Beam-beam working group

On-going SixTrack code development

Javier Barranco Garcia,Tatiana Pieloni

Acknowledgments: D. Banfi, R. Tomas, E. Mcintosh, X. Buffat, R. De Maria, E. Mclean, F. Schmidt...

6D BB implementation in SixTrack

The 6D BB implementation in SixTrack is based in Hirata's BBC code with the option of considering coupled motion. It consists of 5 subroutines:

1. subroutine **stsld**(star,cphi,sphi,sigzs,nsli,calpha,salpha)→Calculates longitudinal/transverse position for all strong beam slices.

2. subroutine **boost**(np,sphi,cphi,tphi,salpha,calpha,track)→It does the Lorentz Boost operation for the weak beam. From crossing angle to head on collisions with longitudinal tilt.

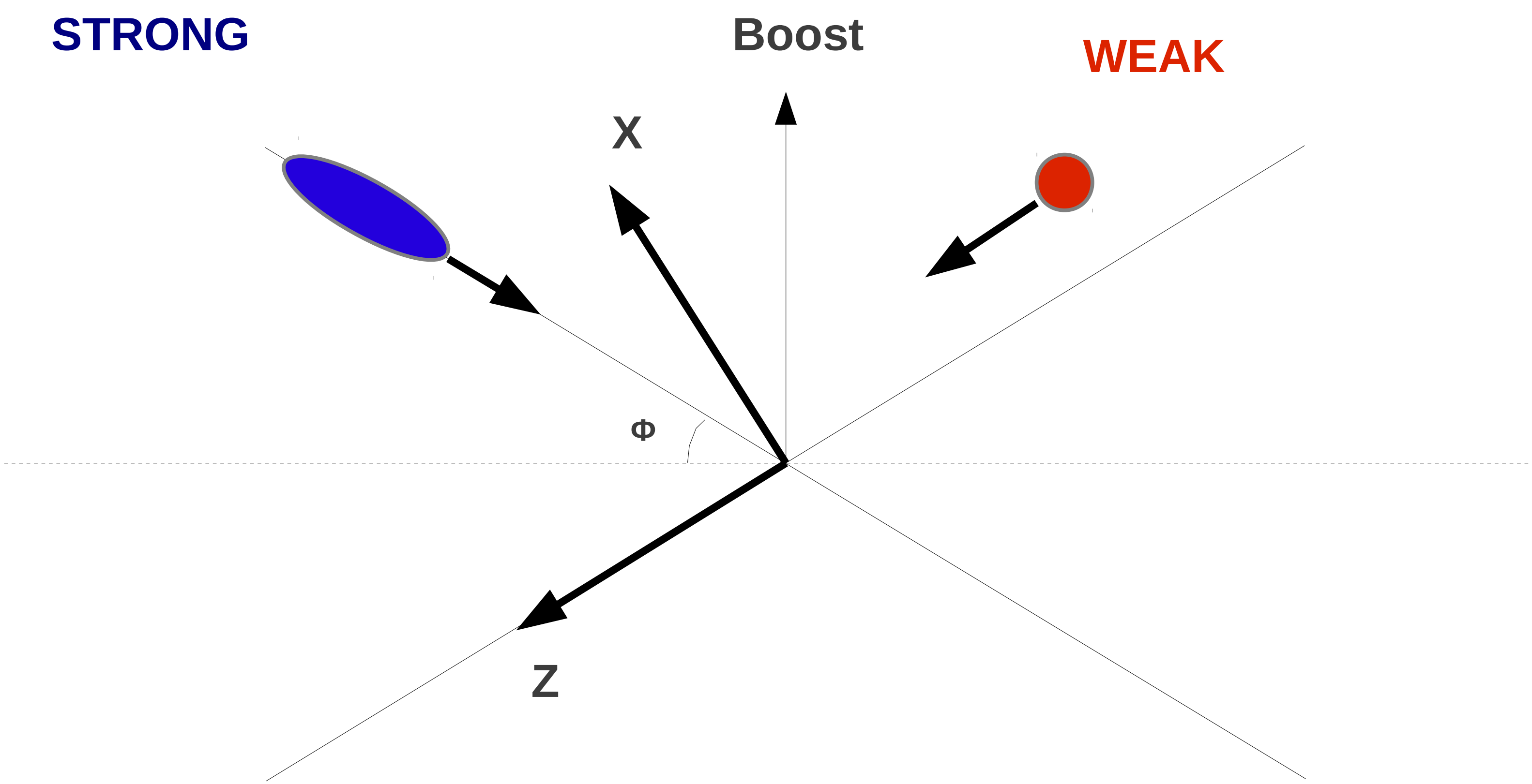
3. subroutine **sbc**(np,star,cphi,nsli,f,ibtyp,ibb,bcu,track,ibbc)→It does the synchro-beam mapping for head on collision for N_{slices}.

4. subroutine **bbf**(sepx,sepy,sx,sy,bbfx,bbfy,bbgx,bbgy,ibtyp)→Calculates the transverse ($f_{x,y}$) and longitudinal ($g_{x,y}$) beam beam kicks.

5. subroutine **boosti**(np,sphi,cphi,tphi,salpha,calpha,track)→It does the inverse Lorentz boost operation for the weak beam. (this ensures the symplecticity of the method).

Lorentz Boost*

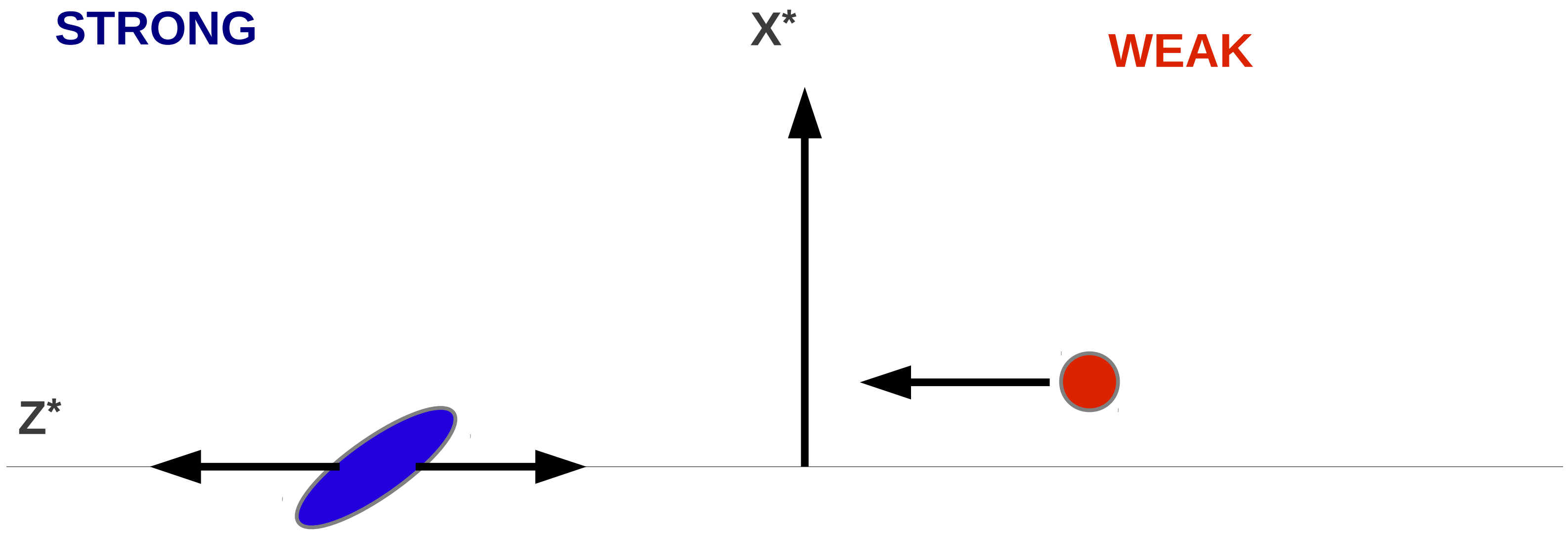
It is a rotation in X-Z plane by an angle Φ/2 and a boost to the direction of the rotated X.



