Form factors for $B_s \to K\ell\nu$ decays in Lattice QCD

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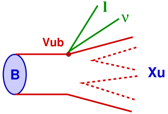
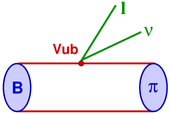
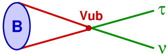
Motivation

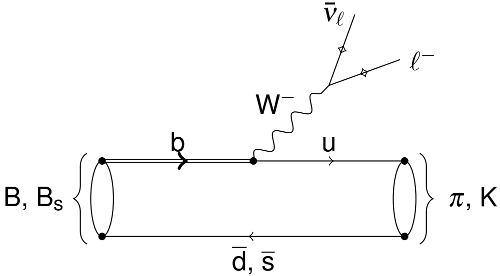
- Determination of $|V_{\rm ub}|$
- $\sim 3\sigma$ discrepancy [PDG] :
  - Inclusive $B \to X_u \ell \nu$: $V_{\rm ub} = (4.41 \pm 0.15^{+0.15}_{-0.17}) \times 10^{-3}$
  - Exclusive $B \to \pi \ell \nu$: $V_{\rm ub} = (3.23 \pm 0.31) \times 10^{-3}$
  - from $B \to \tau \nu$ via f_B: $V_{\rm ub} = (5.10 \pm 0.47) \times 10^{-3}$
- theoretical and experimental input needed
- This talk: Non-perturbative determination of form factors for $B_s \to K \ell \nu$ decay







Semi-leptonic decays $B \to \pi\ell\nu$, $B_s \to K\ell\nu$



$B_s \to K$:

- no experimental data *yet* – predictions
- easier on the lattice (valence $m_K = m_K^{phys}$ computationally less expensive than for the π)
- not far from $B \to \pi$

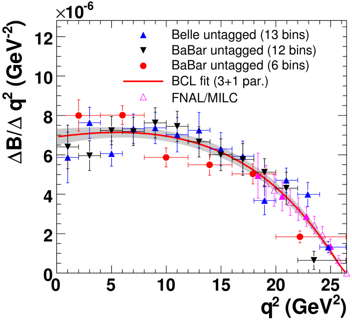
$$\langle K(p_K^\mu)|V^\mu|B_s(p_{B_s}^\mu)\rangle = f_+(q^2)\left[p_{B_s}^\mu + p_K^\mu - \frac{m_{B_s}^2 - m_K^2}{q^2}q^\mu\right] + f_0(q^2)\frac{m_{B_s}^2 - m_K^2}{q^2}q^\mu$$

Experimental decay rates

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{192\pi^3 m_{B_s}^3} \lambda^{3/2}(q^2) |f_+(q^2)|^2$$

$$\lambda(q^2) = (m_{B_s}^2 + m_K^2 - q^2)^2 - 4 m_{B_s}^2 m_K^2$$

- experimentally measured decay rate
- form factor $f_+(q^2)$ computed in LQCD
- $\Rightarrow$ determine V_{ub}



Parameterisation of $f(q^2) \times V_{\mathrm{ub}}$

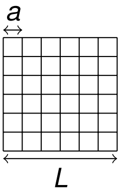
Our ultimate plan:
BCL-Parameterisation [Bourrely, Caprini, Lellouch '09] :

$$f_+(q^2) = \frac{1}{1 - q^2/m^2_{\mathrm{B}_s^*}} \sum_{k=0}^{K-1} b_k \left[z^k(q^2) - (-1)^{k-K} \frac{k}{K} z^K(q^2) \right]$$

- Correlated, combined fit of our data and experimental data
- Minimise $\chi^2 = \chi^2_{\mathrm{th}} + \chi^2_{\mathrm{exp}}$
- fit parameters b_k, V_{ub}

Heavy Quark, HQET expansion of $\langle K | V^\mu | B \rangle$

Problem: $L^{-1} \ll m_\pi \approx 140\,\mathrm{MeV}, \ldots, m_B \approx 5\,\mathrm{GeV} \ll a^{-1}$



Solution: Heavy Quark Effective Theory (HQET) [ALPHA collab. '01-'13]

