QCD Light-Cone Sum Rules for exclusive $b \to u$ transitions

Alexander Khodjamirian

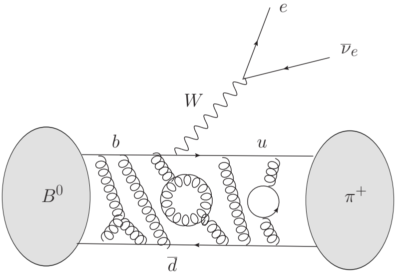
UNIVERSITÄT SIEGEN
Theoretische Physik 1

DFG
RESEARCH UNIT

WG2, CKM 14 Workshop, Vienna, Sept. 8, (2014)

$B \to \pi \ell \nu_\ell$, determination of $|V_{ub}|$

- decay amplitude parametrized by hadronic form factors



$$\langle \pi^+(p)|\bar{u}\gamma_\mu b|B(p+q)\rangle = f^+_{B\pi}(q^2)\Big[...\Big]_\mu + f^0_{B\pi}(q^2)\Big[...\Big]_\mu$$

- $|V_{ub}|$ determination [BaBar,Belle]

$$\left(\frac{1}{\tau_B}\right)\frac{dBR(\bar{B}^0 \to \pi^+ l^- \nu)}{dq^2} = \frac{G_F^2|V_{ub}|^2}{24\pi^3} p_\pi^3 |f^+_{B\pi}(q^2)|^2 + O(m_l^2)$$

$0 < q^2 < (m_B - m_\pi)^2 \sim 26\ \text{GeV}^2$,

- form factors calculated in QCD

QCD techniques for the form factors

- lattice QCD: *see talks at WG2*
  large q^2 (small hadronic recoil) accessible

- QCD LCSRs:
  - form factors at small and intermediate $q^2 \ll m_b^2$
  - full QCD with finite (large) m_b, $1/m_b$ expansion possible
  - both factorizable (hard-scattering) and
    nonfactorizable (soft, "end-point") contributions
    calculated within one method and input

specifics of the method:

– "indirect access" to the form factor
  via hadronic dispersion relation

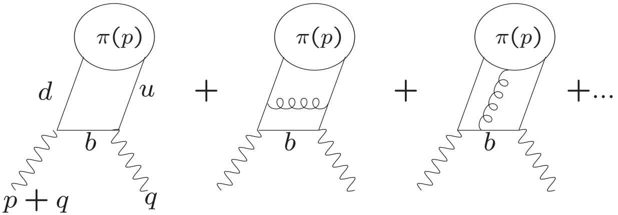
– quark-hadron duality approximation
  introducing some systematic uncertainty.

Light-Cone Sum Rules (LCSR) for $B \to \pi$

- the correlation function, an artifitially “designed” amplitude
- external currents with $(p+q)^2, q^2 \ll m_b^2 \Rightarrow b$-quark virtual,
- factorization: $\mu \sim \sqrt{m_b \chi}$, $\Lambda_{QCD} \ll \chi \ll m_b$

$$F(q^2,(p+q)^2) = \sum_{t=2,3,4,..} \int du\, T^{(t)}(q^2,(p+q)^2, m_b^2, \alpha_s, u, \mu)\, \varphi_\pi^{(t)}(u,\mu)$$

hard scattering amplitudes $\otimes$ pion light-cone DA’s



LO NLO {power corr.}

The OPE result

- current accuracy: (triple expansion: α_s, twist, conformal)

$$F(q^2,(p+q)^2) = \left(T_0^{(2)} + (\alpha_s/\pi)T_1^{(2)} + (\alpha_s/\pi)^2 T_2^{(2)}\right) \otimes \varphi_\pi^{(2)}$$
$$+\frac{\mu_\pi}{m_b}\left(T_0^{(3)} + (\alpha_s/\pi)T_1^{(3)}\right) \otimes \varphi_\pi^{(3)}$$
$$+\frac{\delta_\pi^2}{m_b\chi} T^{(4)} \otimes \varphi_\pi^{(4)} + \ldots$$

- LO twist 2,3,4 $q\bar{q}$ and $\bar{q}qG$ terms:

[V.Belyaev, A.K., R.Rückl (1993); V.Braun, V.Belyaev, A.K., R.Rückl (1996)]

