$|V_{cb}|$ from $B \to D^{(*)}\ell\nu$ on the lattice

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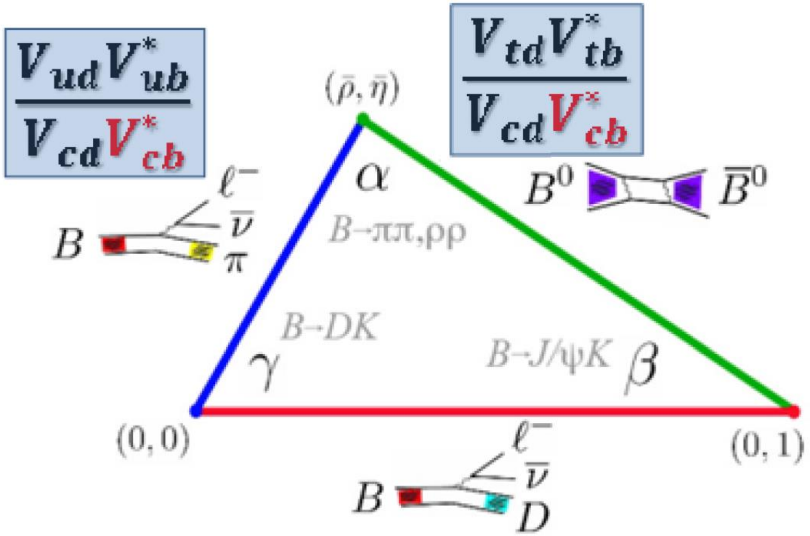
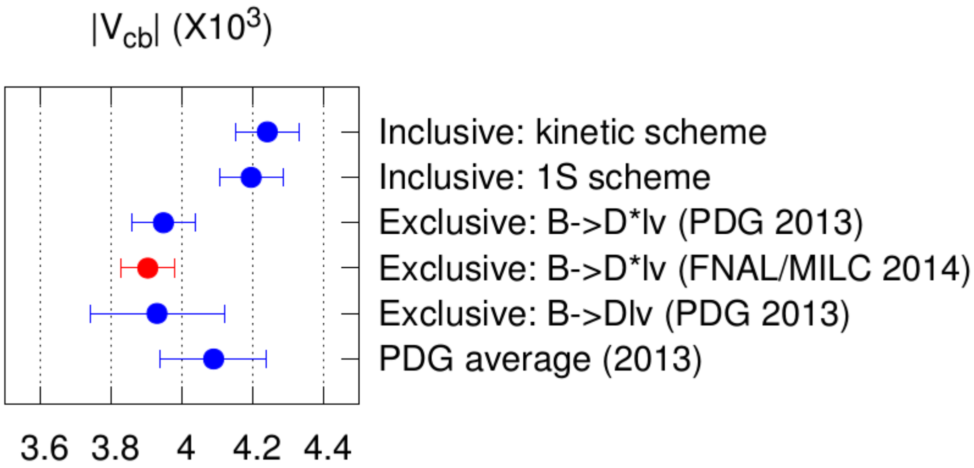
Charmed B semileptonic decays

- CKM matrix element $|V_{cb}|$
  - Supposed to be free from NP effect (dominated by W-exchange)
  - Precision calculation of $|V_{cb}|$ is important to CKM sector of SM. Normalization of the sides of UT triangle.
  - $|V_{ub}/V_{cb}|$ is of high priority in heavy flavor physics.



- Semileptonic decays:
  - The theory is well studied. HQ expansion Λ/m_b works well.
  - Lots of experimental data are available at B factories (BaBar, Belle,…) . Full-reconstruction of B meson from $\Upsilon(4S)$ decays. High-precision measurements enable analysis of shapes of q^2-dependence.
  - Able to probe possible NP contributions lifted by τ mass in processes like $B \to D^{(*)}\tau\nu$.

The determinations of $|V_{cb}|$



- The **inclusive** methods:
  Total decay rate + HQE parameters
  (1S scheme, kinetic scheme)
  Precision : ~2%

- The **exclusive** methods:
  - $B \to D^* \ell \nu$:
    Theory $\mathcal{F}(1)$ + experimental $\mathcal{F}(1)|V_{cb}|$
    Precision: ~2%
    (similar errors from lattice and exp.)
  - $B \to D \ell \nu$:
    Theory $\mathcal{G}(w)$ + experimental $\mathcal{G}(w)|V_{cb}|$
    Precision: ~5% (dominated by experimental error)

- Discrepancy between the exclusive (lattice) and inclusive $|V_{cb}|$ is more than 2σ. A long standing puzzle.

Exclusive methods: $B \to D^{(*)}\ell\nu$ decays

$$\frac{d\Gamma}{dw}(B \to D\ell\nu) = \frac{G_F^2 m_B^5}{48\pi^3}|V_{cb}|^2(1+r)^2 r^3 (w^2-1)^{3/2}[\eta_{\rm ew}\mathcal{G}(w)]^2$$

$w = v \cdot v'$

- Noisy experimental data near zero recoil due to suppression at w=1. Some model dependence in extrapolation.
- Require non-zero recoil lattice calculation even for zero recoil $G(1)$. Combined fit of lattice and experiments.

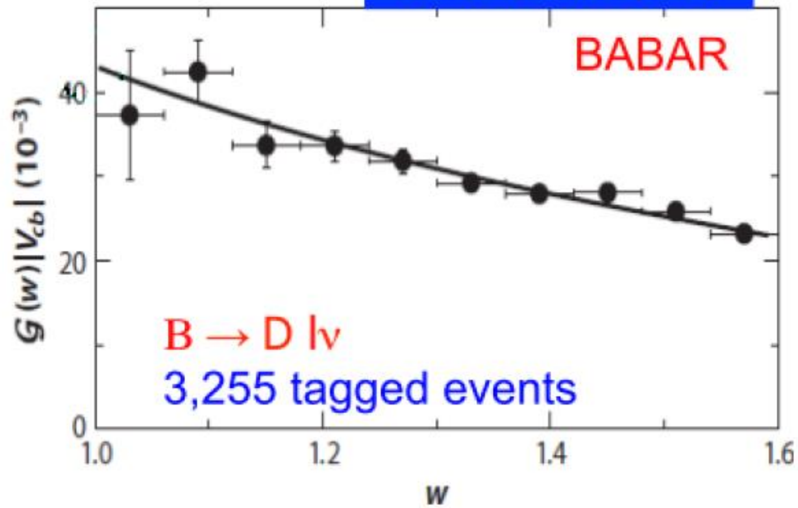
$$\frac{d\Gamma}{dw}(B \to D^*\ell\nu) = \frac{G_F^2 m_B^5}{48\pi^3}|V_{cb}|^2 (w^2-1)^{1/2} P(w)[\eta_{\rm ew}\mathcal{F}(w)]^2$$

