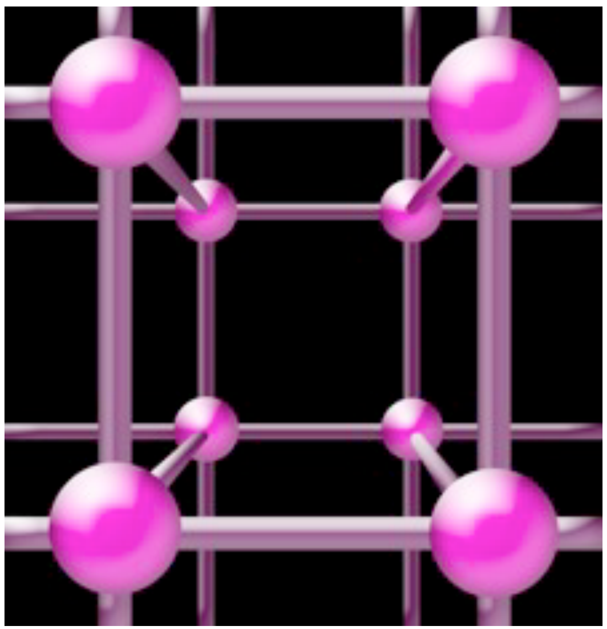
Lattice inputs for the determination of $|V_{cd}|$ and $|V_{cs}|$ from (semi-)leptonic decays



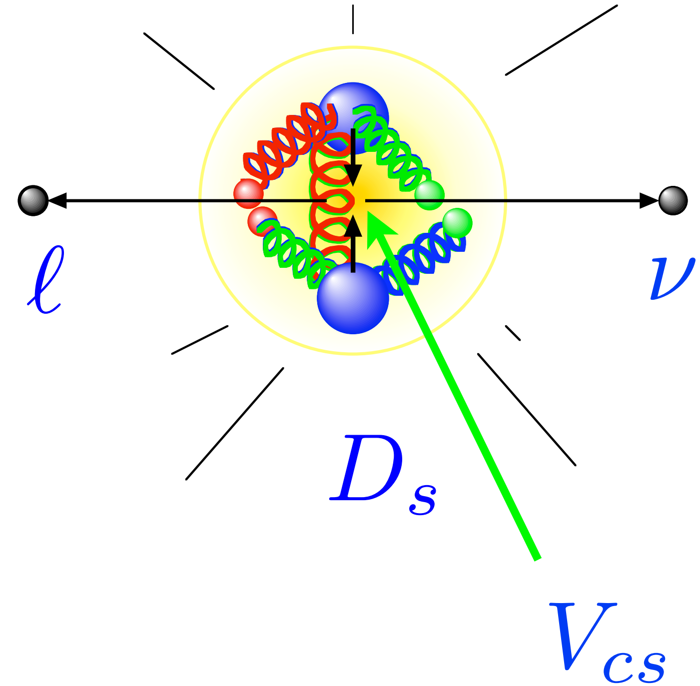
Jonna Koponen
University of Glasgow

CKM 2014
Vienna, Austria
September 2014

Outline

- D and D_s meson leptonic and semileptonic decays
  - motivation, CKM matrix elements
- Results
  - decay constants
  - semileptonic form factors
  - V_{cd} and V_{cs}
- Summary

Leptonic decays



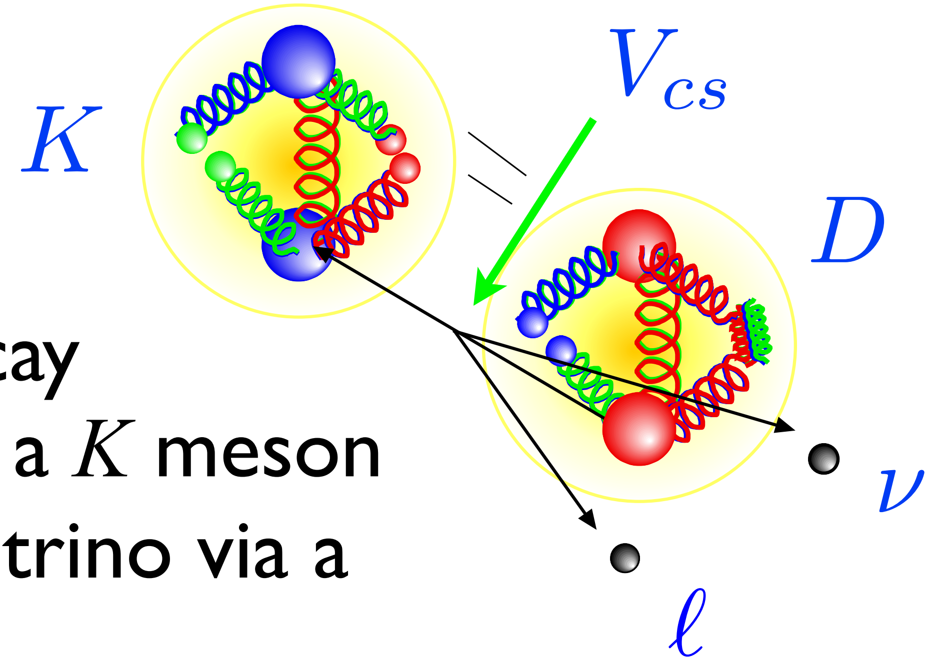
- D or D_s meson decays to a lepton and its neutrino via a virtual W boson

- The branching fraction is given by

$$\mathcal{B}(D_s \to \ell\nu) = \frac{G_F^2}{8\pi} m_\ell^2 M_{D_s} \left(1 - \frac{m_\ell^2}{M_{D_s}^2}\right)^2 f_{D_s}^2 |V_{cs}|^2$$

- Decay constant f_{D_s} (or f_D) depends on the hadronic degrees of freedom only and can be calculated in Lattice QCD $\to$ can determine $|V_{cs}|$ (or $|V_{cd}|$)

Semileptonic decays



- Consider a semileptonic decay where a D meson decays to a K meson (or a π), a lepton and its neutrino via a virtual W boson

- The partial decay rate is (both mesons are pseudoscalars)

$$\frac{\mathrm{d}\Gamma^{D\to K}}{\mathrm{d}q^2} = \frac{G_F^2 p^3}{24\pi^3}|V_{cs}|^2|f_+^{D\to K}(q^2)|^2$$

q^2=four-momentum transfer

p=momentum of daughter meson

- Calculating the form factor f_+ in lattice QCD allows to determine the CKM matrix element $|V_{cs}|$ (or $|V_{cd}|$)

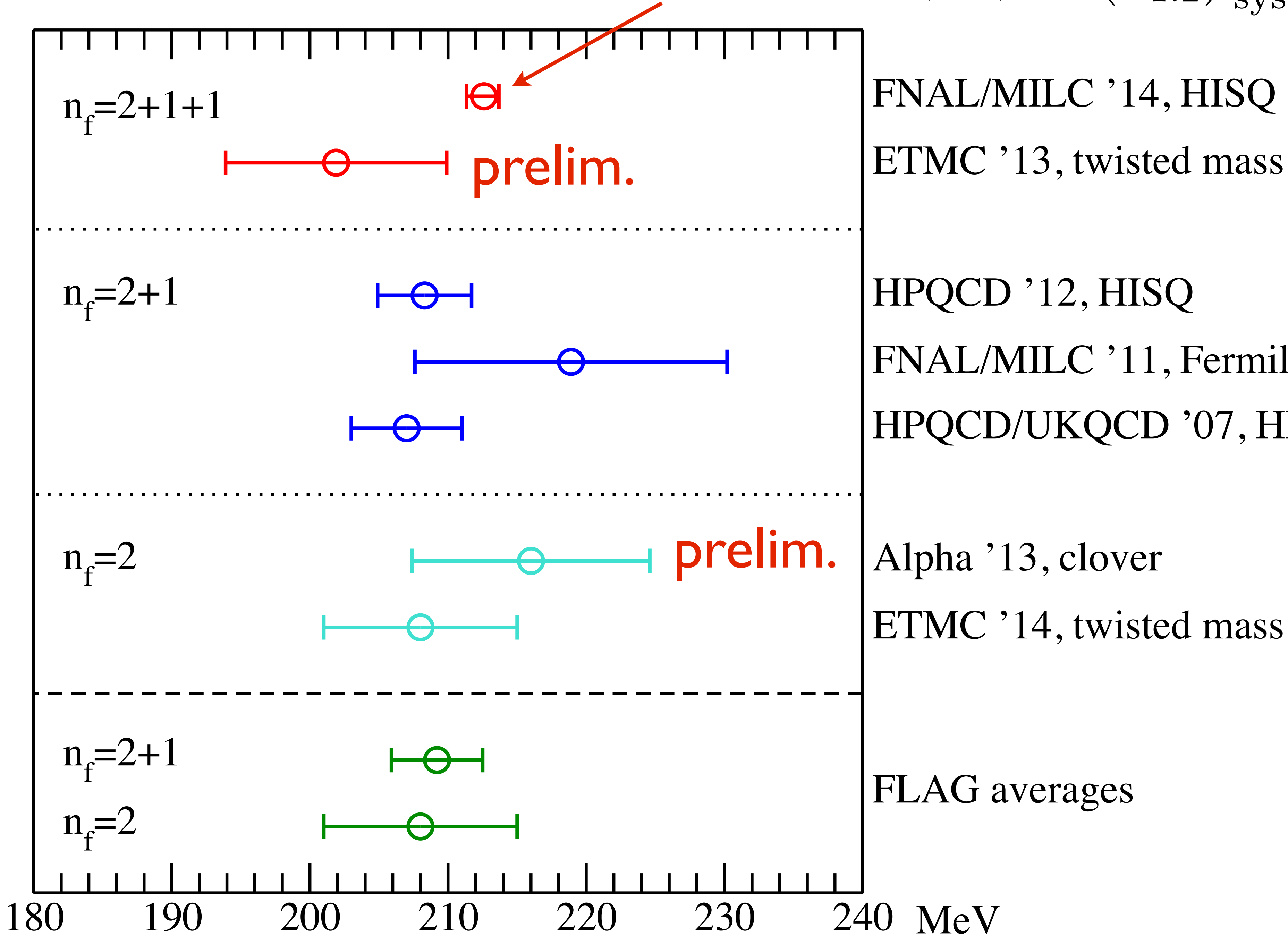
CKM matrix elements

- Several processes can be used to extract the same CKM matrix element involving different experimental and theory input
  - leptonic decays
  - semileptonic decays
  - neutrino experiments
- Allows for cross checks and consistency tests of the Standard Model, constraints/test for new physics (similar methods can be used to study B decays)
- Need non-perturbative theoretical calculation of form factors and decay constants $\rightarrow$ Lattice QCD

Leptonic decays, decay constants

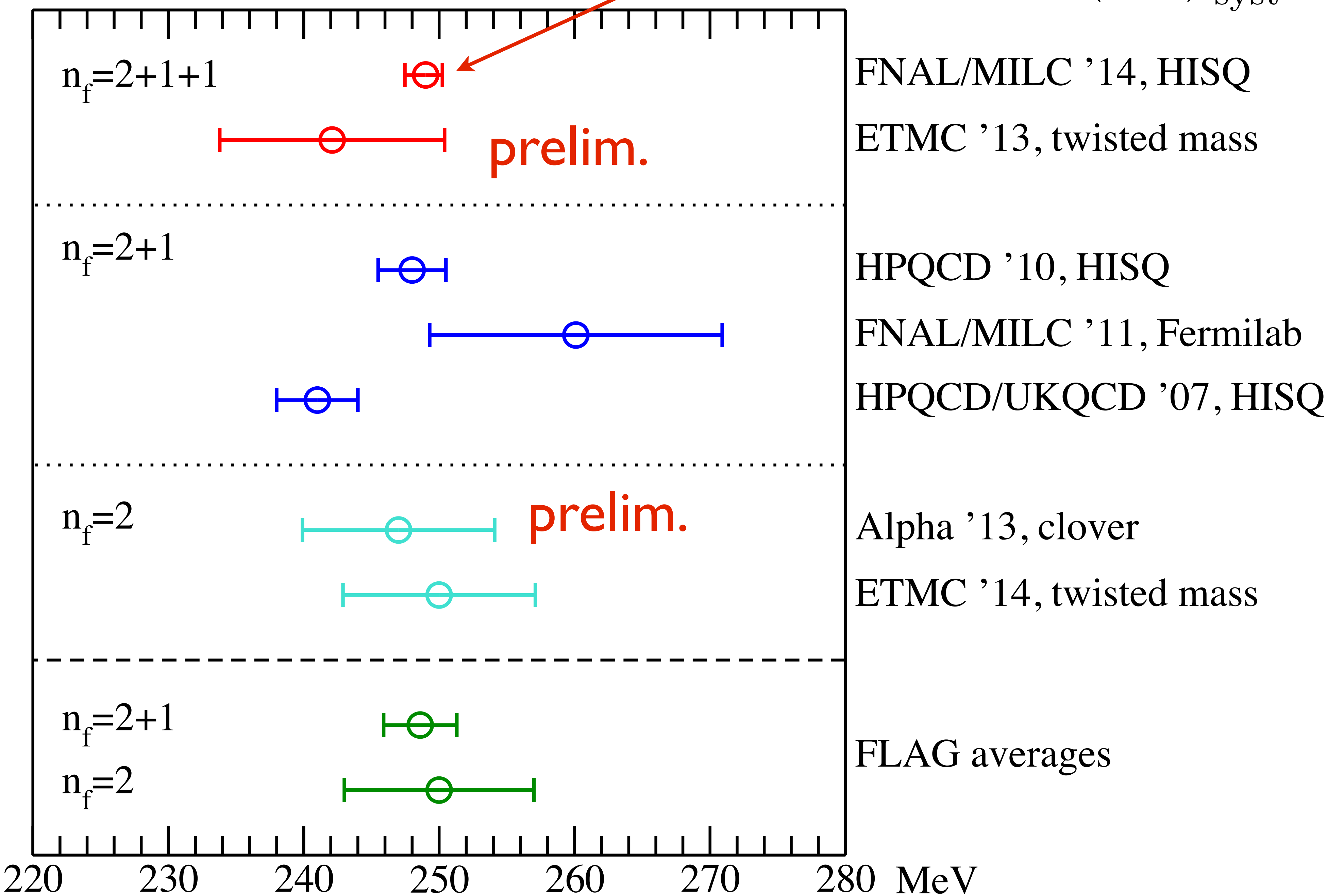
Current status: f_D

$$f_D = 212.6(0.4)_{\rm stat}\left({}^{+1.0}_{-1.2}\right)_{\rm syst}\ \mathrm{MeV}$$

