# CHARMLESS HADRONIC B DECAYS: THEORY STATUS

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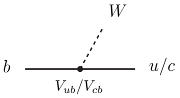
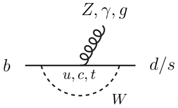
THE ROYAL SOCIETY

# Hadronic B decays

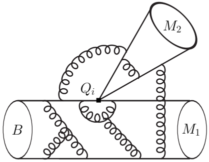
Excellent laboratory to probe the nature of flavour-changing quark transitions




- tree-dominated decays: $B \to \pi\pi$, $B \to \rho\rho$, $B_s \to \pi K$, ...
- penguin-dominated decays: $B \to \pi K$, $B \to \phi K_S$, $B_s \to \phi\phi$, ...

The challenge:




Two strategies:

- flavour symmetries:

  isospin, SU(3), U-spin, V-spin
- heavy-quark expansions:

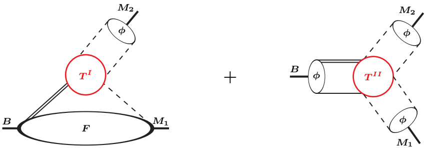
  QCDF, SCET, pQCD

# QCD factorisation

[Beneke, Buchalla, Neubert, Sachrajda 99]

Hadronic matrix elements factorise in heavy quark limit $m_b \gg \Lambda_{QCD}$

$$\langle M_1 M_2 | Q_i | B \rangle \simeq F^{BM_1}(0) \int du \; T_i^I(u) \; \phi_{M_2}(u) + \int d\omega \, du \, dv \; T_i^{II}(\omega, u, v) \; \phi_B(\omega) \; \phi_{M_1}(v) \; \phi_{M_2}(u)$$




vertex corrections $T^I = 1 + \mathcal{O}(\alpha_s)$ spectator scattering $T^{II} = \mathcal{O}(\alpha_s)$

- valid to all orders in $\alpha_s(m_b)$ and to leading power in $\Lambda_{QCD}/m_b$
- strong phases from final-state interactions $\sim \mathcal{O}(\alpha_s), \; \mathcal{O}(1/m_b)$

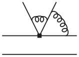
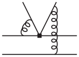
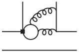
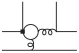
$\Rightarrow$ NNLO is first correction for direct CP asymmetries!

# NNLO calculation in QCDF

Two hard-scattering kernels for each operator insertion

$$\langle M_1 M_2 | Q_i | B \rangle \;\simeq\; F^{BM_1}\, T_i^I \otimes \phi_{M_2} \;+\; T_i^{II} \otimes \phi_B \otimes \phi_{M_1} \otimes \phi_{M_2}$$

and two classes of topological amplitudes (trees vs. penguins)

| Status | 2-loop vertex corrections ($T_i^I$) | 1-loop spectator scattering ($T_i^{II}$) |
|---|---|---|
| Trees |  [GB 07, 09] [Beneke, Huber, Li 09] |  [Beneke, Jäger 05] [Kivel 06] [Pilipp 07] |
| Penguins |  [in progress] |  [Beneke, Jäger 06] [Jain, Rothstein, Stewart 07] |

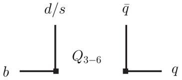
- first NNLO results for tree-dominated observables [GB, Pilipp 09; Beneke, Huber, Li 09]
- no NNLO results for direct CP asymmetries yet
- power-suppressed scalar penguin amplitude known at NLO

# Missing NNLO ingredient

Various contributions to up/charm QCD penguin amplitudes

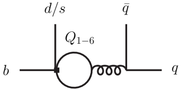
- tree insertions of penguin operators

  2-loop, similar to tree calculation



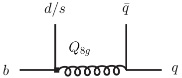

- penguin insertions of current-current and penguin operators

  2-loop, charm quark propagator introduces additional scale



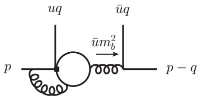

- insertions of magnetic dipole operator

  1-loop, much simpler [Kim, Yoon 11]