- Effective theory: expansion in $1/m_h$
- *Non-perturbatively* renormalisable (order by order in $1/m_h$)
- well-defined continuum limit
- valid for kaon momenta $p_K \ll m_b$
- in practice $p_K \lesssim 1\,\mathrm{GeV} \Rightarrow q^2$ close to q^2_{max}

$$\langle \mathcal{O} \rangle = \langle \mathcal{O} \rangle_{stat} + \omega_{kin} a^4 \sum_x \langle \mathcal{O} \mathcal{O}_{kin}(x) \rangle_{stat} + \omega_{spin} a^4 \sum_x \langle \mathcal{O} \mathcal{O}_{spin}(x) \rangle_{stat}$$

$$\omega_{\{kin,spin\}} \sim 1/m_h$$

$$\mathcal{O}_{kin}(x) = \overline{\psi}_h(x) \vec{D}^2 \psi_h(x), \quad \mathcal{O}_{spin}(x) = \overline{\psi}_h(x) \vec{\sigma} \cdot \vec{B} \psi_h(x)$$

HQET II

$$V_0^{\mathrm{HQET}}(x) = Z_{V_0}^{\mathrm{HQET}} \left[V_0^{\mathrm{stat}}(x) + \sum_{j=1}^{2} c_{\mathrm{V}_{0,j}} V_{0,j}(x) \right]$$

$$V_k^{\mathrm{HQET}}(x) = Z_{V_k}^{\mathrm{HQET}} \left[V_k^{\mathrm{stat}}(x) + \sum_{j=1}^{4} c_{\mathrm{V}_{k,j}} V_{k,j}(x) \right],$$

- HQET parameters $(Z_i, c_{\mathrm{V}_i}, \omega_i)$ determined non-perturbatively:

$$\Phi_i^{\mathrm{QCD}}(L, m_{\mathrm{h}}, 0) = \Phi_i^{\mathrm{HQET}}(L, m_{\mathrm{h}}, a)$$

- Matching HQET and QCD for certain (finite L) “observables” Φ_i [Della Morte et al. ’13]

Extrapolations

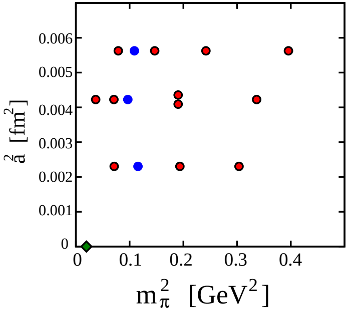
At fixed q^2, achieved by "twisting" [Bedaque '04] the s quark:
$\psi(x+L\hat{k}) = \mathrm{e}^{\mathrm{i}\,\theta_k}\,\psi(x)$

$\vec{p}^{\theta} = (2\pi\vec{n}+\vec{\theta})/L$ freely tuneable $\rightarrow$ heavy quark twisting (keep B_s in rest frame)

- continuum, $a \rightarrow 0$
- chiral, $m_\pi \rightarrow m_\pi^{\mathrm{phys}}$

Ensembles and simulation

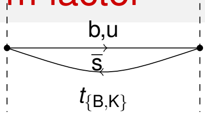
- non-perturbatively $O(a)$ improved Wilson fermions
- $N_f = 2$ CLS ensembles
- scale setting via f_K [Fritzsch et al. '12]
- $m_\pi L \gtrsim 4$
- Error estimates taking into account autocorrelations [Schaefer et al. '12]

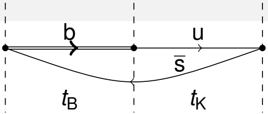


id	$T \times L^3$	a [fm]	m_π [MeV]	$m_\pi L$	# meas.	# target
A5	64×32^3	0.0749(8)	330	4.0	500	500
F6	96×48^3	0.0652(6)	310	5.0	254	500
N6	96×48^3	0.0483(4)	340	4.0	220	500

- keep $m_K / f_K =$ phys.
- for now: one value of q^2 only, $q^2 = 21.23\,\mathrm{GeV}^2$

Obtaining the form factor

$\mathcal{C}^{\{B,K\}} \sim$ 

$\mathcal{C}^3 \sim$ 

Ratio – plateaux

$$\langle K(p_K^\theta)|V^\mu|B_s(0)\rangle = \lim_{T,t_B,t_K\to\infty} \frac{\mathcal{C}^3_\mu(t_K,t_B)}{\sqrt{\mathcal{C}^K(t_K)\mathcal{C}^B(t_B)}} e^{E_K t_K/2} e^{E_B t_B/2} \equiv \lim_{T,t_B,t_K\to\infty} f_\mu^{\text{ratio}}(q^2)$$