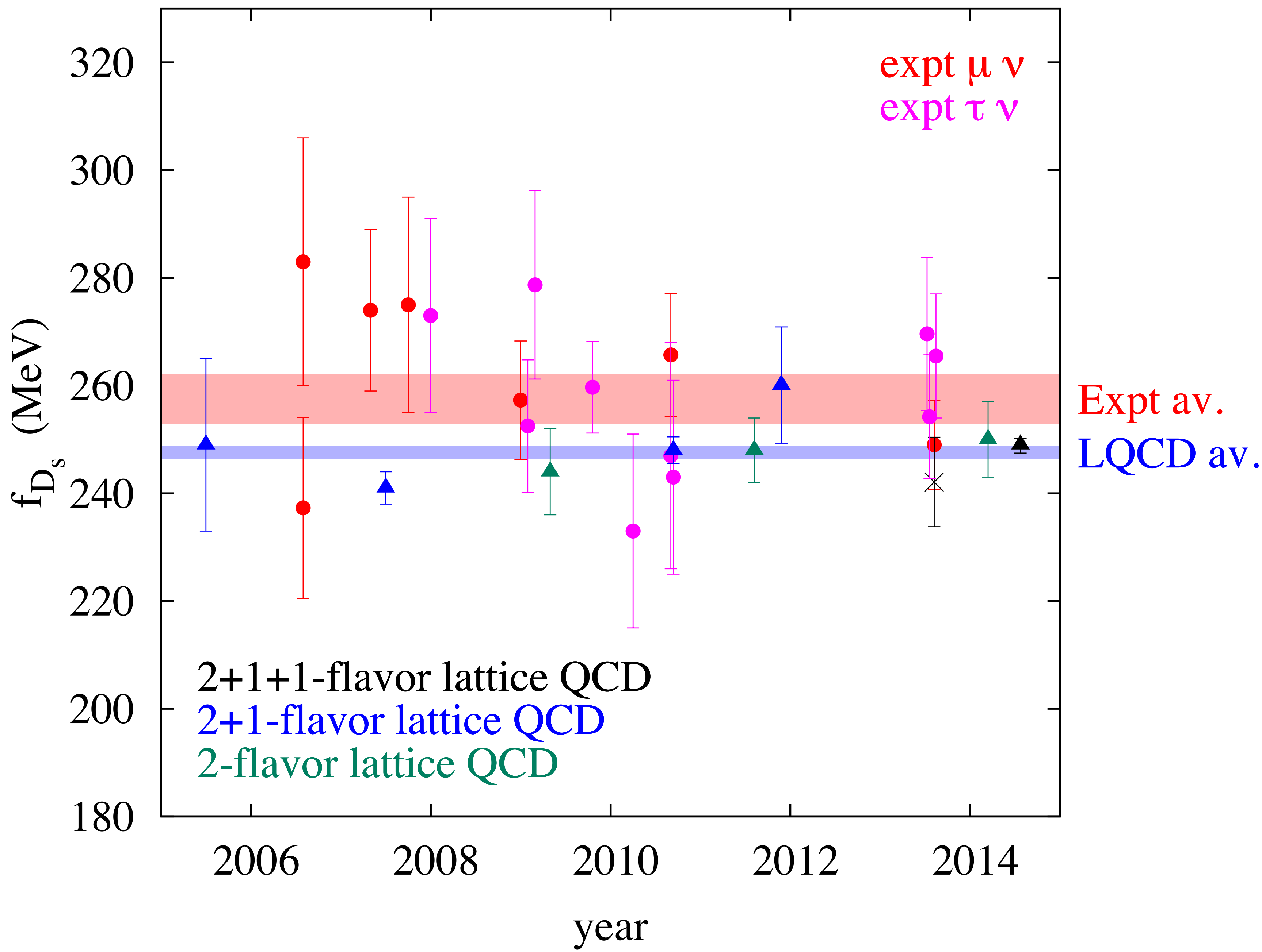


Current status: f_{D_s}

$f_{D_s} = 249.0(0.3)_{\,\mathrm{stat}}\left(^{+1.1}_{-1.5}\right)_{\,\mathrm{syst}}$ MeV

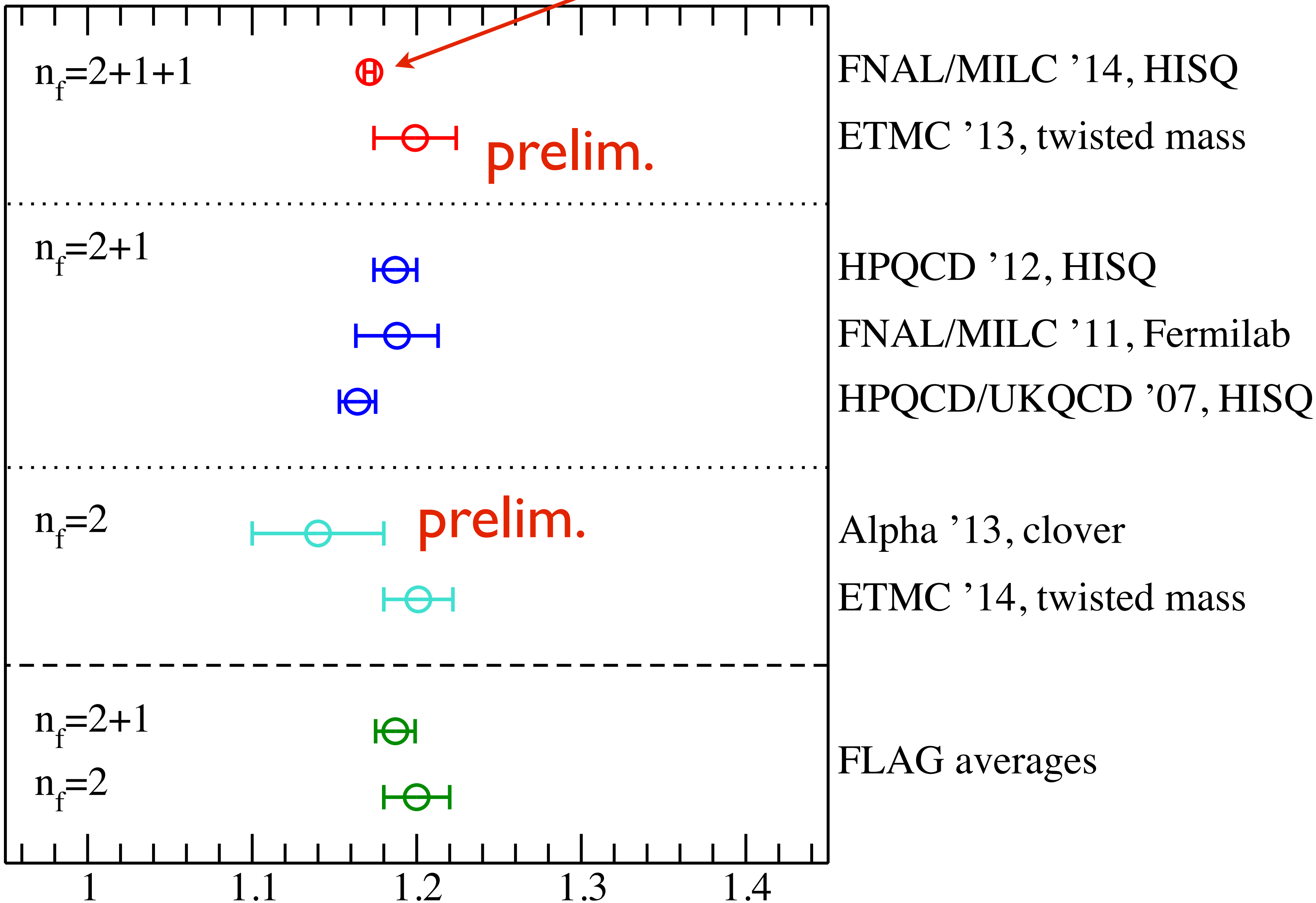


History of D_s decay constant

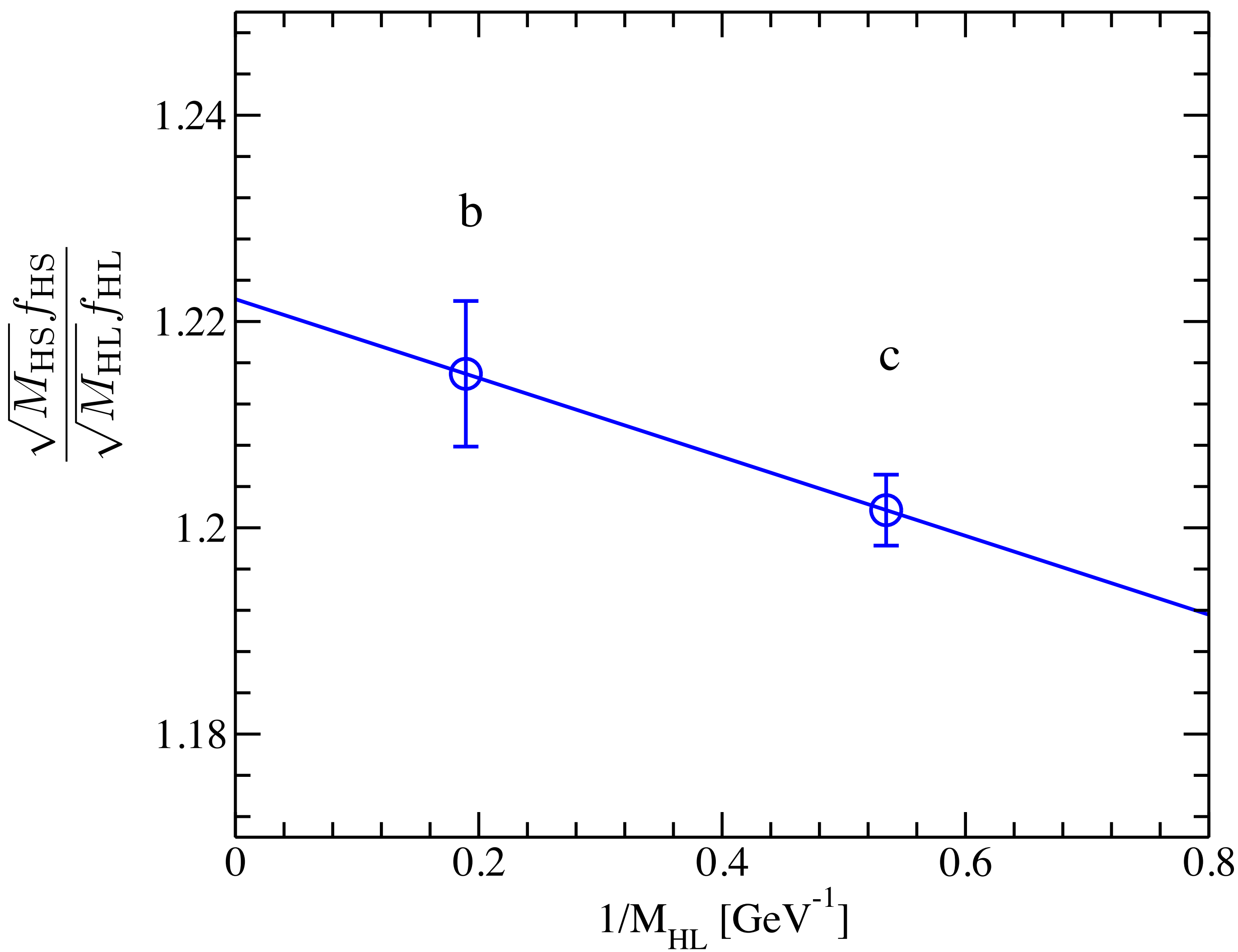


Current status: f_{D_s}/f_D

$$f_{D_s}/f_D = 1.1712(10)_{\rm stat}\left(^{+29}_{-32}\right)_{\rm syst}$$



Ratios of heavy-strange and heavy-light meson decay constants



New determinations of ratios of decay constants (by FNAL/MILC [arXiv:1407.3772] and HPQCD [arXiv:1302.2644], respectively)

$$\frac{f_{D_s}}{f_{D^+}} = 1.1712(10)\left(^{+29}_{-32}\right)$$

$$\frac{f_{B_s}}{f_B} = 1.205(7)$$

Meson decay constants: summary

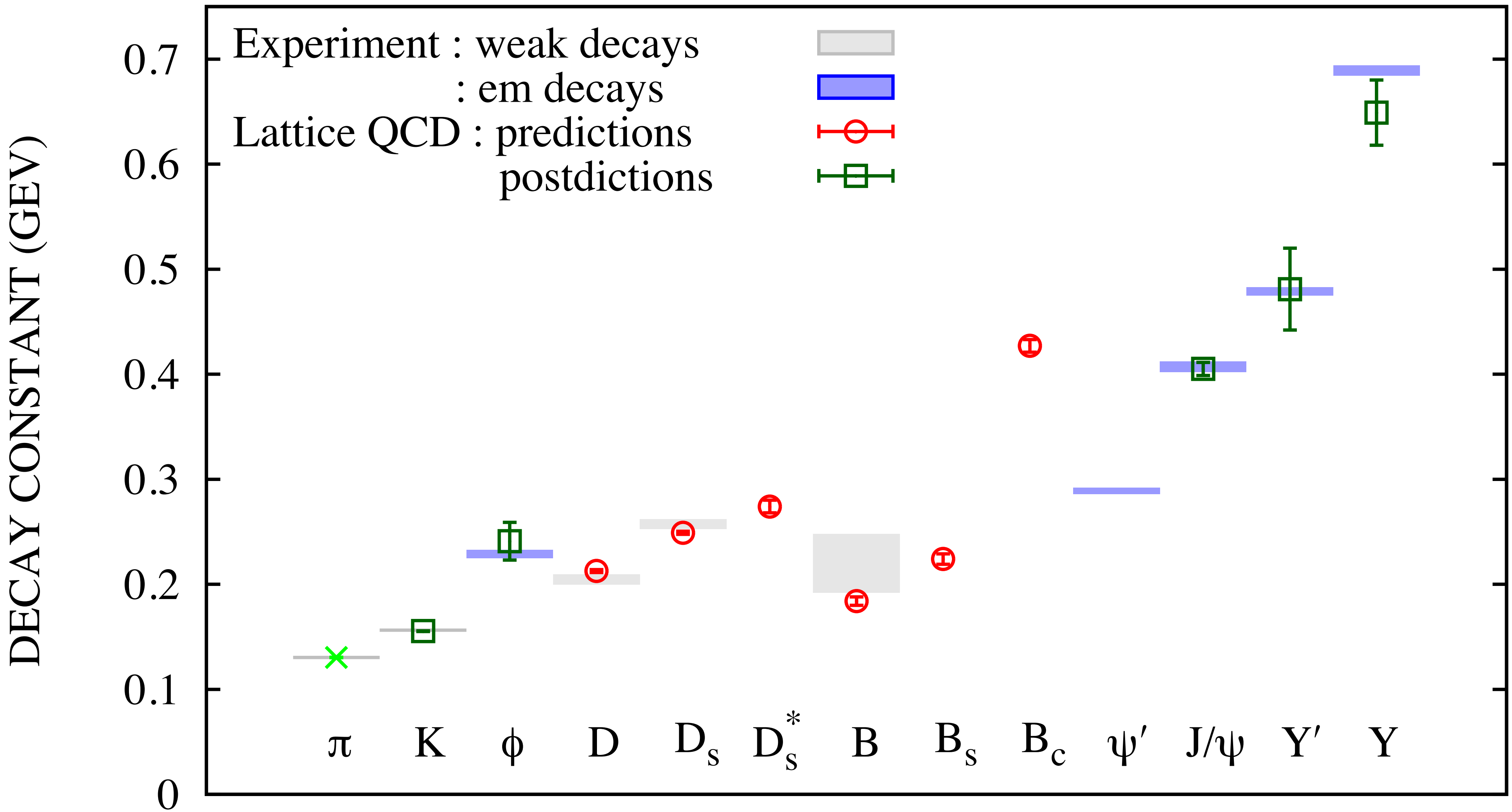
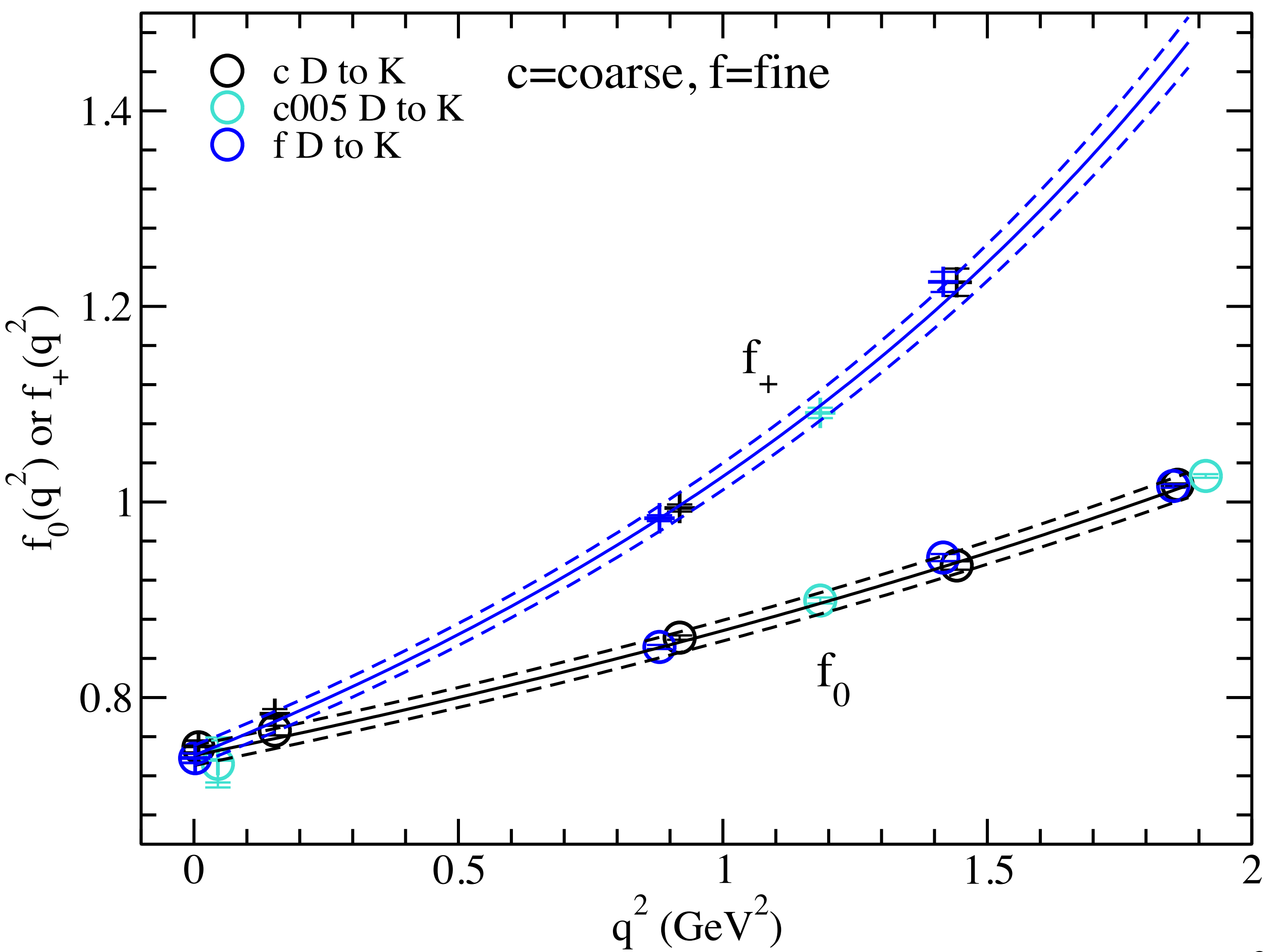


