$\gamma/\phi_3$ measurements at LHCb

CKM2014, Vienna

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(on behalf of the LHCb collaboration)

CERN

11.09.2014
Introduction

- LHCb is a forward spectrometer operated in collider mode.
- Precision measurements of $b$- and $c$-hadron decays
- Rare decays, $CP$ violation, $\gamma$
- Very good $K^\pm/\pi^\pm$ separation: two RICH detectors
- Very good time resolution ($B^0_s$ oscillation): VELO

Dataset:

2011: 1 fb$^{-1}$
2012: 2 fb$^{-1}$
Total: 3 fb$^{-1}$
Introduction

- $\gamma$ is the least well known angle of the unitarity triangle
- it can be measured entirely from tree decays
- the residual uncertainty is negligible: $\delta \gamma / \gamma \approx 10^{-7}$
  see Brod on Tuesday!
  Brod, Zupan 2013 [1]

- **However:** the branching ratios are small, typically at the $10^{-7}$ level
- measure $\gamma$ in different “methods”
  
  **GLW**  $CP$ eigenstates, e.g. $D \rightarrow KK$
  
  **ADS**  flavored states, e.g. $D \rightarrow K\pi$
  
  **GGSZ**  self-conjugate 3-body decays, e.g. $D \rightarrow K_S^0 \pi\pi$
  
  **GLS**  singly Cabibbo suppressed states, e.g. $D \rightarrow K_S^0 K\pi$
  
  **time-dep.**  e.g. $B_S^0 \rightarrow D_s^\mp K^\pm$
Introduction

Figure: Example interference diagram. Coherence factors appear with multibody states.

Example observable:

\[
R^D_{K,K^3\pi} = \frac{\Gamma(B^+ \to [\pi K\pi\pi]_D K^+) \Gamma(B^+ \to [K\pi\pi\pi]_D K^+)}{\Gamma(B^+ \to [\pi K\pi\pi]_D K^+)},
\]

\[
R^D_{K,K^3\pi} = \frac{(r^{DK}_B)^2+(r^{K^3\pi}_D)^2+2\kappa^{K^3\pi}_D r^{DK}_B r^{K^3\pi}_D \cos(\delta^{DK}_B + \delta^{K^3\pi}_D + \gamma)}{1+(r^{DK}_B)^2(r^{K^3\pi}_D)^2+2\kappa^{K^3\pi}_D r^{DK}_B r^{K^3\pi}_D \cos(\delta^{DK}_B - \delta^{K^3\pi}_D + \gamma)}
\]
Introduction

Our new results, LHCb-CONF-2014-004, supersede the previous LHCb combinations.

Table: Summary of results for $\gamma$ from the $B$ factories BaBar and Belle, and from LHCb, and combiners. Errors correspond to 68% confidence or credibility.

<table>
<thead>
<tr>
<th>experiment</th>
<th>result</th>
<th>date</th>
</tr>
</thead>
<tbody>
<tr>
<td>BaBar [2]</td>
<td>$(69^{+17}_{-16})^\circ$</td>
<td>Jan 2013</td>
</tr>
<tr>
<td>Belle [3]</td>
<td>$(68^{+15}_{-14})^\circ$</td>
<td>Jan 2013</td>
</tr>
<tr>
<td>LHCb 3 fb$^{-1}$ preliminary [4]</td>
<td>$(67 \pm 12)^\circ$</td>
<td>Apr 2013 (superseded!)</td>
</tr>
<tr>
<td>LHCb 1 fb$^{-1}$ [5]</td>
<td>$(72.6^{+9.7}_{-17.2})^\circ$</td>
<td>Aug 2013 (superseded!)</td>
</tr>
<tr>
<td>UTfit</td>
<td>$(68.3 \pm 7.5)^\circ$</td>
<td>post Moriond 2014</td>
</tr>
<tr>
<td>CKMfitter</td>
<td>$(70.0^{+7.7}_{-9.0})^\circ$</td>
<td>Moriond / Jun 2014</td>
</tr>
<tr>
<td>CKMfitter</td>
<td>$(73.2^{+6.3}_{-7.0})^\circ$</td>
<td>CKM2014</td>
</tr>
</tbody>
</table>
Inputs

new LHCb results and changes

Two combinations:

