# Lattice QCD study of B-meson decay constants from ETMC


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with

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**ETM Collaboration**


[1308.1851, 1311.2837]


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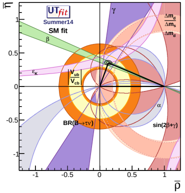
# CKM matrix & lattice QCD

$$V_{\rm CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$




## lattice determinations

| CKM | process | lattice | precision (%) |
|---|---|---|---|
| $\lvert V_{us}\rvert$ | $K \to \ell\nu$ | $f_K$ | $\leq$ 1.5 |
| | $K \to \pi\ell\nu$ | $f_+^{K\pi}(q^2=0)$ | $\leq$ 1.0 |
| $\lvert V_{us}\rvert/\lvert V_{ud}\rvert$ | $K \to \mu\nu / \pi \to \mu\nu$ | $f_K/f_\pi$ | $\leq$ 1.5 |
| $\lvert V_{cd}\rvert$ | $D \to \ell\nu$ | $f_{D_s}/f_D$ | $\leq$ 1.5 |
| | $D \to \pi\ell\nu$ | $f_{+,0}^{D\pi}(0)$ | $\sim$ 5.0 |
| $\lvert V_{cs}\rvert$ | $D_s \to \ell\nu$ | $f_{D_s}$ | $\leq$ 2.5 |
| | $D \to K\ell\nu$ | $f_+^{DK}(0)$ | $\sim$ 3.0 |
| $\lvert V_{ub}\rvert$ | $B \to \ell\nu$ | $f_B$ | $\leq$ 4.0 |
| | | $f_{B_s}/f_B$ | $\leq$ 2.0 |
| | $B \to \pi\ell\nu$ | $f_+^{B\pi}(q^2)$ | $\sim$ 10.0 |
| $\lvert V_{cb}\rvert$ | $B \to D^{(*)}\ell\nu$ | $\mathcal{F}_{B\to D^{(*)}}$ | $\sim$ 1.5 |
| $V_{tq}^* V_{tq'}$ ; $V_{cq}^* V_{cq'}$ | $\epsilon_K$ | $\hat{B}_K$ | $\leq$ 4.0 |
| | | $K \to \pi\pi$ | $\to$ 30.0 |
| $V_{tq}^* V_{tq'}$ | $\Delta m_d$ | $\hat{B}_{B_s}/\hat{B}_{B_d}$ ; $\xi$ | $\leq$ 5.0 |
| | $\Delta m_s$ | $\hat{B}_{B_s}$ | $\leq$ 5.0 |

- quark masses, BSM four-fermion operators, ...

# B-meson decay constants

- definition of decay constants

$$\langle 0|\, A^\mu \,|B_q(p)\rangle \;=\; p_B^\mu\, f_{B_q}$$

with $q = u, d, s$

- $f_B$ , $f_{B_s}$ : $|V_{ub}|$, $\mathcal{B}(B \to \tau\nu)$, rare leptonic decays, $B - \bar{B}$ mixing ...

- $f_B$ : leptonic decays

$$\mathcal{B}(B \to \tau\nu) \;\propto\; G_F^2\, f_B^2\, |V_{ub}|^2$$

BaBar, Belle : [$\sim$ 20%] $\rightsquigarrow$ Belle II : [$\sim$ 5%]

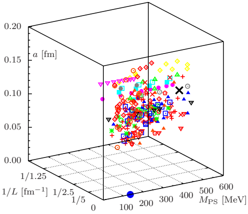
- $f_{B_s}$ : rare leptonic decays

$$\mathcal{B}(B_s \to \mu^+\mu^-) \;\propto\; M_{B_s}\, f_{B_s}^2\, |V_{tb}^* V_{ts}|^2$$

CMS, LHCb : [$\sim$ 25%]

# heavy quarks on the lattice

- lattice spacing : $a$
- lattice size : $L$
- pion masses : $M_{\rm PS}$
- number of flavours : $N_{\rm f}$




$$a \;\in\; [0.05,\, 0.15]\,\mathrm{fm} \qquad \rightsquigarrow \qquad 1/a \;\in\; [1.3,\, 4.0]\,\mathrm{GeV}$$

- to control discretisation effects for heavy quark masses: $m_h < 1/a$

$m_c \approx 1.3\,\mathrm{GeV}$ $\qquad$ $m_b \approx 4.3\,\mathrm{GeV}$ $\qquad$ $am_b > 1$

- for heavy quarks:

$$O[a] \;=\; O|_{\rm C.L.} \;+\; c\cdot(am_h)^2 \;+\; \ldots$$

- how to treat bottom quark on the lattice?

