# Threshold corrections to inclusive jet production at hadron colliders


Meduri C Kumar

*University of Hamburg, Germany*

**In collaboration with**

**S. Moch**

## *Outline*

1. Inclusive jet production at hadron colliders

2. Soft-gluon resummation formalism

3. Cross sections to NNLO at NLL accuracy

4. numerical results

5. Summary

# Inclusive jet production

1. The process is given by

$$P_a + P_b \rightarrow J + X$$

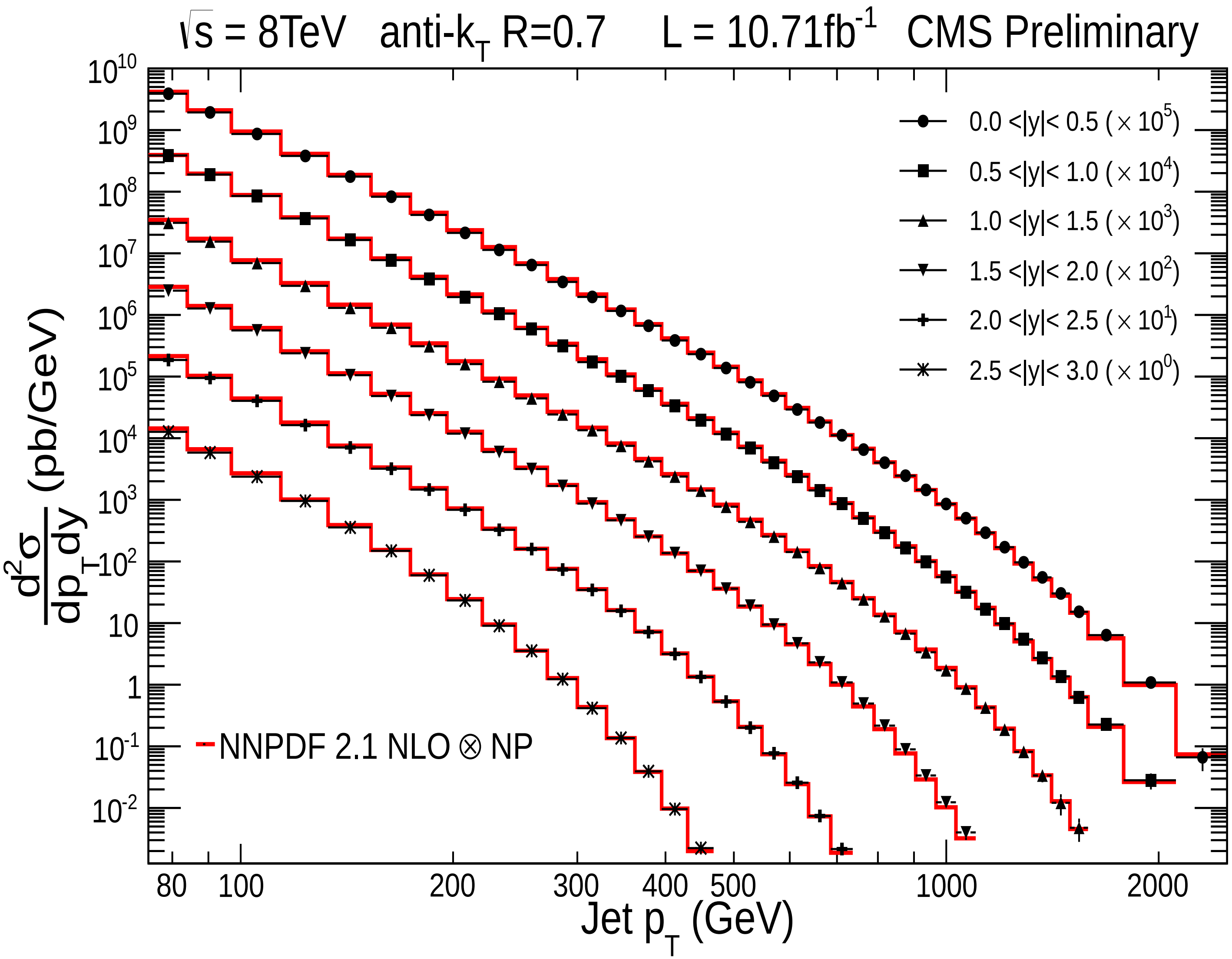
2. Dominant process at hadron colliders
3. Useful in the extraction of parton distribution functions
   - MSTW 2008 PDFs

     **EPJC 63, 189 (2009)**
   - CT10 PDFs

     **PRD 82, 074024 (2010)**
   - NN PDFs

     **NPB 838, 136 (2010)**
4. Useful in the measurement of strong coupling constant $\alpha_s$.
   D0 collaboration **PRD 80, 111107 (2009)**

# Inclusive jet production at 8 TeV LHC




*CMS collaboration;* **CMS-PAS-SMP-12-012**

## State of the art for jet cross sections

1. Three jet production in electron-positron annihilation to NNLO
   **Garland, Gehrmann, Glover, Koukoutsakis, Remiddi,** NPB 642, 227 (2002)
   **Ridder, Gehrmann, Glover, Heinrich,** PRL 100, 172001 (2008)

2. Inclusive jet production to NLO at hadron colliders
   **Ellis, Kunszt and Soper,** PRL 64, 2121 (1990)

3. Di-jet production to NLO at hadron colliders
   **Ellis, Kunszt and Soper**, PRL 69, 1496 (1992)
   **Giele, Glover and Kosower**, PRL 73, 2019 (1994)
   **Catani and Seymour**, NPB485, 291 (1997) (General method for NLO QCD)

4. Threshold corrections to inclusive jet production at hadron colliders
   **Kidonakis and Owens,** PRD 63, 054019 (2001)

5. Di-jet production to NNLO at hadron colliders (only gluonic channel)
   **Ridder, Gehrmann, Glover, Pires** PRL 110 (2013)

# Soft gluon resummation formalism

1. Color basis and color decomposition of QCD amplitudes
2. Soft function
3. Hard function
4. Soft anomalous dimension $\Gamma_S$
5. Jet functions
6. parton level cross section
7. Hadronic cross section

**G. Sterman**, NPB 281 (1987) 310

**N. Kidonakis, G. Oderda and G. Sterman**, NPB531 (1998) 365; NPB525 (1998) 299

**N. Kidonakis and G. Sterman**, Nucl. Phys. B505 (1997) 321

**S. Catani and L. Trentadue**, NPB327 (1989) 323; NPB 353 (1991) 183

**S. Catani, M.L. Mangano, P. Nason and L. Trentadue**, NPB 478 (1996) 273

**R. Bonciani, S. Catani, M.L. Mangano and P. Nason**, NPB 529 (1998) 424

# Color basis for different parton channels

1. The t-channel color basis for a $qq \to qq$ process $i\, j \to k\, l$ is given by

$$c_1 = \delta_{ik}\, \delta_{jl}, \quad c_2 = T^c_{ki}\, T^c_{jl}$$

   The soft function for this basis is given by

$$\begin{bmatrix} N_c^2 & 0 \\ 0 & (N_c^2-1)/4 \end{bmatrix}$$

2. The t-channel color basis for a $qg \to qg$ process $i\, j \to k\, l$ is given by

$$c_1 = \delta_{ik}\, \delta_{jl}, \quad c_2 = d^{jlc} T^c_{ki} \quad c_3 = i\, f^{jlc}\, T^c_{ki}$$

   The soft function for this basis is given by

$$\begin{bmatrix} N_c(N_c^2-1) & 0 & 0 \\ 0 & (N_c^2-4)(N_c^2-1)/(2N_c) & 0 \\ 0 & 0 & N_c(N_c^2-1)/2 \end{bmatrix}$$

3. For the processes that are related to the above by charge conjugation, the basis needs to be defined accordingly.
4. $i, j, k, l = 1, 3$ for quarks and $i, j, k, l = 1, 8$ for gluons.

