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2nd LHC Harmonic Cavity meeting, May 27th ,2013

POWER REQUIREMENTS FOR THE LHC HARMONIC CAVITIES WITH THE FULL-DETUNING SCHEME

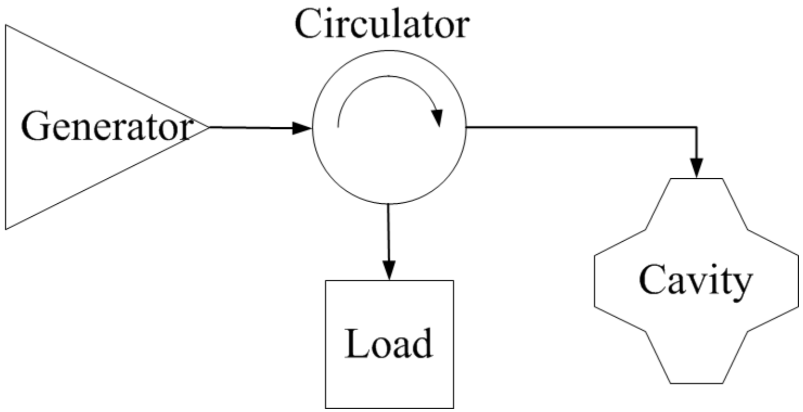
Content

- Beam-Cavity-TX interaction formula
- ACS system with fixed klystron phase and resulting beam phase modulation
- Resulting power requirements for the harmonic system
- Case study 1: Bunch shortening mode
- Case study 2: Bunch lengthening mode
- Conclusions

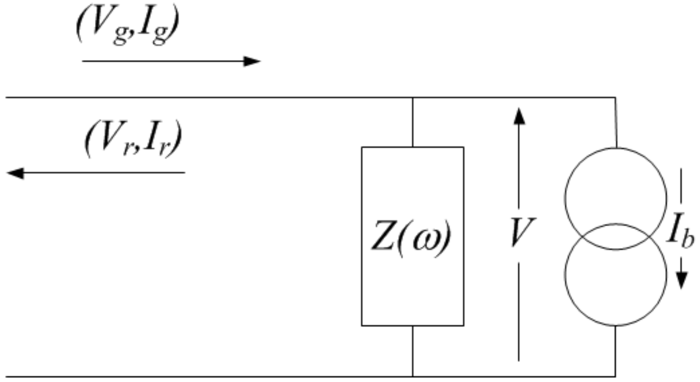
BEAM-CAVITY-TX INTERACTION

(Joachim’s *bible* [1])

- Consider a cavity linked to its generator via a circulator, waveguide and coupler



- Equivalent circuit diagram with incident and reflected waves transformed to the cavity gap impedance



- The TX forward current, cavity voltage and beam current are phasors at the RF frequency with a modulation slow compared to an RF period

$$\overrightarrow{V(t)} = V(t)e^{j\omega t}$$

$$\overrightarrow{I_g(t)} = I_g(t)e^{j\omega t}$$

$$\overrightarrow{I_b(t)} = I_b(t)e^{j\omega t}$$

- We can derive a simple relation between I_g, V, I_b and the cavity detuning $\Delta\omega$ [1]

$$I_g(t) = \frac{V(t)}{2\,R/Q}\left[\frac{1}{Q_L} - 2j\frac{\Delta\omega}{\omega}\right] + \frac{dV(t)}{dt}\frac{1}{\omega\,R/Q} + \frac{I_b(t)}{2} \qquad (1)$$

- The modulation in beam current $I_b(t)$ is imposed by the filling pattern: presence of small gaps for kicker rise time, plus a 3.2 μs minimum gap for the beam dump kicker
- So far we have operated the LHC RF for full compensation of the transient beam loading in the ACS cavities:

$$V(t) = V_0$$

- When proposed in 2007, the power requirement for the harmonic cavities assumed fixed voltage in both ACS and harmonic cavities [2]. After optimization of Q_L, the required power is then simply proportional to voltage and peak RF component of beam current, formula used in the first proposal

$$P_g = \frac{V\ I_{b,pk}}{8}$$

THE ACS (FUNDAMENTAL) SYSTEM AND THE BEAM PHASE MODULATION

Beam phase modulation

- Back to the ACS cavities. We now assume that the **modulus of the voltage is kept constant** but its **phase is allowed to vary** in response to the transient beam loading

$$V(t) = V_0 e^{j\varphi(t)} \qquad (2)$$

- The beam follows the voltage phase modulation. We take the situation in physics: 180 degrees stable phase

$$I_b(t) = i_b(t) e^{j\left[\varphi(t) - \frac{\pi}{2}\right]} = -j\, i_b(t) e^{j\varphi(t)} \qquad (3)$$

  with $i_b(t)$ a scalar representing the beam current modulation

- The smoother will null the phase modulation of the generator current

$$I_g(t) = i_g(t) \qquad (4)$$

  with $i_g(t)$ a scalar representing the (hopefully small) required modulation of the generator current phase. Note that we choose the real axis as aligned to the generator current

