# Mueller-Navelet jets at LHC: matching NLL BFKL with fixed NLO calculations

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CERN, LHCphenoNet meeting December 2-nd, 2013

# Motivation and Outline

- Motivations
  - One of the important longstanding theoretical questions: the behaviour of QCD in the high-energy (Regge) limit $s \gg -t$
  - We expect a new kind of dynamics (BFKL dynamics) beyond fixed order perturbative predictions, with amplitudes and cross section governed by power-like behaviour $s^\omega$
  - For (semi-)hard processes $s \gg -t \gg \Lambda^2_{\mathrm{QCD}}$, P.Th still applicable with all-order resummation of logarithmic coefficients $(\alpha_{\mathrm{s}} \log s)^n$
- Outline
  - Process suited for study of high energy QCD: Mueller-Navelet dijets
  - Review the theoretical description of MN jets within the BFKL approach
  - Comparison with recent CMS analysis (and fixed-order based MC)
  - Improvement by matching fixed NLO with resummed BFKL: method and preliminary results

# How to test QCD in the Regge limit?

Look for high-energy observables

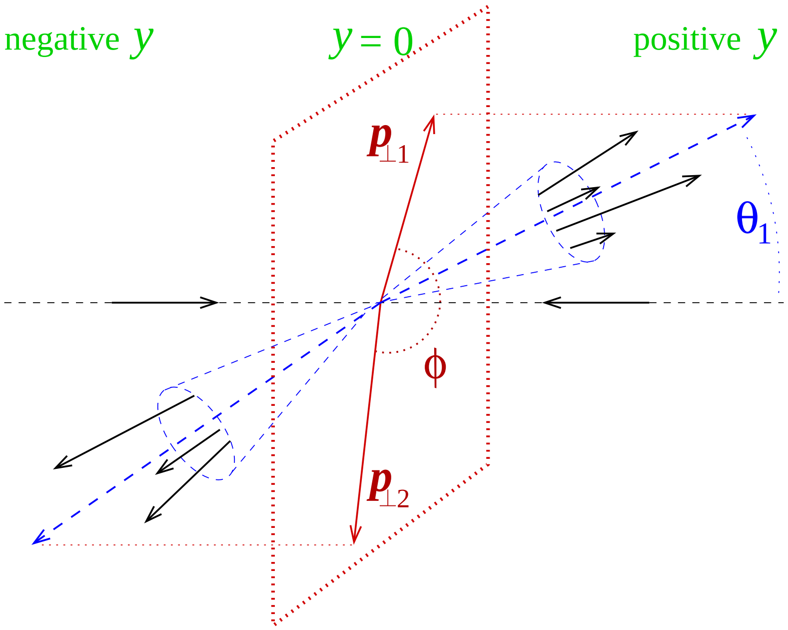
- calculable within perturbative QCD (large scales: hard $\gamma^*$, heavy mesons ($J/\psi$, $\Upsilon$), energetic [forward] jets)
- insensitive to partonic content of hadrons, to hadronization, and to standard collinear evolution (DGLAP)

Criteria met by semi-hard processes with $s \gg \boldsymbol{k}_i^2 \gg \Lambda_{\mathrm{QCD}}^2$, where $\boldsymbol{k}_i^2$ are typical transverse scales, all of the same order.

In particular, we consider Mueller-Navelet dijets

# Mueller-Navelet jets

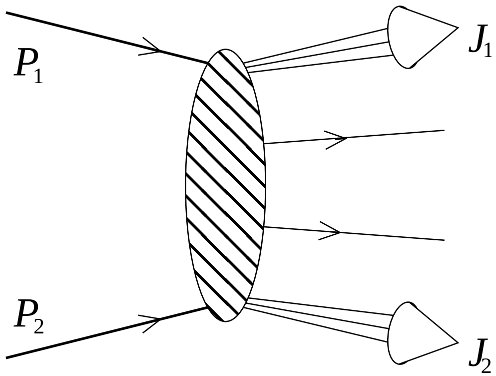
One of most famous testing processes for studying PT high-energy QCD at hadron colliders *[Mueller Navelet 1987]*




Rapidity: $y = \log(\cot(\theta/2))$

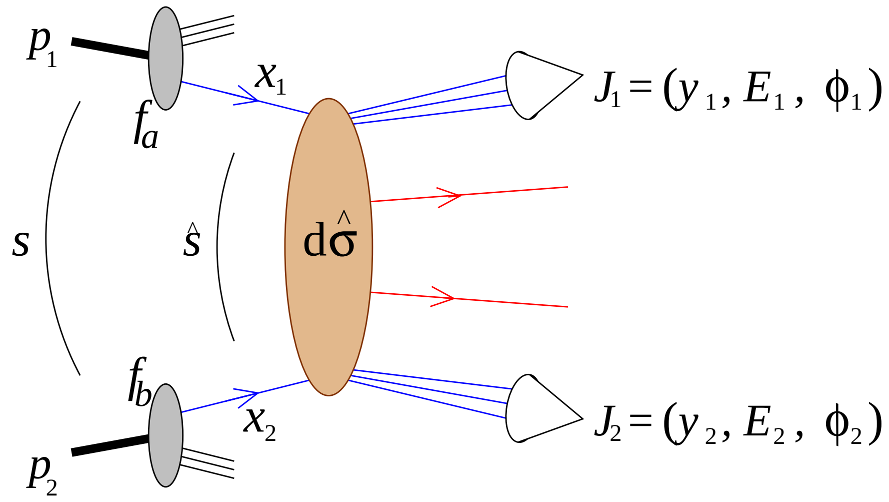
Final states with two jets with similar $E_T$ and large rapidity separation

- Comparable hard scales (jet energies) limit the logarithms of collinear type $\log(E_1/E_2)$
- Big separation in rapidity $Y \equiv y_1 - y_2 \quad \Rightarrow \quad$ large $\log(s/E_J^2) \sim Y$




Anything can be emitted between the jets

# Factorization of NP effects




MN propose use of factorization theorem:

$$\frac{\mathrm{d}\sigma}{(\mathrm{d}y_1\mathrm{d}E_1\mathrm{d}\phi_1)(\mathrm{d}y_2\mathrm{d}E_2\mathrm{d}\phi_2)} =$$

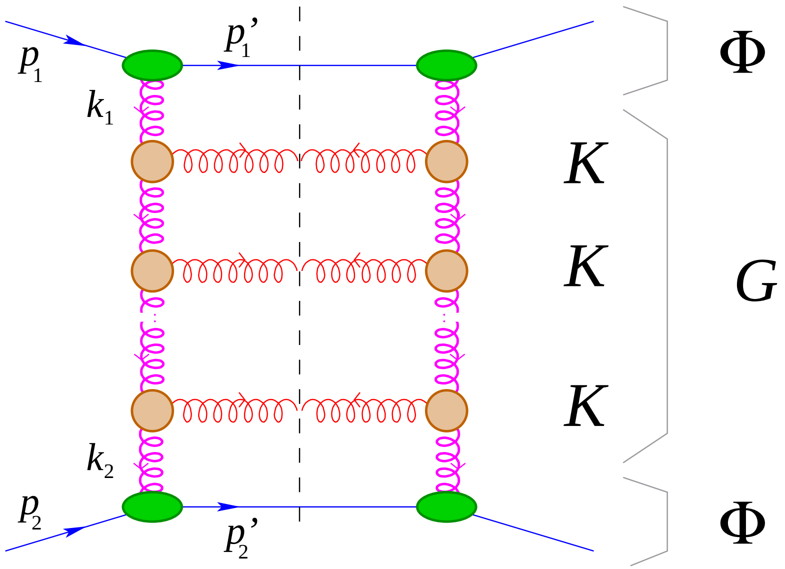
$$\frac{\mathrm{d}\sigma}{\mathrm{d}J_1\mathrm{d}J_2} = \sum_{a,b=g,q,\bar{q}} \int_0^1 \mathrm{d}x_1\mathrm{d}x_2\ f_a(x_1, E_{J1}^2) f_b(x_2, E_{J2}^2) \frac{\mathrm{d}\hat{\sigma}(x_1, x_2)}{\mathrm{d}J_1\mathrm{d}J_2}$$

Factorization formula justified because:

- semi-inclusive observable (jets + anything)
- large transferred momenta ($E_J \gg \Lambda_{\mathrm{QCD}}$)

# High energy factorization and BFKL

At high energy, partonic cross section is factorized in $\boldsymbol{k}_\perp$-dependent factors




Impact factors describe coupling of external particles with (reggeized) gluons

All energy dependence in universal Gluon Green's function is the sum of all ladder diagrams

$$\sigma_{12}(s) = \int \mathrm{d}\boldsymbol{k}_1 \mathrm{d}\boldsymbol{k}_2 \; \Phi_1(\boldsymbol{k}_1) G(s, \boldsymbol{k}_1, \boldsymbol{k}_2) \Phi_2(\boldsymbol{k}_2)$$

$$\frac{\partial}{\partial \log s} G(s, \boldsymbol{k}_1, \boldsymbol{k}_2) = \int \mathrm{d}\boldsymbol{k} \; K(\boldsymbol{k}_1, \boldsymbol{k}) G(s, \boldsymbol{k}, \boldsymbol{k}_2)$$

$$K = \alpha_\mathrm{s} K_0 + \alpha_\mathrm{s}^2 K_1 + \ldots$$

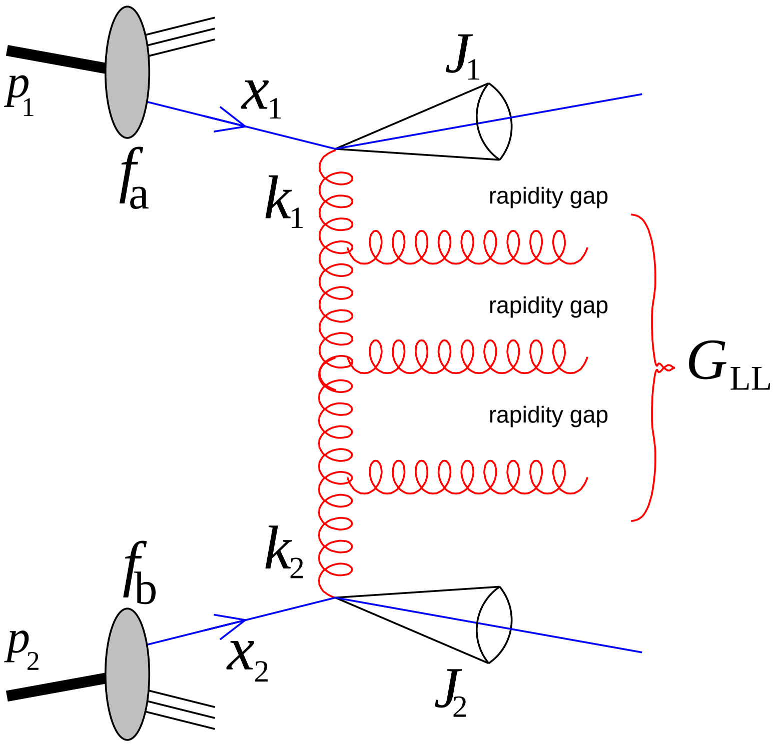
LL resums $\alpha_\mathrm{s}^n \log^n s$
*[Balitski-Fadin -Kuraev-Lipatov '78]*
NLL resums $\alpha_\mathrm{s}^n \log^{n-1} s$
*[Fadin-Lipatov, Camici-Ciafaloni '98]*

# MN Jets in NLL approximation

MN jet factorization formula is a convolution of 5 objects

Starting from LL factorization formula

$$\frac{d\sigma(s)}{dJ_1 dJ_2} = \sum_{a,b} \int_0^1 dx_1 dx_2 \int d\boldsymbol{k}_1 d\boldsymbol{k}_2$$
$$\times\ f_a(x_1)$$
$$\times\ V_a^{(0)}(x_1, \boldsymbol{k}_1; J_1)$$
$$\times\ G_{\text{LL}}(x_1 x_2 s, \boldsymbol{k}_1, \boldsymbol{k}_2)$$
$$\times\ V_b^{(0)}(x_2, \boldsymbol{k}_2; J_2)$$
$$\times\ f_b(x_2)$$




where $V_a^{(0)}(x, \boldsymbol{k}; J) = \alpha_s C_a \delta(\boldsymbol{k} - \boldsymbol{p}_J)\delta(x - x_J)$ and $x_J = |\boldsymbol{p}_J| e^{y_J}/\sqrt{s}$

- At LL level the jet vertex condition is trivial (only 1 parton)
- Kinematics characterized by large rapidity gaps among particles

# MN Jets in NLL approximation

*[Bartels, DC, Vacca '02]* computed NLL calculations of impact factors for Mueller-Navelet jets

Proved NLL factorization formula

$$\frac{\mathrm{d}\sigma(s)}{\mathrm{d}J_1\mathrm{d}J_2} = \sum_{a,b}\int_0^1 \mathrm{d}x_1\mathrm{d}x_2 \int \mathrm{d}\boldsymbol{k}_1\mathrm{d}\boldsymbol{k}_2$$

