Top quark physics

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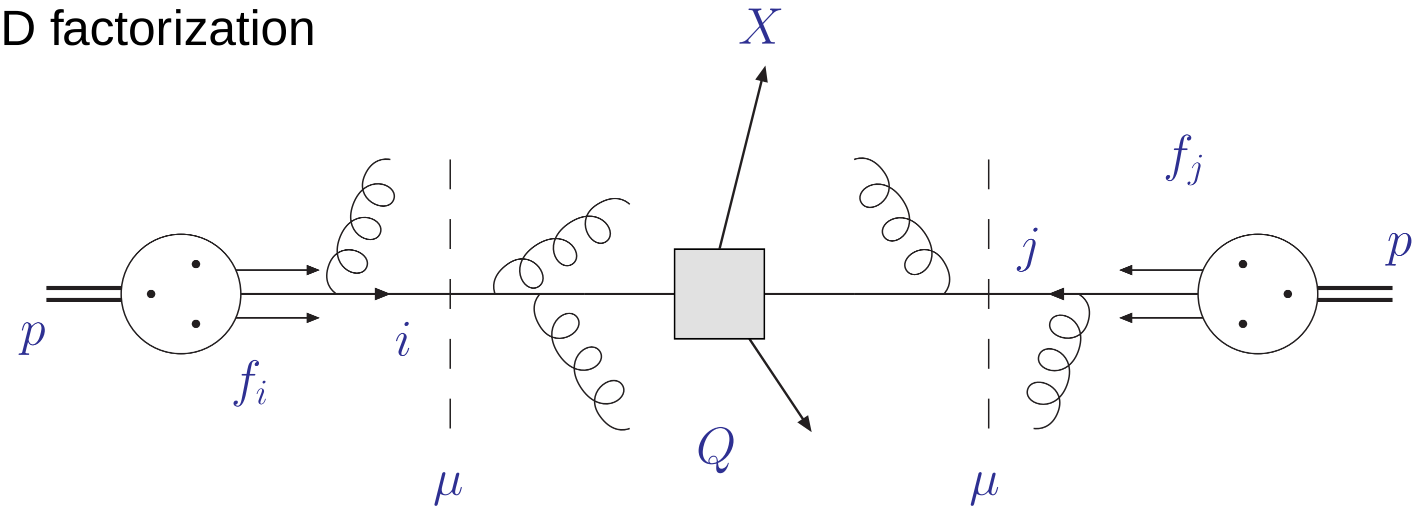
QCD factorization

- QCD factorization



$$\sigma_{pp \to X} = \sum_{ij} f_i(\mu^2) \otimes f_j(\mu^2) \otimes \hat{\sigma}_{ij \to X}\left(\alpha_s(\mu^2), Q^2, \mu^2, m_X^2\right)$$

- Hard parton cross section $\hat{\sigma}_{ij \to X}$ calculable in perturbation theory
  - known to NLO, NNLO, . . . ($\mathcal{O}(\text{few}\%)$ theory uncertainty)
- Non-perturbative parameters: parton distribution functions f_i, strong coupling α_s, particle masses m_X
  - known from global fits to exp. data, lattice computations, . . .

Non-perturbative parameters

Input for collider phenomenology

- Non-perturbative parameters are universal
- Determination from comparision to experimental data
    - masses of heavy quarks m_c, m_b, m_t
    - parton distribution functions $f_i(x, \mu^2)$
    - strong coupling constant $\alpha_s(M_Z)$

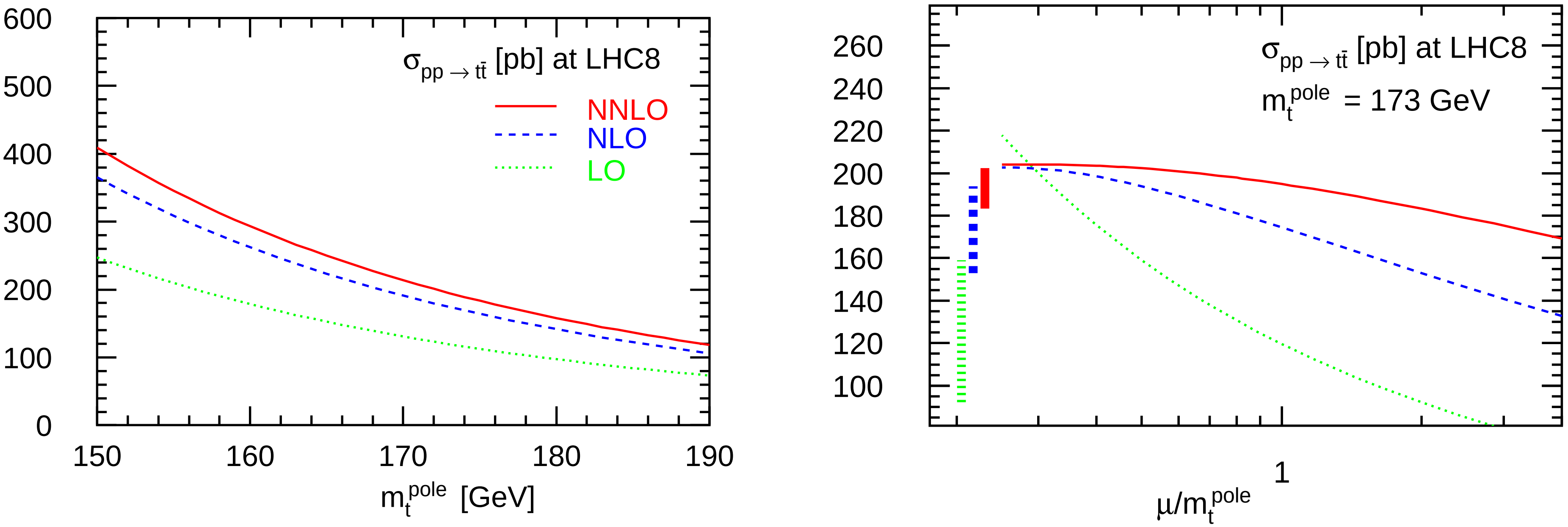
Interplay with perturbation theory

- Accuracy of determination driven by precision of theory predictions
- Non-perturbative parameters sensitive to
    - radiative corrections at higher orders
    - renormalization and factorization scales μ_R, μ_F
    - chosen scheme (e.g., $\overline{MS}$ scheme)
    - $\ldots$

Total cross section

Exact result at NNLO in QCD

Czakon, Fiedler, Mitov ‘13



- NNLO perturbative corrections (e.g. at LHC8)
  - K-factor (NLO → NNLO) of $\mathcal{O}(10\%)$
  - scale stability at NNLO of $\mathcal{O}(\pm 5\%)$

Heavy-quark masses in Standard Model

- Higgs boson gives mass to matter fields via Higgs-Yukawa coupling
  - large top quark mass m_t

QCD

- Classical part of QCD Lagrangian

$$\mathcal{L} \quad = \quad -\frac{1}{4} F_{\mu\nu}^{a} F_{b}^{\mu\nu} + \sum_{\text{flavors}} \bar{q}_i \left(\mathrm{i}\not{D} - m_q\right)_{ij} q_j$$

  - field strength tensor $F_{\mu\nu}^{a}$ and matter fields $q_i, \bar{q}_j$
  - covariant derivative $D_{\mu,ij} = \partial_\mu \delta_{ij} + \mathrm{i} g_s \left(t_a\right)_{ij} A_\mu^a$

- Formal parameters of the theory (no observables)
  - strong coupling $\alpha_s = g_s^2/(4\pi)$
  - quark masses m_q

Challenge

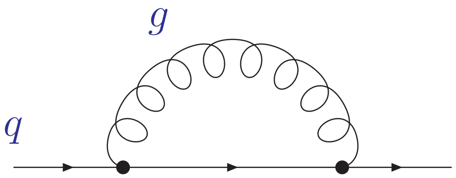
- Suitable observables for measurements of α_s, m_q, . . .
  - comparison of theory predictions and experimental data