*"Don't be afraid of beam-beam interactions with a large crossing angle", K. Hirata, SLAC-PUB-6375.

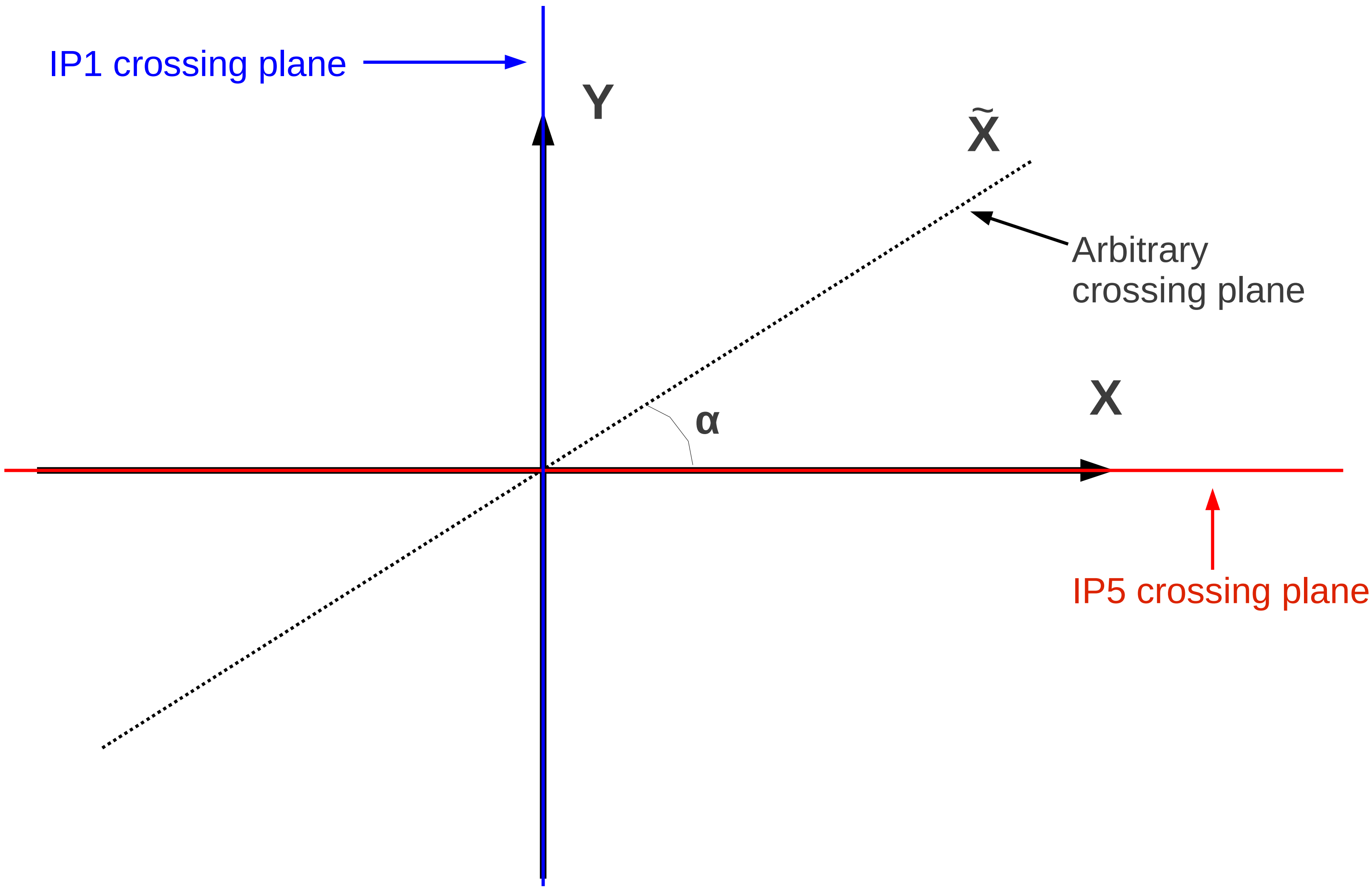
Lorentz Boost

In the boosted frame there are now head on collisions with a longitudinal tilt of the strong beam.



Crossing Plane

Hirata's BBC included only purely horizontal crossings. SixTrack allows to define arbitrary crossing planes*.



*"Six-dimensional beam-beam kick including coupled motion", L.H.A. Leunissen et al., PRSTAB 3 124002.

Lorentz Boost in 6D BB SixTrack

Accordingly, the Lorentz boost performed in SixTrack into the accelerator framework considers the crossing angle (Φ) and the crossing plane (α)*.

$$
\begin{pmatrix}
1/\cos\phi & -\cos\alpha\sin\phi & -\tan\phi\sin\phi & -\sin\alpha\sin\phi \\
-\cos\alpha\tan\phi & 1 & \cos\alpha\tan\phi & 0 \\
0 & -\cos\alpha\sin\phi & \cos\phi & -\sin\alpha\sin\phi \\
-\sin\alpha\tan\phi & 0 & \sin\alpha\tan\phi & 1
\end{pmatrix}
$$

“Six-dimensional beam-beam kick including coupled motion”, L.H.A. Leunissen et al., PRSTAB 3 124002.

Strong beam model

First and second momenta of the particle distribution at the locations of the slices as implemented in SixTrack.

$$
\begin{aligned}
&\mathrm{X}^\dagger = Z^\dagger \cos\alpha \sin\phi, P_X^\dagger = 0, Y^\dagger = Z^\dagger \sin\alpha \sin\phi,\\
&P_Y^\dagger = 0, P_Z^\dagger = 0, \textstyle\sum_{11}^\dagger = \sum_{11},\\
&\textstyle\sum_{22}^\dagger = \dfrac{1}{\cos^2\phi}\sum_{22}, \sum_{33}^\dagger = \sum_{33}, \sum_{44}^\dagger = \dfrac{1}{\cos^2\phi}\sum_{44},\\
&\textstyle\sum_{12}^\dagger = \dfrac{1}{\cos\phi}\sum_{12}, \sum_{13}^\dagger = \sum_{13}, \sum_{14}^\dagger = \dfrac{1}{\cos\phi}\sum_{14},\\
&\textstyle\sum_{23}^\dagger = \dfrac{1}{\cos\phi}\sum_{23}, \sum_{24}^\dagger = \dfrac{1}{\cos^2\phi}\sum_{24}, \sum_{34}^\dagger = \dfrac{1}{\cos\phi}\sum_{34}
\end{aligned}
$$

Problem when crab crossing→ Φ variable is common to strong and weak beams.

