Inclusive $\bar{B} \to X_s \ell^+ \ell^-$ and $\bar{B} \to X_s \gamma$ decays

(Theory)

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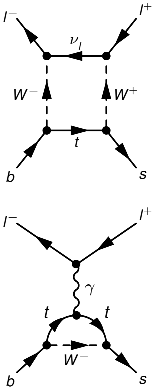
DFG FOR 1873

Based on

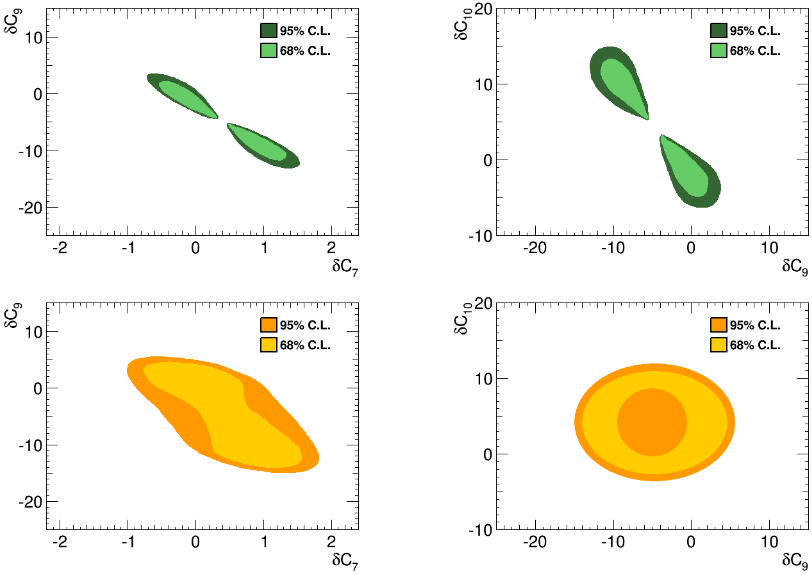
T. Hurth, E. Lunghi, TH	in preparation
M. Poradzinski, J. Virto, TH	in preparation

CKM, Vienna, September 11th, 2014

- Inclusive $\bar{B} \to X_s \ell^+ \ell^-$
  - Rare decay, FCNC process
  - Probes SM directly at the loop level
  - Sensitivity to new physics
- Complementary to $\bar{B} \to X_s \gamma$
  - More observables
  - Box and penguin diagrams
  - Besides C_7, also sensitivity to $C_{9,10}$
- Complementary to $\bar{B} \to K^{(*)} \mu^+ \mu^-$
  - Complementarity in experimental analysis: LHCb vs. BaBar, Belle (II)
  - Handling of power corrections
  - Sensitivity to different (combinations of) operators
  - Probing different theoretical approaches when measuring e.g. C_9



- Inclusive $\bar{B} \to X_s \ell^+ \ell^-$ can serve as a cross-check. [Hurth,Mahmoudi'13]

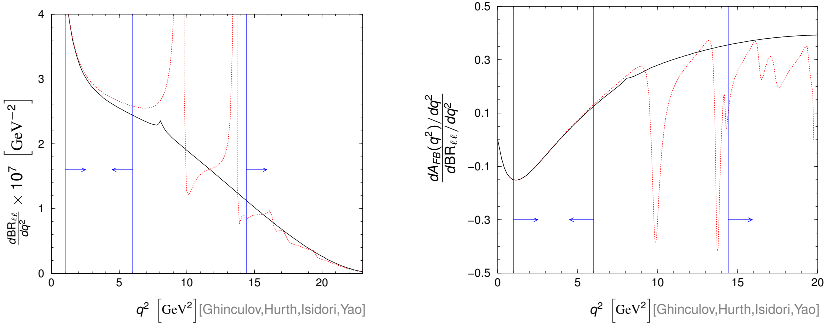


Observables

- Double differential decay width ($z = \cos\theta_\ell$) [Lee,Ligeti,Stewart,Tackmann'06]

$$\frac{d^2\Gamma}{dq^2\,dz} = \frac{3}{8}\left[(1+z^2)\,H_T(q^2) + 2\,z\,H_A(q^2) + 2\,(1-z^2)\,H_L(q^2)\right]$$