<table>
<thead>
<tr>
<th>robust</th>
<th>$B \to DK$-like</th>
</tr>
</thead>
<tbody>
<tr>
<td>full</td>
<td>$B \to DK$-like and $B \to D\pi$</td>
</tr>
</tbody>
</table>

- **updated**: Update the auxiliary inputs
- $B^+ \to Dh^+, D \to hh$, GLW/ADS, 1 fb$^{-1}$ LHCb [6]
- $B^+ \to Dh^+, D \to K\pi\pi\pi$, ADS, 1 fb$^{-1}$ LHCb [7]
- **updated**: $B^+ \to DK^+, D \to K_S^0 hh$, model-ind. GGSZ, 3 fb$^{-1}$ LHCb [8]
- **new**: $B^+ \to DK^+, D \to K_S^0 K\pi$, GLS, 3 fb$^{-1}$ LHCb [9]
- **new**: $B^0 \to D^0 K^{*0}$, $D \to hh$, GLW/ADS, 3 fb$^{-1}$ LHCb [10]
- **new**: $B^0_s \to D_s^{\mp} K^\pm$, 1 fb$^{-1}$ LHCb [11]
new: \( B^+ \rightarrow D K^+ \), \( D \rightarrow K_S^0 K \pi \), GLS, 3 fb\(^{-1}\)

- **GLS method**
  Grossman, Ligeti, Soffer 2002 [12]
- use singly Cabibbo suppressed \( D \) decays to non-\( CP \) eigenstates
- first GLS measurement! LHCb [9]
- no significant \( CP \) violation was observed

we restrict the phase space to the more sensitive \( D \rightarrow K^{*+} K^- \) region

**Figure:** “opposite-sign” decays

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new: $B^0 \to D^0 K^{*0}$, $D \to h h$, GLW/ADS, 3 fb$^{-1}$

- this mode has great potential exploiting the full $D^0 K\pi$ phase space
  - Gershon 2009 [13]
- the $K^{*0}$ decay tags the $B$ flavour at decay—no time dependent measurement
- significance for suppressed decay $B^0 \to D^0 K^{*0}$, $D^0 \to \pi^- K^+$: 2.9$\sigma$
new: $B^0_s \rightarrow D^±_s K^±$, time-dep., 1 fb$^{-1}$

- This new technique gains sensitivity to $\gamma$ through $B^0_s$ mixing.

- first measurement! Different set of systematics, pure “LHCb only”
- $B$ tagging power of $\epsilon_{eff} = 5.07\%$
- Lots of hadronic backgrounds: use multidimensional fit to identify the signal
- 1770 ± 50 signal events

See also talk by Vava Gligorov!
$D^0 - \bar{D}^0$ mixing correction

$D^0 - \bar{D}^0$ mixing has an effect on $\gamma$
Meca, Silva, Atwood, Grossman
[14, 15, 16, 12, 17]

see also Matteo Rama on Tuesday!

$\mathcal{O}(1^\circ)$ on $\gamma$ for $B \to DK$

The effect can be larger for $B \to D\pi$
[18, Rama 2013]:

$$\sqrt{x_D^2 + y_D^2/r_B^{D\pi}} \approx \mathcal{O}(1)$$

The topic was revisited following
FPCP2013 Rama; Bondar et al. [19, 18].

We correct for the effect of $D^0 - \bar{D}^0$ mixing

in leading order in $x_D, y_D$

taking into account individual $D^0$ decay time acceptances

Figure: LHCb $D^0$ decay time acceptance for $B^+ \to DK^+, D \to hh$. 

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Auxiliary input for $D^0 \to K^{\pm \pi^{\mp} \pi^+ \pi^-}$

updated:

- The $D^0 \to K^{\pm \pi^{\mp} \pi^+ \pi^-}$ ADS measurement needs input on the charm system.
- amplitude ratio: $r_{D}^{K3\pi}$
- eff. strong phase diff.: $\delta_{D}^{K3\pi}$
- coherence factor: $\kappa_{D}^{K3\pi}$
- We use the histogrammed likelihood for $\delta_{D}^{K3\pi}$, $\kappa_{D}^{K3\pi}$ from CLEO legacy.
- We use a CLEO [20] and a Belle measurement [21] on $\Gamma(D^0 \to K^- \pi^+ \pi^- \pi^+)$ and $\Gamma(D^0 \to \pi^- K^+ \pi^- \pi^+)$ to constrain $r_{D}^{K3\pi}$.