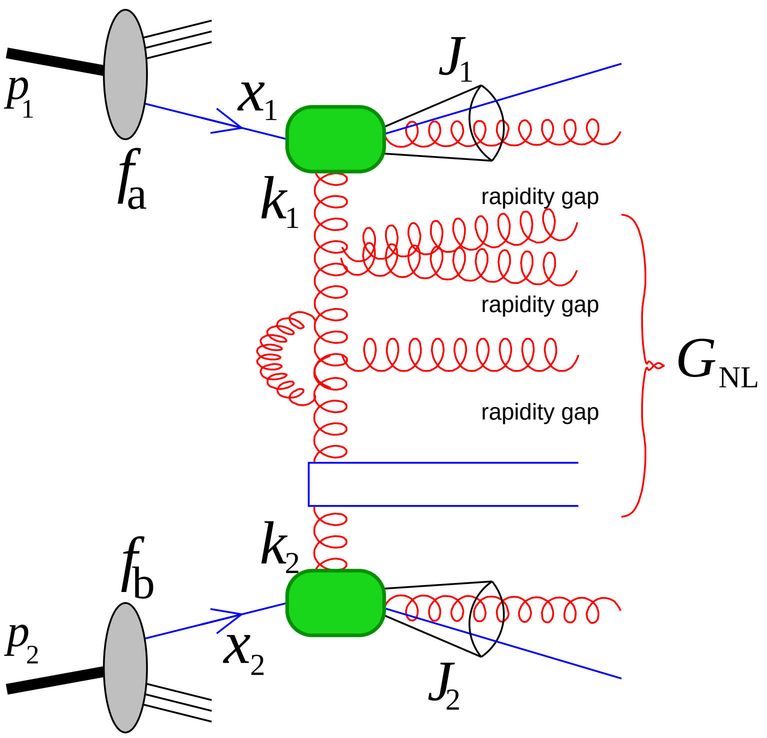
$$\times\ f_a(x_1)$$

$$\times\ \boldsymbol{V_a^{(1)}}(x_1,\boldsymbol{k}_1;J_1)$$

$$\times\ G_{\mathrm{NL}}(x_1x_2s,\boldsymbol{k}_1,\boldsymbol{k}_2)$$

$$\times\ \boldsymbol{V_b^{(1)}}(x_2,\boldsymbol{k}_2;J_2)$$

$$\times\ f_b(x_2)$$

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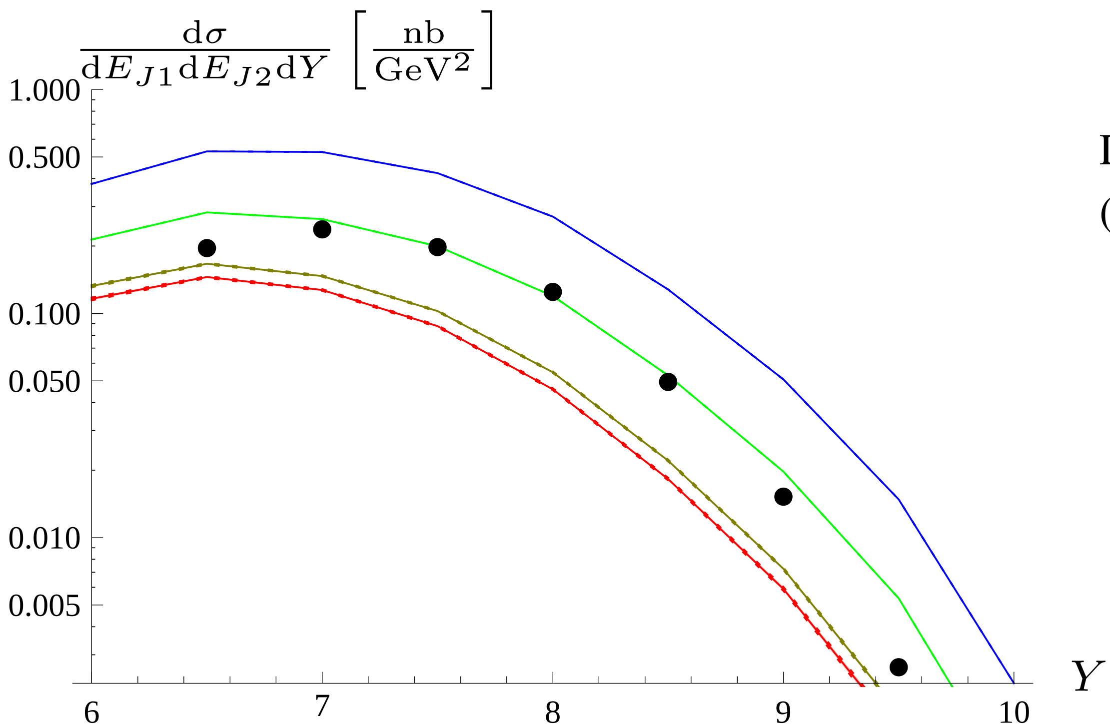
- At NL level the jet vertex condition is non-trivial (e.g. depends on jet radius $R$ and algorithm)
- Pairs of particles can be emitted without rapidity gaps

# Predictions for LHC at 14 TeV

With LHC we can test these ideas




Differential cross-section
$(E_{J1} = 35\,\text{GeV},\, E_{J2} = 50\,\text{GeV})$

pure LL

LL vertices + improved collinear NLL Green's function

NLL vertices + NLL Green's function

NLL vertices + improved collinear NLL Green's function

dots: NLO DGLAP parton generator DIJET *[Fontannaz]*

- NL corrections to jet vertices are sizeable and as important as those to the GGF *[DC, Schwennsen, Szymanowsky, Wallon]*
- Prediction is definitely different from those based on fixed-order MonteCarlos *[Fontannaz]*

# Theoretical uncertainties at 14 TeV

Variations of cross sectin with respect to $\mu_R = \mu_F$ and $s_0$ changes

pure LL

LL vertices + improved collinear NLL Green's function

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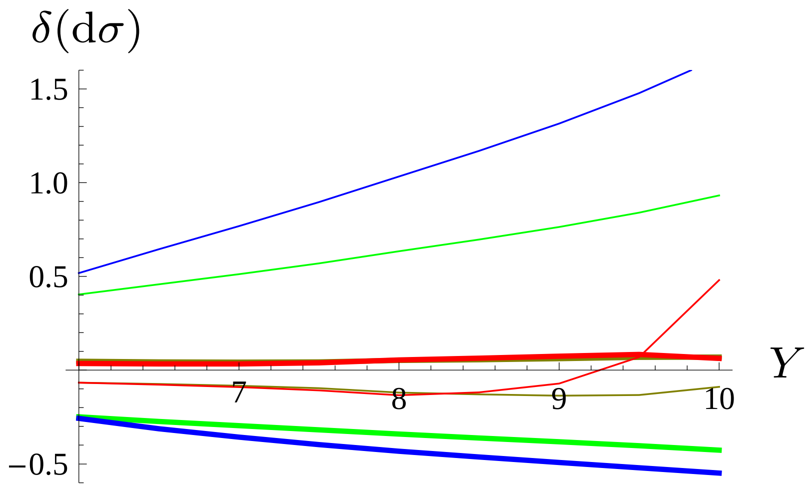
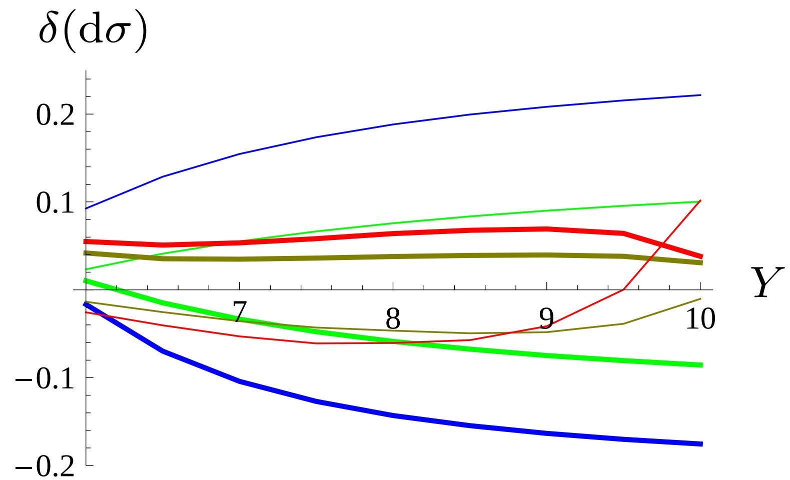



Relative effect of changing $\mu_R = \mu_F$

by factors 2 (thick) and $1/2$ (thin)




Relative effect of changing $\sqrt{s_0}$ (in $\log(\frac{s}{s_0})$)

by factors 2 (thick) and $1/2$ (thin)

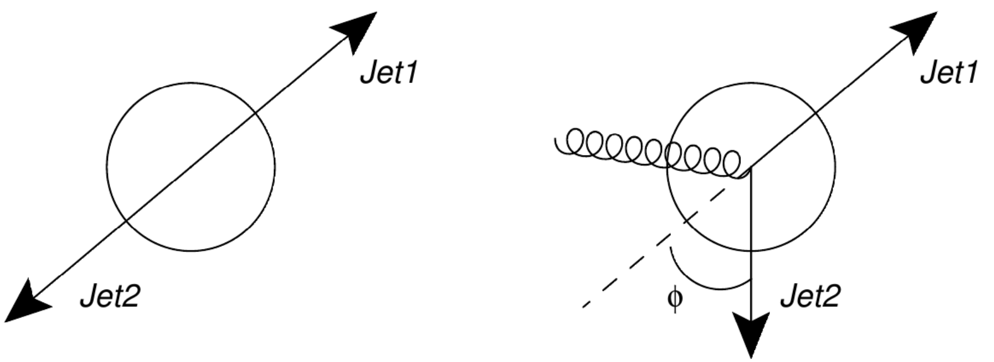
- There is a substantial reduction of scales' dependence from >50% to 10-15%

# CMS analysis of MN jets at 7 TeV

Analysis of the azimuthal decorrelation of the two jets

$$\frac{1}{\sigma}\frac{d\sigma}{d\phi} \qquad \Big|\Big| \qquad \langle\cos(m\phi)\rangle = \frac{C_m(Y)}{C_0(Y)} \equiv \frac{\int d\phi \, \frac{d^2(\sigma\cos(m\phi))}{d\phi dY}}{d\sigma/dY}$$

- Distinguishes BFKL dynamics from fixed order one: they provide different amount of particle emissions between jets, which is responsible for their decorrelation
- $\langle\cos(m\phi)\rangle$ has reduced theoretical scale uncertainties being a ratio of differential cross sections



# CMS analysis of MN jets at 7 TeV

Angular distribution $\frac{1}{\sigma}\frac{\mathrm{d}\sigma}{\mathrm{d}\Delta\phi}$ with $\Delta\phi \equiv \pi - \phi \equiv \phi_1 - \phi_2$

Data selection: $E_{T1,2} > 35\mathrm{GeV}$, $|y_i| < 4.7$

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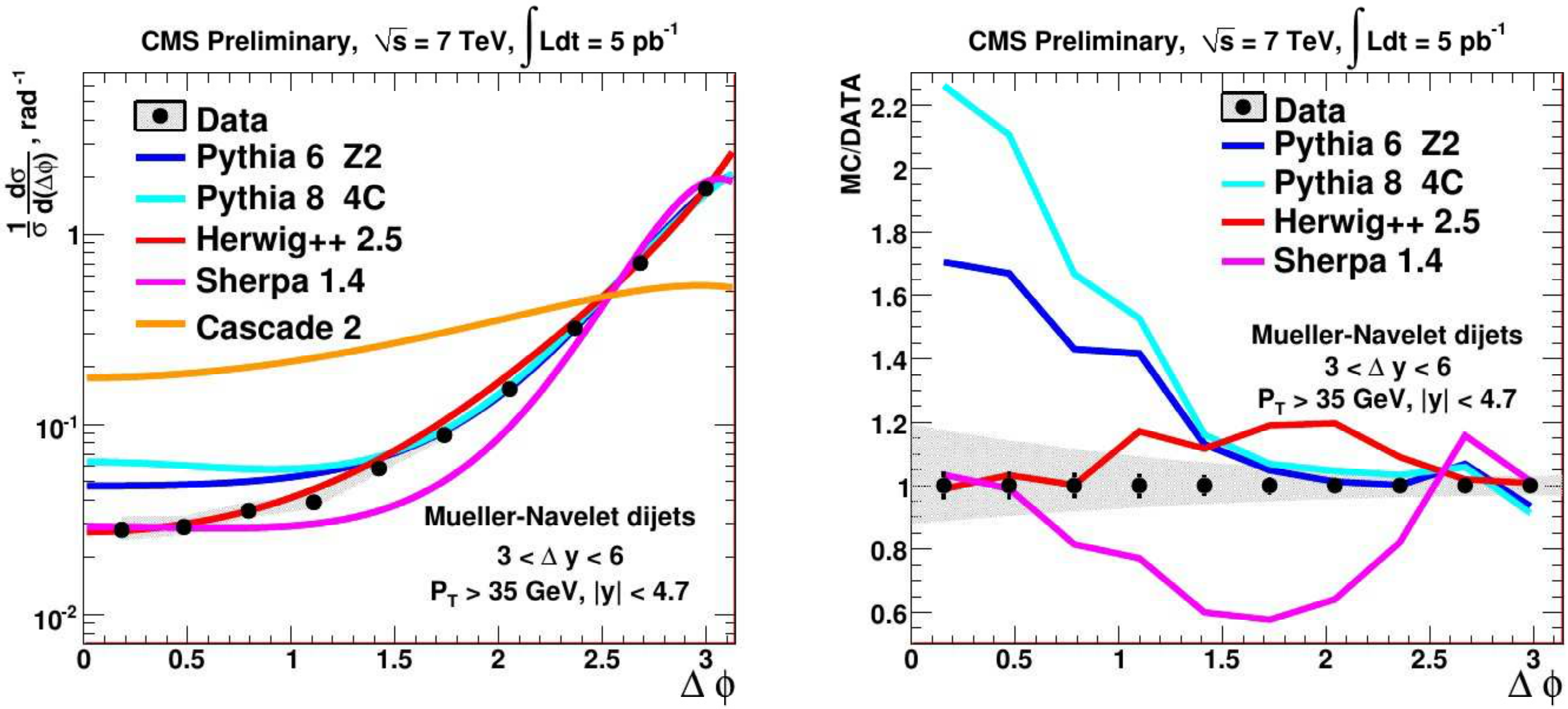


Some MC are close to data (at least for $\phi \lesssim 2$)

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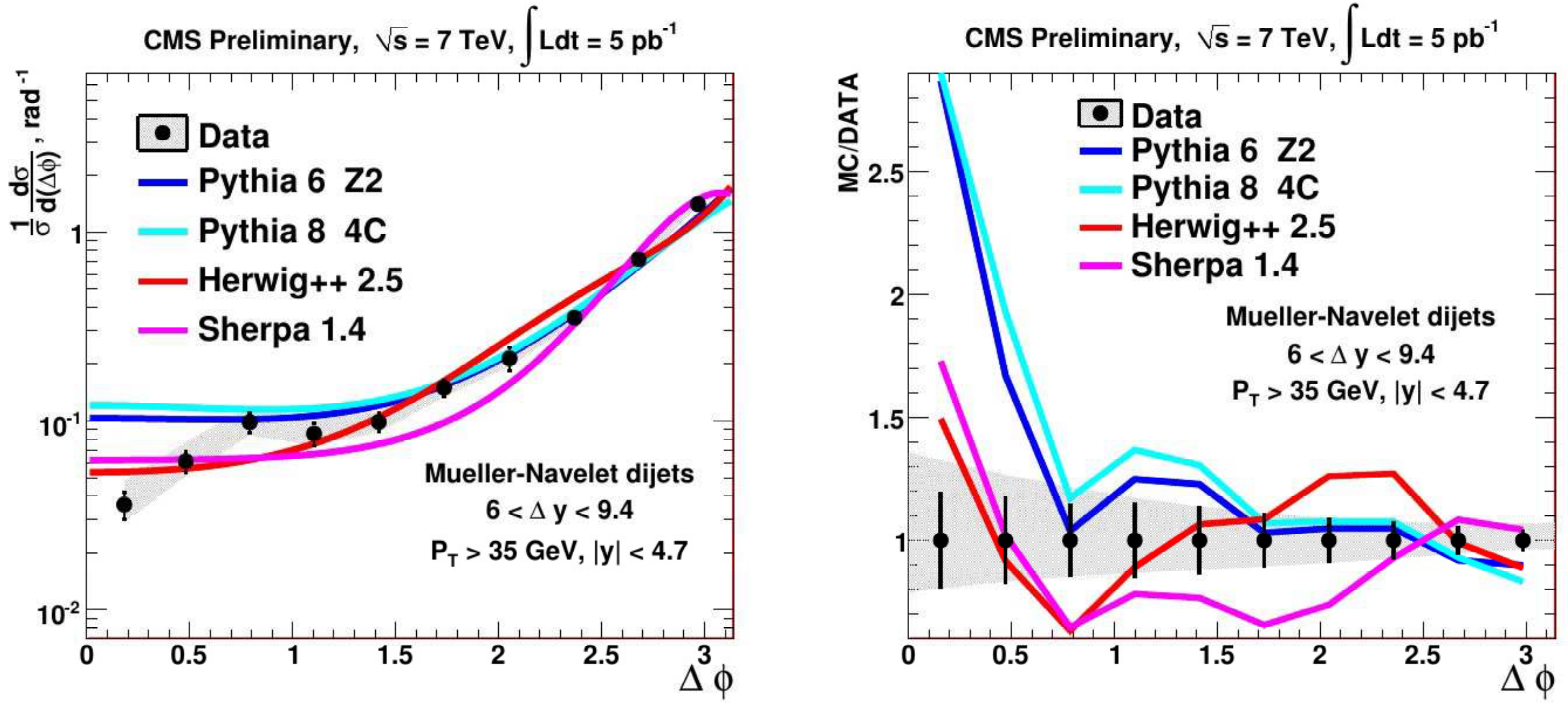


