Extracting V_{us} from Lattice QCD simulations: Recent progress and prospects

Nicolas Garron
School of Maths, Trinity College Dublin

From the unitarity of the CKM triangle in the SM

$$
\begin{aligned}
|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 &= 1 \\
0.97^2 + 0.22^2 + (10^{-4})^2 &= 1
\end{aligned}
$$

How well does that work ? Can one see a deviation from 1 ?

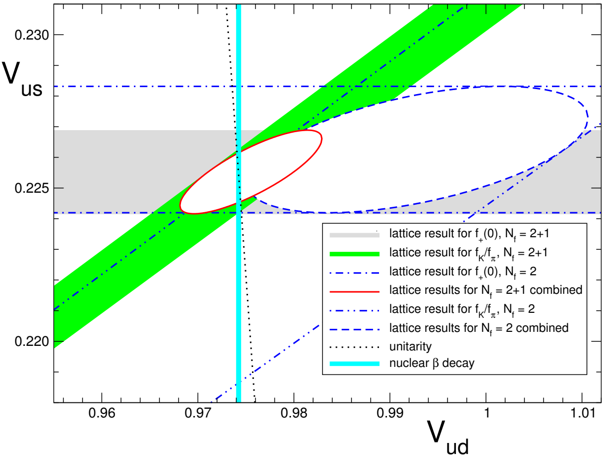
- Very accurate experimental determination of

$$|V_{us} f_+(0)| = 0.2163(5)$$

$$\left|\frac{V_{us}}{V_{ud}}\right| \frac{f_{K^\pm}}{f_{\pi^\pm}} = 0.2758(5)$$

- Compute $f_+(0)$ and $f_{K^\pm}/f_{\pi^\pm}$ on the lattice to obtain V_{us} and V_{ud}

[FLAG (Colangelo et al) '11]



Situation in 2011

Lattice simulation details

- Various discretisations of the Dirac operator on the market: Domain-Wall, Staggered, Twisted-mass, Wilson-type (eg clover), etc.
- Can have very different numerical cost and
- have different properties at finite lattice spacing: flavour, chiral, renormalisation, etc.
- Universality of lattice QCD: results should agree in the continuum limit (and they do)
- Having results from different collaborations/lattice action is very important: check universality, systematic errors, different approaches, etc.

Lattice simulation details

- Various discretisations of the Dirac operator on the market: Domain-Wall, Staggered, Twisted-mass, Wilson-type (eg clover), etc.
- Can have very different numerical cost and
- have different properties at finite lattice spacing: flavour, chiral, renormalisation, etc.
- Universality of lattice QCD: results should agree in the continuum limit (and they do)
- Having results from different collaborations/lattice action is very important: check universality, systematic errors, different approaches, etc.

Results presented here will mainly by obtained using

- Domain-Wall
- HISQ
- Twisted-Mass

Apologies for not covering in details other results

Lattice simulation details

- Various discretisations of the Dirac operator on the market: Domain-Wall, Staggered, Twisted-mass, Wilson-type (eg clover), etc.
- Can have very different numerical cost and
- have different properties at finite lattice spacing: flavour, chiral, renormalisation, etc.
- Universality of lattice QCD: results should agree in the continuum limit (and they do)
- Having results from different collaborations/lattice action is very important: check universality, systematic errors, different approaches, etc.

Results presented here will mainly by obtained using

- Domain-Wall
- HISQ
- Twisted-Mass

Apologies for not covering in details other results

See Plenary talk by Aida X. El-Khadra, Monday @ 11:55

Decay constants f_K, f_π

Reminders

- Define the decay constant f_P of a pseudo-scalar ($P = \overline{\psi}_1 \gamma_5 \psi_2$) meson by

$$\langle 0 | \overline{\psi}_1 \gamma_\mu \gamma_5 \psi_2 | P(p) \rangle = i f_P p_\mu$$