# Color basis for different parton channels contd...

The t-channel color basis for a $gg \to gg$ process $i\ j \to k\ l$ is given by

$$
\begin{aligned}
c_1 &= \frac{i}{4}[f^{ijm}d^{klm} \ - \ d^{ijm}f^{klm}]\,\delta_{ik}\,\delta_{jl}, \\
c_2 &= \frac{i}{4}[f^{ijm}d^{klm} \ + \ d^{ijm}f^{klm}], \\
c_3 &= \frac{i}{4}[f^{ikm}d^{jlm} \ + \ d^{ikm}f^{jlm}], \\
c_4 &= \frac{1}{8}\delta_{ik}\delta_{jl}, \\
c_5 &= \frac{3}{5}d^{ikn}\ d^{jln}, \\
c_6 &= \frac{1}{3}f^{ikn}\ f^{jln}, \\
c_7 &= \frac{1}{2}\left(\delta_{ij}\delta_{kl} - \delta il \delta_{jk}\right) - \frac{1}{3}f^{ikn}\ f^{jln}, \\
c_8 &= \frac{1}{2}\left(\delta_{ij}\delta_{kl} + \delta il \delta_{jk}\right) - \frac{1}{8}\delta_{ik}\delta_{jl} - \frac{3}{5}d^{ikn}\ d^{jln}
\end{aligned}
$$

## Soft function for $gg \to gg$ subprocess

The soft function for this basis is given by

$$S_{8\times 8} = \begin{bmatrix} G_{3\times 3} & 0_{3\times 5} \\ 0_{5\times 3} & G_{5\times 5} \end{bmatrix}$$

where $G_{3\times 3} = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ and $G_{5\times 5} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 8 & 0 & 0 & 0 \\ 0 & 0 & 8 & 0 & 0 \\ 0 & 0 & 0 & 20 & 0 \\ 0 & 0 & 0 & 0 & 27 \end{bmatrix}$

**N. Kidonakis, G. Oderda and G. Sterman**

Nucl. Phys. B. 531 (1998)

# $\Gamma_S$ for different parton subprocesses

1. Soft anomalous function for $qq \to qq$ subprocess is given by

$$\frac{\alpha_s}{\pi}\begin{bmatrix} \frac{-1}{N_c}(T+U)+2C_F U & 2U \\ \frac{C_F}{N_c}U & 2C_F\, T \end{bmatrix}$$

2. Soft anomalous function for $qg \to qg$ subprocess is given by

$$\frac{\alpha_s}{\pi}\begin{bmatrix} (C_F+C_A)T & 0 & U \\ 0 & C_F T+\frac{C_A}{2}U & \frac{C_A}{2}U \\ 2U & \frac{N_c^2-4}{2N_c}U & C_F T+\frac{C_A}{2}U \end{bmatrix}$$

where

$$\begin{aligned} T &= \ln\left(\frac{-t}{s}\right)+i\pi \\ U &= \ln\left(\frac{-u}{s}\right)+i\pi \end{aligned}$$

# $\Gamma_S$ for $gg \to gg$ subprocesses

1. The soft anomalous dimension can be written as

$$\Gamma_S = \begin{bmatrix} G_{3\times 3} & 0_{3\times 5} \\ 0_{5\times 3} & G_{5\times 5} \end{bmatrix} \quad \text{where} \quad G_{3\times 3} = \frac{\alpha_s}{\pi} \begin{bmatrix} 3T & 0 & 0 \\ 0 & 3U & 0 \\ 0 & 0 & 3(T+U) \end{bmatrix}$$

$$G_{5\times 5} = \frac{\alpha_s}{\pi} \begin{bmatrix} 6T & 0 & -6U & 0 & 0 \\ 0 & 3T + \frac{3U}{2} & \frac{-3U}{2} & -3U & 0 \\ -\frac{3U}{4} & -\frac{3U}{2} & 3T + \frac{3U}{2} & 0 & \frac{-9U}{4} \\ 0 & -\frac{6U}{5} & 0 & 3U & -\frac{9U}{5} \\ 0 & 0 & -\frac{2U}{3} & -\frac{4U}{3} & -2T + 4U \end{bmatrix}$$

# Jet functions

1. Jet functions have the information about collinear configurations.
2. The initial state functions $\mathcal{J}_a^I$ are given by

$$\mathcal{J}_a^I = -2\int_{\mu_F}^{2p_a.\zeta}\frac{d\mu}{\mu}C_a\frac{\alpha_s(\mu^2)}{\pi}\ln N_a - \int_0^1 dz\frac{z^{N_a-1}}{1-z}\left[\int_{(1-z)^2}^1\frac{d\lambda}{\lambda}A^{(f_a)}[\alpha_s(\lambda(2p_a.\zeta)^2)] + \frac{1}{2}\nu^{f_i}[\alpha_s((1-z)^2(2p_a.\zeta)^2)]\right]$$

3. The final state functions $\mathcal{J}_a^F$ are given by

$$\mathcal{J}_a^F = \int_0^1 dz\frac{z^{N-1}}{1-z}\left[\int_{(1-z)^2}^{(1-z)}\frac{d\lambda}{\lambda}A^{(f_a)}[\alpha_s(\lambda(p_T^2))] + B_a^{(1)}[\alpha_s((1-z)p_T^2)] + B_a^{(2)}[\alpha_s((1-z)^2p_T^2)]\right]$$

# Parton level cross section

1. The parton level resummed cross section for a generic subprocess is given by

$$
\begin{aligned}
d\hat{\sigma}_{12\,\rightarrow\,34} \quad = \quad & \text{exp}\left\{\sum_{a=1,2} \mathcal{J}_a^I\right\} \times \text{exp}\left\{\sum_{b=3,4} \mathcal{J}_b^I\right\} \\
& \times \text{exp}\left[2\sum_{a=1,2}\int_{\mu_F}^{p_T}\frac{d\mu}{\mu}\gamma_a[\alpha_s(\mu^2)]\right] \times \text{exp}\left[4\int_{\mu_R}^{p_T}\frac{d\mu}{\mu}\beta(\alpha_s(\mu^2))\right] \\
& \times \text{Trace}\left\{H(\alpha_s(\mu_R^2))\ \bar{P}\text{exp}\left[\int_{p_T}^{p_T/N}\frac{d\mu}{\mu}\Gamma_S^\dagger(\alpha_s(\mu^2))\right]\right. \\
& \left.\times\ S(\alpha_s(p_T^2/N^2))\ P\text{exp}\left[\int_{p_T}^{p_T/N}\frac{d\mu}{\mu}\Gamma_S(\alpha_s(\mu^2))\right]\right\}
\end{aligned}
$$

with

$$
\begin{aligned}
H(x) &= H^{(0)}(x) + \frac{\alpha_s}{\pi}\,H^{(1)}(x) \\
S(x) &= S^{(0)}(x) + \frac{\alpha_s}{\pi}\,S^{(1)}(x)
\end{aligned}
$$

## Various subprocesses

$$\begin{aligned}
q(p_1) + q'(p_2) &\longrightarrow q(p_3) + q'(p_4)\,, \\
q(p_1) + \bar{q}(p_2) &\longrightarrow q'(p_3) + \bar{q}'(p_4)\,, \\
q(p_1) + \bar{q}(p_2) &\longrightarrow q(p_3) + \bar{q}(p_4)\,, \\
q(p_1) + q(p_2) &\longrightarrow q(p_3) + q(p_4)\,, \\
q(p_1) + \bar{q}'(p_2) &\longrightarrow q(p_3) + \bar{q}'(p_4)\,, \\
q(p_1) + \bar{q}(p_2) &\longrightarrow g(p_3) + g(p_4)\,, \\
q(p_1) + g(p_2) &\longrightarrow q(p_3) + g(p_4)\,, \\
g(p_1) + g(p_2) &\longrightarrow q(p_3) + \bar{q}(p_4)\,, \\
g(p_1) + g(p_2) &\longrightarrow g(p_3) + g(p_4)\,.
\end{aligned}$$

# Hadron level cross section

1. The hadron level cross section is given by

$$S^2 \frac{d^2\sigma}{dT\,dU} = \int_{x_1-}^{1} \frac{dx_1}{x_1} \int_{x_2-}^{1} \frac{dx_2}{x_2}\, f_1(x_1,\mu_F)\, f_2(x_2,\mu_F)\, s^2 \frac{d^2\hat{\sigma}}{dtdu}$$

$$s = x_1 x_2 S, \quad t = x_1 T, \quad u = x_2 U, \quad x_1- = \frac{-U}{S+T} \quad \text{and} \quad x_2- = \frac{-x_1 T}{x_1 S + U}$$

2. Born kinematics : $s + t + u = 0$

3. Threshold kinematics : $s + t + u = s_4$, with $0 < s_4 < x_1(S+T) + U$

4. The hadron level cross section in terms of $s_4$ variable is given

$$S^2 \frac{d^2\sigma}{dT\,dU} = \int_{x_1-}^{1} \frac{dx_1}{x_1} \int_{s_4-}^{s_4+} \frac{1}{s_4 - x_1 T} ds_4\, f_1(x_1,\mu_F)\, f_2\left[\frac{s_4 - x_1 T}{x_1 S + U}, \mu_F\right]\, s^2 \frac{d^2\hat{\sigma}}{dtdu}$$

5. The total cross section is given as

$$\sigma = \int_0^S d(-T) \int_0^{S+T} d(-U)\, \frac{d^2\sigma}{dT\,dU}$$

## Expansion of the resummed result to NLL

1. At parton level, the resummed result can be expanded to $1$-loop at NLL accuracy as

$$s^2 \frac{d^2\hat{\sigma}}{dtdu} = \frac{\alpha_s}{\pi}\sigma^{(0)} \left\{ c_3 \left[\frac{\ln(s_4/p_T^2)}{s_4}\right]_+ + c_2 \left[\frac{1}{s_4}\right]_+ + c_1 \delta(s_4) \right\}$$

2. The resummed result expanded to $2$-loop level at NLL accuracy is:

$$s^2 \frac{d^2\hat{\sigma}}{dtdu} = \left(\frac{\alpha_s}{\pi}\right)^2 \sigma^{(0)} \left\{ b_3 \left[\frac{\ln^3(s_4/p_T^2)}{s_4}\right]_+ + b_2 \left[\frac{\ln^2(s_4/p_T^2)}{s_4}\right]_+ + b_1 \left[\frac{\ln(s_4/p_T^2)}{s_4}\right]_+ \right\}$$

3. The coefficients of $c_3$ and $b_3$ are leading logarithms (LL)
4. The coefficients of $c_2$ and $b_2$ are next-to-leading logarithms (NLL)
5. To determin the coefficients $c_1$ and $b_1$, we need the hard matching functions $H^{(1)}$.

