Triple collinear splitting amplitudes at NLO

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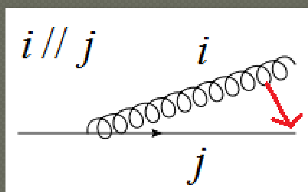
Content

- Introduction
- Collinear limits and DREG
- Double collinear splittings
  - Scheme dependence
- Triple collinear splittings
  - Some results (preliminary)
- Conclusions and perspectives

Based on:
Catani, de Florian,Rodrigo, arXiv:1112.4405 [hep-ph]
De Florian, Rodrigo, GS; arXiv:1310.6841 [hep-ph]
Catani, De Florian, Rodrigo, GS; in preparation

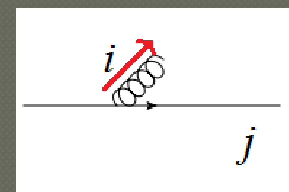
Introduction

- Focusing on IR singularities: general ideas
  - Related with low-energy and collinear configurations



$$k_i \cdot k_j \to 0$$

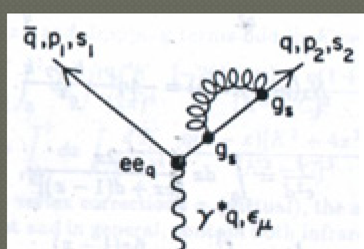
Collinear configuration



$$k_i \to 0$$

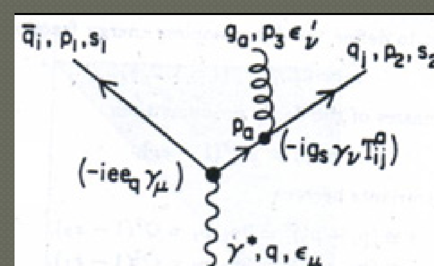
Soft configuration

  - Associated with degenerate states (the experiments are not able to distinguish two particles which are very close neither they can detect low-energy particles)



Virtual corrections

\+



Real corrections

= Finite result

(Classical example extracted from Field, R. – *Applications of perturbative QCD*)

Collinear limits and DREG

- Collinear limit (1->2 processes)
  - It is useful to introduce some kinematical variables to parametrize collinear momenta.

z_i are the momentum fractions

$$p_1^\mu = z_1 \, \tilde{P}^\mu + k_\perp^\mu - \frac{(k_\perp)^2}{2 z_1 n \cdot \tilde{P}} n^\mu$$

$$p_2^\mu = (1 - z_1) \, \tilde{P}^\mu - k_\perp^\mu - \frac{(k_\perp)^2}{2(1 - z_1) n \cdot \tilde{P}} n^\mu$$

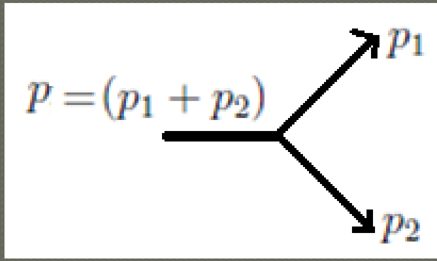
Collinear direction

Transverse component

Approach to the collinear limit

General parametrization

Particles 1 and 2 become collinear. n and p are null-vectors (p is the collinear direction)



$$\tilde{P}^\mu = p_{12}^\mu - \frac{s_{12}}{2n \cdot \tilde{P}} n^\mu$$

Null-vector associated to the parent parton

(For more details see: Catani, de Florian and Rodrigo, arXiv:1112.4405 [hep-ph])

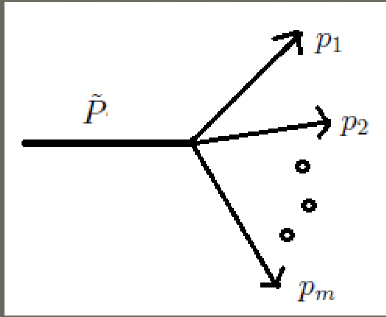
Collinear limits and DREG

- Collinear limit (general)
  - We will work in the LC-gauge (*n* is the quantization vector). This allows to obtain factorization formulas.
  - Generalize kinematics variables

$$p^\mu_{1,m} = p^\mu_1 + \ldots + p^\mu_m \qquad s_{1,m} = p^2_{1,m}$$

$$\tilde{P}^\mu = p^\mu_{1,m} - \frac{s_{1,m}}{2\, n \cdot \tilde{P}}\, n^\mu$$

Null-vector associated to the parent parton



  - Matrix elements have an specific (divergent) behaviour in the collinear limit:

$$\mathcal{M}(p_1, \ldots, p_m, p_{m+1}, \ldots, p_n) \sim (1/\sqrt{s})^{m-1} \text{ mod } (\ln^k s)$$

Involves collinear particles (2 or more)

  - Only keep the most divergent contributions in s (*collinear subscales*)
  - Multiple scales in the multiple collinear limit ➡ Increase difficulty

(For more details see: Catani, de Florian and Rodrigo, arXiv:1112.4405 [hep-ph])

Collinear limits and DREG

- Collinear factorization in color space: We introduce the *splitting matrices* in color+spin space; they describe the divergent behaviour of scattering amplitudes in the collinear limit
  - Tree-level factorization

$$|\mathcal{M}^{(0)}\rangle \simeq \boldsymbol{Sp}^{(0)}(p_1,\ldots,p_m;\widetilde{P})\,|\overline{\mathcal{M}}^{(0)}\rangle$$

  - One-loop level factorization

$$\begin{aligned}|\mathcal{M}^{(1)}\rangle &\simeq \boldsymbol{Sp}^{(1)}(p_1,\ldots,p_m;\widetilde{P};p_{m+1},\ldots,p_n)\,|\overline{\mathcal{M}}^{(0)}\rangle \\ &+ \boldsymbol{Sp}^{(0)}(p_1,\ldots,p_m;\widetilde{P})\,|\overline{\mathcal{M}}^{(1)}\rangle\end{aligned}$$