- Define the decay constant f_P of a pseudo-scalar ($P = \overline{\psi}_1\gamma_5\psi_2$) meson by

$$\langle 0|\overline{\psi}_1\gamma_\mu\gamma_5\psi_2|P(p)\rangle = if_P p_\mu$$

- On the lattice (Euclidean, discrete space-time), compute the 2-pt functions

$$\begin{aligned}\langle \mathcal{P}(x_0)\mathcal{A}_0(0)\rangle &\equiv \sum_{\vec{x}} \langle 0|\big(\overline{\psi}_2\gamma_5\psi_1\big)(x)\big(\overline{\psi}_1\gamma_0\gamma_5\psi_2\big)(0)|0\rangle \\ &\to \sim \langle 0|\mathcal{P}|P(0)\rangle \times \langle P(0)|\mathcal{A}_0|0\rangle \times \big(e^{-m_P x_0} + \dots\big)\end{aligned}$$

  Fit in the asymptotic region to extract the hadronic matrix elements

- Do the same with $\langle \mathcal{P}(x_0)\mathcal{P}_0(0)\rangle$ to extract f_P

Reminders

- Define the decay constant f_P of a pseudo-scalar ($P = \overline{\psi}_1\gamma_5\psi_2$) meson by

$$\langle 0|\overline{\psi}_1\gamma_\mu\gamma_5\psi_2|P(p)\rangle = if_P p_\mu$$

- On the lattice (Euclidean, discrete space-time), compute the 2-pt functions

$$\begin{aligned}\langle \mathcal{P}(x_0)\mathcal{A}_0(0)\rangle &\equiv \sum_{\vec{x}} \langle 0|\left(\overline{\psi}_2\gamma_5\psi_1\right)(x)\left(\overline{\psi}_1\gamma_0\gamma_5\psi_2\right)(0)|0\rangle \\ &\to \sim \langle 0|\mathcal{P}|P(0)\rangle \times \langle P(0)|\mathcal{A}_0|0\rangle \times \left(e^{-m_P x_0} + \dots\right)\end{aligned}$$

  Fit in the asymptotic region to extract the hadronic matrix elements

- Do the same with $\langle \mathcal{P}(x_0)\mathcal{P}_0(0)\rangle$ to extract f_P

- Some renormalisation might have to be done. Details depend on the lattice implementation

Reminders

- Define the decay constant f_P of a pseudo-scalar ($P = \overline{\psi}_1 \gamma_5 \psi_2$) meson by

$$\langle 0 | \overline{\psi}_1 \gamma_\mu \gamma_5 \psi_2 | P(p) \rangle = i f_P p_\mu$$

- On the lattice (Euclidean, discrete space-time), compute the 2-pt functions

$$\begin{aligned} \langle \mathcal{P}(x_0) \mathcal{A}_0(0) \rangle &\equiv \sum_{\vec{x}} \langle 0 | \big(\overline{\psi}_2 \gamma_5 \psi_1 \big)(x) \big(\overline{\psi}_1 \gamma_0 \gamma_5 \psi_2 \big)(0) | 0 \rangle \\ &\to \sim \langle 0 | \mathcal{P} | P(0) \rangle \times \langle P(0) | \mathcal{A}_0 | 0 \rangle \times \left(e^{-m_P x_0} + \dots \right) \end{aligned}$$

  Fit in the asymptotic region to extract the hadronic matrix elements

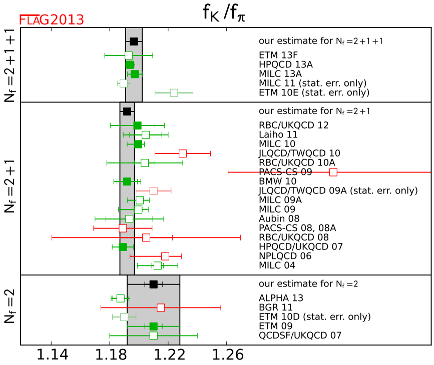
- Do the same with $\langle \mathcal{P}(x_0) \mathcal{P}_0(0) \rangle$ to extract f_P

