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INFN - Roma Tre

8th Workshop on the CKM Unitarity Triangle
Vienna University of Technology
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QCD sum rule results for heavy-light meson decay constants and comparison with Lattice QCD

- importance of accurate determinations of heavy-light meson leptonic decay constants
- lattice averages for f_D, f_{Ds}, f_B and f_{Bs} produced by the Flavor Lattice Averaging Group (FLAG)
- PS and V heavy-meson decay constants from Borel QCD sum rules for heavy-light currents

* excellent agreement in the charm sector, while moderate tensions occur in the beauty sector

work done in collaboration with W. Lucha (HEPHY) and D. Melikhov (HEPHY and Moscow)

Motivations

Leptonic decay constants of PS heavy-mesons are important hadronic quantities relevant for the extraction of **CKM matrix elements** from experimental data on:

* the weak decay of PS heavy-mesons to a lepton-neutrino pair via flavor changing transitions

$$\Gamma\left(H \to \ell \nu_\ell\right) = M_H \frac{G_F^2}{8\pi} f_H^2 \left|V_{hq}\right|^2 m_\ell^2 \left(1 - \frac{m_\ell^2}{M_H^2}\right)^2$$

e.g.: c → d or c → s transitions in the case of the decay of charmed D and D_s mesons

b → u transition in the case of the decay of the B meson

* the rare leptonic decays of neutral PS heavy-mesons to a charged lepton pair via FCNC interactions, e.g.

$$BR\left(B_q \to \ell^+ \ell^-\right) = \tau_{B_q} \frac{G_F^2}{\pi} Y \left(\frac{\alpha}{4\pi \sin^2 \Theta_W}\right)^2 M_{B_q} f_{B_q}^2 \left|V_{tb}^* V_{tq}\right|^2 m_\ell^2 \sqrt{1 - 4\frac{m_\ell^2}{M_{B_q}^2}}$$

q = d, s

Y = loop factor for pQCD and EW corrections

***** evidence for $B_s \to \mu^+ \mu^-$ decay recently seen at LHCb (at the 3.5σ level) [PRL 110 (2013)]

Leptonic decay constants of V heavy-mesons are relevant in the **heavy-quark phenomenology**, e.g. for the contributions of the vector poles coupled to the weak current mediating the semileptonic decays of PS heavy-light mesons

Flavor Lattice Averaging Group

“ ... The scope of the **Flavor Lattice Averaging Group (FLAG)** is to review the current status of lattice results for a variety of physical quantities in low-energy physics ... “

Initially set up in November 2007 [thanks to the initiative of the Bern group in the framework of FlaviaNet], FLAG has published the first review in EPJC 71 (2011) limited to lattice results related to pion and kaon physics:

- light-quark masses (u-, d- and s-flavors),
- the form factor $f_+(0)$ arising in semileptonic $K \to \pi$ transitions at zero momentum transfer and the decay constant ratio f_K/f_π, relevant for the determinations of the CKM matrix elements V_{us} and V_{ud}
- some of the low-energy constants of $SU(2)_L \otimes SU(2)_R$ and $SU(3)_L \otimes SU(3)_R$ Chiral Perturbation Theory
- the B_K parameter of neutral kaon mixing (in the SM)

In the second review [arXiv:1310.8555], besides the update of the above quantities, new ones have been addressed:

- the $D_{(s)}$- and $B_{(s)}$-meson decay constants
- the D- and B-meson semileptonic form factors describing the decays to light flavors and the B-meson decays to charmed mesons
- neutral $B_{(s)}$-meson mixing matrix elements (in the SM)

$\Big\}$ determination of the CKM matrix and UTA

- the QCD coupling α_s

Quality criteria and color coding

Sources of systematic errors in lattice calculations:

- chiral extrapolation in the light-quark mass
- continuum limit
- finite volume effects
- renormalization and running

- heavy-quark treatment (particularly relevant for the b-quark sector)

details in arXiv:1310.8555

see also talk by N. Tantalo

Color coding:

★ the systematic error has been estimated in a satisfactory manner and convincingly shown to be under control;
○ a reasonable attempt at estimating the systematic error has been made, although this could be improved;
■ no or a clearly unsatisfactory attempt at estimating the systematic error has been made.

- Publication status:
  - A published or plain update of published results
  - P preprint
  - C conference contribution

***** only published results without a red tag in the systematic errors can be averaged *****

The FLAG-2 working group on $f_{D(s)}$ and $f_{B(s)}$ is composed by A. El-Khadra, Y. Aoki and M. Della Morte

FLAG-2 closing date: November 30th, 2013

Charm sector

Collaboration	Ref.	N_f	publication status	continuum extrapolation	chiral extrapolation	finite volume	renormalization/matching	heavy quark treatment	f_D	f_{D_s}	f_{D_s}/f_D
ETM 13F	[154]	2+1+1	C	○	○	○	★	✓	202(8)	242(8)	1.199(25)
FNAL/MILC 13$^\nabla$	[328]	2+1+1	C	★	★	★	★	✓	212.3(0.3)(1.0)	248.7(0.2)(1.0)	1.1714(10)(25)
FNAL/MILC 12B	[329]	2+1+1	C	★	★	★	★	✓	209.2(3.0)(3.6)	246.4(0.5)(3.6)	1.175(16)(11)
HPQCD 12A	[330]	2+1	A	○	○	★	★	✓	208.3(1.0)(3.3)	246.0(0.7)(3.5)	1.187(4)(12)
FNAL/MILC 11	[331]	2+1	A	○	○	★	○	✓	218.9(11.3)	260.1(10.8)	1.188(25)
PACS-CS 11	[332]	2+1	A	🟥	★	🟥	○	✓	226(6)(1)(5)	257(2)(1)(5)	1.14(3)
HPQCD 10A	[94]	2+1	A	★	○	★	★	✓	213(4)*	248.0(2.5)	
HPQCD/UKQCD 07	[164]	2+1	A	★	○	★	★	✓	207(4)	241 (3)	1.164(11)
FNAL/MILC 05	[333]	2+1	A	○	○	★	○	✓	201(3)(17)	249(3)(16)	1.24(1)(7)
ETM 13B$^\square$	[334]	2	P	★	○	★	★	✓	208(7)	250(7)	1.20(2)
ETM 11A	[335]	2	A	★	○	★	★	✓	212(8)	248(6)	1.17(5)
ETM 09	[168]	2	A	○	○	★	★	✓	197(9)	244(8)	1.24(3)

$^\nabla$ Update of FNAL/MILC 12B.
* This result is obtained by using the central value for f_{D_s}/f_D from HPQCD/UKQCD 07 and increasing the error to account for the effects from the change in the physical value of r_1.
$^\square$ Update of ETM 11A and ETM 09.

Table 20: Decay constants of the D and D_s mesons (in MeV) and their ratio.

- ■ results included in the average;
- □ results that are not included in the average but pass all quality criteria;
- □ all other results.

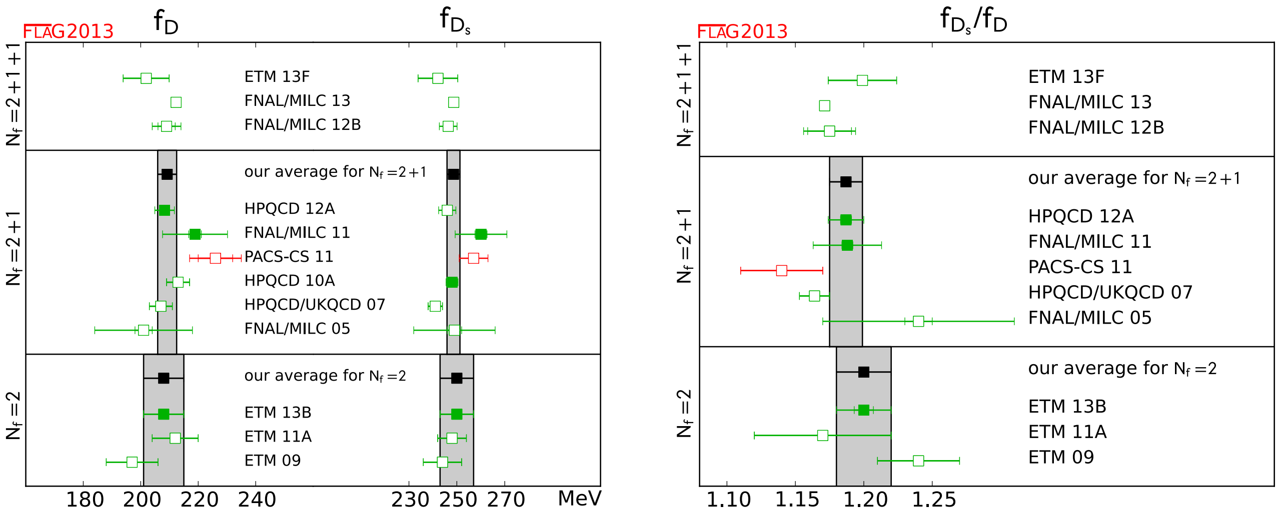


Figure 13: Decay constants of the D and D_s mesons [values in Table 20 and Eqs. (93), (94)]. The significance of the colours is explained in section 2. The black squares and grey bands indicate our averages. Errors in FNAL/MILC 13 are smaller than the symbols.

N_f = number of dynamical flavors in the sea

N_f	f_D (MeV)	f_{D_s} (MeV)	f_{D_s} / f_D
2	208 ± 7	250 ± 7	1.20 ± 0.02
2+1	209.2 ± 3.3	248.6 ± 2.7	1.187 ± 0.012
2+1+1	--	--	--
PDG '13	204.6 ± 5.0	257.5 ± 4.6	1.258 ± 0.038

* few percent accuracy → competitive with experimental errors
* dependence on N_f well within the errors

Beauty sector

Collaboration	Ref.	N_f	publication status	continuum extrapolation	chiral extrapolation	finite volume	renormalization/matching	heavy quark treatment	f_{B^+}	f_{B^0}	f_B	f_{B_s}
ETM 13E	[398]	2+1+1	C	○	○	○	○	✓	–	–	196(9)	235(9)
HPQCD 13	[399]	2+1+1	A	★	★	★	○	✓	184(4)	188(4)	186(4)	224(5)
RBC/UKQCD 13A	[400]	2+1	C	○	○	★	○	✓	–	–	$191(6)^{\diamond}_{\rm stat}$	$233(5)^{\diamond}_{\rm stat}$
HPQCD 12	[401]	2+1	A	○	○	★	○	✓	–	–	191(9)	228(10)
HPQCD 12	[401]	2+1	A	○	○	★	○	✓	–	–	$189(4)^{\triangle}$	–
HPQCD 11A	[365]	2+1	A	★	○	★	★	✓	–	–	–	$225(4)^{\nabla}$
FNAL/MILC 11	[331]	2+1	A	○	○	★	○	✓	197(9)	–	–	242(10)
HPQCD 09	[402]	2+1	A	○	○	★	○	✓	–	–	$190(13)^{\bullet}$	$231(15)^{\bullet}$
ALPHA 13	[403]	2	C	★	★	★	★	✓	–	–	187(12)(2)	224(13)
ETM 13B, 13C	[334, 404]	2	P[†]	★	○	★	○	✓	–	–	189(8)	228(8)
ALPHA 12A	[369]	2	C	★	★	★	★	✓	–	–	193(9)(4)	219(12)
ETM 12B	[392]	2	C	★	○	★	○	✓	–	–	197(10)	234(6)
ALPHA 11	[364]	2	C	★	○	★	★	✓	–	–	174(11)(2)	–
ETM 11A	[335]	2	A	○	○	★	○	✓	–	–	195(12)	232(10)
ETM 09D	[391]	2	A	○	○	○	○	✓	–	–	194(16)	235(12)

$^{\diamond}$Statistical errors only.
$^{\triangle}$Obtained by combining f_{B_s} from HPQCD 11A with f_{B_s}/f_B calculated in this work.
$^{\nabla}$This result uses one ensemble per lattice spacing with light to strange sea-quark mass ratio $m_\ell/m_s \approx 0.2$.
$^{\bullet}$This result uses an old determination of $r_1 = 0.321(5)$ fm from Ref. [379] that has since been superseded.
[†] Update of ETM 11A and 12B.

Collaboration	Ref.	N_f	publication status	continuum extrapolation	chiral extrapolation	finite volume	renormalization/matching	heavy quark treatment	f_{B_s}/f_{B^+}	f_{B_s}/f_{B^0}	f_{B_s}/f_B
ETM 13E	[398]	2+1+1	C	★	○	○	○	✓	–	–	1.201(25)
HPQCD 13	[399]	2+1+1	A	★	★	★	○	✓	1.217(8)	1.194(7)	1.205(7)
RBC/UKQCD 13A	[400]	2+1	C	○	○	★	○	✓	–	–	$1.20(2)^{\diamond}_{\rm stat}$
HPQCD 12	[401]	2+1	A	○	○	★	○	✓	–	–	1.188(18)
FNAL/MILC 11	[331]	2+1	A	○	○	★	○	✓	1.229(26)	–	–
RBC/UKQCD 10C	[405]	2+1	A	🟥	🟥	★	○	✓	–	–	1.15(12)
HPQCD 09	[402]	2+1	A	○	○	★	○	✓	–	–	1.226(26)
ALPHA 13	[403]	2	C	★	★	★	★	✓	–	–	1.195(61)(20)
ETM 13B, 13C	[334, 404]	2	P[†]	★	○	★	○	✓	–	–	1.206(24)
ALPHA 12A	[369]	2	C	★	★	★	★	✓	–	–	1.13(6)
ETM 12B	[392]	2	C	★	○	★	○	✓	–	–	1.19(5)
ETM 11A	[335]	2	A	○	○	★	○	✓	–	–	1.19(5)

$^{\diamond}$Statistical errors only.
[†] Update of ETM 11A and 12B.

Table 25: Ratios of decay constants of the B and B_s mesons (for details see Table 24).