Fig. by C. Davies, HPQCD

Semileptonic decays, form factors

Shape of the form factors: $D \to K \ell \nu$



- Both scalar and vector form factor, f_0 and f_+, as a function of q^2
- Kinematic constraint $f_+(0)=f_0(0)$

q^2=four-momentum transfer

HPQCD, arXiv:1305.1462

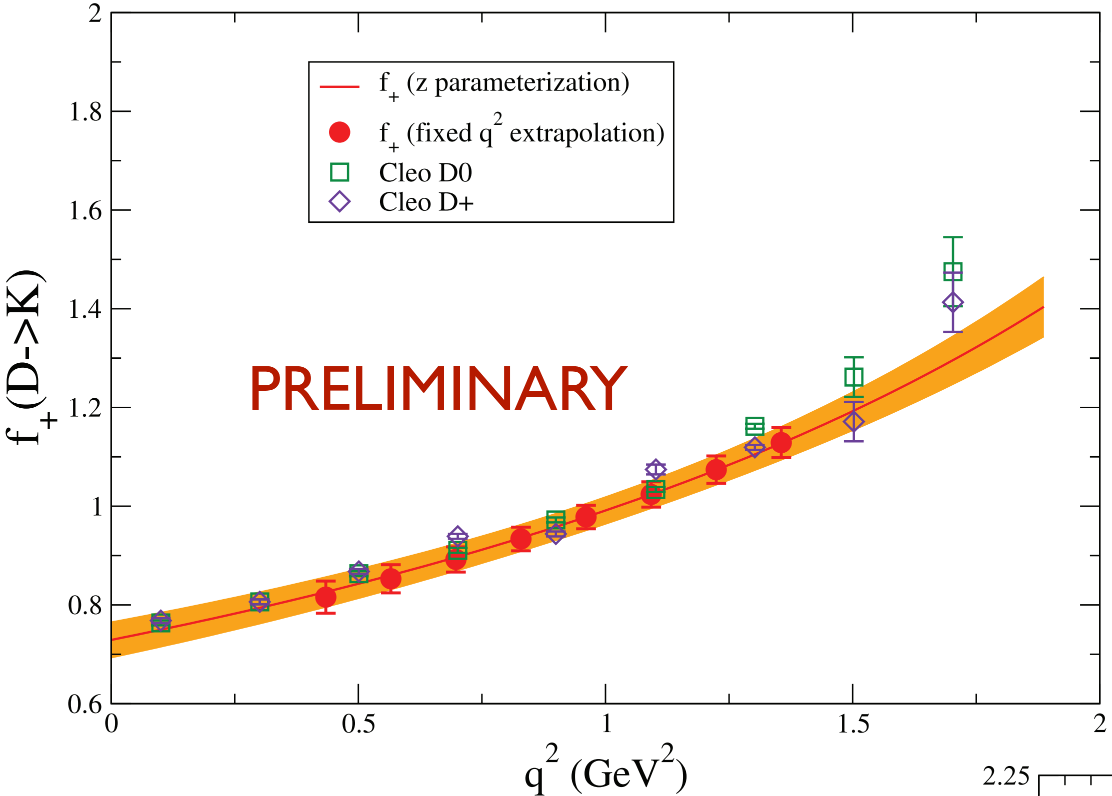


Fig. by F. Sanfilippo, talk at Lattice 2013, ETMC

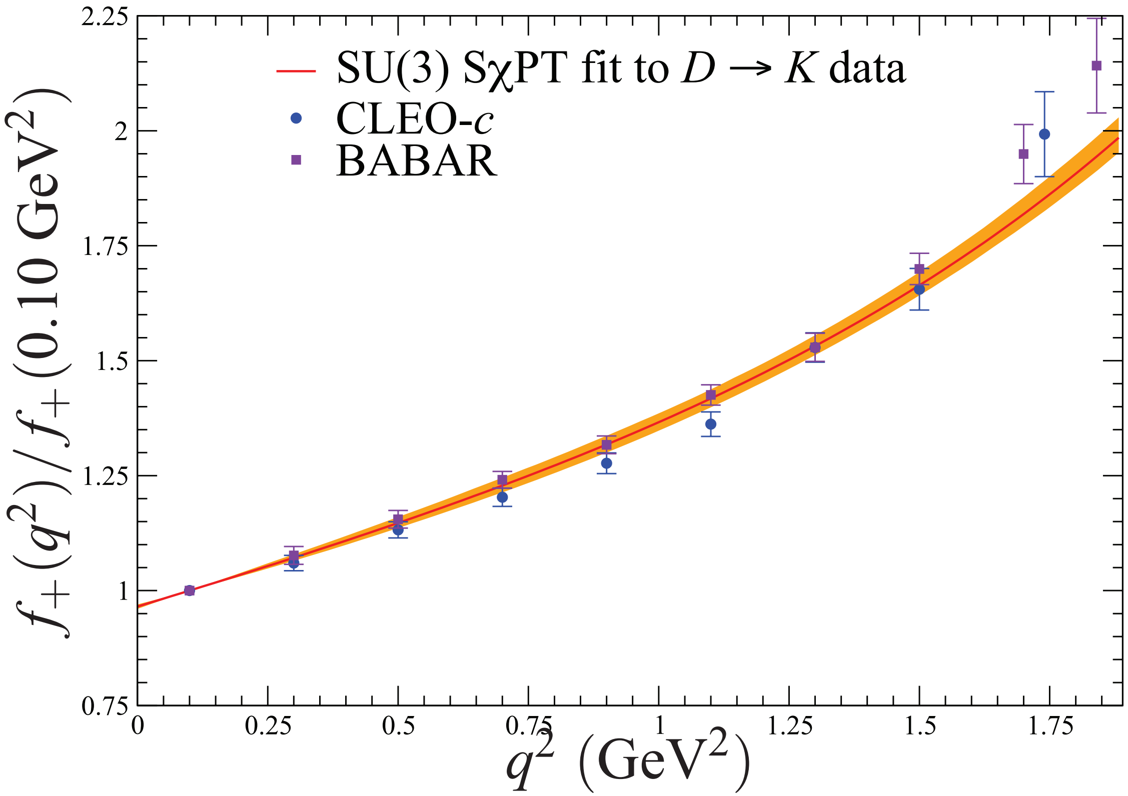
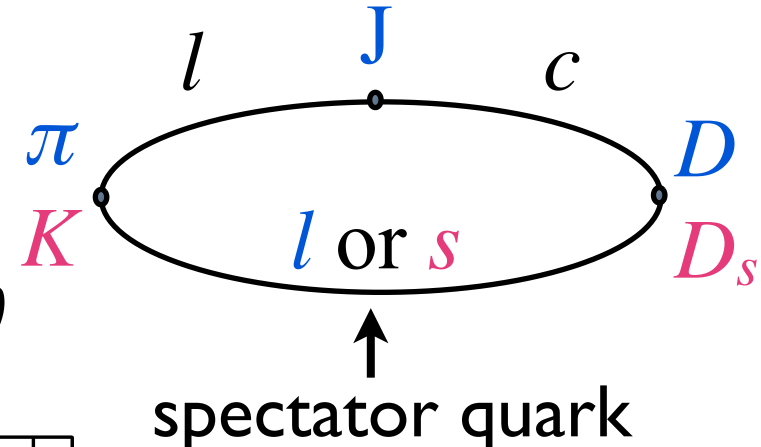
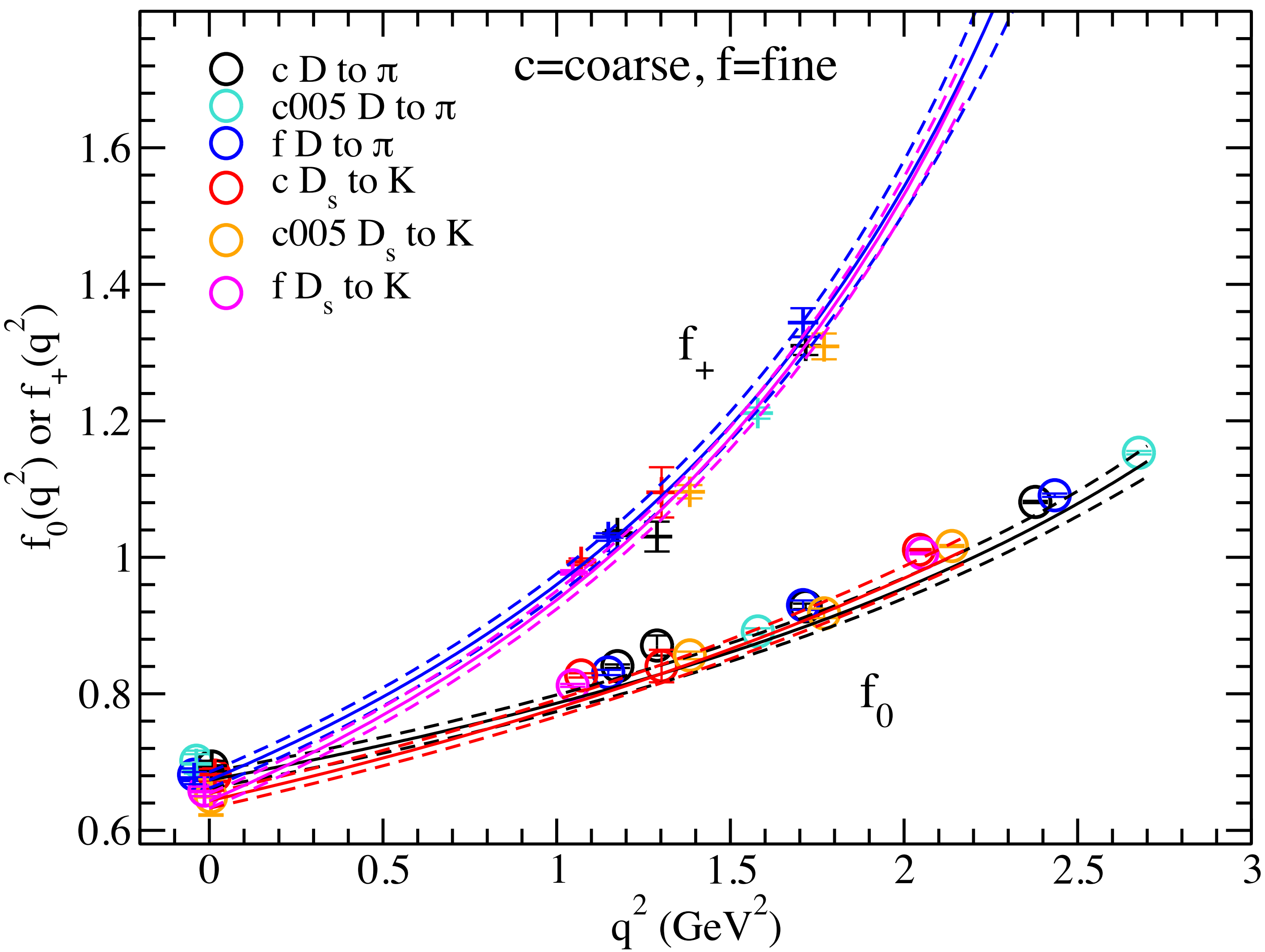


Fig. by FNAL/MILC, PoS(Lattice2012)272

Shape of the FFs: $D \to \pi \ell \nu$ and $D_s \to K \ell \nu$





- Different spectator quark - light vs. strange
- Both decays experimentally accessible
- The shapes of the FFs are same within few %

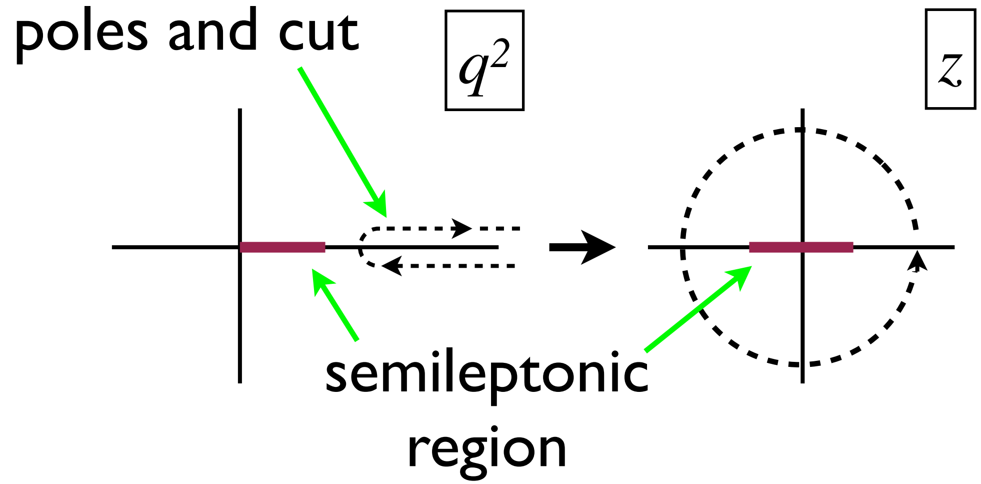
HPQCD, arXiv:1111.0225 and 1208.6242

The z-expansion

- Remove the poles

$$\tilde{f}_0^{D\to K}(q^2) = \left(1 - \frac{q^2}{M^2_{D^*_{s0}}}\right) f_0^{D\to K}(q^2),$$

$$\tilde{f}_+^{D\to K}(q^2) = \left(1 - \frac{q^2}{M^2_{D^*_s}}\right) f_+^{D\to K}(q^2)$$