- Now using (2),(3),(4) in the equation (1), we get the beam phase modulation resulting from operating the ACS cavities with a fixed-phase drive

$$\frac{d\varphi}{dt} + \sigma \tan \varphi(t) = -\Delta\omega_0 \frac{i_b(t) - \overline{I_b}}{\overline{I_b}} \qquad (4)$$

with $\sigma \square \frac{\omega}{2Q_L}$

the component of beam current at the fundamental RF frequency,

$$\overline{I_b} \square \frac{1}{T_{rev}} \int_{t}^{t+T_{rev}} i_b(\tau)\, d\tau$$

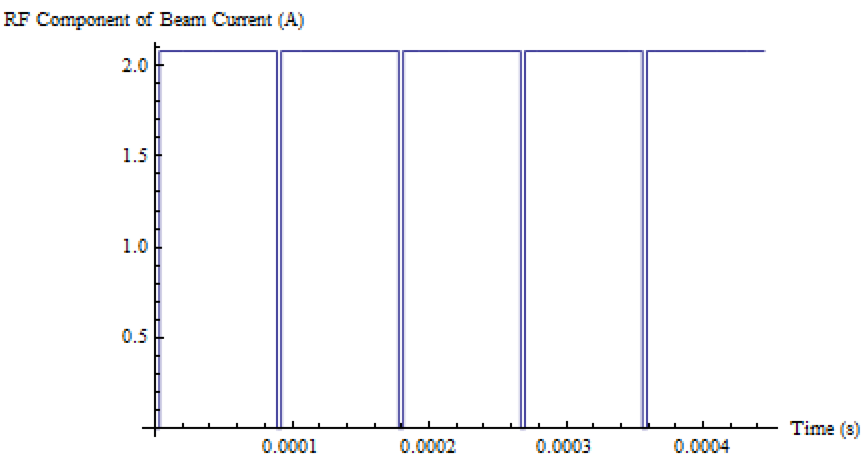
and the optimal (full) detuning $\Delta\omega_0 \square -\frac{1}{2} \frac{\omega R/Q}{V_0} \overline{I_b}$

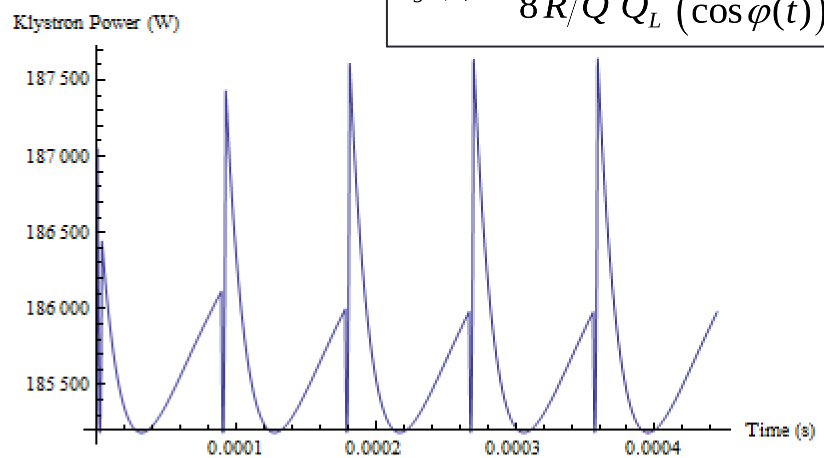
- In 1991 Boussard [3] had derived a simplified formula valid in the limiting case σT_{rev}<<1

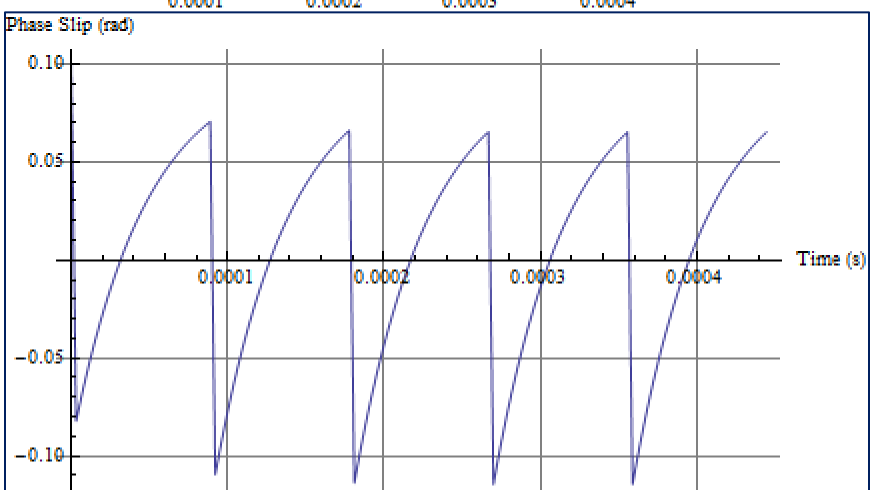
$$\frac{d\varphi}{dt} = -\Delta\omega_0 \frac{i_b(t) - \overline{I_b}}{\overline{I_b}} \qquad \text{Boussard's eq (12)}$$