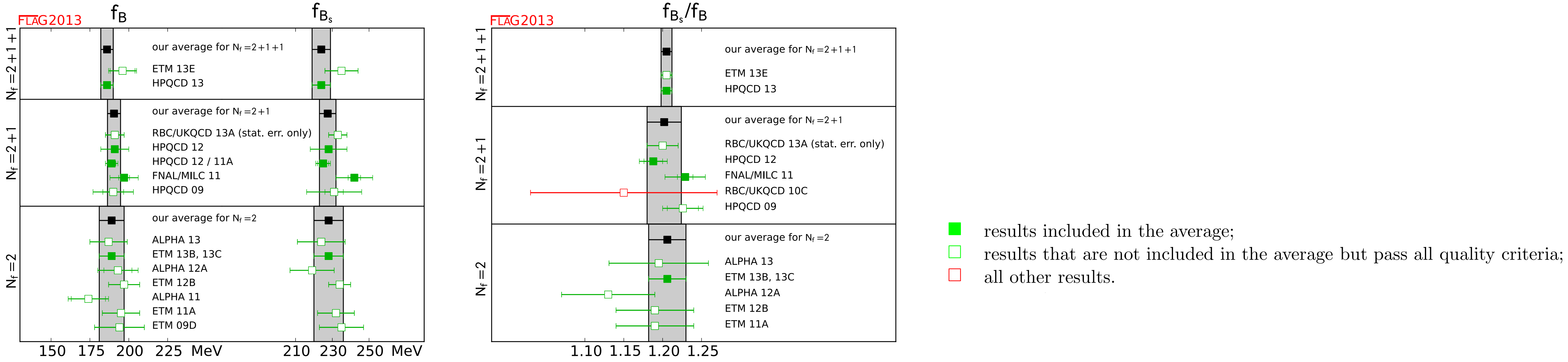
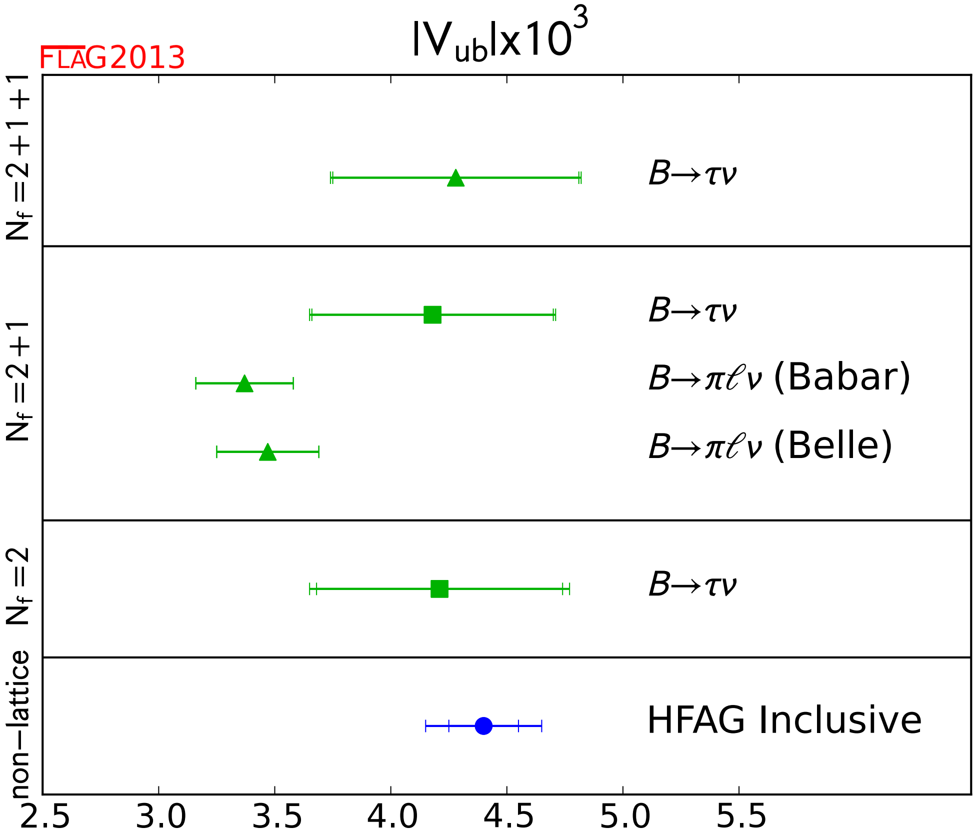


Figure 16: Decay constants of the B and B_s mesons. The values are taken from Table 24 (the f_B entry for FNAL/MILC 11 represents f_{B^+}). The significance of the colours is explained in section 2. The black squares and grey bands indicate our averages in Eqs. (110), (111) and (112).

N_f	f_B (MeV)	f_{B_s} (MeV)	f_{B_s} / f_B
2	189 ± 8	228 ± 8	1.206 ± 0.024
2+1	190.5 ± 4.2	227.7 ± 4.5	1.202 ± 0.022
2+1+1	186 ± 4	224 ± 5	1.205 ± 0.007



* accuracy at the few percent level
* dependence on N_f within the errors

FLAG-2 averages updated up to November 30th, 2013

***** new results: see, e.g., LAT ’14 and this conference *****

* Lattice calculations of heavy-light vector meson decay constant

charm sector

N_f (coll.)	f_{D^*} (MeV)	$f_{D_s^*}$ (MeV)	f_{D^*} / f_D	$f_{D_s^*} / f_{D_s}$	Ref.
2 (ETM)	278 ± 16	311 ± 9	1.28 ± 0.06	1.26 ± 0.03	JHEP 02 (2012)
2 (ETM)			1.197 ± 0.024		arXiv:1407.1019
2+1 (HPQCD)		274 ± 6		1.10 ± 0.02	PRL 112 (2014)

beauty sector

N_f (coll.)	f_{B^*} (MeV)	f_{B^*} / f_B	Ref.
0 (MILC)	177 ± 18	$1.01 \pm 0.01^{+0.04}_{-0.01}$	PRD 65 (2001)
0 (UKQCD)		1.02 ± 0.06	NPB 619 (2001)
2 (ETM)		1.042 ± 0.014	arXiv:1407.1019

QCD Sum Rules

* two-point correlation functions for heavy-light currents:

PS channel:

$$i\int d^4x\, e^{ip\cdot x}\langle 0|T\left[j_5(x)j_5^\dagger(0)\right]|0\rangle = \Pi^{PS}\left(p^2\right) \qquad j_5(x)\equiv\left(m_h+m_q\right)\bar{q}(x)i\gamma_5 h(x)$$

V channel:

$$i\int d^4x\, e^{ip\cdot x}\langle 0|T\left[j_\nu(x)j_{\nu'}^\dagger(0)\right]|0\rangle = \left(-g_{\nu\nu'}+\frac{p_\nu p_{\nu'}}{p^2}\right)\Pi^V\left(p^2\right) \qquad j_\nu(x)\equiv\bar{q}(x)\gamma_\nu h(x)$$

$$+\frac{p_\nu p_{\nu'}}{p^2}\Pi_L^V\left(p^2\right)$$

- after Borelization:

hadronic representation | quark-gluon level (OPE)

$$\Pi^{PS}(\tau) = f_{PS}^2 M_{PS}^4 e^{-M_{PS}^2\tau} + \int_{s_{phys}^{PS}}^{\infty} ds\, e^{-s\tau}\rho_{hadron}^{PS}(s) = \int_{\left(m_h+m_q\right)^2}^{\infty} ds\,\rho_{pert}^{PS}(s,\mu)+\Pi_{power}^{PS}(\tau,\mu)$$

$$\Pi^{V}(\tau) = f_{V}^2 M_{V}^2 e^{-M_{V}^2\tau} + \int_{s_{phys}^{V}}^{\infty} ds\, e^{-s\tau}\rho_{hadron}^{V}(s) = \int_{\left(m_h+m_q\right)^2}^{\infty} ds\,\rho_{pert}^{V}(s,\mu)+\Pi_{power}^{V}(\tau,\mu)$$

$s_{phys}^{PS(V)} =$ physical continuum threshold in the PS (V) channel

$\mu =$ subtraction point

* perturbative part (up to NNLO):

$$\rho_{pert}^{PS(V)}(\tau) = \rho_{LO}^{PS(V)}(\tau) + \frac{\alpha_s}{\pi}\rho_{NLO}^{PS(V)}(\tau) + \frac{\alpha_s^2}{\pi^2}\rho_{NNLO}^{PS(V)}(\tau) + O(\alpha_s^3)$$

NLO: Broadhurst ’81
Generalis ’90
NNLO: Chetyrkin&Steinhauser ’01
Khodjamirian et al. ’13

* power corrections (up to dimension-6):

$$\Pi_{power}^{PS(V)}(\tau) = \Pi_{<\bar{q}q>}^{PS(V)}(\tau) + \Pi_{<GG>}^{PS(V)}(\tau) + \Pi_{<\bar{q}Gq>}^{PS(V)}(\tau) + \Pi_{<\bar{q}q\bar{q}q>}^{PS(V)}(\tau) + O(d>6)$$

– the correlator $\Pi^{PS(V)}(\tau)$ is dominated by the ground-state at large τ, where however the truncated OPE does not converge

$\big\downarrow$ quark-hadron duality [SVZ ’79]

$$\int_{s_{phys}^{PS(V)}}^{\infty} ds\, e^{-s\tau}\rho_{hadron}^{PS(V)}(\tau) = \int_{s_{eff}^{PS(V)}(\tau,\mu)}^{\infty} ds\, e^{-s\tau}\rho_{pert}^{PS(V)}(\tau,\mu)$$

$s_{eff}^{PS(V)}(\tau,\mu)$ = effective threshold, which generally may depend on τ and μ [LMS ’07]

an effective tool for eliminating the excited state contributions at intermediate values of τ

$$f_{PS}^2 M_{PS}^4 e^{-M_{PS}^2\tau} = \Pi_{dual}^{PS}(\tau) = \int_{(m_h+m_q)^2}^{s_{eff}^{PS}(\tau,\mu)} ds\, e^{-s\tau}\rho_{pert}^{PS}(s,\mu) + \Pi_{power}^{PS}(\tau,\mu)$$

$$f_V^2 M_V^2 e^{-M_V^2\tau} = \Pi_{dual}^{V}(\tau) = \int_{(m_h+m_q)^2}^{s_{eff}^{V}(\tau,\mu)} ds\, e^{-s\tau}\rho_{pert}^{V}(s,\mu) + \Pi_{power}^{V}(\tau,\mu)$$

* choice of the Borel window: $\tau_{\min} \le \tau \le \tau_{\max}$

- $\tau > \tau_{\min}$: ground-state provides a sizable contribution (e.g., > 50%)

- $\tau < \tau_{\max}$: power corrections remain sufficiently small numerically

* in the given Borel window the effective threshold $s_{\rm eff}$ is chosen by requiring that the dual mass $M_{\rm dual}$ reproduces the experimental meson mass M_H:

– minimization of $\chi^2 = \frac{1}{N}\sum_{i=1}^{N}\left[M^2_{dual}(\tau_i) - M^2_H\right]^2$

– extraction of f_{dual} : $f^2_{dual} = \frac{1}{N}\sum_{i=1}^{N} M_H^{-4} e^{M_H^2 \tau_i}\,\Pi_{dual}(\tau_i, s_{eff})$

$$M^2_{dual}(\tau_i) \equiv -\frac{d}{d\tau}\log \Pi_{dual}(\tau_i, s_{eff})$$

$$\tau_{\min} \le \tau_i \le \tau_{\max} \qquad (i = 1, \dots, N)$$

* strategies to determine $s_{\rm eff}$: – polynomial Ansatz → $s_{eff}(\tau) = \sum_{j=0}^{n} s_j^{(n)} \tau^j$

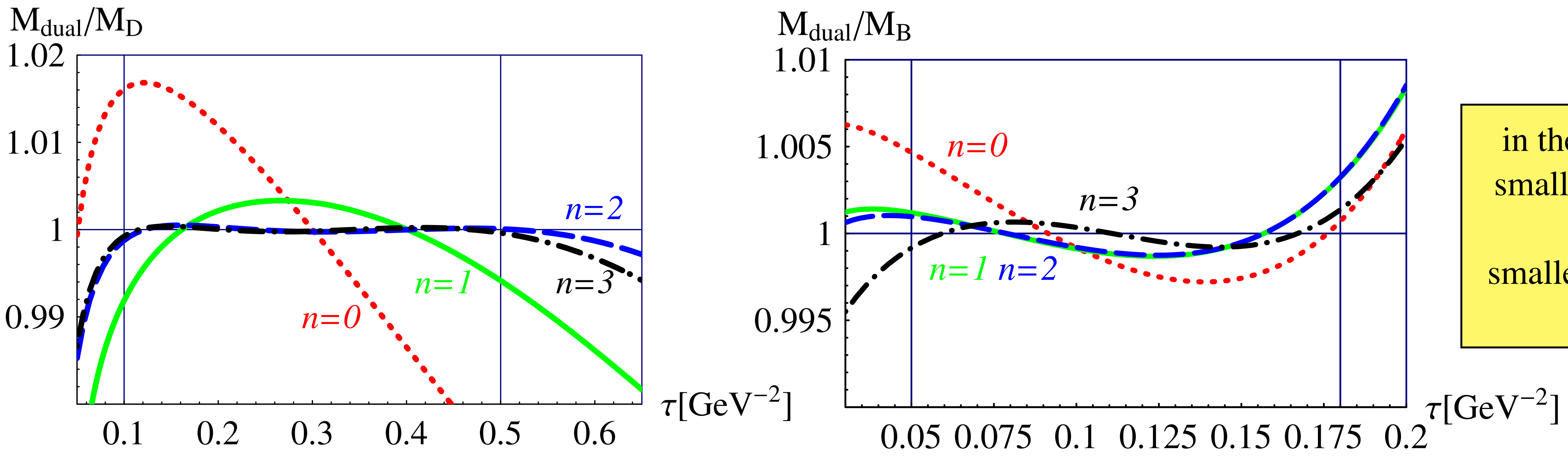
constant threshold: n=0 (widely adopted in literature)
τ-dependent: linear (n=1), quadratic (n=2), ... [LMS ’07]

- $s_{\rm eff}(\tau_i)$ = free parameter at each point τ_i in the Borel window [Khodjamirian et al. ’13]

... note that the calculation of $M_{\rm dual}$ requires the knowledge of the derivative of $s_{\rm eff}(\tau)$

D-meson: $0.1 \le \tau(GeV^{-2}) \le 0.5$

B-meson: $0.05 \le \tau(GeV^{-2}) \le 0.18$



in the given Borel window
smaller deviations from M_H
mean
smaller contaminations from
excited states

***** the τ-dependence of the effective threshold $s_{\rm eff}$ improves the quality of the dual correlator *****

Pole mass versus running mass

- the perturbative spectral density has been calculated in terms of the pole heavy-quark mass m_h^{pole}, but the expansion can be reorganized in terms of the running $\overline{\mathrm{MS}}$ mass $\overline{m_h}(\mu)$

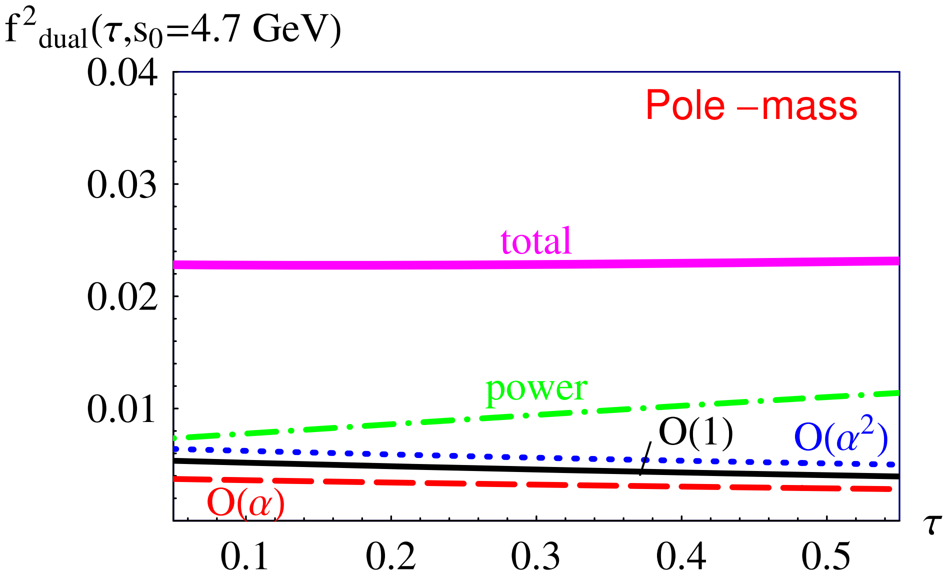
$$m_h^{pole} = \bar{m}_h(\mu)\left[1 + \frac{\alpha_s(\mu)}{\pi} r_1 + \left(\frac{\alpha_s(\mu)}{\pi}\right)^2 r_2 + O\left(\alpha_s^3\right)\right]$$

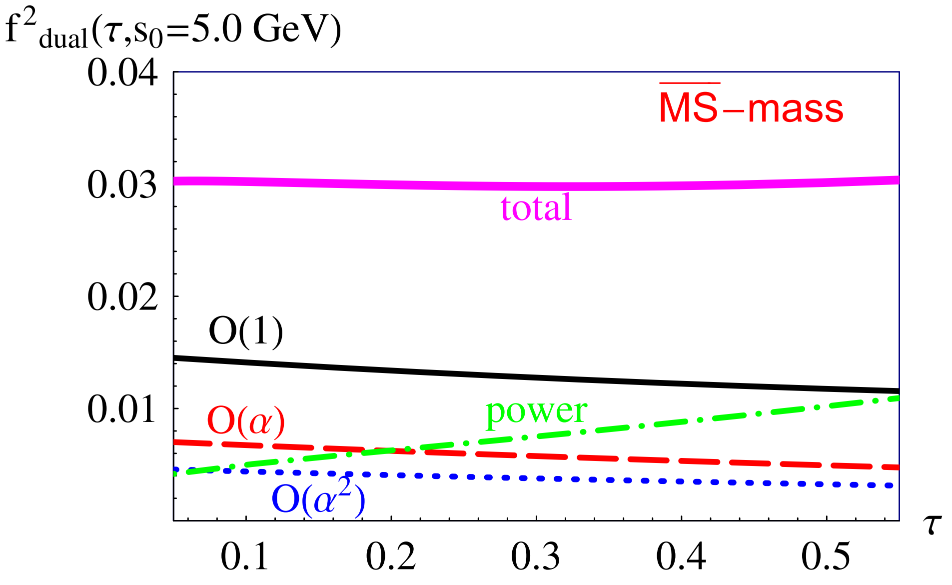
- both the truncated OPE and the hierarchy among the various orders are sensitive to the choice of the scheme [Jamin&Lange '02]