# heavy quarks on the lattice

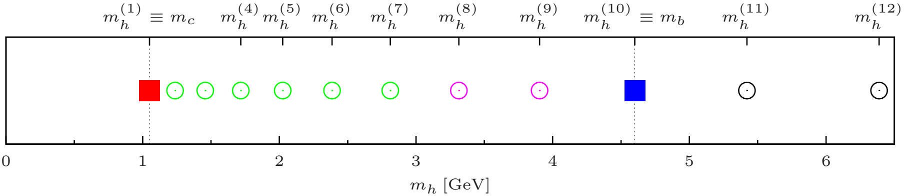
- how to treat bottom quark on the lattice? effective theories
  - heavy quark effective theory (HQET)
  - NRQCD
  - add counter-terms to relativistic action to account for HQET corrections
  - relativistic charm quarks $+$ LO HQET
  - chain of ratios with $m_h \geq m_c$ $+$ scaling laws of HQET $\rightsquigarrow$ ratio method

# ratio method

- series of quark masses : $m_h^{(i+1)} = \lambda\, m_h^{(i)}$

$m_h^{(1)} \approx m_c$

$\lambda \approx 1.18$




- chain equation for observable $O$ :

$$O(m_b) \equiv O(m_h^{(10)}) = O(m_h^{(1)}) \times \frac{O(m_h^{(2)})}{O(m_h^{(1)})} \times \ldots \times \frac{O(m_h^{(10)})}{O(m_h^{(9)})}$$

- ratios $z$ :

$$z(m_h^{(i)}) \equiv \frac{O(m_h^{(i)})}{O(m_h^{(i-1)})}$$

# ratio method

$\lambda \approx 1.18$

- cut-off effects in ratios $z$ :

$$z(m_h^{(i)}) \equiv \frac{O(m_h^{(i)})}{O(m_h^{(i-1)})}$$

$$\approx z(m_h^{(i)})\Big|_{\mathrm{C.L.}} + c \cdot (\lambda^2 - 1) \cdot \left(am_h^{(i)}\right)^2 + \ldots$$

- choice of $O$ : scaling law of HQET e.g. $O = \Phi = f_H \sqrt{M_H}$

$$O(m_h) = O_{\mathrm{stat}} + \frac{d}{m_h} + \ldots$$

- scaling law of ratio $z$

$$z(m_h) = 1 + \frac{\tilde{d} \cdot (\lambda - 1)}{m_h} + \ldots \xrightarrow[m_h \to \infty]{} 1$$

# ratio method: implementation

chain equation for observable $O$ :

$$O(m_b) \equiv O(m_h^{(10)}) = O(m_h^{(1)}) \times \frac{O(m_h^{(2)})}{O(m_h^{(1)})} \times \dots \times \frac{O(m_h^{(10)})}{O(m_h^{(9)})}$$

1. determine $O(m_h^{(1)})$ in the charm region
2. determine $z(m_h^{(i)})$, $i = 2, \dots, 7$, for $\lambda \approx 1.18$
3. use $z(m_h) \xrightarrow[m_h \to \infty]{} 1$ to reach $m_b$
4. apply chain eq. to get $O(m_b)$

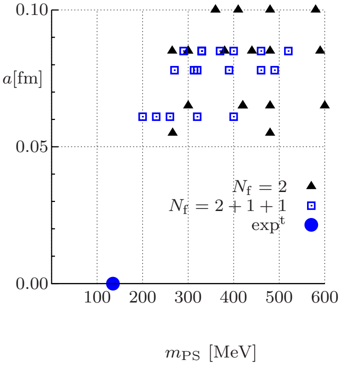
properties:

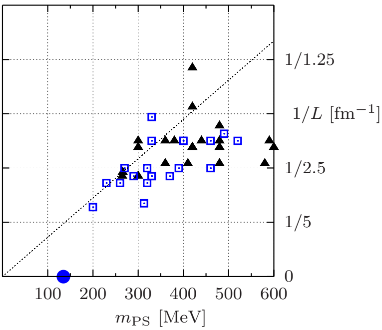
- continuum limit taken at each step : regularisation independent
- does not require explicit computation in an effective theory
- not very computationally costly [multi-mass solver]

# ETMC ensembles

- fermionic lattice action : Wilson twisted-mass
- $N_f = 2$ : $u, d$
- $N_f = 2+1+1$ : $u, d, s, c$

- physical input : $M_\pi$, $M_K$, $f_\pi$




- $m_h \in [m_c, 3m_c]$
- to isolate ground state : smearing $\rightsquigarrow$ GEVP

# ratio method : $f_B$ , $f_{B_s}$

chain equation for observable $O$ :

$$O(m_b) \equiv O(m_h^{(10)}) = O(m_h^{(1)}) \times \frac{O(m_h^{(2)})}{O(m_h^{(1)})} \times \ldots \times \frac{O(m_h^{(10)})}{O(m_h^{(9)})}$$

observables $O$

- $f_{hs} \sqrt{M_{hs}} \rightsquigarrow f_{B_s}$
- $f_{hs} \sqrt{m_h} \rightsquigarrow f_{B_s}$
- $f_{hs}/f_{hl} \rightsquigarrow f_{B_s}/f_B$
- $[\,(f_{hs}/f_{hl})\,/\,(f_{sl}/f_{ll})\,] \times (f_K/f_\pi) \rightsquigarrow f_{B_s}/f_B$

in practice, for $O = f_{hs}\sqrt{M_{hs}}$ :

$$\tilde{z}_s(\overline{\mu}_h, \lambda) = \frac{f_{hs}(\overline{\mu}_h)\sqrt{M_{hs}(\overline{\mu}_h)}}{f_{hs}(\overline{\mu}_h/\lambda)\sqrt{M_{hs}(\overline{\mu}_h/\lambda)}} \cdot \frac{C_A^{\rm stat}(\mu^*, \overline{\mu}_h/\lambda)}{C_A^{\rm stat}(\mu^*, \overline{\mu}_h)}$$

chain equation :

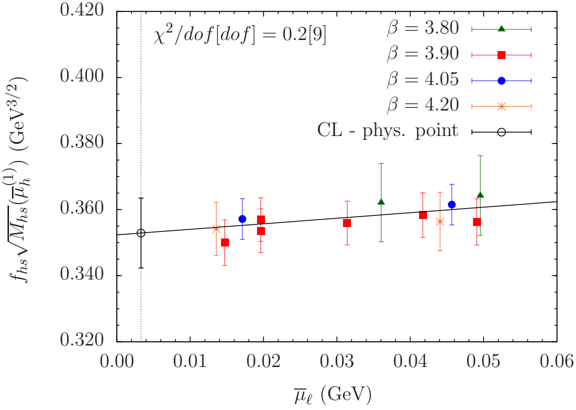
$$\tilde{z}_s(\overline{\mu}_h^{(2)}) \times \tilde{z}_s(\overline{\mu}_h^{(3)}) \times \ldots \times \tilde{z}_s(\overline{\mu}_h^{(10)}) = \frac{f_{hs}(\overline{\mu}_h^{(10)})\sqrt{M_{hs}(\overline{\mu}_h^{(10)})}}{f_{hs}(\overline{\mu}_h^{(1)})\sqrt{M_{hs}(\overline{\mu}_h^{(1)})}} \cdot \left[\frac{C_A^{\rm stat}(\mu^*, \overline{\mu}_h^{(1)})}{C_A^{\rm stat}(\mu^*, \overline{\mu}_h^{(10)})}\right]$$