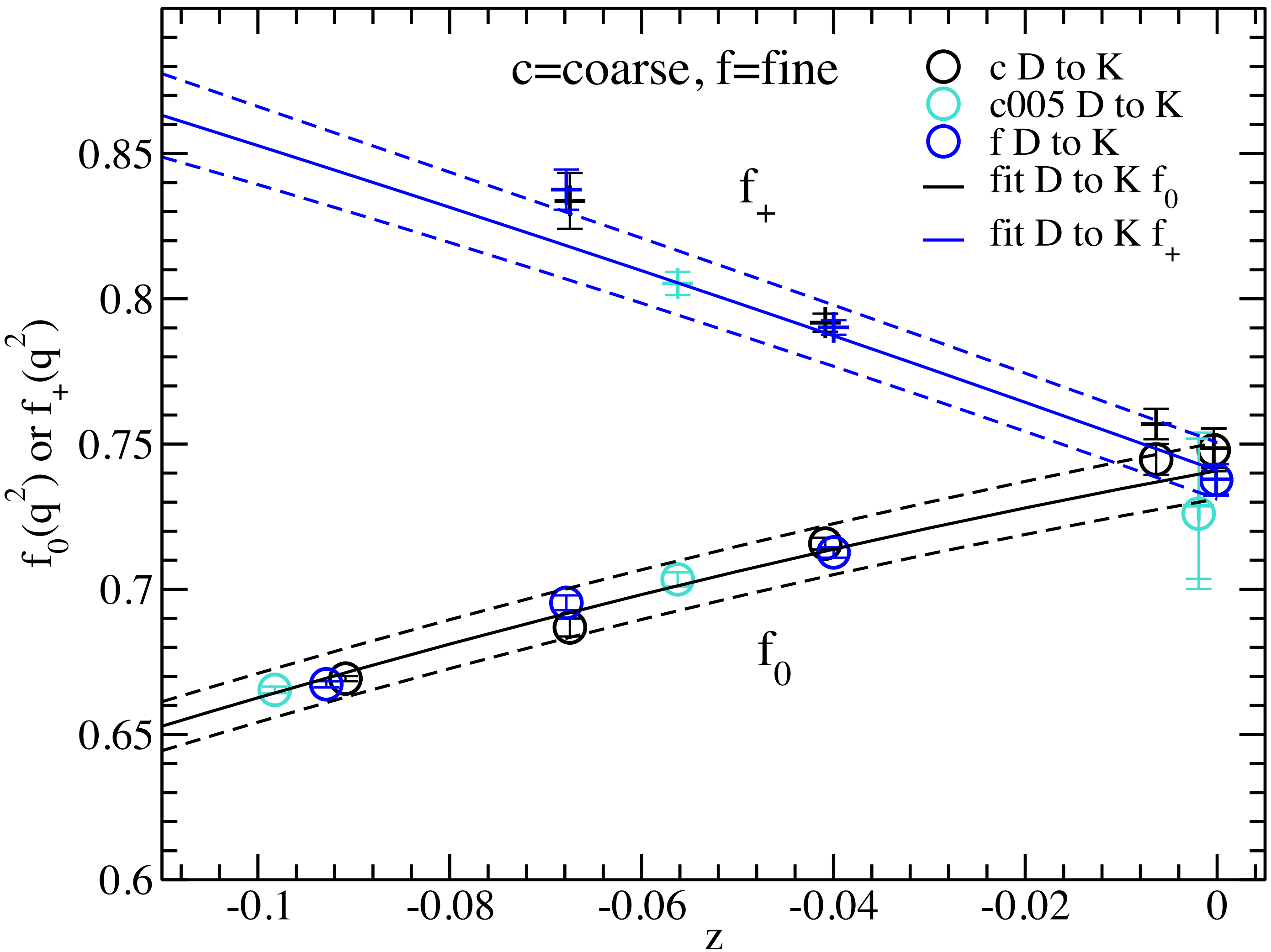
- Convert to z variable and fit $\tilde{f}$ as power series in z

$$z(q^2) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}, \quad t_+ = (m_D + m_K)^2,$$

$$\tilde{f}_0^{D\to K}(z) = \sum_{n\geq 0} c_n(a) z^n, \quad \tilde{f}_+^{D\to K}(z) = \sum_{n\geq 0} b_n(a) z^n, \quad c_0 = b_0$$

C. Bourrely, I. Caprini and L Lellouch, PRD 79, 013008 (2009), arXiv:0807.2722

Continuum and chiral extrapolation

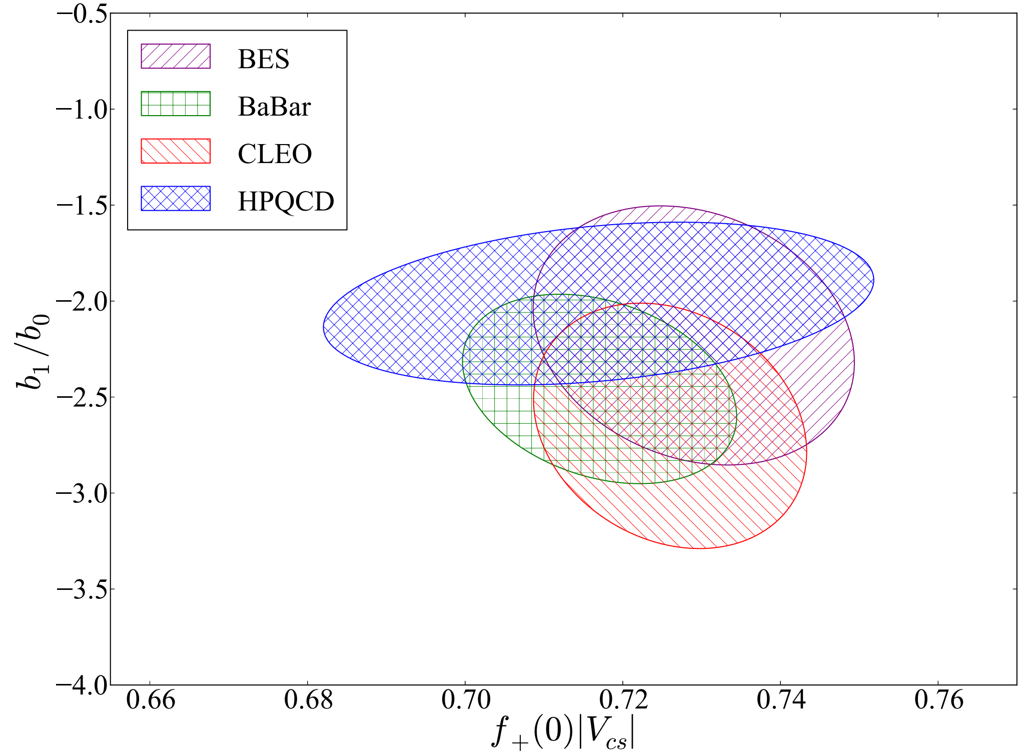


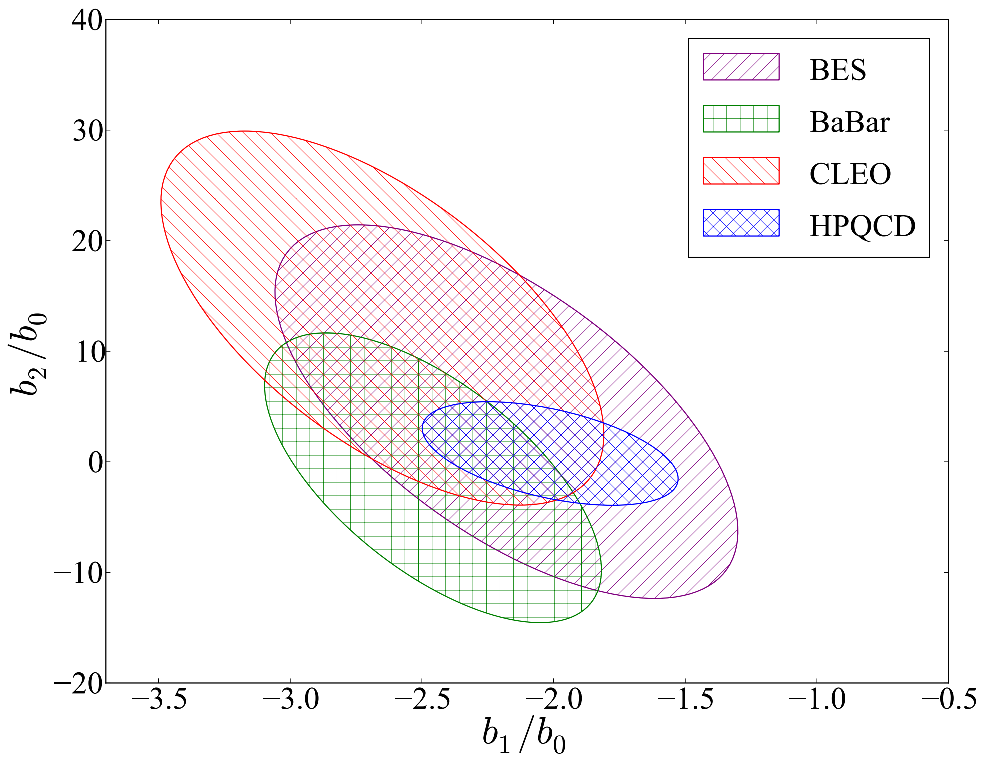
- Fit in z-space, including terms that depend on lattice spacing and quark masses
- Take $a=0$ and $m_q=m_q^{\mathrm{phys}}$