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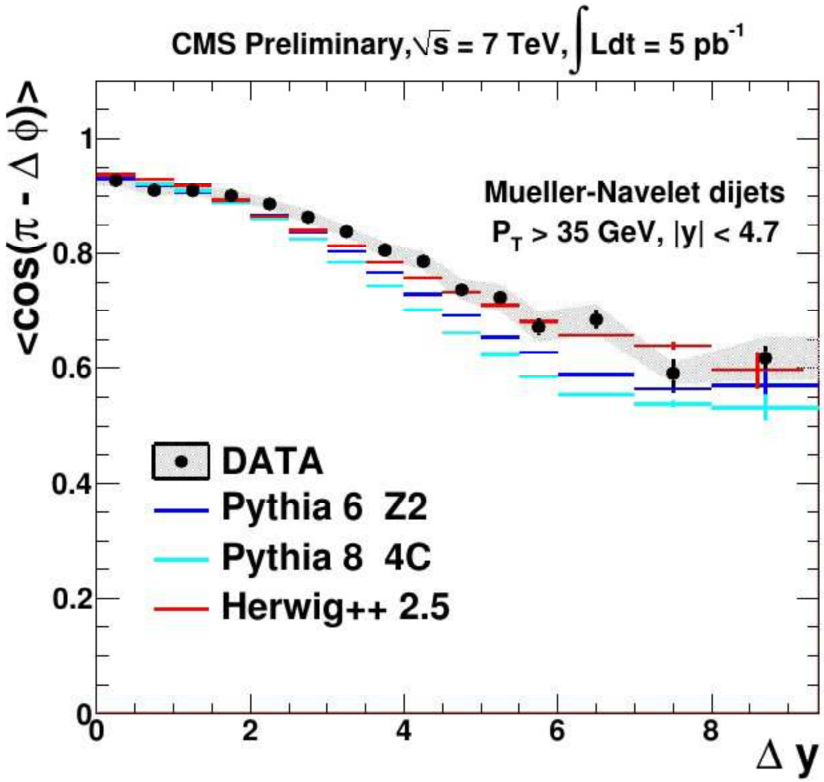
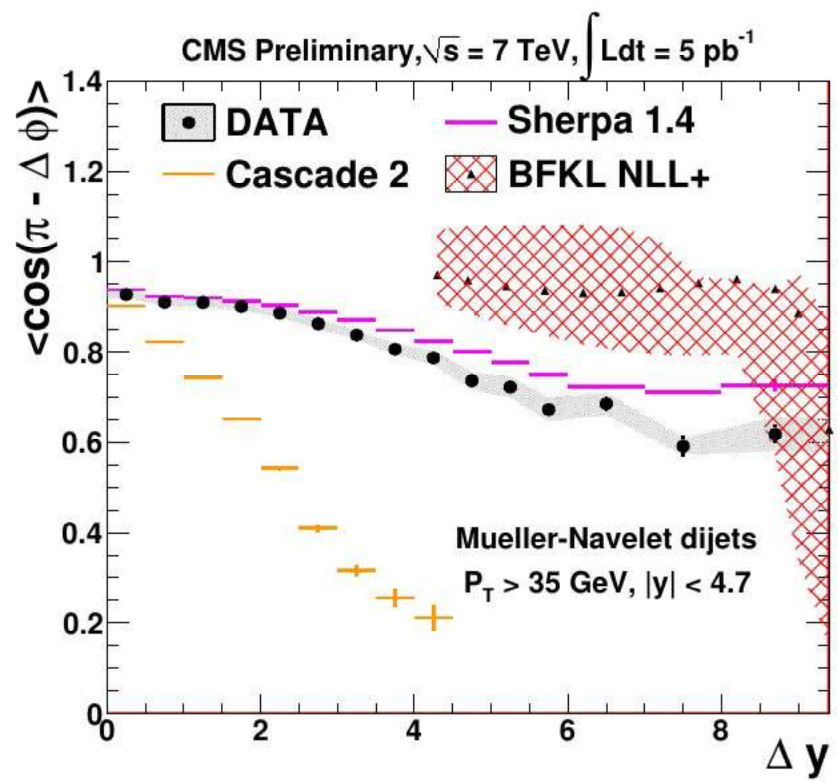
$$\langle\cos(m\phi)\rangle = \frac{C_m(Y)}{C_0(Y)} \equiv \frac{\int \mathrm{d}\phi \, \frac{\mathrm{d}^2(\sigma\cos(m\phi))}{\mathrm{d}\phi\mathrm{d}Y}}{\mathrm{d}\sigma/\mathrm{d}Y}$$

# CMS analysis of MN jets at 7 TeV

Data: $E_{T1,2} > 35\text{GeV}$, $|y_i| < 4.7$ $\Delta y \equiv Y \equiv |y_1 - y_2| < 9.4$ $m = 1$




The larger $Y$, the more radiation and decorrelation

BFKL was expected to predict more radiation than fixed order $\Rightarrow$ more decorrelation

Some MC agree with data

NLL BFKL estimate has problems $\langle \cos\phi \rangle > 1$ for $\mu_R = \mu_F = E_T/2$

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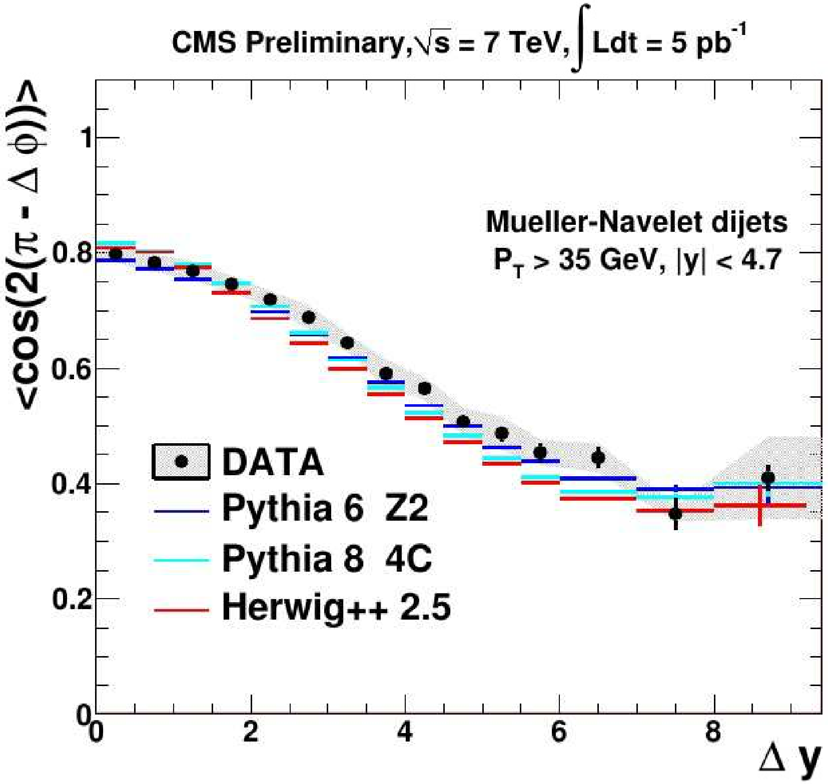


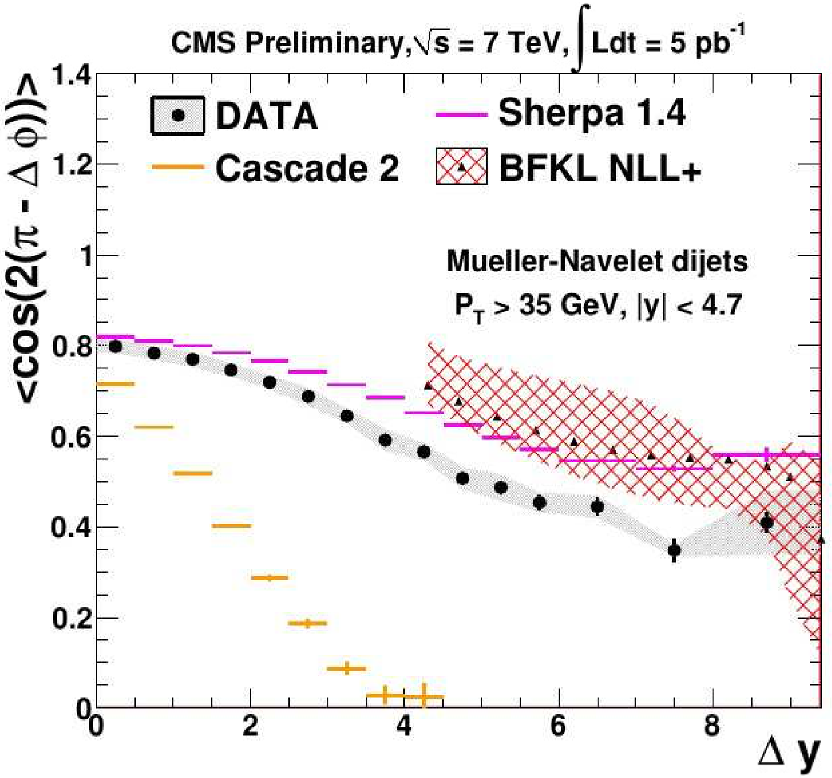


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NLL BFKL still unable to reproduce data

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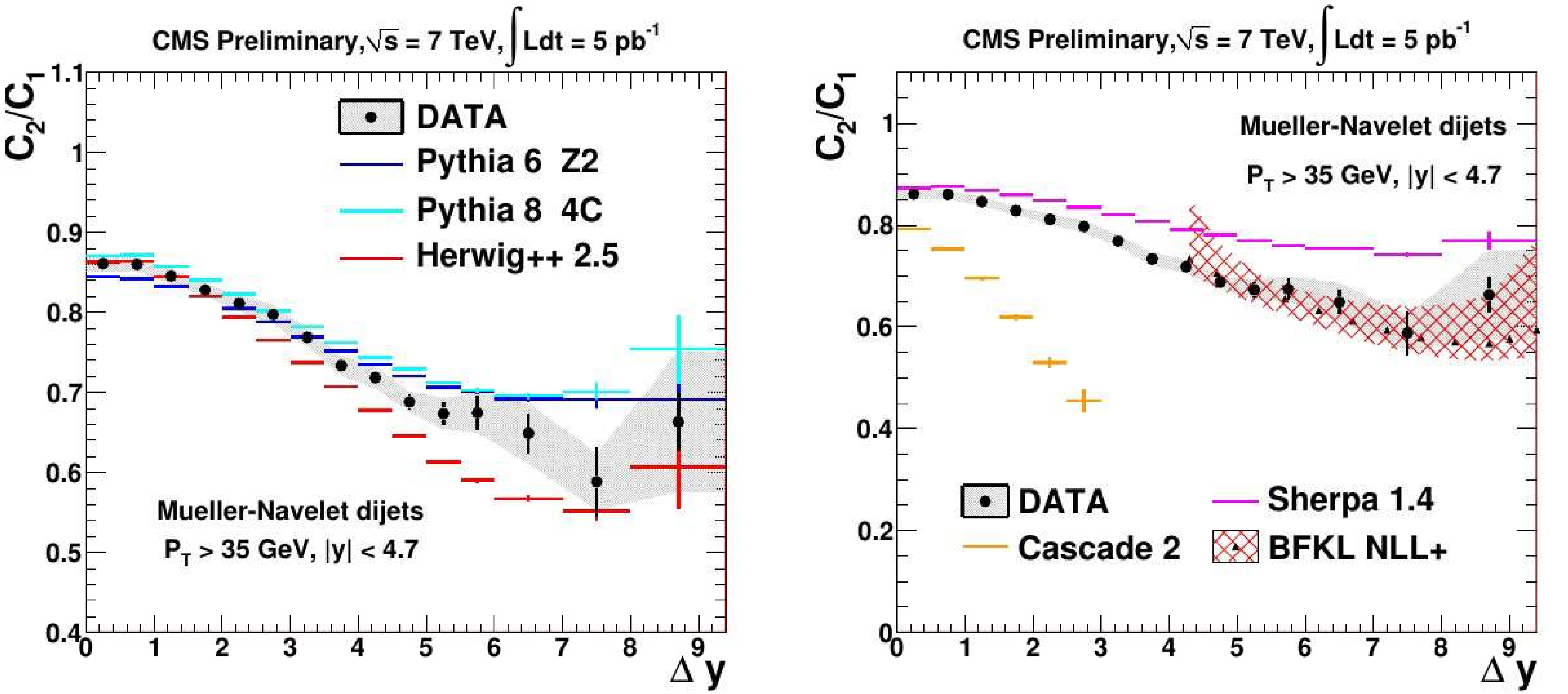


Ratio $\dfrac{C_2}{C_1} = \dfrac{\langle \cos(2\phi) \rangle}{\langle \cos\phi \rangle}$

MC don't agree well with data

NLL BFKL in perfect agreement with data

- Neither BFKL NLL nor fixed order MC give a satisfactory description of data yet

- BFKL NLL still suffers from large scale uncertainties $\sim 10 \div 15\%$

# NLL with BLM scale fixing

*[Ducloué,Szymanowski,Wallon '13]* proposed to tame large scale dependence of BFKL by fixing $\mu_R$ with BLM procedure

$$\mu_R^2 = \exp\left[\frac{1}{2}\chi_0 - \frac{5}{3} + 2\left(1 + \frac{2}{3}I\right)\right] E_{T1}\, E_{T2}$$

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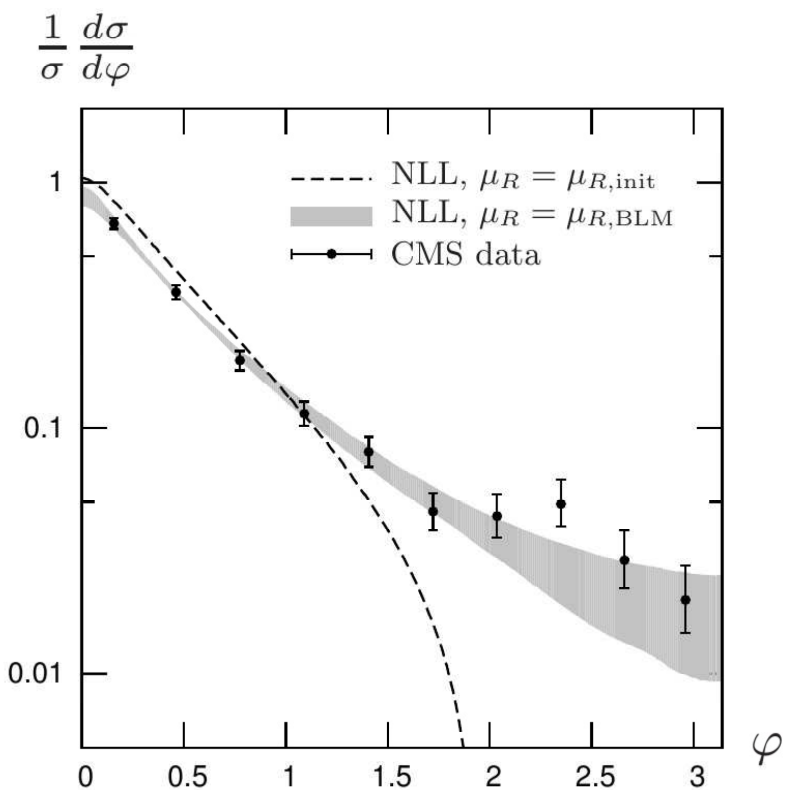