Note: $\displaystyle \frac{d\Gamma}{dq^2} = H_T(q^2) + H_L(q^2)\,, \qquad \frac{dA_{\rm FB}}{dq^2} = 3/4\,H_A(q^2)$



- Low-q^2 region: $1\,\mathrm{GeV}^2 < q^2 < 6\,\mathrm{GeV}^2$
- High-q^2 region: $q^2 > 14.4\,\mathrm{GeV}^2$

Observables

- Dependence of the H_i on WCs

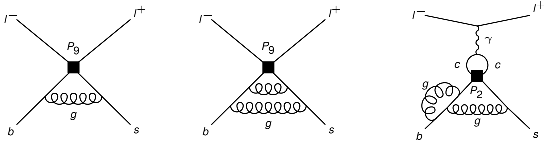
$$H_T(q^2) \propto 2s(1-s)^2 \left[\left| C_9 + \frac{2}{s} C_7 \right|^2 + |C_{10}|^2 \right]$$
$$H_A(q^2) \propto \; -4s(1-s)^2 \;\; \mathrm{Re}\left[C_{10} \left(C_9 + \frac{2}{s} C_7 \right) \right]$$
$$H_L(q^2) \propto (1-s)^2 \left[\left| C_9 + 2\, C_7 \right|^2 + |C_{10}|^2 \right]$$

- Consider integrals of H_i over two bins $1 - 3.5$ GeV^2 and $3.5 - 6$ GeV^2
- Moreover: zero of H_A in low-q^2 region
- High-q^2 region:
- Introduction of the ratio $\mathcal{R}(s_0) = \frac{\int_{\hat{s}_0}^1 \mathrm{d}\hat{s} \;\; d\Gamma(\bar{B} \to X_s \ell^+ \ell^-)/d\hat{s}}{\int_{\hat{s}_0}^1 \mathrm{d}\hat{s} \;\; d\Gamma(\bar{B}^0 \to X_u \ell \nu)/d\hat{s}}$ [Ligeti,Tackmann'07]
  - Normalize to semileptonic $\bar{B}^0 \to X_u \ell \nu$ rate with the same cut
    Need differential semi-leptonic $b \to u$ rate

Perturbative and non-perturbative corrections

$$\Gamma(\bar{B} \to X_s \ell\ell) = \Gamma(b \to X_s \ell\ell) \ + \ \text{power corrections}$$

- Pert. corrections at quark level are known to NNLO QCD + NLO QED
  [Misiak,Buras,Münz,Bobeth,Urban,Asatrian,Asatryan,Greub,Walker,Bobeth,Gambino,Gorbahn,Haisch,Blokland]
  [Czarnecki,Melnikov,Slusarczyk,Bieri,Ghinculov,Hurth,Isidori,Yao,Greub,Pilipp,Schüpbach,Lunghi,TH]
  - Involves diagrams up to three loops



  - Fully differential QCD corrections at NNLO for $P_{9,10}$ also known
    [Brucherseifer,Caola,Melnikov'13]

- $1/m_b^2$, $1/m_b^3$ and $1/m_c^2$ non-pert. corrections
  [Falk,Luke,Savage'93]
  [Ali,Hiller,Handoko,Morozumi'96]
  [Bauer,Burrell'99; Buchalla,Isidori,Rey'97]

- Factorizable $c\bar{c}$ contributions implemented via KS approach [Krüger, Sehgal'96]

