T_F^2-Contributions to 3-Loop Deep-Inelastic Wilson Coefficients and the Asymptotic Representation of Charged Current DIS

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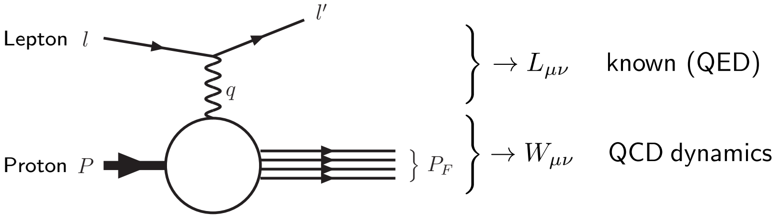
Outline

- Introduction
- Computing Graphs with two massive quark lines of equal masses
- Moments from graphs with two massive lines of unequal mass
- Massive quarks in charged current DIS at 2 loops
- Conclusions

Deep-Inelastic Scattering (DIS)

DIS [inclusive, unpolarized, electromagnetic] gives a clean probe of the proton substructure.



kinematic variables: $Q^2 = -q^2$, $x = \dfrac{Q^2}{2P.q}$, $y = \dfrac{P.q}{P.l}$

parametrization of the hadronic tensor with structure functions

$$W_{\mu\nu}(q,P,s) = \frac{1}{4\pi}\int d^4\xi \exp(iq\xi)\langle P,s \mid [J_\mu^{em}(\xi), J_\nu^{em}(0)] \mid P,s\rangle$$

$$= \frac{1}{2x}\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) F_L(x,Q^2) + \frac{2x}{Q^2}\left(P_\mu P_\nu + \frac{q_\mu P_\nu + q_\nu P_\mu}{2x} - \frac{Q^2}{4x^2} g_{\mu\nu}\right) F_2(x,Q^2)$$

$\rightarrow$ contributions of massive and massless quarks

Light cone expansion of the current commutator:

[Wilson 1969 PR, Zimmermann 1970, Brandt, Preparata 1971 NPB, Frishman 1971 AP]

$$\lim_{\xi^2\to 0}\Big[J(\xi),J(0)\Big] \propto \sum_{i,N,\tau} \underbrace{c_{i,\tau}^{N}(\xi^2,\mu^2)}_{\text{Wilson coefficients}}\,\xi_{\mu_1}\ldots\xi_{\mu_N}\,\underbrace{O_{i,\tau}^{\mu_1\ldots\mu_m}(0,\mu^2)}_{\text{local operators}} + O\left(\frac{\Lambda^2}{Q^2}\right) .$$

At leading twist τ the structure functions factorize

$$F_{(2,L)}(x,Q^2) = \sum_j \quad \mathbb{C}_{j,(2,L)}\left(x,\frac{Q^2}{\mu^2},\frac{m^2}{\mu^2}\right) \quad \otimes \quad f_j(x,\mu^2)$$

into perturbative Wilson coefficients and nonperturbative parton densities (PDFs).

$\otimes$ denotes the Mellin convolution

$$f(x)\otimes g(x) \equiv \int_0^1 dydz\;\delta(x-yz)f(y)g(z)\;,$$

which simplifies to a product upon Mellin transformation

$$\hat{f}(N) := \int_0^1 dx\; x^{N-1} f(x)\;.$$

$\rightarrow$ following computations in Mellin space

Divide the Wilson coefficients into massless and massive parts:

$$\mathbb{C}_{j,(2,L)}\left(N, \frac{Q^2}{\mu^2}, \frac{m^2}{\mu^2}\right) = C_{j,(2,L)}\left(N, \frac{Q^2}{\mu^2}\right) + H_{j,(2,L)}\left(N, \frac{Q^2}{\mu^2}, \frac{m^2}{\mu^2}\right)$$

For $Q^2 \gg m^2$ ($Q^2 \gtrsim 10m^2$ für F_2) the massive Wilson coefficients factorize

$$H_{j,(2,L)}\left(N, \frac{Q^2}{\mu^2}, \frac{m^2}{\mu^2}\right) = \sum_i C_{i,(2,L)}\left(N, \frac{Q^2}{\mu^2}\right) A_{ij}\left(\frac{m^2}{\mu^2}, N\right),$$

[Buza, Matiounine, Smith, van Neerven 1996 NPB]

into massless Wilson coefficients $C_{i,(2,L)}$ and massive operator matrix elements (OMEs); OMEs are local operators O_i sandwiched between partonic states $j = q, g$

$$A_{ij}\left(\frac{m^2}{\mu^2}, N\right) = \langle j \mid O_i \mid j \rangle .$$

The $C_{i,(2,L)}$ are known at 3 loops [Moch, Vermaseren, Vogt, 2005].