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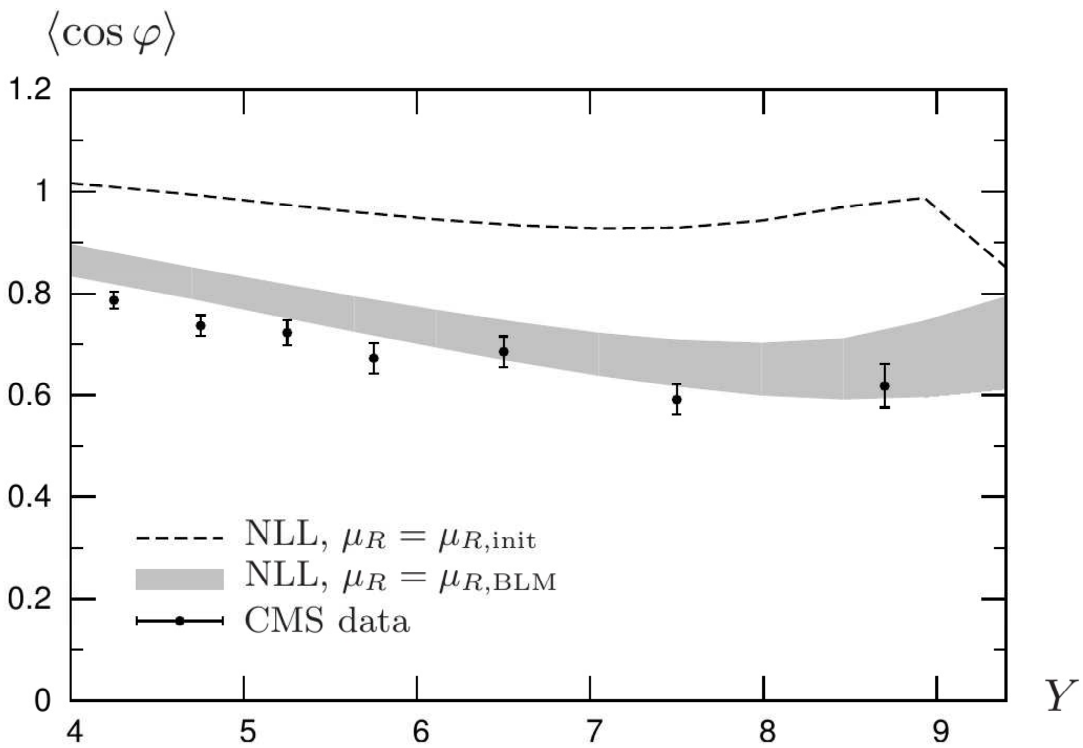


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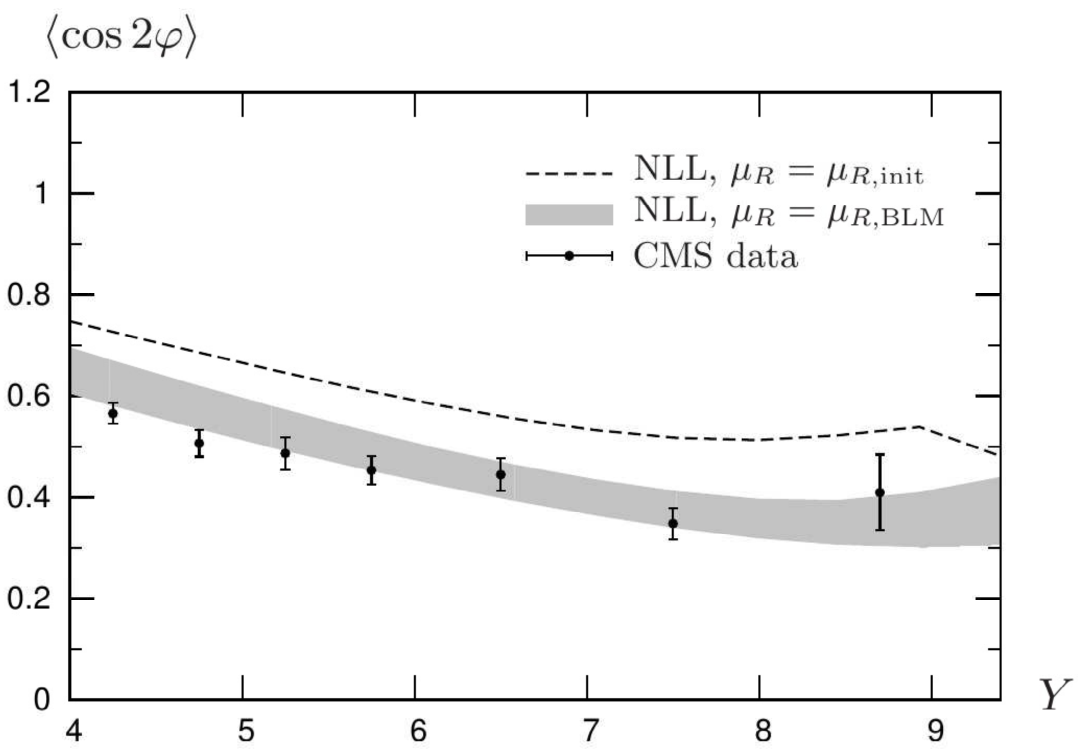


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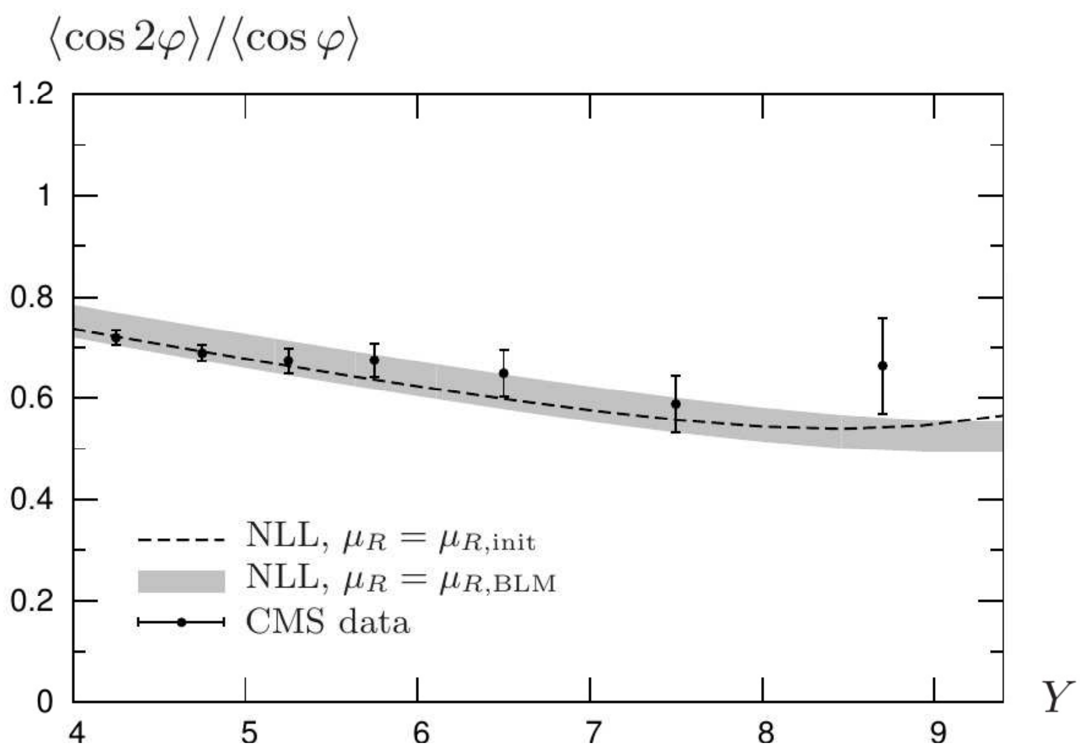
$$\mu_R^2 = \exp\left[\frac{1}{2}\chi_0 - \frac{5}{3} + 2\left(1 + \frac{2}{3}I\right)\right] E_{T1}\, E_{T2} \simeq 503\, E_{T1}\, E_{T2}$$

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# Matching BFKL with Fixed NLO

Our aim is to merge fixed NL order and NLL BFKL resummation to

- improve description of data
- correctly reproduce not only ratios but absolute values

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Standard matching procedure:

- add to BFKL the full perturbative NLO result $\mathcal{O}\left(\alpha_{\rm s}^3\right)$
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Results for cross section and $C_m$ coefficients

- The implementation is still work in progess
- Preliminary results of central values (no error estimate yet)

# LO matching (cross section)

Cross section: LL BFKL + LO pert. $\mathcal{O}\left(\alpha_s\right)^2$ − BFKL $\mathcal{O}\left(\alpha_s^2\right)$

$$\frac{\mathrm{d}\sigma(s)}{\mathrm{d}J_1\mathrm{d}J_2} = \sum_{a,b}\int_0^1 \mathrm{d}x_1\mathrm{d}x_2\; f_a(x_1)f_b(x_2)\Bigg\{$$

$$\int \mathrm{d}\boldsymbol{k}_1\mathrm{d}\boldsymbol{k}_2\left[V_a^{(0)}(x_1,\boldsymbol{k}_1;J_1)\,G_{\mathrm{LL}}(x_1x_2s,\boldsymbol{k}_1,\boldsymbol{k}_2)\,V_b^{(0)}(x_2,\boldsymbol{k}_2;J_2)\right]$$

$$+\frac{\mathrm{d}\hat{\sigma}^{(LO)}(x_1,x_2)}{\mathrm{d}J_1\mathrm{d}J_2}$$

$$-\int \mathrm{d}\boldsymbol{k}_1\mathrm{d}\boldsymbol{k}_2\left[V_a^{(0)}(x_1,\boldsymbol{k}_1;J_1)\,\delta^2(\boldsymbol{k}_1-\boldsymbol{k}_2)\,V_b^{(0)}(x_2,\boldsymbol{k}_2;J_2)\right]\Bigg\}$$

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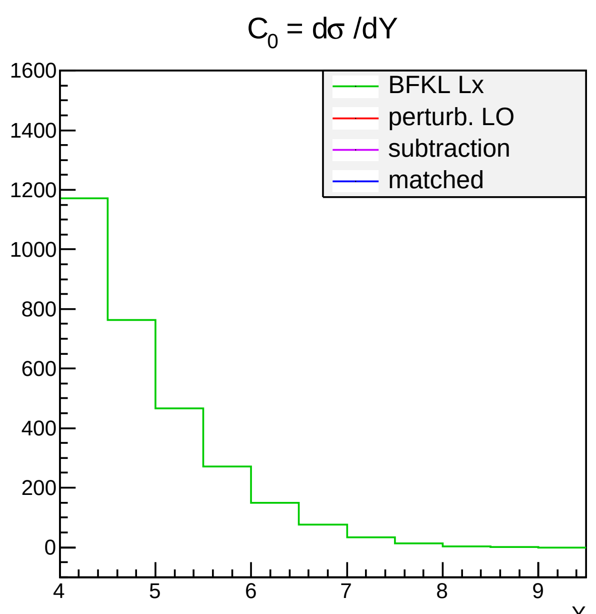


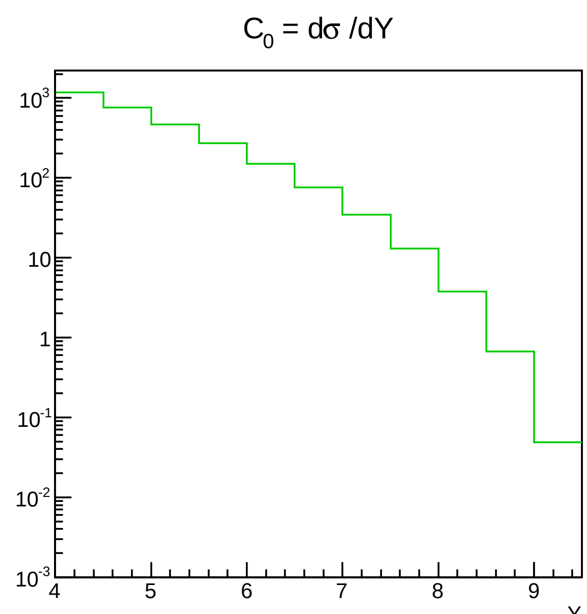

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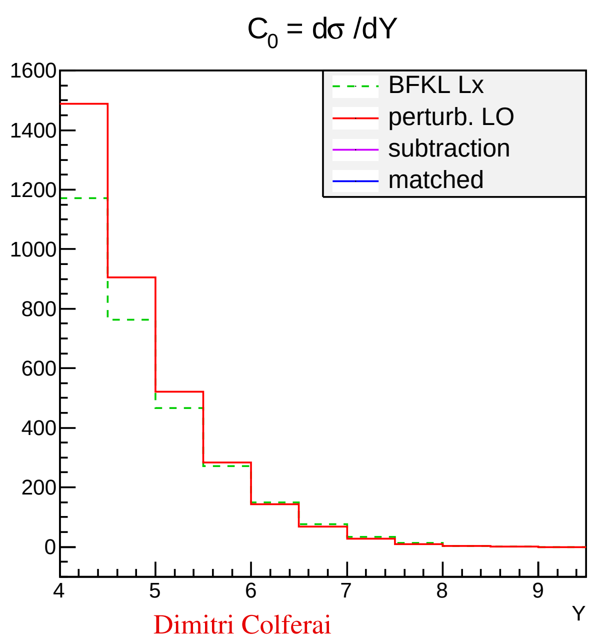