Heavy-quark mass renormalization

Pole mass

- Based on (unphysical) concept of top-quark being a free parton

$$\not{p} - m_q - \Sigma(p, m_q)\Big|_{p^2=m_q^2}$$



  - heavy-quark self-energy $\Sigma(p, m_q)$ receives contributions from regions of all loop momenta – also from momenta of $\mathcal{O}(\Lambda_{QCD})$
- Definition of pole mass ambiguous up to corrections $\mathcal{O}(\Lambda_{QCD})$
  - bound from lattice QCD: $\Delta m_q \geq 0.7 \cdot \Lambda_{QCD} \simeq 200$ MeV

    Bauer, Bali, Pineda '11

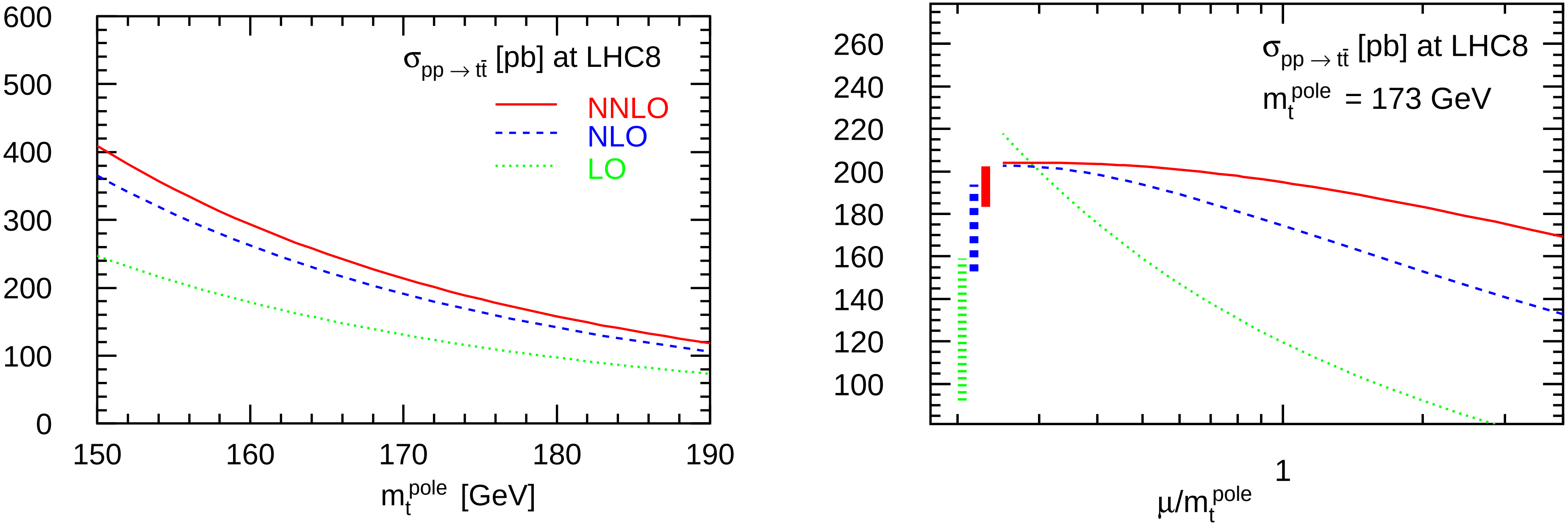
Running quark masses

- $\overline{MS}$ mass definition $m(\mu_R)$ realizes running mass (scale dependence)
  - short distance mass probes at scale of hard scattering

    $m_{\rm pole} = m_{\rm short\ distance} + \delta m$
  - conversion between $m_{\rm pole}$ and $\overline{MS}$ mass $m(\mu_R)$ perturbation theory

Total cross section with running mass

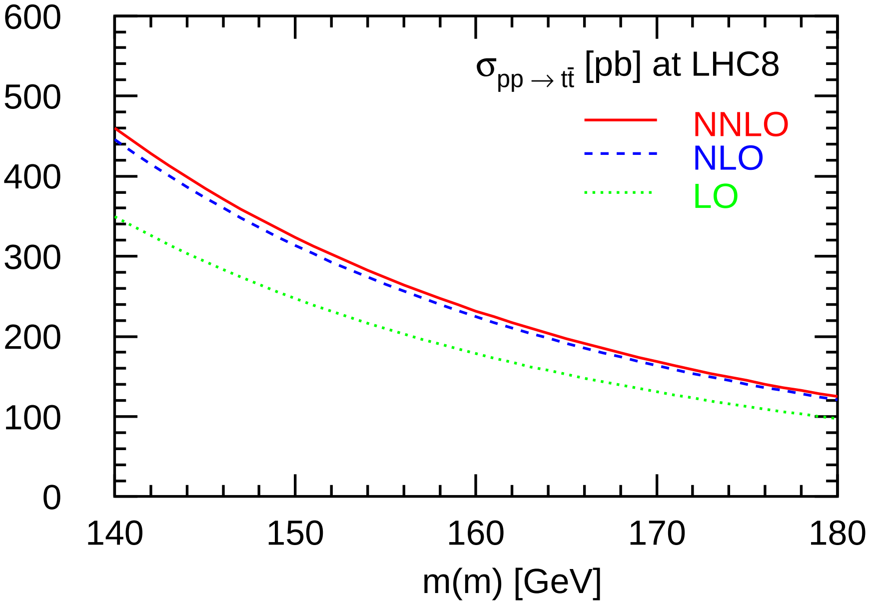
Comparison pole mass vs. $\overline{MS}$ *mass*

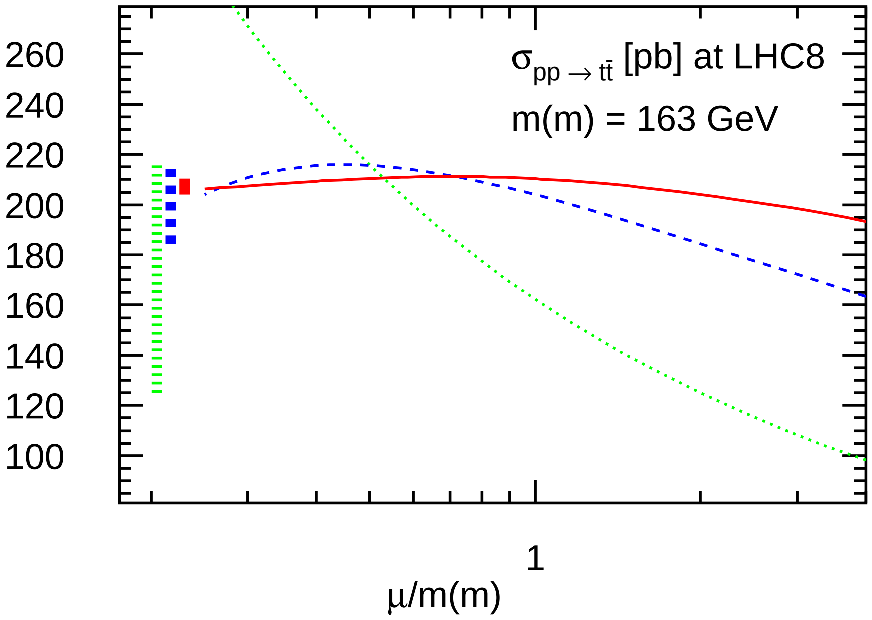


- good apparent convergence of perturbative expansion
- small theoretical uncertainity form scale variation