- Lattice calculation at zero recoil benefits from

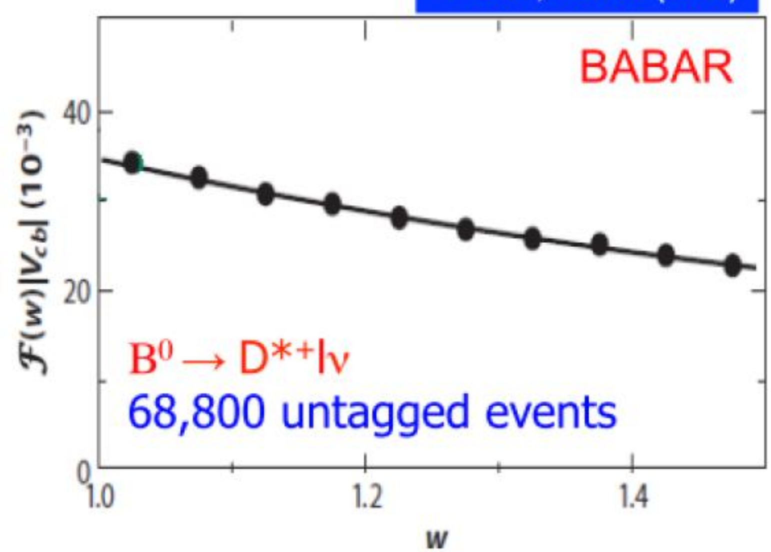
Luke theorem ($\propto 1/m_Q^2$). Experimental extrapolation to $\mathcal{F}(1)|V_{ub}|$ is relatively well under control.

- Non-zero recoil lattice calculation is also desired. A interesting test for consistency of form factor shape.

PRL 104, 011802 (2010)



PRD 77, 032002 (2008)



Lattice QCD calculations and errors

- Gauge fields and sea quarks.
  - Number of gauge configurations → statistical errors.
  - Implementing dynamical sea quarks (unquenched) is expensive, cost ∝ $1/m_\ell^{1\sim 2}$

    Use unphysical heavier masses → chiral extrapolation error
  - Small lattice spacings are expensive, cost ∝ $1/a^{4\sim 6}$ → discretization error
  - Big volumes are expensive, cost ∝ L^5 → finite volume effect

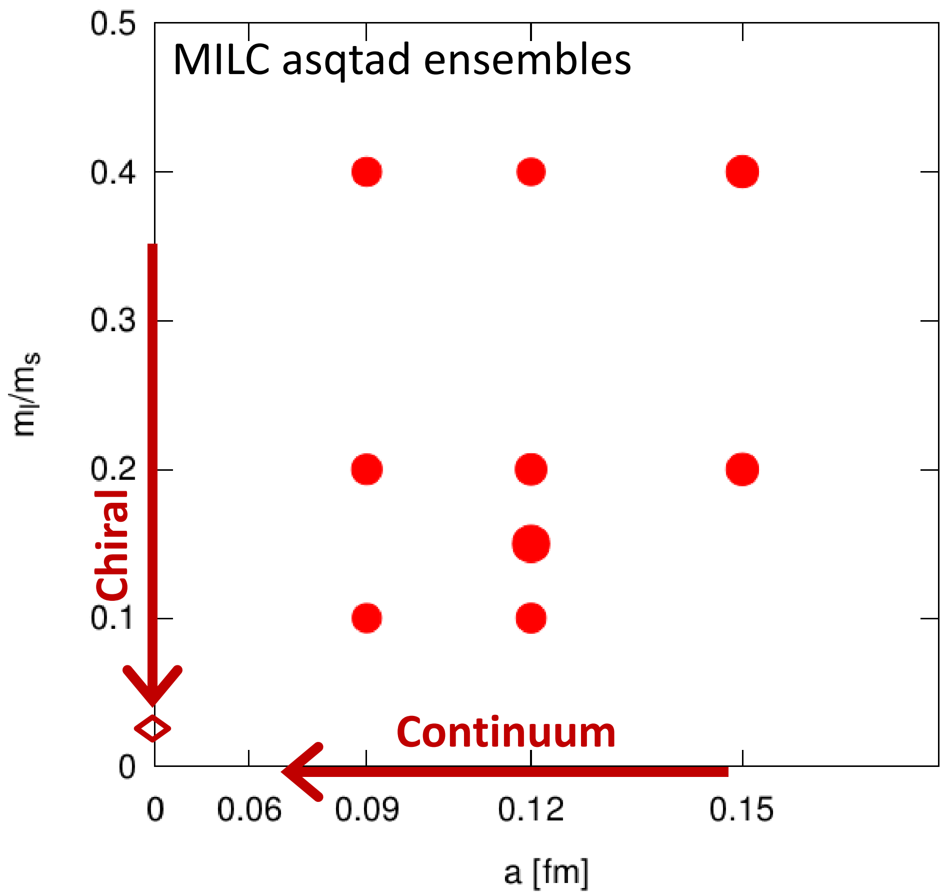
- Valence quarks and currents
  - Light quarks have the same difficulty as the sea quarks
  - Matching to the continuum currents → matching error (renormalization)
  - Input parameters: $g_{DD^*\pi}, ..$ → systematic errors from inputs
  - Heavy quarks: $am_c \lesssim 1, am_b \gtrsim 1$ → heavy quark discretization error

    Fermilab method, NRQCD: error is suppressed $1/m_Q$

    Heavier-than-charm staggered?

Lattice calculations of $B \to D^* \ell \nu$

- 2001, quenched calculation, Hashimoto (FNAL) PRD66(014503)
- 2008, quenched calculation at non-zero recoil, de Divitis et al, NPB807(373)
- 2008, **unquenched** (2+1 sea quarks), Laiho (FNAL/MILC), PRD79(014506)



Circle size is proportional to number of configurations.