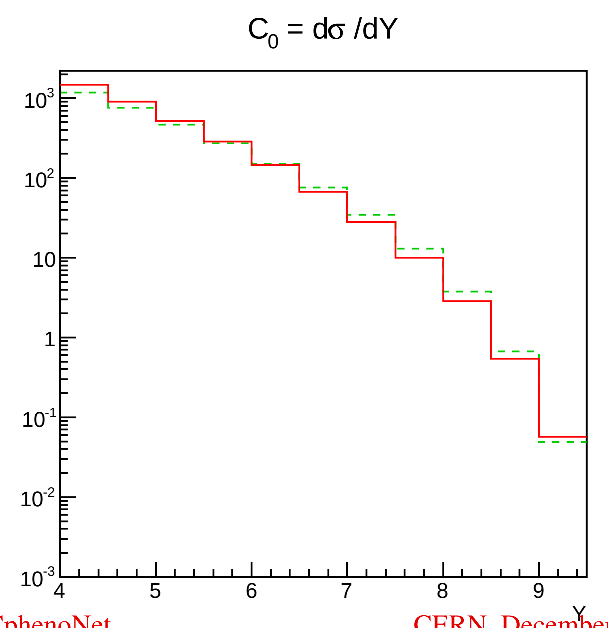

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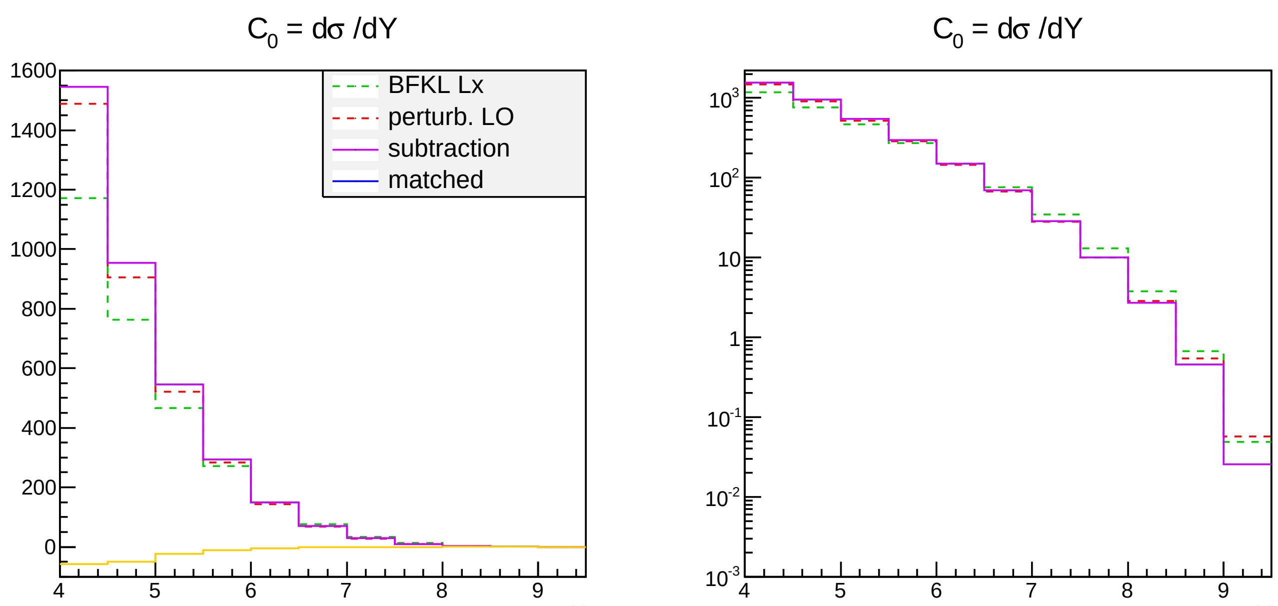
$$\int d\boldsymbol{k}_1 d\boldsymbol{k}_2 \Big[ V_a^{(0)}(x_1, \boldsymbol{k}_1; J_1)\, G_{\rm LL}(x_1 x_2 s, \boldsymbol{k}_1, \boldsymbol{k}_2)\, V_b^{(0)}(x_2, \boldsymbol{k}_2; J_2) \Big]$$

$$+ \frac{d\hat{\sigma}^{(LO)}(x_1, x_2)}{dJ_1 dJ_2}$$

$$- \int d\boldsymbol{k}_1 d\boldsymbol{k}_2 \Big[ V_a^{(0)}(x_1, \boldsymbol{k}_1; J_1)\, \delta^2(\boldsymbol{k}_1 - \boldsymbol{k}_2)\, V_b^{(0)}(x_2, \boldsymbol{k}_2; J_2) \Big] \Big\}$$



# LO matching (cross section)

Cross section: LL BFKL + LO pert. $\mathcal{O}(\alpha_s)^2$ − BFKL $\mathcal{O}(\alpha_s^2)$

$$\frac{d\sigma(s)}{dJ_1 dJ_2} = \sum_{a,b} \int_0^1 dx_1 dx_2 \; f_a(x_1) f_b(x_2) \Big\{$$

$$\int d\boldsymbol{k}_1 d\boldsymbol{k}_2 \left[ V_a^{(0)}(x_1, \boldsymbol{k}_1; J_1)\, G_{\rm LL}(x_1 x_2 s, \boldsymbol{k}_1, \boldsymbol{k}_2)\, V_b^{(0)}(x_2, \boldsymbol{k}_2; J_2) \right]$$

$$+ \frac{d\hat{\sigma}^{(LO)}(x_1, x_2)}{dJ_1 dJ_2}$$

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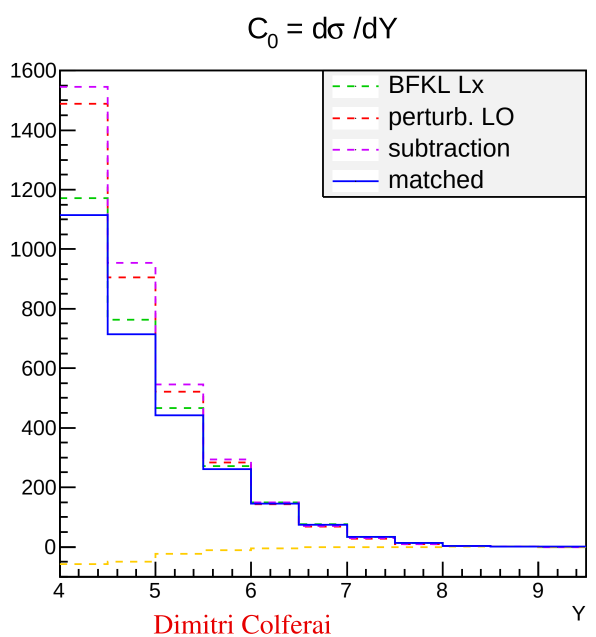


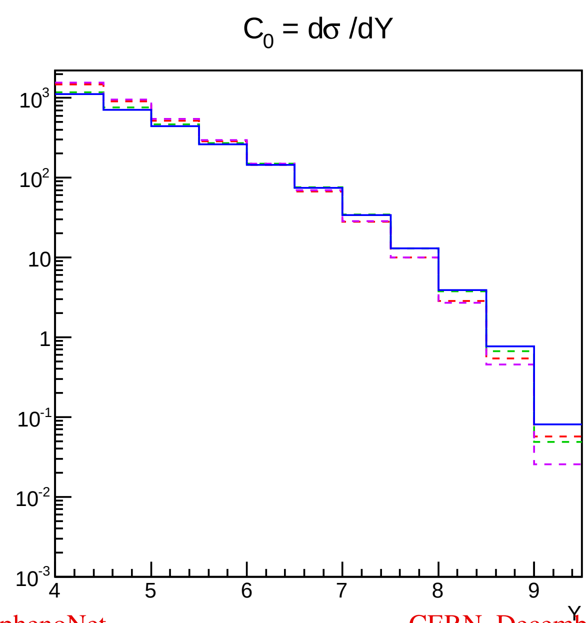


Small but non negligible correction

# NLO matching (cross section)

Cross section: NLL BFKL + NLO pert. $\mathcal{O}(\alpha_s)^3$ − BFKL $\mathcal{O}(\alpha_s^3)$

$$\frac{\mathrm{d}\sigma(s)}{\mathrm{d}J_1\mathrm{d}J_2} = \sum_{a,b}\int_0^1 \mathrm{d}x_1\mathrm{d}x_2\; f_a(x_1)f_b(x_2)\Big\{$$

$$\int \mathrm{d}\boldsymbol{k}_1\mathrm{d}\boldsymbol{k}_2\Big[V_a^{(0+1)}(x_1,\boldsymbol{k}_1;J_1)\,G_{\mathrm{NLL}}(x_1x_2s,\boldsymbol{k}_1,\boldsymbol{k}_2)\,V_b^{(0+1)}(x_2,\boldsymbol{k}_2;J_2)\Big]$$

$$+\frac{\mathrm{d}\hat{\sigma}^{(NLO)}(x_1,x_2)}{\mathrm{d}J_1\mathrm{d}J_2}$$

$$-\int \mathrm{d}\boldsymbol{k}_1\mathrm{d}\boldsymbol{k}_2\Big[V_a^{(0)}(x_1,\boldsymbol{k}_1;J_1)\,\delta^2(\boldsymbol{k}_1-\boldsymbol{k}_2)\,V_b^{(0)}(x_2,\boldsymbol{k}_2;J_2)\Big]$$

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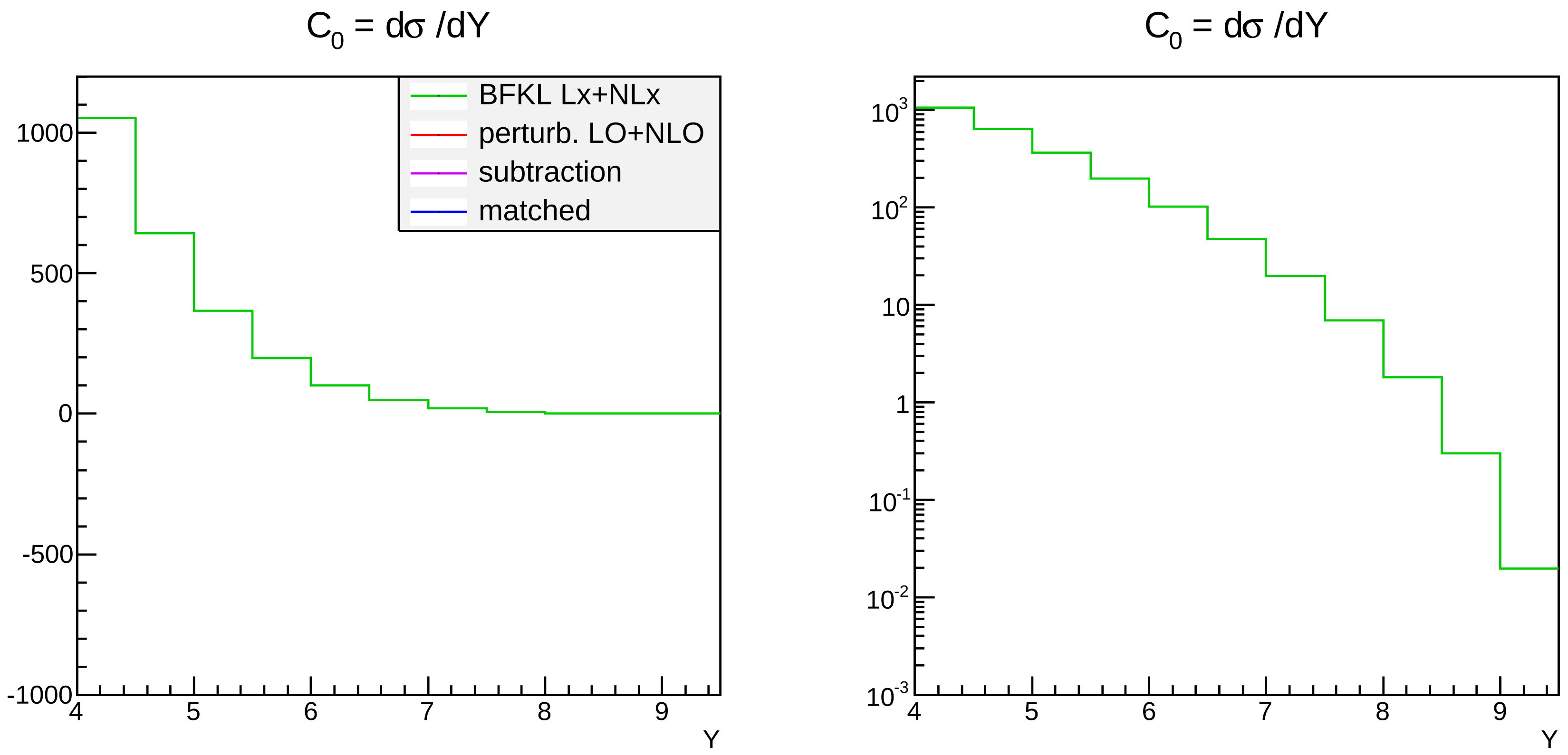
$$-\int \mathrm{d}\boldsymbol{k}_1\mathrm{d}\boldsymbol{k}_2\Big[V_a^{(0)}(x_1,\boldsymbol{k}_1;J_1)\,\delta^2(\boldsymbol{k}_1-\boldsymbol{k}_2)\,V_b^{(1)}(x_2,\boldsymbol{k}_2;J_2)\Big]$$

$$-\int \mathrm{d}\boldsymbol{k}_1\mathrm{d}\boldsymbol{k}_2\Big[V_a^{(0)}(x_1,\boldsymbol{k}_1;J_1)\,\alpha_s\log\frac{\hat{s}}{s_0}K_0(\boldsymbol{k}_1,\boldsymbol{k}_2)\,V_b^{(0)}(x_2,\boldsymbol{k}_2;J_2)\Big]\Big\}$$

# NLO matching (cross section)



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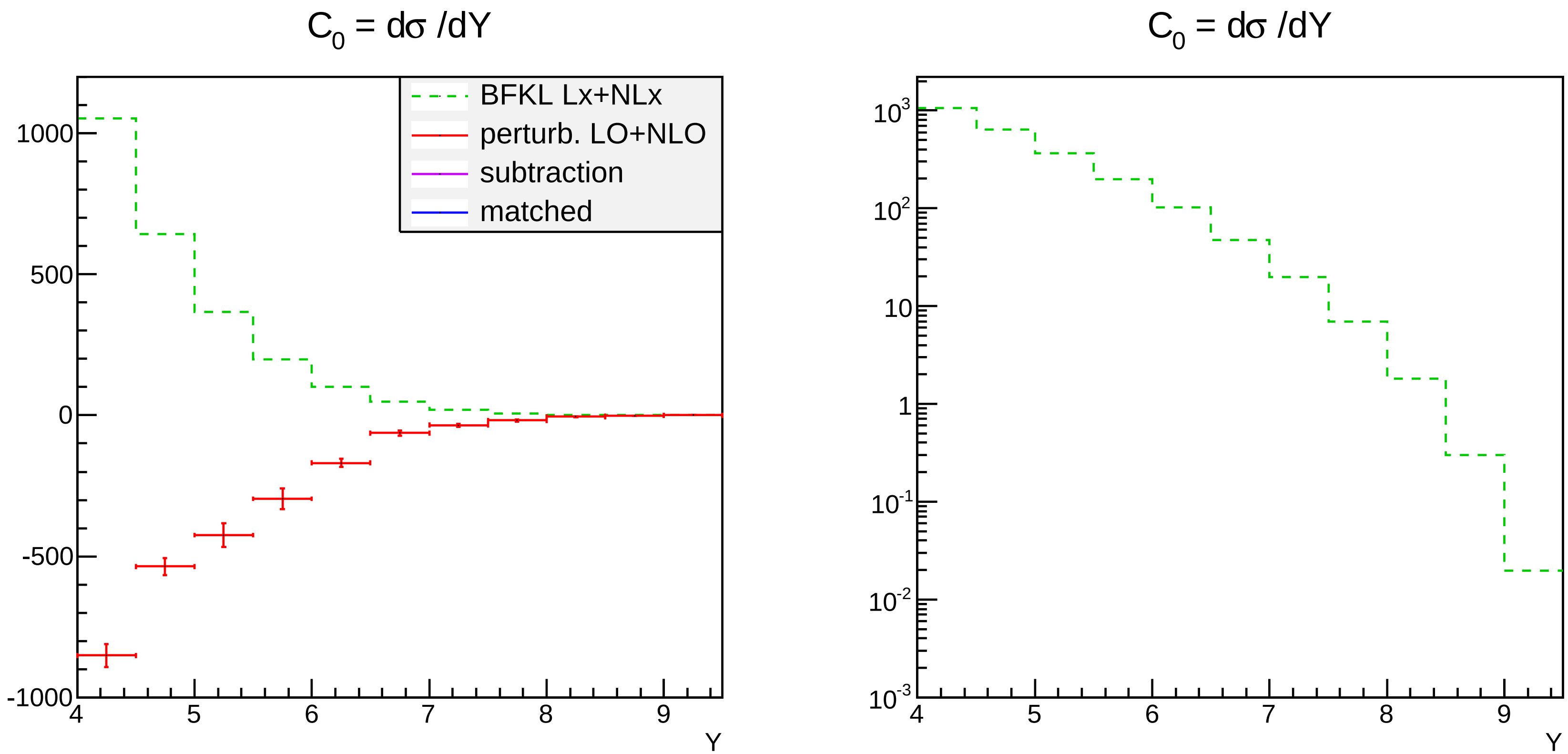


LO+NLO cross section obtained with NLOJET++ *[Nagy]* is negative!