$\mathcal{O}(70)$ diagrams at NNLO

- 2 loops, 3 scales ($m_b$, $um_b$, $m_c$), 4 legs
- charm contribution has non-trivial threshold at $\bar{u}m_b^2 \gtrsim 4m_c^2$



# 2-loop penguin contribution

[GB, Beneke, Huber, Li in progress]

Automated reduction to scalar master integrals (IBP, Laporta)

$\Rightarrow$ $\mathcal{O}(20)$ additional master integrals compared to 2-loop tree calculation

New proposal to choose ”optimal” basis of master integrals [Henn 13]

- simple iterated integrations in each order in $\varepsilon$-expansion
- no systematic construction of such a basis exists so far

$\Rightarrow$ we could find such a basis for all master integrals

Calculation of master integrals

- analytic results in terms of Goncharov polylogarithms
- two independent numerical implementations for cross checks

  Mellin-Barnes + sector decomposition + Feynman parameters $\Rightarrow$ agreement $\sim 10^{-4}$

# Status of calculation

[GB, Beneke, Huber, Li in progress]

Various operator insertions:

- $Q^u_{1,2}$: fully analytic results
- $Q^c_{1,2}$: analytic results for kernels $T^I_i(u)$, numerical results for $\int du\; T^I_i(u)\, \phi_{M_2}(u)$
- $Q_{3\text{-}6}$: 2-loop matrix elements complete, working on UV and IR subtractions ...

Preliminary result for up penguin amplitude

$$
\begin{aligned}
a_4^u(\pi\pi) &= -\,[\,0.029\,]_{V_0} + [\,0.003 - 0.014\,i\,]_{V_1} - [\,0.003 + 0.007\,i\,]_{V_2} \\
&\quad + [\,0.001\,]_{S_1} + [\,0.001 + 0.002\,i\,]_{S_2} + [\,0.001\,]_{1/m_b} \\
&= -\,0.025 - 0.019\,i
\end{aligned}
$$

- includes 2-loop contribution from $Q^u_{1,2}$ only
- $\sim$ 15% correction to real part, $\sim$ 50% to imaginary part

# Status of calculation

[GB, Beneke, Huber, Li in progress]

Various operator insertions:

- $Q^u_{1,2}$: fully analytic results
- $Q^c_{1,2}$: analytic results for kernels $T^I_i(u)$, numerical results for $\int du\; T^I_i(u)\,\phi_{M_2}(u)$
- $Q_{3\text{-}6}$: 2-loop matrix elements complete, working on UV and IR subtractions . . .

Preliminary result for charm penguin amplitude

$$\begin{aligned} a_4^c(\pi\pi) &= -\,[\,0.029\,]_{V_0} \;-\; [\,0.001 + 0.008\,i\,]_{V_1} \;-\; [\,0.008 + 0.005\,i\,]_{V_2} \\ &\quad +\,[\,0.001\,]_{S_1} \;+\; [\,0.001 + 0.001\,i\,]_{S_2} \;+\; [\,0.001\,]_{1/m_b} \\ &= -\,0.034 \;-\; 0.012\,i \end{aligned}$$

- includes 2-loop contribution from $Q^u_{1,2}$ only
- $\sim 30\%$ correction to real part, $\sim 80\%$ to imaginary part