Red: data used in FNAL/MILC2008

"asqtad": a^2 and tadpole-improved

Lattice calculations of $B \to D^* \ell \nu$

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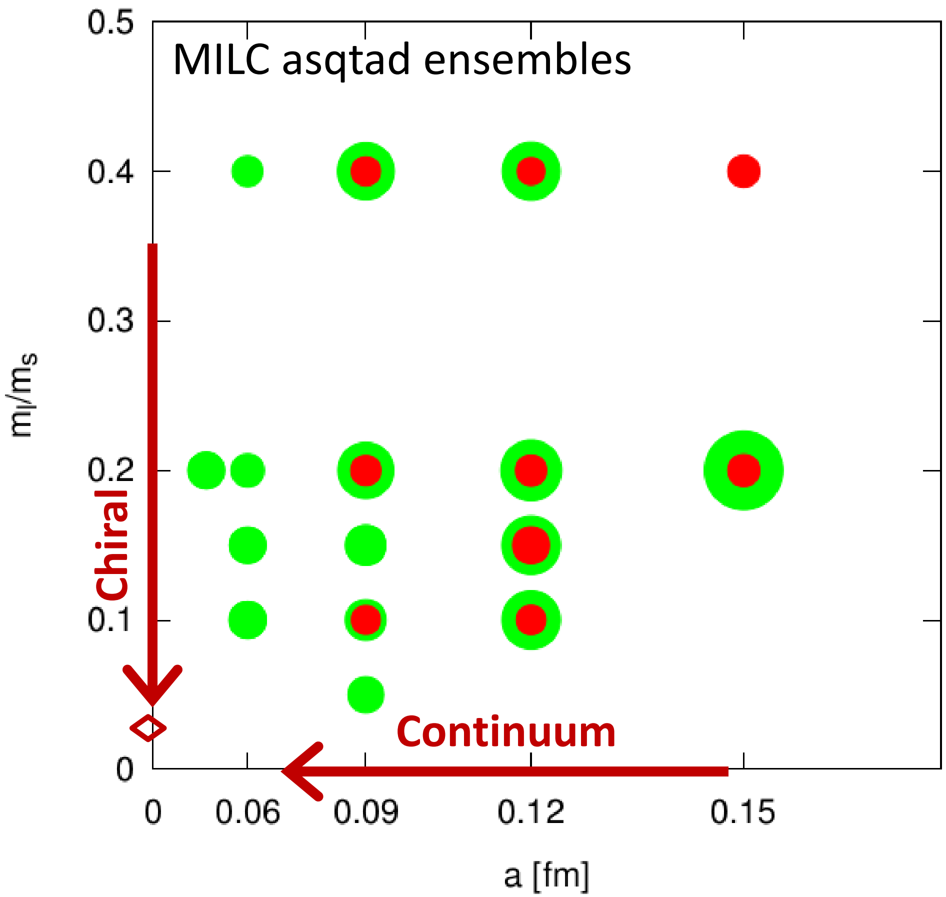
- 2008, quenched calculation at non-zero recoil, de Divitis et al, NPB807(373)
- 2008, **unquenched** (2+1 sea quarks), Laiho (FNAL/MILC), PRD79(014506)
- 2014, update, Laiho (FNAL/MILC) PRD89(114504)



Circle size is proportional to number of configurations.

Red: data used in FNAL/MILC2008
Green: data used in FNAL/MILC2014

Lattice calculations of $B \to D^*\ell\nu$ (FNAL/MILC)

- In the zero recoil limit, only one form factor matters

$$\mathcal{F}(1) = h_{A_1}(1)$$

- Use Fermilab action for heavy b and c quarks.

- Use a double ratio PRD66(014503) that removes all wave-function normalization and the majority of current renormalization

$$\mathcal{R}_{A_1} = \frac{\langle D^*|\bar{c}\gamma^j\gamma^5 b|\bar{B}\rangle\langle\bar{B}|\bar{b}\gamma^j\gamma^5 c|D^*\rangle}{\langle D^*|\bar{c}\gamma^4 c|D^*\rangle\langle\bar{B}|\bar{b}\gamma^4 b|\bar{B}\rangle} = |h_{A_1}(1)|^2$$

- Current renormalization

$$\mathcal{J}_{cb}^{\rm con} = Z_{J^\mu_{cb}} J^{\rm lat}$$

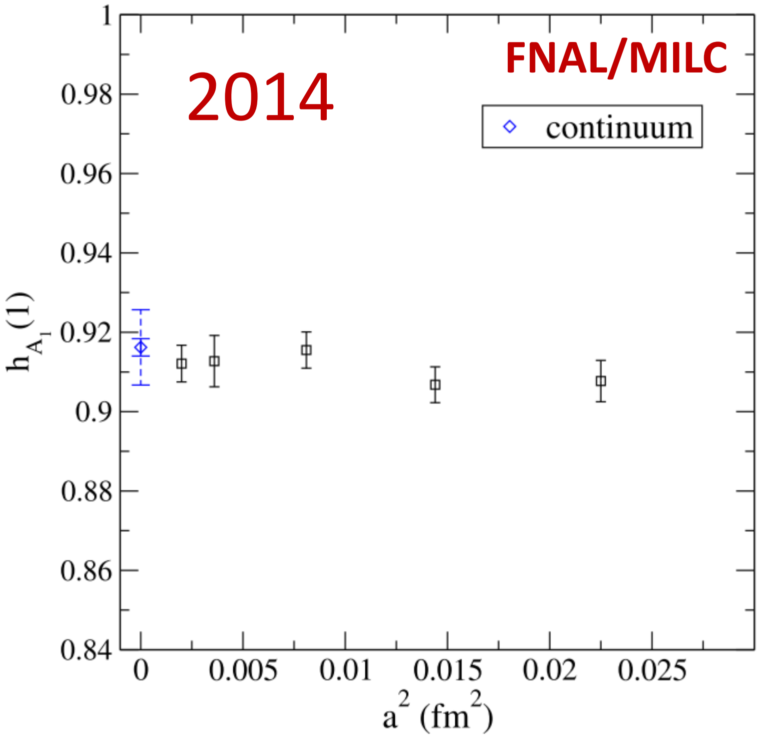
Majority of Z cancels in the ratio;
Small remaining factor is calculated
Using lattice perturbation theory.

Define

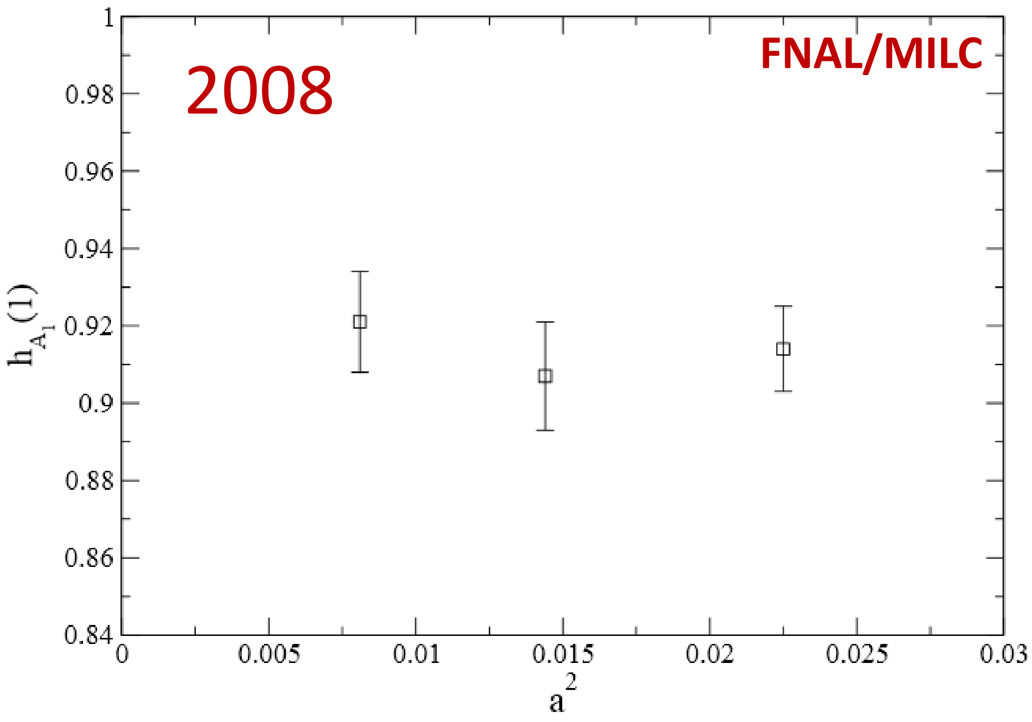
$$\rho_{A^i}^2 = \frac{Z_{A^i_{cb}} Z_{A^i_{bc}}}{Z_{V^4_{cc}} Z_{V^4_{bb}}} \to \left[1 + \sum_\ell \rho_{A^j}^{[\ell]} \alpha_V^\ell(q^*)\right]^2$$