Large errors due to very slow convergence in MC integration

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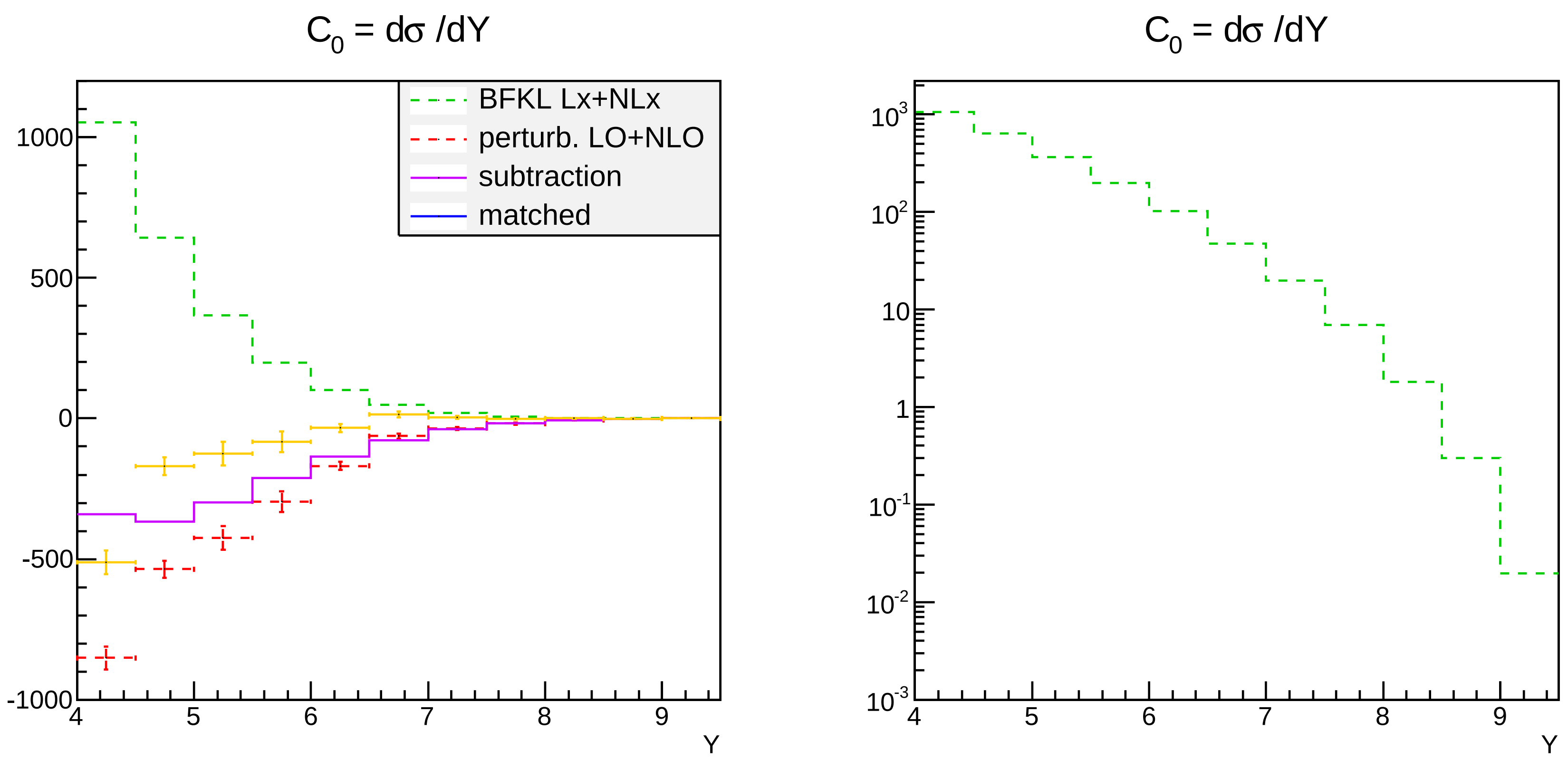


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However, also the subtraction is negative

Their difference is moderate

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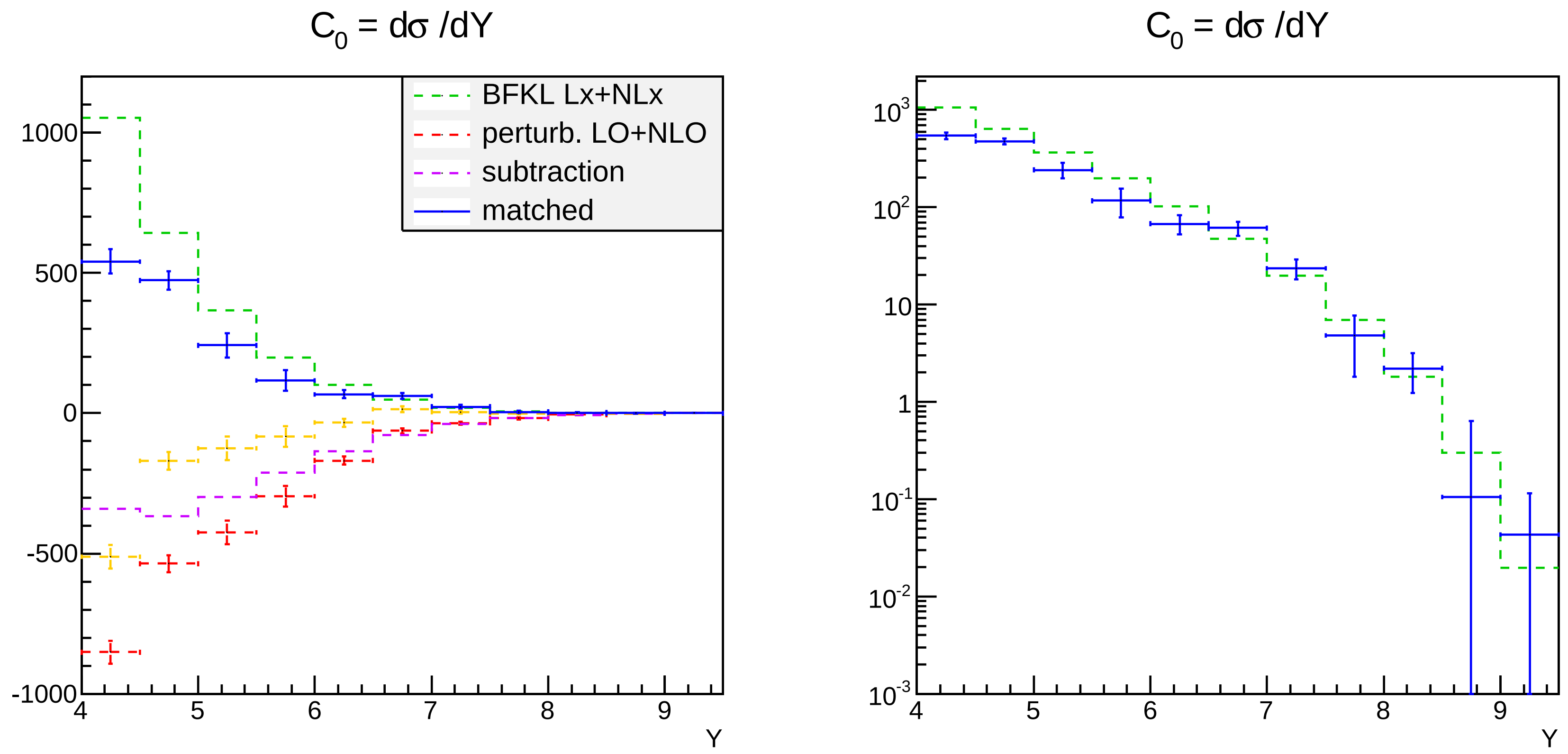


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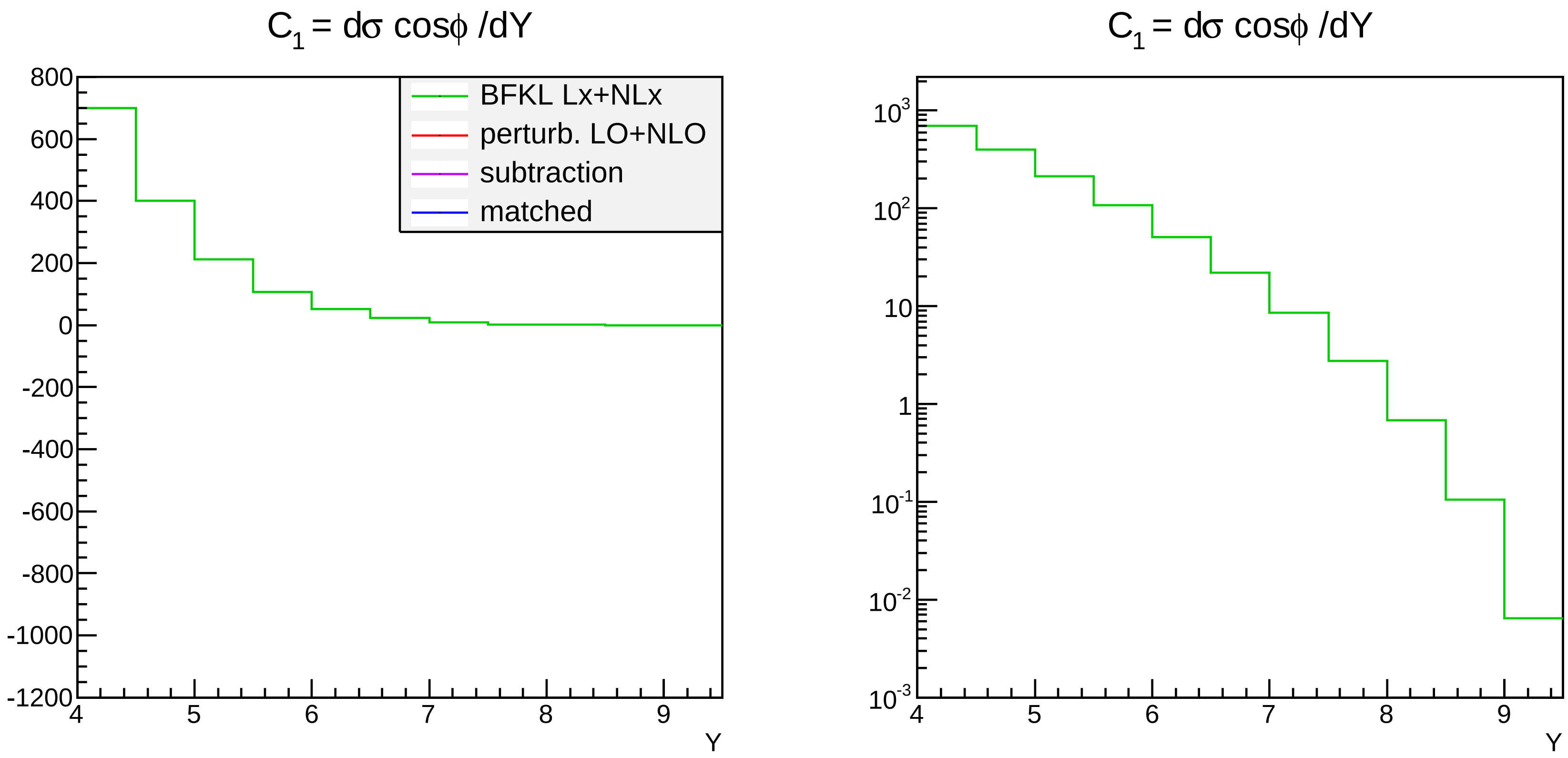
Matched cross section is positive, of the same magnitude of NLL BFKL prediction

# NLO matching (azimuthal coeff. $C_1$)



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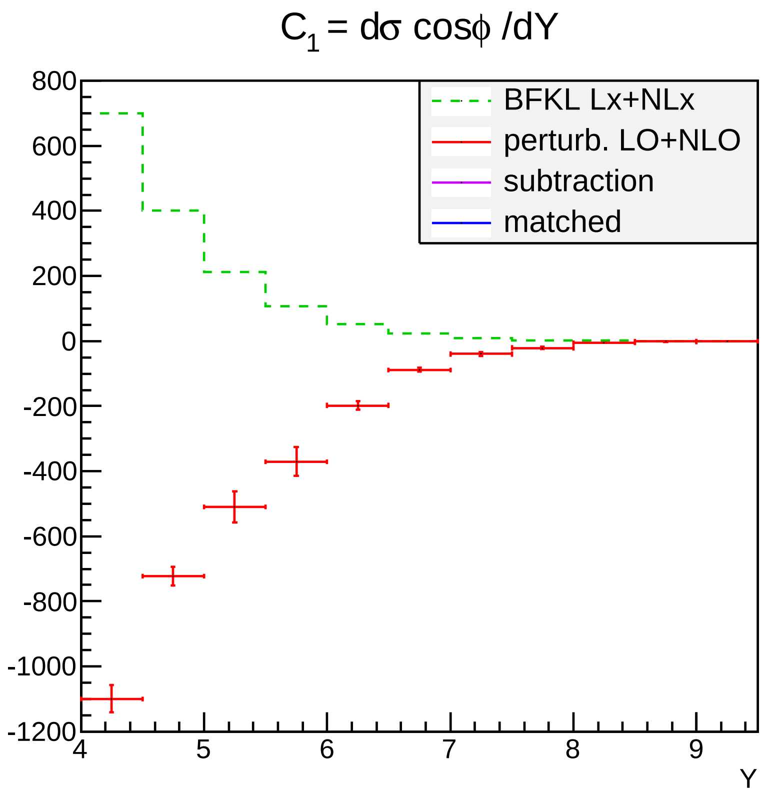