# K-factors

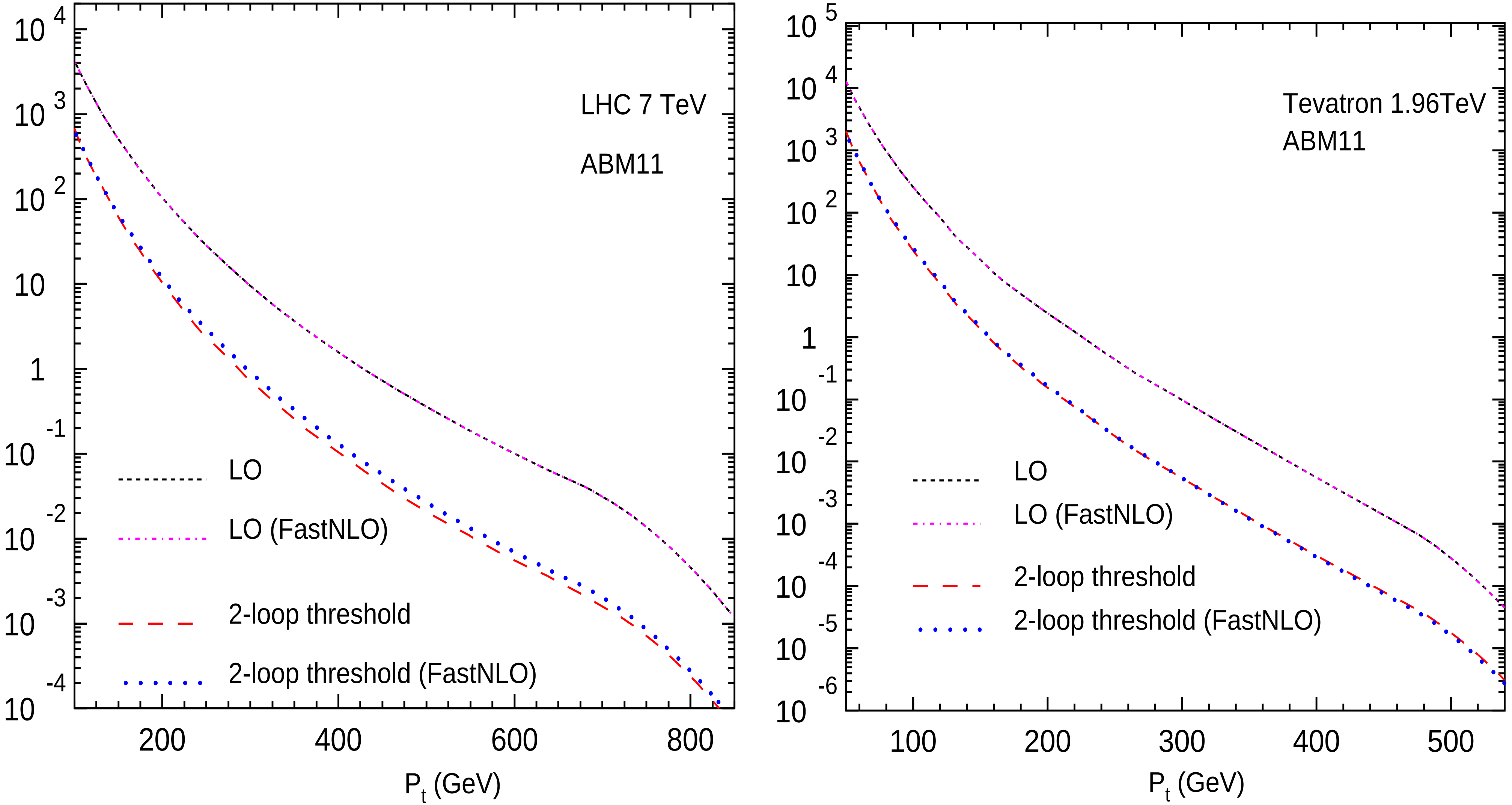
$$K^{(1)} = 1 + \frac{\sigma^{(1)}}{\sigma^{(0)}} \quad \text{(one loop)}$$

$$K^{(2)} = 1 + \frac{\sigma^{(2)}}{\sigma^{(0)}} \quad \text{(two-loop)}$$

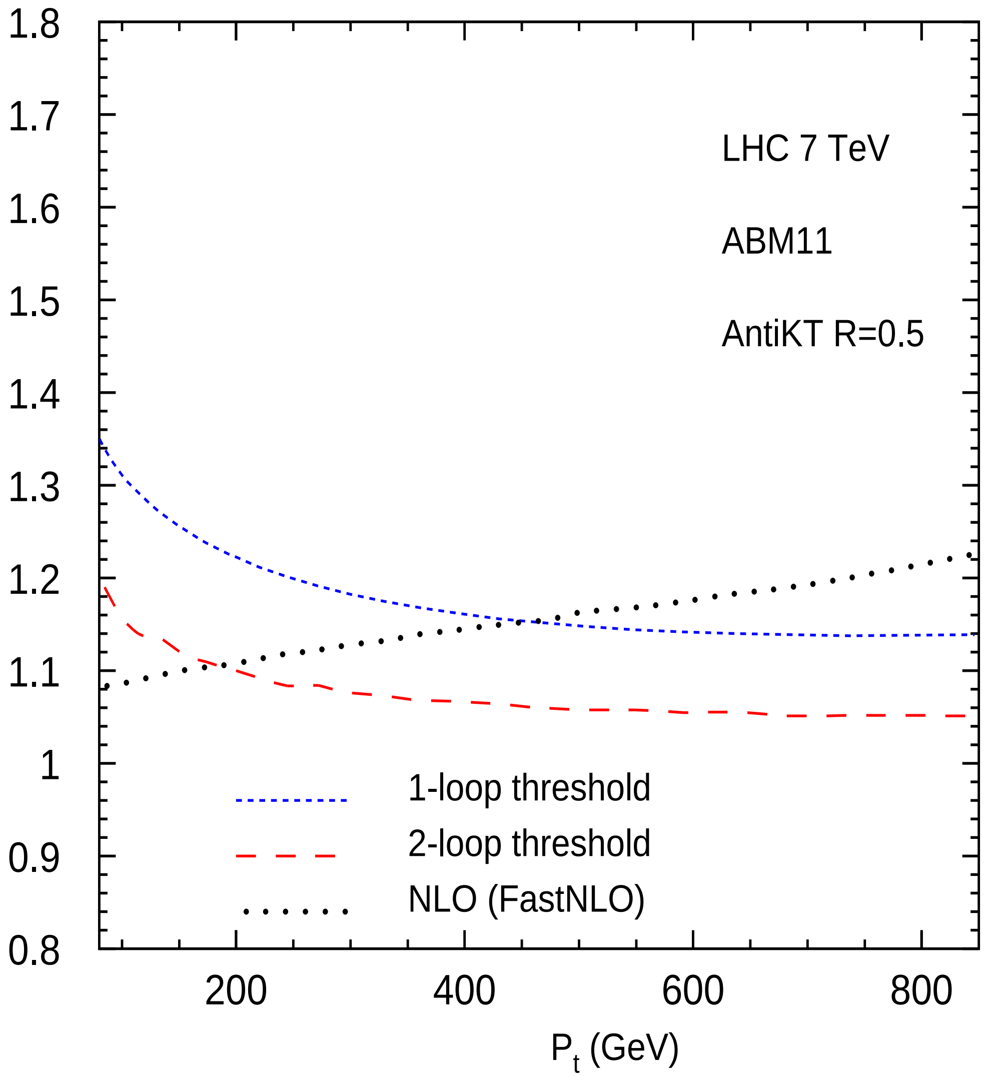
$$K^{(NLO)} = 1 + \frac{\sigma^{(NLO)}}{\sigma^{(0)}} \quad \text{(exact NLO)}$$