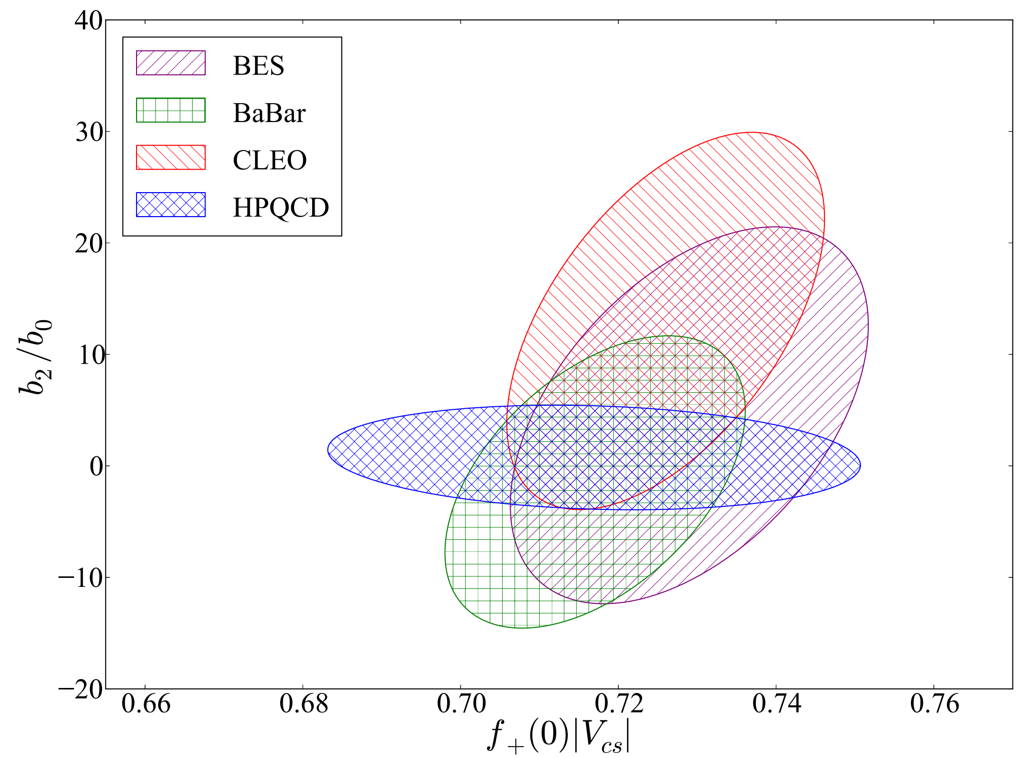
HPQCD, arXiv:1305.1462

f_+ shape parameters: $D \to K \ell \nu$

HPQCD, arXiv:1305.1462



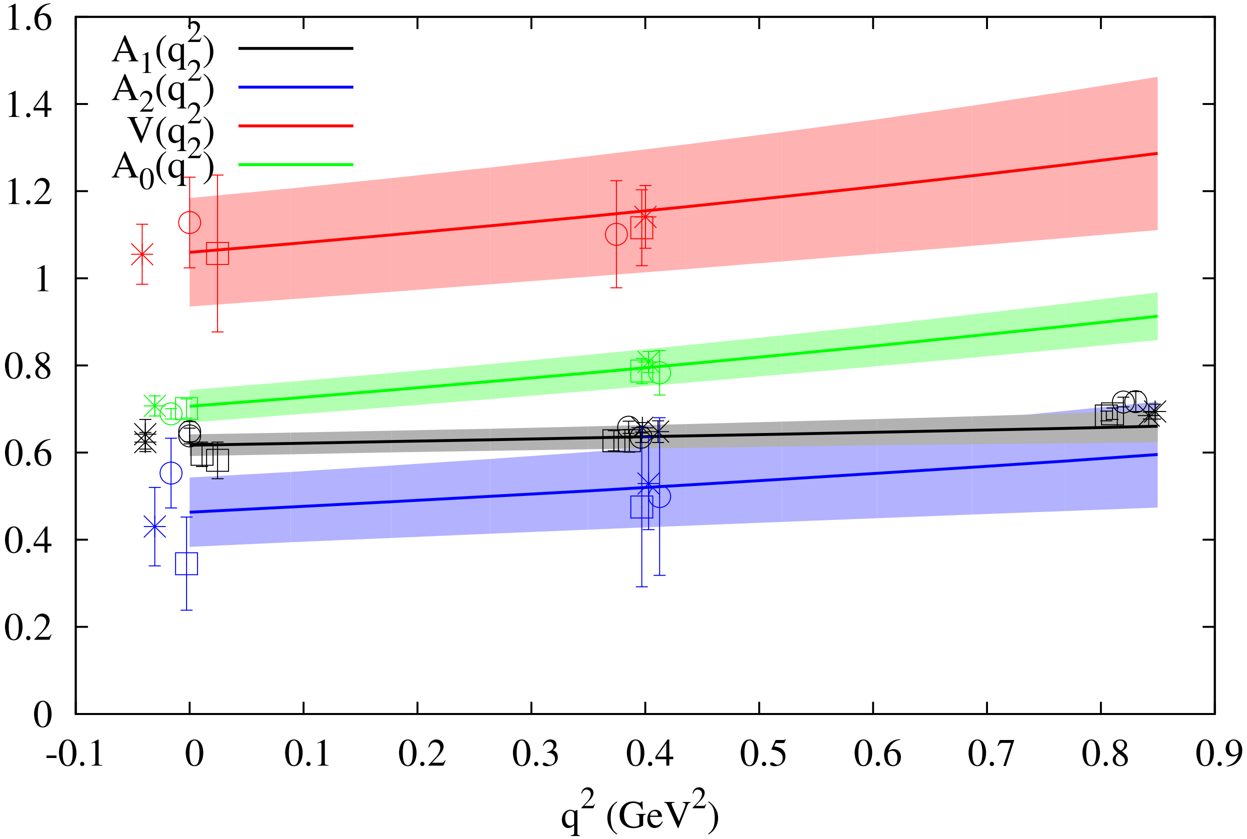




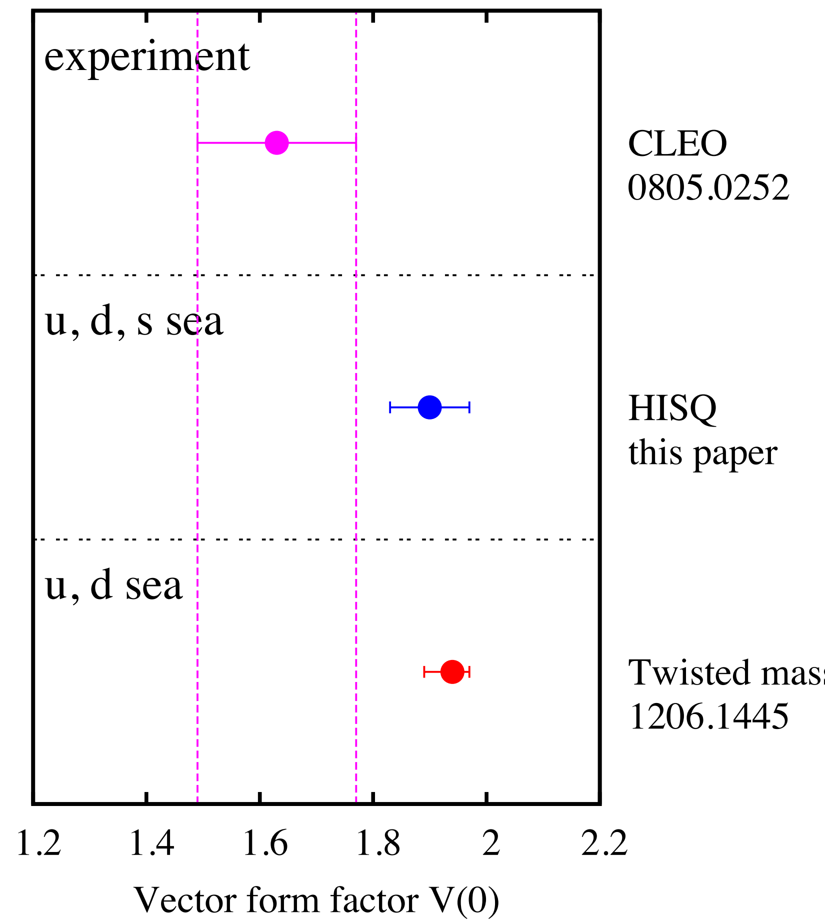
Decays involving vector mesons

- Similar methods can be used to study decays that involve vector mesons

weak decay $D_s \to \phi\ell\nu$

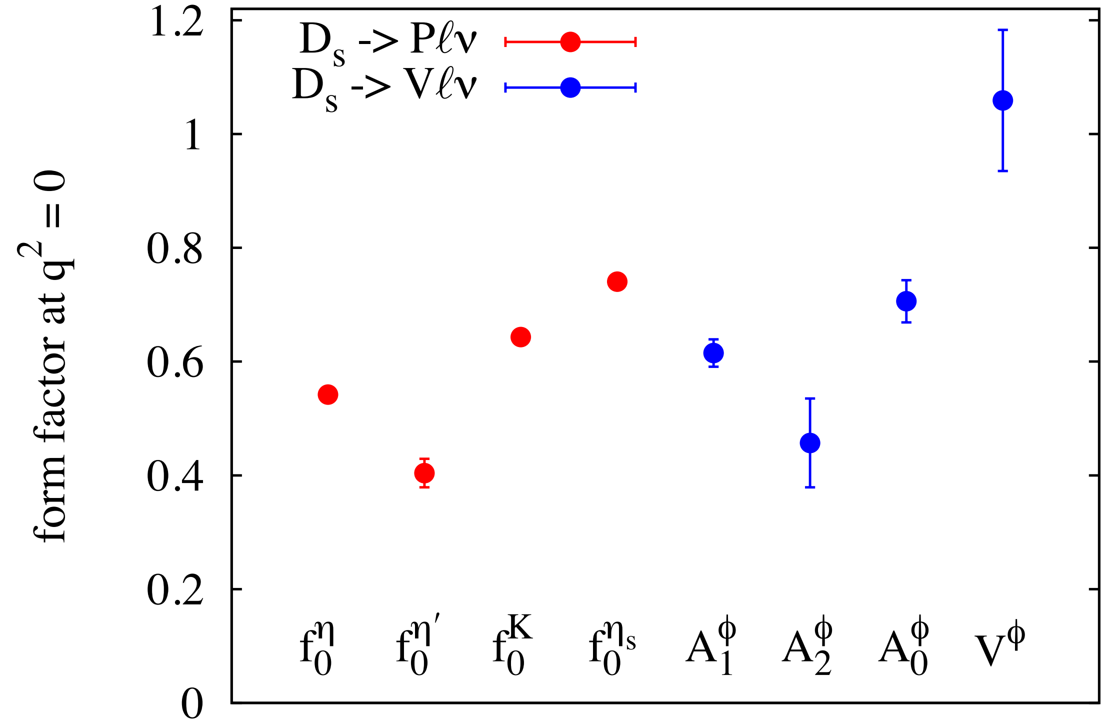


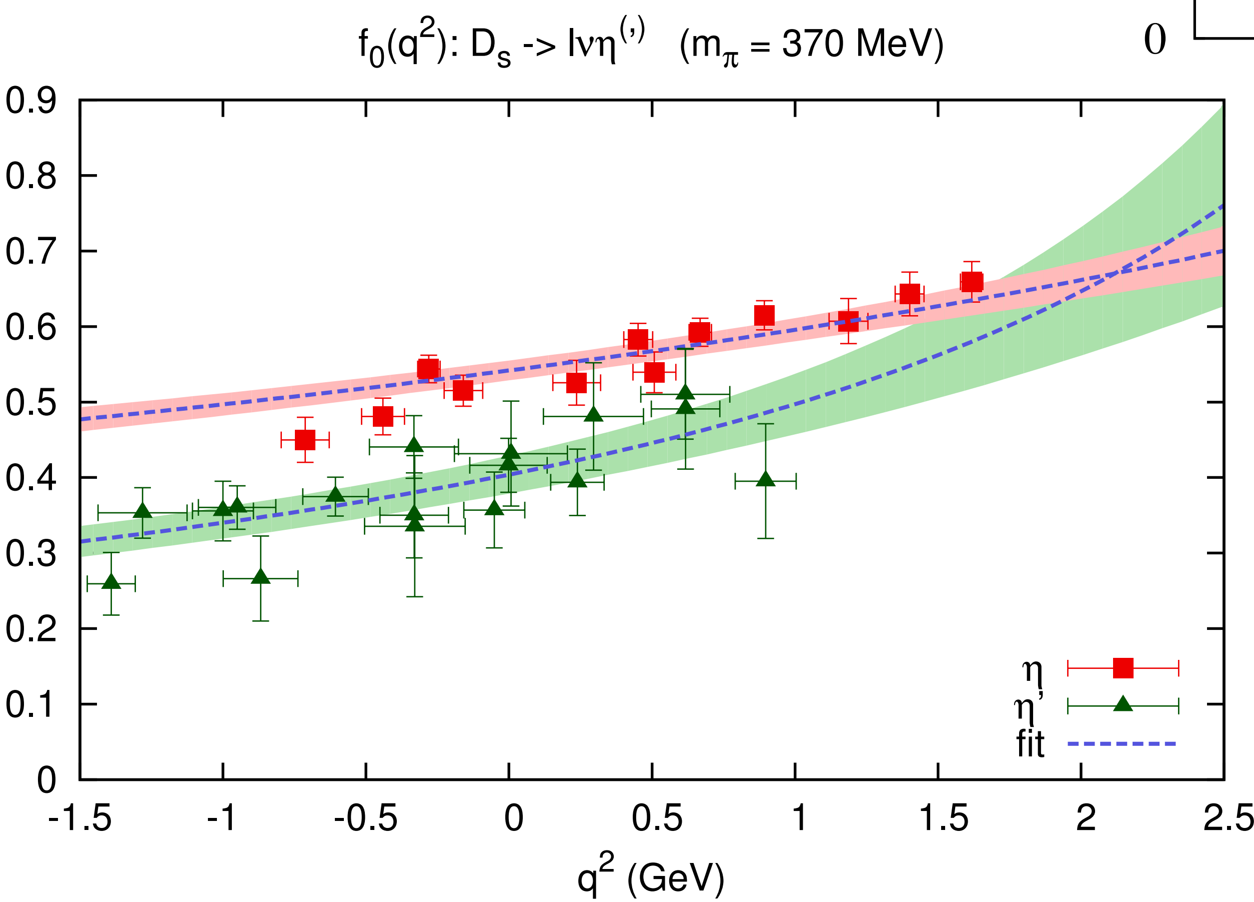
charmonium radiative decay $J/\psi \to \eta_c\gamma$



Figs. by G. Donald, HPQCD, arXiv:1311.6669 and 1208.2855

D_s decay form factors





HPQCD, arXiv:1208.6242,
HPQCD, arXiv:1311.6669,
Bali *et al.*, arXiv:1305.1462

Preliminary

$|V_{cs}|$ and $|V_{cd}|$

CKM elements from leptonic and semileptonic decays

- The branching fraction of D_s meson leptonic decay is

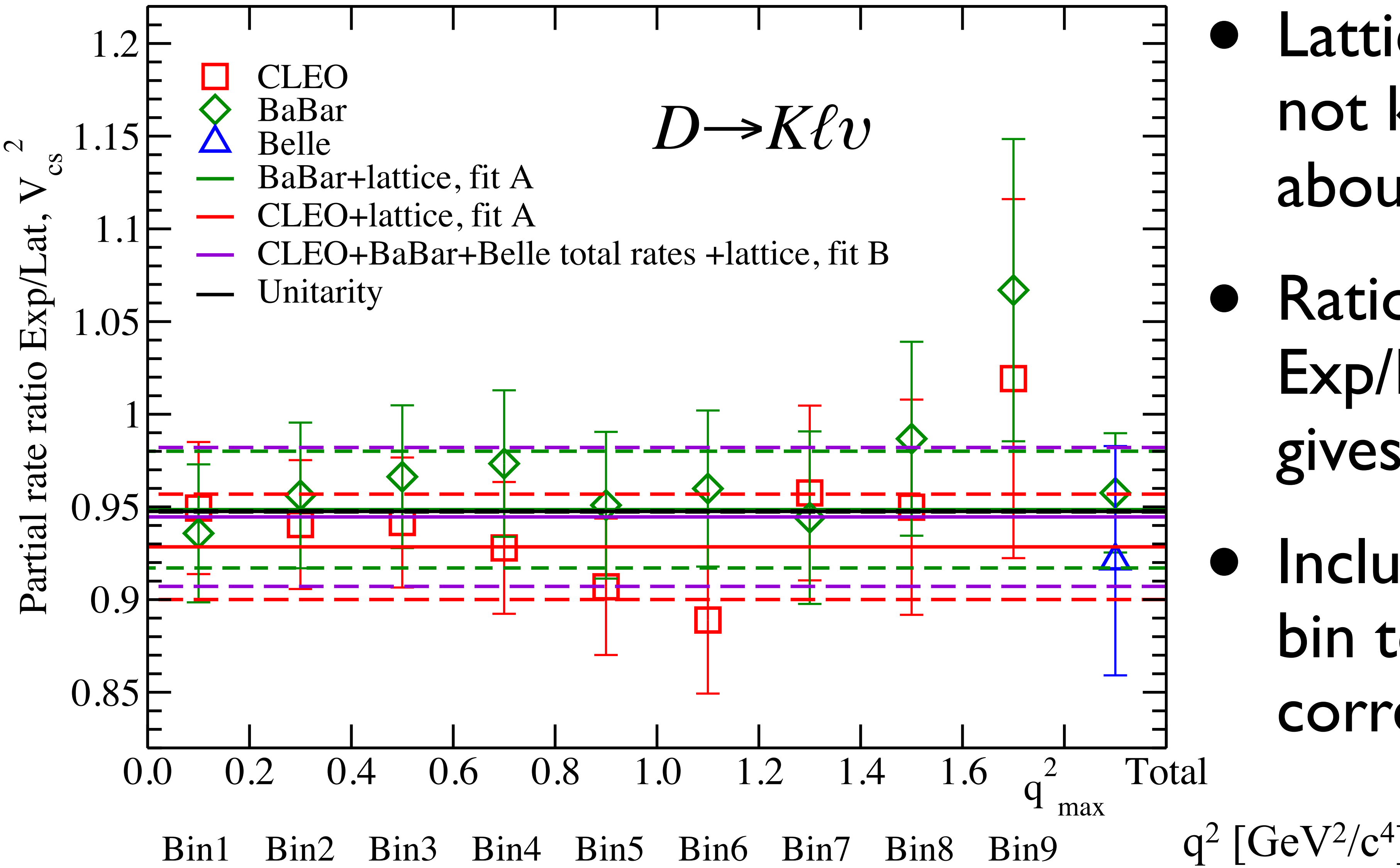
$$\mathcal{B}(D_s \to \ell\nu) = \frac{G_F^2}{8\pi} m_\ell^2 M_{D_s} \left(1 - \frac{m_\ell^2}{M_{D_s}^2}\right)^2 f_{D_s}^2 |V_{cs}|^2$$

- The partial decay rate of $D{\to}K$ semileptonic decay is

$$\frac{\mathrm{d}\Gamma^{D\to K}}{\mathrm{d}q^2} = \frac{G_F^2 p^3}{24\pi^3} |V_{cs}|^2 |f_+^{D\to K}(q^2)|^2$$

- Now we have the needed input from lattice QCD to determine CKM elements

Extracting V_{cs}

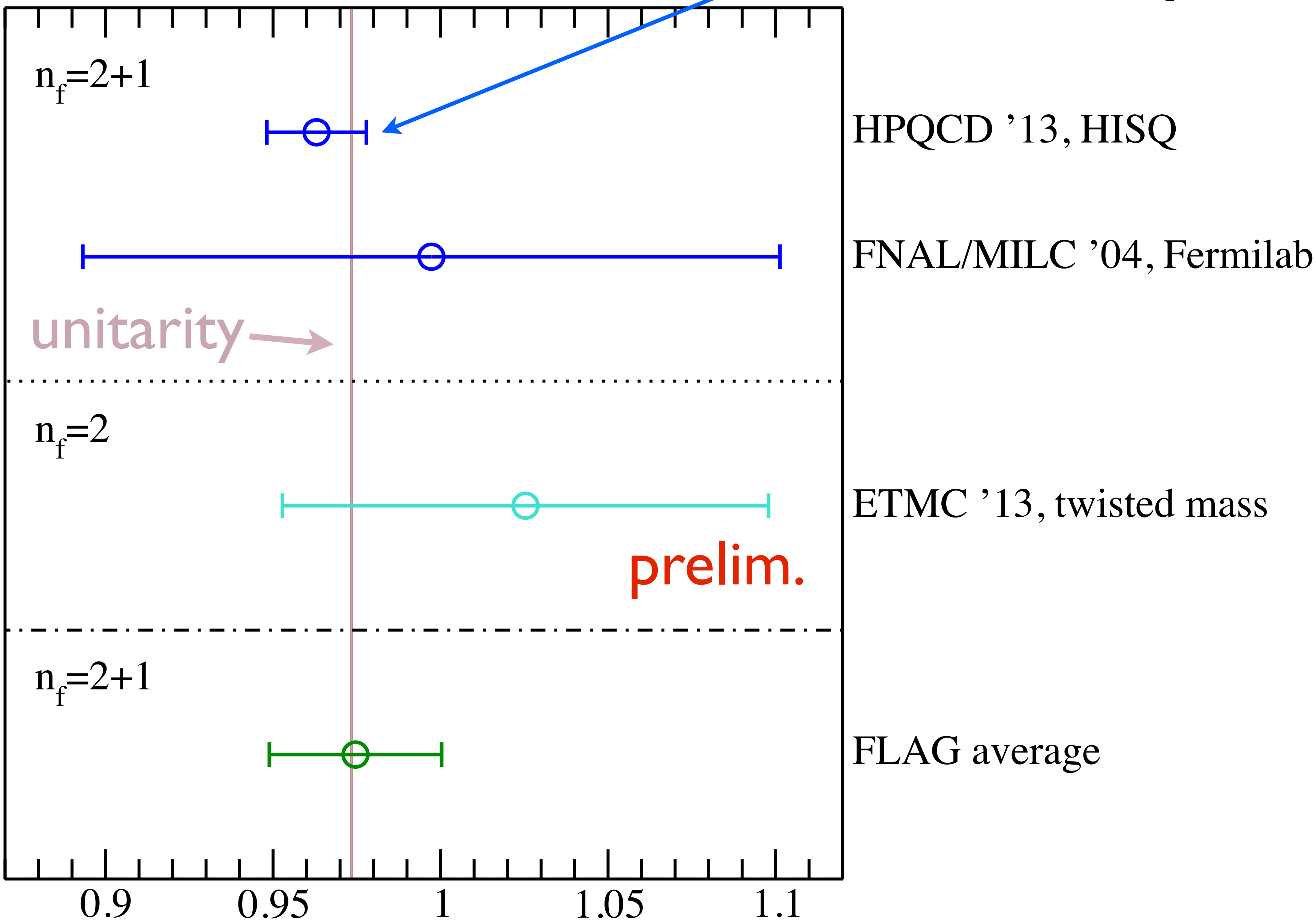


- Lattice does not know about V_{cs}
- Ratio Exp/Lat gives V_{cs}^2
- Include bin to bin correlations