- Some renormalisation might have to be done. Details depend on the lattice implementation

- Renormalisation factor cancels out in the ratio f_K / f_π

FLAG'13

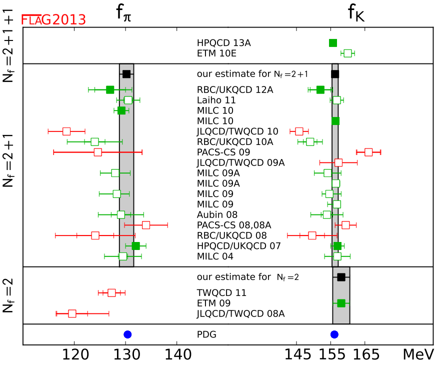
$$f_K/f_\pi = 1.194(5) \qquad n_f = 2+1+1$$
$$f_K/f_\pi = 1.192(5) \qquad n_f = 2+1$$
$$f_K/f_\pi = 1.205(6)(17) \qquad n_f = 2$$



Situation before lattice 2014

FLAG'13

$$f_\pi = 130.2(1.4)\ \mathrm{MeV} \qquad n_f = 2+1$$
$$f_K = 156.3(0.9)\ \mathrm{MeV} \qquad n_f = 2+1$$
$$f_K = 158.1(2.5)\ \mathrm{MeV} \qquad n_f = 2$$



Fermilab/MILC [*A. Bazavov et al.*, Phys.Rev.Lett. 110 (2013) 172003 & PoS LATTICE 2013]

2013 $f_{K^+}/f_{\pi^+} = 1.1947 \quad (26)_{\rm stat}\ (33)_{a^2\ \rm extrap}(17)_{\rm FV}(2)_{\rm EM}$

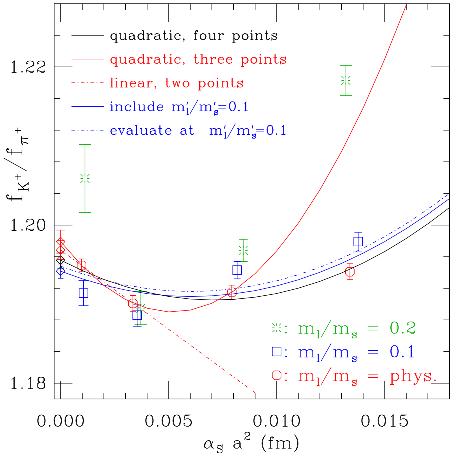
2014 $f_{K^+}/f_{\pi^+} = 1.1956 \quad (10)_{\rm stat}\ {}^{+23}_{-14}|_{a^2\ \rm extrap}(10)_{\rm FV}(5)_{\rm EM}$

- $n_f = 2+1+1$ Highly-Improved Staggered Quark (HISQ)
- $a \sim 0.06, 0.09, 0.12, 0.15$ fm
- $m_\pi \sim 135, 200$ MeV and $m_\pi L > 3.3$

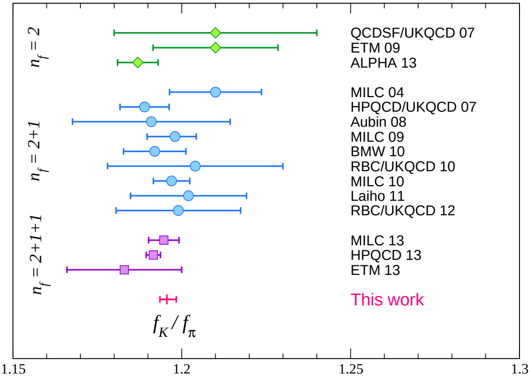
See talk by Javad Komijani @ Lattice'14

Preprint available [arXiv:1407.3772v1, Fermilab Lattice and MILC Collaborations, *Bazavov et al. '14*]