Figure: CLEO legacy likelihood for $\delta_{D}^{K3\pi}$ and $\kappa_{D}^{K3\pi}$ (labeled “$R_{K3\pi}$”) $D \to K3\pi$. Histogram is used in the combination.
Other auxiliary input

**updated:** The GLW/ADS measurements need inputs on the charm system:

- for the $D^0 \rightarrow K^- \pi^+$ and $D^0 \rightarrow \pi^- K^+$ decays: $r_{D}^{K\pi}$, $\delta_{D}^{K\pi}$
- for charm mixing: $x_D$, $y_D$
- for direct $CP$ violation in $D^0 \rightarrow h^+ h^-$ decays: $A_{CP}^{\text{dir}}(KK)$, $A_{CP}^{\text{dir}}(\pi\pi)$
- We use the recent HFAG charm fit (FPCP 2014).

**new:** Input for $D \rightarrow K_S^0 K\pi$ GLS

- We use a CLEO measurement for $r_{D}^{K_S K\pi}$, $\delta_{D}^{K_S K\pi}$, $\kappa_{D}^{K_S K\pi}$ CLEO [23].

**new:** Input for $B^0 \rightarrow D^0 K^{*0}$ GLW/ADS

- We use a MC-based estimate for $\kappa_{B}^{DK^{*0}} = 0.95 \pm 0.03$ LHCb [10].

**new:** Input for $B_s^0 \rightarrow D_s^{\mp} K^{\pm}$ time-dep. to interpret $\gamma - 2\beta_s$

- We use the LHCb measurement of $-2\beta_s \approx \phi_s = 0.01 \pm 0.07 \pm 0.01$ rad LHCb [24].
Evolution from the previous preliminary result

1. previous result
   LHCb preliminary [4]

2. Update the auxiliary inputs

3. Add new $B^+ \rightarrow DK^+$, $D \rightarrow K_S^0 hh$ GGSZ measurement

4. Add new $B^+ \rightarrow DK^+$, $D \rightarrow K_S^0 K\pi$ GLS measurement

5. Add new $B^0 \rightarrow D^0 K^{*0}$, $D \rightarrow hh$ GLW/ADS measurement

6. Add new $B_s^0 \rightarrow D_s^{\mp} K^\pm$ time-dependent measurement

7. robust nominal

Figure: Making one change at a time. Robust combination.
Evolution from the previous preliminary result

1. previous result
   LHCb preliminary [4]

2. Update the auxiliary inputs

3. Add new $B^+ \rightarrow DK^+$, $D \rightarrow K^0 hh$ GGSZ measurement

4. Add new $B^+ \rightarrow DK^+$, $D \rightarrow K^0 K\pi$ GLS measurement

5. Add new $B^0 \rightarrow D^0 K^{*0}$, $D \rightarrow hh$ GLW/ADS measurement

6. Add new $B^0_s \rightarrow D^+_s K^\pm$ time-dependent measurement

7. robust nominal

Figure: Making one change at a time. Robust combination.
Evolution from the previous preliminary result

1. previous result
   LHCb preliminary [4]

2. Update the auxiliary inputs

3. Add new $B^+ \rightarrow DK^+$,
   $D \rightarrow K^0_S hh$ GGSZ measurement

4. Add new $B^+ \rightarrow DK^+$,
   $D \rightarrow K^0_S K\pi$ GLS measurement

5. Add new $B^0 \rightarrow D^0 K^{*0}$,
   $D \rightarrow hh$ GLW/ADS measurement

6. Add new $B^0_s \rightarrow D^\mp_s K^\pm$
   time-dependent measurement

7. robust nominal

Figure: Making one change at a time.
Robust combination.
Evolution from the previous preliminary result

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2. Update the auxiliary inputs

3. Add new $B^+ \rightarrow DK^+$,
   $D \rightarrow K_S^0 hh$ GGSZ measurement

4. Add new $B^+ \rightarrow DK^+$,
   $D \rightarrow K_S^0 K\pi$ GLS measurement

5. Add new $B^0 \rightarrow D^0 K^{*0}$,
   $D \rightarrow hh$ GLW/ADS measurement

6. Add new $B^0_s \rightarrow D^{\mp}_s K^{\pm}$
   time-dependent measurement

7. robust nominal

**Figure:** Making one change at a time.
Robust combination.
Evolution from the previous preliminary result

1. previous result
   LHCb preliminary [4]
2. Update the auxiliary inputs
3. Add new \( B^+ \to D K^+ \),
   \( D \to K_S^0 h h \) GGSZ measurement
4. Add new \( B^+ \to D K^+ \),
   \( D \to K_S^0 K \pi \) GLS measurement
5. Add new \( B^0 \to D^0 K^{*0} \),
   \( D \to h h \) GLW/ADS
   measurement
6. Add new \( B^0_s \to D^+_s K^\pm \)
   time-dependent measurement
7. robust nominal