Total cross section with running mass

Comparison pole mass vs. $\overline{MS}$ *mass*





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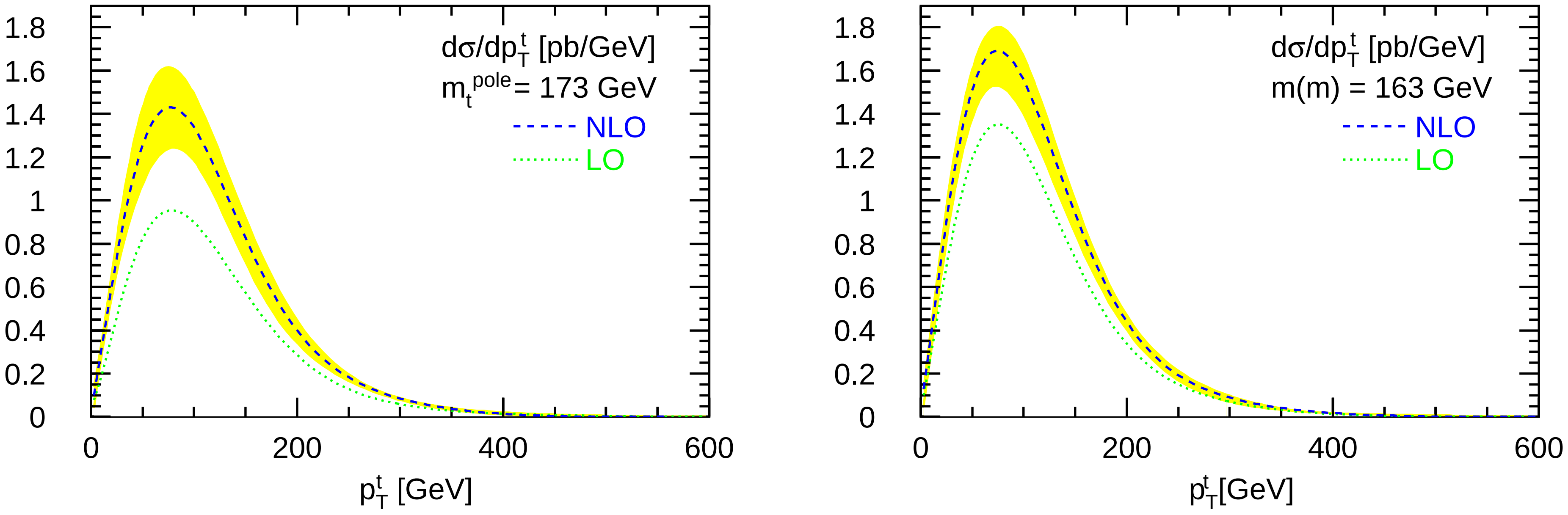
Differential cross sections

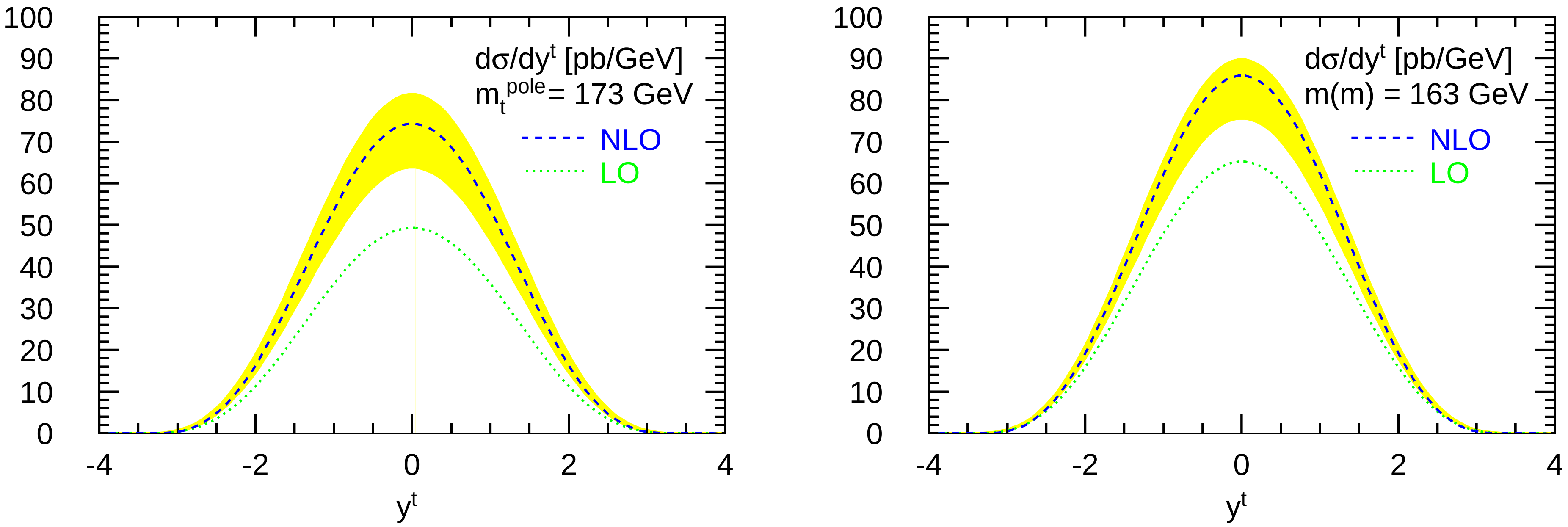
NLO in QCD



- Running mass for differential distributions shows same features, e.g. p_T^t-distribution Dowling, S.M. ‘13

Differential cross sections

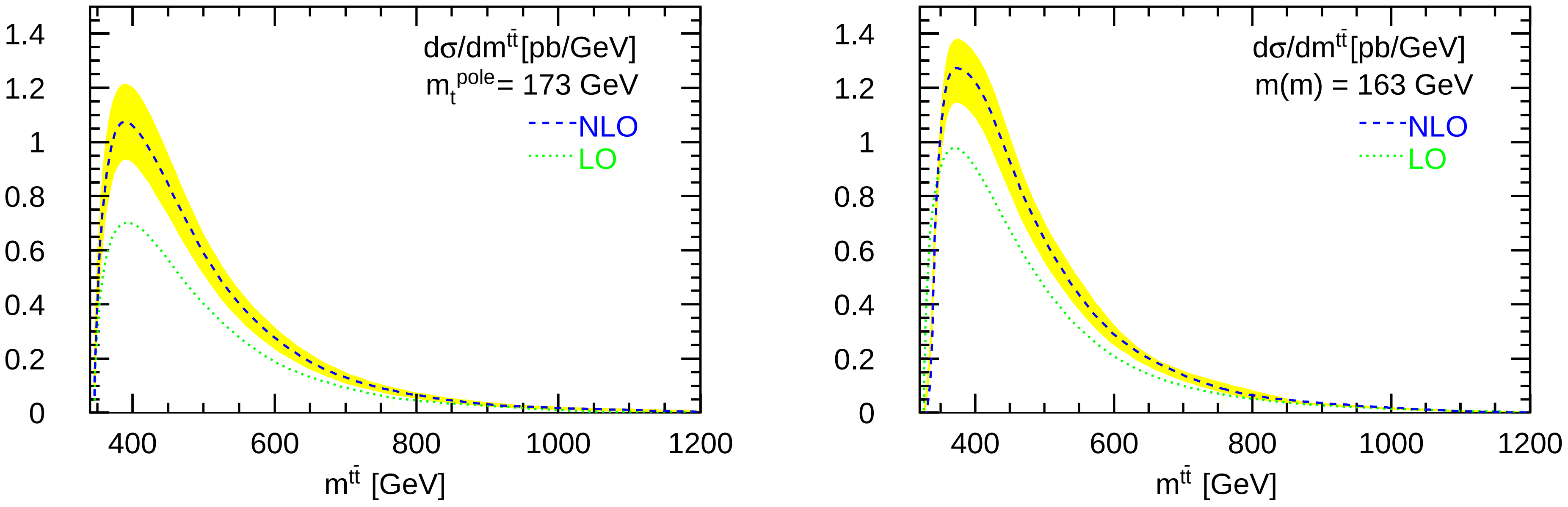
NLO in QCD



- Running mass for differential distributions shows same features, e.g. y^t-distribution Dowling, S.M. ‘13