Y. Sun *et al.* “customized” a version of SixTrack forcing Φ=0 only for the strong beam. Unluckily this version is not available.

$$
\begin{aligned}
&\mathrm{X}^\dagger = 0, P_X^\dagger = 0, Y^\dagger = 0,\\
&P_Y^\dagger = 0, P_Z^\dagger = 0, \textstyle\sum_{11}^\dagger = \sum_{11},\\
&\textstyle\sum_{22}^\dagger = \sum_{22}, \sum_{33}^\dagger = \sum_{33}, \sum_{44}^\dagger = \sum_{44},\\
&\textstyle\sum_{12}^\dagger = \sum_{12}, \sum_{13}^\dagger = \sum_{13}, \sum_{14}^\dagger = \sum_{14},\\
&\textstyle\sum_{23}^\dagger = \sum_{23}, \sum_{24}^\dagger = \sum_{24}, \sum_{34}^\dagger = \sum_{34}
\end{aligned}
$$

Strong beam model

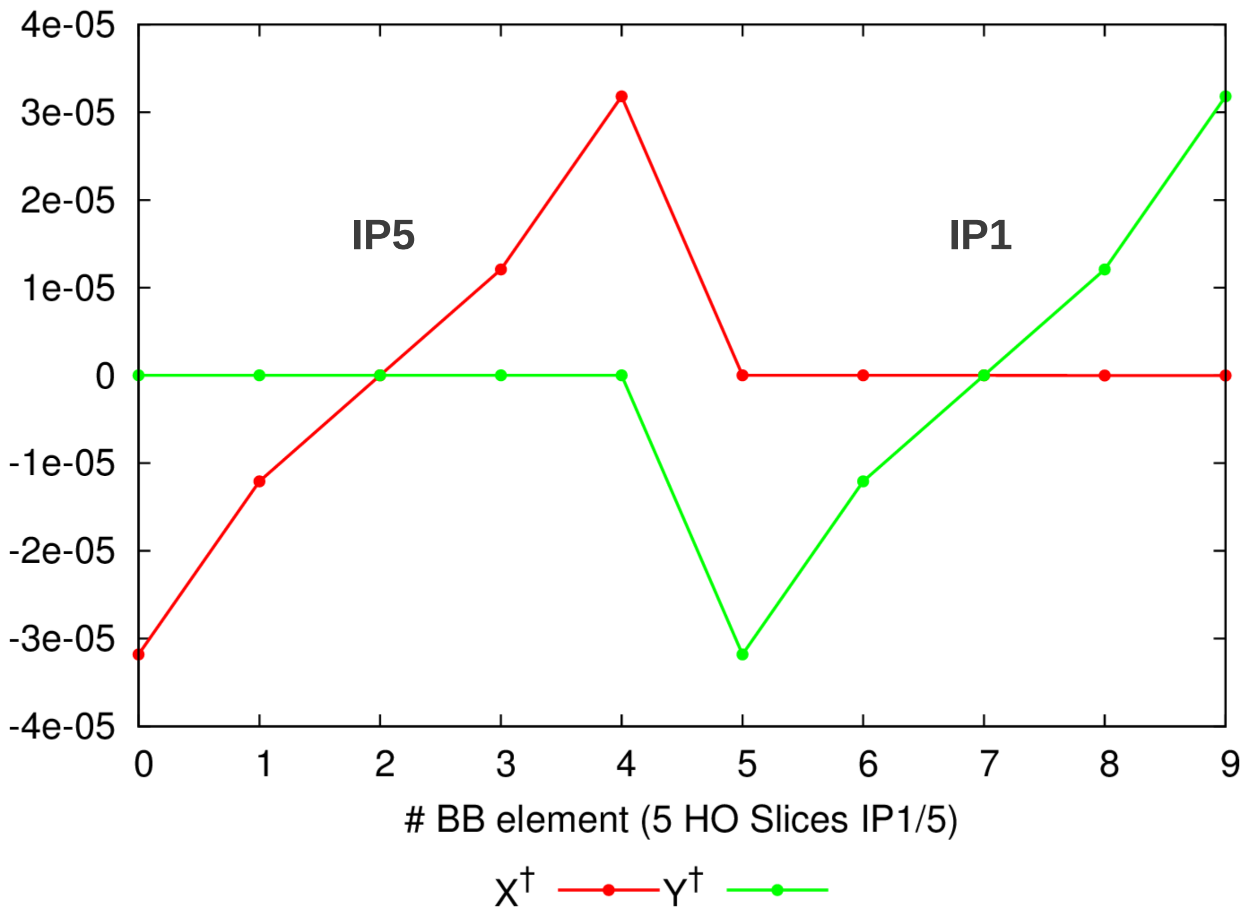
I have just implemented an independent Φ_2 to control the strong bunch tilting w/o affecting the weak beam. Reminder, the crab tilting of the weak particle is given by the single particle tracking itself.

$$
\begin{aligned}
&\mathrm{X}^{\dagger} = Z^{\dagger}\cos\alpha\sin\phi_2, P_X^{\dagger} = 0, Y^{\dagger} = Z^{\dagger}\sin\alpha\sin\phi_2,\\
&P_Y^{\dagger} = 0, P_Z^{\dagger} = 0, \textstyle\sum_{11}^{\dagger} = \sum_{11},\\
&\textstyle\sum_{22}^{\dagger} = \dfrac{1}{\cos^2\phi_2}\sum_{22}, \sum_{33}^{\dagger} = \sum_{33}, \sum_{44}^{\dagger} = \dfrac{1}{\cos^2\phi_2}\sum_{44},\\
&\textstyle\sum_{12}^{\dagger} = \dfrac{1}{\cos\phi_2}\sum_{12}, \sum_{13}^{\dagger} = \sum_{13}, \sum_{14}^{\dagger} = \dfrac{1}{\cos\phi_2}\sum_{14},\\
&\textstyle\sum_{23}^{\dagger} = \dfrac{1}{\cos\phi_2}\sum_{23}, \sum_{24}^{\dagger} = \dfrac{1}{\cos^2\phi_2}\sum_{24}, \sum_{34}^{\dagger} = \dfrac{1}{\cos\phi_2}\sum_{34}
\end{aligned}
$$

Strong beam model

Current SixTrack input for 6D beam-beam lens. 2 angles variables for crossing angle and plane.