Perturbative side, normalisation, inputs

- The organisation of the perturbative expansion is screwed:
  - LO = $\alpha_{\rm em}/\alpha_s$, NLO = $\alpha_{\rm em}$, NNLO = $\alpha_{\rm em}\,\alpha_s$
- Consistent expansion is in α_s and $\kappa = \alpha_{\rm em}/\alpha_s$ [Lunghi,Misiak,Wyler,TH'05]
- Amplitude

$$
\begin{aligned}
A &= \kappa\left[A_{LO} + \alpha_s A_{NLO} + \alpha_s^2 A_{NNLO} + \mathcal{O}(\alpha_s^3)\right] \\
&+ \kappa^2\left[A_{LO}^{em} + \alpha_s A_{NLO}^{em} + \alpha_s^2 A_{NNLO}^{em} + \mathcal{O}(\alpha_s^3)\right] + \mathcal{O}(\kappa^3)
\end{aligned}
$$

- Normalisation

$$
\frac{d\,BR(\bar{B}\to X_s ll)}{d\,\hat{s}} = BR^{exp.}_{b\to c\,e\,\nu}\left|\frac{V_{ub}}{V_{cb}}\right|^2 \frac{1}{C}\,\frac{d\Gamma(\bar{B}\to X_s\,ll)/d\hat{s}}{\Gamma(\bar{B}\to X_u e\bar{\nu})}
$$

$$
C = \left|\frac{V_{ub}}{V_{cb}}\right|^2 \frac{\Gamma(\bar{B}\to X_c e\bar{\nu})}{\Gamma(\bar{B}\to X_u e\bar{\nu})} = 0.574 \pm 0.019
$$

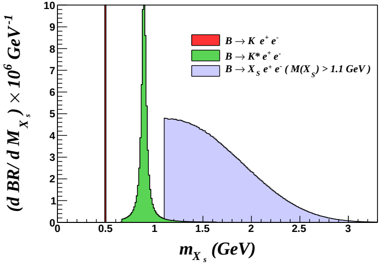
[Gambino,Schwanda'13]

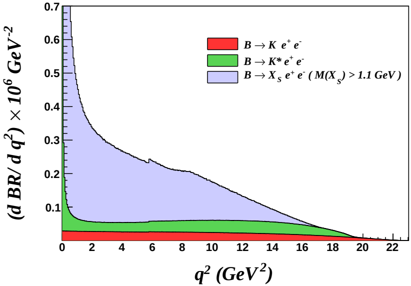
- Key input parameters
  - $m_b^{1S} = (4.691 \pm 0.037)\mathrm{GeV}$, $\overline{m}_c(\overline{m}_c) = (1.275 \pm 0.025)\mathrm{GeV}$
  - $|V_{ts}^* V_{tb}/V_{cb}|^2 = 0.9621 \pm 0.0027$, $BR^{exp.}_{b\to c\,e\,\nu} = (10.51 \pm 0.13)\,\%$

Collinear photons

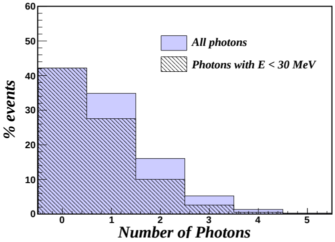
- Rate differential in q^2 is not IR safe w.r.t. energetic, collinear photon radiation off leptons
- Gives rise to log-enhanced QED corrections $\propto \alpha_{\rm em} \log(m_b^2/m_\ell^2)$
- Size of logs depends on experimental setup
  - $q^2 = (p_{\ell^+} + p_{\ell^-})^2$ vs. $q^2 = (p_{\ell^+} + p_{\ell^-} + p_{\gamma,\rm coll})^2$
  - To compare to BaBar electron channel our numbers need to be modified

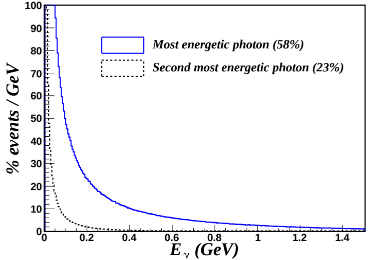
$$\frac{\left[\mathcal{B}_{ee}^{\rm low}\right]_{q=p_{e^+}+p_{e^-}+p_{\gamma_{\rm coll}}}}{\left[\mathcal{B}_{ee}^{\rm low}\right]_{q=p_{e^+}+p_{e^-}}} - 1 = 1.65\% \qquad \frac{\left[\mathcal{B}_{ee}^{\rm high}\right]_{q=p_{e^+}+p_{e^-}+p_{\gamma_{\rm coll}}}}{\left[\mathcal{B}_{ee}^{\rm high}\right]_{q=p_{e^+}+p_{e^-}}} - 1 = 6.8\%$$