$\longrightarrow$ compute $A_{ij}\left(\frac{m^2}{\mu^2}, N\right)$ at 3 loops!

Importance of massive quark contributions at 3 loops

Massive quark contributions:

- amount to 20–30% for small x
- contribute scaling violations which differ in shape from those of massless quarks
- are sensitive to the gluon and sea quark PDFs for small x
- allow for the determination of the strange PDF via charged current DIS
- necessary for the precision determination of α_s at 3 loops
- note: asymptotic representation holds at the 1%-level for F_2

Status of massive quark contributions

Leading Order: [Witten 1976, Babcock, Sivers 1978, Shifman, Vainshtein, Zakharov 1978, Leveille, Weiler 1979, Glück, Reya 1979, Glück, Hoffmann, Reya 1982]

NLO:

[Laenen, van Neerven, Riemersma, Smith 1993]

$Q^2 \gg m^2$: via IBP [Buza, Matiounine, Smith, Migneron, van Neerven 1996]

via ${}_pF_q$'s, more compact [Bierenbaum, Blümlein, Klein, 2007]

$O(\alpha_s^2 \varepsilon)$-contributions (for all-$N$) [Bierenbaum, Blümlein, Klein, Schneider 2008]

[Bierenbaum, Blümlein, Klein 2009]

NNLO: $Q^2 \gg m^2$

moments of F_2: $N = 2...10(14)$ [Bierenbaum, Blümlein, Klein 2009]

contributions to transversity: $N = 1...13$ [Blümlein, Klein, Tödtli 2009]

n_f-contributions to F_2 (all-N): [Ablinger, Blümlein, Klein, Schneider, Wißbrock 2011]

Known at 3 Loop:

- $A_{qq,Q}^{\mathsf{PS}}, A_{qg,Q}$: complete
- $A_{qq,Q}^{\mathsf{NS,(TR)}}$, $A_{gq,Q}$, A_{Qq}^{PS}: complete (→ A. De Freitas' & M. Round's talks)
- A_{Qg}, $A_{gg,Q}$: all $O(n_f T_F^2 C_{A/F})$-contributions known
- $A_{gg,Q}$: $O(T_F^2 C_{A/F})$-contributions $(m_1 = m_2) \longrightarrow$ this talk
- $A_{gg,Q}$: Moments for $(m_1 \neq m_2)$-contributions $\longrightarrow$ this talk

Computing Graphs with two massive quark lines (T_F^2-graphs, $m_1 = m_2$)

Hypergeometric series for T_F^2-graphs

Start from the calculation paradigm:

- rewrite integrals in terms of hypergeometric functions at 1
- represent them in terms of a convergent series
- solve the sums with `Sigma` [C. Schneider]

$\rightarrow$ successful in all 2-loop contributions to the massive OMEs and large classes of 3-loop [cf. page 7]

Graphs with two massive lines of equal masses ($m_1^2 = m_2^2$):

- Feynman parameterization contains

$$(z_1 x + z_2 y(1-x))^{a+b\varepsilon} = \int_{-i\infty}^{i\infty} d\sigma \, \frac{\Gamma(-\sigma)\Gamma(\sigma - a - b\varepsilon)}{2\pi i \Gamma(-a-b\varepsilon)} \frac{[z_2 y(1-x)]^\sigma}{(z_1 x)^{-a-b\varepsilon+\sigma}}$$

- Mellin-Barnes-Representation $\rightarrow B(c+N-\sigma, d+\sigma)$
- unbalanced factor $\Gamma(c+N-\sigma) \rightarrow$ sum of residues diverges

Hypergeometric series for T_F^2-graphs

Solution:

- observe:
  $$B(c+N-\sigma, d+\sigma) = \int_0^1 dx\, x^{c+N-1}(1-x)^{d-1}\left(\frac{1-x}{x}\right)^\sigma$$
  $\rightarrow$ direction for closing the contour depends on x
- split x-integral into two parts $x < \frac{1}{2}$ and $x \geq \frac{1}{2}$
  $\rightarrow$ incomplete beta function, integrate later
- close contour differently $\rightarrow$ convergent sum of residues