-NLO $O(\alpha_s)$ twist 2, (collinear factorization)

[A.K., R.Rückl, S.Weinzierl, O. Yakovlev (1997); E.Bagan, P.Ball, V.Braun (1997);]

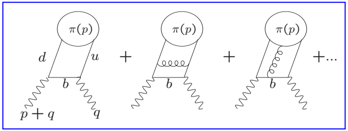
-NLO $O(\alpha_s)$ twist 3 (coll.factorization for asympt. DA)

[P. Ball, R. Zwicky (2001); G.Duplancic,A.K.,B.Melic, Th.Mannel,N.Offen (2007)]

-part of NNLO $O(\alpha_s^2\beta_0)$ twist 2 *[A. Bharucha (2012)]*

Calculating the form factor from LCSR

- {correlator OPE } = {sum over intermediate B states}

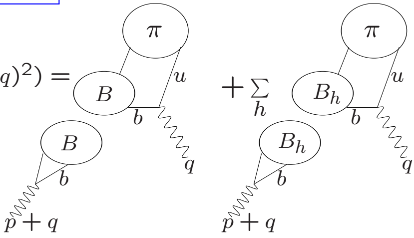


Unitarity, analyticity,

dispersion relation in $(p+q)^2$:

$\Downarrow$

$F(q^2,(p+q)^2) =$



$f_B f^+_{B\pi}(q^2)$ $\qquad$ $\sum_{B_h} \to duality\ (s_0^B)$

- varying flavours and J^{PC} yields LCSR's for $B \to K, \eta$, $D \to \pi, K$, etc.
- LCSR includes "soft" and "hard" contributions to the form factors
- finite m_b

How accurate are LCSR's

- the "raw" sum rule: OPE = dispersion relation :

$$[F((p+q)^2,q^2)]_{OPE} = \frac{m_B^2 f_B f^+_{B\pi}(q^2)}{m_B^2-(p+q)^2} + \int\limits_{s_0^B}^{\infty} ds \frac{[\text{Im}F(s,q^2)]_{OPE}}{s-(p+q)^2}$$

$\Uparrow$ $\overline{m}_b$, α_s, $\varphi_\pi^{(t)}(u)$, t=2,3,4;

$\Uparrow$ QCD SR for f_B

$\Uparrow$ quark-hadron duality

- the input for pion DAs $\varphi_\pi^{(t)}$
  $t=2$ (f_π+ Gegenbauer moments); $t=3$ (3 param.); $t=4$ (2 param.)
- all inputs are "home-made", lattice QCD only for comparison
  dedicated two-point sum rules, LCSR's for the pion form factors
- "systematic error" introduced by quark-hadron duality:
  suppressed with Borel transformation, controlled by the m_B calculation

LCSRs for pion form factors $\to$ pion DAs

$\gamma\gamma^*(Q^2) \to \pi^0$ form factor

[A. K. (1999)], ...

[S. Agaev, V. Braun, N. Offen and F. Porkert, (2011)]

$e^+e^- \to e^+e^- + \pi^0$

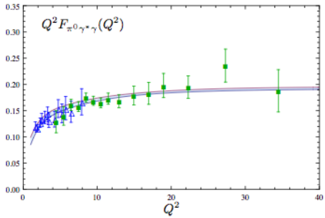


FIG. 2: The pion transition form factor for the model of the pion DA described in the text. The estimated theoretical uncertainty is shown by the shaded area. The experimental data are from [2] (squares) and [4] (open triangles).

● Belle ● CLEO

$\gamma^*(Q^2)\pi^\pm \to \pi^\pm$ form factor

[V.Braun, A.K., M.Maul (2000)]

[AK, T.Mannel, N.Offen, Y-M.Wang (2011)]

$eN \to e\pi N$

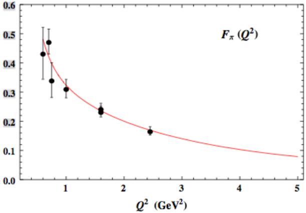


FIG. 1 (color online). The pion e.m. form factor calculated from LCSR [17,18] as a function of Gegenbauer moments a_2^π(1 GeV) and a_4^π(1 GeV) and fitted (solid line) to the experimental data points taken from [19].

