Theory uncertainty of γ from tree decays

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Partially based on

[Martone, Zupan arXiv:1212.0165; Brod, Zupan arXiv:1308.5663]

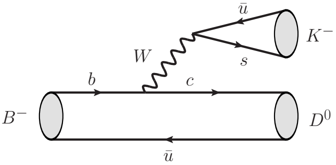
Motivation

- $\gamma \equiv \arg(-V_{ud}V_{ub}^*/V_{cd}V_{cb}^*)$
- Important CPV SM parameter
- Can be determined from tree decays alone
- Expect small contribution of New Physics – can be tested!
- Determination from $B \to DK$ theoretically extremely clean
- Test of NP

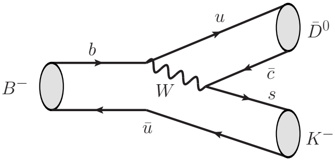
Plan

- γ from tree decays
- Theory uncertainty
  - Including direct CP violation in D decays
  - Electroweak corrections

γ from tree decays – general idea

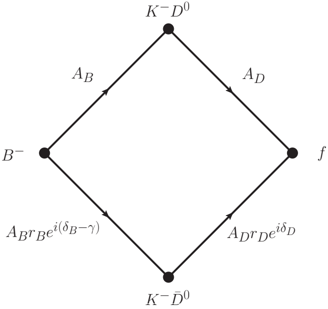


$\propto V_{cb} V_{us}^*$



$\propto V_{ub} V_{cs}^*$

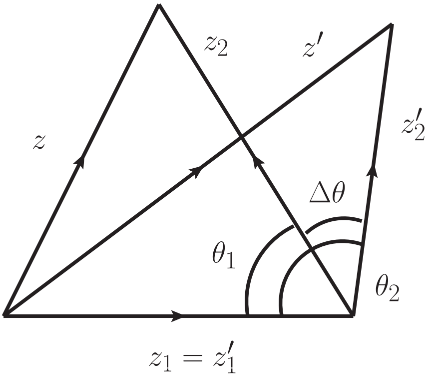
- $b \to c\bar{u}s$, $b \to u\bar{c}s$
- no penguin contribution
- interference from common D^0, $\bar{D}^0$ final states



Example, the GLW method

$$z \equiv A(B^+ \to f_D K^+) = \bar{A}_f A_B + A_f A_B r_B e^{i(\delta_B+\gamma)} \equiv z_1 + z_2 \,,$$
$$z' \equiv A(B^- \to f_D K^-) = A_f A_B + \bar{A}_f A_B r_B e^{i(\delta_B-\gamma)} \equiv z'_1 + z'_2 \,.$$

- decay into CP eigenstate ($\pi^+\pi^-$, K^+K^- etc.)
- $z_1 = A(B^+ \to f_{\bar{D}^0} K^+)$
- $z_2 = A(B^+ \to f_{D^0} K^+)$
- $z'_1 = A(B^- \to f_{D^0} K^-)$
- $z'_2 = A(B^- \to f_{\bar{D}^0} K^-)$
- $\Delta\theta = 2\gamma$



Combine different methods

- CP eigenstates (e.g. $D \to K^+K^-, \pi^+\pi^-; K_S\pi^0$)
  [Gronau, London, Wyler 1990/91]
- Flavor states (e.g. $D \to \pi^-K^+, \pi^+K^-$) [Atwood, Dunietz, Soni 1997]
- Many-body final states (e.g. $D \to K_S K^+K^-, K_S\pi^+\pi^-$)
  [Giri, Grossman, Soffer, Zupan 2003; Poluektov 2004]
- Many variants:
  - Use $D^* \to D\pi^0, D\gamma$ [Bondar, Gershon 2004]
  - Many-body B final states
    [Aleksan, Petersen, Soffer 2002, Gershon 2008; Gershon, Poluektov 2009]
  - Neutral B_d, B_s [Aleksan, Dunietz, Kayser 1992; Kayser, London 2000; Atwood, Soni 2003; Fleischer 2003; Gronau et al. 2004]
  - ...
- Have $\sim n_D n_B$ measurements, $\sim n_D + n_B$ unknowns
- Test for NP by comparing γ from different methods