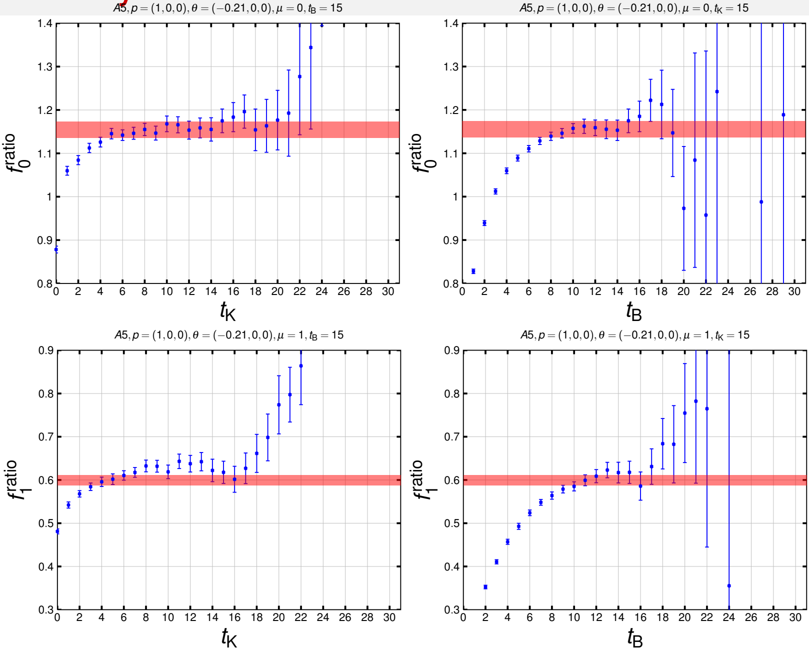
Factorising Fit

Combined fit to ground and first excited state of $\mathcal{C}^3, \mathcal{C}^B$

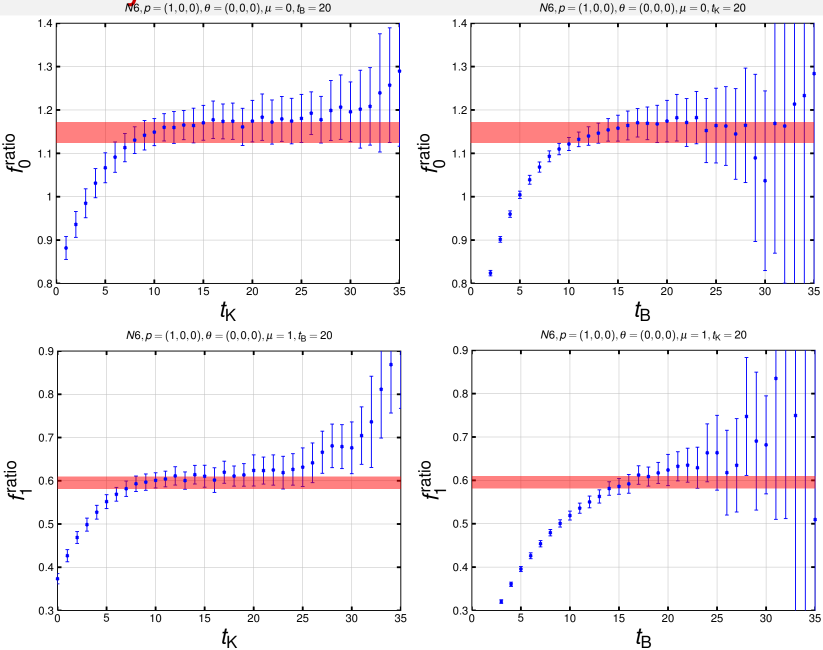
$$\begin{cases} \mathcal{C}^3_{\mu i}(t_B,t_K) & = \sum_{n,m} \beta_i^{(n)} \varphi_\mu^{(n,m)} \kappa^{(m)} e^{-E_B^{(n)} t_B} e^{-E_K^{(m)} t_K}, \qquad \varphi_\mu^{(1,1)} \sim f_+(q^2) \\ \mathcal{C}^B_{ij}(t_B) & = \sum_n \beta_i^{(n)} \beta_j^{(n)} e^{-E_B^{(n)} t_B} \\ \mathcal{C}^K(t_K) & = \sum_m (\kappa^{(m)})^2 e^{-E_K^{(m)} t_K} \end{cases}$$