The discretization effect

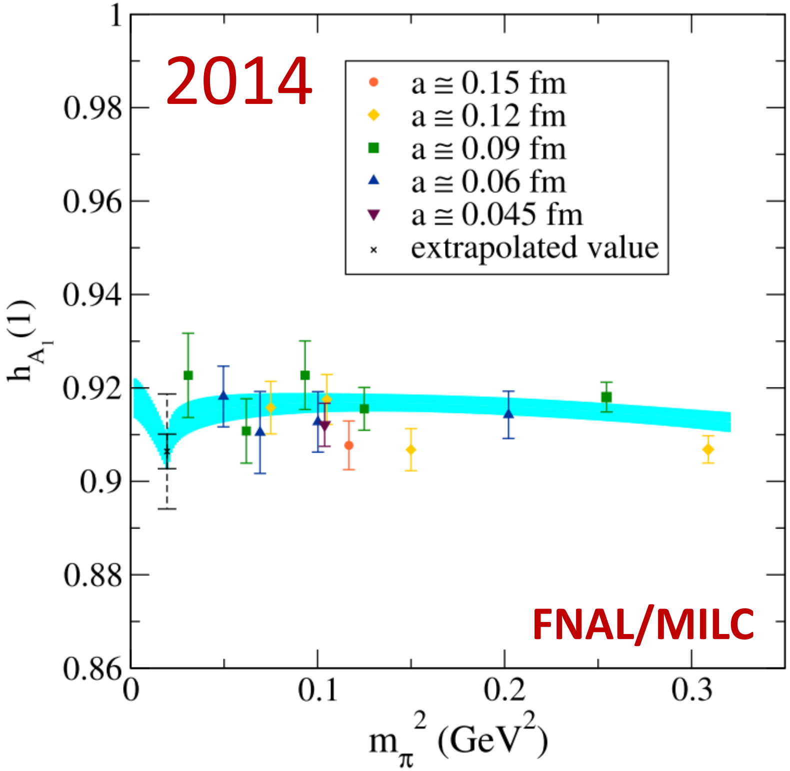
Data with $m_l = 0.2 m_s$

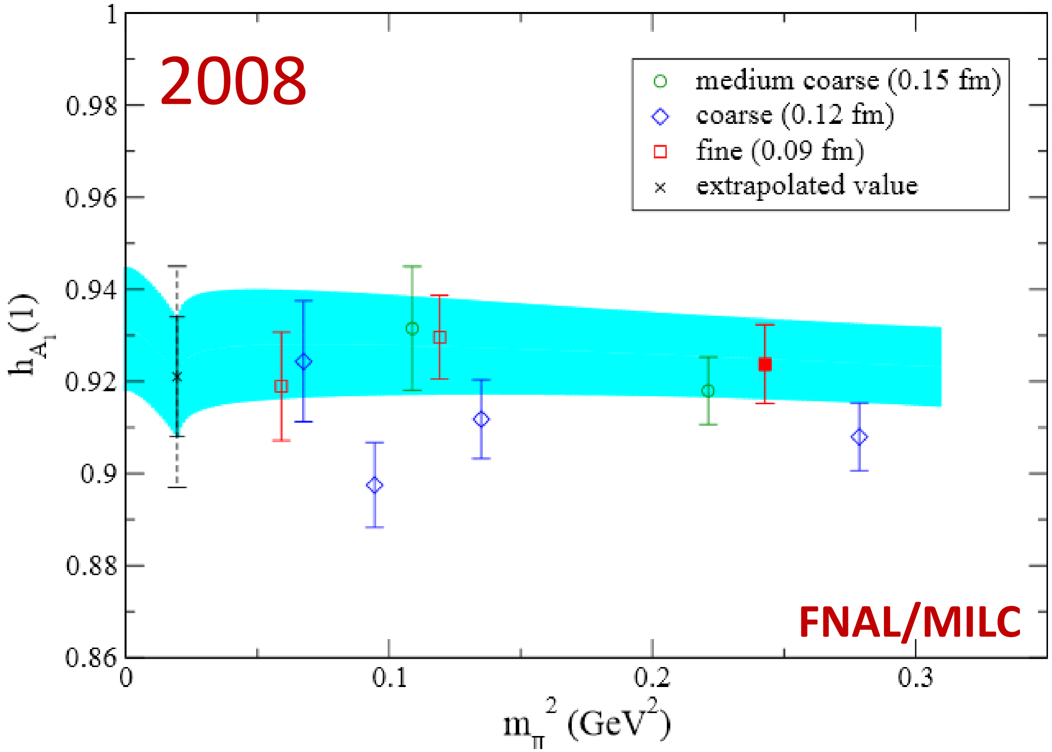


Data with $m_l = 0.2 m_s$



Chiral/continuum extrapolation





Error budget

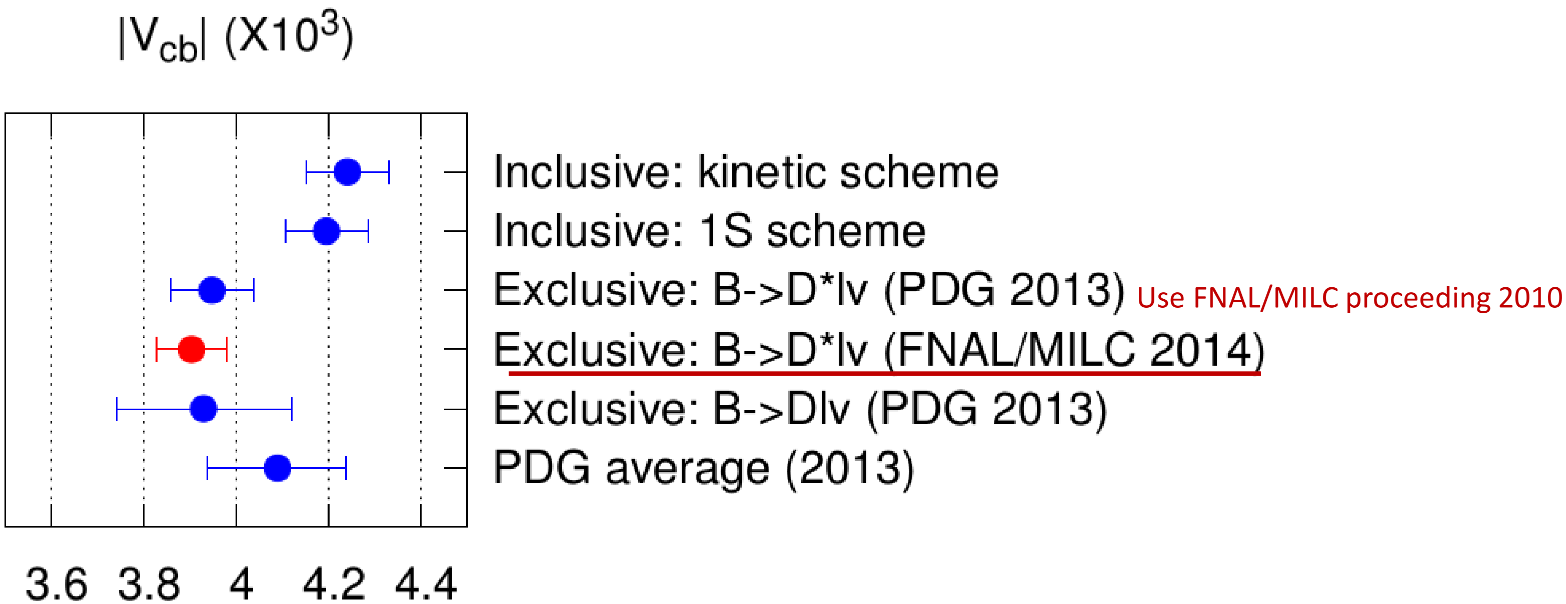
FNAL/MILC

Error of $h_{A_1}(1)$ (%)	2008	2014
Statistics	1.4	0.4
χPT fits	0.9	0.5
$g_{DD^*\pi}$	0.9	0.3
HQ Discretization errors	1.5	1.0
HQ mass tuning	0.7	-
Perturbation theory	0.3	0.4
Isospin	-	0.1
u_0 tuning	0.4	-
Scale r_1	-	0.1
Total	2.6	1.4

- Significant improvement in the statistical uncertainty.
- HQ discretization error is now the dominant source of error.
- The coupling $g_{DD^*\pi}$ used is 0.53(8) based on PLB721(94), PLB719(103)
- HQ mass tuning is included in “Statistics”.
- No “u_0 tuning” error in the newer data.

Update of $|V_{cb}|$ (FNAL/MILC)

- Summary:
  - Full data set analysis is published PRD89(114504)
  - Result: $|V_{cb}| = \left(39.04 \pm 0.49_{expt} \pm 0.53_{QCD} \pm 0.19_{QED}\right) \times 10^{-3}$