# Tree amplitudes

Full NNLO results for colour-allowed and colour-suppressed tree amplitudes

$$
\begin{aligned}
\alpha_1(\pi\pi) &= [\,1.008\,]_{V_0} + [\,0.022 + 0.009\,i\,]_{V_1} + [\,0.024 + 0.026\,i\,]_{V_2} \\
&\quad - [\,0.014\,]_{S_1} - [\,0.016 + 0.012\,i\,]_{S_2} - [\,0.008\,]_{1/m_b} \\
&= 1.015^{+0.020}_{-0.029} + (0.023^{+0.015}_{-0.015})\,i
\end{aligned}
$$

$$
\begin{aligned}
\alpha_2(\pi\pi) &= [\,0.224\,]_{V_0} - [\,0.174 + 0.075\,i\,]_{V_1} - [\,0.029 + 0.046\,i\,]_{V_2} \\
&\quad + [\,0.084\,]_{S_1} + [\,0.037 + 0.022\,i\,]_{S_2} + [\,0.052\,]_{1/m_b} \\
&= 0.194^{+0.130}_{-0.095} - (0.099^{+0.057}_{-0.056})\,i
\end{aligned}
$$

- individual NNLO corrections significant, but cancellations in the sum
- $\alpha_1$: stable under radiative corrections, precise prediction
- $\alpha_2$: sizeable hadronic uncertainties, mainly from $\lambda_B^{-1} = \int \frac{d\omega}{\omega}\,\phi_B(\omega)$
- small relative phase between $\alpha_1$ and $\alpha_2$

# $B \to \pi\pi/\pi\rho/\rho\rho$

[GB, Pilipp 09]

CP-averaged branching ratios in units of $10^{-6}$

| Mode | QCDF | B | Experiment |
|---|---|---|---|
| $\pi^-\pi^0$ | $6.22^{+2.37}_{-2.01}$ | 5.46 | $5.48^{+0.35}_{-0.34}$ |
| $\rho_L^-\rho_L^0$ | $21.0^{+8.5}_{-7.3}$ | 21.3 | $22.5^{+1.9}_{-1.9}$ |
| $\pi^-\rho^0$ | $9.34^{+4.00}_{-3.23}$ | 10.4 | $8.3^{+1.2}_{-1.3}$ |
| $\pi^0\rho^-$ | $15.1^{+5.7}_{-5.0}$ | 11.9 | $10.9^{+1.4}_{-1.5}$ |
| $\pi^+\pi^-$ | $8.96^{+3.78}_{-3.32}$ | 5.21 | $5.10^{+0.19}_{-0.19}$ |
| $\pi^0\pi^0$ | $0.35^{+0.37}_{-0.21}$ | 0.63 | $1.91^{+0.22}_{-0.23}$ |
| $\pi^+\rho^-$ | $22.8^{+9.1}_{-8.0}$ | 13.2 | $15.7^{+1.8}_{-1.8}$ |
| $\pi^-\rho^+$ | $11.5^{+5.1}_{-4.3}$ | 8.41 | $7.3^{+1.2}_{-1.2}$ |
| $\pi^\pm\rho^\mp$ | $34.3^{+11.5}_{-10.0}$ | 21.6 | $23.0^{+2.3}_{-2.3}$ |
| $\pi^0\rho^0$ | $0.52^{+0.76}_{-0.42}$ | 1.64 | $2.0^{+0.5}_{-0.5}$ |
| $\rho_L^+\rho_L^-$ | $30.3^{+12.9}_{-11.2}$ | 22.3 | $23.6^{+3.2}_{-3.2}$ |
| $\rho_L^0\rho_L^0$ | $0.44^{+0.66}_{-0.37}$ | 1.33 | $0.69^{+0.30}_{-0.30}$ |

B: mimics enhanced colour-suppressed amplitude
(with $\lambda_B \to \lambda_B/2$ and smaller form factors)

- theory uncertainties highly correlated (form factors, $|V_{ub}|$)
- colour-suppressed modes $\pi^0\pi^0/\pi^0\rho^0/\rho^0\rho^0$ uncertain ($\lambda_B$ and $1/m_b$)
- overall preference for enhanced colour-suppressed amplitude
- new Belle measurement $\text{Br}(\pi^0\pi^0) = (0.90 \pm 0.16) \cdot 10^{-6}$ about $3\sigma$ below Babar value?