• JLab

both sum rules sensitive to twist-2 pion DA

LCSR for $D \to \pi, K$ form factors

AK, Ch. Klein, Th. Mannel, N.Offen (2009)

- important cross-check of the LCSR method
- $b \to c$ in the correlation function (finite quark masses !)

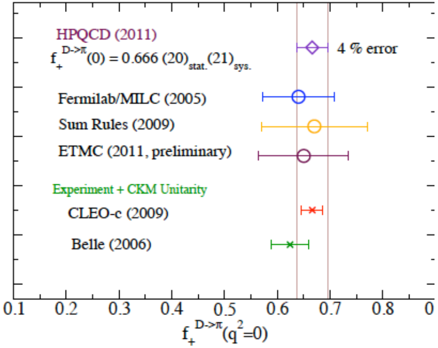


FIG. 6: The $D \to \pi$ form factor $f_+^{D\to\pi}(0)$ from this work an comparisons with other determinations [12, 13, 23–25].

taken from HPQCD 1109. 1501(2011)

$B_{(s)}$ and $D_{(s)}$ decay constants from 2-point SR

see also the talk by S.Simula at this workshop

[P.Gelhausen, AK, A.A.Pivovarov, D.Rosenthal, (2013)]

Decay constant	Lattice QCD [ref.]	this work
f_B[MeV]	196.9 ± 9.1 [1] 186 ± 4 [2]	207^{+17}_{-9}
f_{B_s}[MeV]	242.0 ± 10.0 [1] 224 ± 5 [2]	242^{+17}_{-12}
f_D[MeV]	218.9 ± 11.3 [1] 213 ± 4 [2]	201^{+12}_{-13}
f_{D_s}[MeV]	260.1 ± 10.8 [1] 248.0 ± 2.5 [2]	238^{+13}_{-23}

[1]-Fermilab/MILC, [2]-HPQCD

LCSR results for the $f^+_{B\pi}$

- typical region of validity $0 < q^2 < 12 - 16~\text{GeV}^2$
- previous LCSR results on $|V_{ub}|$:
  using $f^+_{B\pi}(0)$ or integrating $|f^+_{B\pi}(q^2)|^2$ over q^2
  [AK, T.Mannel, N.Offen, Y-M.Wang (2011)] [A. Bharucha (2012)]
- the shape of the form factor remains an important observable, e.g. to constrain the theory $\Rightarrow$ need parametrization
- "standard" one: $q^2 \to z(q^2, t_0)$, mapping SL region to small z, z-series parameterization *the BCL-version [Bourrely, Caprini, Lellouch, (2008)]*

$$f^+_{B\pi}(q^2) = \frac{f^+_{B\pi}(0)}{1 - q^2/m^2_{B*}} \Big\{ 1 + b_1^+ \big[z(q^2, t_0) - z(0, t_0) - \frac{1}{3}\big(z(q^2, t_0)^3 - z(0, t_0)^3\big)\big]$$

$$+ b_2^+ \big[z(q^2, t_0)^2 - z(0, t_0)^2 + \frac{2}{3}\big(z(q^2, t_0)^3 - z(0, t_0)^3\big)\big] \Big\},$$

- used to extrapolate the LCSR form factor to large q^2,
  - correlation between shape and normalization?
  - validity of truncated z-parametrization

Results for $B \to \pi$ form factor (preliminary !):

- first attempt of statistical (Bayesian) analysis:
  updated inputs, taken as priors,
  constructing likelihood by imposing $[m_B]_{SR}$ within 1% of m_B
  [I.Imsong, AK, Th.Mannel, D.van Dyk, work in progress]

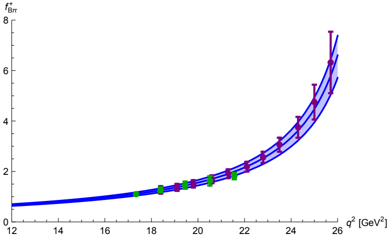
- 6 quantities obtained from LCSR:
  $f^+_{B\pi}(q^2)$ + first + second derivative
  (value, slope, curvature) at $q^2 = 0, 10$ GeV^2 ,
  output resembles gaussian distribution with large correlations

- BCL parameterization fitted to LCSR results,

  $f_{B\pi}(0) = 0.31 \pm 0.02$
  $b_1^+ = -1.28^{+.43}_{-0.58}$
  $b_2^+ = -0.88^{+0.43}_{-0.53}$