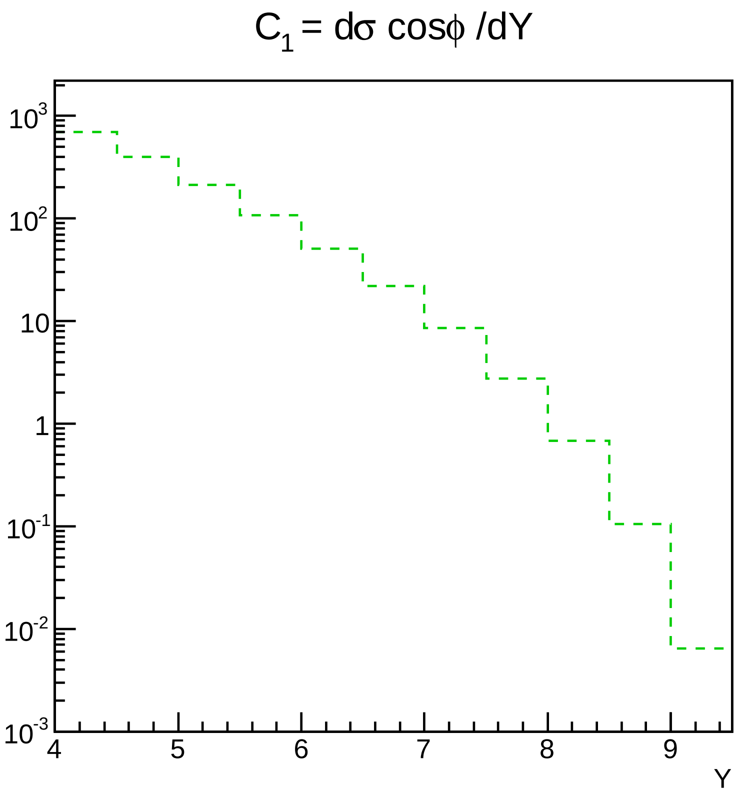


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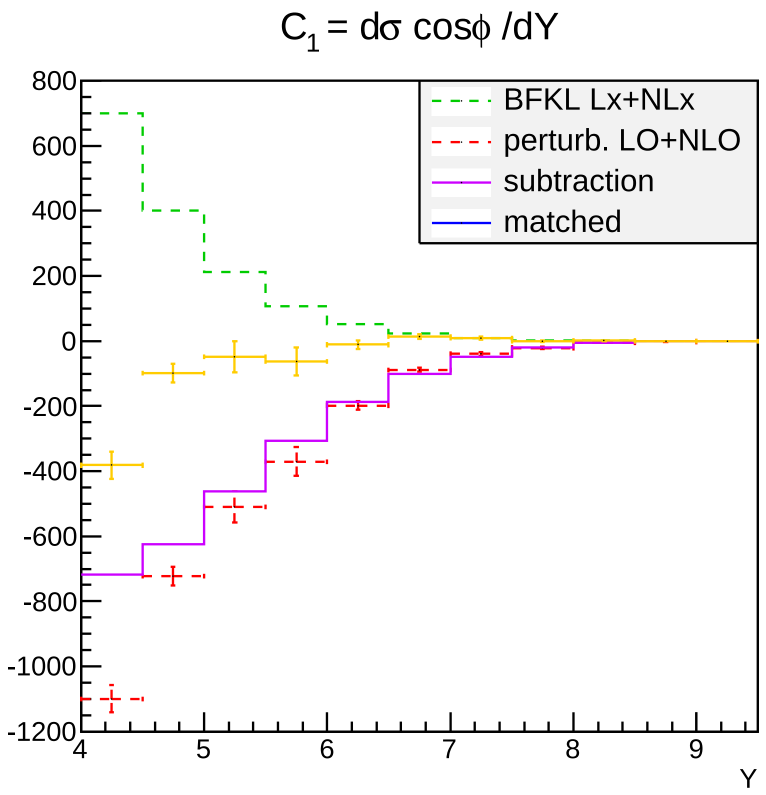


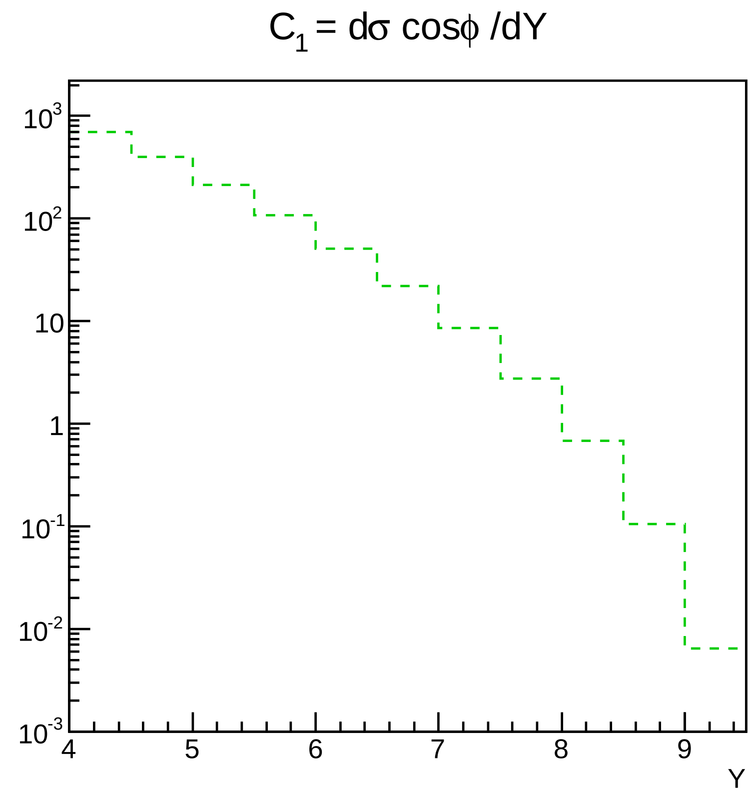


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Moderate difference between NLO and subtraction

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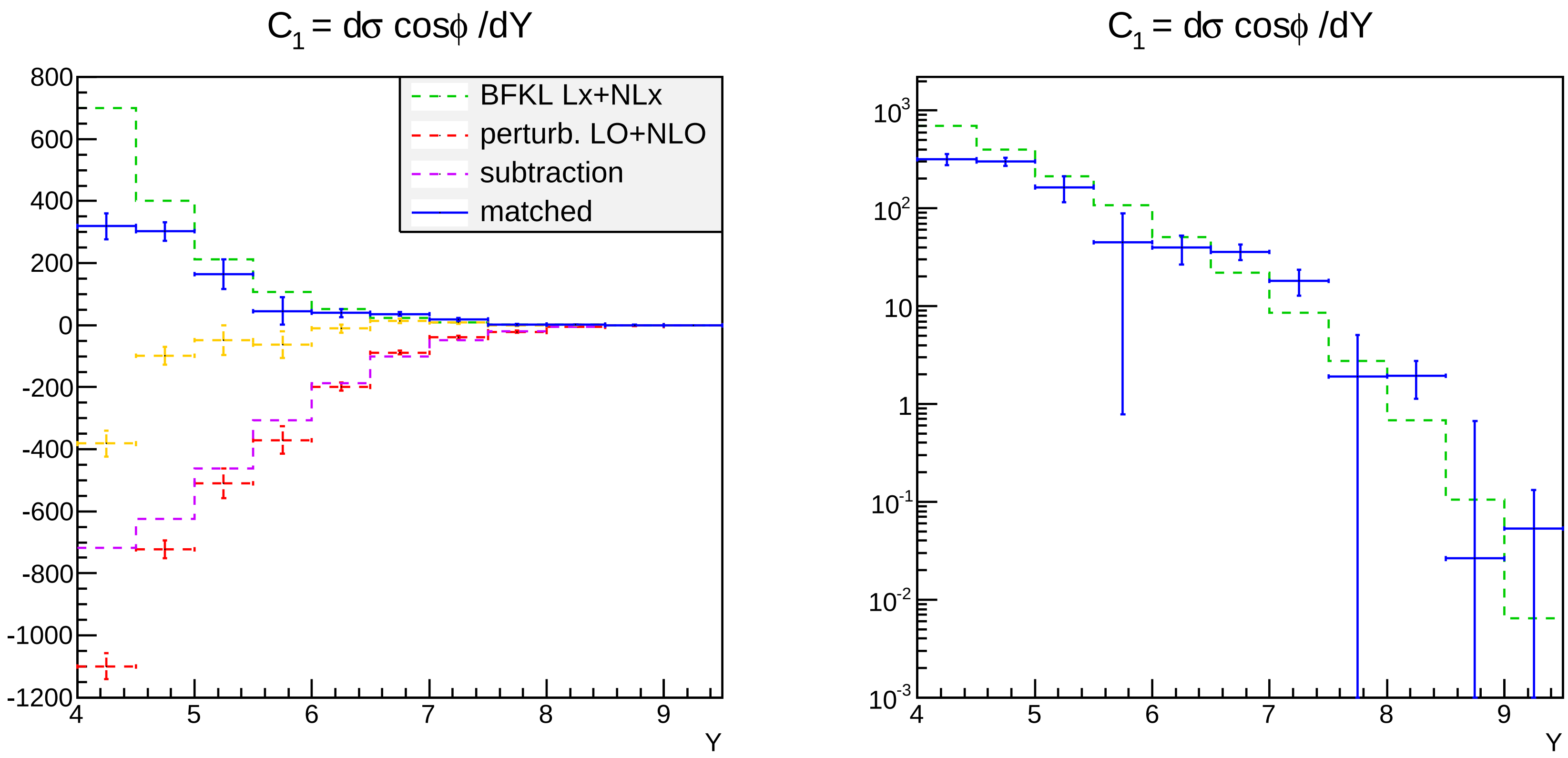


Large errors of NLO calculation due to very slow convergence in MC integration

Moderate difference between NLO and subtraction

Matched $C_1$ of the same magnitude of NLL BFKL prediction

but definitely different at intermediate $Y \simeq 4 \div 6$

# PT instability of symmetric jets

It is well known that cross section of jets at NLO is very sensitive to the asymmetry parameter $D = E_1 - E_2$ *[Frixione,Ridolfi '97]*
The leading collinear singularity for real emission is given by

$$\sigma^{(r)} \propto \int \mathrm{d}\boldsymbol{k}_1 \mathrm{d}\boldsymbol{k}_2 \Theta(|\boldsymbol{k}_1| - E)\Theta(|\boldsymbol{k}_2| - (E + D)) \frac{1}{(\boldsymbol{k}_1 + \boldsymbol{k}_2)^2 + \epsilon^2}$$
$$= A(D, \epsilon) + B \log(\epsilon) - C\,(D + \epsilon) \log(D + \epsilon)$$

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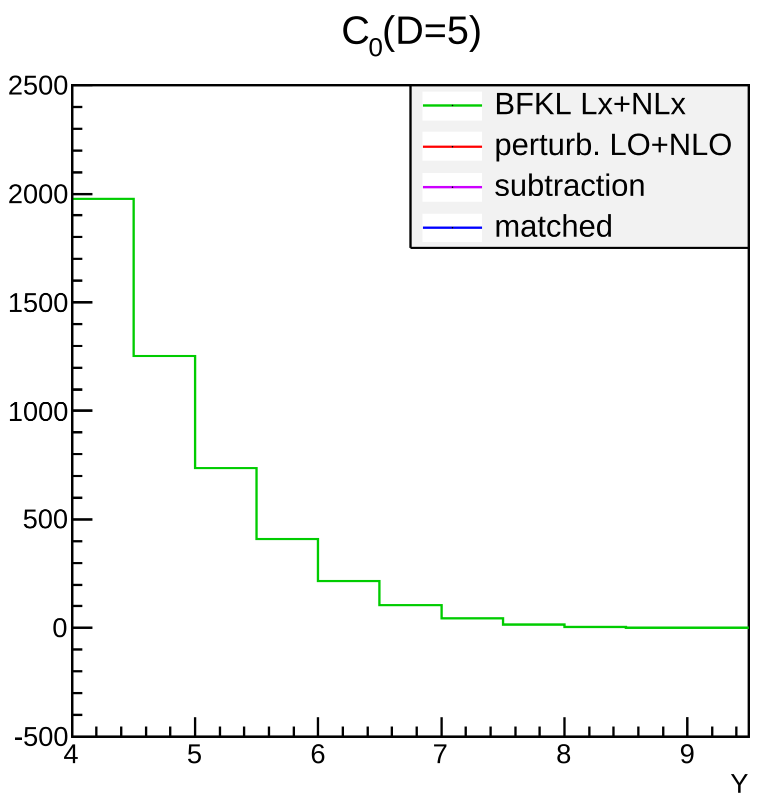
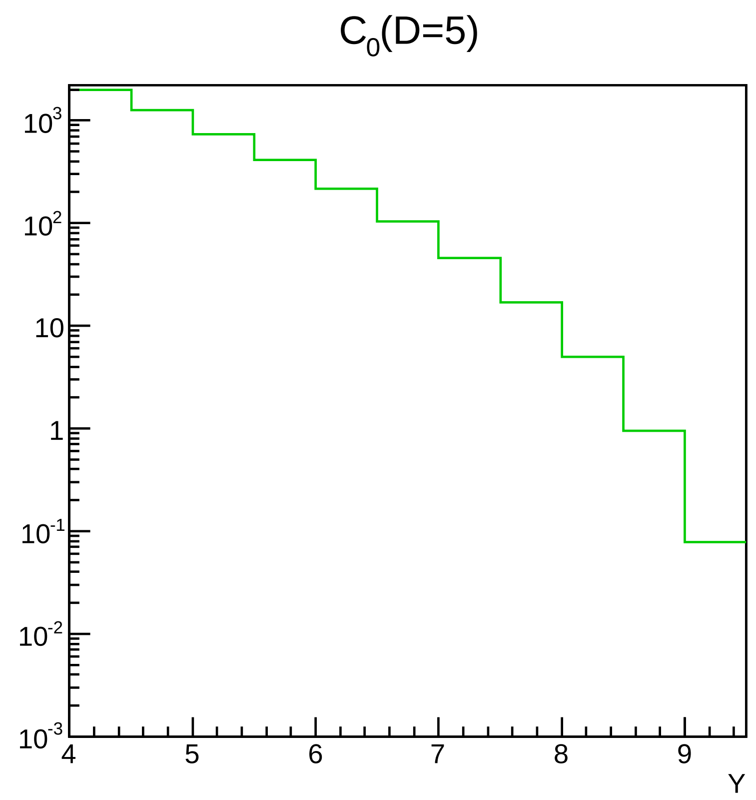
An analogous singularity occurs in the PT expansion of LL BFKL *[Andersen, Del Duca et al. '01]*

$$\sigma_{gg} \propto \frac{1}{(E+D)^2}\left[1 - \alpha_s Y \left( \frac{2ED + D^2}{E^2} \log \frac{2ED + D^2}{(E+D)^2} + 2\log\frac{E}{E+D} \right)\right]$$

In the matching procedure such collinear $D\log(D)$ cancels out to a large extent, therefore the matching procedure should be safe

# Asymmetric jets $E_1 > 30\text{GeV}, E_2 > 35\text{GeV}$



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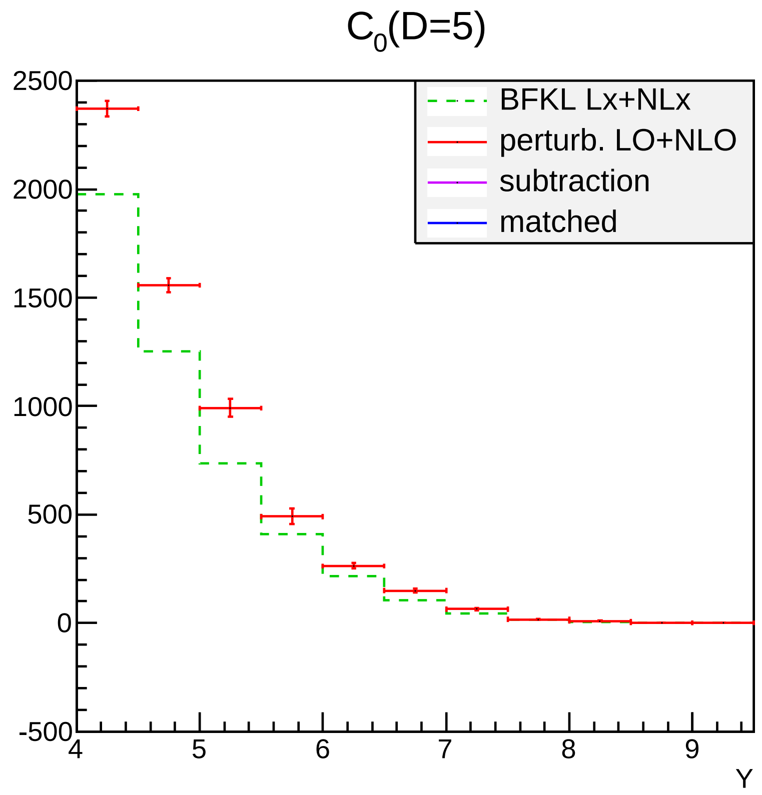