  - The discrepancy between inclusive and exclusive methods remains.



Lattice calculations of $B \to D\ell\nu$

- 1999, quenched, zero recoil, Hashimoto (FNAL), PRD61(014502)
- 2005, unquenched, zero recoil, Okamoto (FNAL/MILC) hep-lat/0409116

  2+1 asqtad (old) + Fermilab b: $\mathcal{G}(1) = 1.074(18)(16)$
- 2012, unquenched, non-zero recoil, Qiu (FNAL/MILC) 1312.0155

  2+1 asqtad(new) + Fermilab b:

  Fit using full w-range with experiments to determine $|V_{cb}|$
- 2012, unquenched, non-zero recoil, Na (HPQCD) Lattice 2014

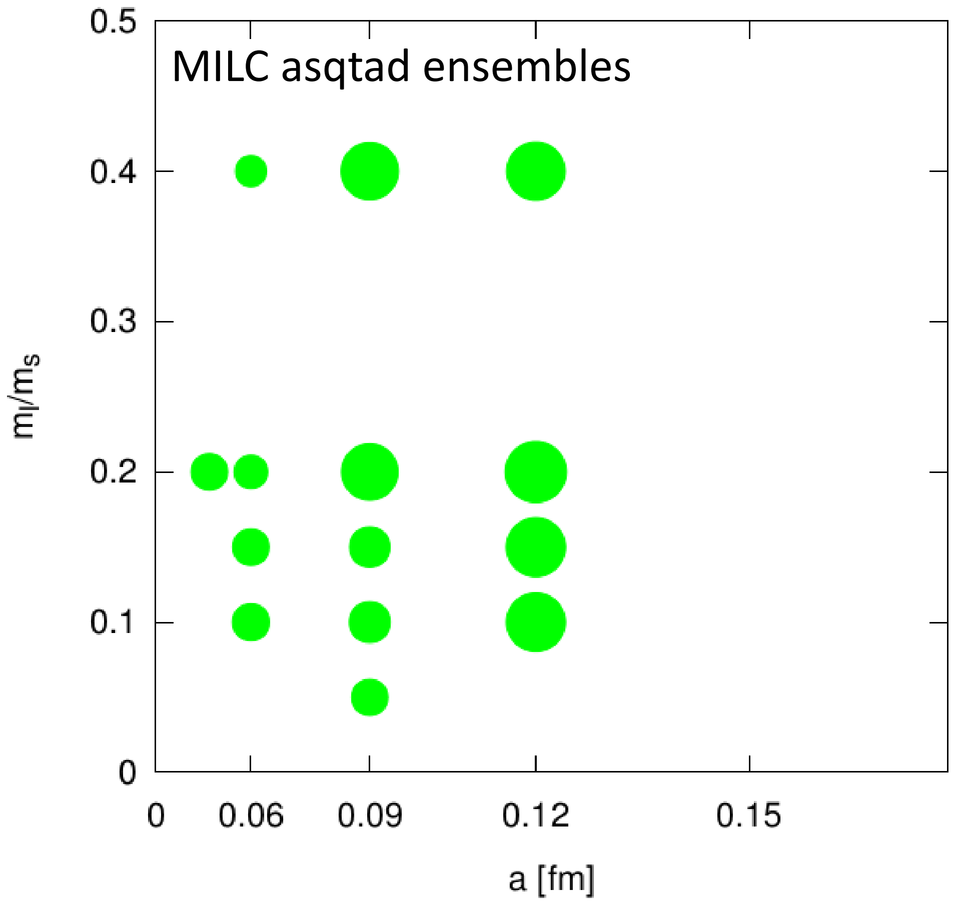
  2+1 asqtad + hisq light valence and charm + NRQCD b:

  Ratio $f_0(B_s \to D_s\ell\nu)/f_0(B \to D\ell\nu)$ and $|V_{cb}|$
- 2013, unquenched, near zero recoil, Atoui et al, 1311.5071

  2+0 twisted mass Wilson, $B_{(s)} \to D_{(s)}\ell\nu$

Lattice calculations of $B \to D\ell\nu$ (FNAL/MILC)

- Use a similar set of lattice configuration as the FNAL/MILC $B \to D^{*}\ell\nu$
- Use Fermilab bottom and charm quarks; vector current
- Four non-zero momenta for the daughter D meson.