D-meson

$m_c^{pole} = 1.68\,GeV$

$\bar{m}_c\left(\bar{m}_c\right) = 1.28\,GeV$

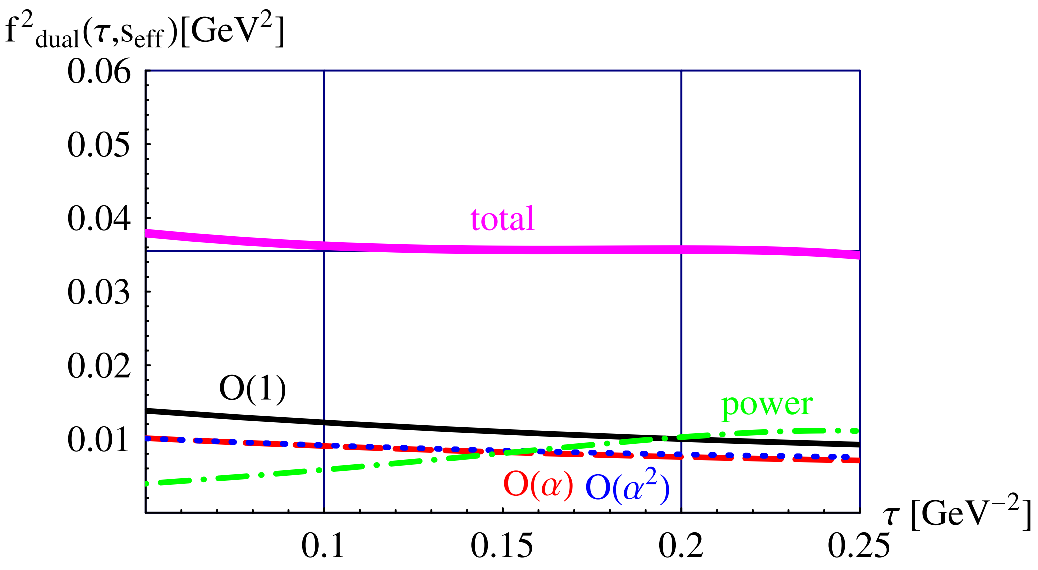


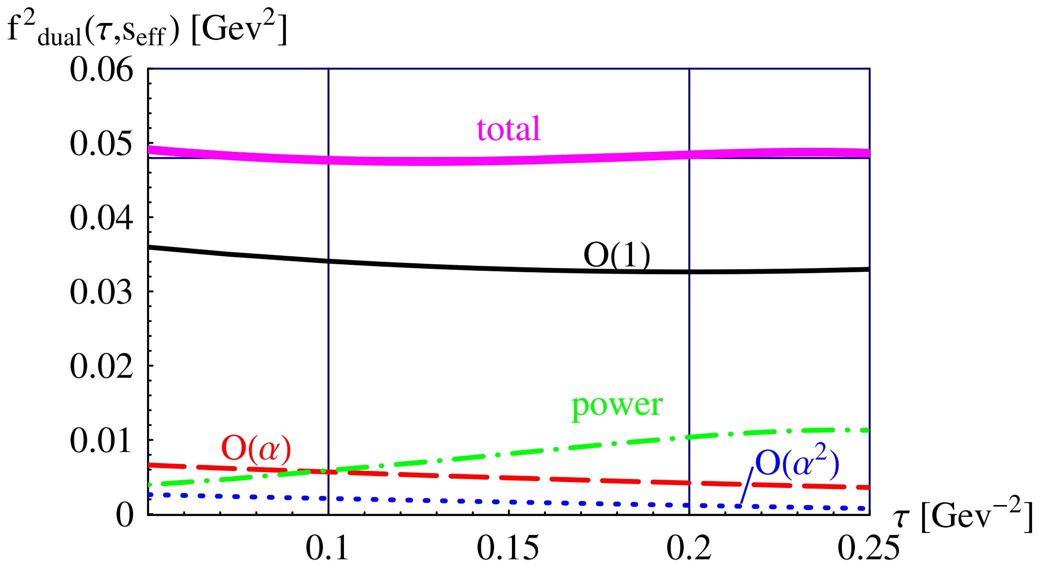


B-meson

$m_b^{pole} = 4.75\,GeV$

$\bar{m}_b\left(\bar{m}_b\right) = 4.16\,GeV$

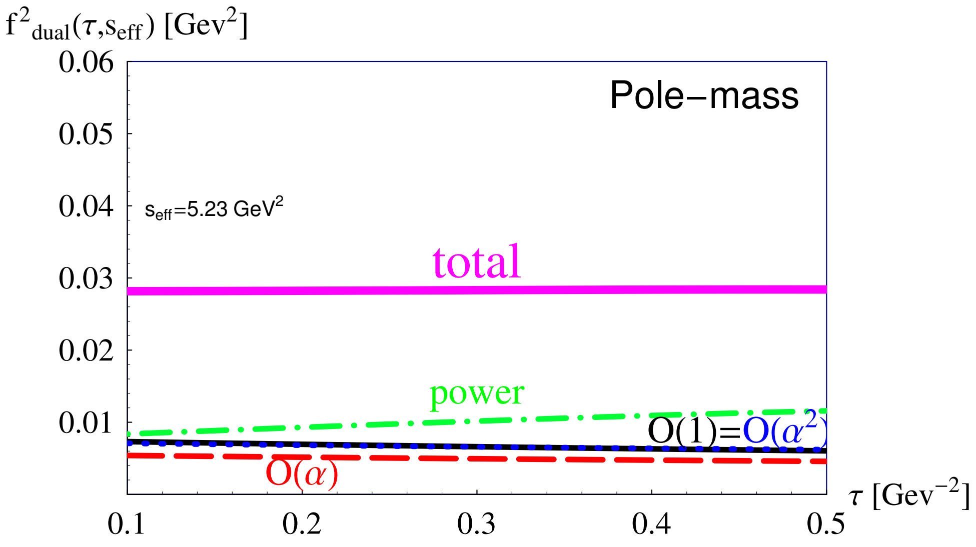


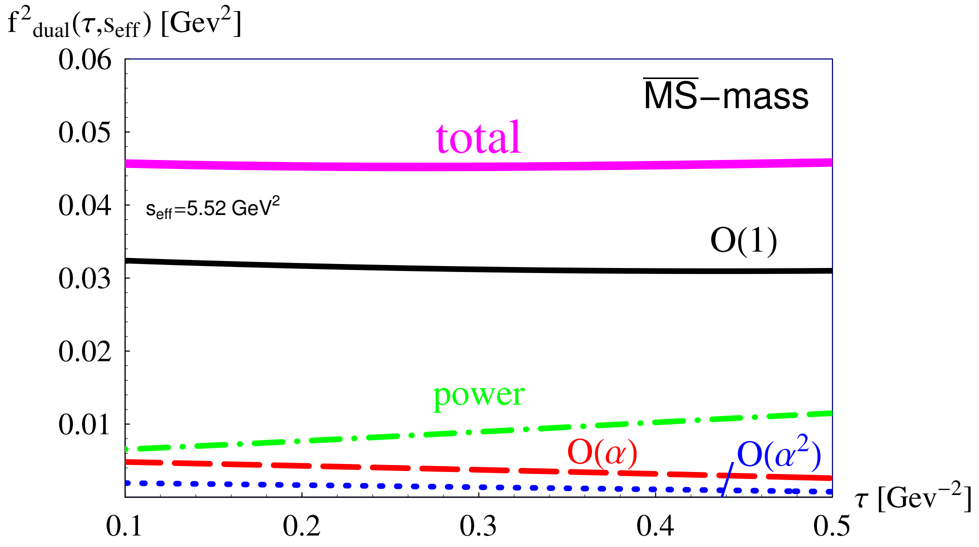


D^*-meson

$m_c^{pole} = 1.68\, GeV$

$\bar{m}_c(\bar{m}_c) = 1.28\, GeV$

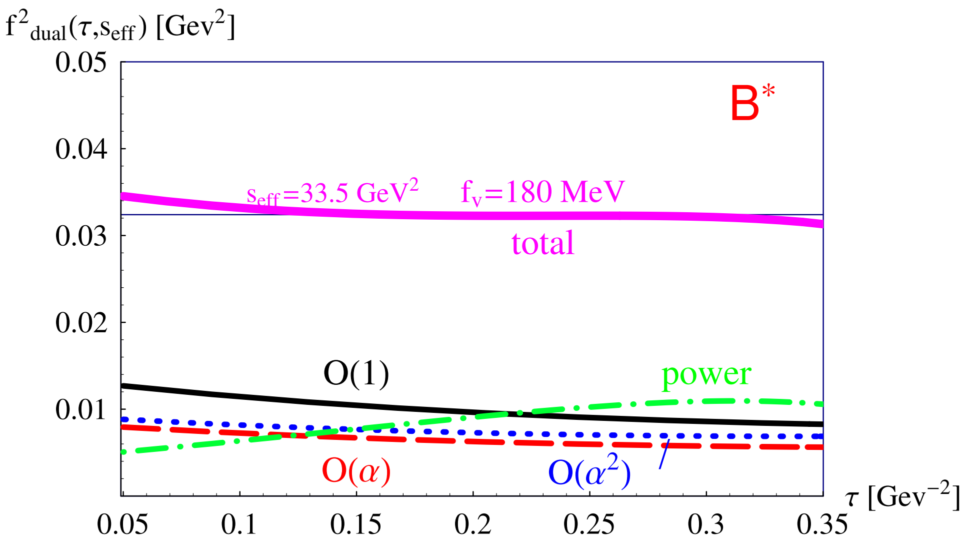


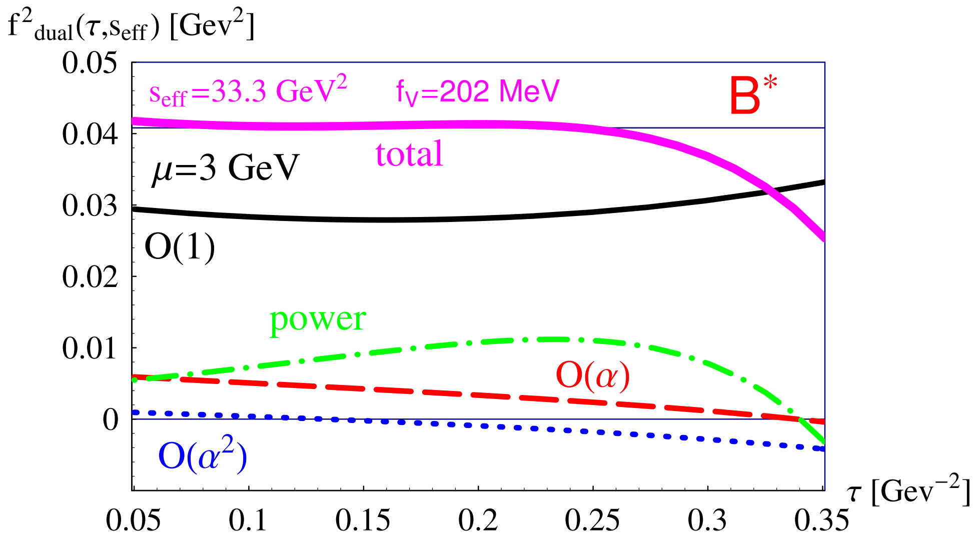


B^*-meson

$m_b^{pole} = 4.80\, GeV$

$\bar{m}_b(\bar{m}_b) = 4.18\, GeV$





*** important messages:**

- the expansion in terms of the pole mass show no sign of convergence

- a reasonable hierarchy among the various perturbative orders is found in terms of the $\overline{\text{MS}}$ running mass (the one usually adopted in SR analyses)

- the decay constants exhibit a nice stability over a wide range of values of τ, but the extracted value from the expansion in terms of the pole mass is around 10% lower than the corresponding one obtained in terms of the $\overline{\text{MS}}$ running mass

******* Borel stability does not guarantee reliability *******

Charmed mesons

*** input parameters**

- quark masses: $\overline{m}_c(\overline{m}_c) = 1.275 \pm 0.025\ GeV$ [PDG '13]

  $\overline{m}_{u/d}(2\ GeV) = 3.42 \pm 0.09\ \ MeV, \quad \overline{m}_s(2\ GeV) = 93.8 \pm 2.4\ \ MeV$ [FLAG '13]

- condensates: $<\bar{q}q> = (-267 \pm 17\ MeV)^3, \quad <\bar{s}s>/<\bar{q}q> = 0.8 \pm 0.3, \quad <\bar{q}q\bar{q}q>/<\bar{q}q>^2 = 0.1 - 1.0$

  $\frac{\alpha_s}{\pi} < GG > = 0.024 \pm 0.012\ GeV^4, \quad <\bar{q}Gq>/<\bar{q}q> = 0.8 \pm 0.2\ GeV^2$

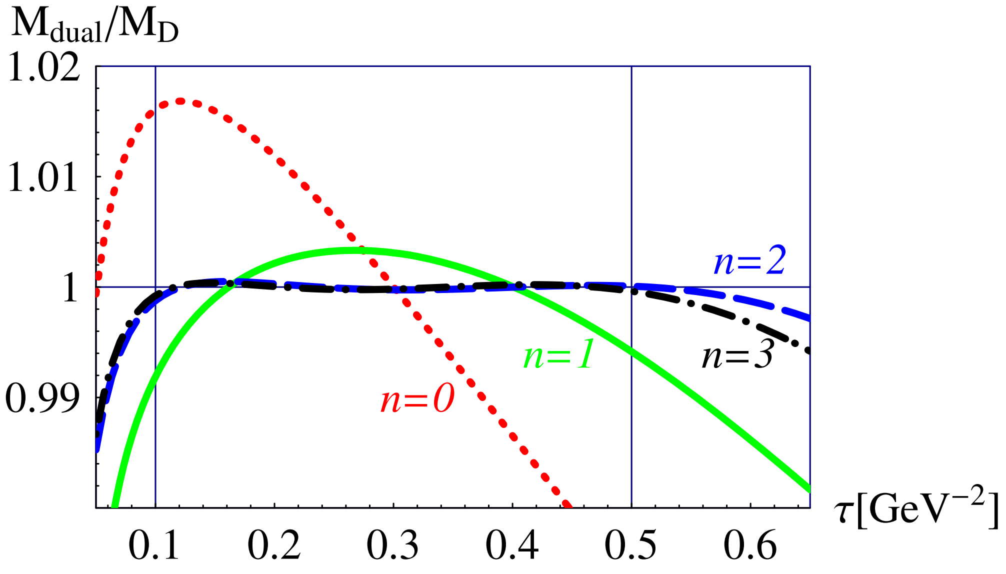
- strong coupling: $\alpha_s(M_Z) = 0.1184 \pm 0.0007$ [PDG '13]