Differential cross sections

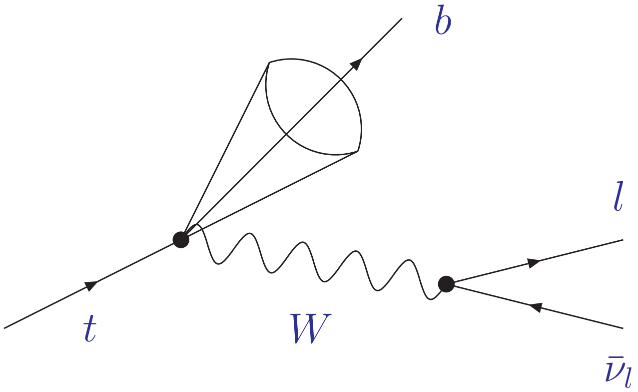
NLO in QCD



- Running mass for differential distributions shows same features, e.g. $m_{t\bar{t}}$-distribution Dowling, S.M. ‘13

Top-quark mass from kinematic reconstruction

- Current methods based on reconstructed physics objects
  - jets, identified charged leptons, missing transverse energy
  - $m_t^2 = (p_{W-\mathrm{boson}} + p_{b-\mathrm{jet}})^2$



Template method

- Distributions of kinematically reconstructed top mass values compared to templates for nominal top mass values
  - distributions rely on parton shower predictions
  - no NLO corrections applied

Matrix element method

- Event-by-event likelihood for kinematic configurations arising from events of a given top mass.
  - tree level matrix elements only
  - combinatorics of assignment of jets to top quarks

Methods based on observables

- Top mass from leptonic decay: m_{lb} distribution
- Top mass from jet rates
- Top mass from total cross section

Top mass from leptonic decay

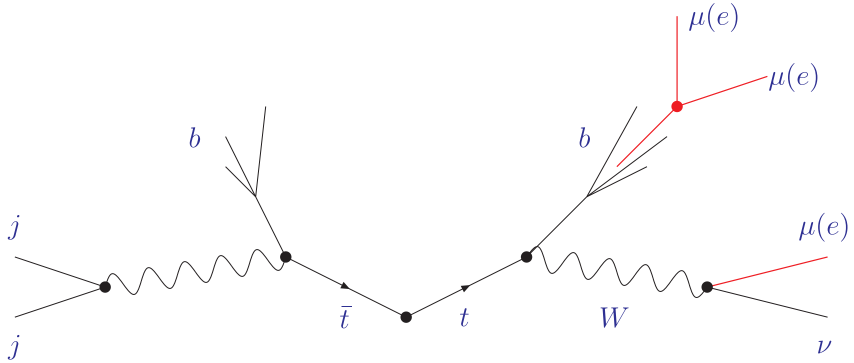
- Top mass from exclusive hadronic states

$$pp \to (t \to W^+ + b \to W^+ + J/\psi) + (\bar{t} \to W^- + \bar{b})$$

  - identification of μ-pair in J/ψ decay; leptonic or hadronic decay of W

    Kharchilava ‘00

    Chierici, Dierlamm ‘06

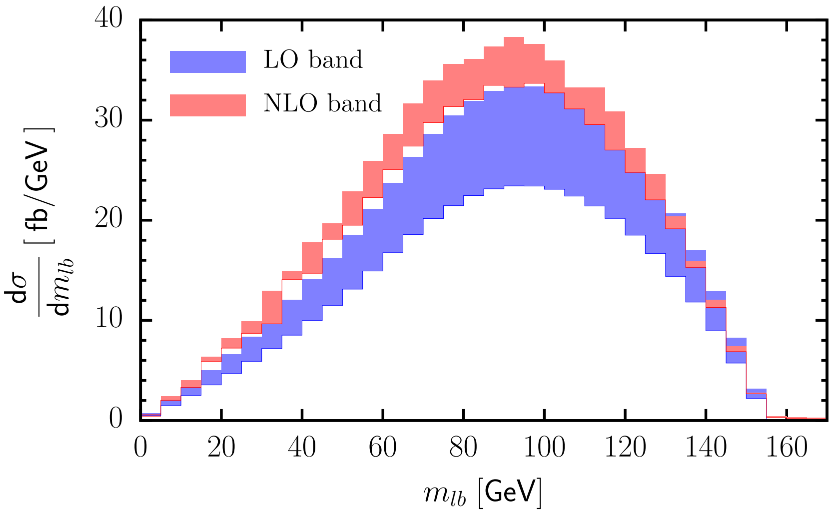


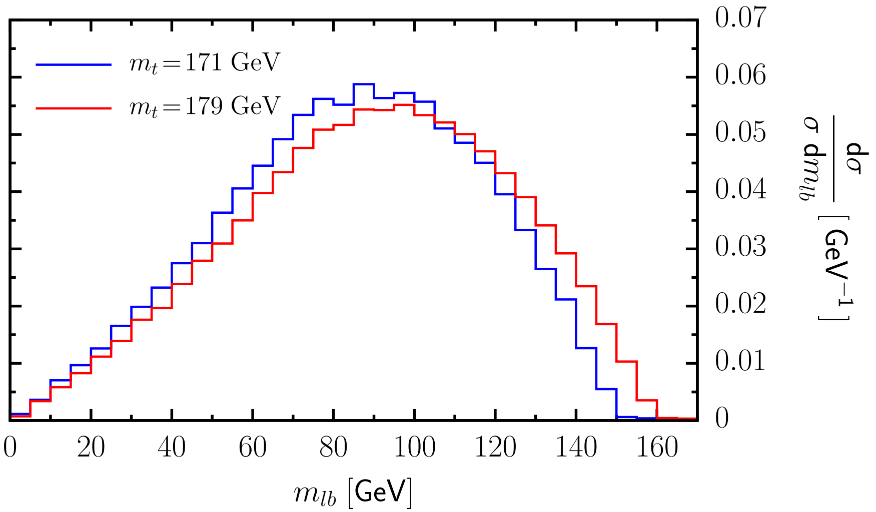
Top mass from leptonic decay

- Top mass from exclusive hadronic states

$$pp \to (t \to W^+ + b \to W^+ + J/\psi) + (\bar{t} \to W^- + \bar{b})$$

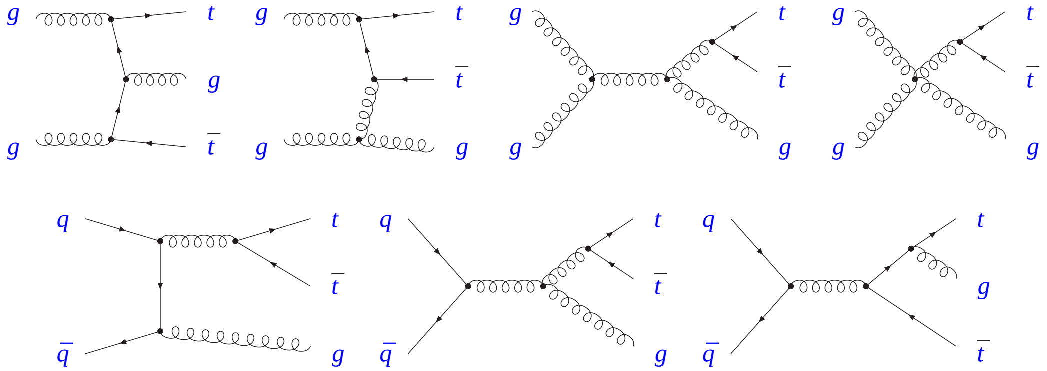
- Study of m_{lb} distribution at NLO in QCD Biswas, Melnikov, Schulze '10
  - NLO QCD corrections to production **and** decay very important for value of m_t (effects of order $\Delta m_t = \mathcal{O}(\text{few})$ GeV
- Invariant mass distribution of lepton and b-jet (LHC14)
  - scale dependence at LO and NLO (left)
  - normalized m_{lb} distributions, $m_t = 171$ GeV and 179 GeV (right)