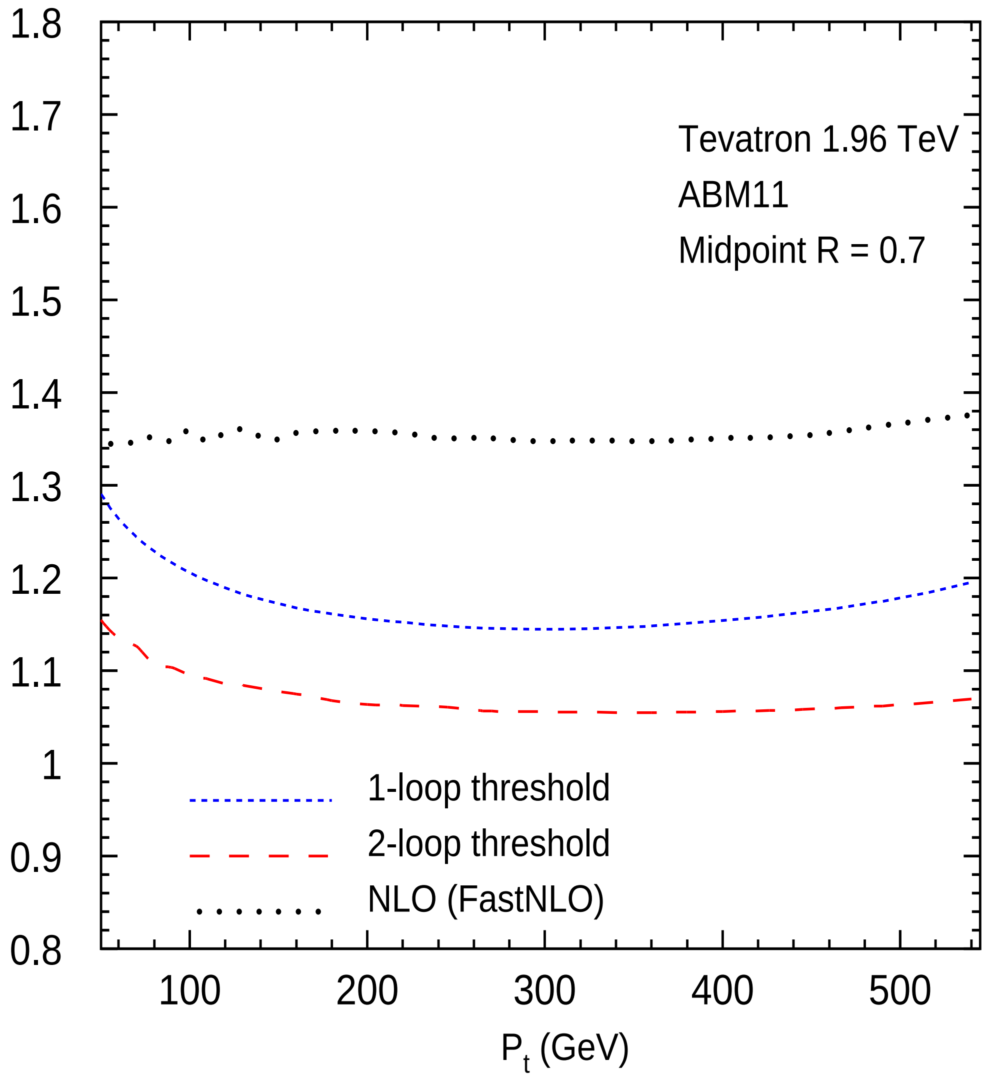
$$K^{(NNLO*)} = 1 + \frac{\sigma^{(NLO)} + \sigma^{(2)}}{\sigma^{(0)}} \quad \text{(exact NLO + two-loop)}$$