$\rightarrow$ ε-expansion and summation: `SumProduction`, `EvaluateMultiSums`, `Sigma` [C. Schneider]

Solving the last Feynman parameter integral

Idea: solve the last integral in the space of cyclotomic HPLs

The occurring cyclotomic HPLs are iterated integrals built from the alphabet:

$$f_0(x) = \frac{1}{x}, \quad f_1(x) = \frac{1}{1-x}, \quad f_{-1}(x) = \frac{1}{1+x},$$
$$f_{(4,0)}(x) = \frac{1}{1+x^2}, \quad f_{(4,1)}(x) = \frac{x}{1+x^2},$$

e.g.: $H_{1,(4,1),0,0}(x) = \int_0^x dx_1\ f_1(x_1) \int_0^{x_1} dx_2\ f_{(4,1)}(x_2)\ \frac{1}{2} \ln^2(x_2).$

Furthermore, generalized letters occur: $f_{[i,\kappa]}(x) = f_i(\kappa x)$

$\rightarrow$ many properties of cycl. HPLs and cycl. S-Sums are known

[Ablinger, Blümlein, Schneider 2011 JMP]

and implemented in the package `HarmonicSums` [J. Ablinger]

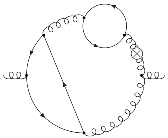
Solving the last Feynman parameter integral

- summation yields cyclotomic S-Sums $\hat{=}$ cycl. HPLs

$$
\begin{aligned}
S_1(-x;\infty) &= -H_{-1}(x) \\
S_{(1,0,1),(2,1,1)}(-x,1;\infty) &= -\frac{4H_{(4,0)}\left(\sqrt{x}\right)}{\sqrt{x}} + \frac{H_{(4,1)}\left(\sqrt{x}\right)}{2x} - \frac{3}{2}H_{(4,1)}\left(\sqrt{x}\right) \\
&\quad - 2H_{(4,0),(4,0)}\left(\sqrt{x}\right) + \frac{15}{4} \\
&\ldots
\end{aligned}
$$

- use generating function: $[f(x)]^N \to \dfrac{1}{1-\kappa f(x)}$,
- integrate
- yield representation $\sum_{\vec{a}} c_{\vec{a}} H_{\vec{a}}(\kappa)$ such that each term has a Taylor expansion for $\kappa \approx 0$
- determine N-th Taylor coefficient ($\to$ `HarmonicSums` [J. Ablinger]: Cauchy products, difference equations $\to$ `EvaluateMultiSums`, `Sigma` [C. Schneider])

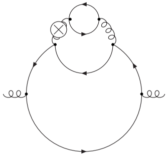
Results: A scalar graph



$$\mathrm{Res} = \frac{(-1)^N+1}{2}\Bigg\{\frac{1}{45\varepsilon^2(N+1)} - \frac{1}{\varepsilon}\left[\frac{S_1(N)}{90(N+1)} + \frac{47N^3+20N^2-67N+40}{1800(N-1)N(N+1)^2}\right]$$
$$+\frac{105N^3-175N^2+56N+96}{13440(N+1)^2(2N-3)(2N-1)4^N}\binom{2N}{N}\left[\sum_{j=1}^{N}\frac{4^j S_1(j)}{\binom{2j}{j}j^2} - \sum_{j=1}^{N}\frac{4^j}{\binom{2j}{j}j^3} - 7\zeta_3\right]$$
$$+\frac{\left(5264N^3-2409N^2-12770N+3528\right)S_1(N)}{100800(N+1)^2(2N-3)(2N-1)} + \frac{S_1(N)^2+S_2(N)+3\zeta_2}{360(N+1)}$$
$$+\frac{S_3(N)-S_{2,1}(N)+7\zeta_3}{420(N+1)} + \frac{P_{13}}{2268000(N-1)^2N^2(N+1)^3(2N-3)(2N-1)}\Bigg\}$$