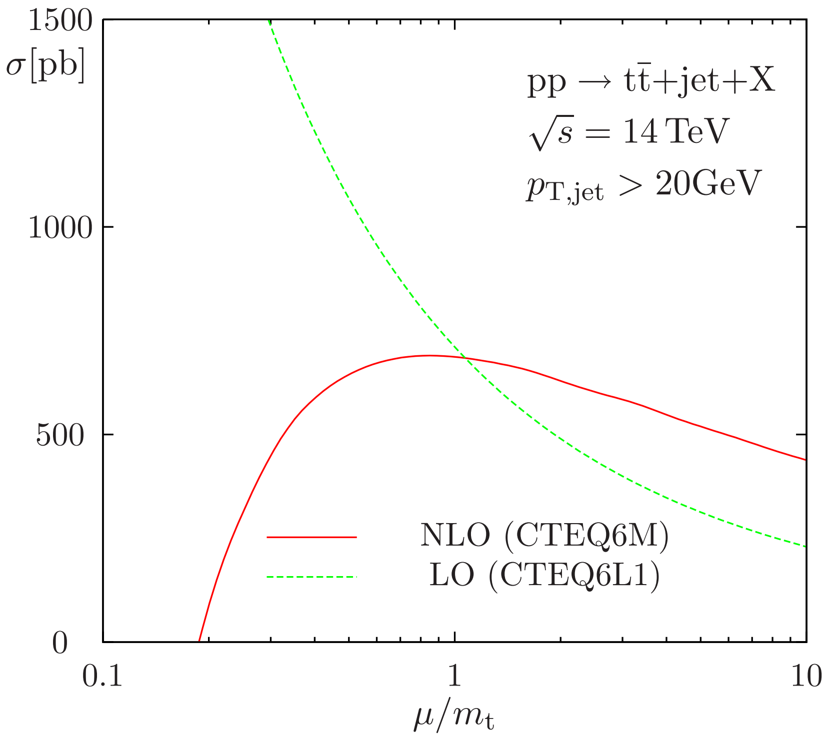




Top-quark pairs with one jet

- LHC: large rates for production of $t\bar{t}$-pairs with additional jets
- NLO QCD corrections for $t\bar{t} + 1\mathrm{jet}$ Dittmaier, Uwer, Weinzierl '07-'08
  - scale dependence greatly reduced at NLO
  - corrections for total rate at scale $\mu_r = \mu_f = m_t$ are almost zero





- Additional jet raises kinematical threshold
  - invariant mass $\sqrt{s_{t\bar{t}+1\mathrm{jet}}}$

Mass measurement with $t\bar{t} + \mathrm{jet}$*-samples*

- Mass measurement with new observable

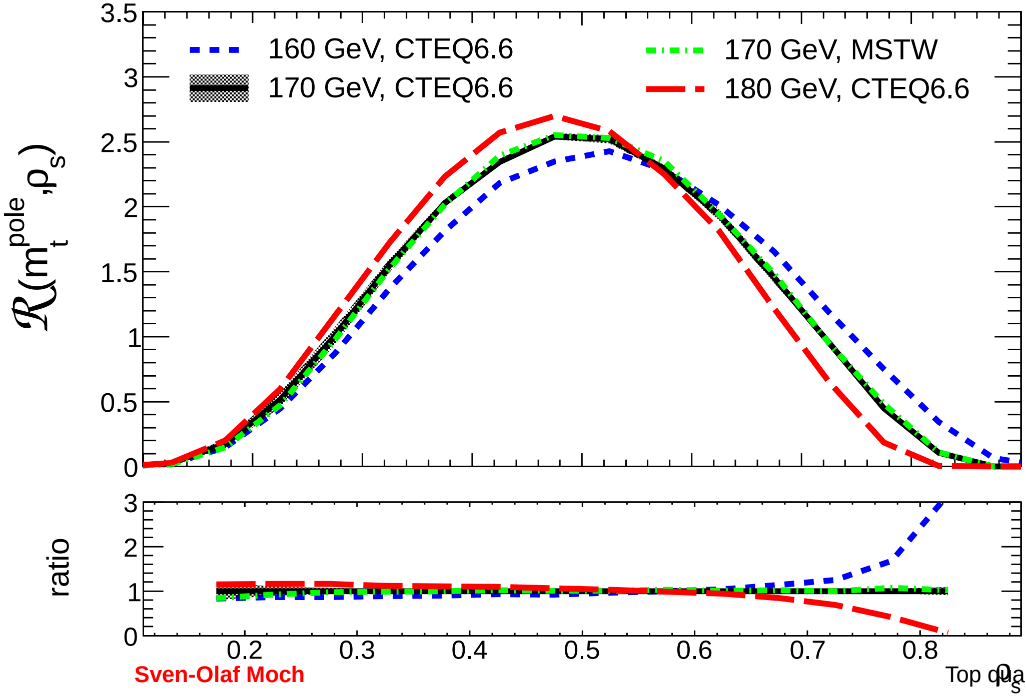
Alioli, Fernandez, Fuster, Irles, S.M., Uwer, Vos ‘13

  - variable $\rho_s = \frac{2 \cdot m_0}{\sqrt{s_{t\bar{t}+1\mathrm{jet}}}}$ with invariant mass of $t\bar{t} + 1\mathrm{jet}$ system and fixed scale $m_0 = 170\,\mathrm{GeV}$

- Normalized-differential $t\bar{t} + \mathrm{jet}$ cross section

$$\mathcal{R}(m_t, \rho_s) = \frac{1}{\sigma_{t\bar{t}+1\mathrm{jet}}} \frac{d\sigma_{t\bar{t}+1\mathrm{jet}}}{d\rho_s}(m_t, \rho_s)$$

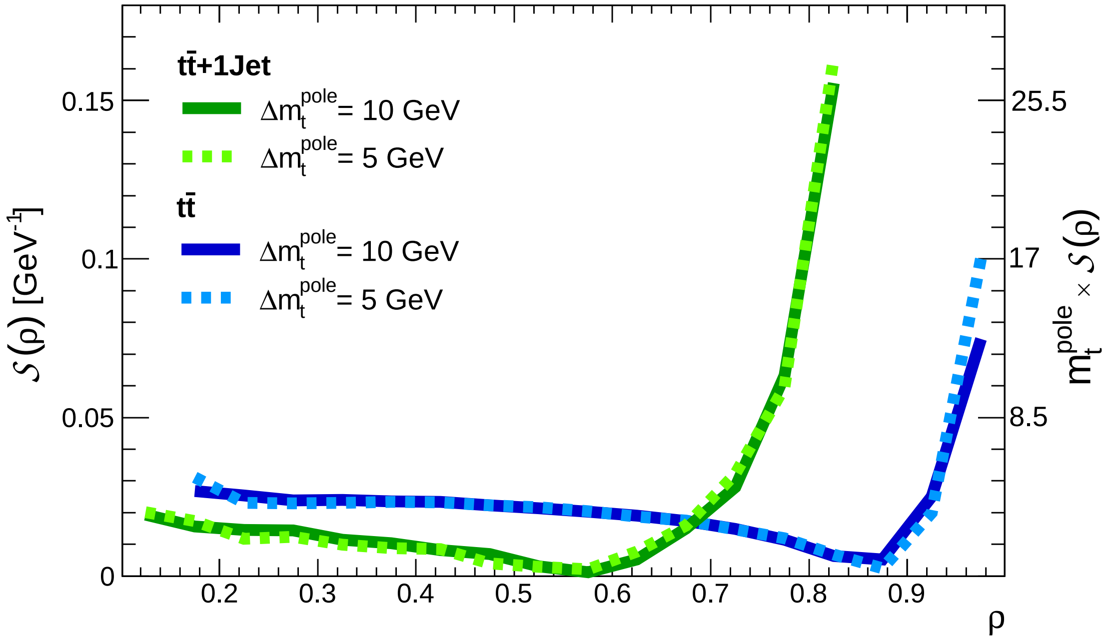
  - significant mass dependence for $0.4 \leq \rho_s \leq 0.5$ and $0.7 \leq \rho_s$



- Differential cross section $\mathcal{R}(m_t, \rho_s)$
  - good pertubative stability, small theory uncertainties, small dependence on experimental uncertainties, . . .
- Sensitivity to top-quark mass very good

$$\left|\frac{\Delta\mathcal{R}}{\mathcal{R}}\right| \simeq (m_t\mathcal{S}) \times \left|\frac{\Delta m_t}{m_t}\right|$$