$f_{B_s}$ $N_f = 2$

chain equation :

$$\tilde{z}_s(\overline{\mu}_h^{(2)}) \times \tilde{z}_s(\overline{\mu}_h^{(3)}) \times \ldots \times \tilde{z}_s(\overline{\mu}_h^{(10)}) = \frac{f_{hs}(\overline{\mu}_h^{(10)}) \sqrt{M_{hs}(\overline{\mu}_h^{(10)})}}{f_{hs}(\overline{\mu}_h^{(1)}) \sqrt{M_{hs}(\overline{\mu}_h^{(1)})}} \cdot \left[ \frac{C_A^{\rm stat}(\mu^*, \overline{\mu}_h^{(1)})}{C_A^{\rm stat}(\mu^*, \overline{\mu}_h^{(10)})} \right]$$

1. triggering point : $f_{hs}(\overline{\mu}_h^{(1)}) \sqrt{M_{hs}(\overline{\mu}_h^{(1)})}$




$\overline{\mu}_h^{(1)} = \overline{\mu}_c$

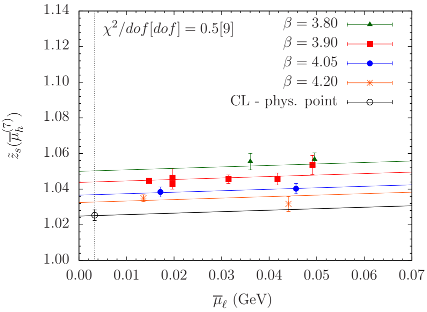
# $f_{B_s}$ $\qquad N_f = 2$

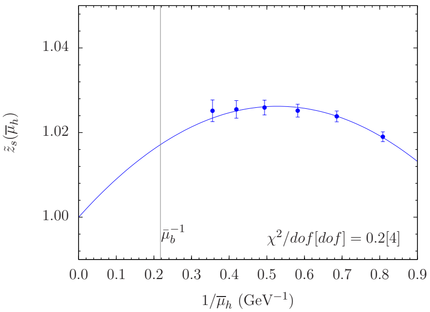
chain equation :

$$\tilde{z}_s(\overline{\mu}_h^{(2)}) \times \tilde{z}_s(\overline{\mu}_h^{(3)}) \times \ldots \times \tilde{z}_s(\overline{\mu}_h^{(10)}) = \frac{f_{hs}(\overline{\mu}_h^{(10)}) \sqrt{M_{hs}(\overline{\mu}_h^{(10)})}}{f_{hs}(\overline{\mu}_h^{(1)}) \sqrt{M_{hs}(\overline{\mu}_h^{(1)})}} \cdot \left[ \frac{C_A^{\mathrm{stat}}(\mu^*, \overline{\mu}_h^{(1)})}{C_A^{\mathrm{stat}}(\mu^*, \overline{\mu}_h^{(10)})} \right]$$

2. ratios : $\tilde{z}_s(\overline{\mu}_h^{(i)})$, $\quad i = 2, \ldots, 7$, for $\lambda = 1.1784$

3. $\tilde{z}_s(\overline{\mu}_h) = 1 + \frac{\eta_1}{\overline{\mu}_h} + \frac{\eta_2}{\overline{\mu}_h^2}$



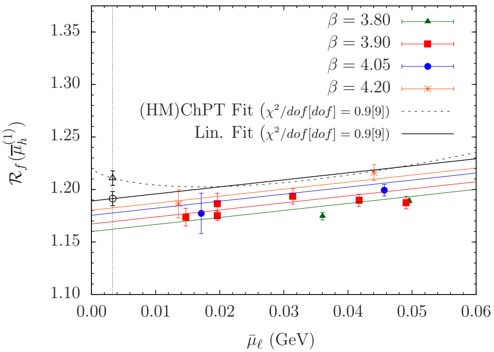




$\overline{\mu}_h^{(7)} \approx 2.7\,\overline{\mu}_c$

4. chain equation : $f_{B_s} = 228(5)(6)$ MeV

# $f_{B_s}/f_B$ $\qquad N_f = 2$

$$O \;=\; \mathcal{R}_f \;\equiv\; [\,(f_{hs}/f_{hl})\,/\,(f_{sl}/f_{ll})\,] \;\times\; (f_K/f_\pi)$$