HPQCD, arXiv:1305.1462

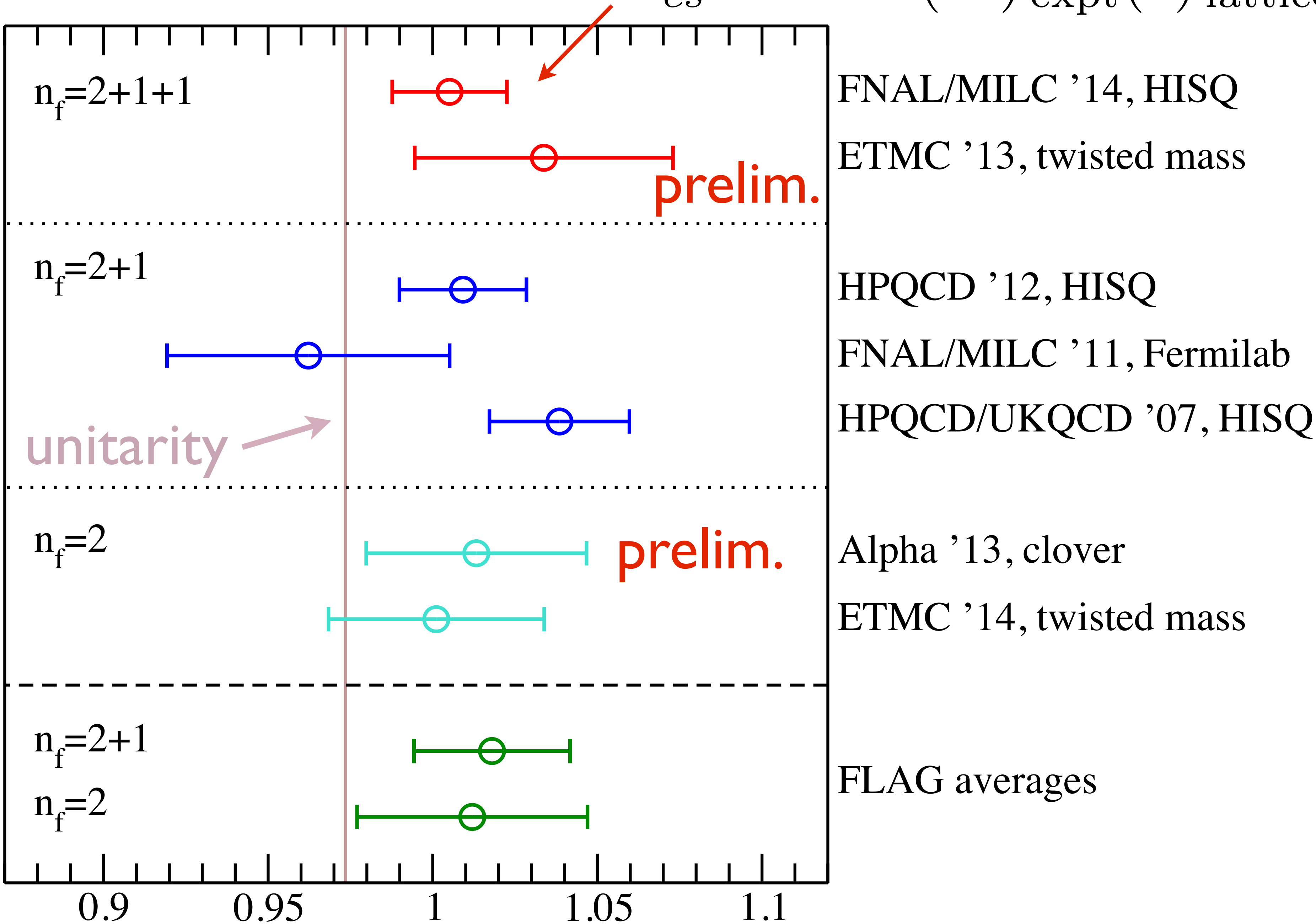
Current status: V_{cs}, semileptonic

$V_{cs} = 0.963(5)_{\rm expt}(14)_{\rm lattice}$

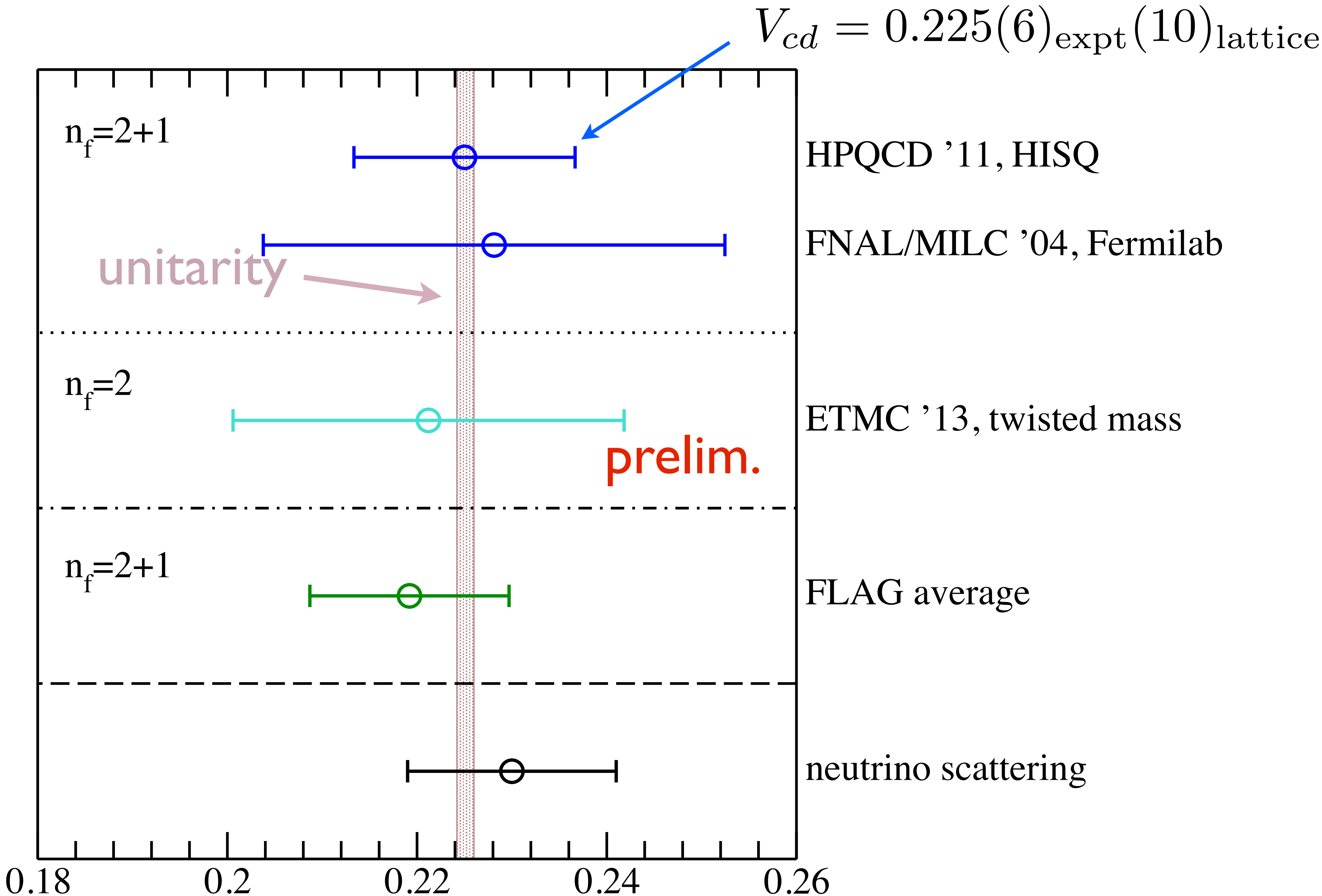


Current status: V_{cs}, leptonic

$$V_{cs} = 1.005(16)_{\text{expt}}(6)_{\text{lattice}}$$

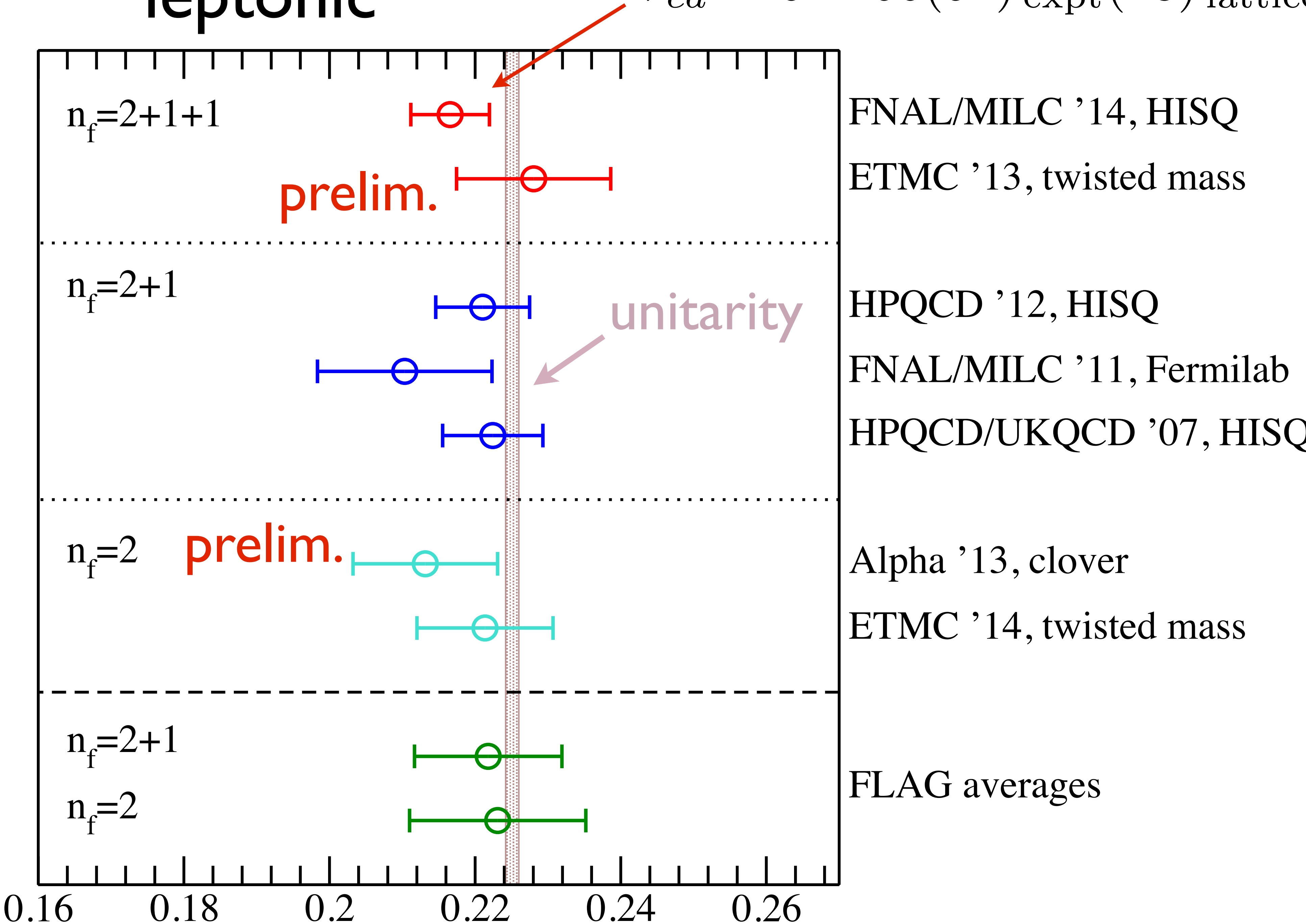


Current status: V_{cd}, semileptonic



Current status: V_{cd}, leptonic

$$V_{cd} = 0.2166(52)_{\rm expt}(13)_{\rm lattice}$$



Summary

- Can do precision tests of SM and search for new physics by extracting CKM elements from D and D_s meson leptonic and semileptonic decays
- Need input from theory (Lattice QCD):
  decay constants f_D, f_{D_s}
  form factors $D \to K\ell\nu, D \to \pi\ell\nu, D_s \to K\ell\nu$
- Also compare the shape of the form factors from Lattice QCD with experimental results $\to$ determine $|V_{cs}|$ using data from all q^2 bins

Summary of latest/best Lattice results

$$f_D = 212.6(0.4)_{\rm stat}\left({}^{+1.0}_{-1.2}\right)_{\rm syst}\ \mathrm{MeV}$$

$$f_{D_s} = 249.0(0.3)_{\rm stat}\left({}^{+1.1}_{-1.5}\right)_{\rm syst}\ \mathrm{MeV}$$

$$f_{D_s}/f_D = 1.1712(10)_{\rm stat}\left({}^{+29}_{-32}\right)_{\rm syst}$$

$$V_{cd} = 0.2166(52)_{\rm expt}(13)_{\rm lattice}\ \text{(leptonic)}$$

$$V_{cs} = 1.005(16)_{\rm expt}(6)_{\rm lattice}\ \text{(leptonic)}$$

$$V_{cd} = 0.225(6)_{\rm expt}(10)_{\rm lattice}\ \text{(semileptonic)}$$

$$V_{cs} = 0.963(5)_{\rm expt}(14)_{\rm lattice}\ \text{(semileptonic)}$$

Decay constants from arXiv:1407.3772, form factors for semileptonic decays taken from PRD84(2011)114505 (arXiv:1109.1501) and arXiv:1305.1462.

References

FNAL/MILC '04: arXiv hep-lat/0408306

FNAL/MILC '05: arXiv hep-lat/0506030

HPQCD/UKQCD '07: arXiv 0706.1726

HPQCD '10: arXiv 1008.4018

FNAL/MILC '11: arXiv 1112.3051

HPQCD '12: arXiv 1206.4936

FNAL/MILC '12: arXiv 1210.8431

HPQCD '13: arXiv 1305.1462

ETMC '14: arXiv 1308.1851

ETMC '13: L. Riggio's talk, Lattice 2013

Alpha '13: J. Heitger's poster, Lattice 2013

FNAL/MILC '14: arXiv 1407.3772

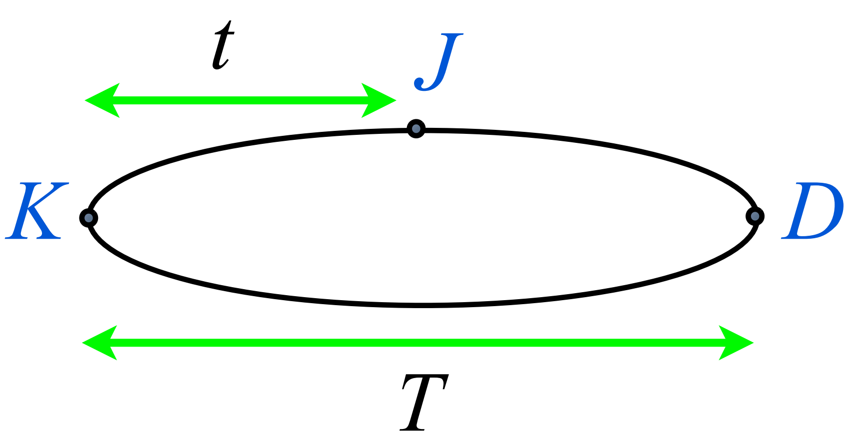
FLAG averages: http://itpwiki/unibe.ch/flag

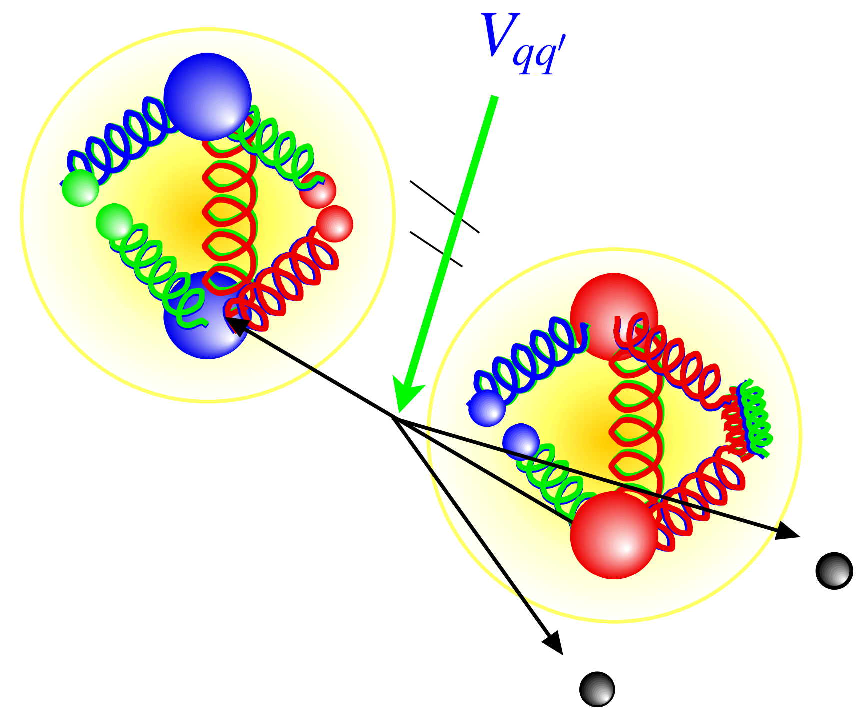
neutrino scattering: PDG, PRD86 (2012) 010001

Thank you!