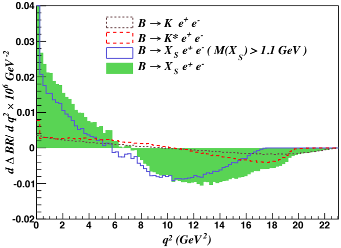


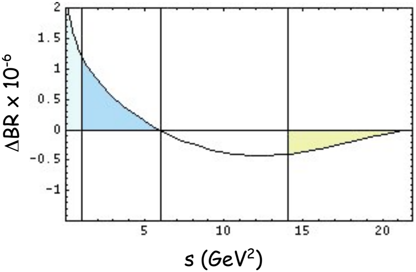


- Validation
  - Generate events (EVTGEN), hadronise (JETSET), add EM radiation (PHOTOS)









H_T

- Results for H_T, integrated over bins in low-q^2 region, in units of 10^{-6}
  - Electron channel (still preliminary)

$$H_T[1,3.5]_{ee} = 0.29 \pm 0.02$$
$$H_T[3.5,6]_{ee} = 0.24 \pm 0.02$$
$$H_T[1,6]_{ee} = 0.53 \pm 0.04$$

  - Muon channel (still preliminary)

$$H_T[1,3.5]_{\mu\mu} = 0.21 \pm 0.01$$
$$H_T[3.5,6]_{\mu\mu} = 0.19 \pm 0.02$$
$$H_T[1,6]_{\mu\mu} = 0.40 \pm 0.03$$

- Total error $\mathcal{O}(5-8\%)$. Still dominated by scale uncertainty.

H_L

- Results for H_L, integrated over bins in low-q^2 region, in units of 10^{-6}
  - Electron channel (still preliminary)

$$H_L[1,3.5]_{ee} = 0.64 \pm 0.03$$
$$H_L[3.5,6]_{ee} = 0.50 \pm 0.03$$
$$H_L[1,6]_{ee} = 1.13 \pm 0.06$$

  - Muon channel (still preliminary)

$$H_L[1,3.5]_{\mu\mu} = 0.68 \pm 0.04$$
$$H_L[3.5,6]_{\mu\mu} = 0.53 \pm 0.03$$
$$H_L[1,6]_{\mu\mu} = 1.21 \pm 0.07$$

- Again total error $\mathcal{O}(5-7\%)$.

Branching ratio, low-q^2 region

- Branching ratio, integrated over bins in low-q^2 region, in units of 10^{-6}
  - Electron channel (still preliminary)

$$\begin{aligned}\mathcal{B}[1,3.5]_{ee} &= 0.93 \pm 0.03_{\text{scale}} \pm 0.01_{m_t} \pm 0.03_{C,m_c} \pm 0.01_{m_b} \pm 0.002_{\alpha_s} \pm 0.003_{\text{CKM}} \pm 0.01_{\text{BR}_{\text{sl}}} \\ &= 0.93 \pm 0.05\end{aligned}$$

$$\begin{aligned}\mathcal{B}[3.5,6]_{ee} &= 0.74 \pm 0.04_{\text{scale}} \pm 0.01_{m_t} \pm 0.03_{C,m_c} \pm 0.01_{m_b} \pm 0.003_{\alpha_s} \pm 0.002_{\text{CKM}} \pm 0.01_{\text{BR}_{\text{sl}}} \\ &= 0.74 \pm 0.05\end{aligned}$$

$$\begin{aligned}\mathcal{B}[1,6]_{ee} &= 1.67 \pm 0.07_{\text{scale}} \pm 0.02_{m_t} \pm 0.06_{C,m_c} \pm 0.02_{m_b} \pm 0.01_{\alpha_s} \pm 0.005_{\text{CKM}} \pm 0.02_{\text{BR}_{\text{sl}}} \\ &= 1.67 \pm 0.10\end{aligned}$$