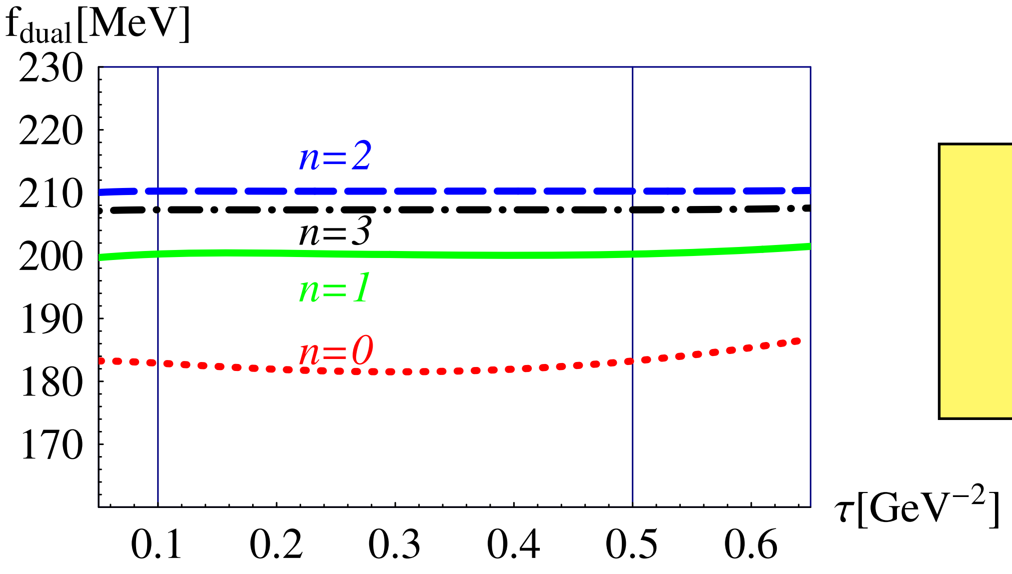
bootstrap analysis [O(1000) events]

- subtraction point: $1 \le \mu(GeV) \le 3$

uniform distribution for μ
gaussian distributions for all the other parameters

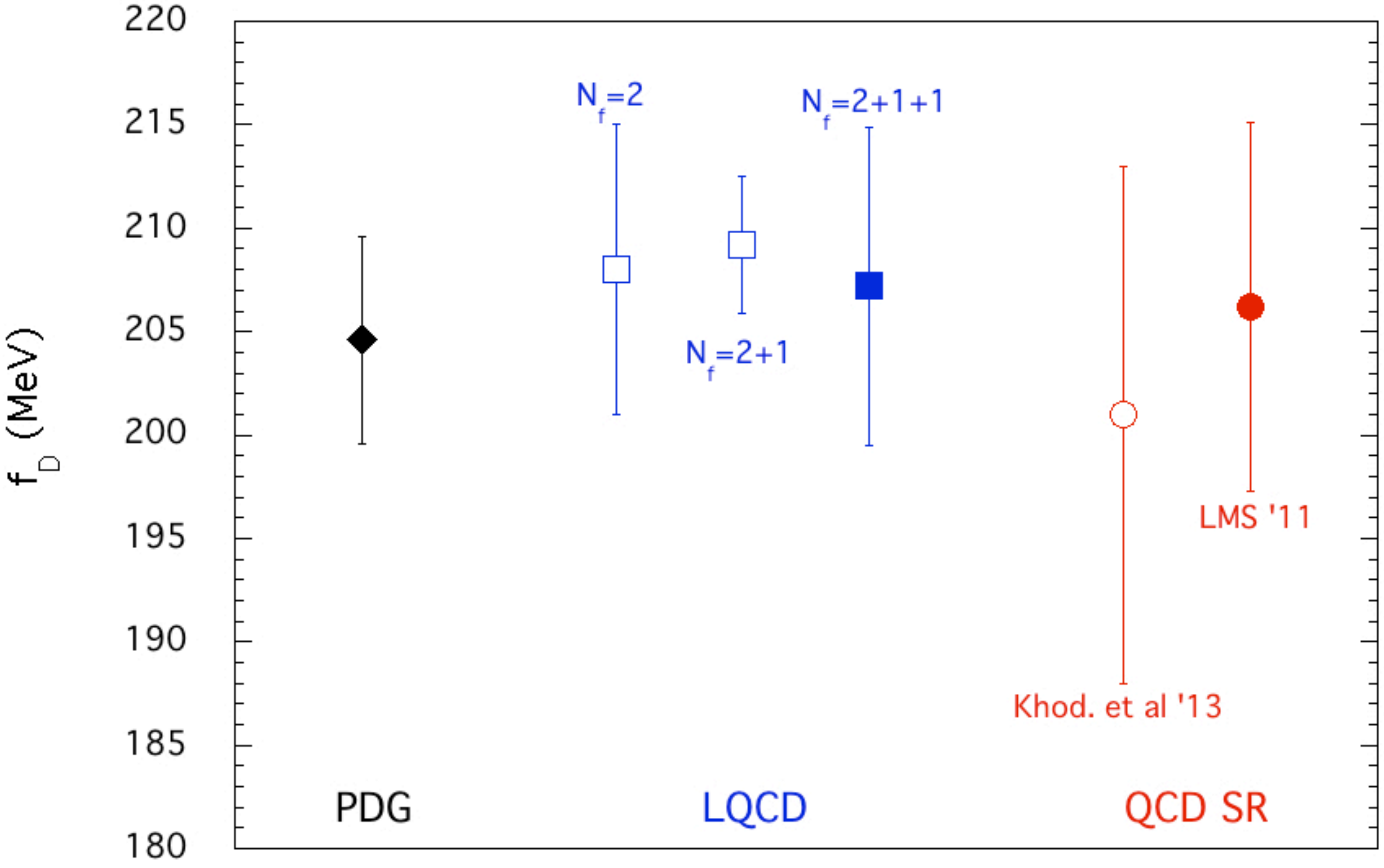
D-meson: $0.1 \le \tau\left(GeV^{-2}\right) \le 0.5$





$$s_{eff}(\tau) = \sum_{j=0}^{n} s_j^{(n)} \tau^j$$

the τ-dependence of s_{eff}
produces
a ~ 10% increase in the
extracted decay constant



LQCD

☐ FLAG '13 averages

■ average of lattice results (unofficial FLAG average)

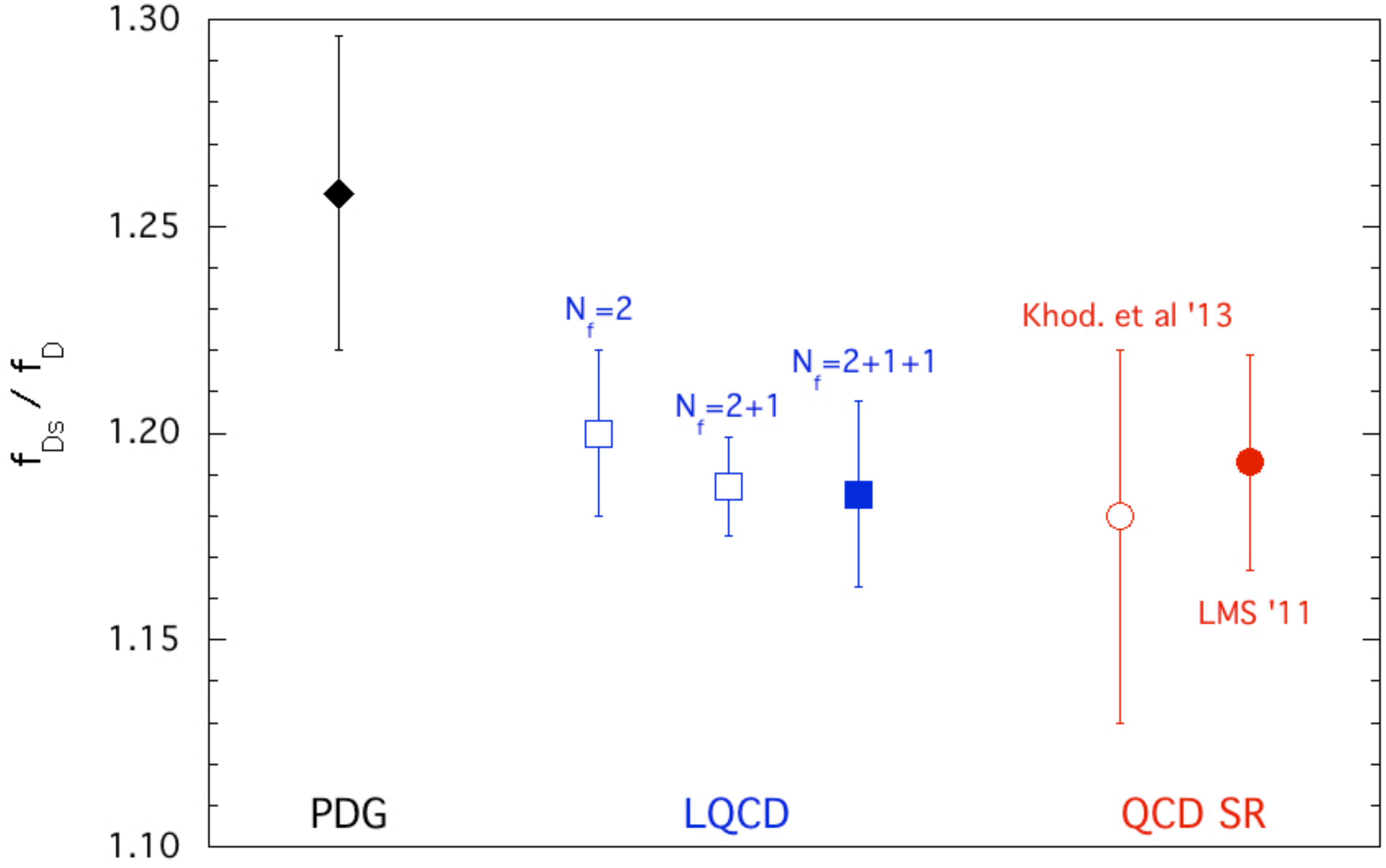
QCD SR

○ Khodjamirian et al., PRD 88 (2013)

● LMS, PLB 701 (2011)

$\bar{m}_c(\bar{m}_c) = 1.275 \pm 0.025\, GeV$ [PDG '13]

τ-dependent s_{eff}



there is an excellent agreement between LQCD and QCD SR (using the PDG value for the charm mass)

$$f_D^{dual} = \left[206.2 - 3.3\left(\frac{\bar{m}_c(\bar{m}_c) - 1.275\,GeV}{0.025\,GeV}\right)\right] MeV$$

$$f_{D_s}^{dual} = \left[245.3 - 4.5\left(\frac{\bar{m}_c(\bar{m}_c) - 1.275\,GeV}{0.025\,GeV}\right)\right] MeV$$

$\boxed{D^{*}\text{-meson: } 0.1 \le \tau\left(GeV^{-2}\right) \le 0.5}$

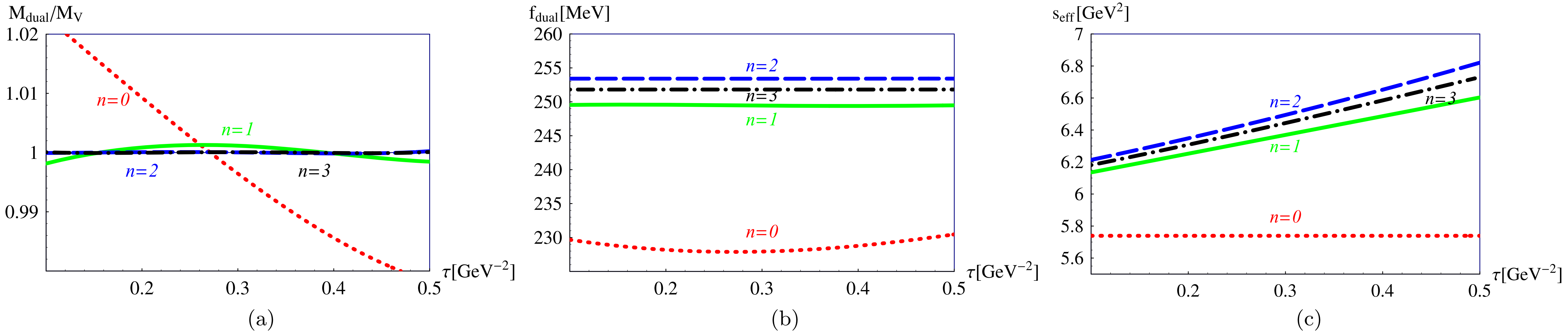
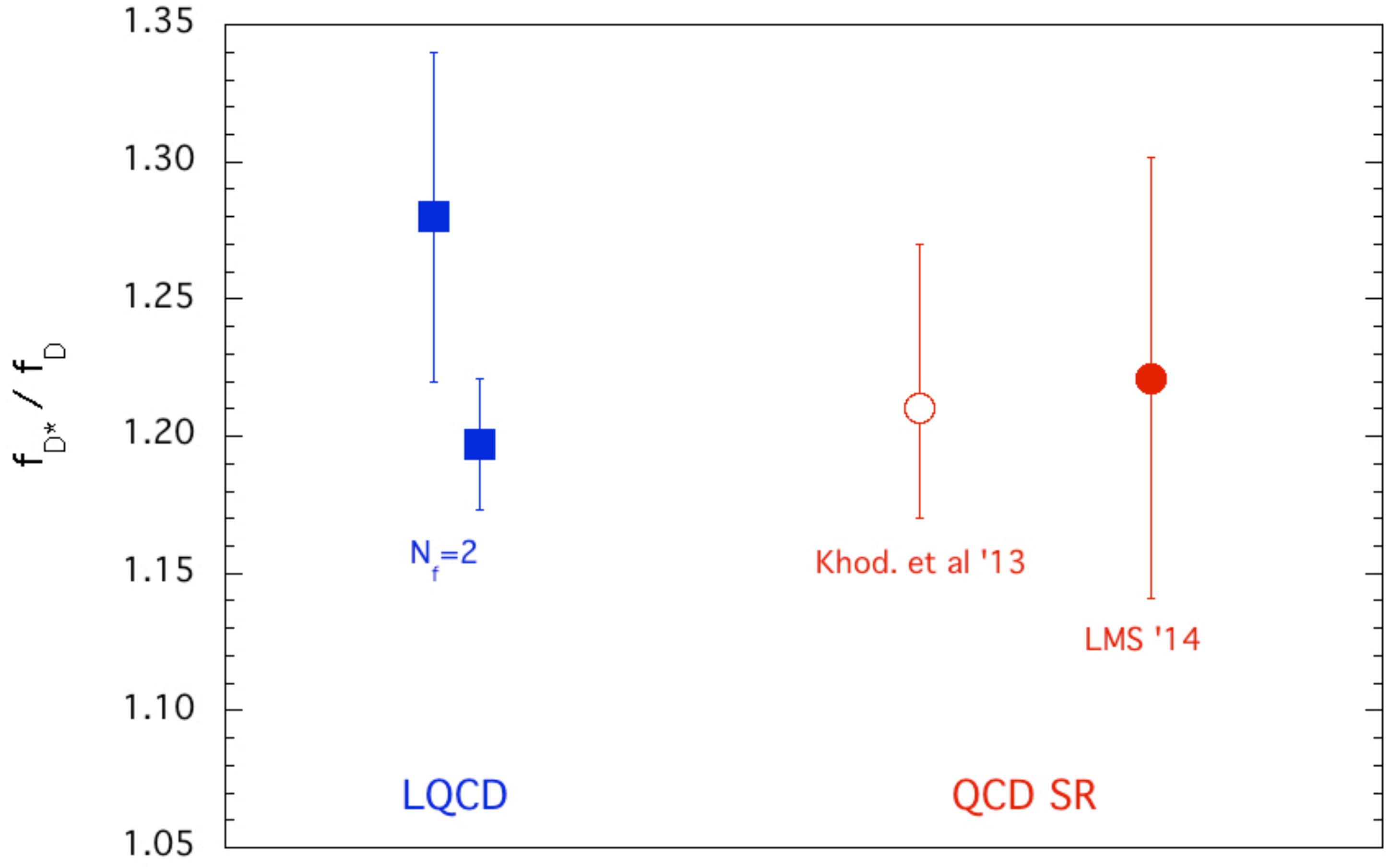


Fig. 2. Dependence on the Borel parameter τ of the dual mass (a) and the dual decay constant (b) of the D^* meson, obtained by employing different Ansätze (3.2) for the effective continuum threshold $s_{\rm eff}(\tau)$ and fixing all thresholds according to (3.3); the results are presented for central values of all OPE parameters and for an average scale $\mu = \mu^* = 1.84$ GeV, where the average scale μ^* is defined by (3.6). (c) Our τ-dependent effective thresholds obtained by the fitting procedure as explained in the text. The integer $n = 0, 1, 2, 3$ is the degree of the polynomial in our Ansatz (3.2) for $s_{\rm eff}(\tau)$: dotted lines (red) — $n = 0$; solid lines (green) — $n = 1$; dashed lines (blue) — $n = 2$; dot-dashed lines (black) — $n = 3$.