→ new nested sums occur

Results: A QCD graph



$$
\begin{aligned}
I_{560} = {} & \frac{2P_4}{3N(N+1)^2(N+2)(2N-5)(2N-3)(2N-1)} \frac{1}{4^N}\binom{2N}{N}\left[\sum_{j=1}^{N}\frac{4^j S_1(j)}{\binom{2j}{j}j^2} - \sum_{j=1}^{N}\frac{4^j}{\binom{2j}{j}j^3} - 7\zeta_3\right] \\
& + \frac{N^2+N+2}{27(N-1)N^2(N+1)}\Big[-144S_{2,1}(N) - 36\zeta_2 S_1(N) - 4S_1(N)^3 + 36S_2(N)S_1(N) + 88S_3(N) + 312\zeta_3\Big] \\
& - \frac{4P_2}{6075(N-2)^2(N-1)^4N^5(N+1)^4(N+2)(2N-5)(2N-3)(2N-1)} - \frac{64\left(N^2+N+2\right)}{9\varepsilon^3(N-1)N^2(N+1)} \\
& + \frac{1}{\varepsilon^2}\left\{-\frac{32P_6}{27(N-1)^2N^3(N+1)^2(N+2)} - \frac{32\left(N^2+N+2\right)}{9(N-1)N^2(N+1)}S_1(N)\right\} \\
& + \frac{1}{\varepsilon}\left\{-\frac{8P_7}{405(N-2)(N-1)^3N^4(N+1)^3(N+2)} - \frac{16P_6}{27(N-1)^2N^3(N+1)^2(N+2)}S_1(N)\right. \\
& \left. - \frac{8\left(N^2+N+2\right)\left(S_1(N)^2 - 3S_2(N) + 3\zeta_2\right)}{9(N-1)N^2(N+1)} - \frac{8\left(55N^3+235N^2-52N+20\right)}{15(N-2)(N-1)N(N+1)^2(N+2)}\right\} \\
& - \frac{4P_5S_1(N)}{81(N-1)^3N^4(N+1)^3(N+2)(2N-5)(2N-3)(2N-1)} - \frac{4P_6\left(S_1(N)^2 - 3S_2(N) + 3\zeta_2\right)}{27(N-1)^2N^3(N+1)^2(N+2)} \\
& - \frac{4P_1}{225(N-2)^2(N-1)^2N^2(N+1)^3(N+2)}
\end{aligned}
$$

T_F^2-contributions to A_{ggQ} for $m_1 = m_2$

- inverse binomial sums occur:

$$\sum_{j=1}^{N} \frac{4^j S_1(j)}{\binom{2j}{j} j^2} - \sum_{j=1}^{N} \frac{4^j}{\binom{2j}{j} j^3} = \int_0^1 dx \, \frac{x^N - 1}{1-x} \int_x^1 dy \, \frac{1}{y\sqrt{1-y}} \times \Big[\ln(1-y) - \ln(y) + 2\ln(2)\Big]$$

  (→ cf. C. Raab's talk)

- removable poles at $N = 1/2$, $N = 3/2$, $N = 5/2$
- used for the calculation of all scalar graphs consistent with $A_{gg,Q}^{(3)}$ (→ proof of principle)
- applied to QCD graphs in the T_F^2-contribution to $A_{gg,Q}^{(3)}$
  → [Ablinger, Blümlein, Hasselhuhn, Round, Schneider in prep.]
- checks with MATAD [M. Steinhauser 2000]
- graphs are similar to the case $m_1 \neq m_2$ → same ideas are applicable with generalization

Moments from graphs with two massive lines of unequal mass (charm and bottom)

Moments for graphs with two massive lines ($m_1 \neq m_2$)

- There are 8 different OMES:

$$\underbrace{A_{Qg},\ A_{qq,Q}^{\mathrm{NS(TR)}},\ A_{Qq}^{\mathrm{PS}},\ A_{gg,Q},\ A_{gq,Q}}_{\text{Contain contributions with b- and c-quarks}}\ \underbrace{A_{qg,Q},\ A_{qq,Q}^{\mathrm{PS}}}_{\text{Completely known}}$$

OME	# diagrams
$A_{Qg}^{(3)}$	272
$A_{Qq}^{(3),\mathrm{PS}}$	16
$A_{qq,Q}^{(3),\mathrm{NS}}$	4
$A_{qq,Q}^{(3),\mathrm{NS,TR}}$	4
$A_{gq,Q}^{(3)}$	4
$A_{gg}^{(3)}$	76
Σ	376

- $m_c^2/m_b^2 \simeq 1/10$ expansion parameter $\rightarrow$ moments $N = 2, 4, 6$ done
- renormalization of the 2-mass case has been performed
- But: Charm cannot be treated massless at the scale $\mu \simeq m_b$.