Continuum/χal extrapolation of f_K/f_π from [arXiv:1407.3772v1, Fermilab Lattice and MILC Collaborations, *Bazavov et al. '14*]



f_K/f_π comparison, [arXiv:1407.3772v1, Fermilab Lattice and MILC Collaborations, *Bazavov et al. '14*]



RBC-UKQCD PRELIMINARY (draft in final stage)

$$\begin{aligned} f_\pi &= 0.1298(9)_{\rm stat}(4)_\chi(2)_{\rm FV}\ \mathrm{GeV} \\ f_K &= 0.1556(8)_{\rm stat}(2)_\chi(1)_{\rm FV}\ \mathrm{GeV} \\ f_K/f_\pi &= 1.199(5)_{\rm stat}(6)_\chi(1)_{\rm FV} \end{aligned}$$

$n_f = 2+1$ Domain-Wall fermions

- New Möbius ensembles [Brower, Neff, Orginos '12] combined with existing Shamir ensembles.
- $a \sim 0.084, 1.144$ fm, $48^3 \times 96 \times 12$ and $64^3 \times 128 \times 12$
- Physical pion masses $m_\pi \sim 130$ MeV and $m_\pi L > 3.5$
- Finer ensemble $a \sim 0.06$, $32^3 \times 64 \times 12$ with $m_\pi \sim 360$ MeV $\Rightarrow m_\pi L \sim 3.8$)

Form factor $f_+(0)$

Obtain $|V_{us} f_+(0)|$ from the experimental rate

$$\Gamma_{K\to\pi l\nu} = C_K^2 \frac{G_F^2 m_K^5}{192\pi^2}\, I\, S_{EW}\ \left[1 + 2\Delta_{SU(2)} + 2\Delta_{EM}\right]\ |V_{us}\, f_+(0)|^2$$

where:

I is the phase space integral evaluated from the shape of the experimental form factor

$\Delta_{SU(2)}$ is the isospin breaking correction

S_{EW} is the short distance electroweak correction

Δ_{EM} is the long distance electromagnetic correction

and $f_+(0)$ is the form factor defined from ($q = p - p'$)

$$\langle \pi(p')|V_\mu|K(p)\rangle \quad = \quad (p_\mu + p'_\mu)\, f_+(q^2) + (p_\mu - p'_\mu)\, f_-(q^2) \qquad \text{with } V_\mu = \bar{s}\gamma_\mu u$$

$\Rightarrow$ determine $f_+(0)$ from the lattice to constrain V_{us}

K_{l3} semileptonic form factor II.

- Introduce the scalar form factor

$$\langle \pi(p')|S|K(p)\rangle = \frac{m_K^2 - m_\pi^2}{m_s - m_l} f_0(q^2)$$

Related to f_+ and f_- by

$$f_0(q^2) = f_+(q^2) + \frac{q^2}{m_K^2 - m_\pi^2} f_-(q^2)$$

So at $q^2 = 0$

$$f_0(0) = f_+(0) = \frac{m_s - m_l}{m_K^2 - m_\pi^2} \langle \pi(p')|S|K(p)\rangle_{q^2=0}$$

$f_0(0) = f_+(0)$ can be extracted from the Vector current or from the Pseudo-scalar density

K_{l3} semileptonic form factor III.

A possible strategy, using the vector current

- Kaon and Pion at rest, compute $f_0(q_{max^2})$ at $q_{max^2} = (m_K - m_\pi)^2$, which can be obtained with high accuracy: [*Hashimoto et al* '00, arXiv:hep-ph/9906376]

$$\frac{\langle \pi|\bar{s}\gamma_0 u|K\rangle}{\langle \pi|\bar{u}\gamma_0 u|\pi\rangle}\frac{\langle K|\bar{u}\gamma_0 u|\pi\rangle}{\langle K|\bar{u}\gamma_0 u|K\rangle} = \left(f_0(q^2_{max})\right)^2 \frac{(m_K + m_\pi)^2}{4 m_K m_\pi}$$