Figure: Making one change at a time.
Robust combination.
Evolution from the previous preliminary result

1. previous result
   LHCb preliminary [4]

2. Update the auxiliary inputs

3. Add new $B^+ \rightarrow DK^+$,
   $D \rightarrow K^0 h\bar{h}$ GGSZ measurement

4. Add new $B^+ \rightarrow DK^+$,
   $D \rightarrow K^0 K\pi$ GLS measurement

5. Add new $B^0 \rightarrow D^0 K^{*0}$,
   $D \rightarrow h\bar{h}$ GLW/ADS measurement

6. Add new $B^0_s \rightarrow D^{\mp} K^\pm$
   time-dependent measurement

7. robust nominal

Figure: Making one change at a time.
Robust combination.
Evolution from the previous preliminary result

1. previous result
   LHCb preliminary [4]

2. Update the auxiliary inputs

3. Add new $B^+ \rightarrow DK^+$,
   $D \rightarrow K^0_Shh$ GGSZ measurement

4. Add new $B^+ \rightarrow DK^+$,
   $D \rightarrow K^0_SK\pi$ GLS measurement

5. Add new $B^0 \rightarrow D^0K^*0$,
   $D \rightarrow hh$ GLW/ADS measurement

6. Add new $B^0_s \rightarrow D^\pm_s K^{\pm}$
   time-dependent measurement

7. robust nominal

Figure: Making one change at a time. Robust combination.
The **method is unchanged** from previous combinations.

- We assume (almost) all observables to be Gaussian distributed.
- We assume Gaussian systematic fluctuations.
- We don’t allow non-physical parameter values.
- Use the **plugin method** for nominal results (Feldman-Cousins based frequentist method nuisances at their best-fit values).
- Use the profile likelihood method for 2D illustration.
- Estimate the frequentist coverage.
- Additional **Bayesian interpretation** with uniform priors.
**Result of the robust combination**

Table: Credibility regions and most probable values for the hadronic parameters extracted from the robust combination. The second part of the table repeats the frequentist results for comparison.

<table>
<thead>
<tr>
<th>Observable</th>
<th>Central value</th>
<th>Intervals</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Frequentist</td>
<td>68%</td>
</tr>
<tr>
<td>$\gamma[^\circ]$</td>
<td>72.9</td>
<td>[63.0, 82.1]</td>
</tr>
<tr>
<td>$\delta_{DK}[^\circ]$</td>
<td>126.8</td>
<td>[115.3, 136.7]</td>
</tr>
<tr>
<td>$r_{DK}^B$</td>
<td>0.0914</td>
<td>[0.0826, 0.0997]</td>
</tr>
<tr>
<td></td>
<td>Bayesian</td>
<td>68%</td>
</tr>
<tr>
<td>$\gamma[^\circ]$</td>
<td>71.9</td>
<td>[61.9, 81.8]</td>
</tr>
<tr>
<td>$\delta_{DK}[^\circ]$</td>
<td>127.4</td>
<td>[115.6, 137.3]</td>
</tr>
<tr>
<td>$r_{DK}^B$</td>
<td>0.091</td>
<td>[0.0826, 0.0984]</td>
</tr>
</tbody>
</table>

The agreement of frequentist and Bayesian results is quite good.
Result of the robust combination

Figure: Graphs for the robust combination (left: frequentist $1 - \text{CL}$ curves, right: Bayesian posterior).
Result of the robust combination

\[ \delta^\text{DK}_B \]

\[ r^\text{DK}_B \]