* the spin splitting $\left(M_{D^*} - M_D\right)$ is properly reproduced in the full parameter space



LQCD ($N_f = 2$, ETM)

Becirevic et al., JHEP 02 (2012)

■

Becirevic et al., arXiv:1407.1019

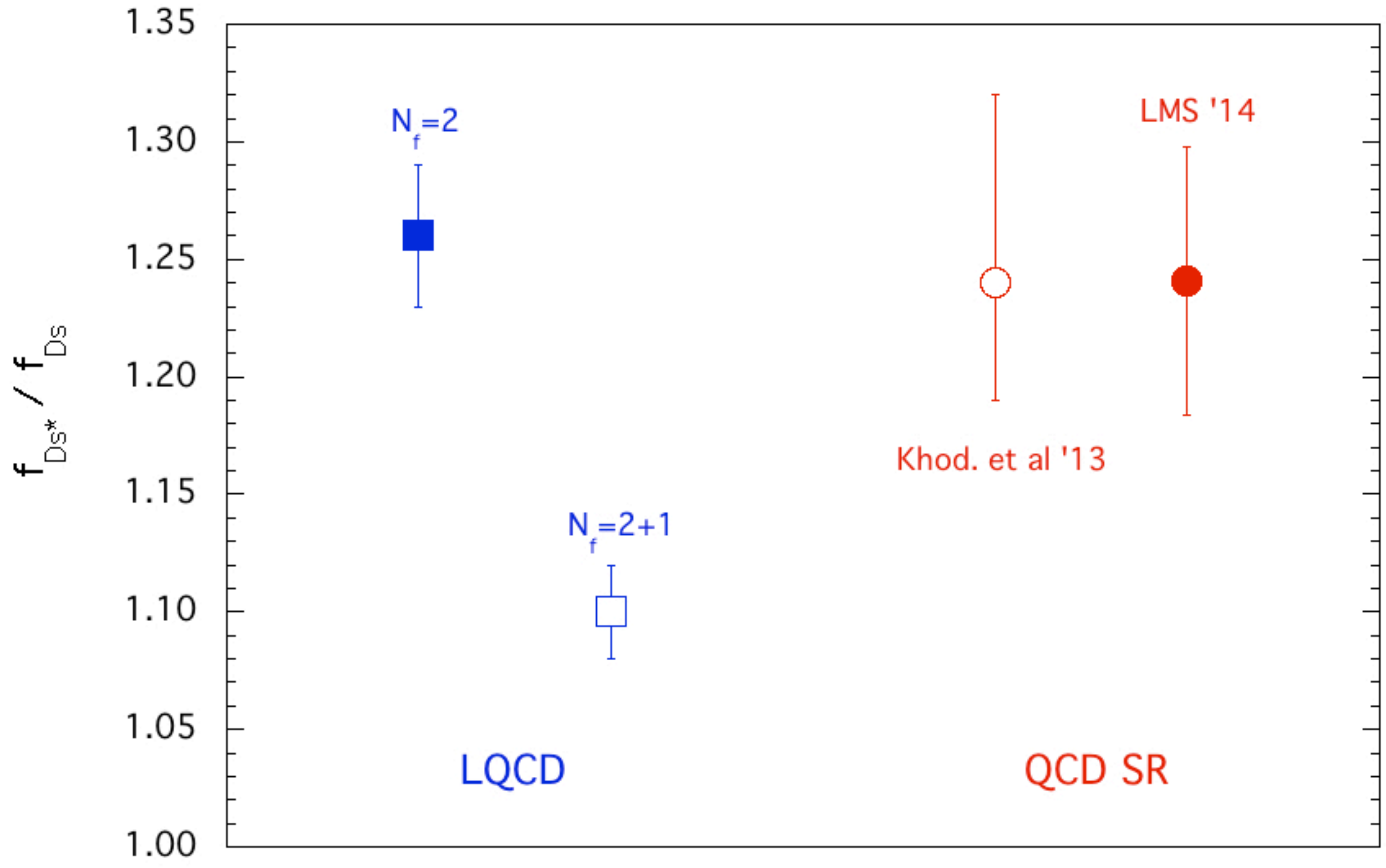
QCD SR

○ Khodjamirian et al., PRD 88 (2013)

● LMS, PLB 735 (2014)

$\overline{m}_c\left(\overline{m}_c\right) = 1.275 \pm 0.025\, GeV$ [PDG '13]

τ-dependent $s_{\rm eff}$



*** ratio $f_{D_s^*} / f_{D_s}$**

LQCD

■ Becirevic et al., JHEP 02 (2012) (ETM)

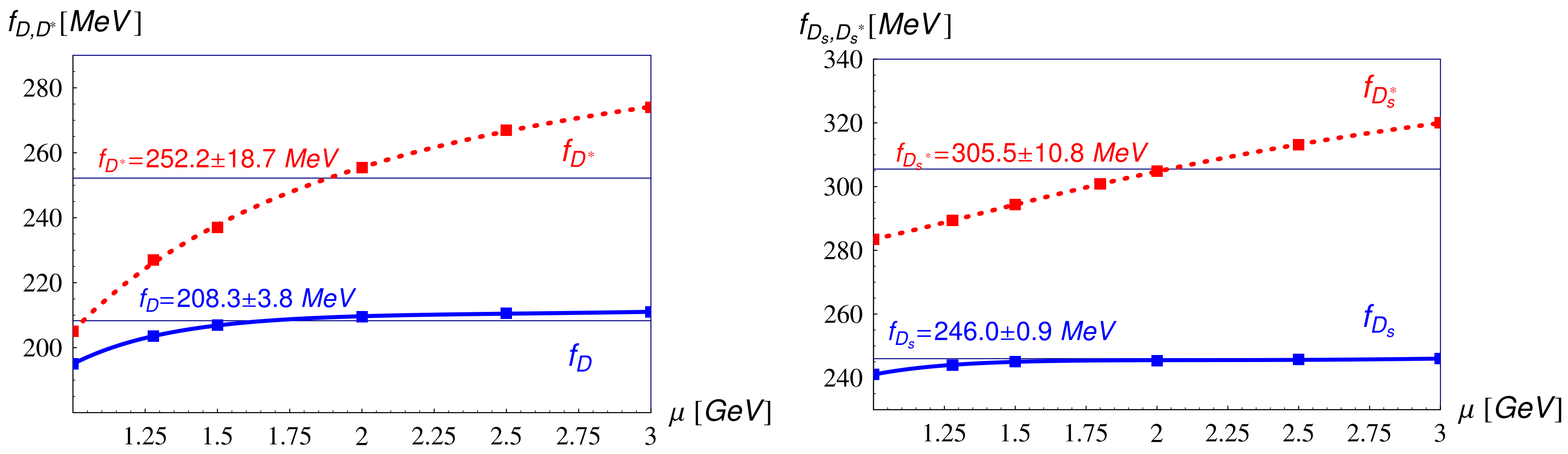
□ Davies et al., PRL 112 (2014) (HPQCD)

QCD SR

○ Khodjamirian et al., PRD 88 (2013)

● LMS, PLB 735 (2014)

* **remarkable sensitivity to the subtraction point μ in the V channel:** $1 \le \mu(GeV) \le 3$

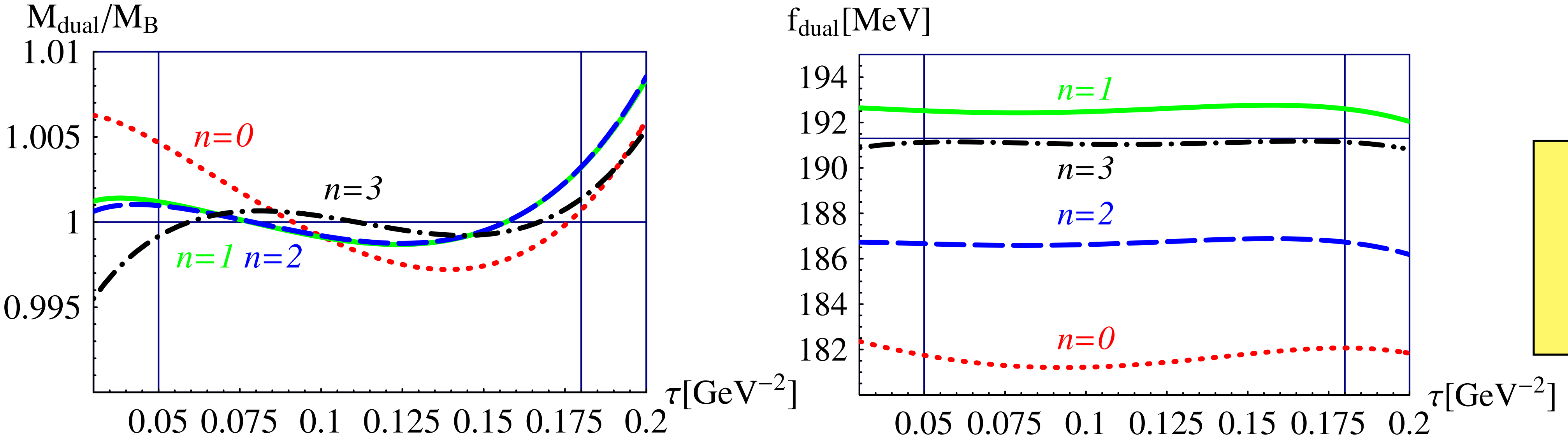


(a) (b)

Fig. 3. Dependence on μ of the dual decay constants: (a) $f_D^{\rm dual}(\mu)$ and $f_{D^*}^{\rm dual}(\mu)$, (b) $f_{D_s}^{\rm dual}(\mu)$ and $f_{D_s^*}^{\rm dual}(\mu)$. The depicted results are obtained as follows: for a fixed value of μ, central values of the OPE parameters in (2.1) and a Borel parameter τ within the window $0.1 < \tau\ ({\rm GeV}^{-2}) < 0.5$, we determine the effective thresholds by our procedure; the presented dual decay constant then is the average of the band formed by the linear, quadratic, and cubic Ansätze for the effective threshold. Clearly, the effective thresholds turn out to depend on the scale μ. Dotted lines (red) — vector mesons; solid lines (blue) — pseudoscalar mesons.

Beauty sector

B-meson: $0.05 \le \tau\left(GeV^{-2}\right) \le 0.18$



$$s_{eff}(\tau) = \sum_{j=0}^{n} s_j^{(n)} \tau^j$$

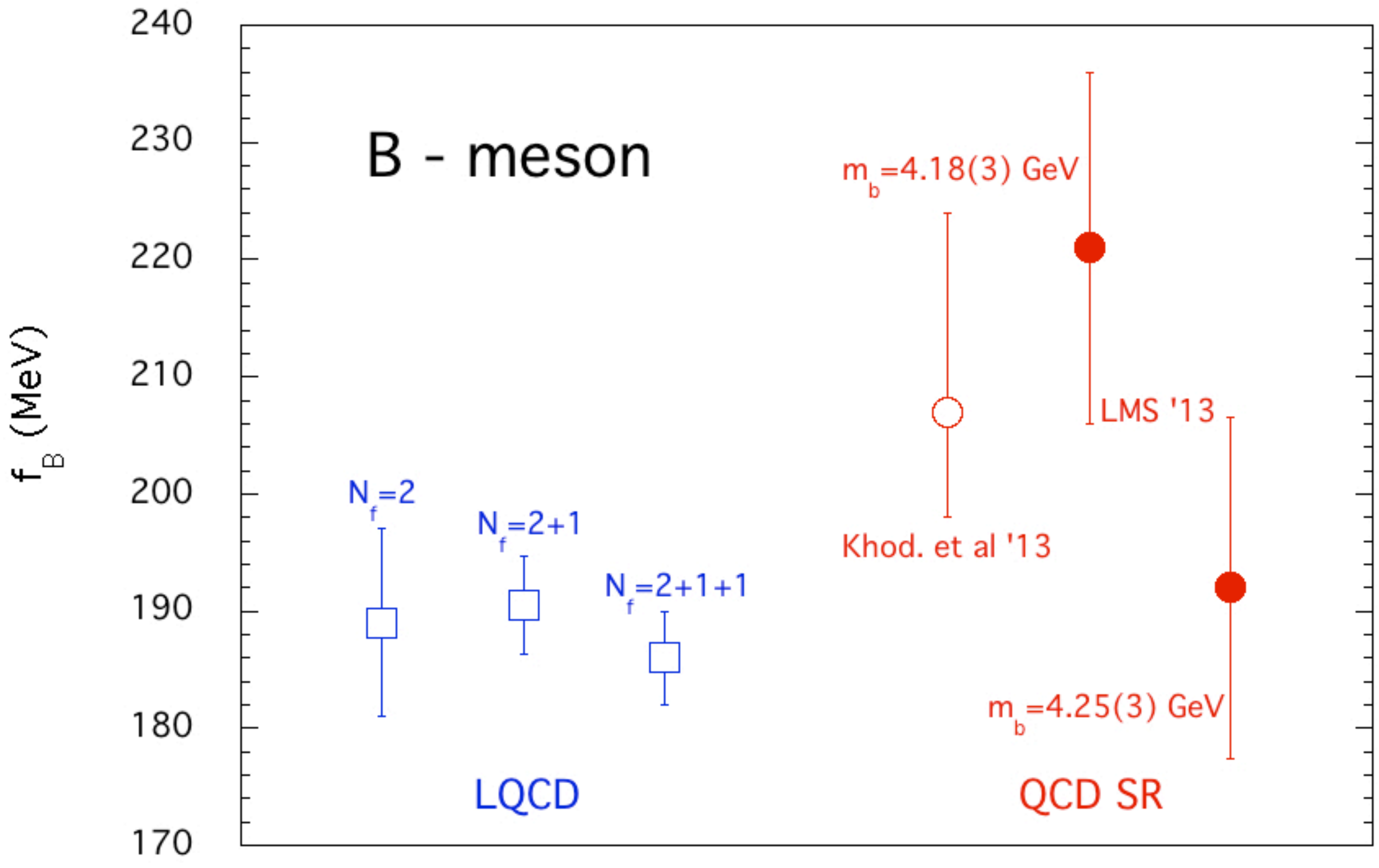
the τ-dependence of s_{eff} produces
a ~ 5% increase in the extracted decay constant

		τ-independent s_{eff}			τ-dependent s_{eff}		finite energy SR
	PDG '13	Narison '01	Jamin&Lange '02	Narison '13	Khod. et al '13	LMS '13	Baker et al. '14
$\overline{m}_b(\overline{m}_b)(GeV)$	4.18 ± 0.03	4.05 ±0.06	4.21 ±0.05	4.236 ± 0.069	4.18 ± 0.03	4.247 ± 0.034	4.18 ± 0.03
f_B (MeV)	--	203 ± 23	210 ± 19	206 ±7	207^{+17}_{-9}	192 ± 15	186 ± 14

*** HQET and potential models:** $f_B\sqrt{M_B} \approx \left|\psi(r=0)\right| \approx \kappa\left(M_B - m_b^{pole}\right)^{3/2}$ $\Rightarrow$ $\delta f_B \approx -0.5\,\delta m_b^{pole}$