  68% probability intervals

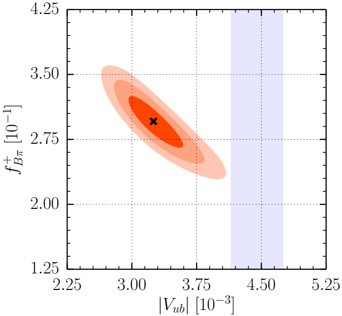
- extrapolating
  beyond the LCSR region
  good agreement with lattice results



● -HPQCD , ● -Fermilab-MILC

Extraction of $|V_{ub}|$ (preliminary !)

using the BaBar (2010) + Belle (2010) data

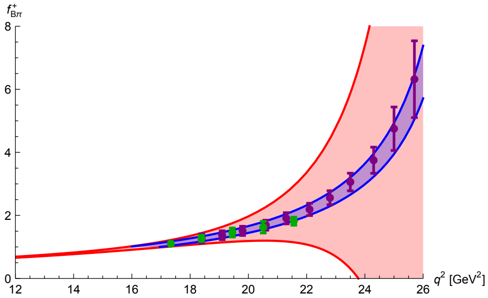


x - best-fit point,
red,orange,light-orange areas:
68%, 95% and 99%
probability regions
blue shaded interval -
inclusive determination (HFAG average)

χ^2= 4.65, and 9 d.o.f. $\Rightarrow$ a good fit with *p*-value: 0.86

Bounds for $B \to \pi$ form factor

- Parameterization-independent bounds following from the analytical properties of the form factor and from the unitarity of two-point correlation function [....., L.Lellouch (1996),...]
- form factor value, slope and curvature at one point yield the best constraints: [Th. Mannel, B.Postler (1998)]
- we use our results of statistical analysis at $q^2 = 10$ GeV^2 (preliminary!)



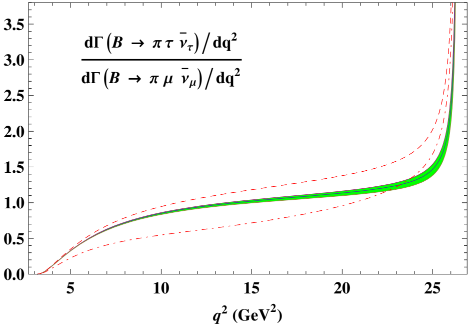
$B \to \pi\tau\nu_\tau$

[AK, T.Mannel, N.Offen, Y-M.Wang (2011)]

- exclusive $b \to u\tau\nu_\tau$, interesting for NP search
- observable independent of V_{ub} : ($\ell = e$ or μ, $m_\ell = 0$)

$$\frac{d\Gamma(B \to \pi\tau\nu_\tau)/dq^2}{d\Gamma(B \to \pi\ell\nu_\ell)/dq^2} = \frac{(q^2 - m_\tau^2)^2}{(q^2)^2}\left(1 + \frac{m_\tau^2}{2q^2}\right)\left\{1 + \frac{3m_\tau^2(m_B^2 - m_\pi^2)^2}{4(m_\tau^2 + 2q^2)m_B^2 p_\pi^2}\frac{|f^0_{B\pi}(q^2)|^2}{|f^+_{B\pi}(q^2)|^2}\right\}$$

a smaller uncertainty ⇑

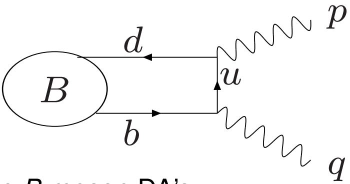


===== SM± uncert

- - - - - ± hypothetical charged Higgs

LCSR's with B-meson distribution amplitudes

- correlation function:
  B -meson is on-shell,
  π interpolated with a current
  [A.K., T. Mannel, N.Offen,(2005) ,
  F.De Fazio, Th. Feldmann and T. Hurth, (2005)]