68.3%
95.5%

126.8^{+9.9}_{-11.5}

0.0914^{+0.0083}_{-0.0088}

LHCb
Preliminary

Probability density

Probability density
## Results full combination

**Let's add** $B^+ \rightarrow D\pi^+$ information

**Table:** Confidence intervals and central values for the full combination.

<table>
<thead>
<tr>
<th>quantity</th>
<th>full</th>
<th>full</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma (^\circ)$</td>
<td>78.9</td>
<td>72.8</td>
</tr>
<tr>
<td>68% CL ($^\circ$)</td>
<td>[71.5, 84.7]</td>
<td></td>
</tr>
<tr>
<td>95% CL ($^\circ$)</td>
<td>[54.6, 91.4]</td>
<td></td>
</tr>
<tr>
<td>$r_{DK}^{B}$</td>
<td>0.0928</td>
<td></td>
</tr>
<tr>
<td>68% CL</td>
<td>[0.0845, 0.1008]</td>
<td></td>
</tr>
<tr>
<td>95% CL</td>
<td>[0.0732, 0.1085]</td>
<td></td>
</tr>
<tr>
<td>$\delta_{BK}^{D\pi}$</td>
<td>128.9</td>
<td></td>
</tr>
<tr>
<td>68% CL ($^\circ$)</td>
<td>[118.9, 137.9]</td>
<td></td>
</tr>
<tr>
<td>95% CL ($^\circ$)</td>
<td>[102.0, 145.9]</td>
<td></td>
</tr>
<tr>
<td>$r_{B}^{D\pi}$</td>
<td>0.027</td>
<td>0.006</td>
</tr>
<tr>
<td>68% CL</td>
<td>[0.016, 0.034]</td>
<td>[0.005, 0.007]</td>
</tr>
<tr>
<td>95% CL</td>
<td>[0.001, 0.040]</td>
<td></td>
</tr>
<tr>
<td>$\delta_{BK}^{D\pi}$</td>
<td>341.8</td>
<td>215.6</td>
</tr>
<tr>
<td>68% CL ($^\circ$)</td>
<td>[328.7, 351.4]</td>
<td>[210.2, 231.5]</td>
</tr>
<tr>
<td>95% CL ($^\circ$)</td>
<td>no constraint</td>
<td></td>
</tr>
</tbody>
</table>
Let's add $B^+ \rightarrow D\pi^+$ information

Figure: $1 - \text{CL}$ curve for the robust and full combinations.

- A second minimum appears.
- The $1\sigma$ errors are **misleadingly small**: highly non-Gaussian!
- At $2\sigma$, the agreement with the robust combination is very good.
Result of the full combination

At $2\sigma$, $r_{D\pi}^{DK}$ is compatible with 0.
Results full combination

Result of the full combination

Figure: Profile likelihood contours, full combination. The contours are the two-dimensional 1σ and 2σ contours.

The high observed value of $r_B^{D\pi}$ also pulls up $\gamma$. 
Coverage test

- We test the frequentist coverage at the minima of the combinations.
- We find that the profile likelihood construction undercovers quite a bit.
- The robust plugin method has good coverage.
- The coverage of the full combination is worse than of the robust. Expected due to the low value of $r_B^{D\pi}$.

<table>
<thead>
<tr>
<th>$\eta = 0.683$</th>
<th>$\alpha$ (prof. LH.)</th>
<th>$\alpha$ (plugin)</th>
</tr>
</thead>
<tbody>
<tr>
<td>robust</td>
<td>0.6158</td>
<td>0.6494</td>
</tr>
<tr>
<td>full (1), $r_B^{D\pi} = 0.027$</td>
<td>0.5593</td>
<td>0.6154</td>
</tr>
<tr>
<td>full (2), $r_B^{D\pi} = 0.006$</td>
<td>0.5454</td>
<td>0.6120</td>
</tr>
</tbody>
</table>
Conclusion

- We updated the LHCb $\gamma$ combination of tree-level measurements.
- The effect of $D^0-\bar{D}^0$ mixing is taken into account, the frequentist coverage was tested.
- Robust Bayesian results are available and agree.
- We explore $B^+ \rightarrow D\pi^+$ channels: they show up early all obstacles on the way to high precision.
- The robust coverage is good.
- Robust frequentist combination:

\[
\gamma = (72.9^{+9.2}_{-9.9})^\circ \quad @ \text{68\% CL ,}
\]
\[
\gamma \in [63.0, 82.1]^\circ \quad @ \text{68\% CL ,}
\]
\[
\gamma \in [52.0, 90.5]^\circ \quad @ \text{95\% CL .}
\]