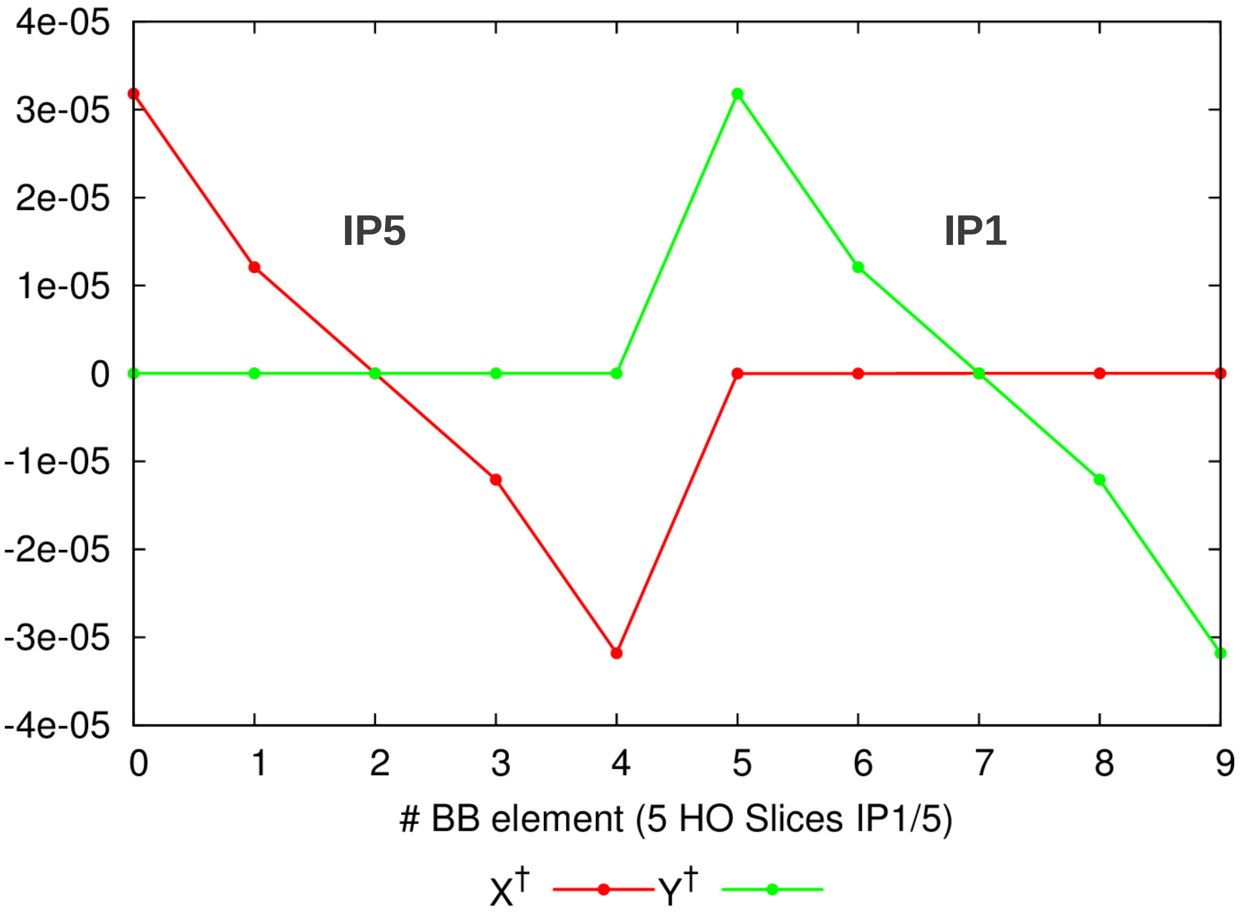
	N_{slices}	Φ	α
bb_ho5b1_0	5	0.295e-3	0.0
bb_ho1b1_0	5	0.295e-3	1.5708



Strong beam model

Possibility of reverting the sign of the crossing angle.

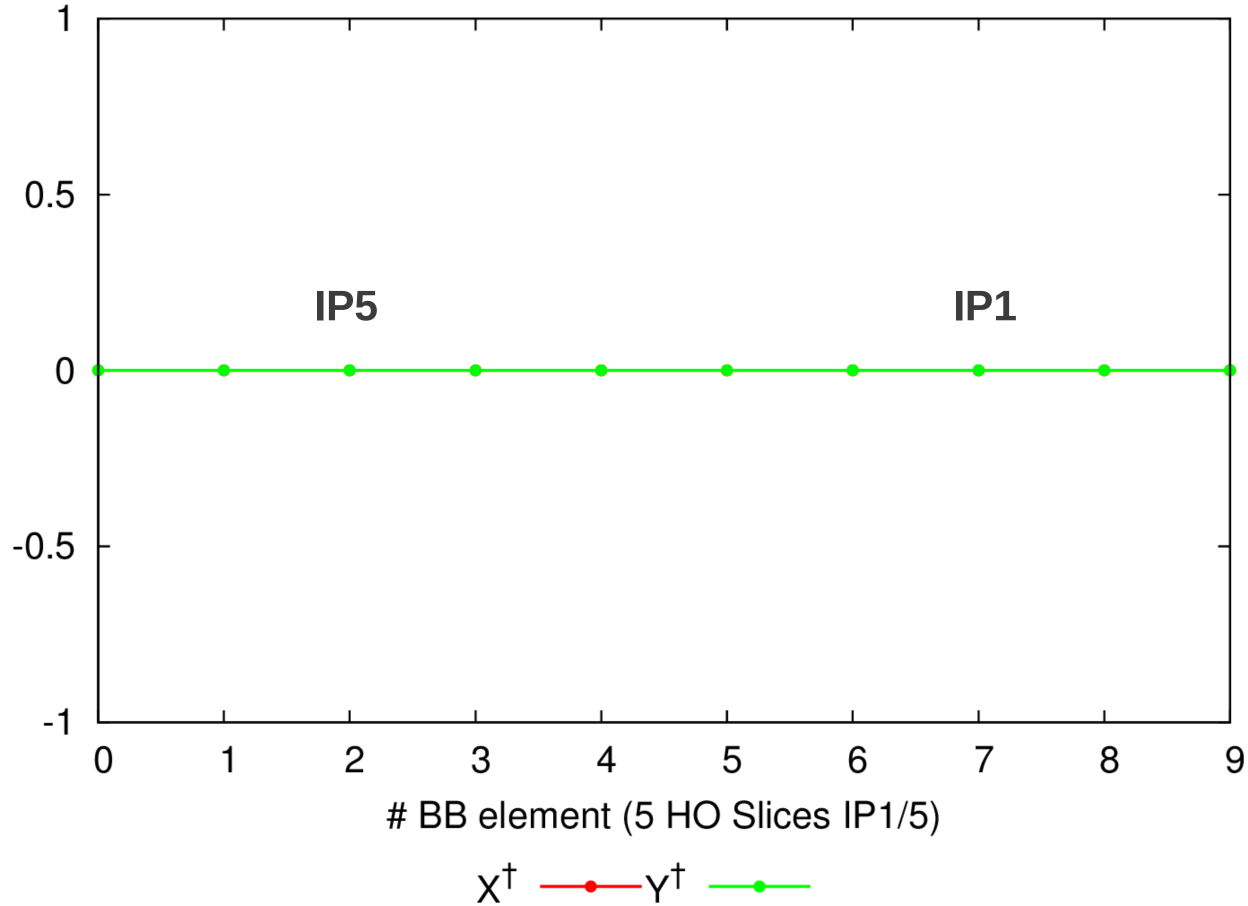
	N_{slices}	Φ	α
bb_ho5b1_0	5	-0.295e-3	0.0
bb_ho1b1_0	5	-0.295e-3	1.5708