- The following figures consider the **HighLumi case**: 2808 bunches, 2.2E11 p/bunch, 1.11 A DC, $\cos^2$ longitudinal bunch profile, 1 ns base length, bunching factor 0.9, 2 MV/cavity, Q_L=60000, R/Q =45 Ω. The cavity is at the optimum detuning (-9039 Hz). We consider the **3.2 μs long abort gap only**.

$$P_g(t) = \frac{V_0^2}{8\,R/Q}\,\frac{1}{Q_L}\,\frac{1}{(\cos\varphi(t))^2}$$







Top left: Component $i_b(t)$ of beam current at 400 MHz. 3.2 μs long abort gap.
Top right: Klystron power, almost independent of beam current
Bottom left: Phase modulation at 400 MHz. We get 0.180 rad pk-pk (10.3 degrees) at 400 MHz equal to 72 ps pk-pk.

THE HARMONIC SYSTEM

- We now consider the harmonic cavities at frequency $\omega_1 = n\,\omega$
- The bunch phase is defined by the Master ACS system (4). The beam current at the frequency of the harmonic cavities is

$$\overrightarrow{I_{b,1}(t)} \triangleq I_{b,1}(t)\,e^{jn\omega t} = i_{b,1}(t)\,e^{jn\left[\varphi(t)-\frac{\pi}{2}\right]}e^{jn\omega t} \qquad (5)$$

with $i_{b,1}(t)$ a scalar representing the beam current modulation at the harmonic frequency. Given the bunch length and resulting bunching factor, it will be significantly smaller than $i_b(t)$

- The harmonic cavity voltage must be locked to the fundamental RF

$$\overrightarrow{V_1(t)} \triangleq V_1(t)\,e^{jn\omega t} = V_1 e^{j\varphi_1} e^{jn\varphi(t)} e^{jn\omega t} \qquad (6)$$

with ϕ_1 a constant phase offset between fundamental and harmonic systems. This parameter will be different for Bunch Lengthening and Bunch Shortening modes.

- Let us consider the RF-beam phase
- For the fundamental we have

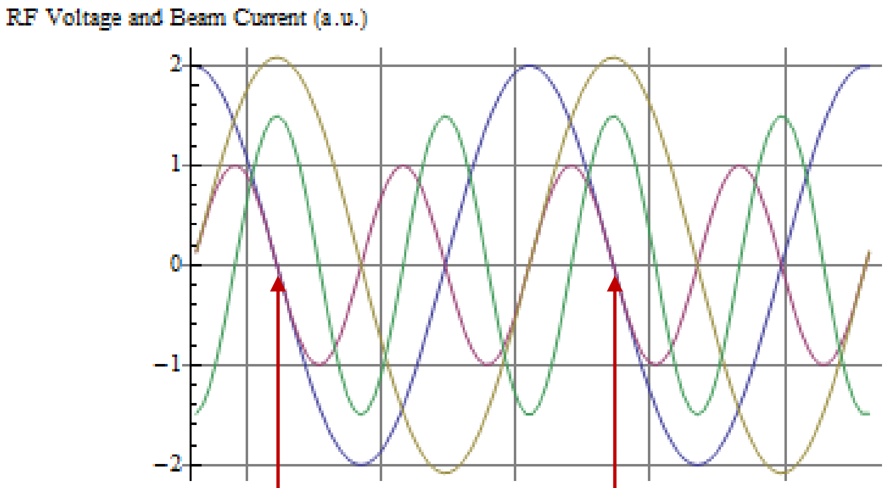
$$Arg\left[\frac{V(t)}{I_b(t)}\right]=\frac{\pi}{2}$$

- For the harmonic system

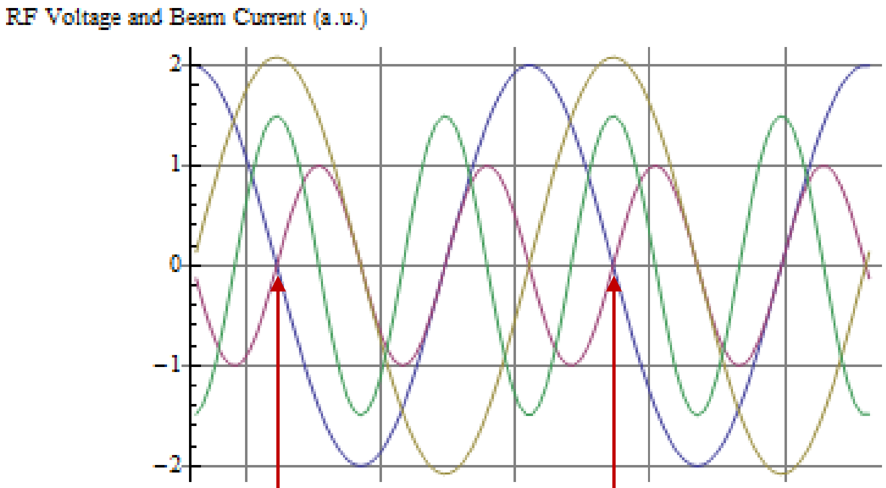
$$Arg\left[\frac{V_1(t)}{I_{b,1}(t)}\right]=\varphi_1+n\frac{\pi}{2}$$