- key nonperturbative objects: $\phi_B^{\pm}$, the B-meson DA's defined in HQET *[A.Grozin, M.Neubert (1997); M.Beneke,Th.Feldmann (2001)]*
- advantage: no need for DA's of final-state mesons, treated via quark-hadron duality approximation
- uncertainty in $\phi_B^{\pm}$ will be removed measuring $B \to \gamma \ell \nu_\ell$
  ... [M. Beneke, J. Rohrwild, 2011], [V.Braun, AK, 2012]
- NLO corrections in HQET have to be calculated
- $B \to \pi, K$ form factors from both LCSRs agree
- nonlocal matrix elements in $B \to K \ell^+ \ell^-$ *[AK, Th.Mannel, Y.M.Wang (2010)]*
- form factors of $\Lambda_b \to \Lambda \ell \ell$ *[Th.Feldmann, M.Yip (2012)]*

Other $b \to u$ exclusive transitions

- $\Lambda_b \to p$ form factors, LCSRs with nucleon DAs
  [AK, Ch.Klein, Th. Mannel and Y.M.Wang (2011)]
- $B \to \rho \ell \nu_\ell \Rightarrow |V_{ub}|$ LCSR's for $B \to \rho$ form factors
  [P. Ball, V.Braun (1998), P.Ball, R. Zwicky (2004,...)]
  in $\Gamma_V = 0$ approximation ("quenched")
- $B \to \pi\pi\ell\nu_\ell$ form factors,
  partial wave expansion & resonances: ρ (P-wave) , f_0 (S-wave)
  defining regions of Dalitz plot with specific QCD dynamics
  [S. Faller, T. Feldmann, A. Khodjamirian, T. Mannel and D. van Dyk, (2013)]
- calculating $B \to \pi\pi\ell\nu_\ell$ at low 2-pion mass and small q^2
  from LCSR with 2-pion DAs
  timelike pion form factors (including resonances) replacing Gegenbauer coeffs
  (in progress)

Conclusions

- LCSR's with pion or B-meson DA's provide $b \to u$ exclusive transition form factors at large hadronic recoil (small q^2)
- experiment can help to improve the accuracy of DA's: $\pi\gamma^*\gamma$, $F_\pi^{em}(Q^2)$, $B \to \gamma\ell\nu_\ell$
- first attempt of statistical analysis of LCSR for $f_{B\pi}^+$
- $B_s \to K$, $B \to K$ form factors follow
- LCSR's for $B \to \pi\pi\ell\nu_\ell$ form factors

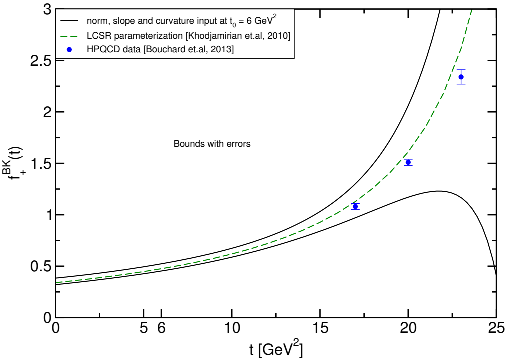
BACKUP SLIDES

$B \to K$ form factor from LCSR

- $\pi \to K$ in the correlation function, $O(m_s)$ effects included
- last update *[A.K, Th.Mannel, A.Pivovarov, Y-M. Wang (2010)]*
- using the unitarity bounds for the z-transformed form factor, *[L.Lellouch (1996)], [Th.Mannel,B.Postler(1998)]*

(PRELIMINARY), S.Imsong, AK, Th.Mannel, work in progress

input:
$f^{+}_{BK}(q^2 = 6.0~\text{GeV}^2)$
$\oplus$ slope $\oplus$ curvature

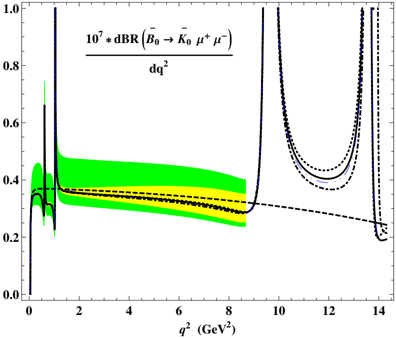


$d\mathrm{BR}(B \to K\mu^+\mu^-)/dq^2$ and bins

solid (dotted) lines - central input, default (alternative) parametrization for the dispersion integrals.

long-dashed line -the width calculated without nonlocal hadronic effects.

The green (yellow) shaded area indicates the uncertainties including (excluding) the one from the $B \to K$ FF normalization.