Reducible theory uncertainty

- $K - \bar{K}$, $D - \bar{D}$, $B_{(s)} - \bar{B}_{(s)}$ mixing

  [E.g. Bondar et al., 1004.2350; Rama 1307.4384; Gronau, Grossman, Surujon, Zupan 0702011]

- Dalitz plot errors
- Direct CP violation in D decays

Including direct CP violation

[Martone, Zupan 1212.0165;

see also Wang 2012, Bhattacharya et al. 2013, Bondar et al., 2013]

$$\Delta\mathcal{A}_{CP} = (-0.253 \pm 0.104)\%$$

[HFAG May 2014]

- Modify amplitudes D decay amplitudes.
- E.g. SCS decays

$$A_f \equiv A(D^0 \to f) = A_f^T \left[1 + r_f e^{i(\delta_f + \phi_f)}\right],$$
$$\bar{A}_f \equiv A(\bar{D}^0 \to f) = A_f^T \left[1 + r_f e^{i(\delta_f - \phi_f)}\right].$$

$$a_f^{\rm dir} \approx -2 r_f \sin\delta_f \sin\phi_f\,, \quad r_f = \mathcal{O}\left(10^{-3}\right)$$

Including direct CP violation

- B decay amplitude gets modified:

$$A(B^\pm \to f_D K^\pm) = A_B A_f^T \left[1 + r_B^\pm e^{i(\delta'_B \pm \gamma \pm \delta\gamma)}\right]$$

- What is the size of the effect?

$$\delta\gamma = \mathcal{O}(r_f/r_B)\,, \quad \delta'_B - \delta_B = \mathcal{O}(r_f/r_B)\,, \quad r_B^\pm - r_B = \mathcal{O}(r_f)$$

$$A_{CP}(B \to f_D K) = 2 r_B \sin\delta_B \sin\gamma - a_f^{\mathrm{dir}}$$

- Shift in branching ratio depends on strong phase

Including direct CP violation

- Unknowns: $2n_{\mathrm{SCS}} + 3n_B + 1$
- Observables: $2n_B(n_{\mathrm{CA}} + n_{\mathrm{SCS}})$
- Shift symmetry $\gamma \to \gamma + \phi$, $\alpha_f \to \alpha_f - \phi$:

$$|A(B^\pm \to f_D K^\pm)|^2 = |A_B|^2 \left[|A_f|^2 + 2r_B |A_f||\bar{A}_f| \cos(\delta_B \pm \gamma \pm \alpha_f) + \ldots \right]$$

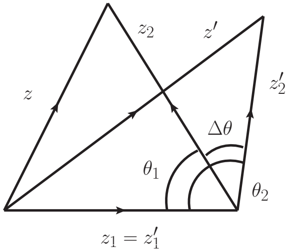
- $\alpha_f \equiv \arg(A_f/\bar{A}_f) = -a_f^{\mathrm{dir}} \cot \delta_f$
- Cannot extract γ from $B \to DK$ alone without assumptions
- Measure δ_f at charm factories

Example: Modification of GLW

- Two modifications:
  - $\theta_1 - \theta_2 = 2(\gamma + \alpha_f)$
  - $|z_1| \neq |z_1'|$
- Numerically,

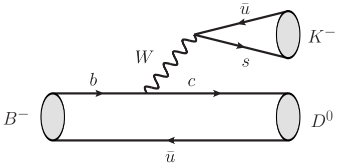
$$\gamma = \gamma^{\text{naive}} + a_f^{\text{dir}} \left[\cot \delta_f - 0.36/r_B \times \tilde{\mathcal{C}} \right]$$