- We have bunch shortening when the two gradients add, i.e. when the two arguments are equal. Bunch lengthening corresponds to opposite gradients, i.e. when the two arguments differ by π

$$\begin{cases}\varphi_1=(1-n)\dfrac{\pi}{2} & \text{bunch shortening (BS)}\\ \varphi_1=(3-n)\dfrac{\pi}{2} & \text{bunch lengthening (BL)}\end{cases}$$



Possible buckets in BS mode: the beam current peaks on the zero crossing, **negative** slope of **both** fundamental and harmonic RF



Possible buckets in BL mode: the beam current peaks on the zero crossing, **negative** slope of **fundamental** and **positive** slope of the **harmonic** RF

Real parts of the fundamental (blue) and harmonic (red) voltages given by equations (2) and (21), plus real parts of the component of beam current at fundamental (yellow) and harmonic (green). Harmonic at twice fundamental frequency (n=2).

Left: Bunch shortening mode ($\phi 1 = -\pi/2$)

Right: Bunch lengthening mode ($\phi 1 = \pi/2$)

Assumes a **positive bunching factor** at the harmonic frequency.

- We now apply Joachim's formula to the harmonic cavity in order to derive the required generator current

$$I_{g1}(t) = \frac{V_1(t)}{2(R/Q)_1}\left[\frac{1}{Q_{L1}} - 2j\frac{\Delta\omega_1}{\omega_1}\right] + \frac{dV_1(t)}{dt}\frac{1}{\omega_1(R/Q)_1} + \frac{I_{b1}(t)}{2}$$

- With $I_{b1}(t)$ following the phase modulation $\phi(t)$ imposed by the fundamental cavities (4),(5) and $V_1(t)$ locked to the fundamental system (6) at optimal detuning $\Delta\omega_0$, we get, after some calculations

$$I_{g_1}(t)e^{-j[n\varphi(t)+\varphi_1]} = \frac{V_1}{2(R/Q)_1 Q_{L1}} + j\frac{\overline{I_{b1}}}{2}\left[\frac{\Delta\omega_1 - n\Delta\omega_0}{\Delta\omega_{01}} + \frac{n\sigma\tan\varphi(t)}{\Delta\omega_{01}} + \left(\frac{n\Delta\omega_0}{\Delta\omega_{01}} \mp 1\right)\frac{i_b(t)}{\overline{I_b}}\right]$$

the minus sign in BS mode, the plus sign in BL mode. $\Delta\omega_1$ is the detuning of the harmonic cavity, and

$$\overline{I_{b1}} \square \frac{1}{T_{rev}}\int_t^{t+T_{rev}} i_{b1}(\tau)\,d\tau$$ component of beam current at the harmonic frequenc

$$\Delta\omega_{01} \square -\frac{1}{2}\frac{\omega_1 (R/Q)_1}{V_1}\overline{I_{b1}}$$ full detuning for the harmonic cavity

- We now want to minimize the peak modulus of $I_{g1}(t)$ over a turn

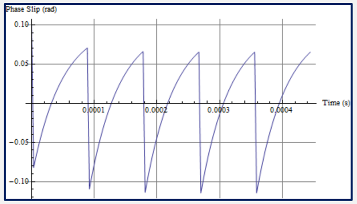
$$I_{g_1}(t)e^{-j[n\varphi(t)+\varphi_1]} = \frac{V_1}{2(R/Q)_1 Q_{L1}} + j\frac{\overline{I_{b1}}}{2}\left[\frac{\Delta\omega_1 - n\Delta\omega_0}{\Delta\omega_{01}} + \frac{n\sigma\tan\varphi(t)}{\Delta\omega_{01}} + \left(\frac{n\Delta\omega_0}{\Delta\omega_{01}}\mp 1\right)\frac{i_b(t)}{\overline{I_b}}\right]$$

Real-valued term. Independent of beam.

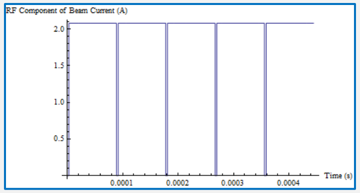
The cavity tune can be chosen to minimize peak generator current. After some calculations we get

$$\Delta\omega_{1,opt} = n\Delta\omega_0 - \left(n\Delta\omega_0 \mp \Delta\omega_{01}\right)\frac{I_{b1,pk}}{2\overline{I_{b1}}}$$

This term makes a small excursion, symmetric around zero, in both beam and no-beam segment



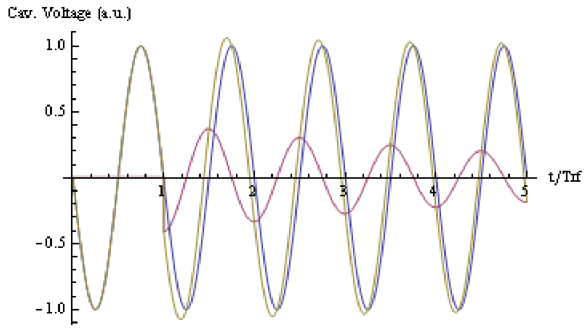
This term toggles between beam and no-beam segment.