Splitting matrix at LO

Splitting matrix at NLO

with the reduced scattering amplitude $\overline{\mathcal{M}} = \mathcal{M}(\widetilde{P},p_{m+1},\ldots,p_n)$

- Some remarks:
  - *It is important to note that these expressions only consider the most divergent contributions in the collinear limit*
  - *Splitting functions are recovered if we project over color and remove color dependence*

(For more details see: Catani, de Florian and Rodrigo, arXiv:1112.4405 [hep-ph])

Collinear limits and DREG

- Collinear factorization in color space
  - General structure of one-loop splitting matrices (1->2)

$$\boldsymbol{Sp}^{(1)}(p_1,p_2;\widetilde{P};p_3,\ldots,p_n) = \boldsymbol{Sp}_H^{(1)}(p_1,p_2;\widetilde{P}) + \boldsymbol{I}_C(p_1,p_2;p_3,\ldots,p_n)\,\boldsymbol{Sp}^{(0)}(p_1,p_2;\widetilde{P})$$

One-loop splitting matrix — Finite contribution — Singular contribution — Tree-level splitting matrix

  - More details:
    - **$\mathbf{Sp_H}$** contains only rational functions of the momenta and only depends on collinear particles.
    - **$\mathbf{I_C}$** contains trascendental functions and can depend of *non-collinear* particles (through colour correlations). This contribution introduces a violation of *strict* collinear factorization.
    - It can be extended to the multiple collinear limit.

(For more details see: Catani, de Florian and Rodrigo, arXiv:1112.4405 [hep-ph])

Collinear limits and DREG

- Collinear factorization in color space
  - Divergent structure of one-loop splitting matrices (1->2)
    - Time-like region (TL): $s_{ij}>0$ for all particles
    - Space-like region (SL): $s_{ij}<0$ for some i,j

$$\boldsymbol{I}_C(p_1,p_2;p_3,\dots,p_n) = g_S^2\, c_\Gamma \left(\frac{-s_{12}-i0}{\mu^2}\right)^{-\epsilon} \times \left\{ \frac{1}{\epsilon^2}\left(C_{12}-C_1-C_2\right) + \frac{1}{\epsilon}\left(\gamma_{12}-\gamma_1-\gamma_2+b_0\right) + \frac{2}{\epsilon}\sum_{j=3}^{n} \boldsymbol{T}_j \cdot \left[\boldsymbol{T}_1\, f(\epsilon; z_1 - i0 s_{j1}) + \boldsymbol{T}_2\, f(\epsilon; z_2 - i0 s_{j2})\right]\right\}$$

Diagonal in color space

Color correlations

Sum over all partons

Explicit correlation with non-collinear particles in SL region

$$f(\epsilon;1/x) \equiv \frac{1}{\epsilon}\left[\,{}_2F_1(1,-\epsilon;1-\epsilon;1-x) - 1\right] = \ln x - \epsilon\left[\mathrm{Li}_2(1-x) + \sum_{k=1}^{+\infty}\epsilon^k\, \mathrm{Li}_{k+2}(1-x)\right]$$

Presence of branch-cuts!

(For more details see: Catani, de Florian and Rodrigo, arXiv:1112.4405 [hep-ph])

Collinear limits and DREG

- Dimensional regularization (DREG): Introduction
  - Change space-time dimension to allow integrals to converge (both loop and phase space) $4 \Rightarrow D_{ST}$
  - Regulates both UV and IR singularities using the same procedure (but not necessarily the same regulator) ⇒ Make divergences explicit!

Structure involving physical momenta and metric tensors

$$\int_q \frac{q^{\mu_1} q^{\mu_2} \ldots q^{\mu_m}}{D_1^{\alpha_1} D_2^{\alpha_2} \ldots D_n^{\alpha_n}} = \sum_A F_A^{\mu_1 \ldots \mu_m}\left(\{p_i\}, \{\alpha_i\}, \eta^{D_{ST}}\right) I_A^{\text{scalar}}\left(\{p_i \cdot p_j\}, D_{ST}\right)$$

Tensor-type integrals

D_{ST}-dimensional metric tensor

Scalar integrals (depend on scalar products and D_{ST})

  - When applied to virtual corrections, only loop momenta must be D-dimensional; the external momenta and the number of polarizations (both internal and external) can be chosen

⇓

DREG scheme definition

Collinear limits and DREG

- Dimensional regularization (DREG): Schemes definitions

  We can play with many parameters. Each choice defines a DREG scheme.

  - Dirac algebra dimension:

$$\{\gamma^\mu, \gamma^\nu\} = 2(\eta^{D_{\rm Dirac}})^{\mu\nu}\,{\rm Id} \quad \longrightarrow \quad D_{\rm Dirac} = 4 - 2\delta\epsilon$$

  - Trace of identity matrix in Dirac's matrices space:

External particles ← $\mathrm{Tr}^{\mathrm{Ext}}(\mathrm{Id}) = 2n_q$

Fermions in loops ← $\mathrm{Tr}^{\mathrm{Int}}(\mathrm{Id}) = 2h_q$

→ Number of fermion polarizations

$$n_q = 2 - 2\beta_{\rm R}\epsilon \;\rightarrow\; \mathrm{Tr}^{\mathrm{Ext}}(\mathrm{Id}) = 4 - 4\beta_{\rm R}\epsilon$$

$$h_q = 2 - 2\beta\epsilon \;\rightarrow\; \mathrm{Tr}^{\mathrm{Int}}(\mathrm{Id}) = 4 - 4\beta\epsilon$$

For more details see: Catani, Seymour and Trócsányi, arXiv:hep-ph/9610553
de Florian, Rodrigo and GS, arXiv:1310.6841 [hep-ph]

Collinear limits and DREG

- Dimensional regularization (DREG): Schemes definitions

  We can play with many parameters. Each choice defines a DREG scheme.