A possible strategy, using the vector current

- Kaon and Pion at rest, compute $f_0(q_{max^2})$ at $q_{max^2} = (m_K - m_\pi)^2$, which can be obtained with high accuracy: [*Hashimoto et al* '00, arXiv:hep-ph/9906376]

$$\frac{\langle\pi|\bar{s}\gamma_0 u|K\rangle}{\langle\pi|\bar{u}\gamma_0 u|\pi\rangle}\frac{\langle K|\bar{u}\gamma_0 u|\pi\rangle}{\langle K|\bar{u}\gamma_0 u|K\rangle} = \left(f_0(q^2_{max})\right)^2 \frac{(m_K + m_\pi)^2}{4 m_K m_\pi}$$

- Compute $f_0(q^2)$ for several negative values of q^2
- Interpolate to $q^2 = 0$

A possible strategy, using the vector current

- Kaon and Pion at rest, compute $f_0(q_{max^2})$ at $q_{max^2} = (m_K - m_\pi)^2$, which can be obtained with high accuracy: [*Hashimoto et al* '00, arXiv:hep-ph/9906376]

$$\frac{\langle\pi|\bar{s}\gamma_0 u|K\rangle}{\langle\pi|\bar{u}\gamma_0 u|\pi\rangle}\frac{\langle K|\bar{u}\gamma_0 u|\pi\rangle}{\langle K|\bar{u}\gamma_0 u|K\rangle} = \left(f_0(q^2_{max})\right)^2 \frac{(m_K + m_\pi)^2}{4 m_K m_\pi}$$

- Compute $f_0(q^2)$ for several negative values of q^2
- Interpolate to $q^2 = 0$
- Or use twisted boundary conditions to compute $f_0(0)$ directly [RBC-UKQCD, *Boyle et al* '10]

A possible strategy, using the vector current

- Kaon and Pion at rest, compute $f_0(q_{max^2})$ at $q_{max^2} = (m_K - m_\pi)^2$, which can be obtained with high accuracy: [*Hashimoto et al* '00, arXiv:hep-ph/9906376]

$$\frac{\langle \pi | \bar{s}\gamma_0 u | K\rangle}{\langle \pi | \bar{u}\gamma_0 u | \pi\rangle} \frac{\langle K | \bar{u}\gamma_0 u | \pi\rangle}{\langle K | \bar{u}\gamma_0 u | K\rangle} = \left(f_0(q^2_{max})\right)^2 \frac{(m_K + m_\pi)^2}{4 m_K m_\pi}$$

- Compute $f_0(q^2)$ for several negative values of q^2

- Interpolate to $q^2 = 0$

- Or use twisted boundary conditions to compute $f_0(0)$ directly [RBC-UKQCD, *Boyle et al* '10]

Another strategy, compute $f_+(0)$ from [Fermilab Lattice and MILC Collaborations *Bazavov, et al. '13*]

$$f_+(0) = f_0(0) = \frac{m_s - m_l}{m_K^2 - m_\pi^2} \langle \pi(p') | S | K(p) \rangle$$

using directly the Pseudo-scalar density.

A possible strategy, using the vector current

- Kaon and Pion at rest, compute $f_0(q_{max^2})$ at $q_{max^2} = (m_K - m_\pi)^2$, which can be obtained with high accuracy: [*Hashimoto et al* '00, arXiv:hep-ph/9906376]

$$\frac{\langle\pi|\bar{s}\gamma_0 u|K\rangle}{\langle\pi|\bar{u}\gamma_0 u|\pi\rangle}\frac{\langle K|\bar{u}\gamma_0 u|\pi\rangle}{\langle K|\bar{u}\gamma_0 u|K\rangle} = \left(f_0(q^2_{max})\right)^2 \frac{(m_K + m_\pi)^2}{4 m_K m_\pi}$$

- Compute $f_0(q^2)$ for several negative values of q^2

- Interpolate to $q^2 = 0$

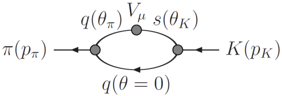
- Or use twisted boundary conditions to compute $f_0(0)$ directly [RBC-UKQCD, *Boyle et al* '10]

Another strategy, compute $f_+(0)$ from [Fermilab Lattice and MILC Collaborations *Bazavov, et al. '13*]

$$f_+(0) = f_0(0) = \frac{m_s - m_l}{m_K^2 - m_\pi^2}\langle\pi(p')|S|K(p)\rangle$$

using directly the Pseudo-scalar density.