Strong beam model

Added an additional variable **Φ_2** in the BEAM block of the fort.3 input to control the tilting of the strong beam

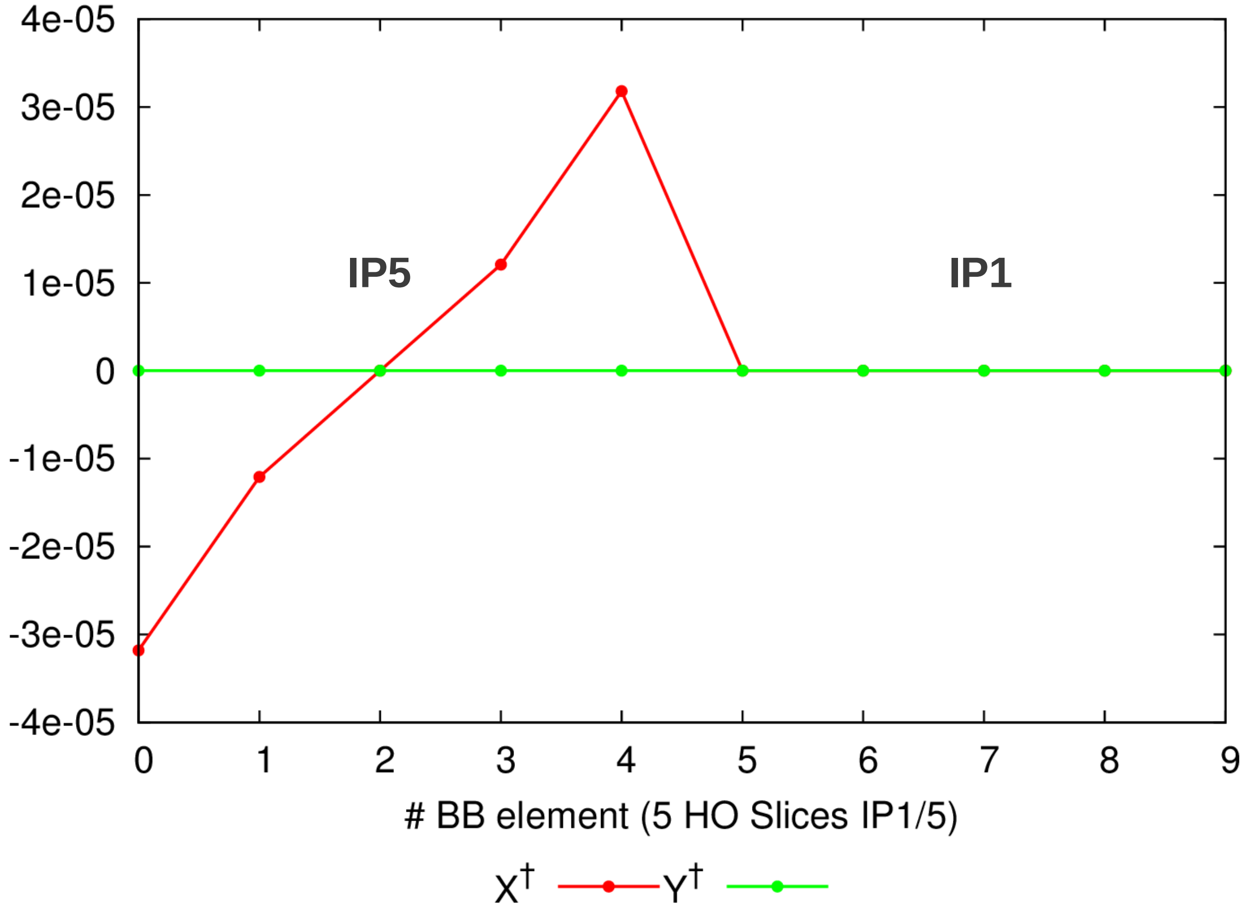
	N_{slices}	Φ	α	Φ_2	
bb_ho5b1_0	5	0.295e-3	0.0	**0.0**	**Full crab crossing IP1**
bb_ho1b1_0	5	0.295e-3	1.5708	**0.0**	**Full crab crossing IP5**



Strong beam model

New option of crabbing the strong beam in a single IP only.

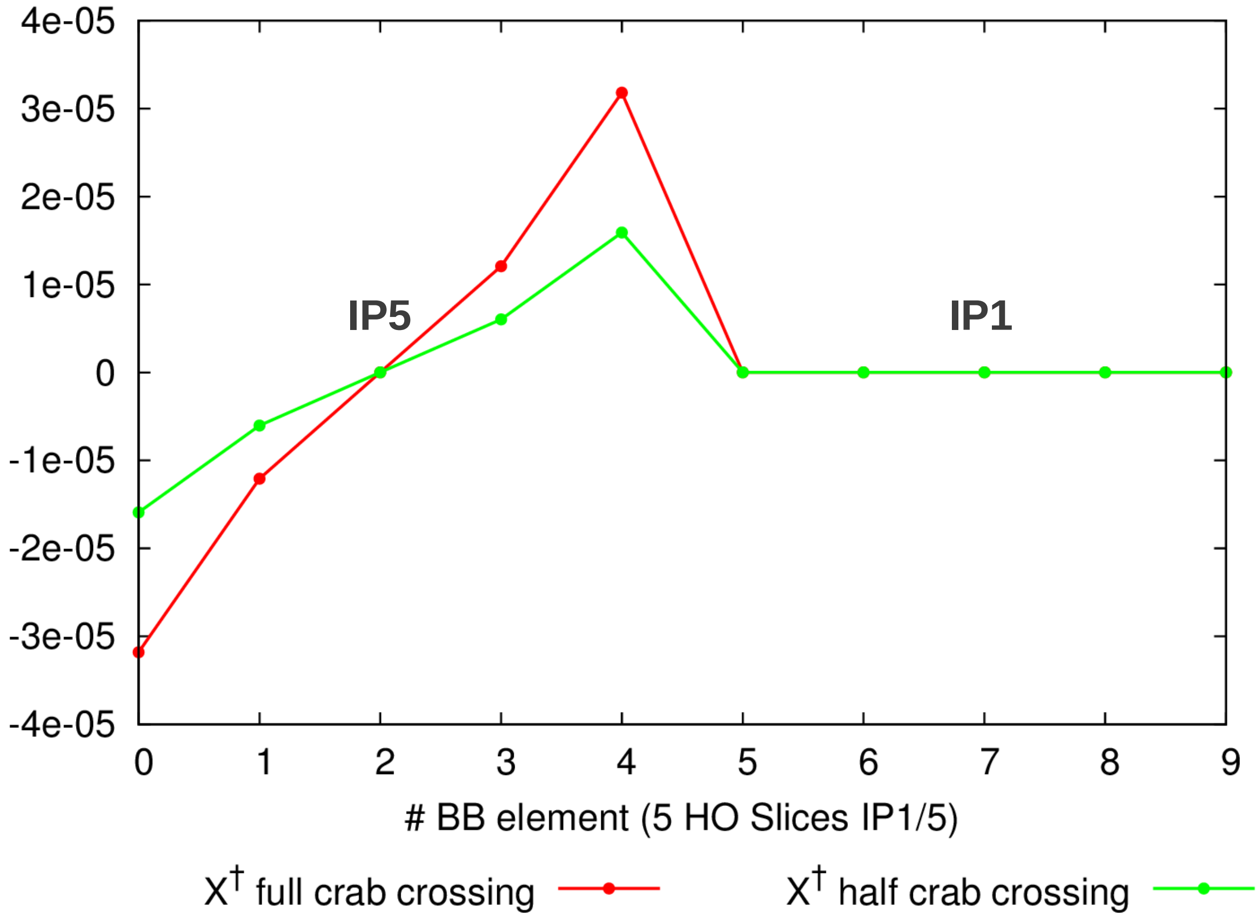
	N_{slices}	Φ	α	Φ_2	
bb_ho5b1_0	5	0.295e-3	0.0	**0.295e-3**	**Full crossing angle at IP1**
bb_ho1b1_0	5	0.295e-3	1.5708	**0.0**	**Full crab crossing at IP5**