- More precise than the $B$ factory legacy.
Backup
Auxiliary input for $D \rightarrow K^0_SK\pi$

**new:**

- Similar input is needed as in the case of $D \rightarrow K\pi\pi\pi$.
- rate ratio: $R_D^{K_SK\pi}$
- eff. strong phase diff.: $\delta_D^{K_SK\pi}$
- coherence factor: $\kappa_D^{K_SK\pi}$
- We use a single CLEO measurement [23].

\[
R_D^{K_SK\pi} = 0.356 \pm 0.034 \pm 0.007 , \\
\delta_D^{K_SK\pi} = 0.46 \pm 0.28 , \\
\kappa_D^{K_SK\pi} = 1.00 \pm 0.16 .
\]

**Figure:** CLEO likelihood for $\delta_D^{K_SK\pi}$ and $\kappa_D^{K_SK\pi}$ (labeled “$R_{K^*K}$”) [23]. Histogram is not used in the combination.
Auxiliary input from HFAG

taking a step back

Comparing the old auxiliary inputs to the new ones:

old:

- we needed input on the $D \rightarrow K\pi\pi\pi$ system from CLEO
- that measurement had some influence on the charm mixing parameters $x_D, y_D$
- it was obtained using the HFAG status as of 2009 as input
- ...so we took the 2009 CLEO input and added the new LHCb $D^0$ wrong sign mixing measurement
- we also added "$\Delta A_{CP}$"
Auxiliary input from HFAG

taking a step back

Comparing the old auxiliary inputs to the new ones:

**new:**
- the input for $D \rightarrow K\pi\pi\pi$ was updated using more CLEO-c data
- the new measurement no longer impacts $x_D, y_D$, as the world average is much stronger now
- it was obtained using the HFAG status as of 2014 as input
- ...so now we can use HFAG directly, including up-to-date charm mixing, and "$\Delta A_{CP}$"
Auxiliary input from HFAG

comparing old and new

**Figure:** Profile likelihood contours: The “old” contour corresponds to what was used in the previous (2013) combination (the 2009 CLEO input [25] together with the 2013 LHCb charm mixing measurement [26]). The “new” contour is what is used in this combination (HFAG 2014). The contours are two-dimensional $1-4\sigma$ contours.
Auxiliary input from HFAG

The parameter $R_{K\pi}^{D}$ is the squared ratio of the doubly-Cabibbo-suppressed amplitude $D^0 \rightarrow \pi^- K^+$ to the favored one $D^0 \rightarrow K^- \pi^+$. It is not the ratio of branching ratios. It gets often measured in time-dependent wrong-sign $D^0$ mixing measurements:

$$R_{WS} = R_{K\pi}^{D} + \sqrt{R_{K\pi}^{D}} \left( x \cos(\delta_{K\pi}^{D}) \pm y \sin(\delta_{K\pi}^{D}) \right) \frac{t}{\tau} + \frac{x^2_D + y^2_D}{4} \left( \frac{t}{\tau} \right)^2$$

Figure: Evolution of HFAG results on $R_{K\pi}^{D}$. 

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Auxiliary input for $D^0 \rightarrow K^{\pm} \pi^\mp \pi^+ \pi^-$

**updated:** We use an updated CLEO-c measurement of $\kappa_D^{K3\pi}$ and $\delta_D^{K3\pi}$ (histogrammed likelihood). For the amplitude ratio

$$r_D^{K3\pi} = \left| A(D^0 \rightarrow \pi^- K^+ \pi^- \pi^+)/A(D^0 \rightarrow K^- \pi^+ \pi^- \pi^+) \right|$$  \hspace{1cm} (1)

we use

- a CLEO measurement [20] of $\Gamma(D^0 \rightarrow K^- \pi^+ \pi^- \pi^+)$,

  \[ \Gamma(D^0 \rightarrow K^- \pi^+ \pi^- \pi^+) = 0.08290 \pm 0.00043 \pm 0.00200, \]  \hspace{1cm} (2)

- a Belle measurement [21] of $\Gamma(D^0 \rightarrow \pi^- K^+ \pi^- \pi^+)$,

  \[ \Gamma(D^0 \rightarrow \pi^- K^+ \pi^- \pi^+) = 0.00026 \pm 0.00001 \pm 0.00001. \]  \hspace{1cm} (3)