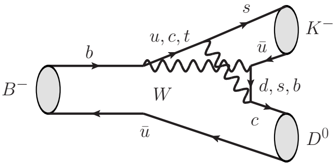
- $\tilde{\mathcal{C}}$ is a known $\mathcal{O}(1)$ function of θ_1, θ_2
- $r_B(DK) = \mathcal{O}(10\%)\,, \quad \delta\gamma = \mathcal{O}(\text{few}\,\%)$
- $r_B(D\pi) = \mathcal{O}(0.5\%)\,, \quad \delta\gamma = \mathcal{O}(1)$



Irreducible theory uncertainty

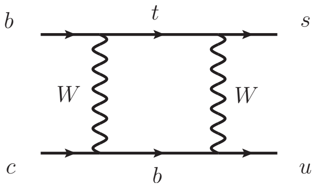
- Many effects (mixing, CP violation) can be incorporated by modifying equations and extracted from data
- QED radiative corrections $\rightarrow$ CP conserving
- Electroweak corrections
  - No effect from Z exchange
  - No effect from vertex corrections
  - Box diagrams can change CKM structure



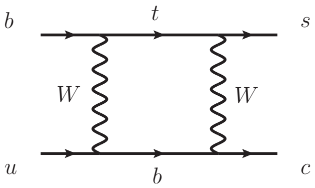


Box diagrams

- $b \to u\bar{c}s$:
  - tree level $\sim V_{ub} V_{cs}^*$
  - box diagram $\sim (V_{tb} V_{ts}^*)(V_{ub} V_{cb}^*)$
- same weak phase, no shift in γ



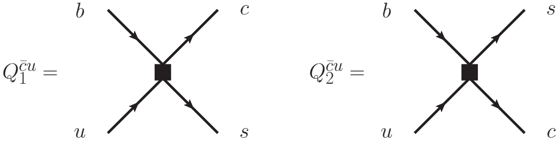
- $b \to c\bar{u}s$:
  - tree level $\sim V_{cb} V_{us}^*$
  - box diagram $\sim (V_{tb} V_{ts}^*)(V_{cb} V_{ub}^*)$
- different weak phase, induces $\delta\gamma$



Effective Hamiltonians at scale m_b

$$\mathcal{H}^{(0)}_{\bar{c}u} = \frac{G_F}{\sqrt{2}} V_{cb} V^*_{us} \left[C_1(\mu_b) Q_1^{\bar{c}u} + C_2(\mu_b) Q_2^{\bar{c}u} \right] ,$$

$$\mathcal{H}^{(0)}_{\bar{u}c} = \frac{G_F}{\sqrt{2}} V_{ub} V^*_{cs} \left[C_1(\mu_b) Q_1^{\bar{u}c} + C_2(\mu_b) Q_2^{\bar{u}c} \right] .$$



- Electroweak corrections induce a shift:

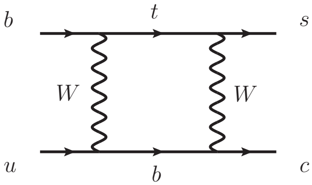
$$\mathcal{H}^{(1)}_{\bar{c}u} = \frac{G_F}{\sqrt{2}} V_{cb} V^*_{us} \left[\left(C_1 + \Delta C_1 \right)(\mu_b) Q_1^{\bar{c}u} + \left(C_2 + \Delta C_2 \right)(\mu_b) Q_2^{\bar{c}u} \right]$$

Direct Matching

- Integrate out W, top, bottom simultaneously

$$\Delta C_2 = -\left|\frac{\alpha}{4\pi \sin^2\theta_w}\frac{V_{tb}V_{ts}V_{ub}}{V_{us}}\right| F(x_t, y_b)e^{i\gamma}$$

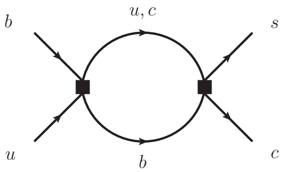
$$\Delta C_1 = 0$$

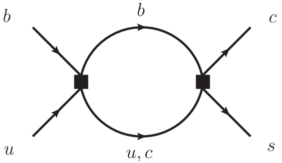


- $F(x_t, y_b)$ function of $x_t = m_t^2/M_W^2$, $y_b = m_b^2/M_W^2$
- $F(x_t, y_b) \to 0$ for $x_t \to 0$, $y_b \to 0$
- $F(x_t, y_b) \simeq 2y_b \log y_b$
- $|\Delta C_2| = (5.3 \pm 0.3) \times 10^{-8}$ $\qquad \propto \lambda^4 \times \alpha/4\pi \times F$