[for a similar analysis cf. Beneke, Huber, Li 09]

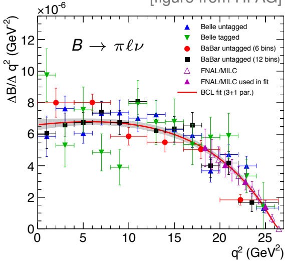
# Semileptonic ratios

Can eliminate dependence on form factors and $|V_{ub}|$ via

$$\mathcal{R}_{M_3}(M_1 M_2) = \frac{\Gamma(B \to M_1 M_2)}{d\Gamma(B \to M_3 \ell\nu)/dq^2|_{q^2=0}}$$

$\Rightarrow$ requires to measure semileptonic decay spectrum and extrapolation to $q^2 = 0$

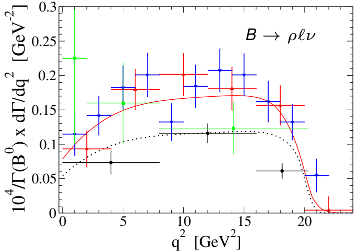
[figure from HFAG]




$$|V_{ub}|F_+^{B\pi}(0) = (9.1 \pm 0.7) \cdot 10^{-4}$$

[Ball 06]

[figure from Albertus et al 14]




$$|V_{ub}|A_0^{B\rho}(0) = (11.4 \pm 1.5) \cdot 10^{-4}$$

[Albertus, Hernandez, Nieves 14]

# Semileptonic ratios

[GB, Pilipp 09; CKM 2014 update]

Can eliminate dependence on form factors and $|V_{ub}|$ via

$$\mathcal{R}_{M_3}(M_1 M_2) = \frac{\Gamma(B \to M_1 M_2)}{d\Gamma(B \to M_3 \ell\nu)/dq^2|_{q^2=0}}$$

| Mode | QCDF | B | Experiment |
|---|---|---|---|
| $\mathcal{R}_\pi(\pi^- \pi^0)$ | $0.70^{+0.12}_{-0.08}$ | 0.95 | $0.79 \pm 0.13$ |
| $\mathcal{R}_\rho(\rho_L^- \rho_L^0)$ | $1.91^{+0.32}_{-0.23}$ | 2.38 | $2.21 \pm 0.61$ |
| $\mathcal{R}_\rho(\pi^- \rho^0)$ | $0.85^{+0.22}_{-0.14}$ | 1.16 | $0.82 \pm 0.25$ |
| $\mathcal{R}_\pi(\pi^0 \rho^-)$ | $1.71^{+0.27}_{-0.24}$ | 2.07 | $1.58 \pm 0.33$ |
| $\mathcal{R}_\pi(\pi^+ \pi^-)$ | $1.09^{+0.22}_{-0.20}$ | 0.97 | $0.80 \pm 0.13$ |
| $\mathcal{R}_\pi(\pi^+ \rho^-)$ | $2.77^{+0.32}_{-0.31}$ | 2.46 | $2.45 \pm 0.47$ |
| $\mathcal{R}_\rho(\pi^- \rho^+)$ | $1.12^{+0.20}_{-0.14}$ | 1.01 | $0.77 \pm 0.24$ |
| $\mathcal{R}_\rho(\rho_L^+ \rho_L^-)$ | $2.95^{+0.37}_{-0.35}$ | 2.68 | $2.50 \pm 0.74$ |
| $R(\rho_L^- \rho_L^0 / \rho_L^+ \rho_L^-)$ | $0.65^{+0.16}_{-0.11}$ | 0.89 | $0.88 \pm 0.14$ |
| $R(\pi^- \pi^0 / \pi^+ \pi^-)$ | $0.65^{+0.19}_{-0.14}$ | 0.98 | $1.00 \pm 0.07$ |