Strong beam model

Possibility of simulating different degrees of crabbing. For example for failure scenarios.

	N_{slices}	**Φ**	**α**	**$Φ_2$**	
bb_ho5b1_0	5	0.295e-3	0.0	**0.1475e-3**	**Half crossing angle at IP1**
bb_ho1b1_0	5	0.295e-3	1.5708	**0.0**	**Full crab crossing at IP5**

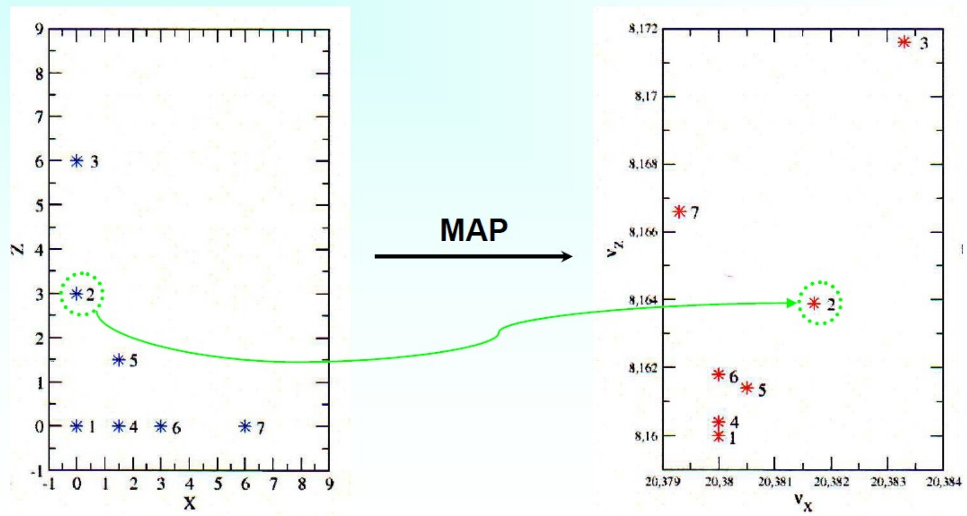


Frequency Maps Analysis

- Frequency Maps Analysis (FMA) is a powerful tool to study the stability of a dynamical system. Fast convergence ($1/T^4$) compared to FFT ($1/T$), with T defined a finite time span, i.e. number of turns.
- FMA was originally applied to the dynamics of planets by J. Laskar in 1989. Afterwards it was Laskar and Dumas who applied it for the first time to accelerators (PRL 1993). Later on, different people and codes did similar studies in different machines.
- The SixDesk enviroment has been modified to allow FMA studies in a similar automatized way of the dynamic aperture ones.
- Tracking done by SixTrack and frequency spectrum calculated with SUSSIX.
- This will allow us to provide consistent benchmark against Lifetrack (much faster convergence that DA studies).

Frequency Maps Analysis

Brief explanation of the FMA procedure.



1000 turns	1000 turns
$\Downarrow$	$\Downarrow$
$(Q_{x1}$ & $Q_{y1})$	$(Q_{x2}$ & $Q_{y2})$

If $(Q_{x1}$ & $Q_{y1})_{0\rightarrow1000} \neq (Q_{x2}$ & $Q_{y2})_{1000\rightarrow2000}$ we have diffusion