Moments for graphs with two massive lines ($m_1 \neq m_2$)

$$
\begin{aligned}
a_{Qg}^{(3)}(N=6) = T_F^2 C_A \Bigg\{ & \frac{69882273800453}{367569090000} - \frac{395296}{19845}\zeta_3 + \frac{1316809}{39690}\zeta_2 + \frac{832369820129}{14586075000}x + \frac{1511074426112}{624023544375}x^2 - \frac{84840004938801319}{690973782403905000}x^3 \\
& + \ln\left(\frac{m_2^2}{\mu^2}\right)\left[\frac{11771644229}{194481000} + \frac{78496}{2205}\zeta_2 - \frac{1406143531}{69457500}x - \frac{105157957}{180093375}x^2 + \frac{2287164970759}{7669816654500}x^3\right] \\
& + \ln^2\left(\frac{m_2^2}{\mu^2}\right)\left[\frac{2668087}{79380} + \frac{112669}{661500}x - \frac{49373}{51975}x^2 - \frac{31340489}{34054020}x^3\right] + \ln^3\left(\frac{m_2^2}{\mu^2}\right)\frac{324148}{19845} + \ln^2\left(\frac{m_2^2}{\mu^2}\right)\ln\left(\frac{m_1^2}{\mu^2}\right)\frac{156992}{6615} \\
& + \ln\left(\frac{m_2^2}{\mu^2}\right)\ln\left(\frac{m_1^2}{\mu^2}\right)\left[\frac{128234}{3969} - \frac{112669}{330750}x + \frac{98746}{51975}x^2 + \frac{31340489}{17027010}x^3\right] + \ln\left(\frac{m_2^2}{\mu^2}\right)\ln^2\left(\frac{m_1^2}{\mu^2}\right)\frac{68332}{6615} \\
& + \ln\left(\frac{m_1^2}{\mu^2}\right)\left[\frac{83755534727}{583443000} + \frac{78496}{2205}\zeta_2 + \frac{1406143531}{69457500}x + \frac{105157957}{180093375}x^2 - \frac{2287164970759}{7669816654500}x^3\right] \\
& + \ln^2\left(\frac{m_1^2}{\mu^2}\right)\left[\frac{2668087}{79380} + \frac{112669}{661500}x - \frac{49373}{51975}x^2 - \frac{31340489}{34054020}x^3\right] + \ln^3\left(\frac{m_1^2}{\mu^2}\right)\frac{412808}{19845}\Bigg\} \\
+ T_F^2 C_F \Bigg\{ & -\frac{3161811182177}{71471767500} + \frac{447392}{19845}\zeta_3 + \frac{9568018}{4862025}\zeta_2 - \frac{64855635472}{2552563125}x + \frac{1048702178522}{97070329125}x^2 + \frac{1980566069882672}{2467763508585375}x^3 \\
& + \ln\left(\frac{m_2^2}{\mu^2}\right)\left[\frac{1786067629}{204205050} - \frac{111848}{15435}\zeta_2 - \frac{128543024}{24310125}x - \frac{22957168}{3361743}x^2 - \frac{2511536080}{2191376187}x^3\right] \\
& + \ln^2\left(\frac{m_2^2}{\mu^2}\right)\left[\frac{3232799}{4862025} + \frac{752432}{231525}x + \frac{177944}{40425}x^2 + \frac{127858928}{42567525}x^3\right] - \ln^3\left(\frac{m_2^2}{\mu^2}\right)\frac{111848}{19845} - \ln^2\left(\frac{m_2^2}{\mu^2}\right)\ln\left(\frac{m_1^2}{\mu^2}\right)\frac{223696}{46305} \\
& + \ln\left(\frac{m_2^2}{\mu^2}\right)\ln\left(\frac{m_1^2}{\mu^2}\right)\left[\frac{22238456}{4862025} - \frac{1504864}{231525}x - \frac{355888}{40425}x^2 - \frac{255717856}{42567525}x^3\right] + \ln\left(\frac{m_2^2}{\mu^2}\right)\ln^2\left(\frac{m_1^2}{\mu^2}\right)\frac{223696}{46305} \\
& + \ln\left(\frac{m_1^2}{\mu^2}\right)\left[-\frac{24797875607}{1021025250} - \frac{111848}{15435}\zeta_2 + \frac{128543024}{24310125}x + \frac{22957168}{3361743}x^2 + \frac{2511536080}{2191376187}x^3\right] \\
& + \ln^2\left(\frac{m_1^2}{\mu^2}\right)\left[\frac{3232799}{4862025} + \frac{752432}{231525}x + \frac{177944}{40425}x^2 + \frac{127858928}{42567525}x^3\right] - \ln^3\left(\frac{m_1^2}{\mu^2}\right)\frac{1230328}{138915}\Bigg\} + O\left(x^4\ln^3(x)\right)
\end{aligned}
$$