  - Internal gluon's polarization tensor:

$$d_{\mu\nu}(p,n) = -\left(\eta^{4}_{\mu\nu} + \alpha_R \eta^{D_{\rm Dirac}-4}_{\mu\nu}\right) + \frac{p_\mu n_\nu + n_\mu p_\nu}{p\cdot n}$$

Light-cone gauge reference vector

Number of internal gluons' polarizations

$$h_g = d_{\mu\nu}(p,n)(\eta^{D_{\rm ST}})^{\mu\nu} \Rightarrow \begin{cases} \alpha_R = 0 \Rightarrow h_g = 2 \\ \alpha_R = 1 \Rightarrow h_g = D_{\rm Dirac} - 2 \end{cases}$$

  - Sum over external gluon's polarizations:

$$d^{\rm Ext}_{\mu\nu}(p,Q) = \sum_{\rm phys.pol.} \epsilon^*_\mu(p)\epsilon_\nu(p) = -\left(\eta^{4}_{\mu\nu} + \alpha \eta^{D_{\rm ST}-4}_{\mu\nu}\right) + \frac{p_\mu Q_\nu + Q_\mu p_\nu}{p\cdot Q}$$

Reference vector (consider Q=n)

Number of external gluons' polarizations

$$n_g = d^{\rm Ext}_{\mu\nu}(p,n)(\eta^{D_{\rm ST}})^{\mu\nu} = 2 - 2\alpha\epsilon$$

For more details see: Catani, Seymour and Trócsányi, arXiv:hep-ph/9610553
de Florian, Rodrigo and GS, arXiv:1310.6841 [hep-ph]

Collinear limits and DREG

- Dimensional regularization (DREG): Schemes definition

Commonly used

- CDR*: All momenta and polarizations in 4-2ε dimensions (δ=1,α_R=1,α=1)*
- HV*: Internal polarizations in 4-2ε dimensions but external particles with physical 4-dimensional polarizations (δ=1,α_R=1,α=0)*
- FDH*: All particles have physical 4-dimensional polarizations (δ=0,α_R=1)*

Other choices

- HSA/HSB *(«hybrid-schemes»): Momenta in 4-2ε dimensions but α_R=0 (metric tensor inside gluon polarization tensor is 4-dimensional)*
- TSC *(«toy scheme» introduced in hep-ph/9610553): Like CDR, but fermions are in 4-2ε dimensions.*

For more details see: Catani, Seymour and Trócsányi, arXiv:hep-ph/9610553
de Florian, Rodrigo and GS, arXiv:1310.6841 [hep-ph]

Collinear limits and DREG

- Dimensional regularization (DREG): Schemes definition

Scheme	n_g	h_g	δ	α_R	α
CDR	$2-2\epsilon$	$2-2\epsilon$	1	1	1
HV	2	$2-2\epsilon$	1	1	0
FDH	2	2	0	1	0
HSA	$2-2\epsilon$	2	1	0	1
HSB	2	2	1	0	0

Common schemes (CDR, HV, FDH)

Other choices (with usual fermion polarizations) (HSA, HSB)

$\beta = 0 = \beta_R$

CDR with 2-2ε fermionic DF

Scheme	n_g	h_g	δ	α_R	α
TSC	$2-2\epsilon$	$2-2\epsilon$	1	1	1

$\beta = 1 = \beta_R$

For more details see: Catani, Seymour and Trócsányi, arXiv:hep-ph/9610553
de Florian, Rodrigo and GS, arXiv:1310.6841 [hep-ph]

Collinear limits and DREG

- Relation between splitting amplitudes and Altarelli-Parisi kernels:
  - Altarelli-Parisi kernels are related to the collinear behaviour of squared matrix elements. They also control the evolution of PDF's and FF's (through DGLAP equations).
    - LO contribution

$$\langle \hat{P}^{(0)}_{a_1 \cdots a_m} \rangle = \left(\frac{s_{1\ldots m}}{2\,\mu^{2\epsilon}} \right)^{m-1} \overline{|\boldsymbol{Sp}^{(0)}_{a_1\ldots a_m}|^2}$$

Normalization (depends on the number of collinear particles)

    - NLO contribution

$$\langle \hat{P}^{(1)}_{a_1 \cdots a_m} \rangle = \left(\frac{s_{1\ldots m}}{2\,\mu^{2\epsilon}} \right)^{m-1} \left(\overline{\boldsymbol{Sp}^{(1)}_{a_1\ldots a_m} \left(\boldsymbol{Sp}^{(0)}_{a_1\ldots a_m} \right)^{\dagger}} + \text{h.c.} \right)$$

  - They are a key component in the dipole formalism, parton shower algorithms, evolution of PDF's and FF's, etc.

See: Catani, de Florian and Rodrigo, arXiv:hep-ph/0312067 and arXiv:1112.4405 [hep-ph]
Catani, Seymour and Trócsányi, arXiv:hep-ph/9610553

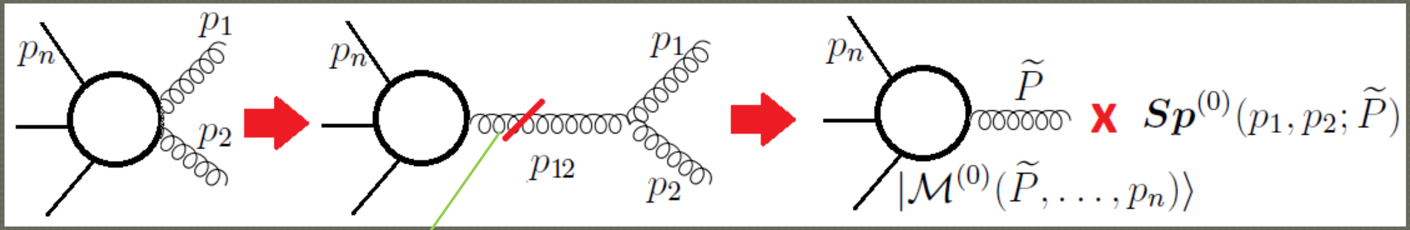
Double collinear splittings

- Motivation and objectives
  - Double collinear splittings are known up to NNLO
  - Both amplitude and squared-amplitude results are available
  - Deep study of scheme dependence performed at LO
  - We extended the analysis to NLO (in QCD+QED) and studied the consistency of some DREG configurations (HSA,HSB,TSC…)
  - *Main motivation:* understand transition rules between schemes and apply them to higher-order splittings