- These are related to $r_D^{K3\pi}$ through the equation

  \[ R_{WS}(D \rightarrow K\pi\pi\pi) = \]

  \[ (r_D^{K3\pi})^2 - \kappa_D^{K3\pi} r_D^{K3\pi} (y_D \cos \delta_D^{K3\pi} - x_D \sin \delta_D^{K3\pi}) + \frac{1}{2} (x_D^2 + y_D^2). \]  \hspace{1cm} (5)
Goodness of fit

**Table**: Fit probabilities of the best fit values of the two combinations.

<table>
<thead>
<tr>
<th>Combination</th>
<th>$P$ (PROB)</th>
<th>$P$ (toy-based)</th>
</tr>
</thead>
<tbody>
<tr>
<td>robust</td>
<td>89.7%</td>
<td>(89.4 ± 0.5)%</td>
</tr>
<tr>
<td>full</td>
<td>97.2%</td>
<td>(96.8 ± 0.3)%</td>
</tr>
</tbody>
</table>
Result of the robust combination

Figure: Profile likelihood contours, robust combination. The contours are the two-dimensional $1\sigma$ and $2\sigma$ contours.
Result of the full combination

Figure: Profile likelihood contours, full combination. The contours are the two-dimensional $1\sigma$ and $2\sigma$ contours.
Result of the full combination

Figure: Profile likelihood contours, full combination. The contours are the two-dimensional $1\sigma$ and $2\sigma$ contours.
## Coverage test

**Table:** Measured coverage $\alpha$ of the confidence intervals for $\gamma$. The expected coverage is denoted as $\eta$.

<table>
<thead>
<tr>
<th>robust</th>
<th>$\alpha$ (profile likelihood)</th>
<th>$\alpha$ (PLUGIN)</th>
<th>$\eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta = 0.6827$</td>
<td>$0.6158 \pm 0.0089$</td>
<td>$0.6494 \pm 0.0087$</td>
<td>$0.6158 \pm 0.0089$</td>
</tr>
<tr>
<td>$\eta = 0.9545$</td>
<td>$0.9200 \pm 0.0050$</td>
<td>$0.9358 \pm 0.0045$</td>
<td>$0.9200 \pm 0.0050$</td>
</tr>
<tr>
<td>$\eta = 0.9973$</td>
<td>$0.9923 \pm 0.0016$</td>
<td>$0.9919 \pm 0.0016$</td>
<td>$0.9923 \pm 0.0016$</td>
</tr>
<tr>
<td>full, $r_{B}^{D\pi} = 0.027$</td>
<td>$\alpha$ (profile likelihood)</td>
<td>$\alpha$ (PLUGIN)</td>
<td>$\eta$</td>
</tr>
<tr>
<td>$\eta = 0.6827$</td>
<td>$0.5593 \pm 0.0091$</td>
<td>$0.6154 \pm 0.0089$</td>
<td>$0.5593 \pm 0.0091$</td>
</tr>
<tr>
<td>$\eta = 0.9545$</td>
<td>$0.9160 \pm 0.0051$</td>
<td>$0.9402 \pm 0.0043$</td>
<td>$0.9160 \pm 0.0051$</td>
</tr>
<tr>
<td>$\eta = 0.9973$</td>
<td>$0.9929 \pm 0.0015$</td>
<td>$0.9929 \pm 0.0015$</td>
<td>$0.9929 \pm 0.0015$</td>
</tr>
<tr>
<td>full, $r_{B}^{D\pi} = 0.006$</td>
<td>$\alpha$ (profile likelihood)</td>
<td>$\alpha$ (PLUGIN)</td>
<td>$\eta$</td>
</tr>
<tr>
<td>$\eta = 0.6827$</td>
<td>$0.5454 \pm 0.0091$</td>
<td>$0.6120 \pm 0.0089$</td>
<td>$0.5454 \pm 0.0091$</td>
</tr>
<tr>
<td>$\eta = 0.9545$</td>
<td>$0.9026 \pm 0.0054$</td>
<td>$0.9314 \pm 0.0046$</td>
<td>$0.9026 \pm 0.0054$</td>
</tr>
<tr>
<td>$\eta = 0.9973$</td>
<td>$0.9906 \pm 0.0018$</td>
<td>$0.9933 \pm 0.0015$</td>
<td>$0.9906 \pm 0.0018$</td>
</tr>
</tbody>
</table>


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