  - increased sensitivity for system $t\bar{t} + \mathrm{jet}$ compared to $t\bar{t}$

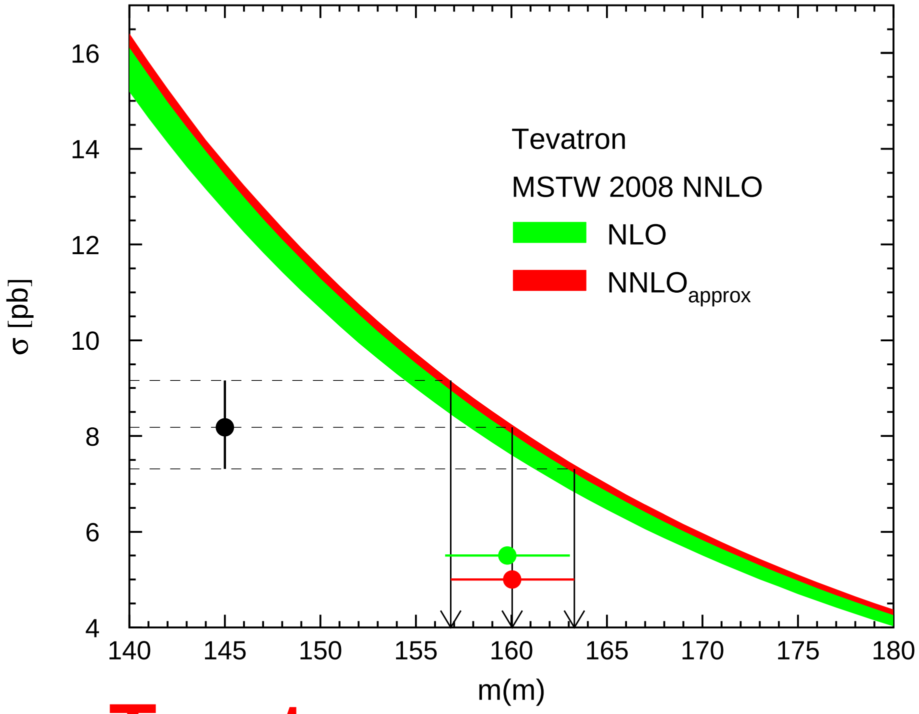


Upshot

- Precision determination of well-defined top-quark mass m_t possible
  - alternative to inclusive cross sections

Top mass from total cross section

- Total top quark cross section as function of $\overline{MS}$ mass
  Langenfeld, S.M., Uwer '09
- Illustration of idea



Tevatron

The fine print

- Intrinsic limitation of sensitivity in total cross section

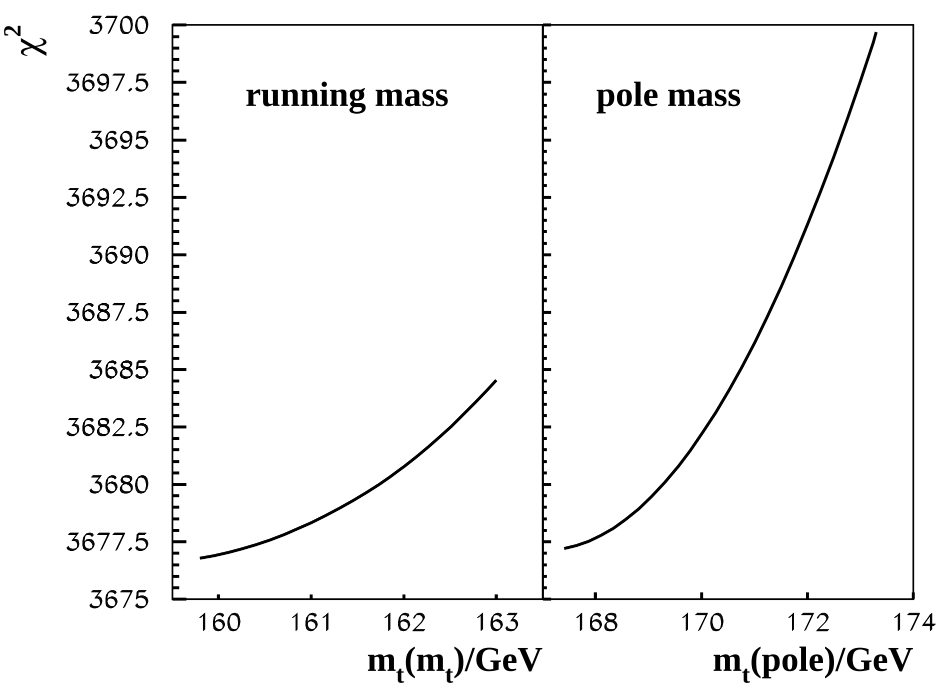
$$\left|\frac{\Delta\sigma_{t\bar{t}}}{\sigma_{t\bar{t}}}\right| \simeq 5 \times \left|\frac{\Delta m_t}{m_t}\right|$$

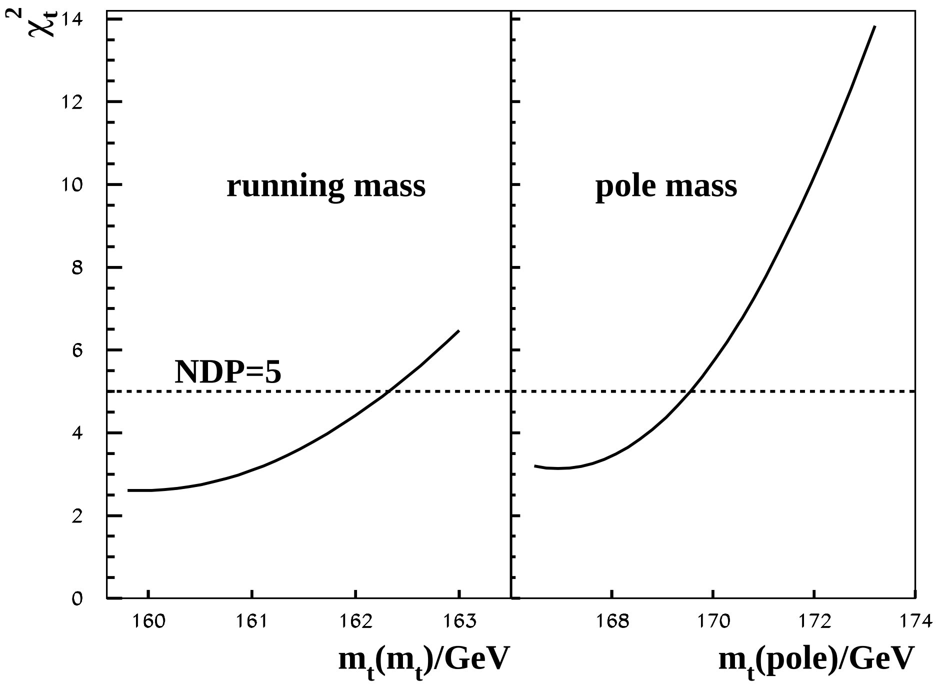
Correlations are essential

- Cross section at LHC has correlation of m_t, $\alpha_S(M_Z)$ and gluon PDF
  $\sigma_{t\bar{t}} \;\sim\; \alpha_s^2\, m_t^2\, g(x) \otimes g(x)$
  - effective parton $\langle x \rangle \sim 2m_t/\sqrt{s} \sim 2.5 \ldots 5 \cdot 10^{-2}$
  - fit with fixed values of m_t and $\alpha_S(M_Z)$ carries significant bias
    Czakon, Mangano, Mitov, Rojo '13
  - likewise, fit with PDF re-weighting and for fixed values of m_t insufficient Beneke, Falgari, Klein, Piclum, Schwinn, Ubiali, Yan '12