  - Muon channel (still preliminary)

$$\begin{aligned}\mathcal{B}[1,3.5]_{\mu\mu} &= 0.89 \pm 0.03_{\text{scale}} \pm 0.01_{m_t} \pm 0.03_{C,m_c} \pm 0.01_{m_b} \pm 0.002_{\alpha_s} \pm 0.002_{\text{CKM}} \pm 0.01_{\text{BR}_{\text{sl}}} \\ &= 0.89 \pm 0.05\end{aligned}$$

$$\begin{aligned}\mathcal{B}[3.5,6]_{\mu\mu} &= 0.73 \pm 0.04_{\text{scale}} \pm 0.01_{m_t} \pm 0.03_{C,m_c} \pm 0.01_{m_b} \pm 0.003_{\alpha_s} \pm 0.002_{\text{CKM}} \pm 0.01_{\text{BR}_{\text{sl}}} \\ &= 0.73 \pm 0.05\end{aligned}$$

$$\begin{aligned}\mathcal{B}[1,6]_{\mu\mu} &= 1.62 \pm 0.07_{\text{scale}} \pm 0.02_{m_t} \pm 0.05_{C,m_c} \pm 0.02_{m_b} \pm 0.01_{\alpha_s} \pm 0.005_{\text{CKM}} \pm 0.02_{\text{BR}_{\text{sl}}} \\ &= 1.62 \pm 0.09\end{aligned}$$

- Again total error $\mathcal{O}(5-7\%)$, dominated by scale uncertainty.

H_A

- Results for H_A, integrated over bins in low-q^2 region, in units of 10^{-6}
  - Electron channel (still preliminary)

$$H_A[1,3.5]_{ee} = -\,0.103 \pm 0.005$$
$$H_A[3.5,6]_{ee} = +\,0.073 \pm 0.012$$
$$H_A[1,6]_{ee} = -\,0.029 \pm 0.016$$

  - Muon channel (still preliminary)

$$H_A[1,3.5]_{\mu\mu} = -\,0.110 \pm 0.005$$
$$H_A[3.5,6]_{\mu\mu} = +\,0.067 \pm 0.012$$
$$H_A[1,6]_{\mu\mu} = -\,0.042 \pm 0.016$$

- Single bins much better behaved than entire low-$\hat{s}$ region, owing to cancellations due to zero crossing

Zero of H_A (FBA)

- Forward-backward asymmetry (or H_A) has a zero in low-q^2 region
- Electron channel (still preliminary)

$$(q_0^2)_{ee} = (3.46 \pm 0.10_{\rm scale} \pm 0.001_{m_t} \pm 0.02_{C,m_c} \pm 0.06_{m_b} \pm 0.02_{\alpha_s})\ {\rm GeV}^2$$
$$= (3.46 \pm 0.11)\ {\rm GeV}^2$$

- Muon channel (still preliminary)

$$(q_0^2)_{\mu\mu} = (3.58 \pm 0.10_{\rm scale} \pm 0.001_{m_t} \pm 0.02_{C,m_c} \pm 0.06_{m_b} \pm 0.02_{\alpha_s})\ {\rm GeV}^2$$
$$= (3.58 \pm 0.12)\ {\rm GeV}^2$$

High-q^2 region

- Branching ratio, integrated over high-q^2 region, in units of 10^{-7}
  - Electron channel (still preliminary)

$$\begin{aligned}\mathcal{B}[>14.4]_{ee} =& 2.20 \pm 0.30_{\rm scale} \pm 0.03_{m_t} \pm 0.06_{C,m_c} \pm 0.16_{m_b} \pm 0.003_{\alpha_s} \pm 0.01_{\rm CKM} \pm 0.03_{\rm BR_{sl}} \\ & \pm 0.12_{\lambda_2} \pm 0.48_{\rho_1} \pm 0.36_{f_s} \pm 0.05_{f_u} \\ =& 2.20 \pm 0.70\end{aligned}$$