Moving to non-zero recoil

- The form factor

$$\mathcal{G}(w) = h_+(w) + \frac{1-r}{1+r} h_-(w)$$

$h_-(1) \to 0$, in HQ limit, but not in general
$h_-(1)$ has to be extrapolated from $h_-(w)$

- Double ratio for $h_+(1)$

$$\mathcal{R}_+ = \rho^2_{V^4}(1) \frac{\langle D|\bar{c}\gamma^4|\bar{B}\rangle\langle\bar{B}|\bar{b}\gamma^4 c|D\rangle}{\langle D|\bar{c}\gamma^4 c|D\rangle\langle\bar{B}|\bar{b}\gamma^4 b|\bar{B}\rangle} = |h_+(1)|^2$$

- Single ratios for $h_+(w)$ and $h_-(w)$

$$h_+(w(\boldsymbol{p})) = \sqrt{\mathcal{R}_+}\mathcal{Q}_+(\boldsymbol{p})\left[1 - \boldsymbol{\mathcal{R}}_-(\boldsymbol{p}) \cdot \boldsymbol{x}_f(\boldsymbol{p})\right],$$

$$h_-(w(\boldsymbol{p})) = \sqrt{\mathcal{R}_+}\mathcal{Q}_+(\boldsymbol{p})\left[1 - \frac{\boldsymbol{\mathcal{R}}_-(\boldsymbol{p}) \cdot \boldsymbol{x}_f(\boldsymbol{p})}{\boldsymbol{x}_f^2(\boldsymbol{p})}\right],$$

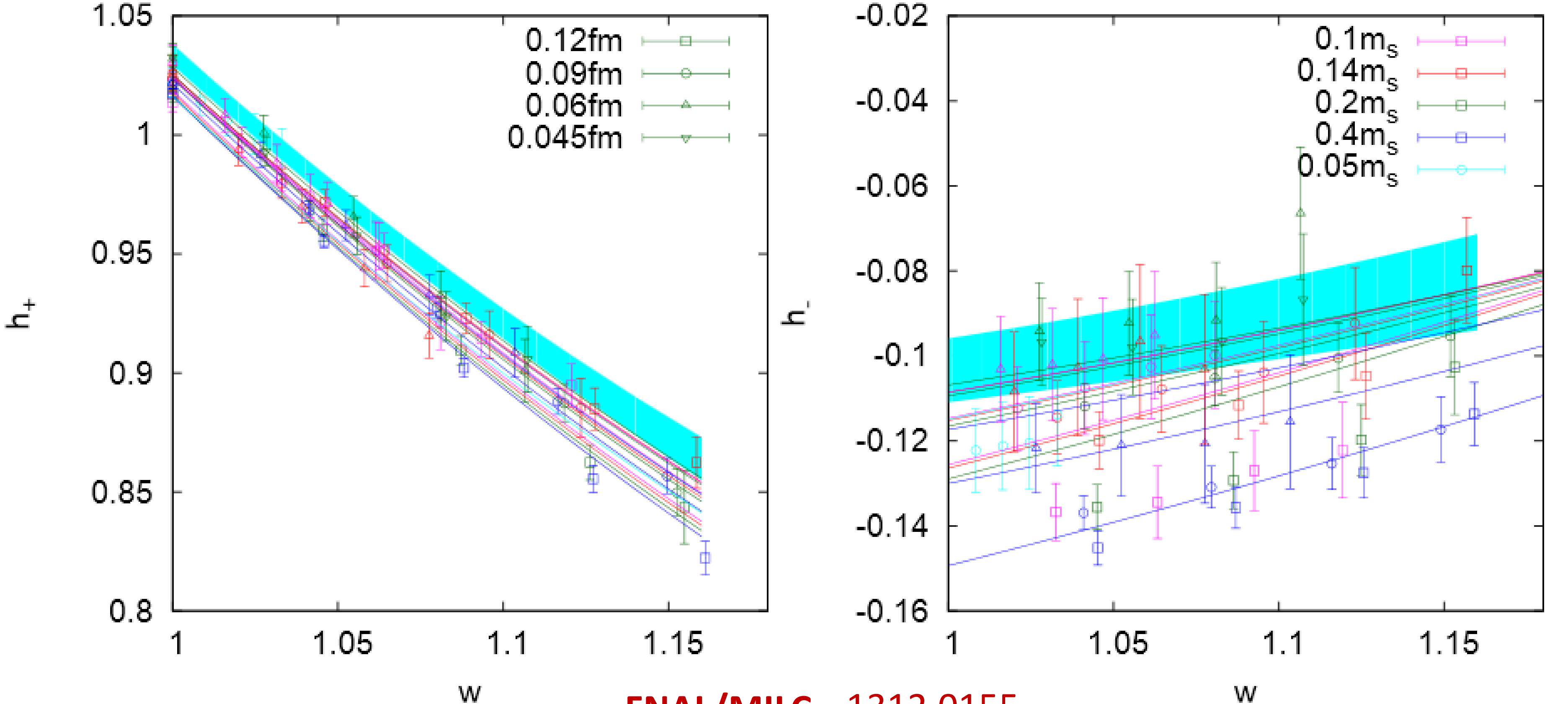
Renormalized Q_+, R_-, x_f

$$Q_+(\boldsymbol{p}) \equiv \frac{\langle D(\boldsymbol{p})|V^4|B(\boldsymbol{0})\rangle}{\langle D(\boldsymbol{0})|V^4|B(\boldsymbol{0})\rangle}$$

$$x_f^i(\boldsymbol{p}) \equiv \frac{\langle D(\boldsymbol{p})|V^i|D(\boldsymbol{0})\rangle}{\langle D(\boldsymbol{p})|V^4|D(\boldsymbol{0})\rangle}$$

$$R_-^i(\boldsymbol{p}) \equiv \frac{\langle D(\boldsymbol{p})|V^i|B(\boldsymbol{0})\rangle}{\langle D(\boldsymbol{p})|V^4|B(\boldsymbol{0})\rangle}$$