# Two-loop threshold corrections (LHC & Tevatron)

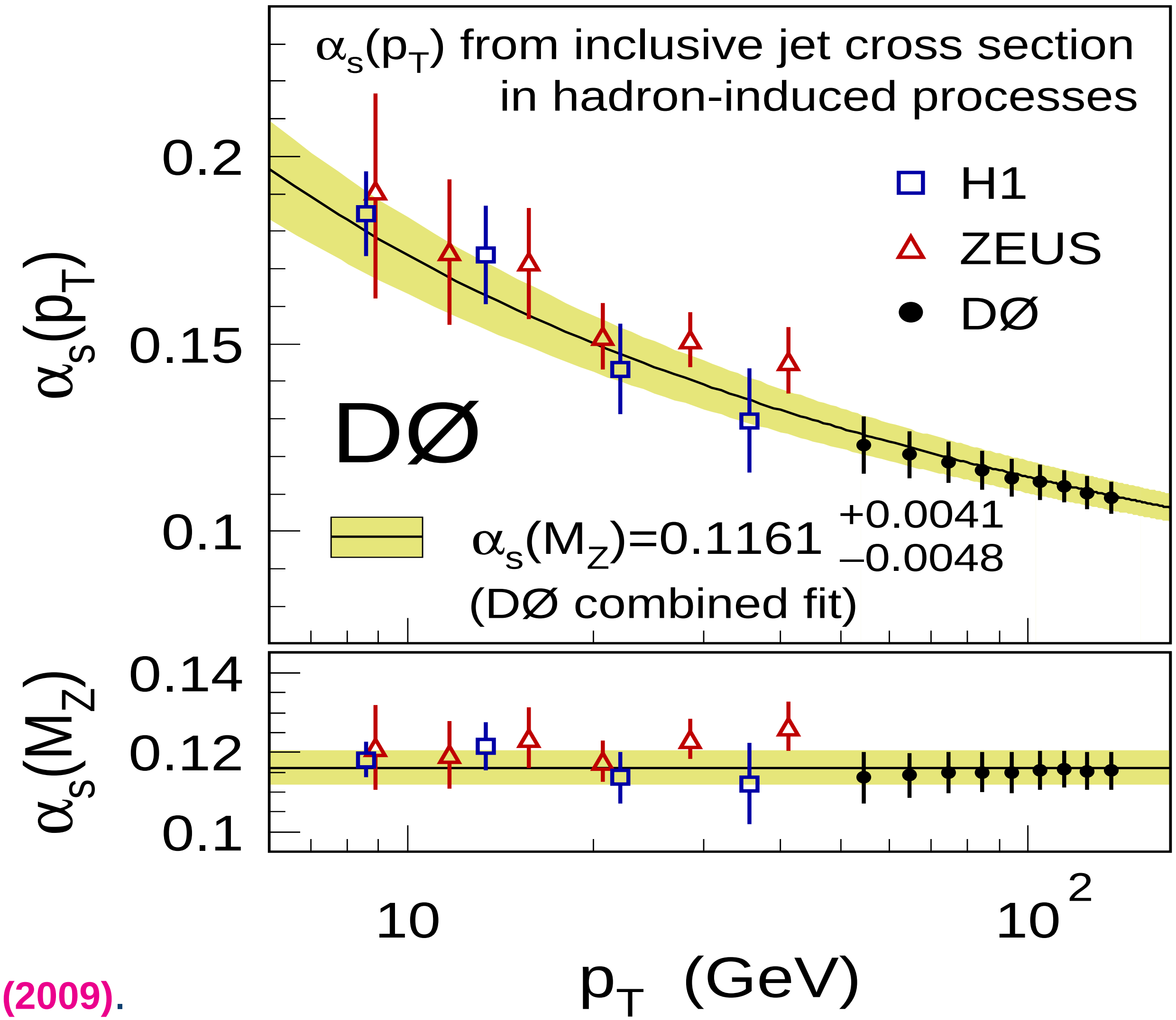


# K-factors for LHC and Tevatron



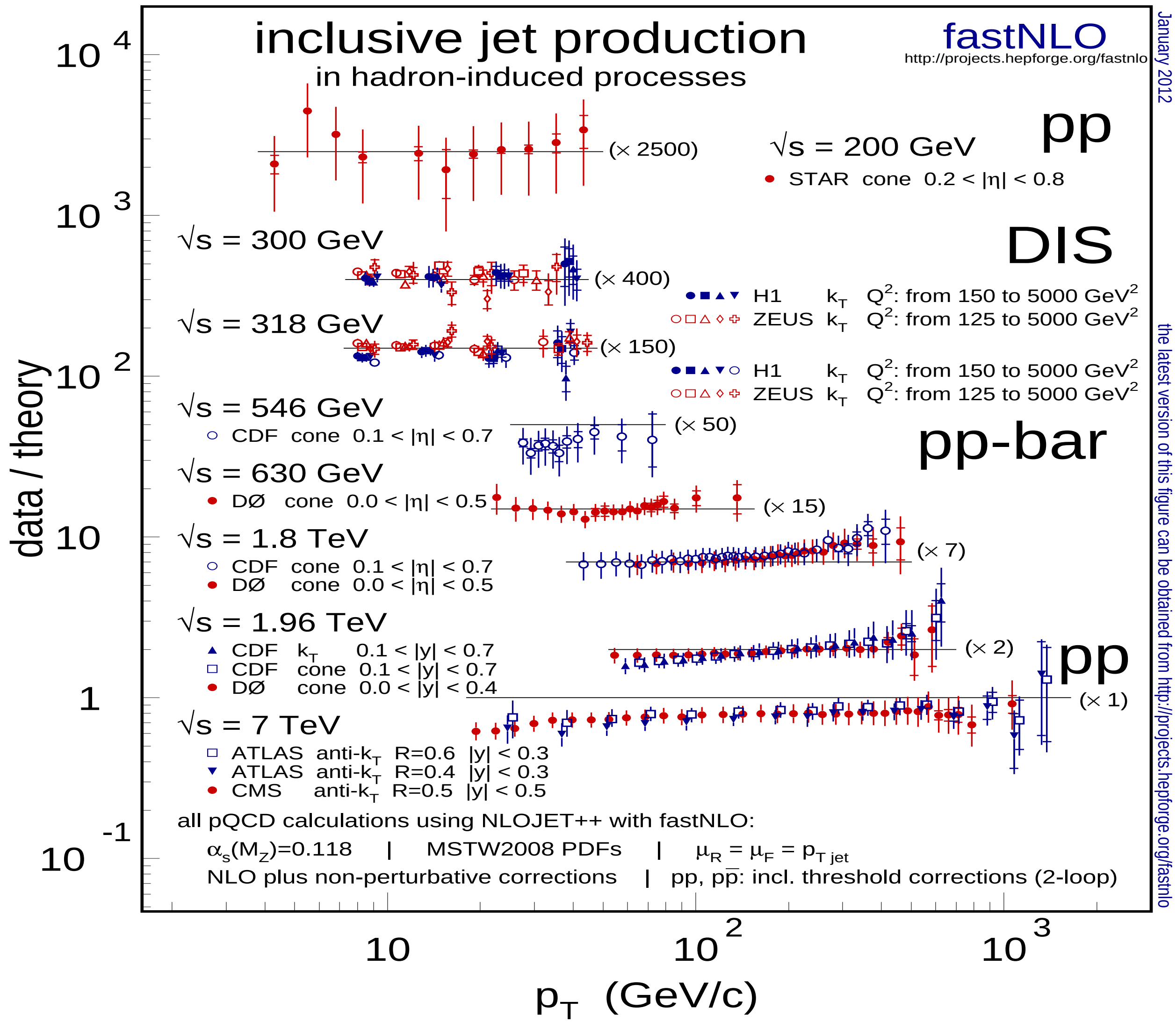
# Determination of $\alpha_s$ from Tevatron data




**PRD 80, 111107 (2009)**.

Without $2$-loop th. corrections: $\alpha_s(M_Z)$ = 0.1202 (MSTW 2008 NLO PDFs)

# Comparison to the data




*D. Britzger, K. Rabbertz, F. Stober and M. Wobisch;* **arXiv:1208.3641**.

# Fixed order results vs threshold corrections

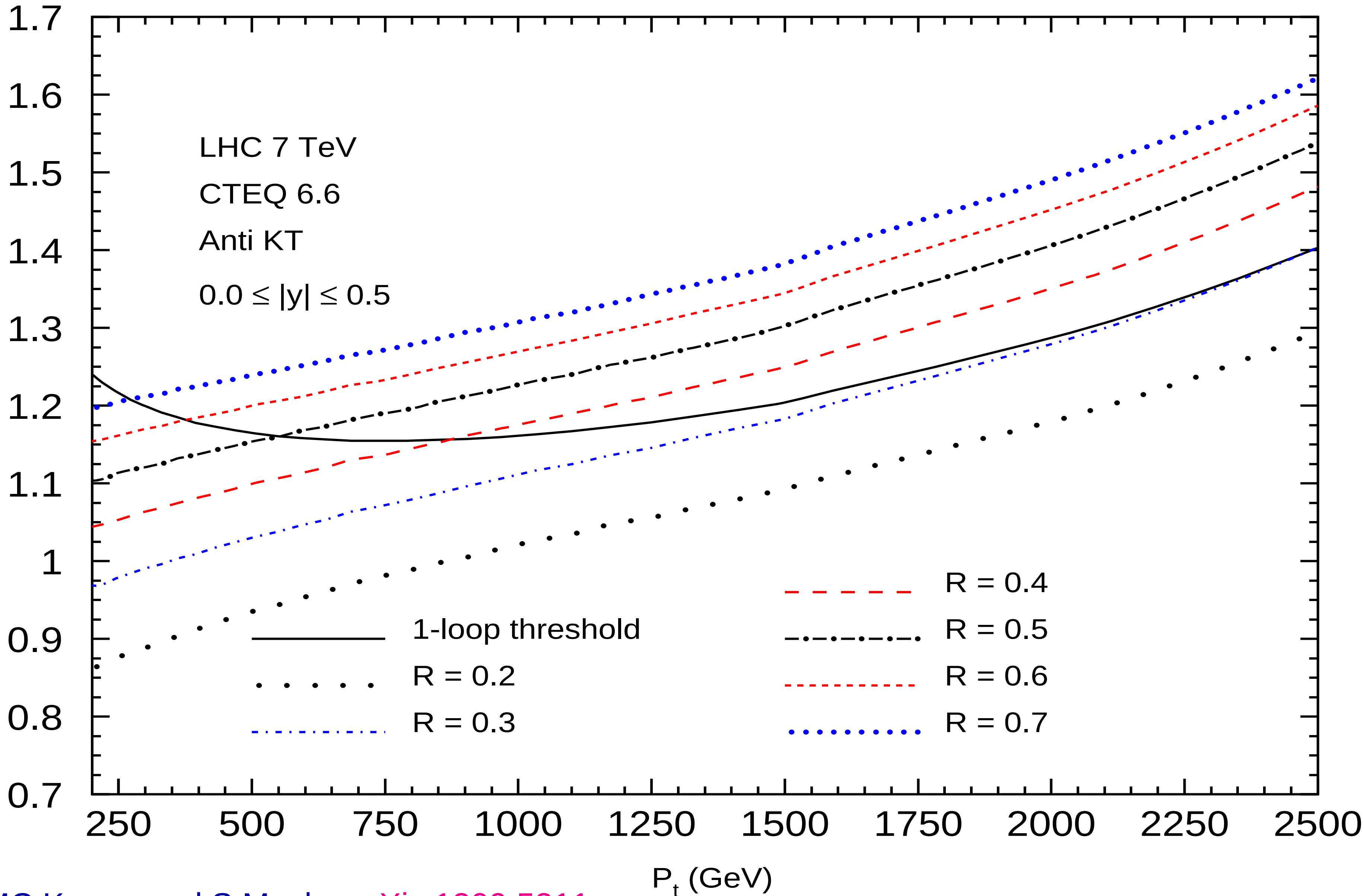
- Fixed order results

1. Complete real and virtual contributions
2. Jet definition
3. Jet algorithm, recombination scheme, dependence on $R$
4. `NLOJET++` : Z. Nagy; PRD68, 094002 (2003)
5. `MEKS`: For inclusive jet cross sections to NLO (interface with `LHAPDF`)
   Gao, Liang, Soper, Lai, Nadolsky, Yuan; Comp. Phys. Comm. 184, 1626 (2013)