B: mimics enhanced colour-suppressed amplitude (with $\lambda_B \to \lambda_B/2$ and smaller form factors)

- theory uncertainties largely reduced
- satisfactory description of clean observables
- $B \to \rho\ell\nu$ ratios not yet competitive

# Tree-dominated $B_s$ decays

[GB, CKM 2010]

CP-averaged branching ratios in units of $10^{-6}$

| Mode | QCDF | B | Experiment |
|---|---|---|---|
| $\pi^- K^+$ | $8.73^{+5.77}_{-4.60}$ | 4.88 | $5.4^{+0.6}_{-0.6}$ |
| $\pi^0 K^0$ | $0.50^{+0.71}_{-0.35}$ | 1.12 | n.a. |
| $\pi^- K^{*+}$ | $15.4^{+8.6}_{-7.0}$ | 11.0 | n.a. |
| $\pi^0 K^{*0}$ | $0.39^{+0.58}_{-0.26}$ | 0.90 | n.a. |
| $\rho^- K^+$ | $22.4^{+14.7}_{-11.6}$ | 12.5 | n.a. |
| $\rho^0 K^0$ | $0.73^{+1.28}_{-0.58}$ | 2.24 | n.a. |
| $\rho_L^- K_L^{*+}$ | $40.7^{+22.4}_{-18.3}$ | 29.1 | n.a. |
| $\rho_L^0 K_L^{*0}$ | $0.70^{+1.07}_{-0.54}$ | 1.87 | n.a. |
| $\rho^- K^+ / \pi^- K^+$ | $2.57^{+0.31}_{-0.26}$ | 2.57 | n.a. |
| $\rho_L^- K_L^{*+} / \pi^- K^{*+}$ | $2.64^{+0.31}_{-0.33}$ | 2.65 | n.a. |

B: mimics enhanced colour-suppressed amplitude
(with $\lambda_{B_s} \to \lambda_{B_s}/2$ and smaller form factors)

- hadronic parameters less well known (form factors, $\lambda_{B_s}$)
- simpler pattern of annihilation contributions
- clean ratios of colour-allowed modes can be used to test charming penguins [Zhu 10]

# B-meson LCDA

Recent progress in theoretical understanding of $\phi_B(\omega;\mu)$

- expansion in eigenfunctions of 1-loop evolution kernel [GB, Feldmann, Wang, Yip 13]
- formulation in terms of conformal symmetry generators [Braun, Manashov 14]
- consistent implementation of perturbative constraints [Feldmann, Lange, Wang 14]

Most important for phenomenology $\lambda_B^{-1}(\mu) = \int \frac{d\omega}{\omega}\, \phi_B(\omega;\mu)$

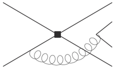
- QCD sum rule estimate $\lambda_B(1\text{GeV}) \simeq (460 \pm 110)$ MeV [Braun, Ivanov, Korchemsky 03]
- but $\pi\pi/\pi\rho/\rho\rho$ branching ratios seem to prefer $\sim$ 200 MeV ?

$\lambda_B$ can be measured in $B \to \gamma\ell\nu$ decays

- state-of-the-art analyses (NLL, tree-level $1/m_b$) [Beneke, Rohrwild 11; Braun, Khodjamirian 12]
- Babar 09 data $\Rightarrow$ $\lambda_B(1\text{GeV}) > 115$ MeV

# Weak annihilation

Power-suppressed annihilation amplitudes introduce model dependence



$\Rightarrow \quad X_A = \left(1 + \rho_A \, e^{i\phi_A}\right) \ln \frac{m_B}{\Lambda_h}$ IR-cutoff $\Lambda_h = 0.5$ GeV

Two-parameter model: $\rho_A \leq 1$ and arbitrary universal soft-rescattering phase $\phi_A$