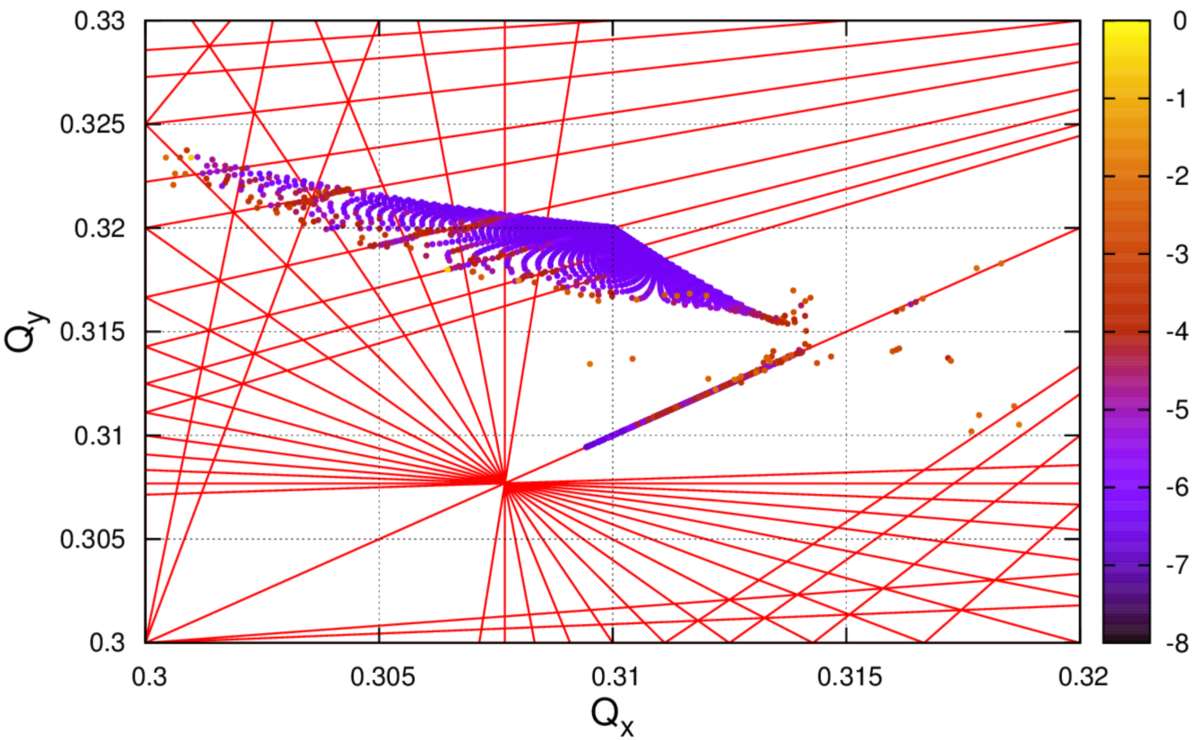
Define

$$D = \log_{10}\sqrt{[(Q_{x2}-Q_{x1})^2+(Q_{y2}-Q_{y1})^2]}$$

Slide from C. Bocchetta, CAS, Brunnen, 2003

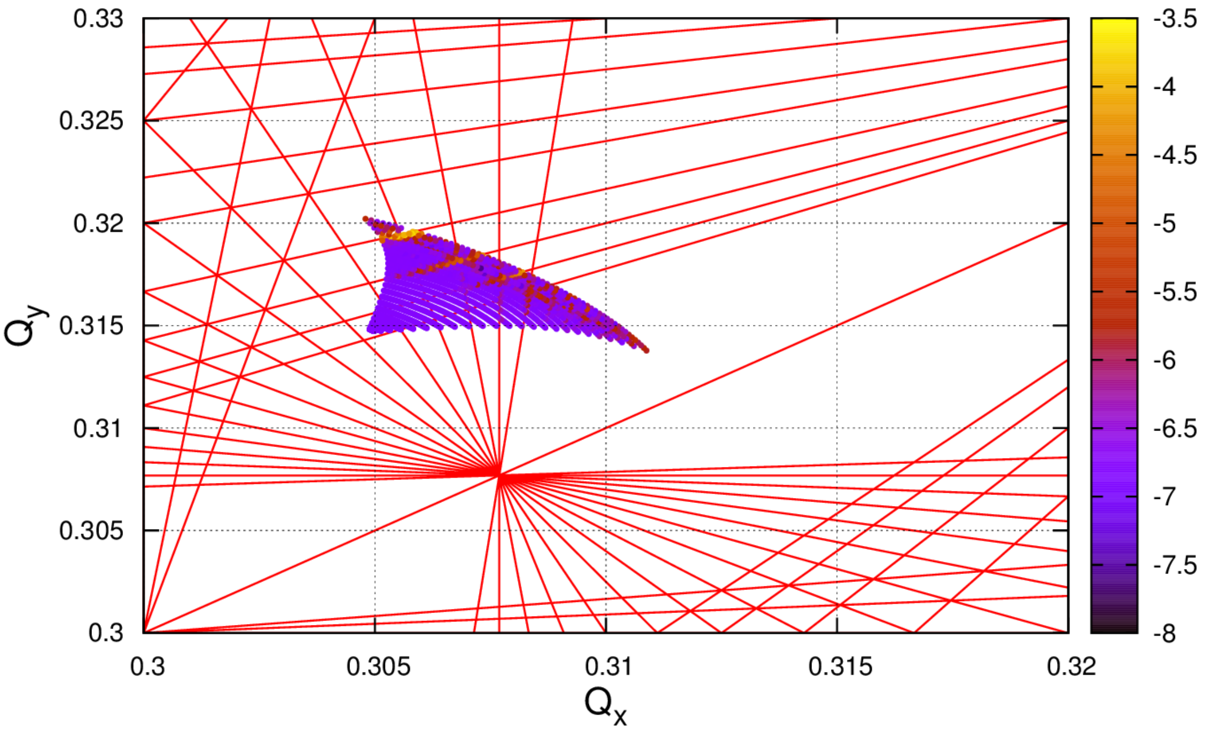
Frequency Maps Analysis

Example HL-LHC lattice w/o BB. Resonaces up to 15th order shown.



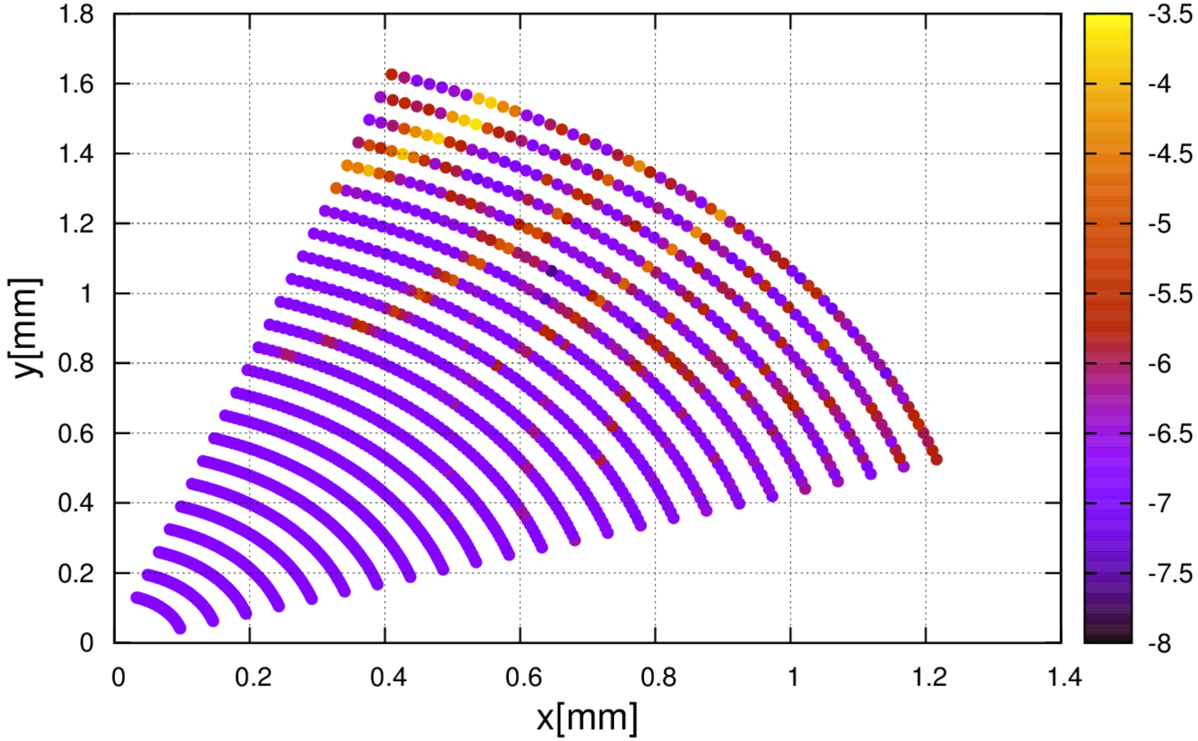
Frequency Maps Analysis

Preliminary results for HL-LHC lattice w BB in IP1/5 4D 5 slices HO+LR. Tracking done up to 6σ. Resonances up to 15th order shown.