- Threshold corrections

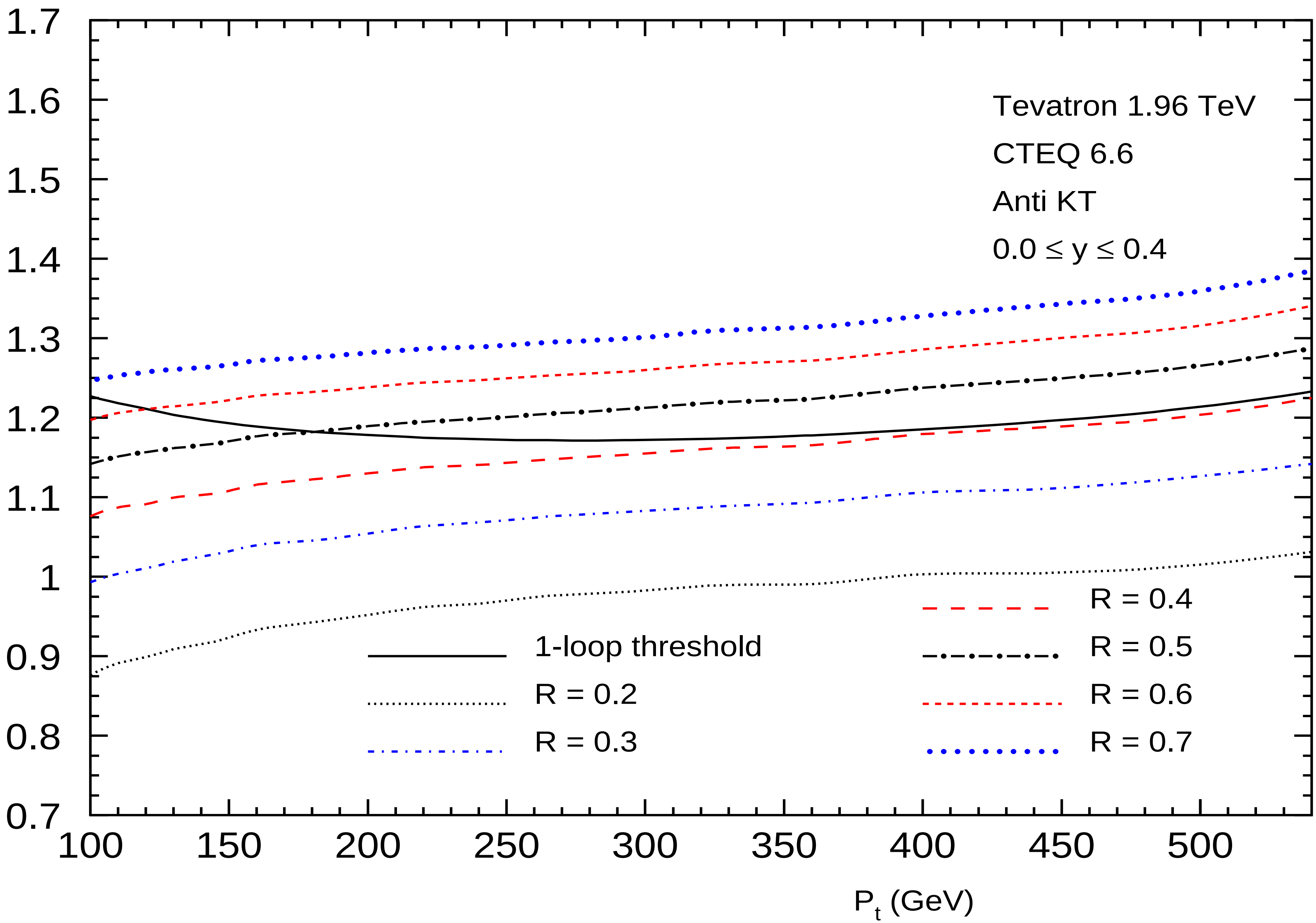
1. Soft and collinear contributions only, no matching functions yet.
2. No jet definition
3. No dependence on $R$