- Twisted boundary conditions used to reach $q^2 = 0$



- Define a three-point function

$$\langle 0|\pi(p_\pi)V_\mu(0)K(p_K)|0\rangle \quad \longrightarrow \quad \sim \langle 0|\pi|\pi(p_\pi)\rangle \times \langle\pi(p_\pi)|V_\mu(0)|K(p_K)\rangle \times \langle K(p_K)|K|0\rangle$$

- In practise, take appropriate ratios with two-point functions to cancel the unwanted matrix elements $\langle 0|\pi|\pi\rangle$, $K(p_K)|K|0\rangle$ and kinematic factors

- Fit in the appropriate time range to obtain $\langle\pi(p_\pi)|V_\mu(0)|K(p_K)\rangle$

- Renormalise the vector current (if needed)

- Control the chiral extrapolation through

$$f_+(0) \stackrel{m_s \to m_l}{\longrightarrow} = 1$$

and χal Perturbation Theory

$$f_+(0) = 1 + f_2(f, m_\pi^2, m_K^2, m_\eta^2) + \Delta_f$$

NLO [*Ademollo & Gatto* '85 , *Gasser & Leutwyler* '85],

NNLO [*Bijnens & Talavera* '03 , *Bernard & Passemar* '10]

- Control the chiral extrapolation through

$$f_+(0) \stackrel{m_s \to m_l}{\longrightarrow} = 1$$

and χal Perturbation Theory

$$f_+(0) = 1 + f_2(f, m_\pi^2, m_K^2, m_\eta^2) + \Delta_f$$

NLO [*Ademollo & Gatto* '85 , *Gasser & Leutwyler* '85],

NNLO [*Bijnens & Talavera* '03 , *Bernard & Passemar* '10]

- Lattice simulations are reaching the physical point

- Control the chiral extrapolation through

$$f_+(0) \overset{m_s \to m_l}{\longrightarrow} = 1$$

and χal Perturbation Theory

$$f_+(0) = 1 + f_2(f, m_\pi^2, m_K^2, m_\eta^2) + \Delta_f$$

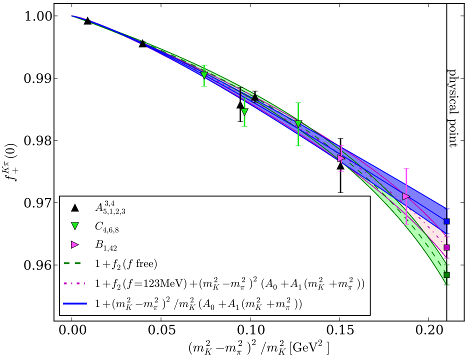
NLO [*Ademollo & Gatto* '85 , *Gasser & Leutwyler* '85],

NNLO [*Bijnens & Talavera* '03 , *Bernard & Passemar* '10]

- Lattice simulations are reaching the physical point
- Nevertheless, it is useful to also include the non-physical masses in the analysis as it increases the statistical precision.
- Also provides a test of previous computations and can be used to compute LOC's

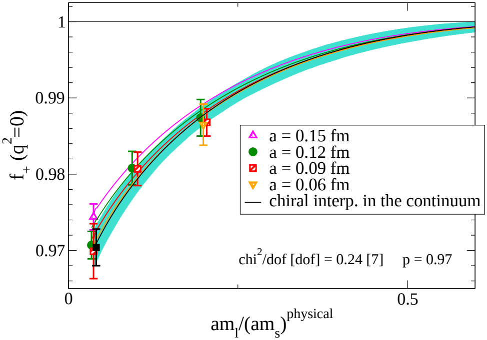
Light quark mass dependence

[RBC-UKQCD, JHEP 1308 (2013)]