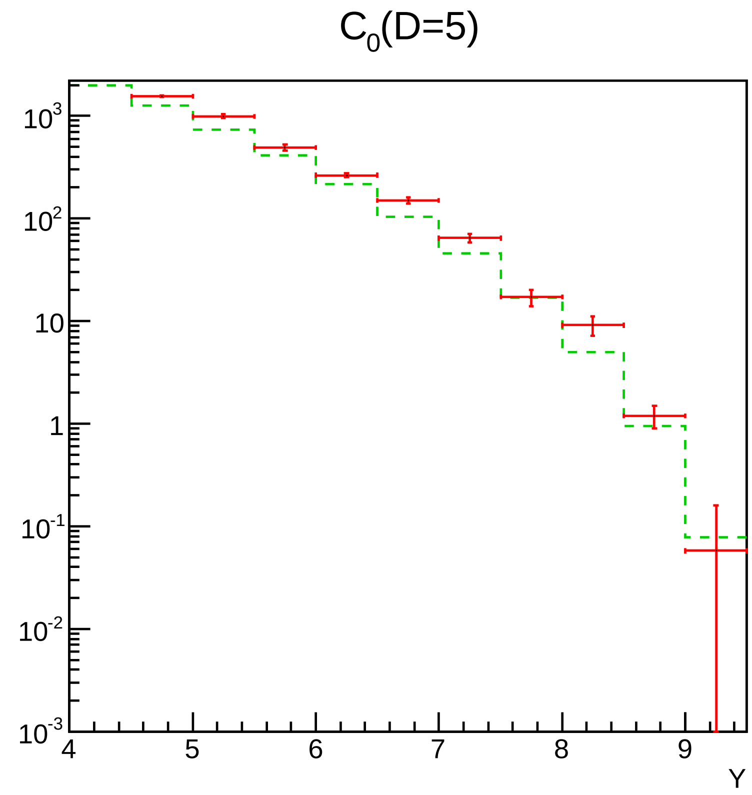


LO+NLO cross section obtained with NLOJET++ *[Nagy]* now is positive

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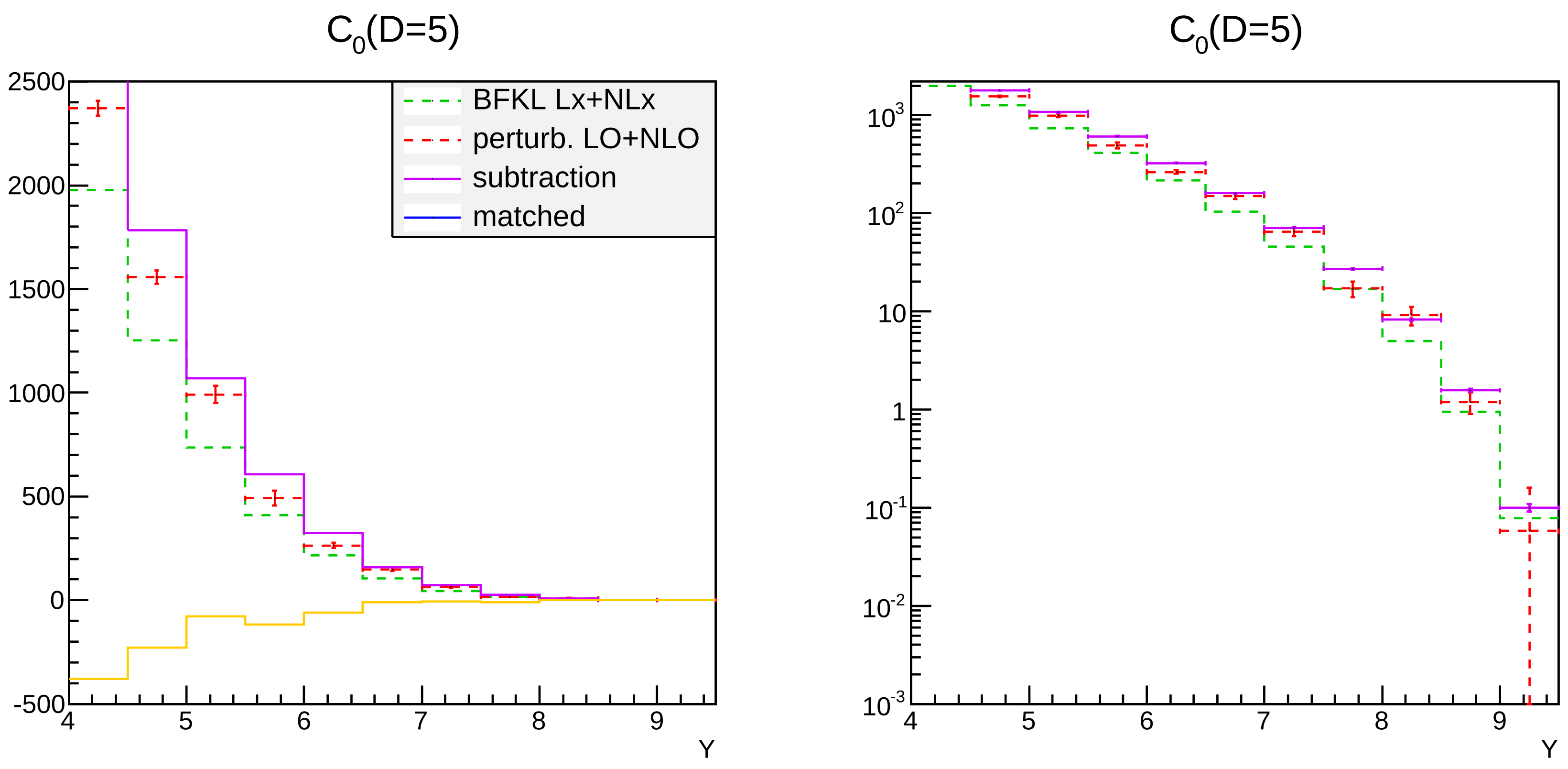


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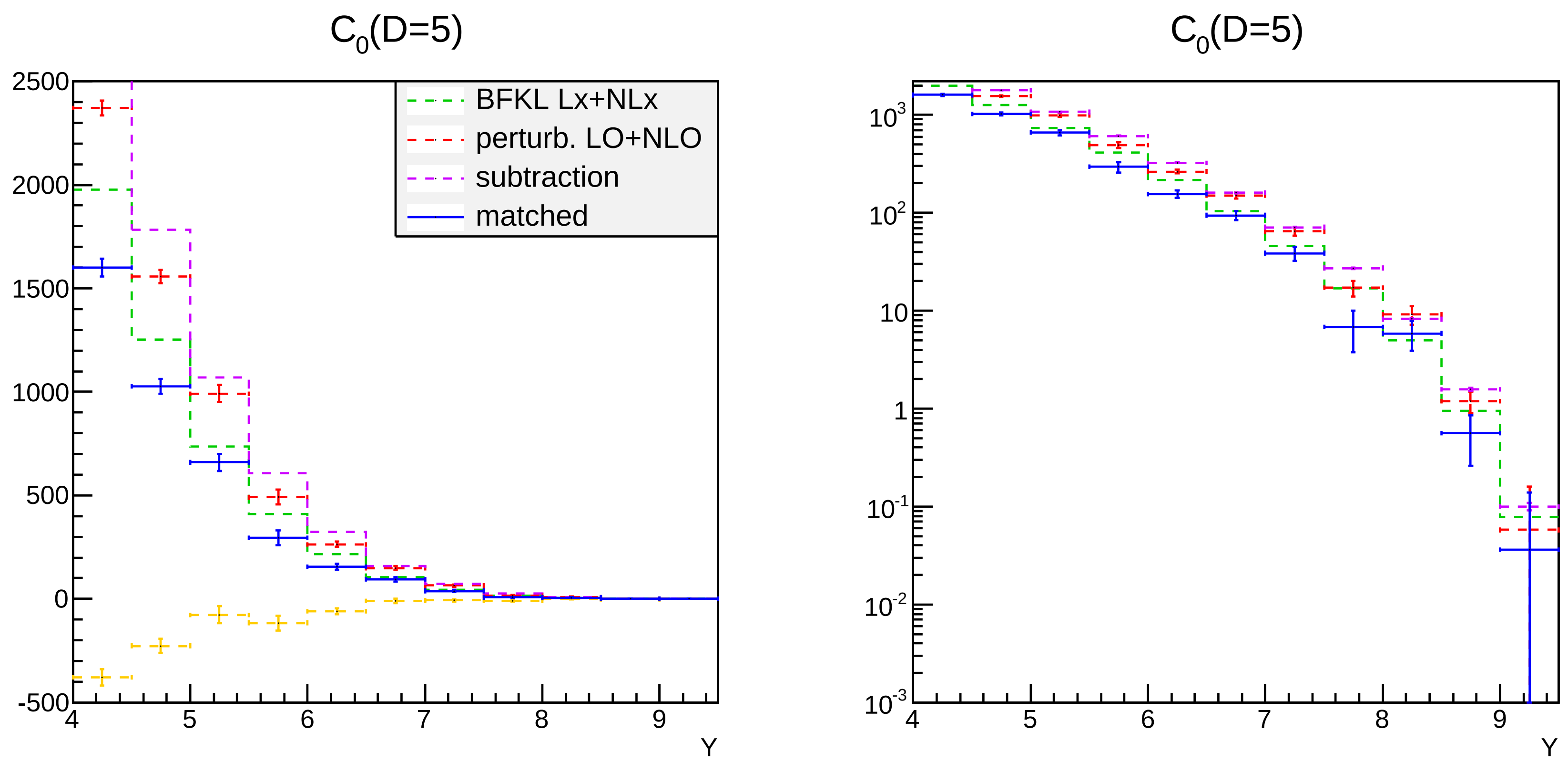


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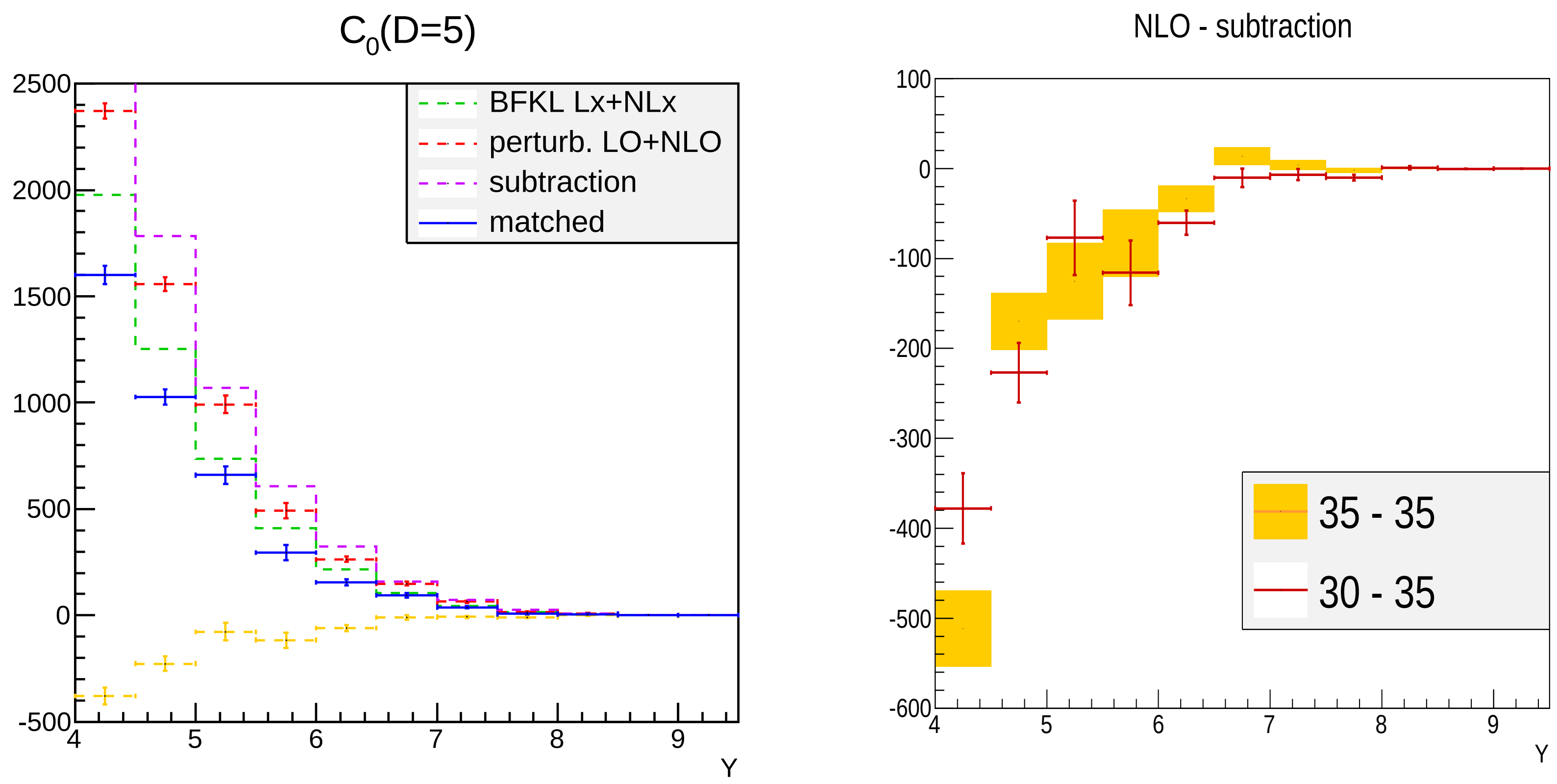


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Their difference is moderate, similar to the symmetric case

Matched cross section is positive, different from BFKL at $Y \lesssim 6$

# Conclusions and outlook

- Mueller-Navelet jets appear to be a good observable for demonstrating presence of BFKL dynamics at high energy
- some fixed order MC and NLL resummation are close to the data, but yield different results
- NLL predictions suffers scale uncertainties $\sim 15\%$
  it may be cured by a scale-fixing at very large scale $\mu_R^2 \simeq 500 E_J^2$

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- NLL predictions suffers scale uncertainties $\sim 15\%$
  it may be cured by a scale-fixing at very large scale $\mu_R^2 \simeq 500 E_J^2$
- We propose to match fixed order and resummed calculations in order to (hopefully) obtain more precise and stable predictions
- Preliminary results are encouraging,
  despite the fact that symmetric jets suffer perturbative instability at NLO
- More accurate NLO calculation are needed
- Full analysis with estimate of errors