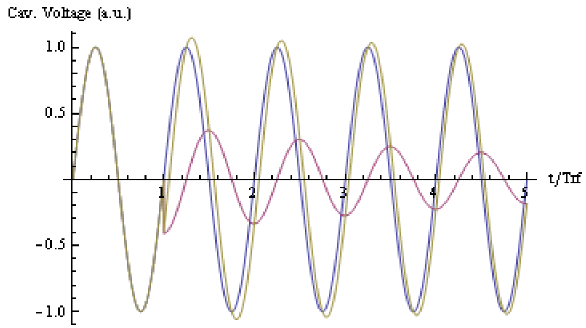


In BL mode it will be large (+ sign). In BS mode it can be made small by choosing

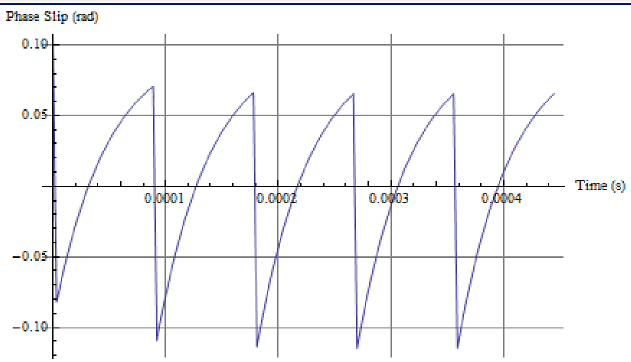
$$\Delta\omega_{01} \approx n\Delta\omega_0$$

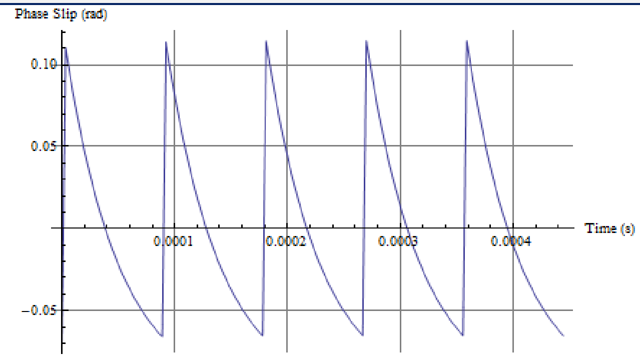
$$\frac{(R/Q)_1}{V_1}\overline{I_{b1}} \approx \frac{R/Q}{V}\overline{I_b}$$





Left: Beam induced voltage caused by a bunch passage at **180 degree** stable phase. Generator driven voltage in blue, bunch induced voltage in red, total voltage in yellow. The bunch causes a positive phase shift (phase advance). **Right**: Idem with a bunch passage at **0 degree** stable phase resulting in a negative phase shift (phase lag).



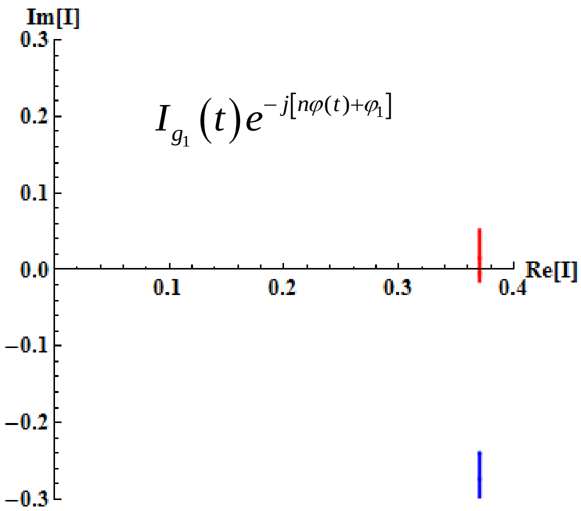


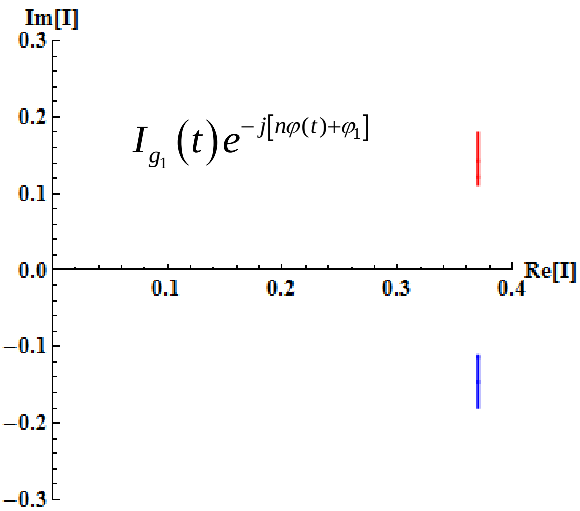
Left: Phase modulation caused by abort gap in a cavity at **180 degree** stable phase, with fixed klystron drive. **Right**: idem with **0 degree** stable phase.