$n_f = 2+1$ Domain-Wall, $m_\pi \to 170$ MeV

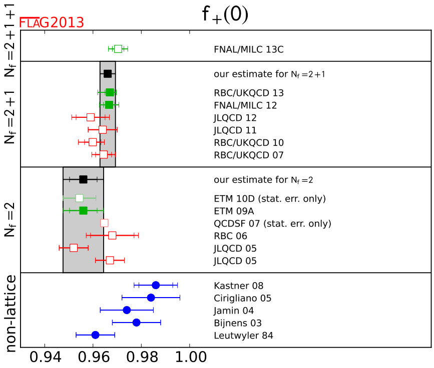
[*A. Bazavov et al.*, Phys.Rev.Lett. 110 (2013)]



$n_f = 2+1+1$ HISQ down to the physical mass

FLAG'13

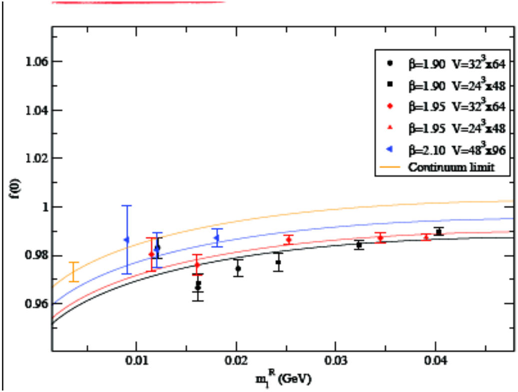
$$f_+(0) = 0.9661(32) \qquad n_f = 2+1$$
$$f_+(0) = 0.9560(57)(62) \qquad n_f = 2$$



ETMc

- $2+1+1$ Twisted Mass / Osterwalder-Seiler fermions
- Results obtained from the vector current
- Preliminary result

$$f_+(0) = 0.9683(50)_{\text{stat+fit}}(42)_{\text{chiral}}$$



See talk by Lorenzo Riggio @ Lattice'14

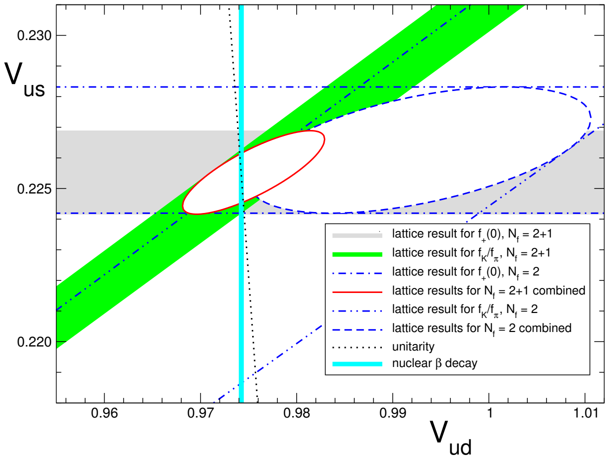
RBC-UKQCD

- New ensembles 48^3 and 64^3 at the physical point
- Results obtained from the vector current