  - Muon channel (still preliminary)

$$\begin{aligned}\mathcal{B}[>14.4]_{\mu\mu} =& 2.53 \pm 0.29_{\rm scale} \pm 0.03_{m_t} \pm 0.07_{C,m_c} \pm 0.18_{m_b} \pm 0.003_{\alpha_s} \pm 0.01_{\rm CKM} \pm 0.03_{\rm BR_{sl}} \\ & \pm 0.12_{\lambda_2} \pm 0.48_{\rho_1} \pm 0.36_{f_s} \pm 0.05_{f_u} \\ =& 2.53 \pm 0.70\end{aligned}$$

- Total error $\mathcal{O}(30\%)$
- Significantly lower values compared to earlier works [Greub,Pilipp,Schüpbach'08]
  - Main reaons: Power corrections, QED corrections, different $q^2_{\rm min}$
  - To lesser extend: Input parameters, normalisation
  - Perfect agreement if we switch to prescription by Greub et. al.

High-q^2 region

- Ratio $\mathcal{R}(q^2_{\rm min})$, integrated over high-$q^2$ region, in units of 10^{-3}
  - Electron channel (still preliminary)

$$\begin{aligned}\mathcal{R}(14.4)_{ee} =& 2.25 \pm 0.12_{\rm scale} \pm 0.03_{m_t} \pm 0.02_{C,m_c} \pm 0.01_{m_b} \pm 0.01_{\alpha_s} \pm 0.20_{\rm CKM} \\ & \pm 0.02_{\lambda_2} \pm 0.14_{\rho_1} \pm 0.08_{f_u^0+f_s} \pm 0.12_{f_u^0-f_s} \\ =& 2.25 \pm 0.31\end{aligned}$$

  - Muon channel (still preliminary)

$$\begin{aligned}\mathcal{R}(14.4)_{\mu\mu} =& 2.62 \pm 0.09_{\rm scale} \pm 0.03_{m_t} \pm 0.01_{C,m_c} \pm 0.01_{m_b} \pm 0.01_{\alpha_s} \pm 0.23_{\rm CKM} \\ & \pm 0.0002_{\lambda_2} \pm 0.09_{\rho_1} \pm 0.04_{f_u^0+f_s} \pm 0.12_{f_u^0-f_s} \\ =& 2.62 \pm 0.30\end{aligned}$$

- Total error $\mathcal{O}(10-15\%)$.
  - Uncertainties due to power corrections significantly reduced
  - Largest source of error are CKM elements (V_{ub})

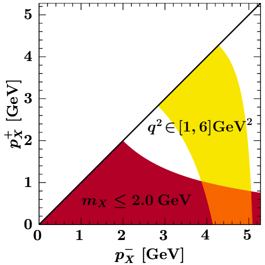
Cuts on M_{X_s}

- The suppression of background from $b \to c\ (\to s\ell\nu)\ \ell\nu$ requires a cut on M_{X_s}. Have $M_{X_s} < 1.8\ (2.0)$ GeV at BaBar (Belle).
- Usually taken into account on experimental side

- This puts kinematics at low-q^2 into the shape function region

  $\Rightarrow$ SCET applicable, define
  $p_X^{\pm} = E_X \mp |\vec{p}_X|$ [Lee,Ligeti,Stewart,Tackmann'06]

- High-q^2 region hardly affected by the cut

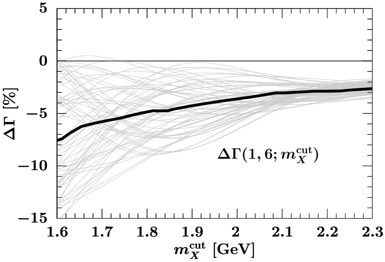


Cuts on M_{X_s}

- Compute non-perturbative corrections of leading and subleading order in $\Lambda_{\rm QCD}/m_b$ [Lee,Tackmann'08]