$[q^2_{min}, q^2_{max}]$	Belle	CDF	LHCb	LHCb	this work
[0.05, 2.0]	$0.81^{+0.18}_{-0.16} \pm 0.05$	$0.33 \pm 0.10 \pm 0.02$	$0.21^{+0.27}_{-0.23}$	$0.56 \pm 0.05 \pm 0.03$	$0.71^{+0.22}_{-0.08}$
[2.0, 4.3]	$0.46^{+0.14}_{-0.12} \pm 0.03$	$0.77 \pm 0.14 \pm 0.05$	$0.07^{+0.25}_{-0.21}$	$0.57 \pm 0.05 \pm 0.02$	$0.80^{+0.27}_{-0.11}$
[4.3, 8.68]	$1.00^{+0.19}_{-0.08} \pm 0.06$	$1.05 \pm 0.17 \pm 0.07$	1.2 ± 0.3	$1.00 \pm 0.07 \pm 0.04$	$1.39^{+0.53}_{-0.22}$
[1.0, 6.0]	$1.36^{+0.23}_{-0.21} \pm 0.08$	$1.29 \pm 0.18 \pm 0.08$	$0.65^{+0.45}_{-0.35}$	$1.21 \pm 0.09 \pm 0.07$	$1.76^{+0.60}_{-0.23}$

Extraction of $|V_{ub}|$

- integrate over the region of validity

$$\Delta\zeta\,(0, q^2_{max} = 12 GeV^2) \equiv \frac{G_F^2}{24\pi^3} \int\limits_0^{q^2_{max}} dq^2 p_\pi^3 |f^+_{B\pi}(q^2)|^2 = \frac{1}{|V_{ub}|^2 \tau_{B^0}} \int\limits_0^{q^2_{max}} dq^2 \frac{d\mathcal{B}(B \to \pi \ell \nu_\ell)}{dq^2},$$

from [Belle Collab 1306.2781 [hep-ex]]

TABLE XII: Values of the CKM matrix element $|V_{ub}|$ based on rates of exclusive $\bar{B} \to X_u \ell^- \bar{\nu}_\ell$ decays and theoretical predictions of form factors within various q^2 ranges. The first uncertainty is statistical, the second is experimental systematic and the third is theoretical. The theoretical uncertainty for the ISGW2 model is not available.

X_u	Theory	q^2 GeV/c^2	$N^{\rm fit}$	$N^{\rm MC}$	$\Delta\mathcal{B}$ 10^{-4}	$\Delta\zeta$ ps^{-1}	$\|V_{ub}\|$ 10^{-3}
π^0	LCSR [33]	< 12	119.6 ± 16.2	116.5	0.423 ± 0.057	$4.59^{+1.00}_{-0.85}$	$3.35 \pm 0.23 \pm 0.09^{+0.36}_{-0.31}$
	LCSR [34]	< 16	168.2 ± 18.9	153.5	0.588 ± 0.066	$5.44^{+1.43}_{-1.43}$	$3.63 \pm 0.20 \pm 0.10^{+0.60}_{-0.40}$
	HPQCD [35]	> 16	58.6 ± 10.5	57.6	0.196 ± 0.035	$2.02^{+0.55}_{-0.55}$	$3.44 \pm 0.31 \pm 0.09^{+0.59}_{-0.39}$
	FNAL [36]					$2.21^{+0.47}_{-0.42}$	$3.29 \pm 0.30 \pm 0.09^{+0.37}_{-0.30}$
π^+	LCSR [33]	< 12	247.2 ± 18.9	233.1	0.808 ± 0.062	$4.59^{+1.00}_{-0.85}$	$3.40 \pm 0.13 \pm 0.09^{+0.37}_{-0.32}$
	LCSR [34]	< 16	324.2 ± 22.6	305.1	1.057 ± 0.074	$5.44^{+1.43}_{-1.43}$	$3.58 \pm 0.12 \pm 0.09^{+0.59}_{-0.39}$
	HPQCD [35]	> 16	141.3 ± 16.0	116.1	0.445 ± 0.050	$2.02^{+0.55}_{-0.55}$	$3.81 \pm 0.22 \pm 0.10^{+0.66}_{-0.43}$
	FNAL [36]					$2.21^{+0.47}_{-0.42}$	$3.64 \pm 0.21 \pm 0.09^{+0.40}_{-0.33}$
	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]