# Cone size dependence of jet distributions for LHC 7 TeV

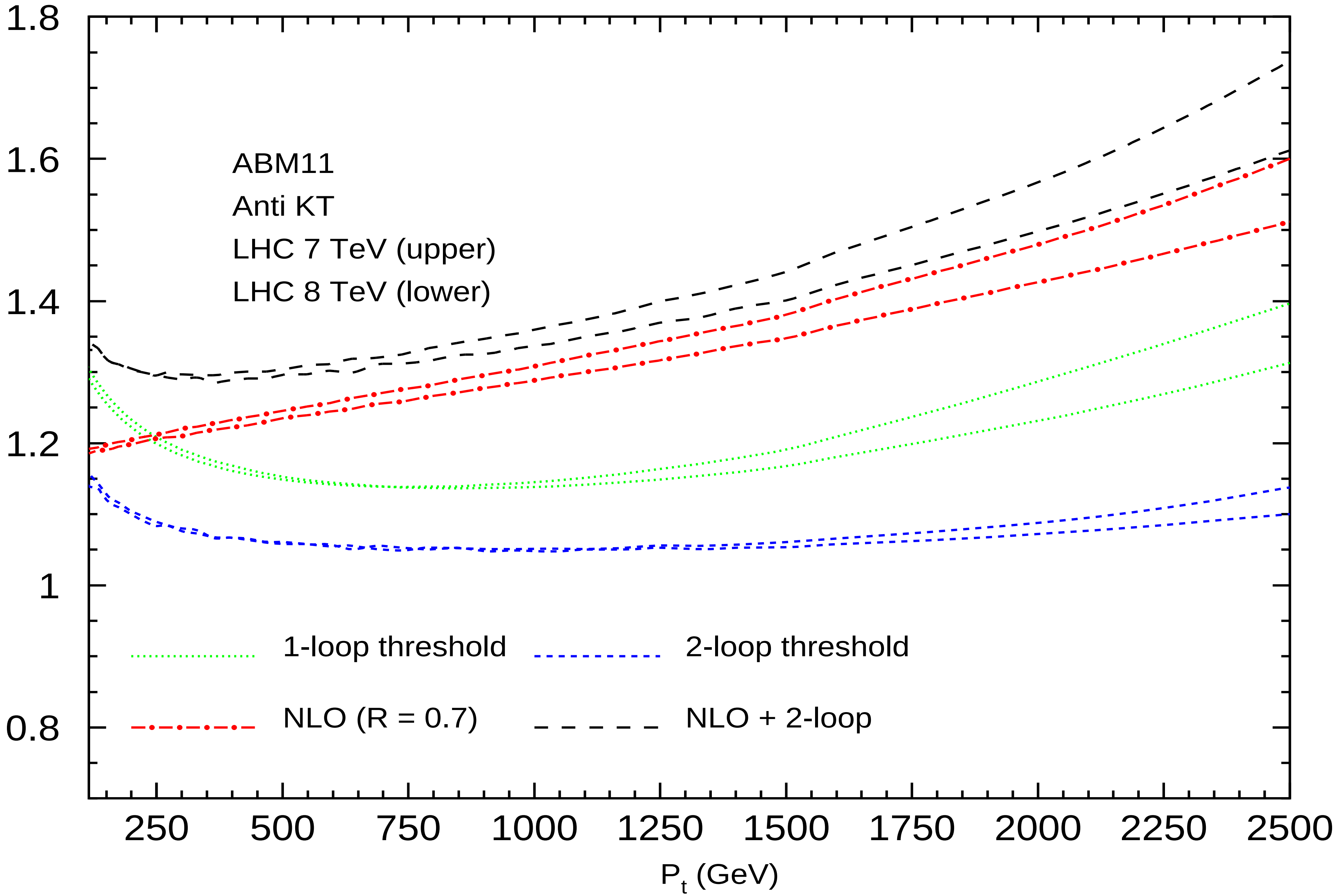



MC Kumar and S Moch; arXiv:1309.5311

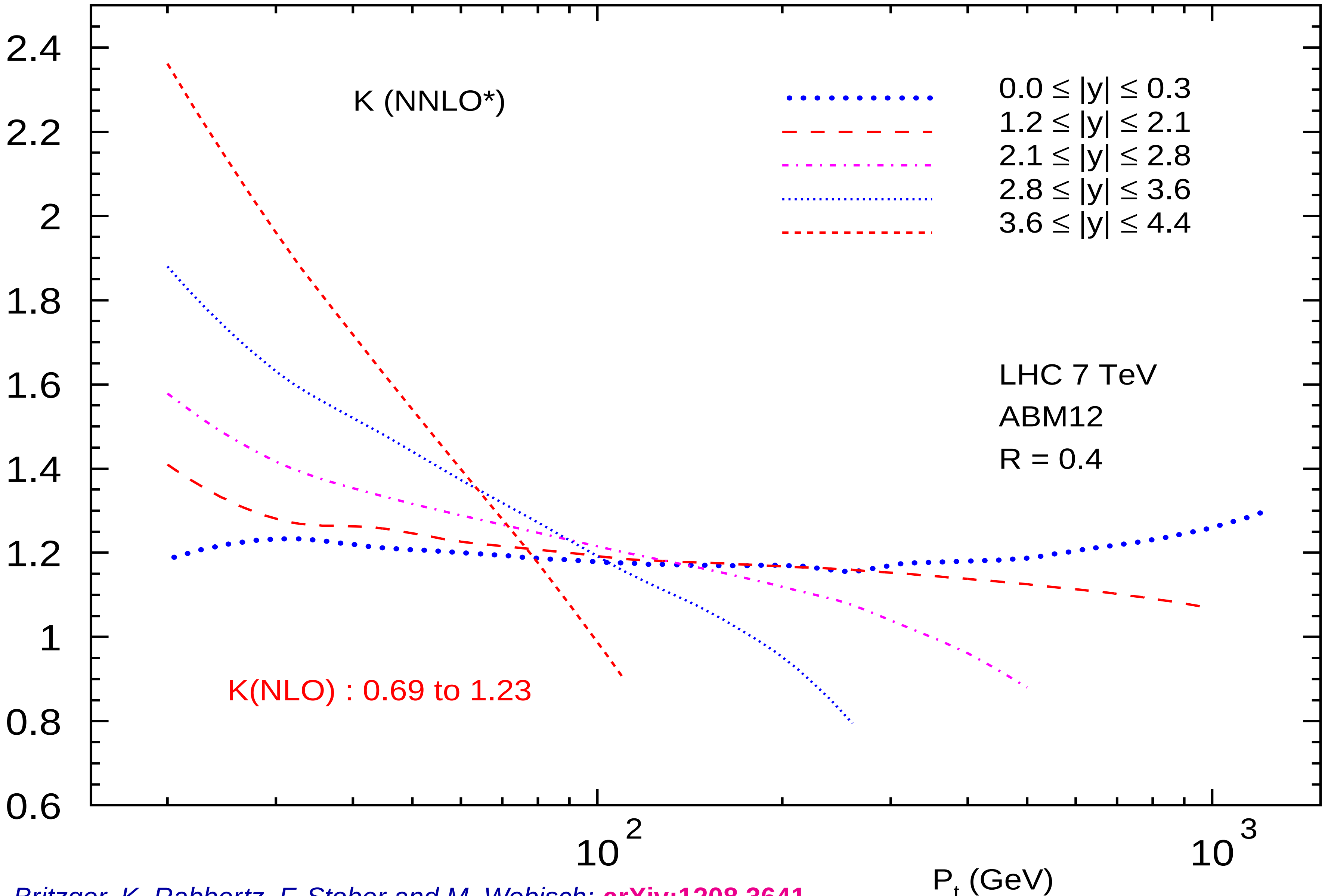
# Cone size dependence of jet distributions for Tevatron



# K-factors for LHC 7 TeV and 8 TeV

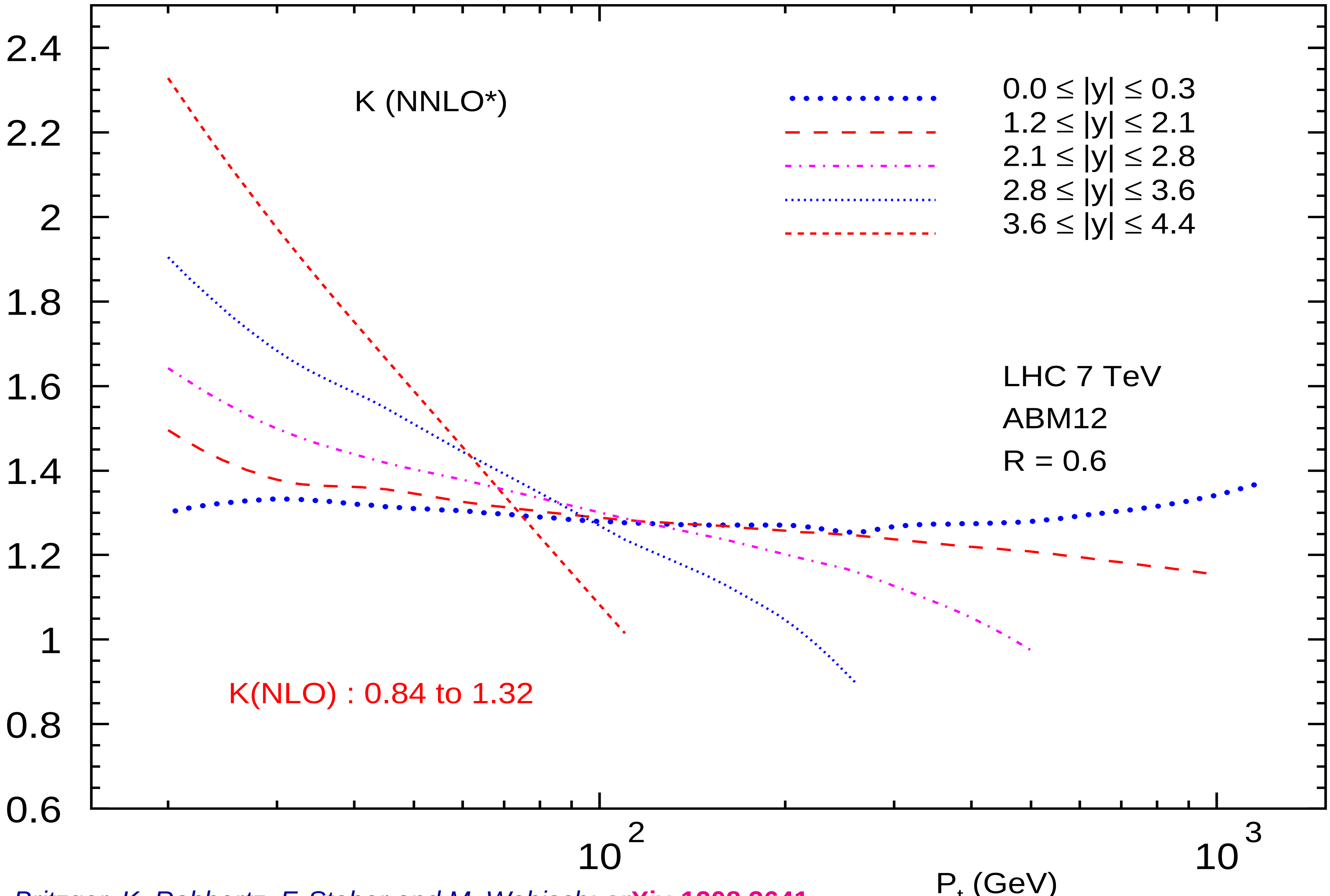


# NNLO* K-factors for ATLAS 7 TeV




*D. Britzger, K. Rabbertz, F. Stober and M. Wobisch;* **arXiv:1208.3641**.

# NNLO* K-factors for ATLAS 7 TeV



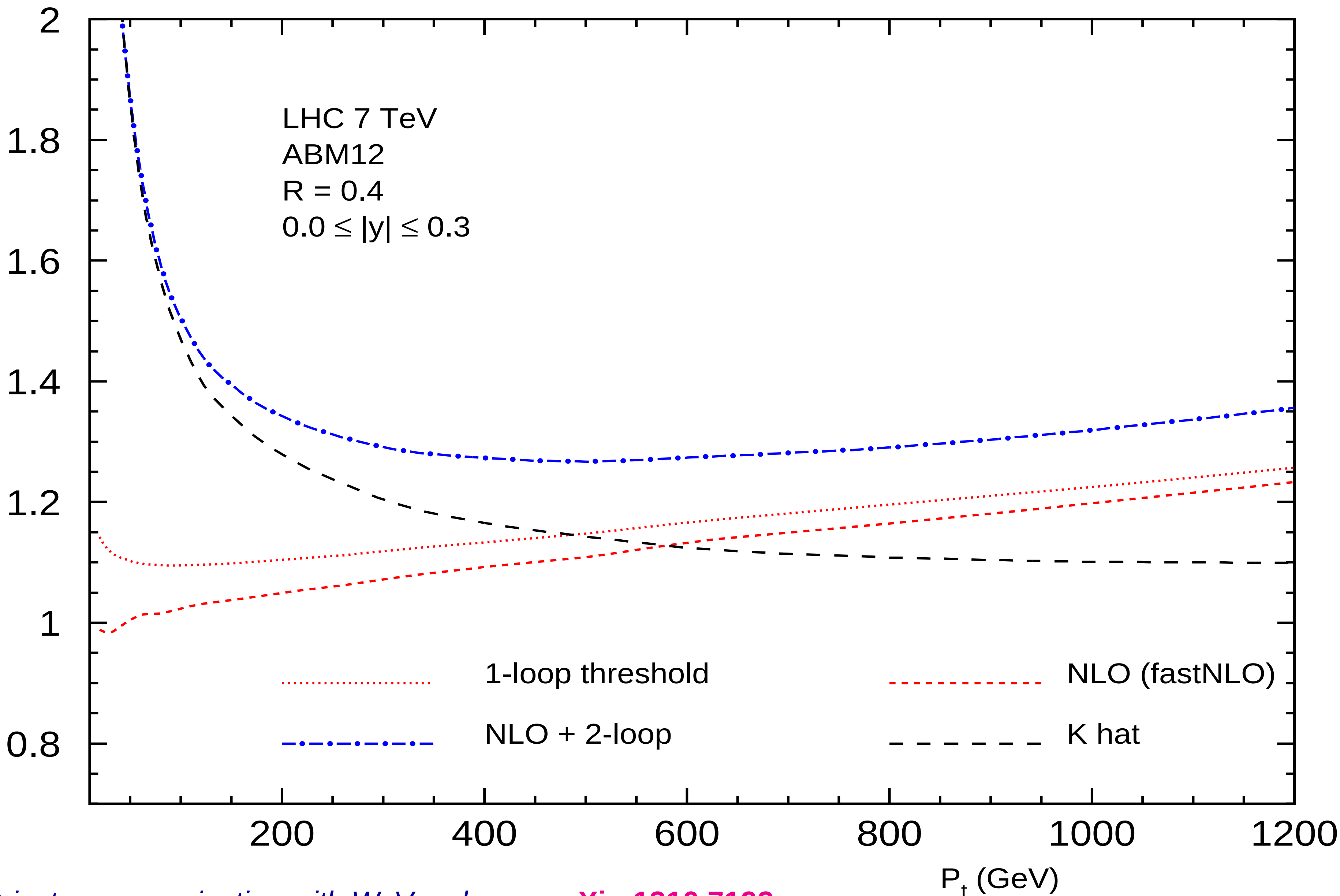

*D. Britzger, K. Rabbertz, F. Stober and M. Wobisch;* **arXiv:1208.3641**.