$f_B \approx 200\ MeV$
$m_b^{pole} \approx 4.6\ GeV$
$\kappa \approx 1$

*** QCD SR:** $\delta f_B \approx -0.37\,\delta\overline{m}_b\left(\overline{m}_b\right)$ [LMS '13]



LQCD

□ FLAG '13 averages

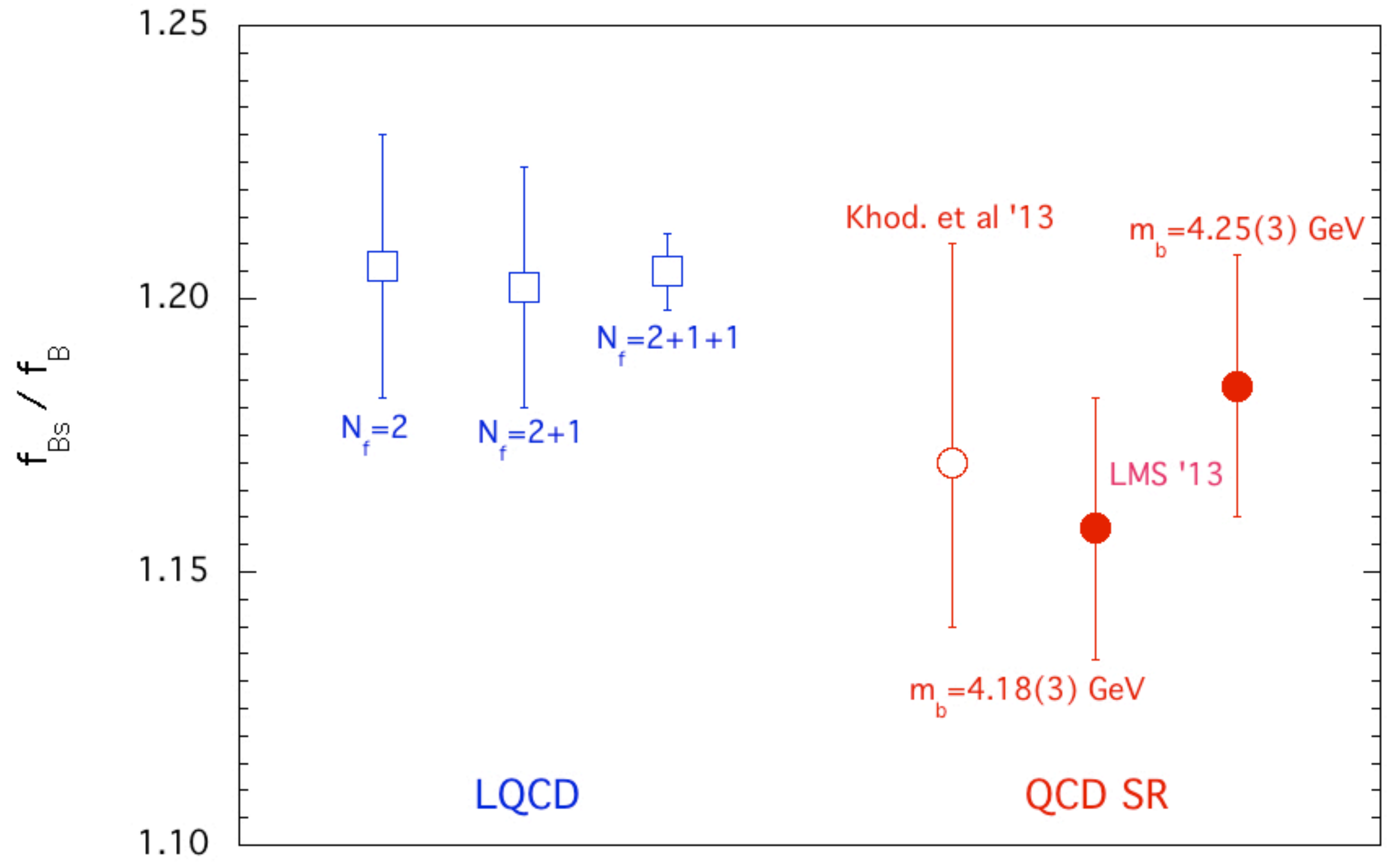
QCD SR

○ Khodjamirian et al., PRD 88 (2013)

$0.154 < \tau\ (\mathrm{GeV}^{-2}) < 0.222$
$3 < \mu\ (\mathrm{GeV}) < 5$

● LMS, PRD 88 (2013)

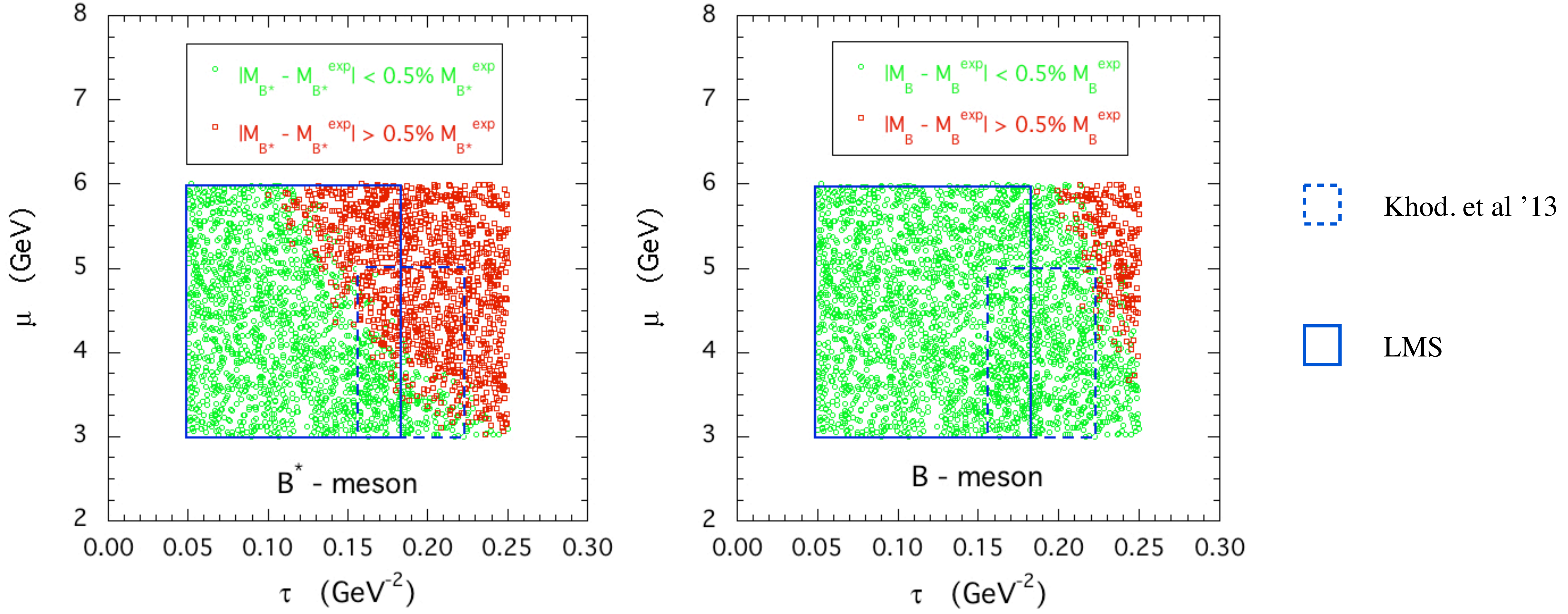
$0.05 < \tau\ (\mathrm{GeV}^{-2}) < 0.18$
$3 < \mu\ (\mathrm{GeV}) < 6$

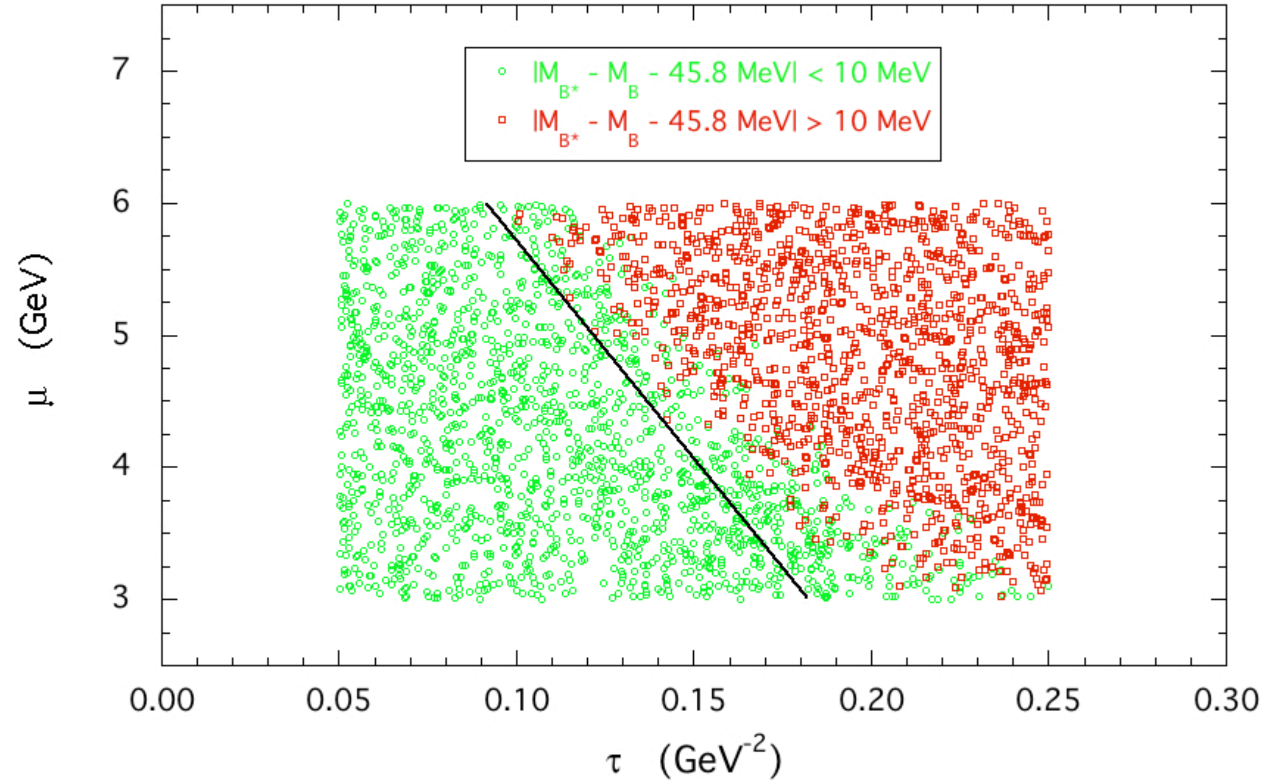


*** the decay constant f_B obtained using the PDG value of the b-quark mass is ~ 10% higher than the FLAG averages**

*** using the lattice average as input for f_B, the b-quark mass obtained with Borel QCD sum rules is ~ 2% higher than the PDG value (~ 1.5 standard deviations)**

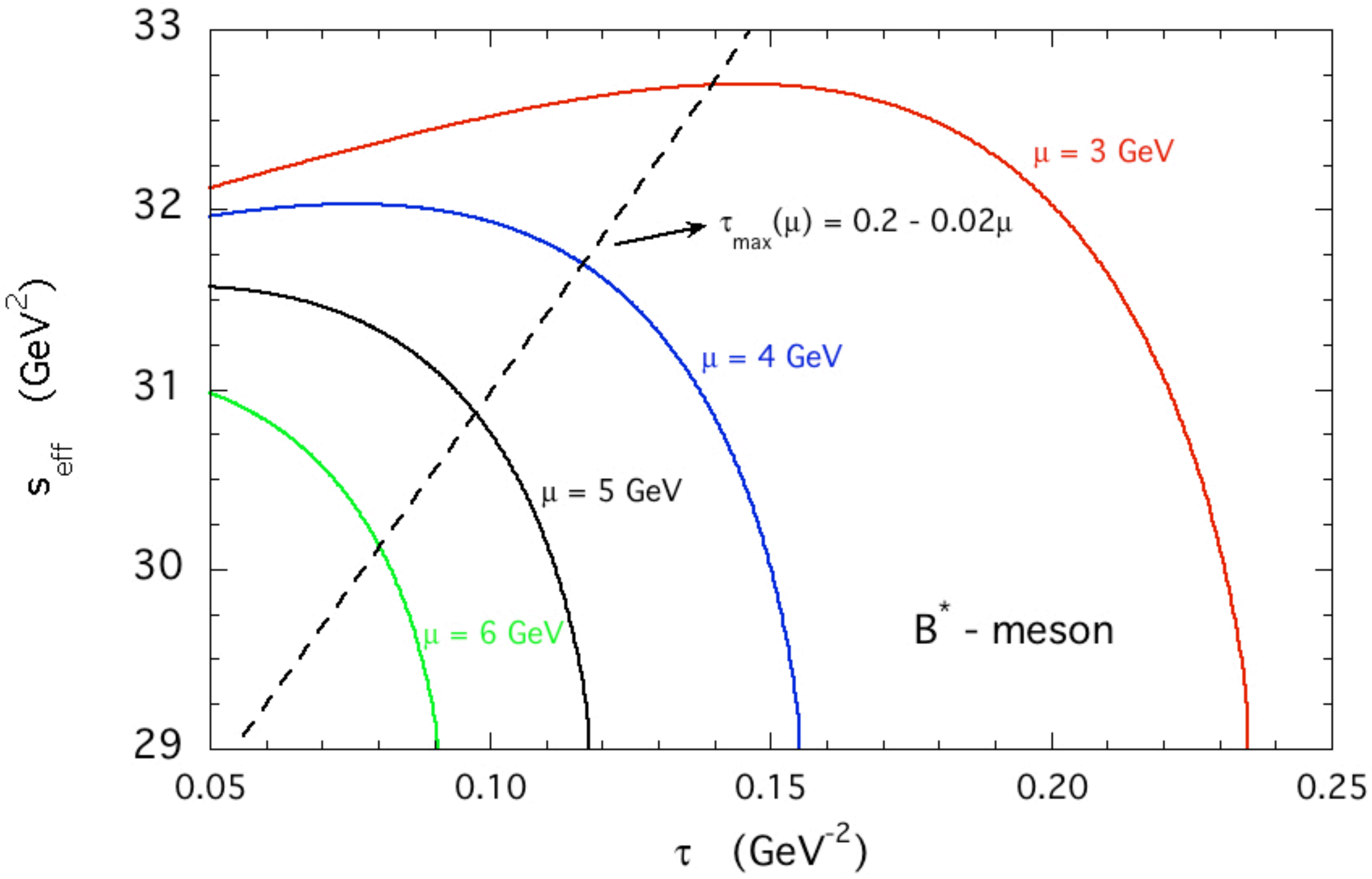
- treat τ as a (uniformly distributed) variable in the bootstrap analysis
- for each bootstrap event the value of s_{eff} is determined by minimizing $[(M_{dual} - M_H^{exp}) / \delta M_H^{exp}]^2$ with $\delta = 0.5\%$ ($\to \Delta M_H = 25$ MeV)





- **at large values of μ and τ the B*-meson mass is not properly reproduced (at variance with the B-meson case)**
- **the spin splitting $M_{B^*} - M_B$ is the key quantity to be reproduced and this requires a μ-dependent Borel window**

$$\tau_{\min} \leq \tau \leq \tau_{\max}(\mu)$$



- the value of $s_{eff}(\tau)$ obtained in this way cannot include the effect of the derivative of $s_{eff}(\tau)$ in the calculation of M_{dual}