Top cross section data in ABM12 fit

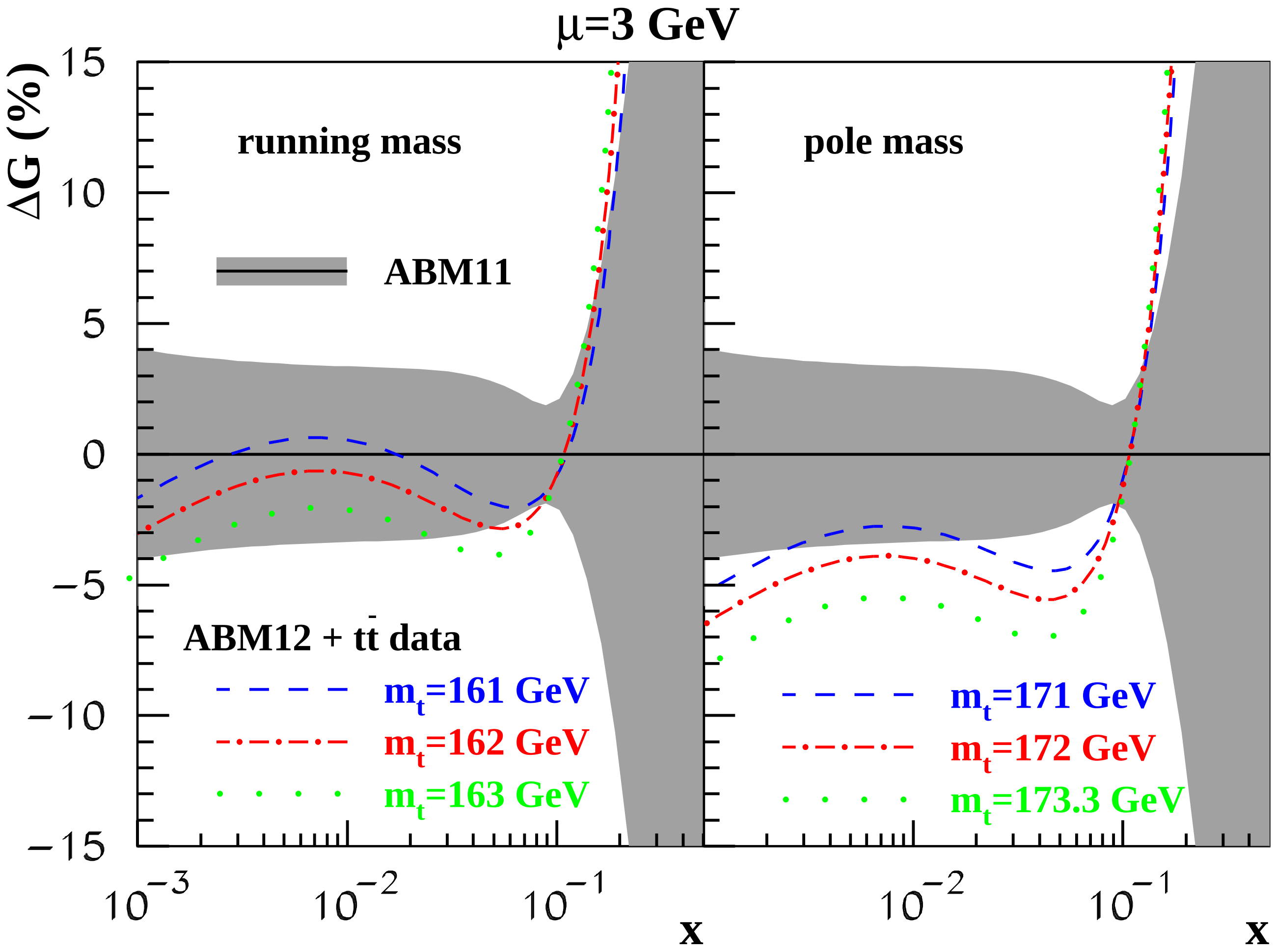
- Fit with correlations
  - $g(x)$ and $\alpha_s(M_Z)$ already well constrained by global fit (no changes)
  - for fit with $\chi^2/NDP = 5/5$ obtain value of $m_t(m_t) = 162.3 \pm 2.3$ GeV (equivalent to pole mass $m_t = 171.2 \pm 2.4$ GeV) Alekhin, Blümlein, S.M. ‘13
  - χ^2-profile steeper for pole mass (bigger impact of top-quark data and greater sensitivity to theoretical uncertainty at NNLO)





Top cross section data in ABM12 fit

- Fit with correlations
  - $g(x)$ and $\alpha_s(M_Z)$ already well constrained by global fit (no changes)
  - correlation of gluon PDF with value of m_t
    (illustration of bias in recent analysis Czakon, Mangano, Mitov, Rojo ‘13)



Higgs potential

Renormalization group equation

- Quantum corrections to Higgs potential $V(\Phi) = \lambda \left|\Phi^\dagger\Phi - \frac{v}{2}\right|^2$
- Radiative corrections to Higgs self-coupling λ
  - electro-weak couplings g and g' of $SU(2)$ and $U(1)$
  - top-Yukawa coupling y_t

$$16\pi^2 \frac{d\lambda}{dQ} = 24\lambda^2 - \left(3g'^2 + 9g^2 - 12y_t^2\right)\lambda + \frac{3}{8}g'^4 + \frac{3}{4}g'^2 g^2 + \frac{9}{8}g^4 - 6y_t^4 + \ldots$$

Higgs potential

Triviality

- Large mass implies large λ
  - renormalization group equation dominated by first term

$$16\pi^2 \frac{d\lambda}{dQ} \simeq 24\lambda^2 \qquad \longrightarrow \qquad \lambda(Q) = \frac{m_H^2}{2v^2 - \frac{3}{2\pi^2} m_H^2 \ln(Q/v)}$$

- $\lambda(Q)$ increases with Q
- Landau pole implies cut-off Λ
  - scale of new physics smaller than Λ to restore stability
  - upper bound on m_H for fixed Λ

$$\Lambda \leq v \exp\left(\frac{4\pi^2 v^2}{3m_H^2}\right)$$

- Triviality for $\Lambda \to \infty$
  - vanishing self-coupling $\lambda \to 0$ (no interaction)

Higgs potential

Vacuum stability

- Small mass
  - renormalization group equation dominated by y_t

$$16\pi^2 \frac{d\lambda}{dQ} \simeq -6y_t^4 \qquad \longrightarrow \qquad \lambda(Q) = \lambda_0 - \frac{\frac{3}{8\pi^2} y_0^4 \ln(Q/Q_0)}{1 - \frac{9}{16\pi^2} y_0^2 \ln(Q/Q_0)}$$

- $\lambda(Q)$ decreases with Q
- Higgs potential unbounded from below for $\lambda < 0$
- $\lambda = 0$ for $\lambda_0 \simeq \frac{3}{8\pi^2} y_0^4 \ln(Q/Q_0)$
- Vacuum stability

$$\Lambda \leq v \exp\left(\frac{4\pi^2 m_H^2}{3y_t^4 v^2}\right)$$

  - scale of new physics smaller than Λ to ensure vacuum stability
  - lower bound on m_H for fixed Λ