- In **BS mode, both systems have 180 degree stable phase.** The beam phase modulation produces a voltage transient that "helps" the harmonic cavity track the voltage of the fundamental cavity -> **required power is reduced**
- In **BL mode**, the beam phase modulation with the 0 degree stable phase in the harmonic system produces a **voltage slip opposite to the desired RF phase modulation** imposed by the ACS system -> **much larger power required**

- In both modes, the detuning must be chosen to minimize the peak generator current

$$I_{g_1}(t)e^{-j[n\varphi(t)+\varphi_1]} = \frac{V_1}{2(R/Q)_1 Q_{L1}} + j\frac{\overline{I_{b1}}}{2}\left[\frac{\Delta\omega_1 - n\Delta\omega_0}{\Delta\omega_{01}} + \frac{n\sigma\tan\varphi(t)}{\Delta\omega_{01}} + \left(\frac{n\Delta\omega_0}{\Delta\omega_{01}} \mp 1\right)\frac{i_b(t)}{\overline{I_b}}\right]$$





The amplitude of the vector is proportional to the klystron current. Beam-on segment in red, beam-off segment in blue. The figure on the right corresponds to the optimal detuning (-15.4 kHz), minimizing the peak klystron current. On the left the detuning (-13 kHz) is almost perfect for the beam segment but will make the klystron power significantly larger in the no-beam segment.

- We now optimize the main coupling to minimize generator peak power

$$P_{g,1}(t) = \frac{1}{2}(R/Q)_1 Q_{e,1}\left|I_{g,1}(t)\right|^2 \approx \frac{1}{2}(R/Q)_1 Q_{L,1}\left|I_{g,1}(t)\right|^2$$

Power needed with half-detuning scheme

- We get

$$Q_{L1,opt} = \frac{V_1}{2(R/Q)_1\left(\left|\frac{n\,\Delta\omega_0}{\Delta\omega_{01}} \mp 1\right|\frac{I_{b1,pk}}{4} + \left|\frac{n\left(\sigma\,T_{gap}\right)\Delta\omega_0}{\Delta\omega_{01}}\right|\frac{\overline{I_{b1}}}{4}\right)} \qquad (7)$$

Small for high Q_L

$$P_{1pk,opt} = \left(\left|\frac{n\,\Delta\omega_0}{\Delta\omega_{01}} \mp 1\right| + \left|\frac{n\left(\sigma\,T_{gap}\right)\Delta\omega_0}{\Delta\omega_{01}}\right|\frac{\overline{I_{b1}}}{I_{b1,pk}}\right)\frac{V_1\,I_{b1,pk}}{8} \approx \left(\left|\frac{n\,\Delta\omega_0}{\Delta\omega_{01}} \mp 1\right|\right)\frac{V_1\,I_{b1,pk}}{8} \qquad (8)$$

- BS mode: minus sign
- A good design selects (8)

$$\Delta\omega_{01} \approx n\Delta\omega_0$$

$$\frac{(R/Q)_1}{V_1}\overline{I_{b1}} \approx \frac{R/Q}{V}\overline{I_b}$$

- A large loaded Q_{L1} is favorable (7)
- The required power can be much smaller than with half detuning scheme

- BL mode: plus sign
- A good design selects a large $\Delta\omega_{01}$, that is large $(R/Q)_1$ and low voltage per cavity

$$\Delta\omega_{01} \approx -\frac{1}{2}\frac{\omega_1 (R/Q)_1}{V_1}\overline{I_{b1}}$$

- A small loaded Q_{L1} is favorable
- The required power will always be larger than with the half detuning scheme