More details in De Florian, Rodrigo and GS, arXiv:1310.6841 and references therein

Double collinear splittings

- Collinear factorization in color space: Graphical motivation for double splittings (gluon parent)



$$\frac{\imath d_{\mu\nu}(p_1+p_2)}{s_{12}} \approx \frac{\imath d_{\mu\nu}(\tilde{P})}{s_{12}} + \mathcal{O}(s_{12}^0) = \sum_{\text{phys.pol.}} \epsilon^*_\mu(\tilde{P},\sigma)\frac{\imath \epsilon_\nu(\tilde{P},\sigma)}{s_{12}}$$

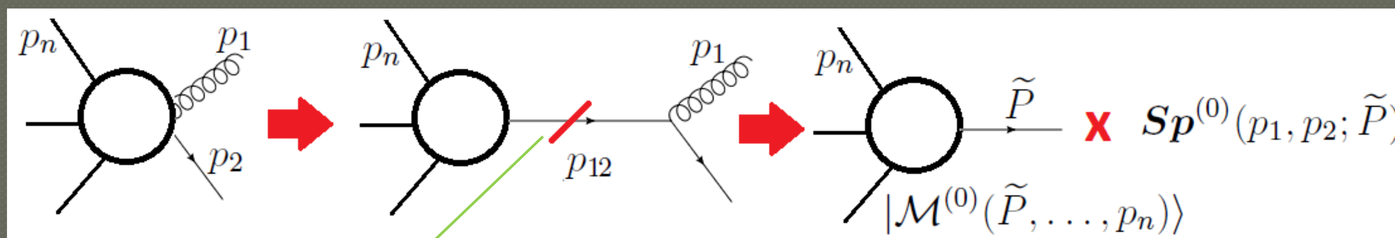
Intermediate particle propagator in the collinear limit (dominant contribution)

- Important remarks:
  - *Splitting functions and matrices are computed using a* ***on-shell massless parent*** *particle, but* ***off-shell*** *kinematics*

$$\boldsymbol{Sp}_{g\to a_1 a_2} = \frac{1}{s_{12}} \left|\mathcal{A}^{\mu}_{g,a_1,a_2}(p_{12},p_1,p_2)\right\rangle \epsilon_\mu(\tilde{P})$$

Double collinear splittings

- Collinear factorization in color space: Graphical motivation for double splittings (quark parent)



$$\frac{\imath(\not p_1 + \not p_2)}{s_{12}} \approx \frac{\imath \tilde{\not P}}{s_{12}} + \mathcal{O}(s_{12}^0) = \sum_{\text{phys.pol.}} \frac{\imath u_\sigma(\tilde{P})}{s_{12}} \bar{u}_\sigma(\tilde{P})$$

Intermediate particle propagator in the collinear limit (dominant contribution)

- Important remarks:
  - *Splitting functions and matrices are computed using a **on-shell massless parent** particle, but **off-shell** kinematics*

$$Sp_{g\to a_1 a_2} = \frac{1}{s_{12}} \left|\mathcal{A}^{\mu}_{g,a_1,a_2}(p_{12}, p_1, p_2)\right\rangle \epsilon_\mu(\tilde{P})$$

Double collinear splittings

- Scheme dependence: Consistency
  - *Divergent structure must be compatible with Catani's formula for IR singularities*

$$\boldsymbol{I}_C\left(p_1,p_2;\tilde{P}\right) = c_\Gamma g_s^2 \left(\frac{-s_{12}-\imath 0}{\mu^2}\right)^{-\epsilon} \left\{\frac{1}{\epsilon^2}\left(C_{12}-C_1-C_2\right)+\frac{1}{\epsilon}\left(\gamma_{12}-\gamma_1-\gamma_2+b_0\right) - \frac{1}{\epsilon}\left[\left(C_{12}+C_1-C_2\right) f(\epsilon,z_1)+\left(C_{12}+C_2-C_1\right) f(\epsilon,1-z_1)\right]\right\}$$

Divergent factor in TL kinematics (scheme-independent)

  - *Gluon polarization tensor must fulfill some physical requirements*

1 $d_{\mu\nu}(p,n)n^\mu = 0 = d_{\mu\nu}^{\rm Ext}(p,n)n^\mu$ (orthogonality to n)

2 $d_{\mu\nu}^{\rm Ext}(p,n)p^\mu = 0$ (orthogonality to external momenta p)

Always valid if n and external momenta are null extensions of 4-vectors (implicit in our choices)

3 $d_{\mu\nu}(p,n)p^\mu \propto p^2$

Potential problems if α_R=0…

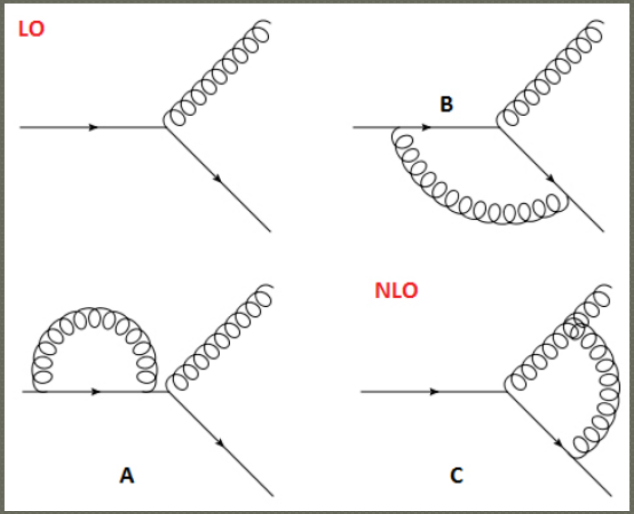
Scheme	n_g	h_g	δ	α_R	α
HSA	$2-2\epsilon$	2	1	0	1
HSB	2	2	1	0	0

$$d_{\mu\nu}(p,n)p^\mu = -\left(p_\nu^{(4)} + \delta\alpha_R p_\nu^{(D_{\rm ST}-4)}\right) + \frac{p^2\, n_\nu + (p\cdot n)\, p_\nu}{p\cdot n} = p^2 \frac{n_\nu}{p\cdot n} + (1-\delta\alpha_R) p_\nu^{(D_{\rm ST}-4)}$$