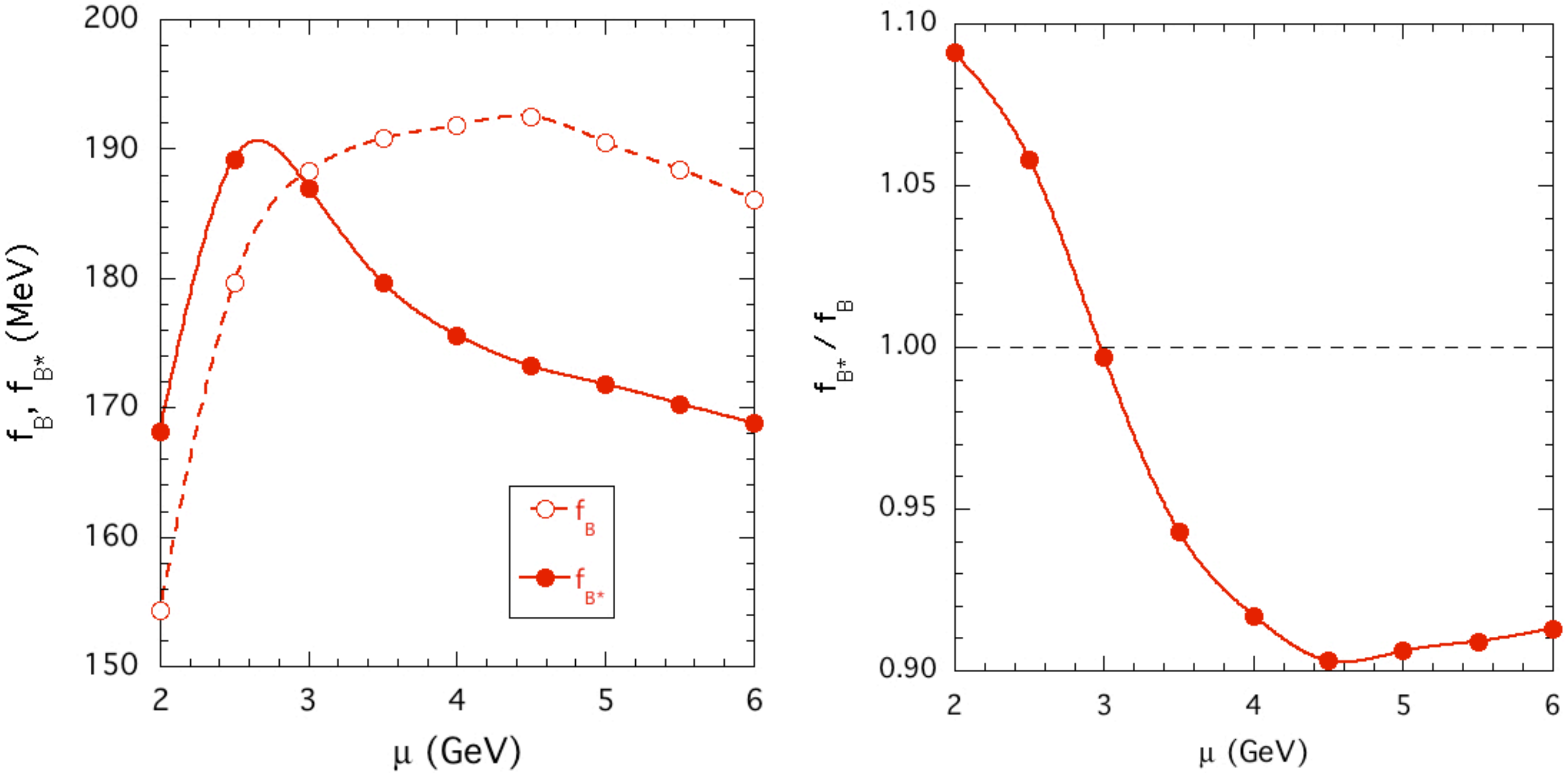
- in order to include the effect of the derivative of $s_{eff}(\tau)$ we adopt a polynomial Ansatz

$$s_{eff}(\tau) = \sum_{j=0}^{n} s_j^{(n)} \tau^j$$

- to work well a quadratic or cubic form for $s_{eff}(\tau)$ requires a μ-dependent Borel window

$$\tau_{\min} \leq \tau \leq \tau_{\max}(\mu)$$

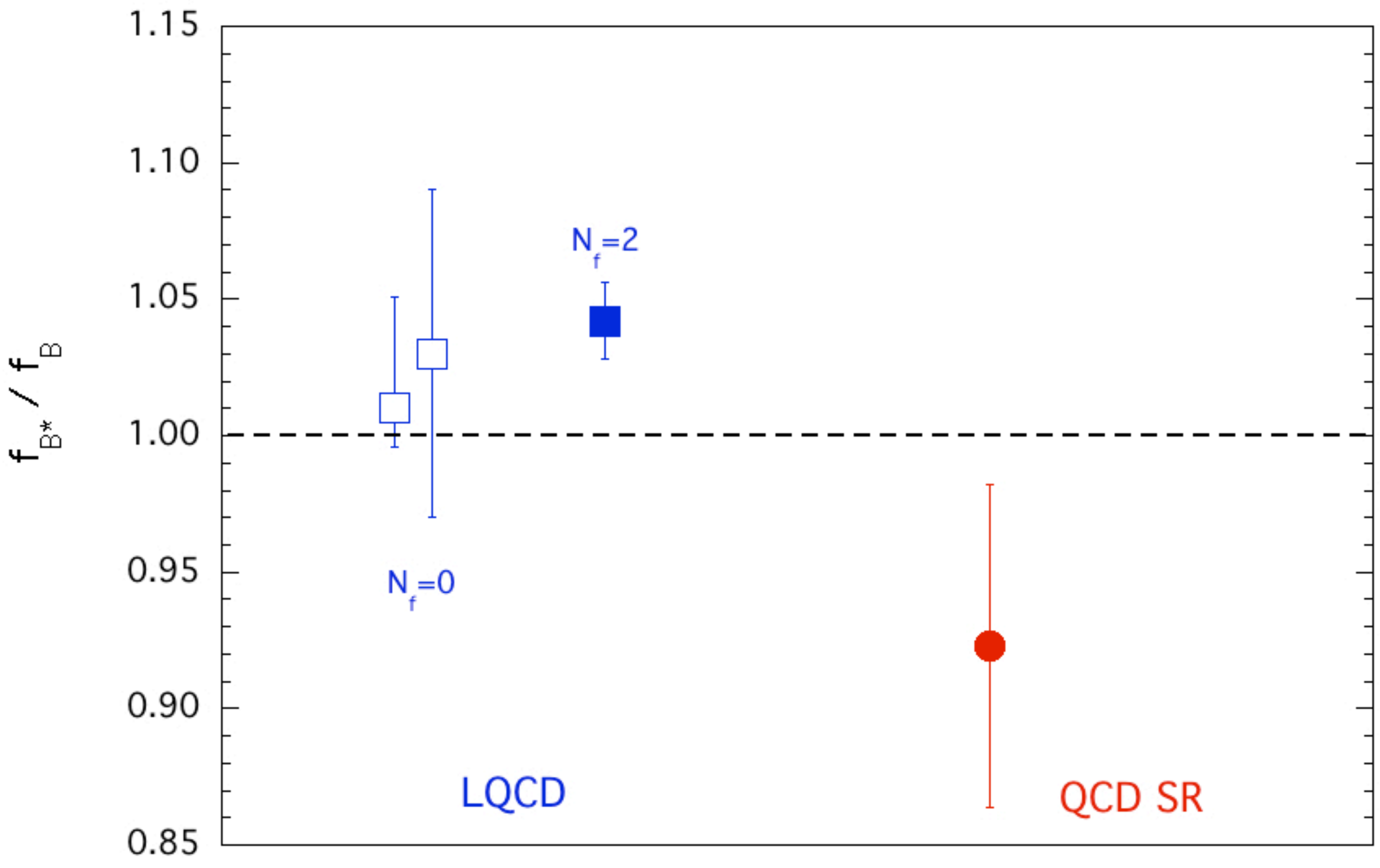
preliminary results



- **strong μ-dependence** around μ = 3 GeV

- for **μ > 3 GeV** the ratio is definitely below 1

- for **μ < 3 GeV** the ratio is above 1, but f_B is definitely lower than 190 MeV (FLAG averages) and the hierarchy between the perturbative orders deteriorates



LQCD

□ Bernard et al., PRD 65 (2001) (MILC)

Bowler et al., NPB 619 (2001) (UKQCD)

■ Becirevic et al., arXiv:1407.1019 (ETM)

QCD SR

● LMS, **preliminary**: $f_{B^*} = 175 \pm 12\ MeV$

$$\frac{f_{B^*}}{f_B} = 0.923 \pm 0.059$$

$\tau_{\min} \le \tau \le \tau_{\max}(\mu)$ and $3 \le \mu(GeV) \le 6$

$$M_{B^*} - M_B = 46.5 \pm 0.9\ MeV$$

$$\left[M_{B^*} - M_B\right]^{\mathrm{exp.}} = 45.78 \pm 0.35\ MeV$$

*** some (moderate) tension present between the predictions for the ratio f_{B^*} / f_B obtained from LQCD and Borel QCD-SR**

*** in the case of the B_s^*-meson we got the <u>preliminary</u> results:** $f_{B_s^*} = 207 \pm 16\ MeV$, $\dfrac{f_{B_s^*}}{f_{B_s}} = 0.932 \pm 0.047$

Conclusions

* Heavy-light meson leptonic decay constants are important hadronic quantities that can be calculated from Borel QCD Sum Rules for heavy-light currents both in the PS and V channels.

* During the past years important improvements in the quality of the QCD-SR predictions have been reached. Among them:
  - perturbative spectral densities known up to NNLO;
  - condensate contributions known up to dimension 6;
  - Borel-dependent effective threshold as a tool to lower excited state contaminations.

* The updated results for f_D, f_{Ds}, f_B and f_{Bs} have been compared with the predictions of Lattice QCD analyzed by **F**lavor **L**attice **A**veraging **G**roup. Note that:
  - during the past decade there has been a remarkable improvement in the quality of LQCD calculations of the decay constants of heavy-light mesons;
  - in the case of f_D and f_{Ds} LQCD has reached a precision competitive with the experimental errors.

* In the **charm sector**, both for the PS and V channels, there is full agreement between the predictions of LQCD and Borel QCD-SR, adopting in the latter the PDG value for the charm quark mass.

* In the **beauty sector** the decay constant f_B, extracted from the Borel QCD-SR using the PDG value of the b-quark mass, is ~ 10% higher than the FLAG averages (f_B^{latt} ~ 190 MeV). Correspondingly, using the lattice average as input for f_B, the b-quark mass obtained from the Borel QCD sum rule turn out to be ~ 2% higher than the PDG value (~ 1.5 standard deviations).

* In the **vector channel** the reproduction of the B*-meson mass is problematic in some parts of the parameter space and a μ-dependent Borel window has to be considered for a reliable extraction, in which the experimental spin splitting (M_{B^*} - M_B) is reproduced. While current LQCD predictions for the ratio f_{B^*} / f_B suggest a value slightly larger than 1, the Borel QCD-SR is remarkably sensitive to the value of the subtraction point μ and favors values of the ratio less than 1 in the range $3 < \mu$ (GeV) < 6.

* The presence of the above tensions in the beauty sector and their absence in the charm one are open issues to be further investigated.

backup slides

FLAG-2 members

The current list of FLAG members and their Working Group assignments is:

- Advisory Board (AB): S. Aoki, C. Bernard, C. Sachrajda
- Editorial Board (EB): G. Colangelo, H. Leutwyler, A. Vladikas, U. Wenger
- Working Groups (WG)
  (each WG coordinator is listed first):

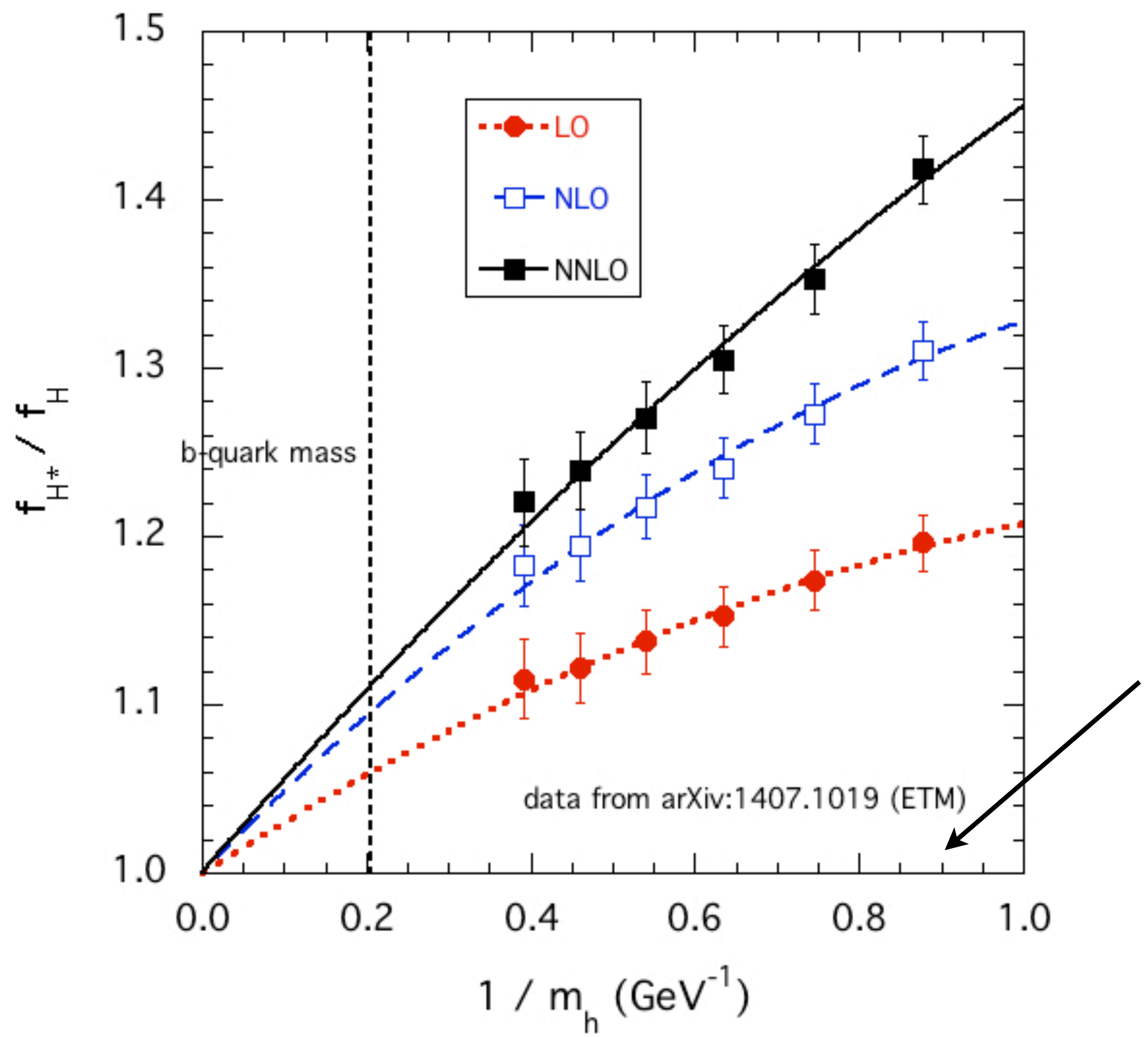
– Quark masses	L. Lellouch, T. Blum, V. Lubicz
– V_{us}, V_{ud}	A. Jüttner, T. Kaneko, S. Simula
– LEC	S. Dürr, H. Fukaya, S. Necco
– B_K	H. Wittig, J. Laiho, S. Sharpe
– $f_{B_{(s)}}$, $f_{D_{(s)}}$, B_B	A. El-Khadra, Y. Aoki, M. Della Morte
– $B_{(s)}$, D semileptonic and radiative decays	R. Van de Water, E. Lunghi, C. Pena, J. Shigemitsu[3]
– α_s	R. Sommer, R. Horsley, T. Onogi

[3] J. Shigemitsu has withdrawn from FLAG, immediately after completion of the first version of the present paper (arXiv:1310.8555 [hep-lat]), of which she is a co-author. She is listed here in recognition of her full involvement in the review of B(s) and D semileptonic and radiative decays, as well as for her valuable contribution of the whole FLAG effort.

$$\frac{f_{H^*}}{f_H} = \left(1+\frac{\alpha_1}{m_h}+\frac{\alpha_2}{m_h^2}+\dots\right)_{HQET} C_{matching}\left(\mu=m_h\right)$$

$$C_{matching}(\mu) = 1 - \frac{2}{3}\frac{\alpha_s(\mu)}{\pi} - \left[-\frac{1}{9}\zeta_3+\frac{2}{27}\pi^2\log 2+\frac{4}{81}\pi^2+\frac{145-6n_f}{36}\right]\left(\frac{\alpha_s(\mu)}{\pi}\right)^2 + O\left(\alpha_s^3\right)$$

[Broadhurst&Grozin '95, Campanario et al. '03]