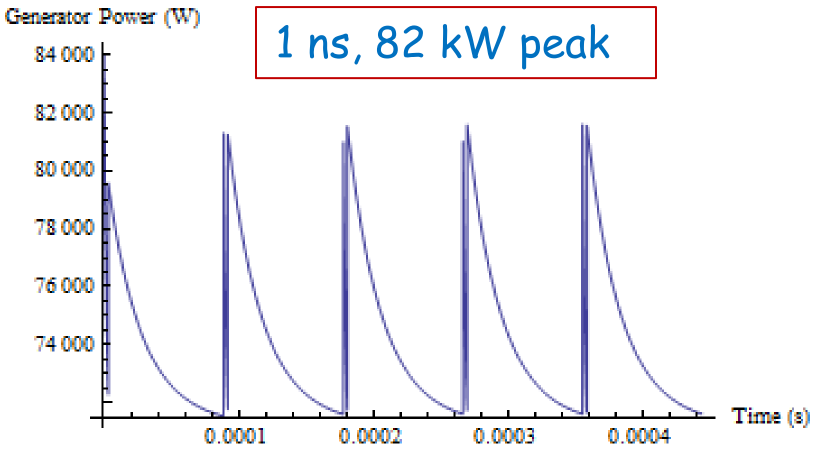
CASE STUDY 1

Bunch shortening mode, 800 MHz cavity

- Beam and fundamental cavity parameters: 3.2 μs long abort gap, 1.11 A DC , 1 ns Cosine square line density bunch, 2 MV per ACS cavity (16 MV total)
- 2 A RF component of beam current at 400 MHz and 1.44 A at 800 MHz
- Assuming 8 MV total at 800 MHz, and 45 Ω $(R/Q)_1$, the **beam phase modulation matches the harmonic cavity perfect** for

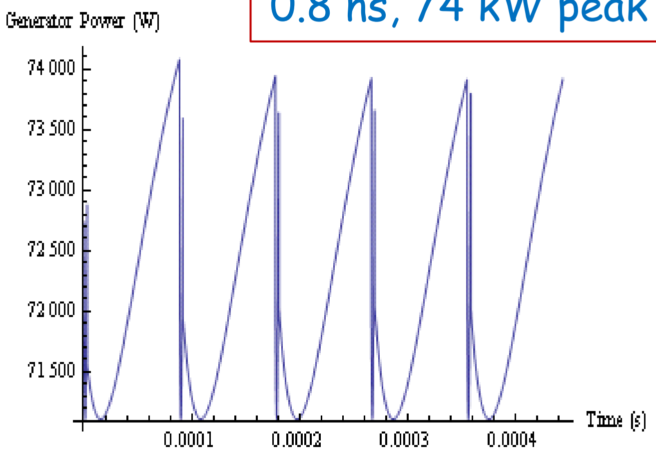
$$\frac{(R/Q)_1}{V_1}\overline{I_{b1}} = \frac{R/Q}{V}\overline{I_b}$$

- We get 1.44 MV ideal voltage per harmonic cavity. I consider 1.6 MV/cavity (5 cavities total)
- The absolute minimal required power is **55 kW** with a $Q_{L1,opt}$=260000. Much reduced from the **300 kW** (1/8 $V_1\ I_{b,pk1}$) required without beam phase modulation
- Let us use a Q_{L1}=100000 instead to make beam loading compensation easier when deviating from the 1 ns bunch length



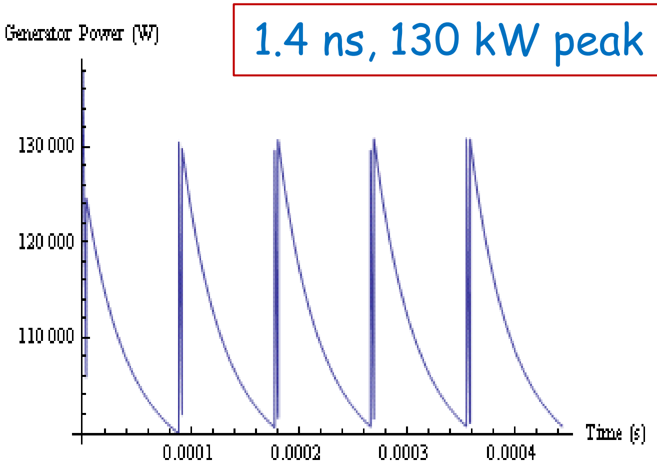
1 ns, 82 kW peak

0.8 ns, 74 kW peak



1.6 MV/cav, Q_{L1}=100000
Top left: Generator power with the 1 ns bunch length
Top right: Idem with 0.8 ns bunch length
Bottom right: Idem with 1.4 ns bunch length

1.4 ns, 130 kW peak

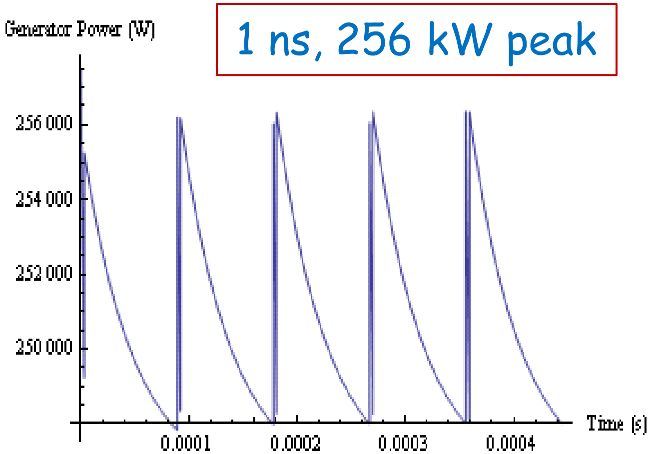


- In BS mode, phase modulation significantly reduces the needed RF power if the cavity parameters (*R/Q* and *V*) are selected properly
- The scheme is not very sensitive to bunch length