(See De Florian, Rodrigo and GS, arXiv:1310.6841 [hep-ph])

Double collinear splittings

- Scheme dependence: q->gq (example)
  - Replace the incoming polarization spinor by a massless physical one, associated with the light-like vector $\widetilde{P}$.
  - Use the usual Feynman rules to write the amplitude.
  - *Use off-shell kinematics (parent parton with a tiny virtuality).*



$$q(\tilde{P}) \to g(p_1)q(p_2)$$

LO splitting amplitude

$$\boldsymbol{Sp}^{(0)}_{q\to gq} = \frac{g_s \mu^{\epsilon}}{s_{12}} \boldsymbol{T}^a \left[\bar{u}(p_2)\tilde{\gamma}^{\mu} u(\tilde{P}) + \bar{u}(p_2)\hat{\gamma}^{\mu} u(\tilde{P}) \right] \epsilon_{\mu}(p_1)$$

$$\boldsymbol{Sp}^{(1,\mathrm{STD})}_{q\to gq} = \frac{c_\Gamma g_s^3 \mu^{\epsilon}}{2 s_{12} \epsilon^2} \left(\frac{-s_{12} - \imath 0}{\mu^2} \right)^{-\epsilon} \boldsymbol{T}^a \left[C^{(\mathrm{STD},1)}_{q\to gq}\, \bar{u}(p_2) \not{\epsilon}(p_1) u(\tilde{P}) + C^{(\mathrm{STD},2)}_{q\to gq} \frac{1}{nP} \bar{u}(p_2) \not{n} u(\tilde{P}) p_2 \cdot \epsilon(p_1) + \delta_{\alpha,1}\, C^{(\mathrm{STD},3)}_{q\to gq}\, \bar{u}(p_2) \hat{\gamma}^{\mu} u(\tilde{P}) \hat{\epsilon}_{\mu}(p_1) \right]$$

NLO correction

(See De Florian, Rodrigo and GS, arXiv:1310.6841 [hep-ph])

Double collinear splittings

- Scheme dependence: q->gq (example)
  - Explicit results:

$$C^{(\mathrm{STD},1)}_{q\to gq} = 2(C_A-2C_F)\,{}_2F_1\left(1,-\epsilon,1-\epsilon,\frac{z_1}{z_1-1}\right) - 2C_A\,{}_2F_1\left(1,-\epsilon,1-\epsilon,\frac{z_1-1}{z_1}\right)$$
$$-\,2\frac{C_A\left(\epsilon(\delta\epsilon^2+\epsilon-3)+1\right)-C_F\left(\delta\epsilon^3+3\epsilon^2-6\epsilon+2\right)}{(\epsilon-1)(2\epsilon-1)}$$
$$+\,(1-\alpha_R)\delta\epsilon^2\frac{C_A\left(2\epsilon+1+\alpha_R\right)-2C_F\epsilon}{(\epsilon-1)(2\epsilon-1)}$$

$$C^{(\mathrm{STD},2)}_{q\to gq} = \frac{2\epsilon^2(C_A-C_F)(\delta\epsilon-1)}{(\epsilon-1)(2\epsilon-1)} + \frac{\delta(1-\alpha_R)\epsilon}{2(1-z_1)^2(\epsilon-1)(2\epsilon-1)}\Big[2(1-z_1)^2\epsilon^2\left(2C_F-C_A(\alpha_R+2)\right)$$
$$+\,C_A(1-z_1)^2\epsilon\,{}_2F_1\left(1,1-\epsilon,2-2\epsilon,\frac{1}{z_1}\right)+C_A z_1(\epsilon-1)\,{}_2F_1\left(1,-\epsilon,1-\epsilon,\frac{z_1-1}{z_1}\right)$$
$$+\,C_A\left((z_1^2-4z_1+2)\epsilon+z_1\right)\Big]$$

$$C^{(\mathrm{STD},3)}_{q\to gq} = 2(1-\alpha_R)C_A\left[{}_2F_1\left(1,-\epsilon,1-\epsilon,\frac{z_1-1}{z_1}\right)+\frac{(1-z_1)\epsilon}{z_1(2\epsilon-1)^2}\,{}_2F_1\left(1,1-\epsilon,2-2\epsilon,\frac{1}{z_1}\right)\right]$$
$$+\,\frac{(1-\alpha_R)\left[2C_F(1-2\epsilon)\epsilon-C_A\left(\epsilon((1+\epsilon\delta)(1-\alpha_R)-6\epsilon+7)-4\right)\right]}{(\epsilon-1)(2\epsilon-1)}$$

Additional pole in HSA configuration!

$$C^{(\mathrm{STD},3)}_{q\to gq} = 6(1-\alpha_R)C_A + \mathcal{O}(\epsilon)$$

IR divergences don't agree with expected structure!