# “$R$” dependent threshold corrections

- Analytical computation using Small Cone Approximation
- Soft and collinear contributions
- Hard matching functions included.
- Cone size “$R$” dependence included by modifying the final state jet functions.
- Jets are massive

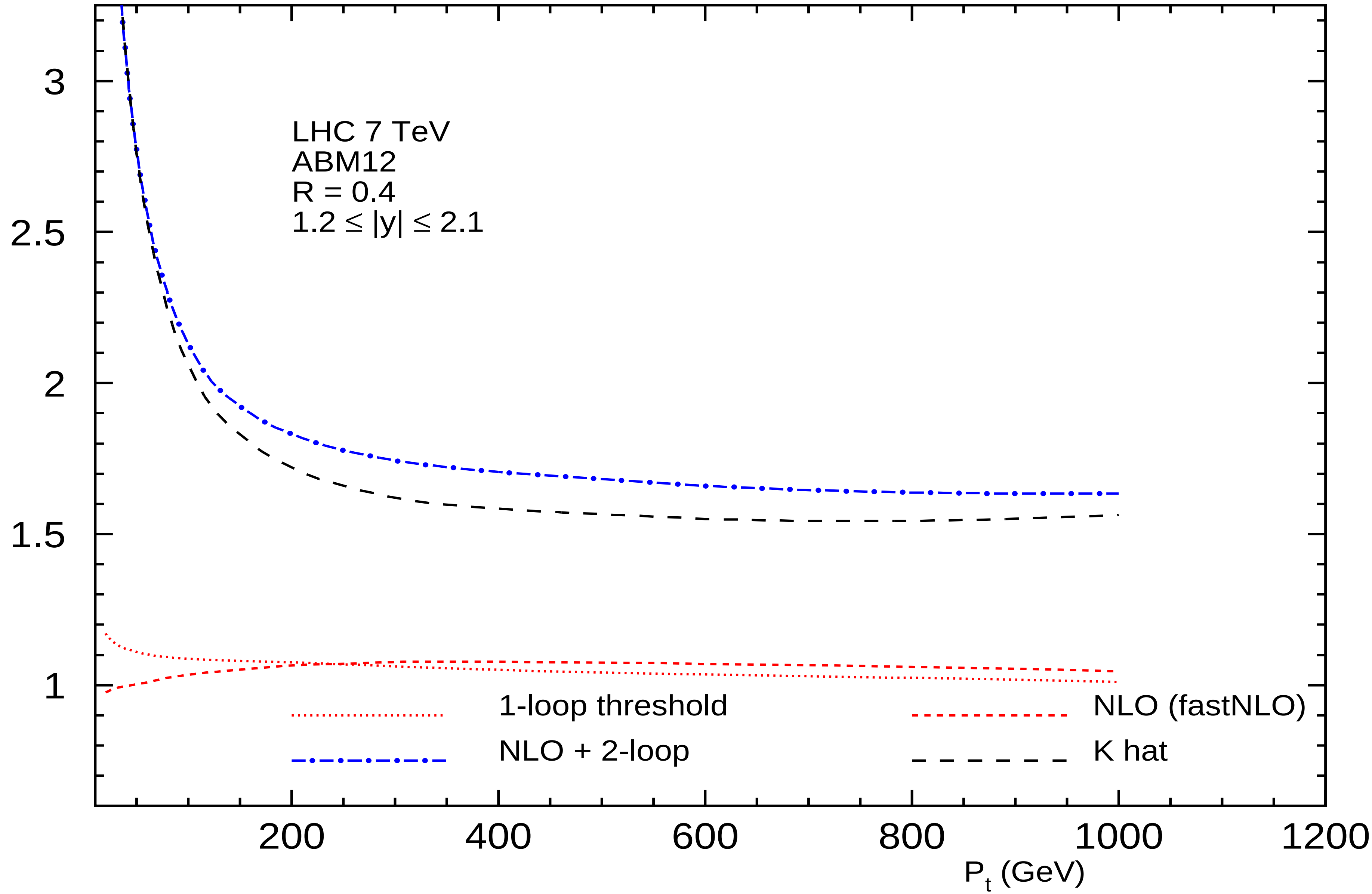
*Daniel de Florian, P. Hinderer, A. Mukherjee, F. Ringer, W Vogelsang;* **arXiv:1310.7192**.

# NNLO* K-factors for ATLAS 7 TeV




*Private communication with W. Vogelsang;* **arXiv:1310.7192**.

# NNLO* K-factors for ATLAS 7 TeV




*Private communication with W. Vogelsang;* **arXiv:1310.7192**.

## Summary

1. Threshold corrections for inclusive jet process have been re-calculated to NNLO at NLL accuarcy.

2. Transverse momentum distributions of the jet are presented for LHC and Tevatron, and the kinematical range of the validity of the threshold corrections has been investigated.
3. For large $p_T$ values of the jet, threshold corrections have similar behaviour to that of fixed order NLO results but underestimate them for the current values of $R$ used in the jet analysis.

4. Large theory uncertainties for smaller $p_T$ values of the jet, at LHC.

5. Need for improvement of the threshold corrections (with and without R) , e.g. in the high rapidity region.

# NNLO calculations ...

1. Higgs production

   **C. Anastasiou and K. Melnikov,** Nucl.Phys. B646 (2002) 220-256,

   **M. Spira, A. Djouadi, D. Graudenz, P.M. Zerwas,** NPB453 (1995) 17-82,

   **Robert V. Harlander and William B. Kilgore,** PRL 88 (2002) 201801,

   **V. Ravindran, J. Smith, W. L. van Neerven,** NPB665, 325 (2003)

2. Drell-Yan production

   **R. Hamberg, W. L. van Neerven and T. Matsuura,** NPB359, 343 (1991)

   **R. V. Harlander and W. B. Kilgore,** Phys. Rev. Lett. 88 (2002) 201801

3. Deep-Inelastic Scattering

   E.B. Zijlstra and W.L. van Neerven, Phys. Lett. B272 (1991) 12

   E.B. Zijlstra and W.L. van Neerven, Nucl. Phys. B383 (1992) 525

4. Diphoton production

   **S. Catani, L. Cieri, Daniel de Florian, G. Ferrera, M. Grazzini,**

   Phys.Rev.Lett. 108 (2012) 072001

5. Top pair production at hadron colliders

   **Michal Czakon, Paul Fiedler, Alexander Mitov,** Phys.Rev.Lett. 110 (2013) 252004

# Color basis, Soft function, Hard function ....

1. The t-channel color basis for a process $i\ j \to k\ l$ is given by

$$c_1 = \delta_{ik}\,\delta_{jl}, \quad c_2 = T^c_{ki}\,T^c_{jl}$$

2. The soft function for this basis is

$$S_{IJ} = |C_I >< C_J|, \quad \text{with} \quad |C_I >= \{c_1, c_2\}$$

3. If $\mathcal{M}$ denotes the QCD amplitude for a given subprocess, then the color decomposed matrix elements $H_I$ are obtained from

$$|H_I >= \mathcal{M}|P_I > \quad \text{and} \quad < H_I| =< P_I|\mathcal{M}^*$$

   where $|P_I >= \{p_1, p_2\}$ such that $p_i = c_i/S_{ii}$

4. The hard function for that given subprocess is then given as

$$H_{IJ} = |H_I >< H_J|$$

5. The squared matrix elements at Born level are then simply obtained from

$$|M|^2 = S_{IJ}H_{JI} = Tr[S.H]$$

   and the trace is taken in color space.