$\rightarrow$ `q2e/exp` [Harlander, Seidensticker, Steinhauser 1999] & `MATAD` [M. Steinhauser 2000]

Massive quarks in charged current DIS at 2 loops

Structure functions of charged current DIS

The cross section is parameterized introducing 3 structure functions.
For symmetry reasons consider the combinations

$$\frac{d\sigma^{\nu}}{dxdy} \pm \frac{d\sigma^{\bar\nu}}{dxdy} = \frac{G_F^2 s}{4\pi}\left\{(1+(1-y)^2)F_2^{W^+\pm W^-} - y^2 F_L^{W^+\pm W^-} + (1-(1-y)^2)xF_3^{W^+\pm W^-}\right\}.$$

The structure functions factorize in PDFs and Wilson coefficients

$$\begin{aligned}
F_2^{W^+ + W^-} = 2\Big\{ & \left(|V_{du}|^2(d+\bar d) + |V_{su}|^2(s+\bar s) + V_u(u+\bar u)\right)(C_{2,q}^{W^+ + W^-,\mathrm{NS}} + L_{2,q}^{W^+ + W^-,\mathrm{NS}}) \\
& + \left(|V_{dc}|^2(d+\bar d) + |V_{sc}|^2(s+\bar s)\right) H_{2,q}^{W^+ + W^-,\mathrm{NS}} + 2V_c\left[H_{2,q}^{W,\mathrm{PS}}\Sigma + H_{2,g}^{W} G\right] \\
& + 2V_u\left[(C_{2,q}^{W,\mathrm{PS}} + L_{2,q}^{W,\mathrm{PS}})\Sigma + (C_{2,g}^{W} + L_{2,g}^{W})G\right]\Big\}, \\
F_3^{W^+ + W^-} = 2\Big\{ & \left(|V_{du}|^2(d+\bar d) + |V_{su}|^2(s+\bar s) - V_u(u+\bar u)\right)(C_{3,q}^{W^+ + W^-,\mathrm{NS}} + L_{3,q}^{W^+ + W^-,\mathrm{NS}}) \\
& + \left(|V_{dc}|^2(d+\bar d) + |V_{sc}|^2(s+\bar s)\right) H_{3,q}^{W^+ + W^-,\mathrm{NS}} + 2V_c\left[H_{3,q}^{W,\mathrm{PS}}\Sigma + H_{3,g}^{W} G\right]\Big\}.
\end{aligned}$$

where $V_i := |V_{id}|^2 + |V_{is}|^2$, $i = u, c$, and $\Sigma = \sum_q (q + \bar q)$.

factors (-1) due to charge antisymmetry of corresponding contributions.

2-Loop corrections in the region $Q^2 \gg m^2$

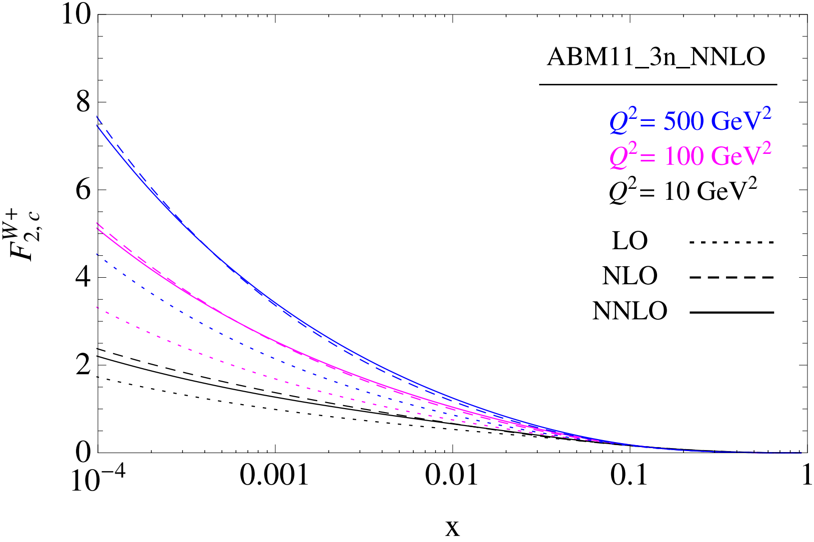
At 2-Loop order we derived the asymptotic representation using the transition to the variable flavor number scheme:

$$
\begin{aligned}
L_{2,q}^{W^+\pm W^-,\mathrm{NS},(2)} &= A_{qq,Q}^{\mathrm{NS},(2)} + C_{2,q}^{W^+\pm W^-,\mathrm{NS},(2)}(n_f+1) - C_{2,q}^{W^+\pm W^-,\mathrm{NS},(2)}(n_f)\,,\\
H_{2,q}^{W^+\pm W^-,\mathrm{NS},(2)} &= A_{qq,Q}^{\mathrm{NS},(2)} + C_{2,q}^{W^+\pm W^-,\mathrm{NS},(2)}(n_f+1)\,,\\
L_{2,q}^{W,\mathrm{PS},(2)} &= C_{2,q}^{W,\mathrm{PS},(2)}(n_f+1) - C_{2,q}^{W,\mathrm{PS},(2)}(n_f) = 0\,,\\
H_{2,q}^{W,\mathrm{PS},(2)} &= \frac{1}{2} A_{Qq}^{\mathrm{PS},(2)} + C_{2,q}^{W,\mathrm{PS},(2)}(n_f+1)\,,\\
L_{2,g}^{W,(2)} &= A_{gg,Q}^{(1)} C_{2,g}^{W,(1)}(n_f+1) + C_{2,g}^{W,(2)}(n_f+1) - C_{2,g}^{W,(2)}(n_f)\,,\\
H_{2,g}^{W,(2)} &= A_{gg,Q}^{(1)} C_{2,g}^{W,(1)}(n_f+1) + C_{2,g}^{W,(2)}(n_f+1)\\
&\quad + \frac{1}{2}\left(A_{Qg}^{(2)} + A_{Qg}^{(1)} C_{2,q}^{W^+ + W^-,\mathrm{NS},(1)}(n_f+1)\right)\,,\\
L_{3,q}^{W^+\pm W^-,\mathrm{NS},(2)} &= A_{qq,Q}^{\mathrm{NS},(2)} + C_{3,q}^{W^+\pm W^-,\mathrm{NS},(2)}(n_f+1) - C_{3,q}^{W^+\pm W^-,\mathrm{NS},(2)}(n_f)\,,\\
H_{3,q}^{W^+\pm W^-,\mathrm{NS},(2)} &= A_{qq,Q}^{\mathrm{NS},(2)} + C_{3,q}^{W^+\pm W^-,\mathrm{NS},(2)}(n_f+1)\,,\\
H_{3,q}^{W,\mathrm{PS},(2)} &= -\frac{1}{2} A_{Qq}^{\mathrm{PS},(2)}\,,\\
H_{3,g}^{W,(2)} &= \frac{1}{2}\left(-A_{Qg}^{(2)} - A_{Qg}^{(1)} C_{3,q}^{W^+ + W^-,\mathrm{NS},(1)}(n_f+1)\right)\,.
\end{aligned}
$$

2-Loop corrections in the region $Q^2 \gg m^2$

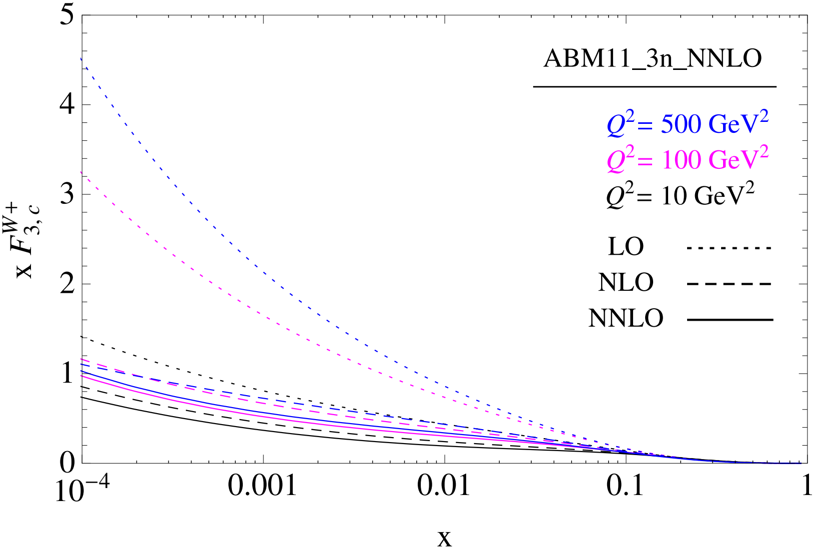
- A previous drivation [Buza, van Neerven 1997 NPB] was corrected and completed.
- Using OMEs and massless Wilson coefficients from the literature, the massive Wilson coefficients in Mellin space and x-space have been constructed.
- The Mellin transformation implemented in `HarmonicSums` [J. Ablinger] was used.
- A `FORTRAN` implementation for the application to experimental data will be published soon. [Blümlein, Hasselhuhn, Pfoh 2013]
- The asymptotic representation is accurate since experimentally $Q^2 \gtrsim 100$ GeV^2.