Chiral/continuum extrapolation



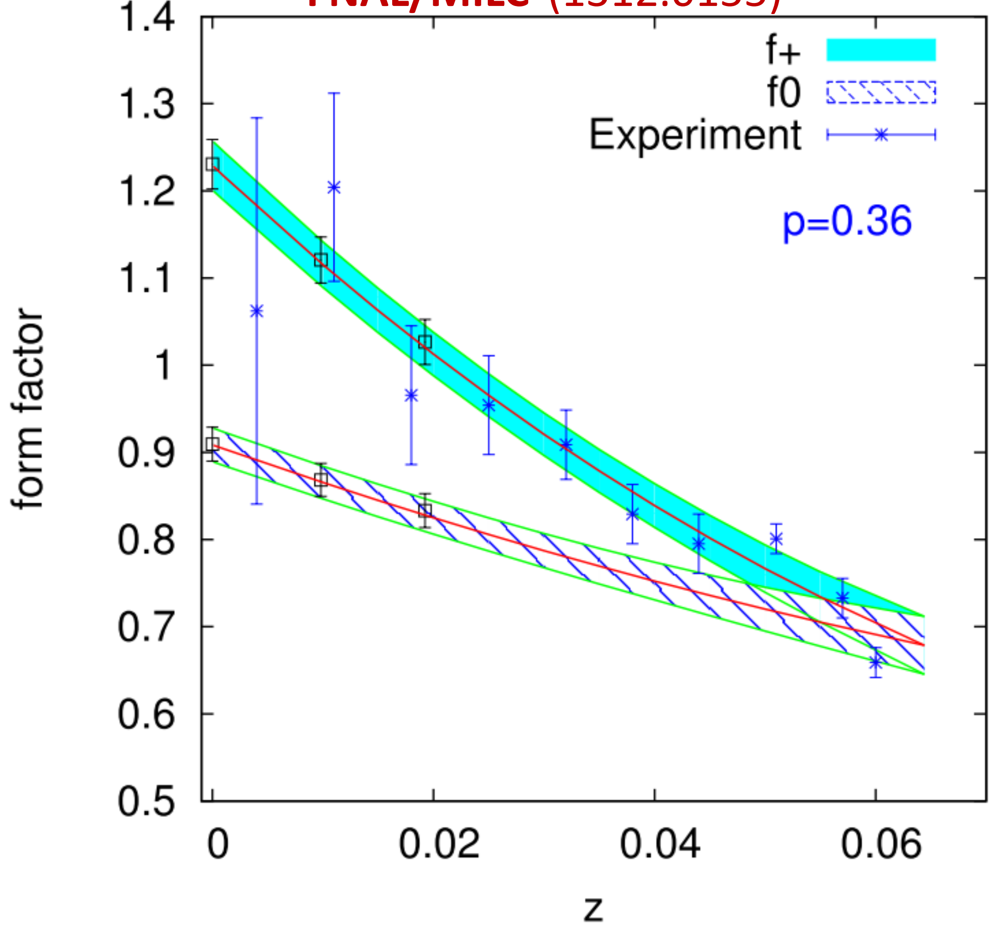
FNAL/MILC 1312.0155

Combined fit with experiments in full w-range

- Use model-independent z-parameterization (with order z^2)
- Combined fit with BaBar's measurements PRL104(011802)
- Fit is constrained with kinematic condition $f_+(q^2=0) = f_0(q^2=0)$.

$$z = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_-}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_-}}$$

FNAL/MILC (1312.0155)



BaBar PRL104(011802)

Lattice error budget (FNAL/MILC, preliminary)

- The dominant source of errors:

  statistical, HQ discretization and (perturbative) renormalization .
- The perturbative renormalization ρ for the spatial component of the vector current is difficult to calculate (in progress).
- Result (2013 lattice conference, preliminary):

$$|V_{cb}| = (38.5 \pm 1.9_{\mathrm{expt+lat}} \pm 0.2_{\mathrm{QED}}) \times 10^{-3}$$

- Uncertainty is dominated by experimental uncertainty.

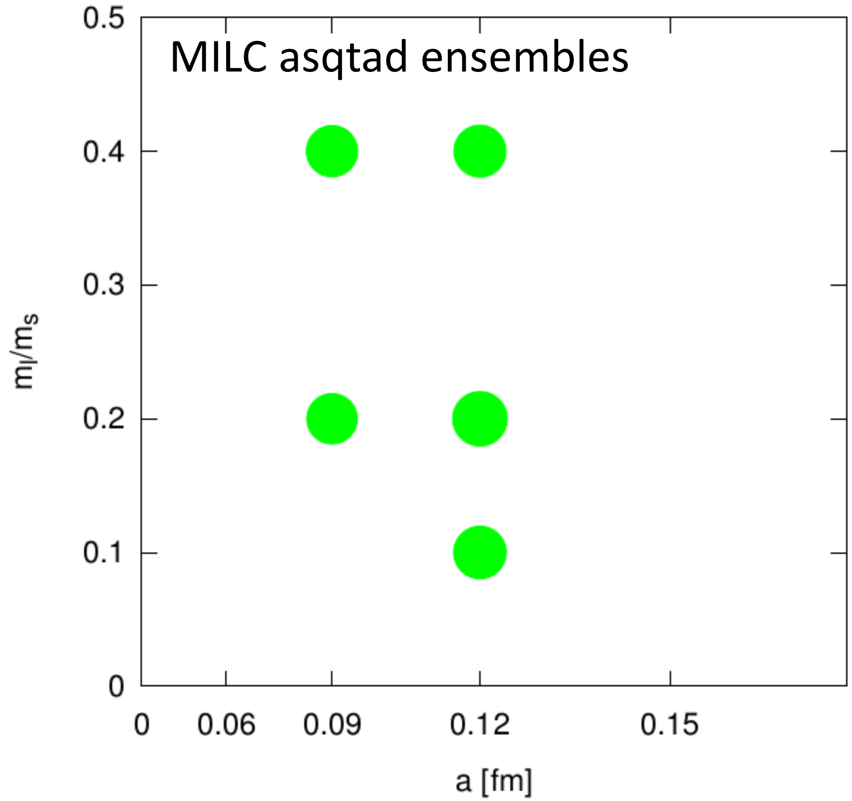
Take only ~5% in $G(w)$, $w \sim 1$

FNAL/MILC 1312.0155

source	$h_+(\%)$	$h_-(\%)$
Statistics+χPT	≤ 1	≤ 11
κ-tuning adjustment	≤ 0.1	1.4
Lattice scale r_1	0.2	≤ 0.1
heavy-quark discretization	2.0	10.
ρ factor	0.4	20.
Net systematic error	2.1	22.