(See De Florian, Rodrigo and GS, arXiv:1310.6841 [hep-ph])

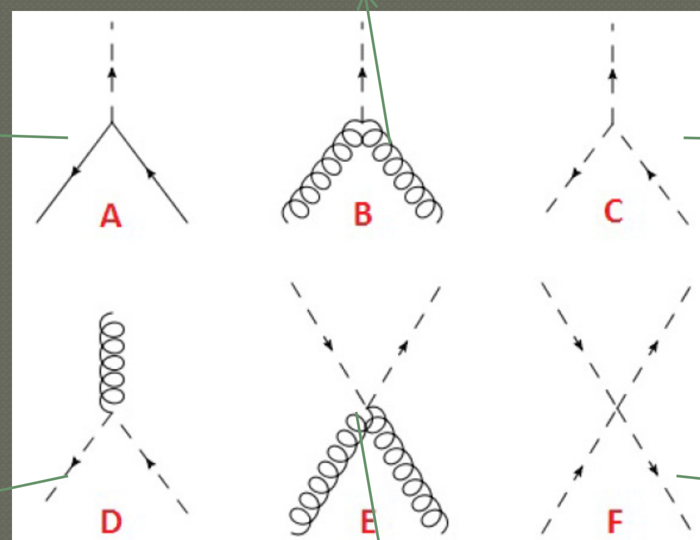
Double collinear splittings

- Scheme dependence: q->gq (example)
  - Introduce scalar-gluons (associated with ε–polarizations)
  - Typical scalar-particle propagator and Feynman rules

$$g_s^{\rm scalar}\mu^\epsilon f_{abc}\eta^{\rm 4D}_{\nu\sigma}(p_2-p_3)_{\hat\mu}$$

$$-\imath g_s^{\rm scalar}\mu^\epsilon T^a\tilde\gamma^\mu$$

$$g_s^{\rm scalar}\mu^\epsilon f_{abc}V_{3g}^{\rm Cin}(p_1,p_2,p_3;\hat\mu,\hat\nu,\hat\rho)$$



$$g_s^{\rm scalar}\mu^\epsilon f_{abc}\eta^\epsilon_{\hat\nu\hat\sigma}(p_2-p_3)_\mu$$

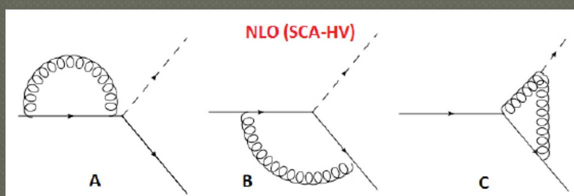
$$-\imath\left(g_s^{\rm scalar}\right)^2\mu^{2\epsilon}V_{4g}(\hat\mu,\hat\nu,\hat\sigma,\hat\rho,a,b,c,d)$$

$$-2\imath\left(g_s^{\rm scalar}\right)^2\mu^{2\epsilon}(f_{ace}f_{bde}+f_{ade}f_{bce})\eta^{\rm 4D}_{\sigma\rho}\eta^\epsilon_{\mu\nu}$$

(See De Florian, Rodrigo and GS, arXiv:1310.6841 [hep-ph])

Double collinear splittings

- Scheme dependence: q->gq (example)
  - Compute $q(\tilde{P}) \to \phi(p_1) q(p_2)$ with the previous rules
  - Add this contribution to q->gq in HSA configuration



$$Sp^{(1,\mathrm{SCA-HV})}_{q\to\phi q} = \frac{c_\Gamma g_s^3 \mu^\epsilon}{2 s_{12} \epsilon^2} \left(\frac{-s_{12} - \imath 0}{\mu^2}\right)^{-\epsilon} \boldsymbol{T}^a C^{(\mathrm{STD},1)}_{q\to gq}(\alpha_R = 1, \delta = 1)\, \bar{u}(p_2) \hat{\gamma}^\mu u(\tilde{P}) \hat{\epsilon}_\mu(p_1)$$

  - Additional poles cancel, but we recover CDR results…
  - *CDR is the consistent version of HSA*
  - In fact, we show that

$$P^{CDR}_{q\to gq} = P^{HV}_{q\to gq} + P_{q\to\phi q}$$

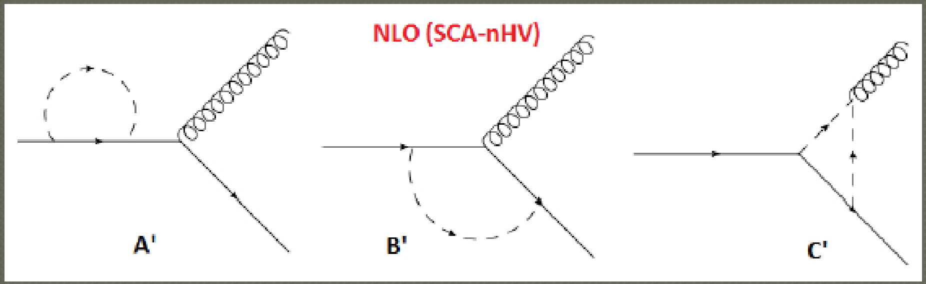
which is compatible with the interpretation of extra-polarizations as scalar-particles

(See De Florian, Rodrigo and GS, arXiv:1310.6841 [hep-ph])

Double collinear splittings

- Scheme dependence: q->gq (example)
  - Starting from FDH we can recover HV results by adding (internal) scalar-gluons contributions which are compatible with helicity-conservation

$$
\boldsymbol{Sp}^{(1,\mathrm{SCA-nHV})}_{q\to gq} = c_\Gamma \left(\frac{-s_{12}-\imath 0}{\mu^2}\right)^{-\epsilon} \boldsymbol{T}^a \, \frac{g_s^3\mu^\epsilon}{s_{12}} \, \frac{\epsilon\,(C_F - C_A)}{(\epsilon-1)(2\epsilon-1)} \left[\bar{u}(p_2)\not{\epsilon}(p_1)u(\tilde{P}) - \frac{1}{nP}\bar{u}(p_2)\not{n}u(\tilde{P})\, p_2\cdot\epsilon(p_1)\right]
$$



$$
\boldsymbol{Sp}^{(1,\mathrm{STD},HV)}_{q\to gq} = \boldsymbol{Sp}^{(1,\mathrm{STD},FDH)}_{q\to gq} + \boldsymbol{Sp}^{(1,\mathrm{SCA-nHV})}_{q\to gq}
$$

(See De Florian, Rodrigo and GS, arXiv:1310.6841 [hep-ph])

Double collinear splittings

- Scheme dependence: Conclusions
  - HSA/HSB schemes are incomplete versions of CDR/HV
  - Scalar-gluons can be used to connect different schemes
  - Playing with the number of fermion's polarizations introduces $O(\varepsilon^0)$ differences among results
  - TSC scheme seems to be compatible with supersymmetric Ward's identity at NLO

$$P_{g\to gg}(z) + P_{g\to q\bar{q}}(z) = P_{q\to qg}(1-z) + P_{q\to qg}(z)$$

when considering $C_A = C_F = T_R = N_f$

(See De Florian, Rodrigo and GS, arXiv:1310.6841 [hep-ph])

