- Momentum shuffling for the matching

$$p_{\mathcal{J}_{l\perp}}^{B} = p_{\mathcal{J}_{l\perp}} + q_\perp * \frac{|p_{\mathcal{J}_{l\perp}}|}{P_\perp}$$

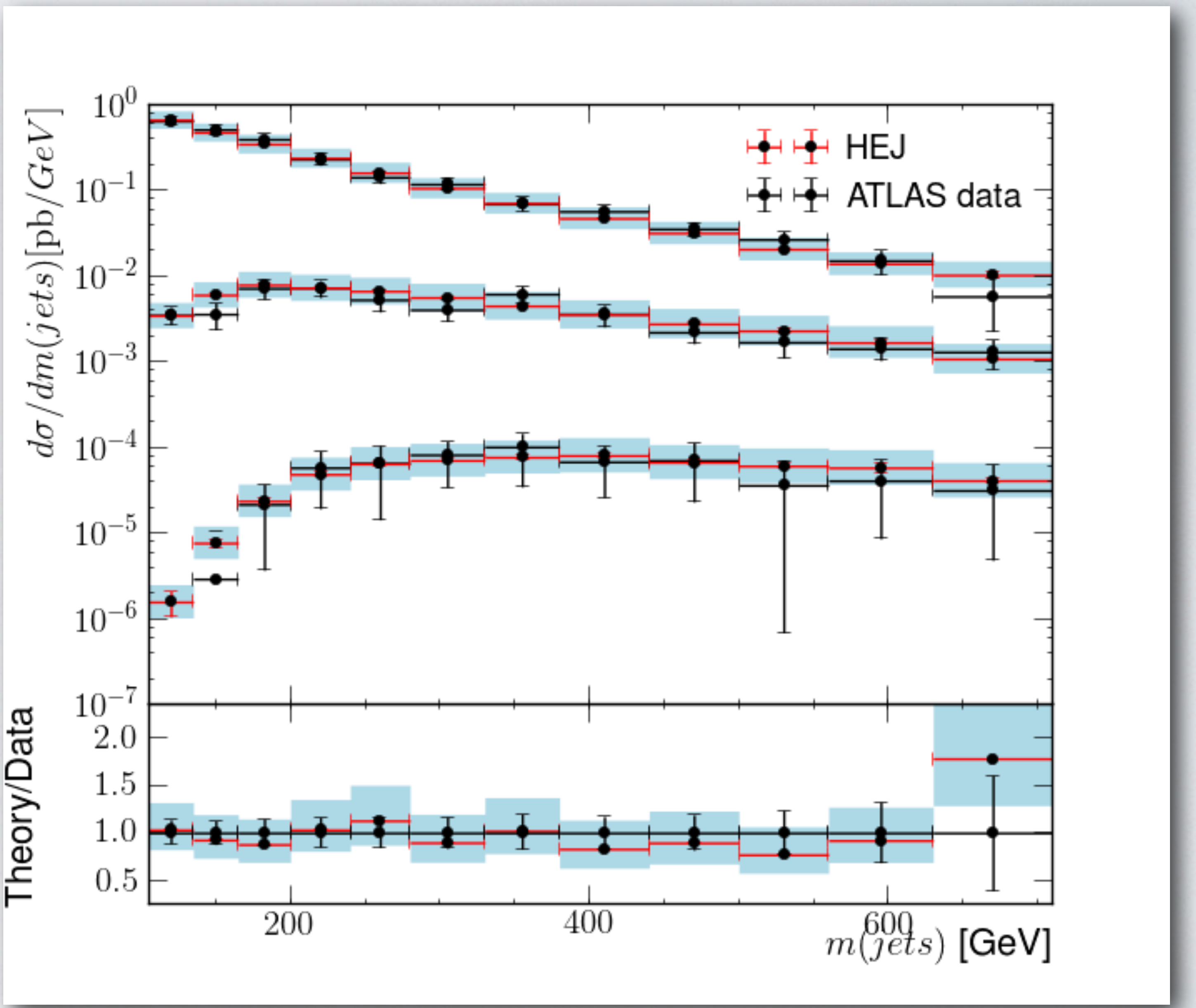
$q_\perp$ pt of the sum of non-jet momenta

$P_\perp$ Scalar sum of the jet pt

SOME RESULTS



arXiv:1302.6508



10.1103/PhysRevD.85.092002

http://hepdata.cedar.ac.uk/view/ins1083318

FUTURE DEVELOPMENTS

- Unordered emission
  - Done for the Higgs case
- NLO matching
  - Inverting HEJ formalism currently
- Matching with the parton shower(s)
  - solution exists: 10.1007/JHEP07(2011)110

CONCLUSIONS

- HEJ designed to describe multi-jet events with large rapidity separation
- Simple form of the matrix elements allow effective phase space integration
- Possible to compare different description agains the data