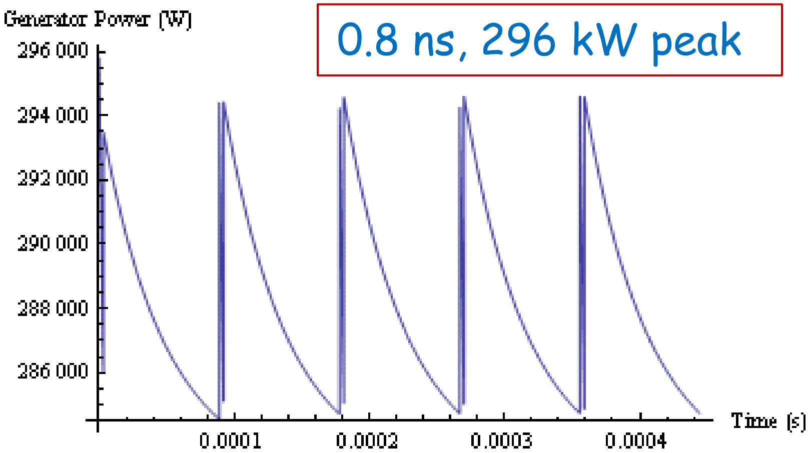
CASE STUDY 2

Bunch lengthening mode, 800 MHz cavity

- Beam and fundamental cavity parameters as before
- To minimize the required power we want to maximize $\Delta\omega_{01}$, that is a large $(R/Q)_1$ and low voltage per cavity (8)
- With 90 Ω $(R/Q)_1$, and 1 MV per cavity (8 cavities total), the absolute minimal required power is **260 kW** with Q_{L1}=11000
- Alternatively, with 45 Ω $(R/Q)_1$ we need 330 kW with an optimal Q_{L1}=17000
- Without modulation, we would need **190 kW** per cavity (1/8 V_1 $I_{b,pk1}$)



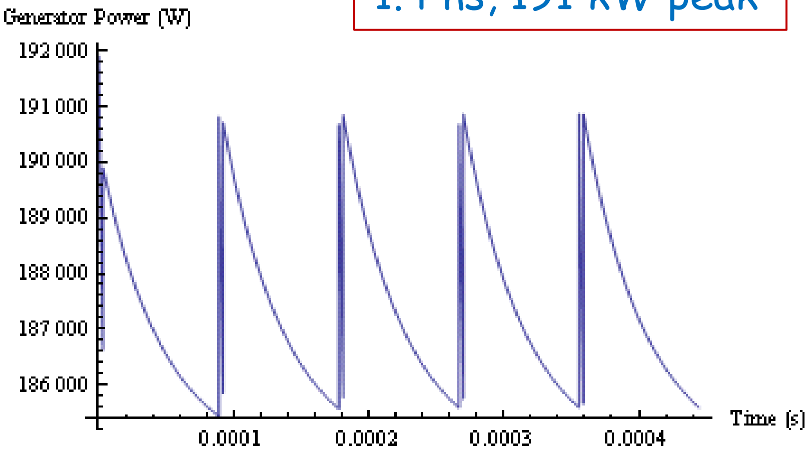
1 ns, 256 kW peak



0.8 ns, 296 kW peak

1 MV/cav , Q_{L1}=11000
Top left: Generator power with the 1 ns bunch length
Top right: Idem with 0.8 ns bunch length
Bottom right: Idem with 1.4 ns bunch length

1.4 ns, 191 kW peak



- In BL mode, phase modulation significantly increases the needed RF power
- The scheme is not very sensitive to bunch length

CONCLUSIONS

- In bunch shortening mode (fundamental and harmonic RF in phase at bunch passage), the phase modulation scheme can much reduce the power requirement on the generators of the harmonic cavities, with a proper choice of the RF parameters: R/Q, Q_L and voltage per cavity. With this optimal choice a good part of the voltage is induced by the beam passage
- In bunch lengthening mode (fundamental and harmonic in phase opposition at bunch passage), the scheme will significantly increase the required RF power, compared to the situation with equispaced bunches, while still keeping it manageable if the voltage per cavity is reduced
- The optimal main coupling differs much between the two applications: high Q_L in bunch shortening mode, where the needed voltage is induced by the beam, and low Q_L in bunch lengthening mode where the beam induces the wrong voltage change and much power must flow through the main coupler to compensate for that and track the fundamental cavity phase change
- In both cases the scheme is not very sensitive to bunch length
- Formulas have been derived for the required power and optimal parameters, and can be used to evaluate candidate scenario
- Must use more detailed beam current /several gaps)

THANK YOU FOR YOUR ATTENTION

REFERENCES

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[3] D. Boussard, RF Power Requirements for a High Intensity Proton Collider, CERN SL/91-16 (RFS), US PAC, San Francisco, May 1991