Triple collinear splittings

- Motivation and objectives
  - We are interested in computing triple-collinear splitting functions (at amplitude level and AP kernels) at NLO
  - Only partial results available at NLO

$$
\begin{aligned}
\langle \hat{P}^{(1)\,(\mathrm{an.})}_{q_1\bar{Q}_2Q_3}\rangle = {}& \frac{\Gamma(1+\epsilon)\,\Gamma^2(1-\epsilon)}{(4\pi)^{-\epsilon}\,\Gamma(1-2\epsilon)}\, C_F\, T_R\, \frac{N_c^2-4}{4N_c} \left(\frac{-s_{123}-i0}{\mu^2}\right)^{-\epsilon} \\
\times & \left(\left\{ \frac{1}{\epsilon^2} \left[\left(\left(\frac{s_{23}}{s_{123}}\right)^{-\epsilon} + 1 \right) \left(\left(\frac{z_2}{z_2+z_3}\right)^{-\epsilon} - \left(\frac{z_3}{z_2+z_3}\right)^{-\epsilon} + \left(\frac{s_{13}}{s_{123}}\right)^{-\epsilon} - \left(\frac{s_{12}}{s_{123}}\right)^{-\epsilon} \right) \right.\right.\right. \\
+ & \left.\left. \left(\frac{s_{12}}{s_{123}}\right)^{-\epsilon} \left(\frac{z_3}{z_2+z_3}\right)^{-\epsilon} - \left(\frac{s_{13}}{s_{123}}\right)^{-\epsilon} \left(\frac{z_2}{z_2+z_3}\right)^{-\epsilon} \right] \right\} \langle \hat{P}^{(0)}_{q_1\bar{Q}_2Q_3}\rangle/(C_F T_R) \\
+ & \left\{ \left(\hat{a} - \frac{s_{13}}{s_{123}-s_{13}}\, \hat{b} \right) \ln\left(\frac{s_{13}}{s_{123}}\right) + \left(\frac{s_{23}}{s_{123}-s_{23}}\, \hat{a} - \hat{b} \right) \ln\left(\frac{s_{23}}{s_{123}}\right) \right. \\
+ & \frac{s_{123}\; s_{12}}{s_{12}+s_{13}-z_1\; s_{123}} \left(\frac{z_2\;\hat{a} - z_1\;\hat{b}}{s_{12}(1-z_1)} + \frac{\hat{a}+\hat{b}}{s_{123}} \right) \ln\left(\frac{s_{12}\; z_2}{s_{13}\; z_3}\right) \ln\left(\frac{s_{23}}{s_{123}(z_2+z_3)}\right) \\
+ & \frac{(2s_{12}+s_{23})\;\hat{a} + (s_{12}-s_{13})\;\hat{b}}{s_{12}} \\
\times & \left. \left. \left[\ln\left(\frac{s_{13}}{s_{123}}\right) \ln\left(\frac{s_{23}}{s_{123}}\right) + \mathrm{Li}_2\left(1-\frac{s_{13}}{s_{123}}\right) + \mathrm{Li}_2\left(1-\frac{s_{23}}{s_{123}}\right) - \frac{\pi^2}{6} \right] - (2\leftrightarrow 3) \right\} + \mathcal{O}(\epsilon) \right) \\
+ & \text{complex conjugate}\,,
\end{aligned}
$$

$$
\begin{aligned}
\hat{a} &= \frac{s_{123}}{s_{23}} \left(\frac{z_1\; s_{13}}{s_{123}} + \frac{z_1(z_1\; s_{23} - z_2\; s_{13} - z_3\; s_{12})}{2(z_2+z_3)\; s_{12}} + \frac{z_1\; s_{23} + z_2\; s_{13} - z_3\; s_{12}}{2s_{123}} \right) \\
\hat{b} &= \frac{s_{123}}{s_{23}} \left(\frac{z_2\; s_{23}}{s_{123}} + \frac{z_2(z_1\; s_{23} - z_2\; s_{13} + z_3\; s_{12})}{2(z_2+z_3)\; s_{12}} + \frac{z_1\; s_{23} + z_2\; s_{13} - z_3\; s_{12}}{2s_{123}} \right)
\end{aligned}
$$

Antisymmetric contribution to *q->qQQbar* AP kernel at NLO (Catani, de Florian, Rodrigo; Phys.Lett. B586, 323-331,2004)

  - Presence of different scales → Difficult computation!
  - Start to work with QCD+QED and splittings with photons (simpler color structure)

(See Catani, de Florian and Rodrigo, arXiv:hep-ph/0312067 and arXiv:1112.4405 [hep-ph])

Triple collinear splittings

- Divergent behaviour of multiple-collinear splitting amplitudes
  - Extension for QCD+QED in TL kinematics (strict factorization)

$$
\begin{aligned}
\boldsymbol{Sp}^{(1)\,\mathrm{div.}}(p_1,\ldots,p_m) = c_\Gamma g_s^2 \left(\frac{-s_{1\ldots m}-i0}{\mu^2}\right)^{-\epsilon} \Bigg\{ & \frac{1}{\epsilon^2}\sum_{i,j=1(i\neq j)}^{\bar m} \boldsymbol{T}_i\cdot\boldsymbol{T}_j \left(\frac{-s_{ij}-i0}{-s_{1\ldots m}-i0}\right)^{-\epsilon} \\
& + \frac{1}{\epsilon^2}\sum_{i,j=1}^{\bar m} \boldsymbol{T}_i\cdot\boldsymbol{T}_j \left(2-(z_i)^{-\epsilon}-(z_j)^{-\epsilon}\right) \\
& - \frac{1}{\epsilon}\left(\sum_{i=1}^{\bar m}\left(\gamma_i-\epsilon\tilde\gamma_i^{\mathrm{RS}}\right)-\left(\gamma_a-\epsilon\tilde\gamma_a^{\mathrm{RS}}\right)-\frac{\bar m-1}{2}\left(\beta_0-\epsilon\tilde\beta_0^{\mathrm{RS}}\right)\right)\Bigg\} \\
& \times \boldsymbol{Sp}^{(0)}(p_1,\ldots,p_m)
\end{aligned}
$$