Summation of the log

- Logarithmic term dominates $|\Delta C_2|$
- Summation of the leading-log (QCD) effects
- Matching in two steps:
  1. At scale m_t integrate out W, top
  2. At scale m_b match to four-flavor effective theory





Solving the RGE

- Resulting RGE can be solved with standard methods (cf. ϵ_K)

$$\mu \frac{d}{d\mu} \tilde{C}_k = \sum_l \tilde{C}_l \gamma_{lk} + \sum_{ij} C_i C_j \hat{\gamma}_{ij,k} \,,$$

- Matching to effective four-flavor theory yields

$$\Delta C_k(\mu_b) = 2 m_b^2 \frac{\sqrt{2} G_F}{16\pi^2} \left| \frac{V_{tb} V_{ts} V_{ub}}{V_{us}} \right| e^{i\gamma} \tilde{C}_k^{(0)}(\mu_b) \,.$$

- Expansion about $\mu = M_W$ reproduces the leading log
- Numerically
  - $|\Delta C_1| = (4.5 \pm 0.2) \times 10^{-9}$
  - $|\Delta C_2| = (4.3 \pm 0.2) \times 10^{-8}$

The shift $\delta\gamma$

- Need the ratio of matrix elements

$$r_A = \frac{\langle K^- D^0 | Q_2^{\bar{c}u} | B^- \rangle}{\langle K^- D^0 | Q_1^{\bar{c}u} | B^- \rangle} = \frac{f_D F_0^{B\to K}(0)}{f_K F_0^{B\to D}(0)} = 0.4$$

- Setting $\gamma = 68^\circ$, this yields

$$\delta\gamma = \frac{\text{Im}\Delta C_1}{C_1 + C_2 r_A} + \frac{\text{Im}\Delta C_2}{C_1/r_a + C_2} = 2.0 \times 10^{-8}$$

- Nonlocal contributions expected to lead to uncertainty of a factor of a few

$$\delta\gamma/\gamma \lesssim \mathcal{O}(10^{-7})$$

Summary

- γ from $B \to DK$ is theoretically extremely clean,

$$\delta\gamma/\gamma \lesssim \mathcal{O}(10^{-7})$$

- Important SM input parameter
- Search for new physics

Backup

Summation of the log

- Using GIM and keeping only relevant terms, can write

$$\mathcal{H}_{\text{eff}}^{f=5} = \frac{G_F}{\sqrt{2}} \sum_{\substack{u_{1,2}=u,c \\ d_{1,2}=s,d,b}} V_{u_1 d_2} V^*_{u_2 d_1} \sum_{i,j=1}^{2} C_i(\mu) Z_{ij} Q_j^{(u_1 d_2; d_1 u_2)}$$

$$+ 2 G_F^2 V_{cb} V^*_{us} \cdot \left| \frac{V_{tb} V_{ts} V_{ub}}{V_{us}} \right| e^{i\gamma} \left[\sum_{i,j,k=1}^{2} C_i C_j \hat{Z}_{ij,k} \tilde{Q}_k + \sum_{l,k=1}^{2} \tilde{C}_l \tilde{Z}_{lk} \tilde{Q}_k \right]$$

- Effective dim. 8 operators

$$\tilde{Q}_1 = \frac{m_b^2}{\mu^{2\epsilon} g_s^2} (\bar{s} u)_{V-A} (\bar{c} b)_{V-A}\,, \quad \tilde{Q}_2 = \frac{m_b^2}{\mu^{2\epsilon} g_s^2} (\bar{s} b)_{V-A} (\bar{c} u)_{V-A}\,.$$

- Double insertions of dim. 6 operators generate $\tilde{C}$