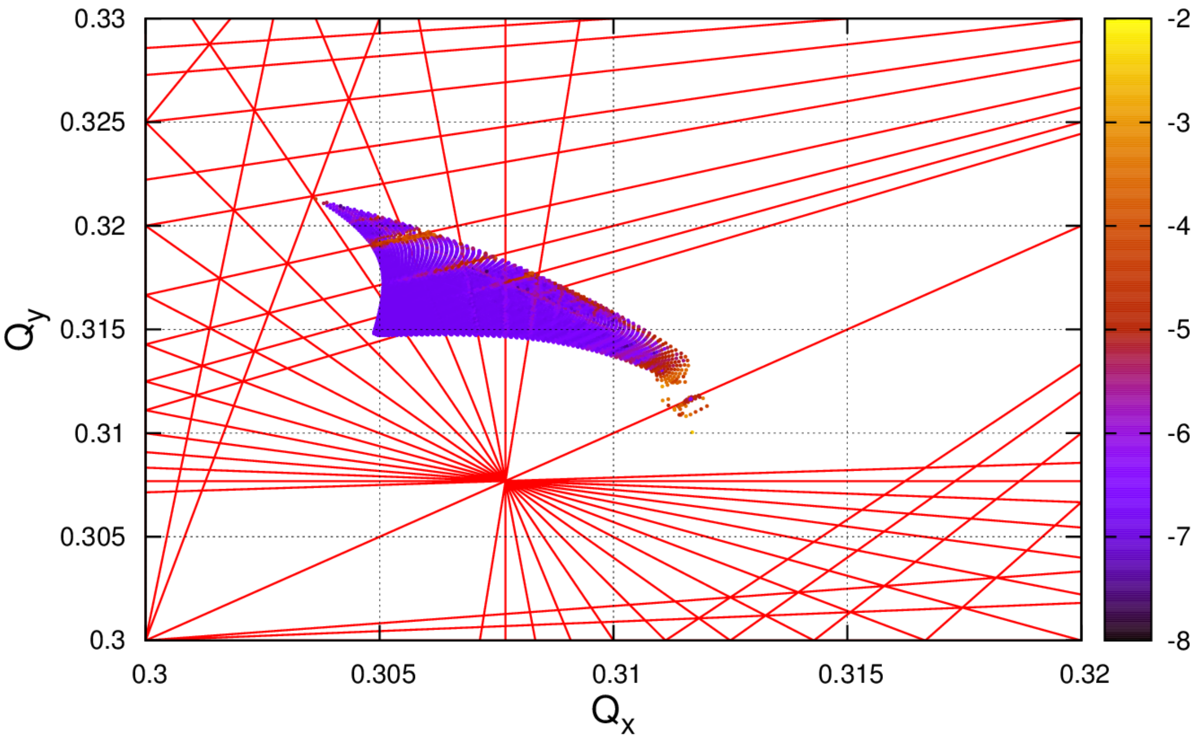
Frequency Maps Analysis

Preliminary results for HL-LHC lattice w BB 4D 5 slices HO+LR. Tracking done up to 6σ ever



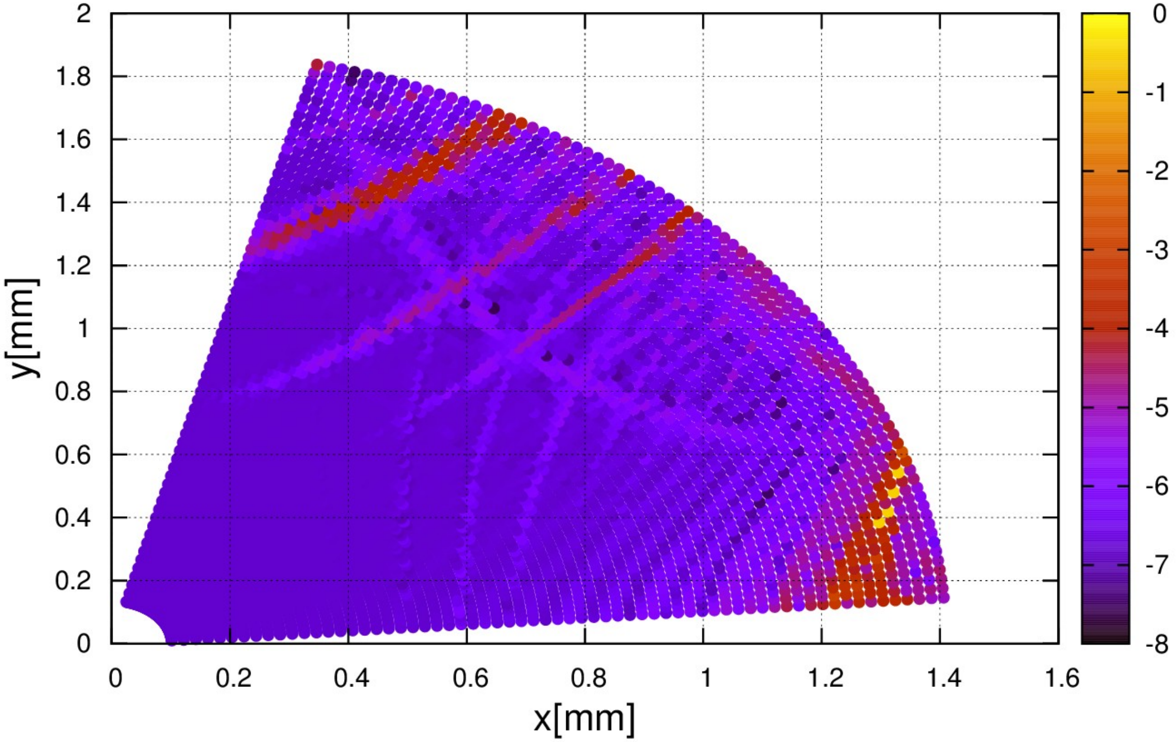
Frequency Maps Analysis

Preliminary results for HL-LHC lattice w BB 4D 5 slices HO+LR. Tracking done up to 7σ every 0.25σ and extended angles. Resonances up to 15th order shown.



Frequency Maps Analysis

Preliminary results for HL-LHC lattice w BB 4D 5 slices HO+LR. Tracking done up to 7σ every 0.25σ and extended angles. Resonances up to 15th order shown.



Summary

- Crab crossing for the strong beam now implemented in SixTrack by means of an additional crossing angle variable.
- 6D beam-beam lens checks have already started but not reported here.
- The SixDesk environment has been adapted to perform FMA in a similar way of dynamic aperture studies.

Future Work

- Finalize the 6D beam-beam lens checks (see Danilo's talk).
- Benchmark FMA results between SixTrack and Lifetrack (Valishev and Shatilov). FMA much faster convergence. This work contributes to HL-LHC WP2 Task Force 2.3.
- Start massive DA simulations campaigns with BB and CCs with Danilo's improved environment and analysis tools.