Multiple scales

Scheme-dependence (up to ε^0)

  - Useful to test our results (only the divergent structure…)
  - Transition rules well known

(See Catani, de Florian and Rodrigo, arXiv:hep-ph/0312067 and arXiv:1112.4405 [hep-ph])

Triple collinear splittings

- Computation strategy
  - Use Feynman diagram approach and follow the recipe applied to double collinear splittings
  - Work at squared-amplitude level (i.e. AP kernels), in TL region and in CDR → Easier calculation! → Transition rules to change scheme!
  - Use IBP and other techniques to simplify loop-integrals in the LCG
  - Expand the result up to O(ε^0) → Transcendentality

Divergent contribution (also includes O(ε^0) terms)

$$\langle \hat{P}^{(1)}_{a_1\cdots a_m}\rangle = \left(\frac{s_{1\ldots m}}{2\,\mu^{2\epsilon}}\right)^{m-1} \left(\overline{\boldsymbol{Sp}^{(1)}_{a_1\ldots a_m}\left(\boldsymbol{Sp}^{(0)}_{a_1\ldots a_m}\right)^\dagger} + \text{h.c.}\right)$$
$$= I^{(1)}_{a_1\cdots a_m}(p_1,\ldots,p_m;\tilde{P})\,\langle \hat{P}^{(0)}_{a_1\cdots a_m}\rangle + R^{(1)}_{a_1\cdots a_m} + \text{h.c.}$$

Finite remainder

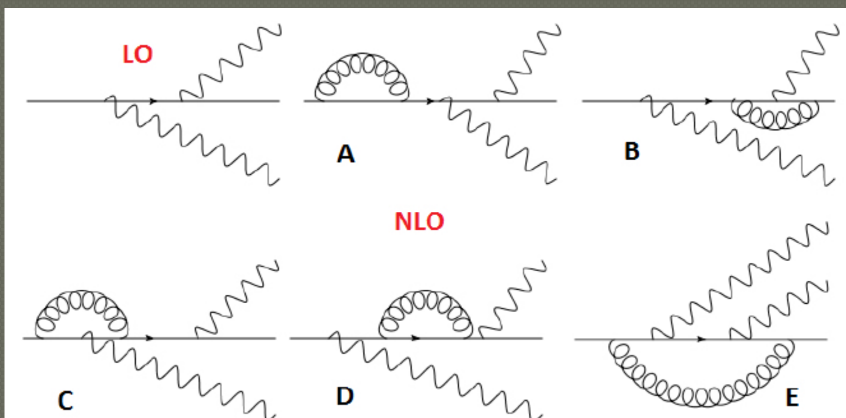
$$R^{(1)}_{a_1\cdots a_m} = \left(\frac{s_{1\ldots m}}{2\,\mu^{2\epsilon}}\right)^{m-1} \overline{\boldsymbol{Sp}^{(1)\,\text{fin.}}_{a_1\ldots a_m}\left(\boldsymbol{Sp}^{(0)}_{a_1\ldots a_m}\right)^\dagger}$$
$$= c^{a_1\cdots a_m}_{\text{Factor}}\left[C_0 + \sum_{i=1}^{2}\sum_{j\in\mathcal{F}_i} C^i_j F^i_j(\{s_{kl},z_k\}) + \mathcal{O}(\epsilon)\right]$$

Set of functions of transcendentality i

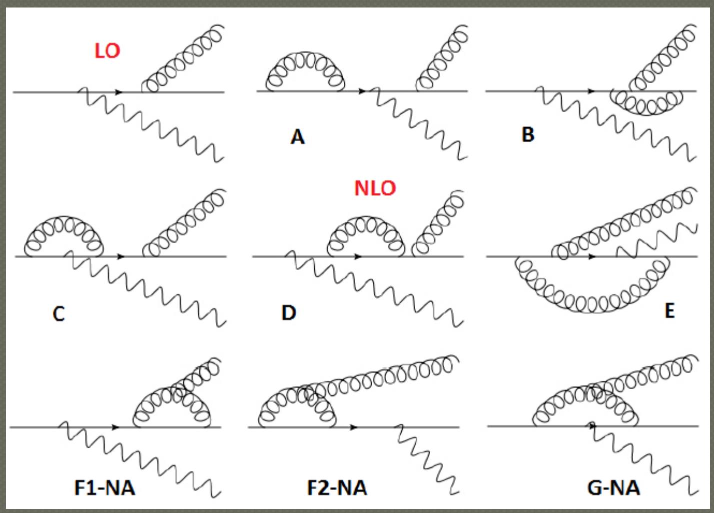
(Catani, de Florian, Rodrigo and GS, in preparation)

Triple collinear splittings

- Status:
  - Splitting functions with photons fully computed at NLO



$q \to q\gamma\gamma$



$q \to qg\gamma$ → $g \to q\bar{q}\gamma$

  - Divergent structure compatible with Catani's formula
  - Symmetries used to both reduce and check results
  - Very lengthy expressions → Manual simplification…

(Catani, de Florian, Rodrigo and GS, in preparation)

Conclusions and perspectives

- Splitting amplitudes describe collinear factorization properties and are process-independent quantities (except, maybe, in some kinematical configurations).
- Changing (*well-defined*) DREG scheme introduces $O(\varepsilon^0)$ differences among results. They can be interpreted in terms of *scalar-gluons* contributions.
- DREG scheme consistency studied for all QCD and QCD+QED double-splittings at NLO.
- Triple collinear splitting functions with photons fully computed at NLO.
- We are computing triple collinear splittings in pure QCD at NLO (including amplitude level results)

THANK YOU!

References

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