Backup slides

Semileptonic form factors
= 3pt amplitudes





$$\langle K|S|D\rangle = f_0^{D\to K}(q^2)\frac{M_D^2 - M_K^2}{m_{0c} - m_{0s}}$$

$$\langle K|V^\mu|D\rangle = f_+^{D\to K}(q^2)\left[p_D^\mu + p_K^\mu - \frac{M_D^2 - M_K^2}{q^2}q^\mu\right] \qquad q^\mu = p_D^\mu - p_K^\mu$$

$$+f_0^{D\to K}(q^2)\frac{M_D^2 - M_K^2}{q^2}q^\mu \quad \to \quad f_0(0) = f_+(0)$$

Form factors: $D_s \rightarrow \phi \ell \upsilon$

$$\langle \phi(p',\epsilon)|V^\mu - A^\mu|D(p)\rangle = \frac{2i\epsilon^{\mu\nu\alpha\beta}}{M+m_\phi}\epsilon^*_\nu p'_\alpha p_\beta V(q^2) + (M+m_\phi)\epsilon^{*\mu}A_1(q^2)$$

$$+\frac{\epsilon^*\cdot q}{M+m_\phi}(p+p')^\mu A_2(q^2) + 2m_\phi \frac{\epsilon^*\cdot q}{q^2}q^\mu A_3(q^2)$$

$$-2m_\phi\frac{\epsilon^*\cdot q}{q^2}q^\mu A_0(q^2),$$

where $V^\mu = \bar{q}'\gamma^\mu Q, A^\mu = \bar{q}'\gamma^\mu\gamma_5 Q,$

and $A_3(q^2) = \frac{M+m_\phi}{2m_\phi}A_1(q^2) - \frac{M-m_\phi}{2m_\phi}A_2(q^2)$ with $A_0(0) = A_3(0)$

$D_s \to \phi \ell \upsilon$ differential decay rate

$$\frac{\mathrm{d}\Gamma(P \to V\ell\nu, V \to P_1P_2)}{\mathrm{d}q^2\mathrm{d}\cos\theta_V\mathrm{d}\cos\theta_\ell\mathrm{d}\chi} = \frac{3}{8(4\pi)^4}G_F^2|V_{q'Q}|^2\frac{p_V q^2}{M^2}\mathcal{B}(V \to P_1P_2)$$
$$\times \{(1-\eta\cos\theta_\ell)^2\sin^2\theta_V|H_+(q^2)|^2$$
$$+ (1+\eta\cos\theta_\ell)^2\sin^2\theta_V|H_-(q^2)|^2$$
$$+ 4\sin^2\theta_\ell\cos^2\theta_V|H_0(q^2)|^2$$
$$- 4\eta\sin\theta_\ell(1-\eta\cos\theta_\ell)\sin\theta_V\cos\theta_V\cos\theta_\chi H_+(q^2)H_0(q^2)$$
$$+ 4\eta\sin\theta_\ell(1+\eta\cos\theta_\ell)\sin\theta_V\cos\theta_V\cos\theta_\chi H_+(q^2)H_0(q^2)$$
$$- 2\sin^2\theta_\ell\sin^2\theta_V\cos 2\chi H_+(q^2)H_-(q^2)\},$$

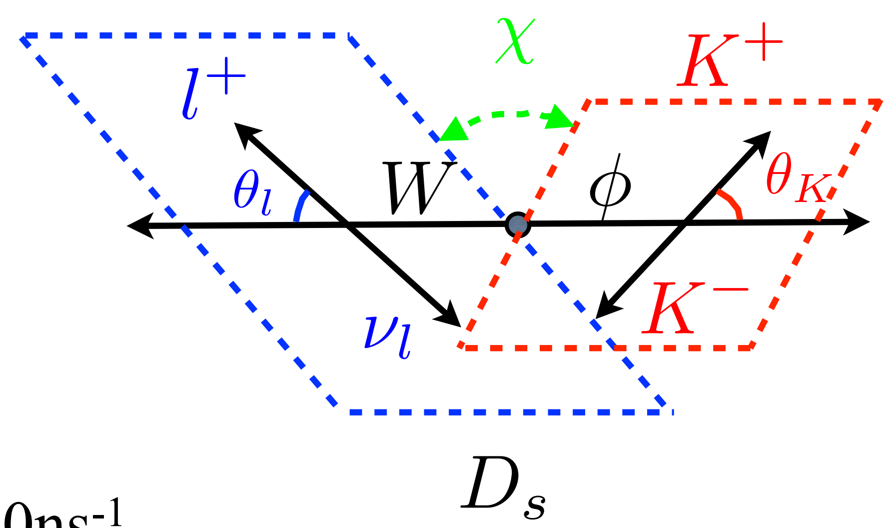
where the helicity amplitudes are

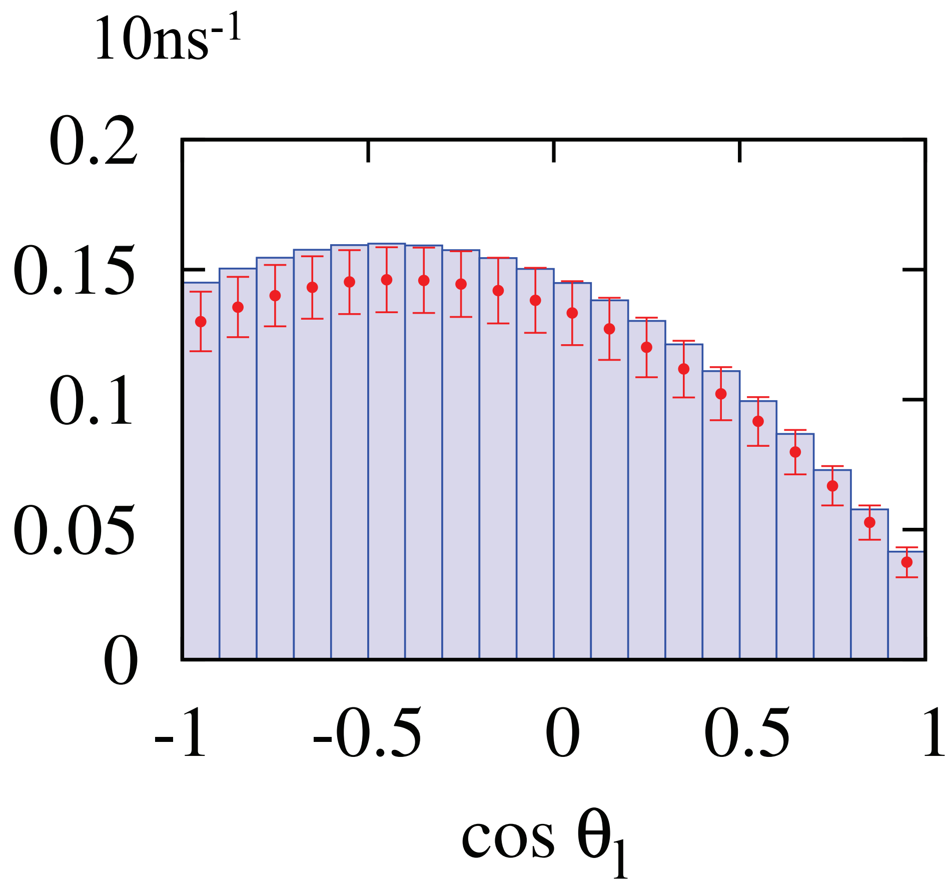
$$H_0(q^2) = \frac{1}{2m_\phi\sqrt{q^2}}\left[(M^2 - m_\phi^2 - q^2)(M + m_\phi)A_1(q^2) - 4\frac{M^2p_\phi^2}{M + m_\phi}A_2(q^2)\right]$$

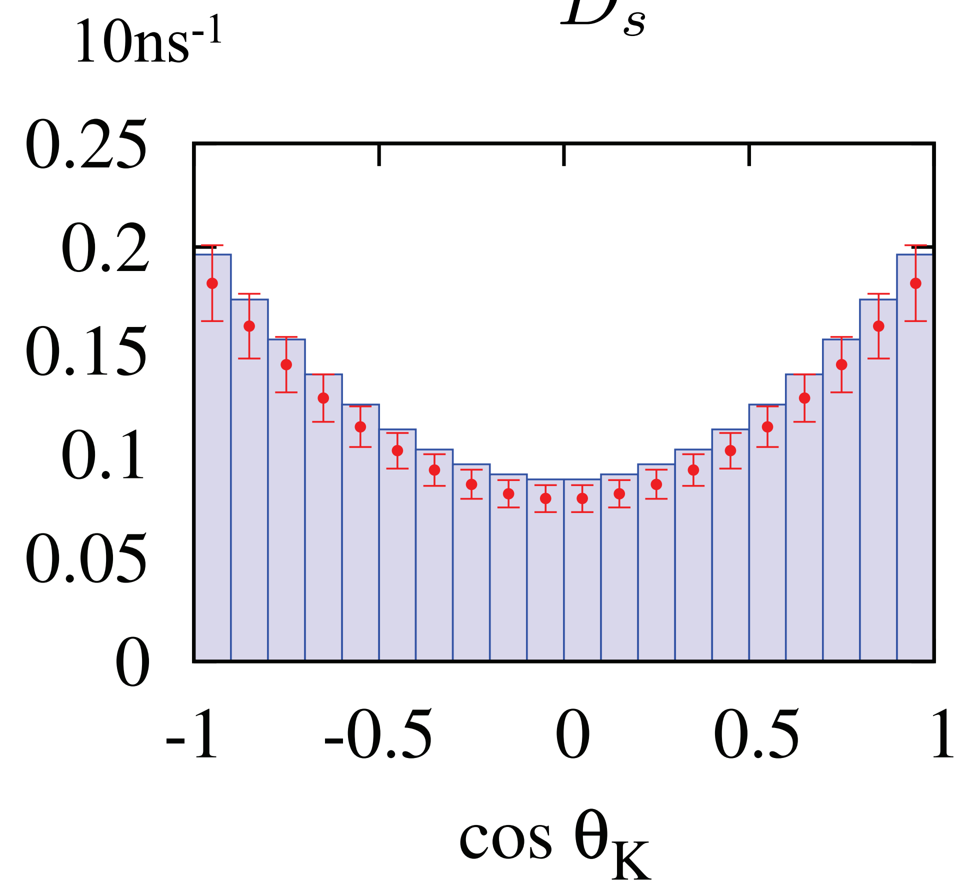
$$H_\pm(q^2) = (M + m_\phi)A_1(q^2) \mp \frac{2Mp_\phi}{M + m_\phi}V(q^2)$$

$D_s \to \phi \ell \nu$ angular distributions

G. Donald, HPQCD, Lattice 2013

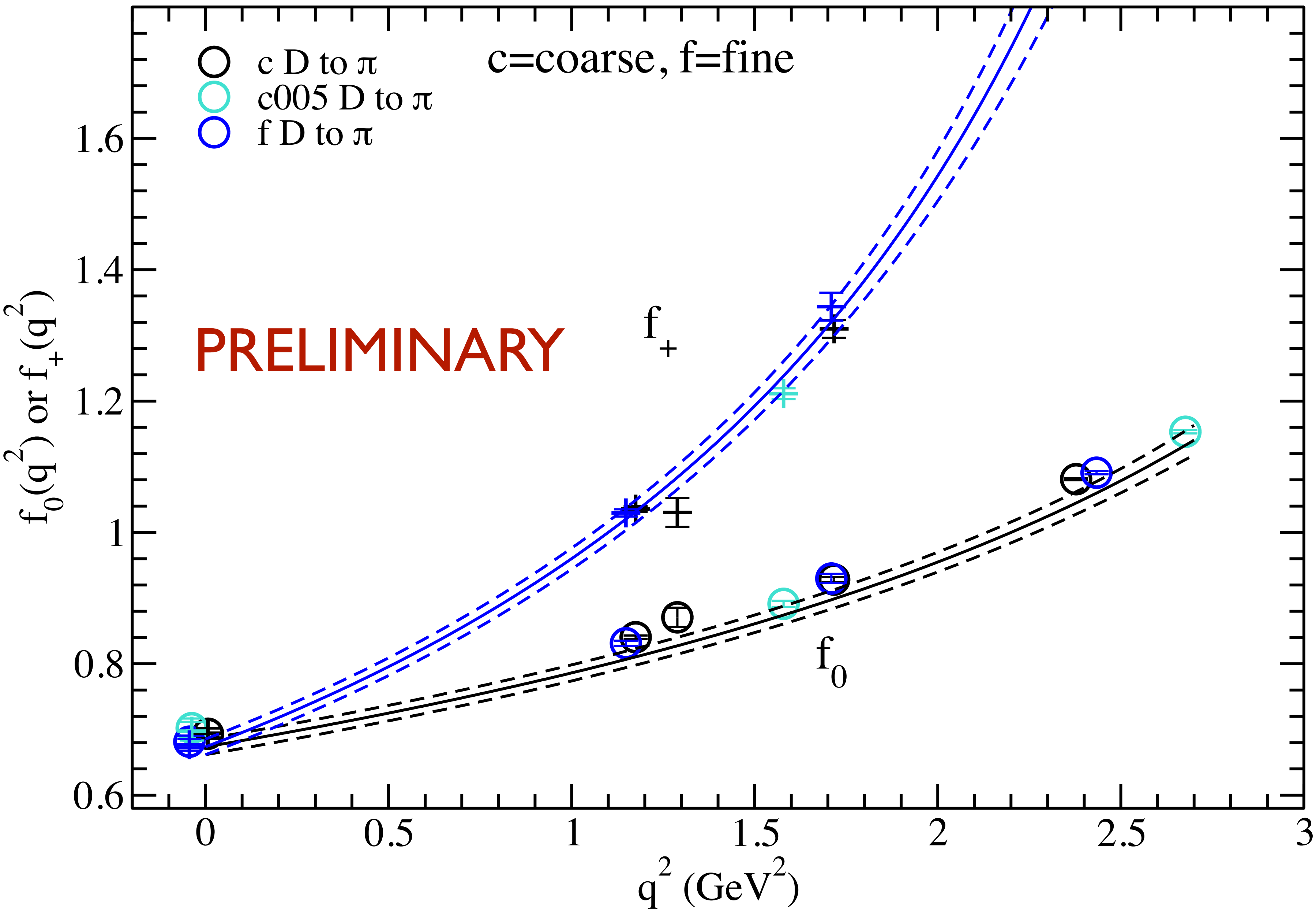






Experimental data from BaBar, PRD 78, 051101(R) (2008)