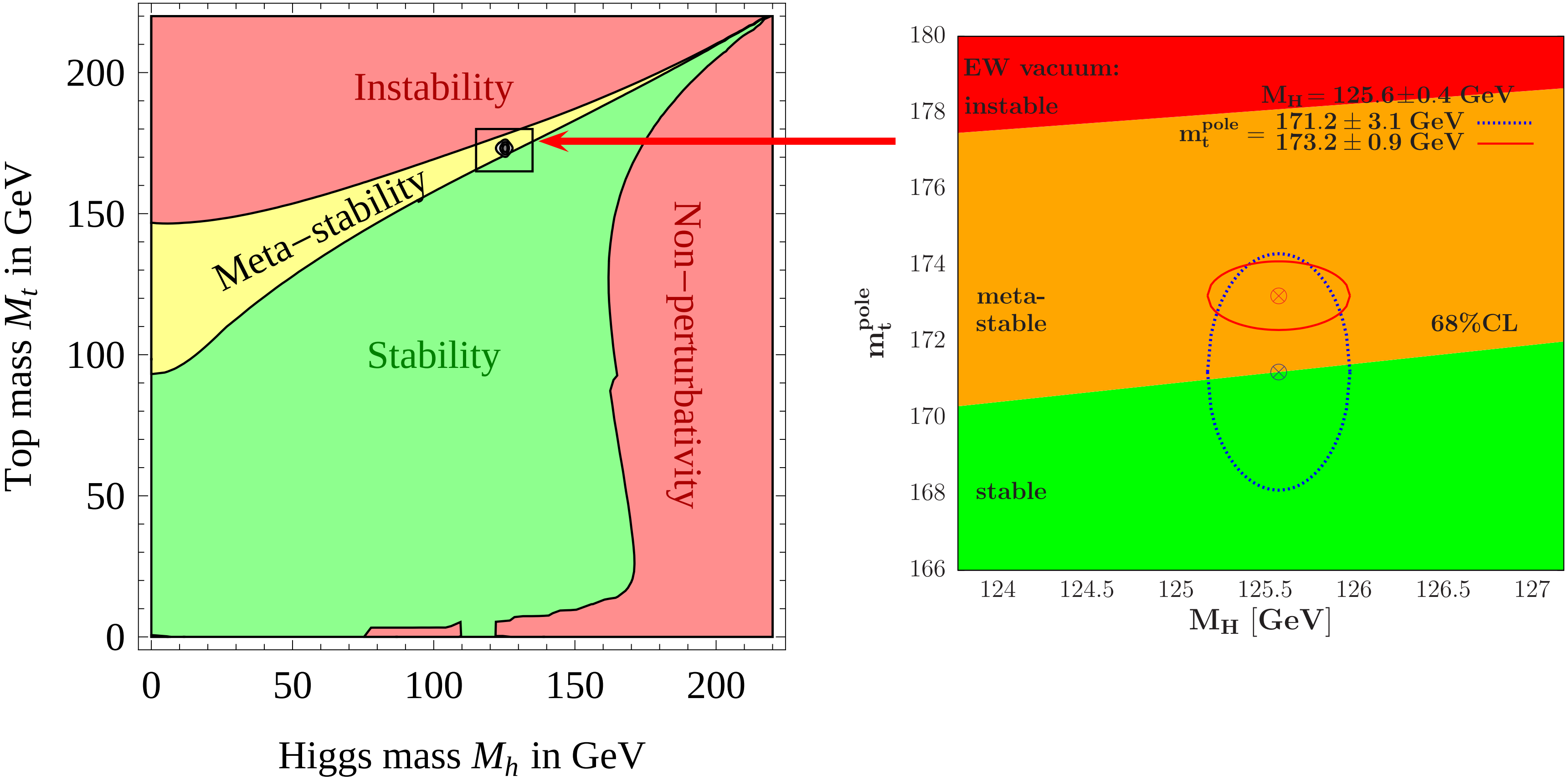
Implications on electroweak vacuum

- Relation between Higgs mass m_H and top quark mass m_t
  - condition of absolute stability of electroweak vacuum $\lambda(\mu) \geq 0$
  - extrapolation of Standard Model up to Planck scale M_P
  - $\lambda(M_P) \geq 0$ implies lower bound on Higgs mass m_H

$$m_H \; \geq \; 129.2 + 1.8 \times \left(\frac{m_t^{\rm pole} - 173.2 \text{ GeV}}{0.9 \text{ GeV}} \right) - 0.5 \times \left(\frac{\alpha_s(M_Z) - 0.1184}{0.0007} \right) \pm 1.0 \;\; \text{GeV}$$

  - recent NNLO analyses Bezrukov, Kalmykov, Kniehl, Shaposhnikov '12; Degrassi, Di Vita, Elias-Miro, Espinosa, Giudice et al. '12
  - uncertainity in results due to α_s and m_t (pole mass scheme)
- Top quark mass from Tevatron in well-defined scheme
  - $m_t^{\overline{\rm MS}}(m_t) \; = \; 162.3 \pm 2.3 \;\; \pm 0.7 \text{ GeV}$ implies in pole mass scheme $m_t^{\rm pole} \; = \; 171.2 \pm 2.4 \pm 0.7 \text{ GeV}$
  - good consistency of mass value between different PDF sets

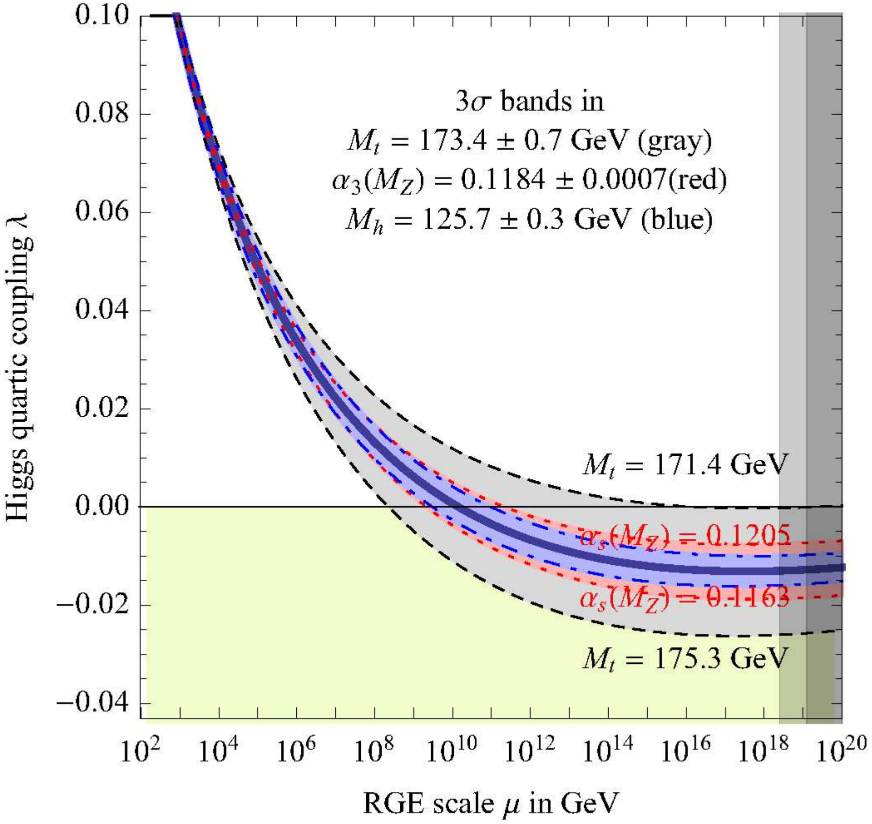
Fate of the universe



Degrassi, Di Vita, Elias-Miro, Espinosa, Giudice et al. ‘12; Alekhin, Djouadi, S.M. ‘12; Masina ‘12

- Uncertainty in Higgs bound due to m_t from in $\overline{MS}$ scheme
  - bound relaxes $m_H \geq 125.2 \pm 6.2$ GeV
  - “fate of universe” still undecided

Higgs self-coupling



Buttazzo, Degrassi, Giardino, Giudice, Sala, Salvio, Strumia ‘13

- Renormalization group evolution of λ with uncertainties in m_H, m_t and α_s
  - top-quark mass least precise parameter
- Vacuum stability bound at M_P in terms of m_t

$$m_t \;\leq\; (171.36 \pm 0.15 \pm 0.25_{\alpha_3} \pm 0.17_{m_h})\;\mathrm{GeV} \;=\; (171.36 \pm 0.46)\;\mathrm{GeV}$$

Summary

Top quark physics

- Top quark physics is becoming precision science
- Correlations in data analysis are important, e.g. with α_s and PDFs

Top quark mass

- Top quark mass is parameter of Standard Model Lagrangian
- Measurements of m_t require careful definition of observable
- Radiative corrections at higher orders mandatory for scheme definition

LHC measurements

- Inclusive and differential observables
- Very precise data and well-defined renormalization scheme definition
- $\overline{\mathrm{MS}}$ scheme for mass exhibits better convergence

Future challenge

- Study of new observables for top-quark mass determination
- Joint effort theory and experiment