$$\mathcal{R}_f \;=\; a_h^{(1)} \;+\; b_h^{(1)}\overline{\mu}_\ell \;+\; D_h^{(1)}a^2$$

$$\mathcal{R}_f \;=\; a_h^{(2)}\left[1 \;+\; b_h^{(2)}\overline{\mu}_\ell \;+\; \left[\frac{3(1+3\hat{g}^2)}{4} - \frac{5}{4}\right]\frac{2B_0\overline{\mu}_\ell}{(4\pi f_0)^2}\log\left(\frac{2B_0\overline{\mu}_\ell}{(4\pi f_0)^2}\right)\right] \;+\; D_h^{(2)}a^2$$

$$\rightsquigarrow \quad \frac{f_{B_s}}{f_B} \;=\; 1.206(10)(22)$$

# statistical and systematic uncertainties

$N_f = 2$

| source of uncertainty [%] | $f_{B_s}$ | $f_{B_s}/f_B$ | $f_B$ |
|---|---|---|---|
| stat. + fit (C.L. and chiral) | 2.2 | 0.8 | 2.1 |
| lat. scale | 2.0 | - | 2.0 |
| discr. effects | 1.3 | 0.4 | 1.7 |
| $1/\mu_h$ | 1.0 | 0.1 | 1.1 |
| chiral extr. trig. point | - | 1.7 | 1.7 |
| total | 3.4 | 2.0 | 4.0 |

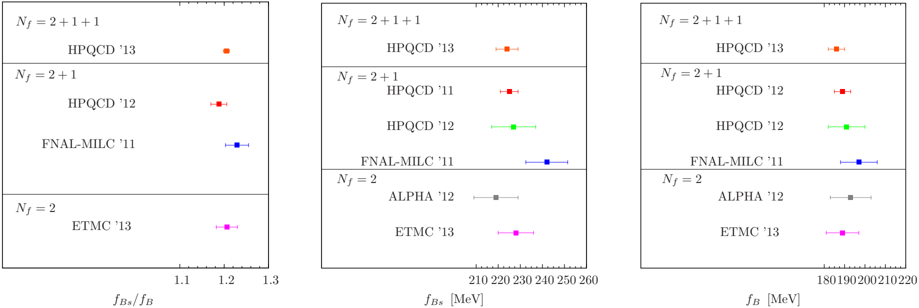
- $N_f = 2$

$$f_{B_s} = 228(8)\ \mathrm{MeV} \quad , \quad f_B = 189(8)\ \mathrm{MeV} \quad , \quad \frac{f_{B_s}}{f_B} = 1.206(24)$$

- $N_f = 2+1+1$ : [PRELIMINARY]

$$f_{B_s} = 235(9)\ \mathrm{MeV} \quad , \quad f_B = 196(9)\ \mathrm{MeV} \quad , \quad \frac{f_{B_s}}{f_B} = 1.201(25)$$

# non exhaustive comparison : $f_{B_s}/f_B,\ f_{B_s},\ f_B$




ETMC $N_f = 2+1+1$ : [PRELIMINARY]

$$\frac{f_{B_s}}{f_B} = 1.201(25) \quad , \quad f_{B_s} = 235(9) \ \ \mathbf{MeV} \quad , \quad f_B = 196(9) \ \ \mathbf{MeV}$$

reviews : [FLAG & talk by S. Simula]
[talk by A. El-Khadra]
[talk by C. Bouchard, lat2014]

# conclusion

- currently, lattice QCD can determine B-meson decay constants with a few % accuracy
- no significant effect of $s$ and $c$ sea quarks with current level of accuracy for the decay constants
- ratio method :
  profits from partial cancellation of systematic effects and from scaling-laws of HQET to approach $b$-quark sector
- has also been applied to $m_b$, bag parameters, form factors of $B$ decays, ...