- Effect on H_i and Γ is ~ -5 to -10%

- Shift of zero of FBA is ~ -0.05 to -0.10 GeV^2



- Add NNLO QCD-corrections to heavy-light currents in shape function region [Bell,Beneke,Li,TH'10]

- Zero of FBA

$$q_0^2 = \left[(3.34 \ldots 3.40)^{+0.22}_{-0.25}\right] \text{GeV}^2 \qquad \text{for} \qquad m_X^{\rm cut} = (2.0 \ldots 1.8)\,\text{GeV}$$

  - In same region as inclusive result
  - Significantly smaller than exclusive result [Beneke,Feldmann,Seide'01]

Inclusive $\bar{B} \to X_s\,\gamma$

- Current experimental world average [HFAG'13]

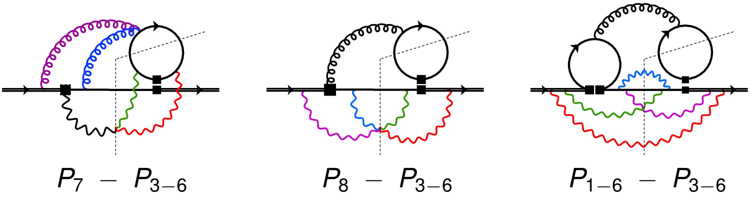
$$\mathcal{B}(\bar{B} \to X_s\gamma)^{E_0>1.6\,\text{GeV}}_{\text{exp}} = (3.43 \pm 0.22) \times 10^{-4}$$

- Standard Model prediction [Misiak et.al.'06]

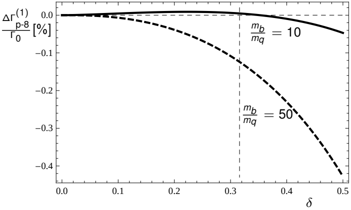
$$\mathcal{B}(\bar{B} \to X_s\gamma)^{E_0>1.6\,\text{GeV}}_{\text{SM}} = (3.15 \pm 0.23) \times 10^{-4}$$

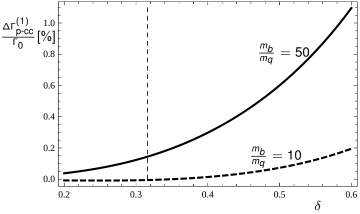
- Agreement is at the 1σ level
- Both uncertainties are at the $\pm 7\%$ level
  - $\pm 3\%$ of which stem from unknown higher order corrections

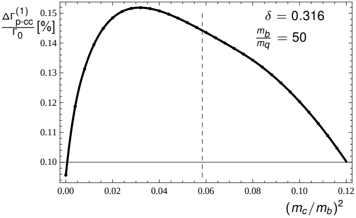
    $\Longrightarrow$ Here: Four-body contributions $b \to s\, q\bar{q}\,\gamma$ at NLO
- Several interferences



- Technicalities
  - Integration over four-body phase space in D dimensions
  - Dependence on charm mass m_c and photon energy cut $\delta = 1 - 2E_\gamma/m_b$
  - Again enhancement $\propto \ln(m_b/m_q)$ from energetic collinear photons
- Preliminary results: The corrections stay within $\lesssim 1\%$ of the LO rate







Conclusion

- Inclusive $\bar{B} \to X_s \, \ell^+ \ell^-$ is an unsung hero
  - Complementarity to $\bar{B} \to X_s \, \gamma$ and $\bar{B} \to K^{(*)} \, \mu^+ \mu^-$ can help in the search for NP
- Pheno analysis to NNLO QCD + NLO QED for all angular observables is almost complete
  - Careful investigation of treatment of energetic collinear photons
  - Most observables have parametric + perturbative errors of $\mathcal{O}(5 - 10\%)$
- Four-body contributions to $\bar{B} \to X_s \, \gamma$ at NLO stay within $\lesssim 1\%$ of the LO rate

Backup slides

WC sensitivity

- Data extrapolated to 1ab^{-1}. Constraints in C_9-C_{10} plane [Lee,Ligeti,Stewart,Tackmann'06]