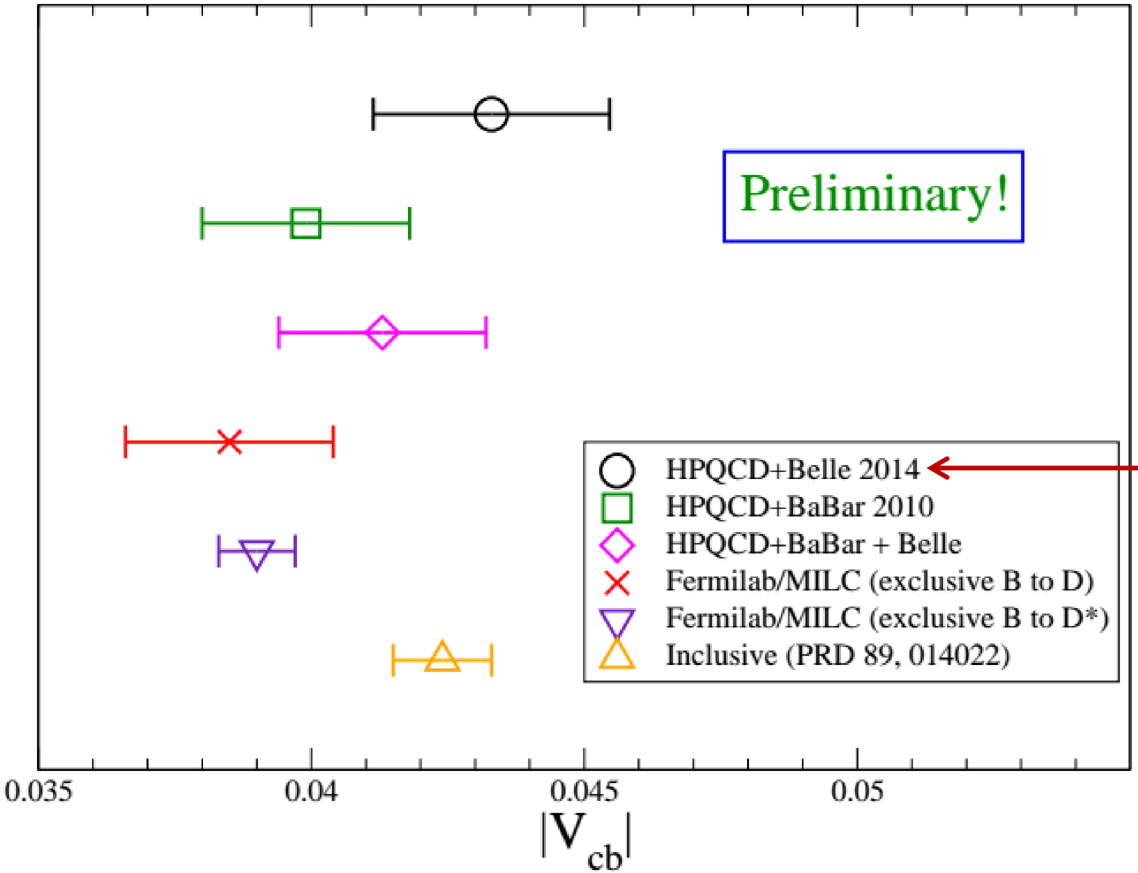
Lattice calculations of $B \to D\ell\nu$ (HPQCD)

- Use a subset of the MILC asqtad ensembles
- NRQCD bottom quark, hisq light (valence) and charm quarks (relativistic).
- Lattice vector current correction: $O(\alpha_S, \Lambda/m_b, \alpha_S/am_b)$ at one-loop perturbation.



Preliminary result and errors

- Dominant sources of error:
  Current matching error (~4%), statistics, discretization, z-expansion



Robin Glattauer, ICHEP 2014

Summary and outlook

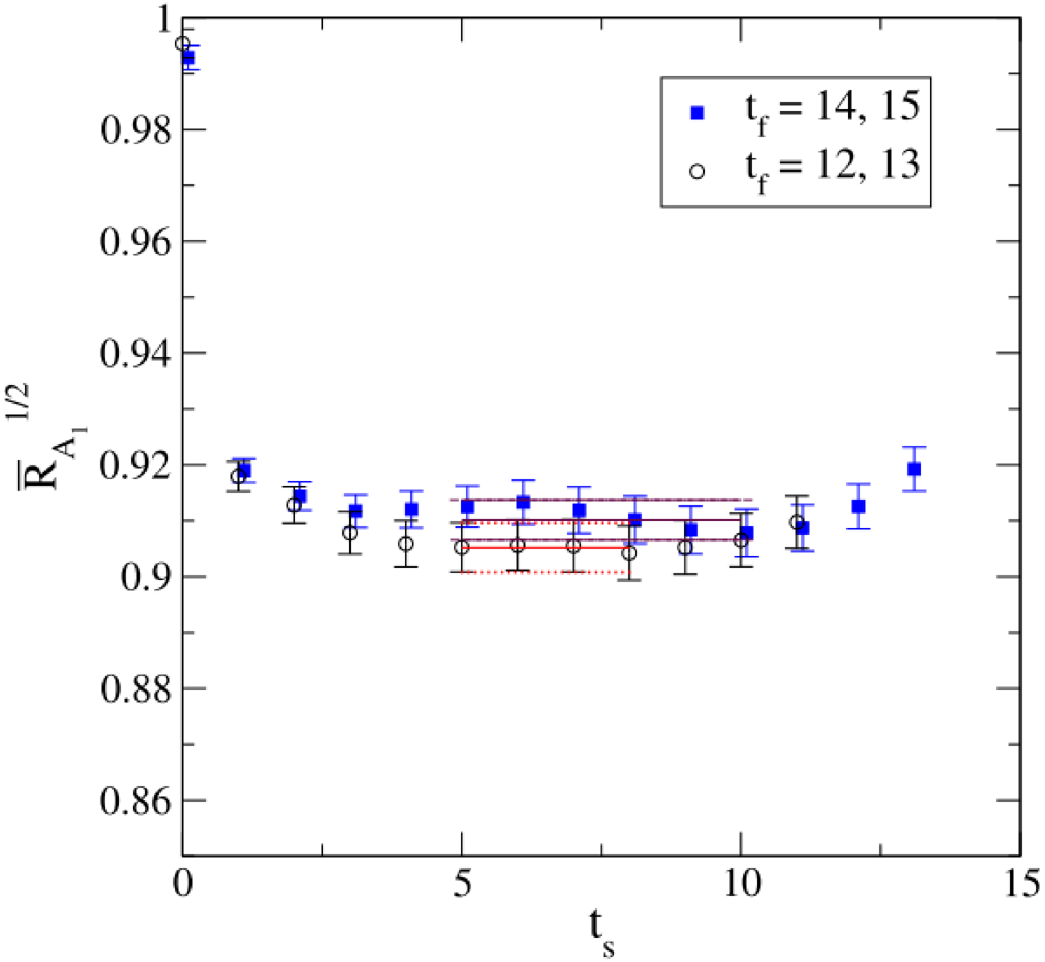
- The exclusive result of $|V_{cb}|$ determination is dominated by the $B \to D^* \ell \nu$ decay channel. Lattice error is commensurate with the experimental error. Using the FNAL/MILC lattice result, the $|V_{cb}|$ from $B \to D^* \ell \nu$ decay is 0.03905(49)(53)(19), with a total uncertainty of 1.9%.

- The determination of $|V_{cb}|$ using $B \to D \ell \nu$ decay at non-zero recoil is in progress. The preliminary results from FNAL/MILC and HPQCD are consistent with the determination from $B \to D^* \ell \nu$ decay.

- New analysis results from Belle is important for the crosscheck.

- A lattice calculation of $B \to D^* \ell \nu$ decay at non-zero recoil is desired.

Backup: excited state effect is small



Backup: hyperfine splittings