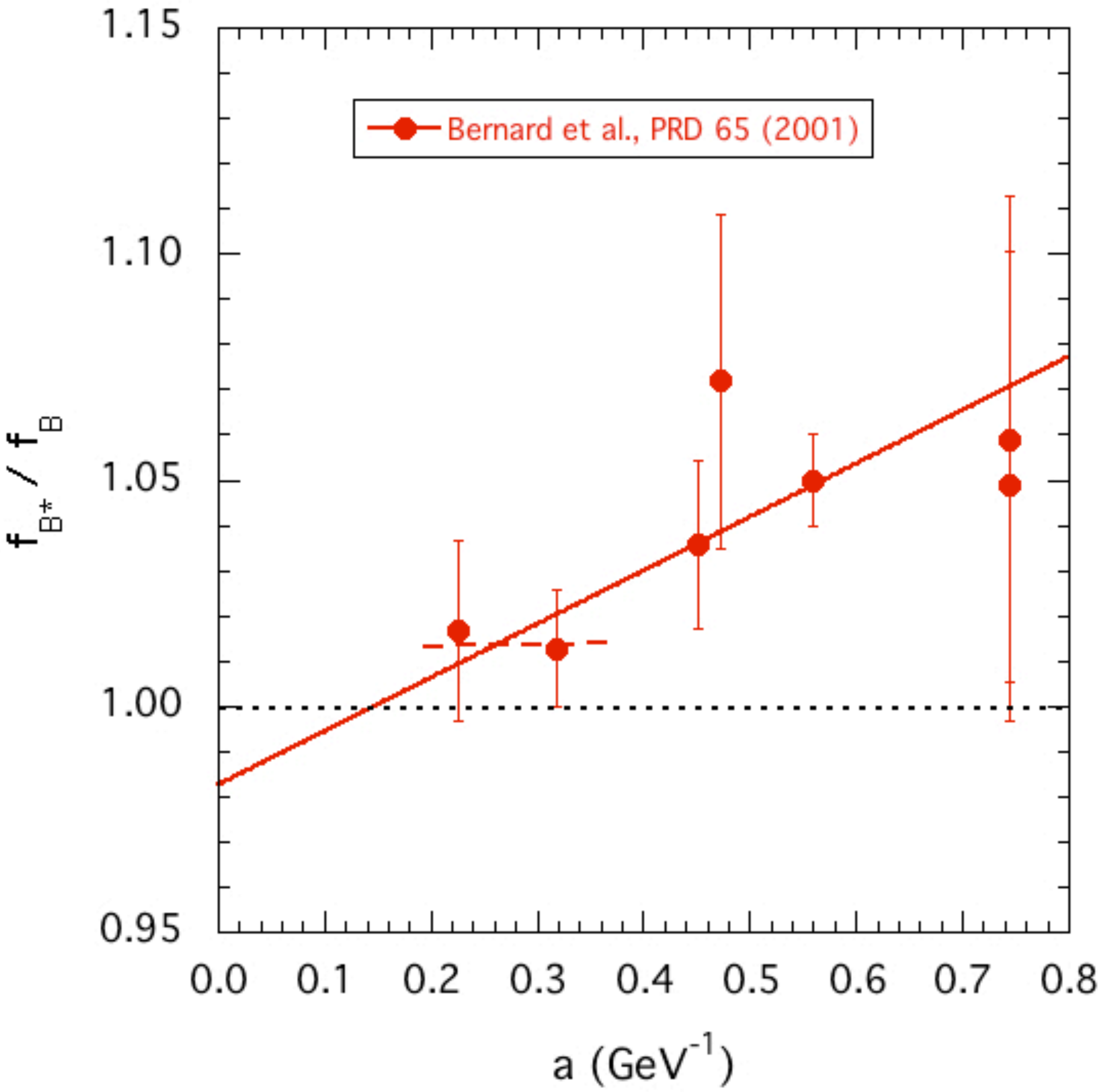
at μ ~ m_b: NLO ~ -4% and NNLO -2%

at μ ~ m_c: NLO ~ -10% and NNLO -8%

around m_c

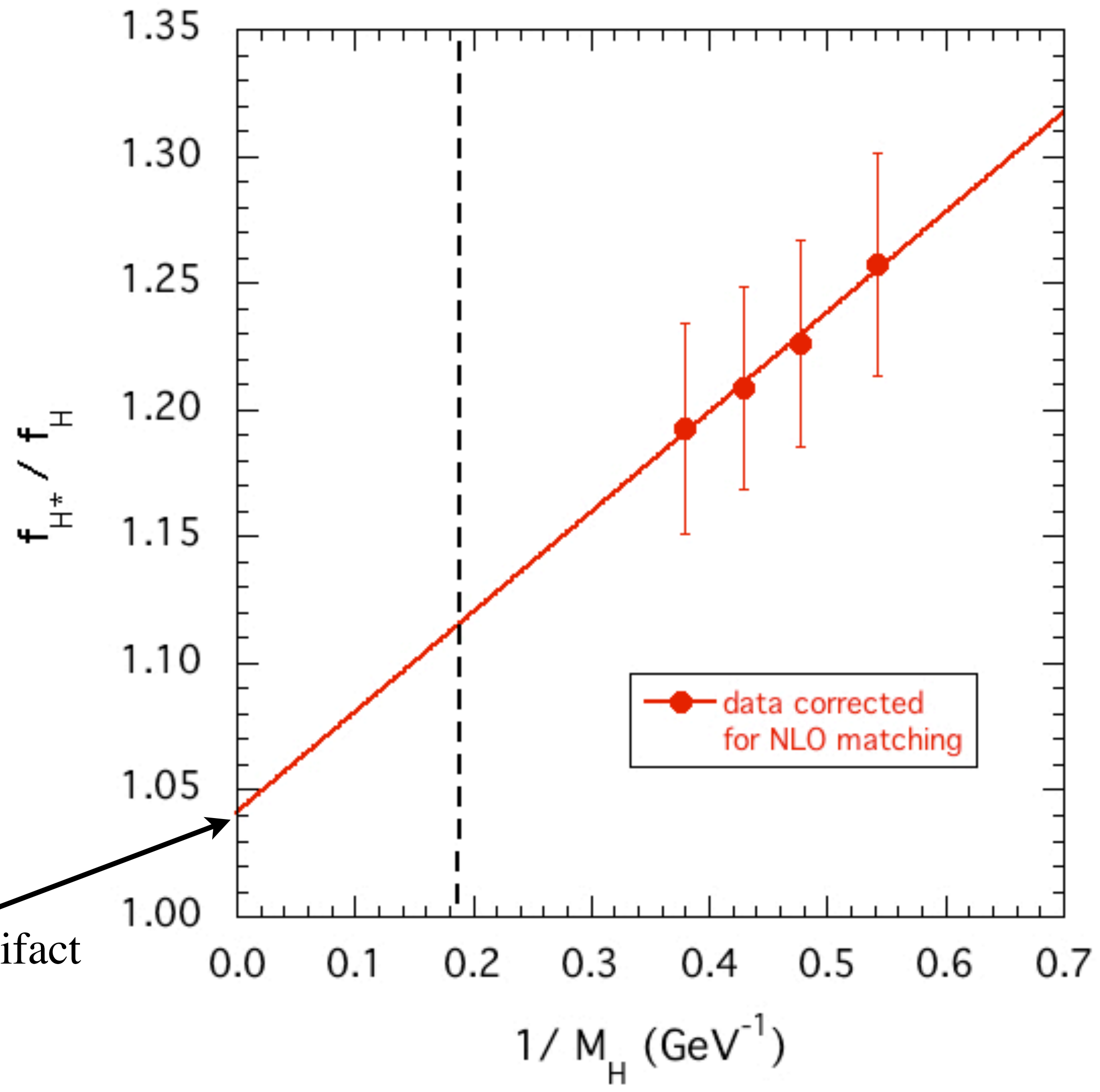
* quenched calculations with unimproved Wilson valence quarks [Bernard et al., PRD 65 (2001)]

$$\frac{f_{B^*}}{f_B} = 1.01 \pm 0.01^{+0.04}_{-0.01}$$



* quenched calculations with Clover fermions at a single lattice spacing (a ~ 0.07 fm) [Becirevic et al., PRD 60 (1999)]

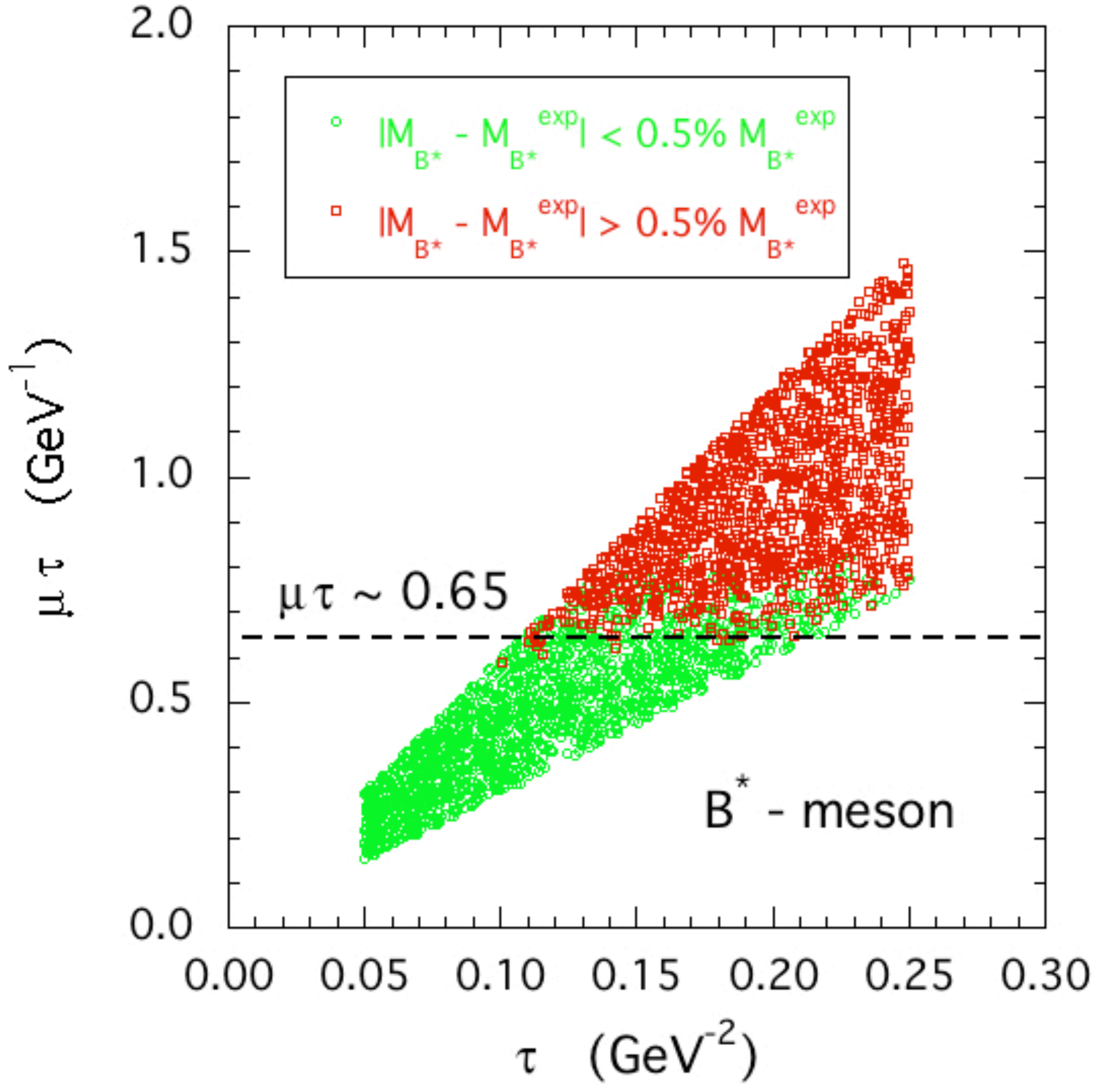
$$\frac{f_{B^*}}{f_B} = 1.07 \pm 0.05$$ (no matching between full QCD and HQET)



lattice artifact

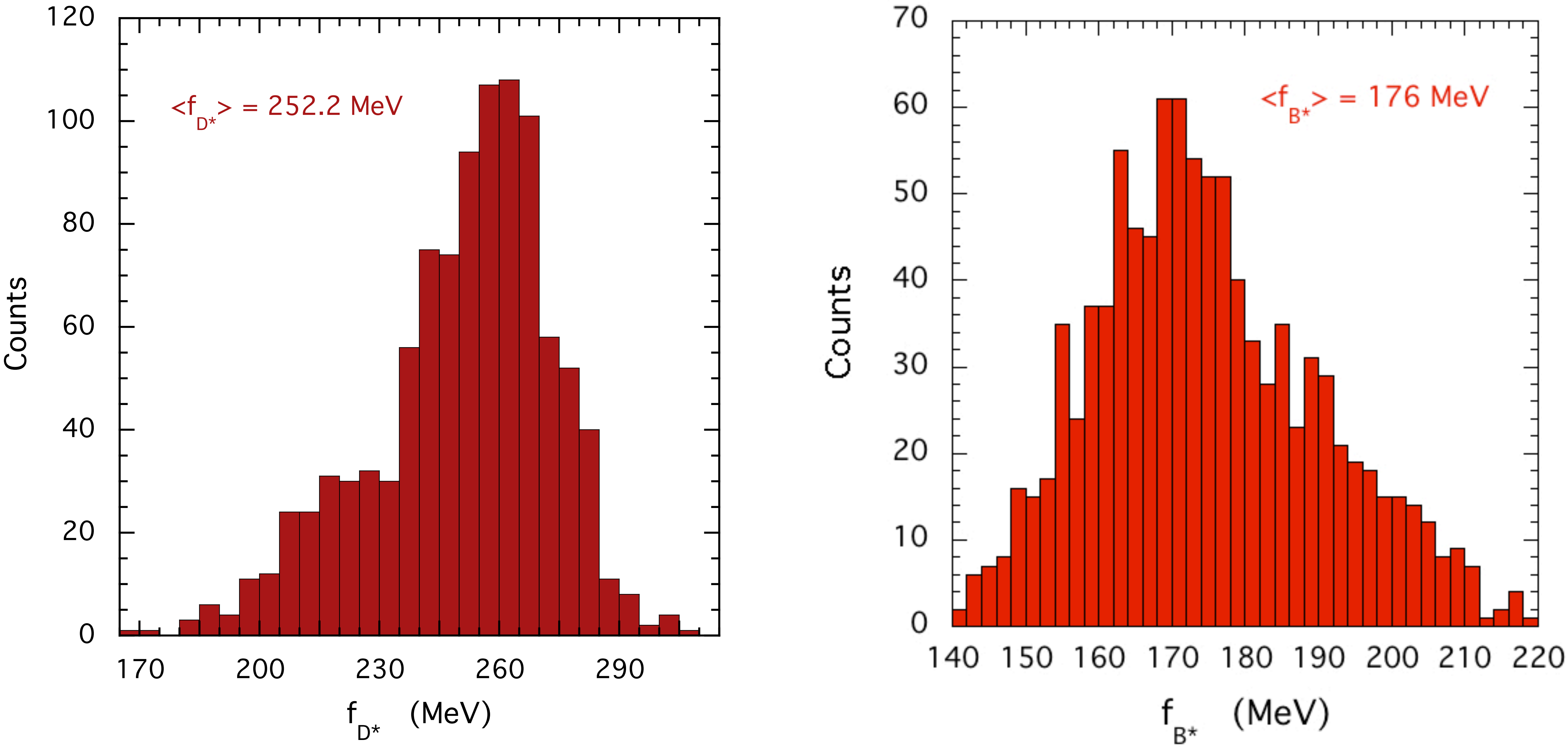
* taking into account a NLO matching between full QCD and HQET:

$$\frac{f_{B^*}}{f_B} = 1.03 \pm 0.05$$



* at large values of $\mu\tau$ ($\mu\tau > 0.6$ GeV^{-1}) the B*-meson mass is not properly reproduced (at variance with the B-meson case)

*** distributions of the bootstrap analyses for f_{D^*} and f_{B^*}**



*** distribution of the bootstrap analysis for f_{B*} / f_B**