See talk by David Murphy @ Lattice'14

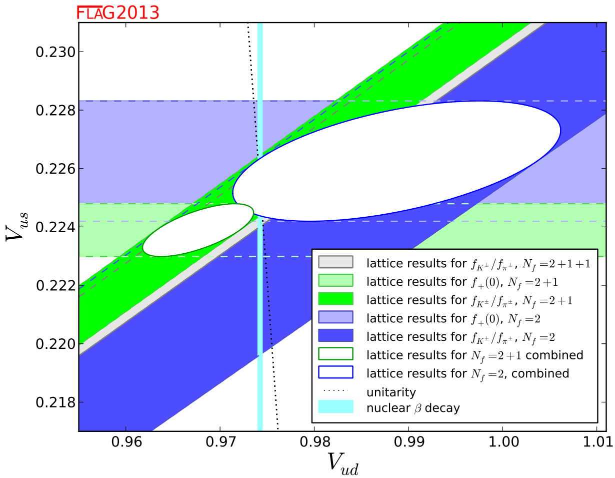
Lattice	m_π (MeV)	$f_+^{K\pi}(0)$	Stat. error
24I	678	0.9992(1)	0.01%
24I	563	0.9956(4)	0.04%
24I	422	0.9870(9)	0.09%
24I	334	0.9760(43)	0.4%
24I	334	0.9858(28)	0.3%
48I (PRELIMINARY)	**139**	**0.9727(25)**	**0.3%**
32ID	248	0.9771(21)	0.2%
32ID	171	0.9710(45)	0.5%
32I	398	0.9904(17)	0.2%
32I	349	0.9845(23)	0.2%
32I	295	0.9826(35)	0.4%
64I (PRELIMINARY)	**139**	**0.9701(22)**	**0.2%**

Preliminary No continuum limit yet



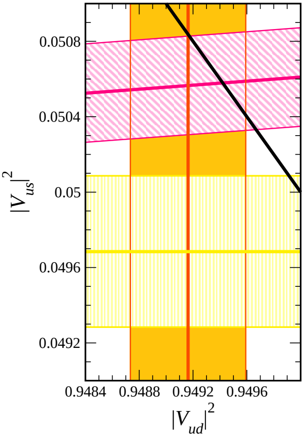
[FLAG *Colangelo et al.* '11]

Comparison



[FLAG *Aoki et al.* '13]

[arXiv:1407.3772v1, Fermilab Lattice and MILC Collaborations, *Bazavov et al. '14*]



$n_f = 2 + 1 + 1$

Conclusions and Outlook

- Combine f_K/f_π and $f_+(0)$ to obtain V_{ud}, V_{us}, test the unitarity relation for the first row
- Standard Model and lattice QCD passes a non-trivial precision test

- f_K/f_π and $f_+(0)$ can now be extracted from Lattice QCD at physical quark masses: $n_f = 2+1+1$ with HISQ, $n_f = 2+1$ with Domain-Wall (in progress), ...

- At the level of precision, we have to keep an eye on all the approximations, eg

$$\Gamma_{K\to\pi l\nu} = C_K^2 \frac{G_F^2 m_K^5}{192\pi^2} I\, S_{EW} \left[1 + 2\Delta_{SU(2)} + 2\Delta_{EM}\right] |V_{us}\, f_+(0)|^2$$

can the estimate of the corrections be improved ?

- We Reach a level of precision where electromagnetic and isospin effect have to be accounted for. Huge effort by the lattice community (BMW, PACS-CS, QCDSF, RBC-UKQCD, ...)

Conclusions and Outlook

- Combine f_K/f_π and $f_+(0)$ to obtain V_{ud}, V_{us}, test the unitarity relation for the first row
- Standard Model and lattice QCD passes a non-trivial precision test

- f_K/f_π and $f_+(0)$ can now be extracted from Lattice QCD at physical quark masses: $n_f = 2+1+1$ with HISQ, $n_f = 2+1$ with Domain-Wall (in progress), ...

- At the level of precision, we have to keep an eye on all the approximations, eg

$$\Gamma_{K\to\pi l\nu} = C_K^2 \frac{G_F^2 m_K^5}{192\pi^2} \, I \, S_{EW} \;\left[1 + 2\Delta_{SU(2)} + 2\Delta_{EM}\right] \, |V_{us}\, f_+(0)|^2$$

can the estimate of the corrections be improved ?

- We Reach a level of precision where electromagnetic and isospin effect have to be accounted for. Huge effort by the lattice community (BMW, PACS-CS, QCDSF, RBC-UKQCD, ...)

See next talk by A.Portelli