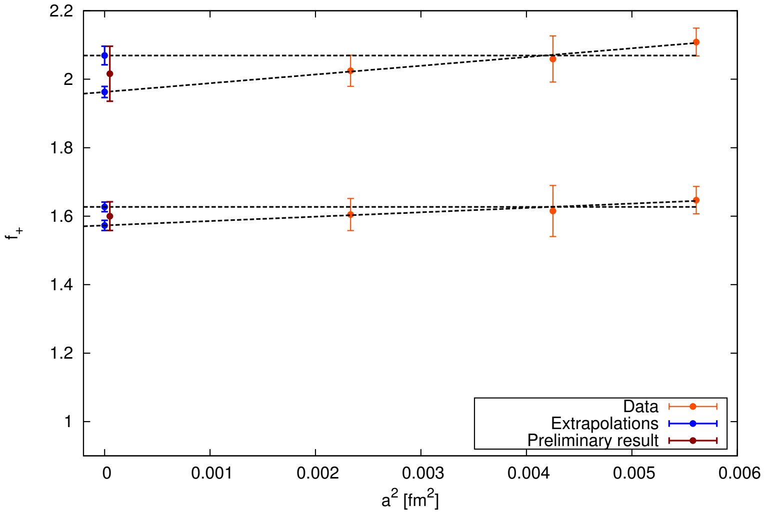
- Gaussian smearing, $\psi_l^{\text{sm}}(x) = (1+\kappa\Delta)^{N_{\text{it}}} \psi_l(x)$, $N_{\text{it}} \leftrightarrow$ wavefunctions
- random noise sources, full time dilution

Preliminary results



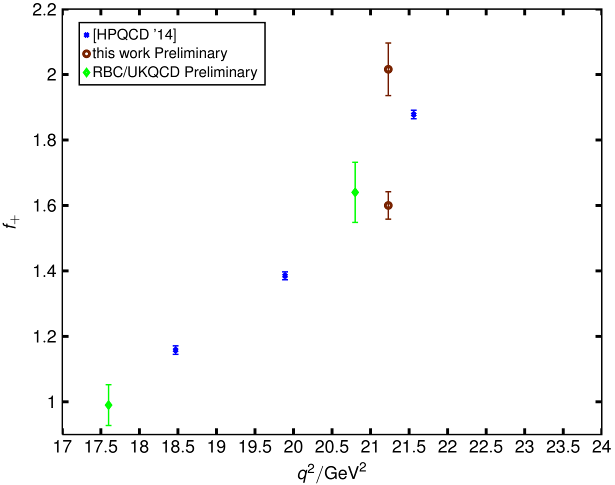
Preliminary results





Note: Different points reflect $O(1/m_{\mathrm{h}})$ ambiguity in conversion from HQET form factors $f_\perp, f_\parallel$ to f_+, f_0 (including / excluding $f_\parallel$)

A little comparison



- blue: [HPQCD '14] , $a = 0.09$ fm, $m_\pi = 320$ MeV Pert. renormalisation
- brown: this work, continuum, static, $m_\pi = 340$ MeV, NP renormalisation [Della Morte et al. '07] . Preliminary.
- green: RBC/UKQCD Preliminary, chiral, continuum. Pert. Renormalisation

Error budget – rough estimates

- extraction of FF through fits / ratios ($\approx 2\%$)
- lattice spacing (scale setting): determination of q^2 ($\approx 1\%$)
- continuum extrapolations ($2\dots5\%$)
- chiral extrapolations (seems flat: small)
- BCL parameterisation, experimental data (none yet, for $\mathrm{B} \to \pi \approx 10\%$)
- $N_\mathrm{f} = 2$ (*"To date, no significant differences between results with different values of* $\mathrm{N_f}$ *have been observed."* [FLAG '13])
- HQET truncation (static: $\sim 10\%$, at $O(1/m_\mathrm{h})$: $\sim 1\%$; [$< 1\%$ for $f_{\mathrm{B_s}}$ [Bernardoni et al. '14]])

Conclusions and Outlook

Conclusions

- $f_+(q^2)$ for $\mathrm{B_s} \to \mathrm{K}$ in HQET
- *fully non-pertubative* renormalisation setup (at LO, soon at NLO in $1/m_\mathrm{h}$)
- *small discretisation errors*
- rough agreement with recent HPQCD results

Outlook

- Chiral extrapolation: $m_\pi \to m_\pi^\mathrm{phys}$
- Inclusion of $O(1/m_\mathrm{b})$ effects in analysis (matching to be done, large volume measurements available)
- Measure at one or two more q^2
- $N_\mathrm{f} = 2+1$
- $\mathrm{B} \to \pi$