Results



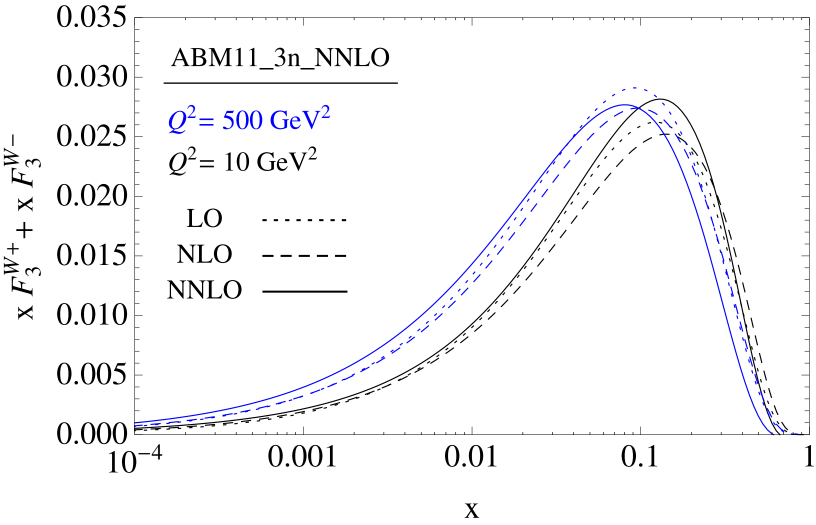
ABM11_3n_NNLO
Q^2= 500 GeV^2
Q^2= 100 GeV^2
Q^2= 10 GeV^2
LO
NLO
NNLO
$F_{2,c}^{W+}$
x
0
2
4
6
8
10
10^{-4}
0.001
0.01
0.1
1

Results



ABM11_3n_NNLO
Q^2= 500 GeV^2
Q^2= 100 GeV^2
Q^2= 10 GeV^2
LO
NLO
NNLO
x $F_{3,c}^{W+}$
x
5
4
3
2
1
0
10^{-4}
0.001
0.01
0.1
1

Results



ABM11_3n_NNLO
Q^2= 500 GeV^2
Q^2= 10 GeV^2
LO
NLO
NNLO
0.035
0.030
0.025
0.020
0.015
0.010
0.005
0.000
$x\,F_3^{W+} + x\,F_3^{W-}$
10^{-4}
0.001
0.01
0.1
1
x

Conclusions

- Heavy flavor contributions are needed at 3-loop order for the precise determination of α_s from DIS data.
- 6 out of 8 massive operator matrix elements have been completed recently.
- A method for the calculation of graphs with two massive lines of equal masses and operator insertions was presented, and applied to the corresponding contributions to $A_{gg,Q}^{(3)}$. The results and methods will be published $\rightarrow$ [Ablinger, Blümlein, Hasselhuhn, Round]
- The method can be generalized to the case of unequal masses.
- The moments for $N = 2, 4, 6$ for all graphs with two quark lines of unequal masses are now known, as well as the renormalization of these contributions.

Conclusions

- On the phenomenological side, the complete 2-loop heavy flavor contributions to charged current DIS were derived from light flavor Wilson coefficients and massive OMEs. They are implemented into a Mellin space program in `FORTRAN` and will be published soon
  $\rightarrow$ [Blümlein, Hasselhuhn, Pfoh 2013].
- The calculations of new 3-loop corrections can take advantage of rigorous computer algebra packages of our friends at RISC; at the same time techniques known to physicists or newly developed on practical problems lead to improvements in general purpose programs.