Shape of the form factors: $D \to \pi \ell \upsilon$



HPQCD, arXiv:1111.0225 and 1208.6242

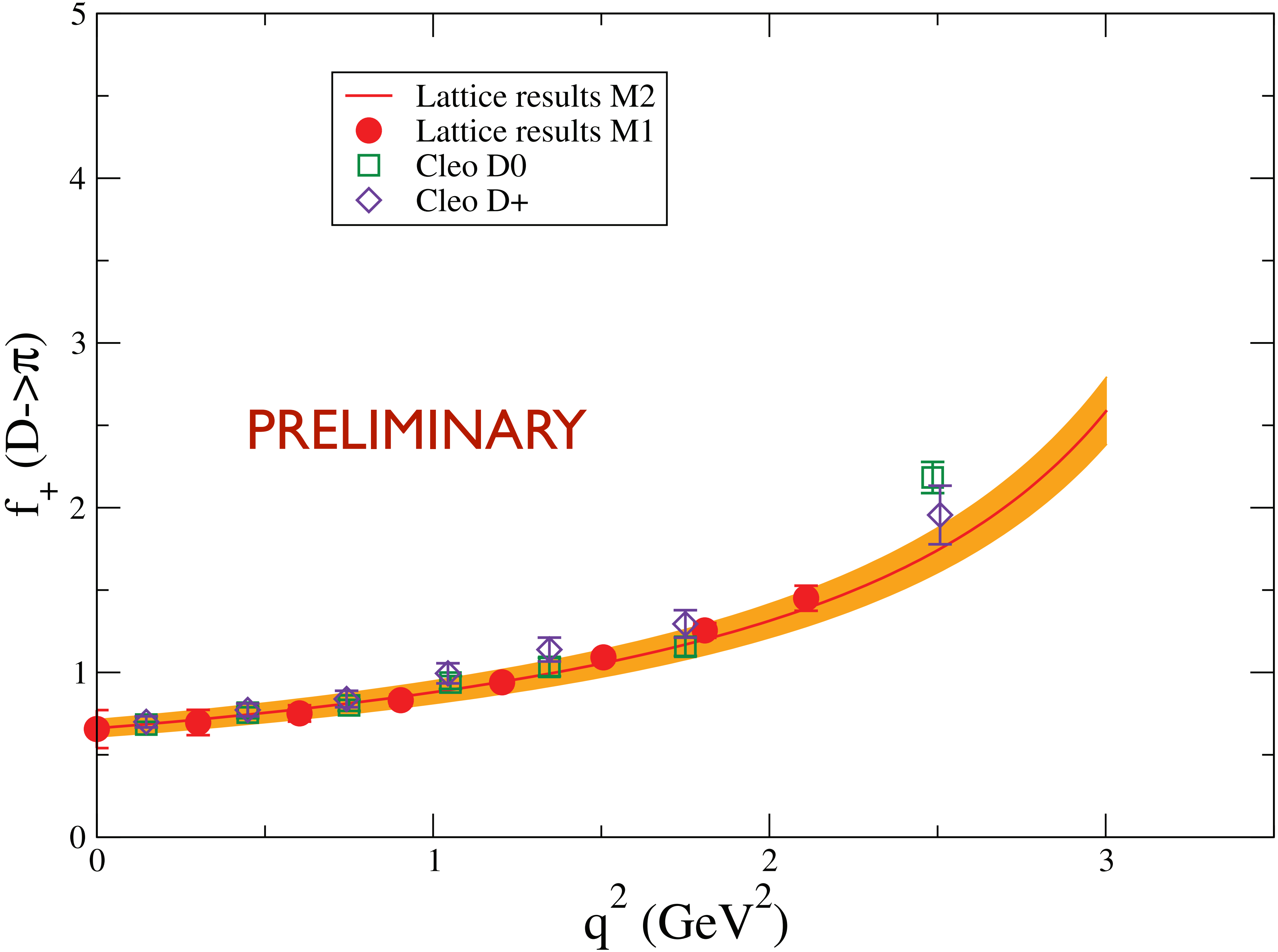
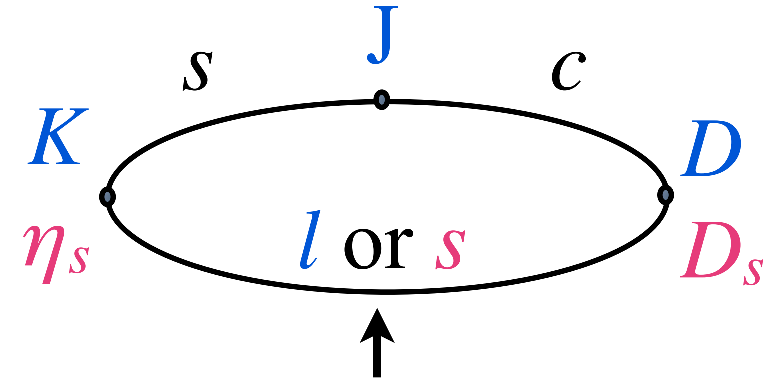
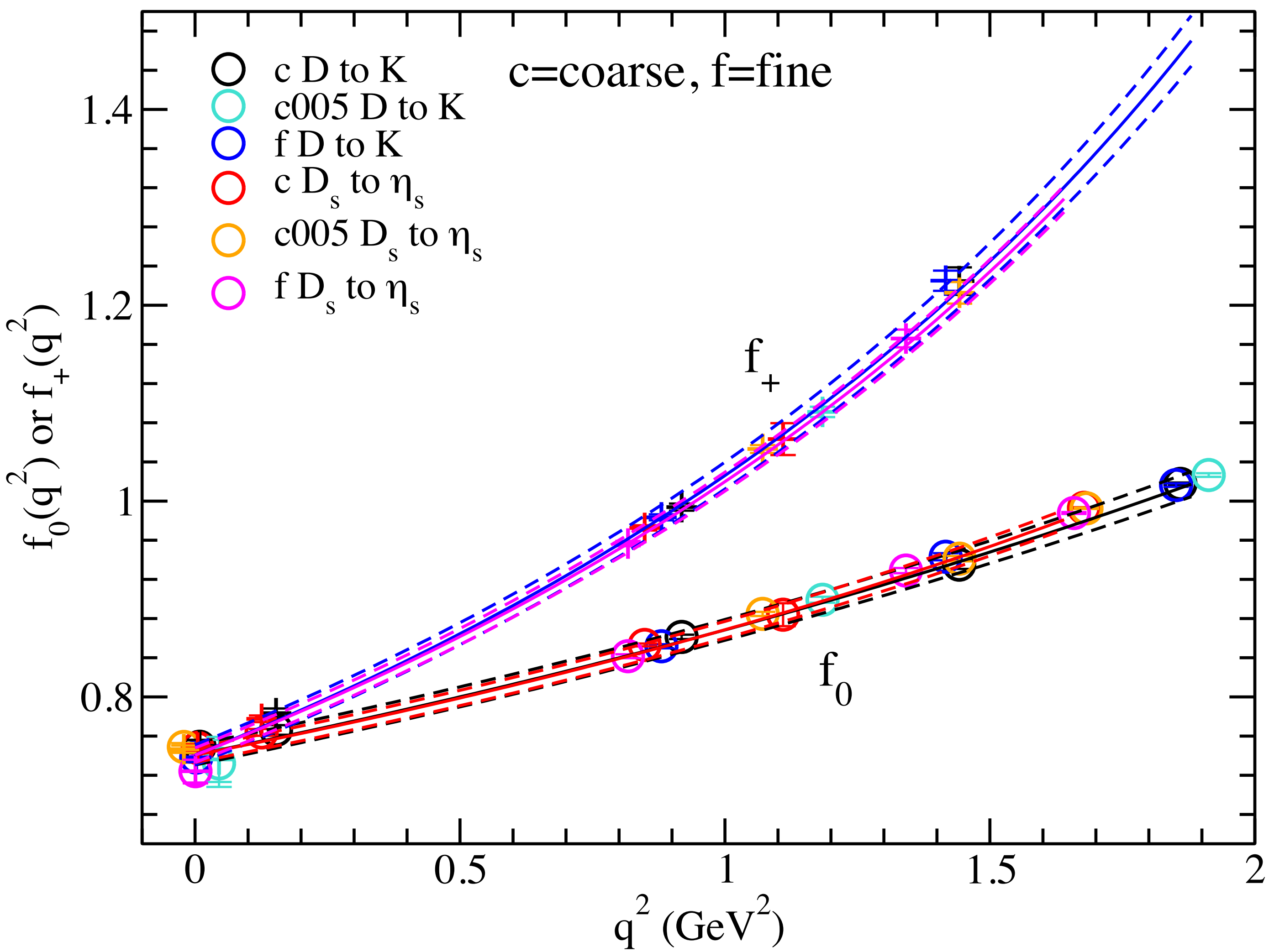


Fig. by F. Sanfilippo, talk at Lattice 2013, ETMC

Shape of the FFs: spectator quark



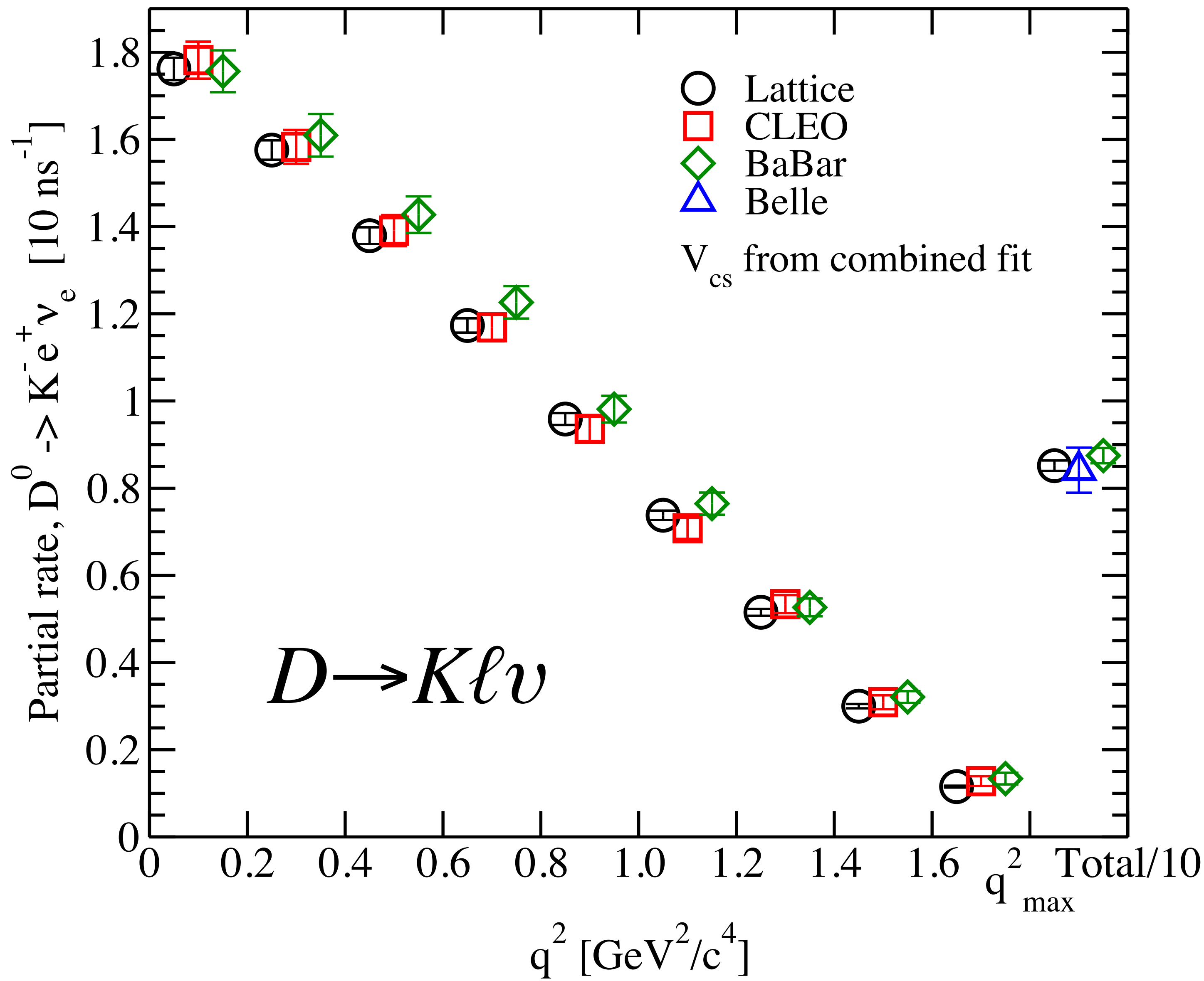
HPQCD, arXiv:1305.1462



- The difference between $D \to K\ell\upsilon$ and $D_s \to \eta_s \ell\upsilon$ is the spectator quark - light vs. strange
- The shapes of the form factors are same within *3%*

Decay rates in q^2 bins

HPQCD, arXiv:1305.1462



Experimental data: CLEO, PRD 80, 032005 (2009); Belle, PRL 97, 061804 (2006); BaBar, PRD 76, 052005 (2007) and PRD 78, 051101(R) (2008)

Lattice averages (FLAG)

Latest results from Lattice 2013 NOT included here!
Only results from 2012 and earlier.

$$f_D = 209.2(3.3)\ \mathrm{MeV}$$

$$f_{D_s} = 248.6(2.7)\ \mathrm{MeV}$$

$$f_{D_s}/f_D = 1.187(12)$$

$$V_{cd} = 0.2218(35)(95)\ \mathrm{(leptonic)}$$

$$V_{cs} = 1.018(11)(21)\ \mathrm{(leptonic)}$$

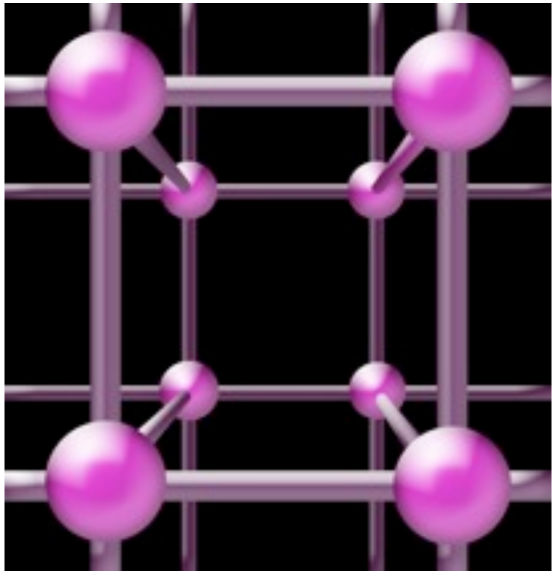
$$V_{cd} = 0.2192(95)(45)\ \mathrm{(semileptonic)}$$

$$V_{cs} = 0.9746(248)(67)\ \mathrm{(semileptonic)}$$

http://itpwiki/unibe.ch/flag

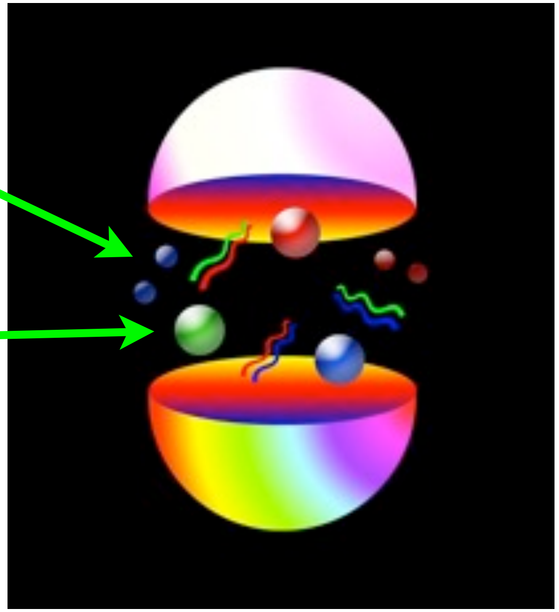
Lattice QCD

= fully nonperturbative QCD calculation



RECIPE

- Generate sets of gluon fields for Monte Carlo integration of path integral (including effects of u, d, s and c sea quarks)
- Calculate averaged "hadron correlators" from valence quark propagators
- Fit as a function of time to obtain masses and simple matrix elements



Lattice QCD RECIPE
continued

- Determine lattice spacing a and fix m_q using experimental information (often meson masses) to get results in physical units
- extrapolate to $a=0$, physical u/d quark mass for real world
  - lattices with physical $m_{u,d}$ now appearing: chiral extrapolation becoming a small correction