Can gain insights from pure annihilation decays

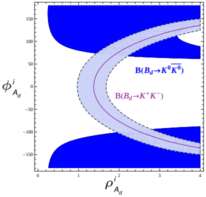
| | BBNS (S4) | |
|---|---|---|
| $10^6 \, \mathrm{Br}(B_d \to K^+K^-) = 0.12 \pm 0.05$ | 0.07 | ($\Delta D = 1$, exchange topology) |
| $10^6 \, \mathrm{Br}(B_s \to \pi^+\pi^-) = 0.73 \pm 0.14$ | 0.16 | ($\Delta S = 1$, penguin annihilation) |

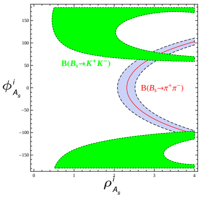
$\Rightarrow$ challenges universal annihilation model

global $\pi\pi/\pi K/KK$ analysis gives

$\rho_A^f \sim 1.6, \; \rho_{A_d}^i \sim 2.5, \; \rho_{A_s}^i \sim 3.0$

[Wang, Zhu 13]



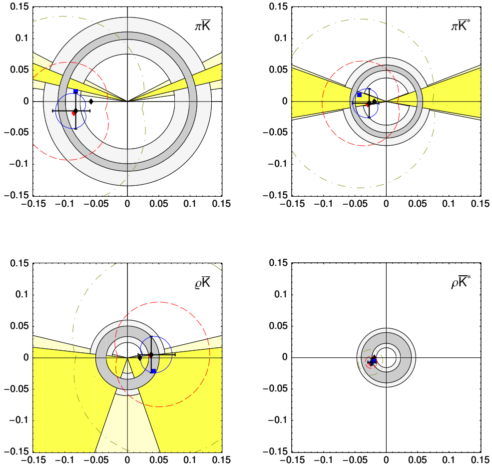
# Scalar penguin amplitude

Calculable power correction from insertion of Fierz-transformed penguin operators $\Rightarrow$ $a_6^p(M_1 M_2)$

Distinct final state dependence

- PP $\sim$ $a_4^p + r_\chi\, a_6^p$
- PV $\sim$ $a_4^p$
- VP $\sim$ $a_4^p - r_\chi\, a_6^p$
- VV $\sim$ $a_4^p$

$\Rightarrow$ pattern of $P/T$ ratios in good agreement with data




[figures from Beneke, Jäger 06]

# Conclusion

- We are about to complete the NNLO calculation in hadronic $B$ decays –

  phenomenlogocial update of all $B \to PP/PV/VV$ observables largely overdue

- Lattice and QCD sum rule calculations provide valuable input for our predictions –

  some hadronic parameters can also be constrained by experimental measurements

- Main limitation of QCDF is poor understanding of power corrections –

  improved modelling based on flavour symmetries possible

# Backup slides

# Comparison

[BBNS: Beneke, Buchalla, Neubert, Sachrajda 99]
[BPRS: Bauer, Pirjol, Rothstein, Stewart 04]
[pQCD: Keum, Li, Sanda 00]

| | BBNS (QCDF) | BPRS (SCET) | pQCD |
|---|---|---|---|
| $\alpha_s(\sqrt{\Lambda m_b})$ | perturbative | non-perturbative | perturbative |
| charm loops | perturbative (small phase) | non-perturbative (large phase from fit to data) | perturbative (small phase) |
| weak annihilation (power correction) | non-perturbative (model, arbitrary phase) | perturbative (zero bins, small phase) | perturbative (large phase) |
| perturbative calculation | towards NNLO | NLO | essentially LO |
| hadronic input | from lattice + QCD sum rules | from QCD sum rules + data, model $\xi_J^{BM}(z)$ | from QCD sum rules + data, model $\phi_B(x, b)$ |

- conceptually QCDF $=$ SCET $\neq$ pQCD

  but phenomenological implementations of BBNS and BPRS differ
- some open problems in BPRS (zero-bin subtractions) and pQCD (glauber gluons)