Backup – form factor

B rest frame:

$$\langle \mathrm{K}|V^0|\mathrm{B_s}\rangle = \sqrt{2m_{\mathrm{B_s}}}f_\parallel$$
$$\langle \mathrm{K}|V^j|\mathrm{B_s}\rangle = \sqrt{2m_{\mathrm{B_s}}}p^j_{\mathrm{K}}f_\perp$$

$$f_+ = \frac{1}{\sqrt{2m_{\mathrm{B_s}}}}f_\parallel + \frac{1}{\sqrt{2m_{\mathrm{B_s}}}}(m_{\mathrm{B_s}} - E_{\mathrm{K}})f_\perp$$
$$f_0 = \frac{\sqrt{2m_{\mathrm{B_s}}}}{m^2_{\mathrm{B_s}} - m^2_{\mathrm{K}}}[(m_{\mathrm{B_s}} - E_{\mathrm{K}})f_\parallel + (E^2_{\mathrm{K}} - m^2_{\mathrm{K}})f_\perp]$$

static:

$$f_+(q^2) = \sqrt{\frac{m_{\mathrm{B_s}}}{2}}f_\perp(q^2)$$

corrections $O(10\%)$

Backup – correlators

$$\mathcal{C}^{\mathrm{K}}(t;\vec{p}) = \sum_{\vec{x}_f,\vec{x}_i} \mathrm{e}^{-\mathrm{i}\vec{p}\cdot(\vec{x}_f-\vec{x}_i)} \langle P_{\mathrm{su}}(x_f) P_{\mathrm{us}}(x_i) \rangle$$

$$\mathcal{C}^{\mathrm{B_s}}(t;\vec{0}) = \sum_{\vec{x}_f,\vec{x}_i} \langle P_{\mathrm{sb}}(x_f) P_{\mathrm{bs}}(x_i) \rangle$$

$$\mathcal{C}^{3}_{\mu}(t_{\mathrm{K}}, t_{\mathrm{B_s}};\vec{p}) = \sum_{\vec{x}_f,\vec{x}_v,\vec{x}_i} \mathrm{e}^{-\mathrm{i}\vec{p}\cdot(\vec{x}_f-\vec{x}_v)} \langle P_{\mathrm{su}}(x_f) V^{\mu}(x_v) P_{\mathrm{bs}}(x_i) \rangle,$$

$$P_{\mathrm{q_1 q_2}}(x) = \overline{\psi}_{\mathrm{q_1}}(x)\gamma_5 \psi_{\mathrm{q_2}}(x),$$

$$V^{\mu}(x) = \overline{\psi}_{\mathrm{u}}(x)\gamma_{\mu} \psi_{\mathrm{b}}(x).$$

Backup – fit

Factorising Fit

Combined fit to ground and first excited state of $\mathcal{C}^3, \mathcal{C}^{\mathrm{B}}$

$$\begin{cases} \mathcal{C}^3_{\mu i}(t_\mathrm{B}, t_\mathrm{K}) & = \sum_{n,m} \beta_i^{(n)} \varphi^{(n,m)} \kappa^{(m)} \mathrm{e}^{-E_\mathrm{B}^{(n)} t_\mathrm{B}} \mathrm{e}^{-E_\mathrm{K}^{(m)} t_\mathrm{K}}, \qquad \varphi^{(1,1)} \sim f_+(q^2) \\ \mathcal{C}^\mathrm{B}_{ij}(t_\mathrm{B}) & = \sum_n \beta_i^{(n)} \beta_j^{(n)} \mathrm{e}^{-E_\mathrm{B}^{(n)} t_\mathrm{B}} \\ \mathcal{C}^\mathrm{K}(t_\mathrm{K}) & = \sum_m (\kappa^{(m)})^2 \mathrm{e}^{-E_\mathrm{K}^{(m)} t_\mathrm{K}} \end{cases}$$

Factorising Fit

Combined fit to ground and first excited state of $\mathcal{C}^3, \mathcal{C}^{\mathrm{B}}$, assuming fixed t_K

$$\begin{cases} \mathcal{C}^3_{\mu i}(t_\mathrm{B}) & = \sum_n \beta_i^{(n)} \tilde{\varphi}^{(n)} \mathrm{e}^{-E_\mathrm{B}^{(n)} t_\mathrm{B}}, \qquad \tilde{\varphi} = \varphi \kappa^{(1)} \mathrm{e}^{-E_\mathrm{K}^{(1)} t_\mathrm{K}} \\ \mathcal{C}^\mathrm{B}_{ij}(t_\mathrm{B}) & = \sum_n \beta_i^{(n)} \beta_j^{(n)} \mathrm{e}^{-E_\mathrm{B}^{(n)} t_\mathrm{B}} \end{cases}$$