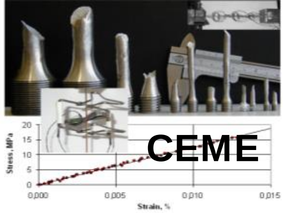

ESTIMATION OF UNCERTAINTY FOR FATIGUE GROWTH RATE AT CRYOGENIC TEMPERATURES

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- Overview about motivation & necessity of cyclic performance
- FCGR measurement its evaluation and possible errors
- Concept of uncertainty and its basics (true value ?)
- Application of uncertainty in case of FCGR
- Paris law region as model equation
- Combined uncertainty equation & Partial differentials
- Definition and estimation of particular uncertainties of variables
- Evaluation of the best estimate and its uncertainty term
- Conclusions

Background of motivation: Uncertainty in mechanical testing / IEC-standard

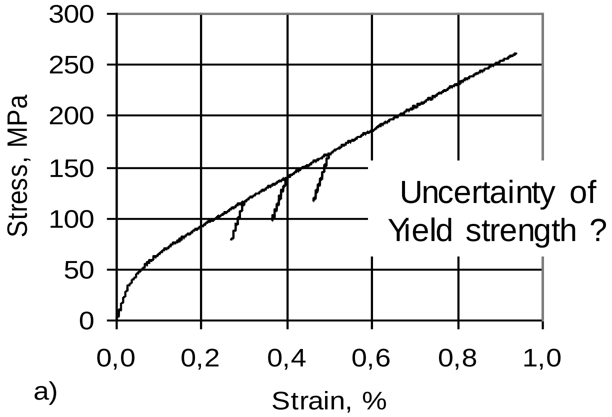
61788-19 Ed1/CDV ©
IEC(E)

Superconductivity -
Part 19: Mechanical properties measurement Room temperature tensile test of reacted Nb_3Sn composite superconductors

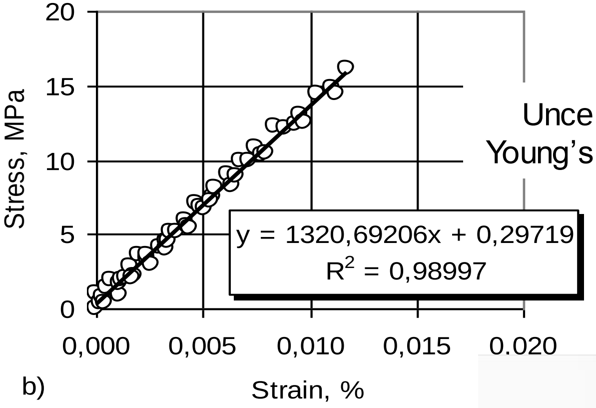
Annex C (informative)

Specific examples related to mechanical tests C.1 Scope

These are specific examples to illustrate techniques of uncertainty estimation. The inclusion of these examples does not imply that users must complete a similar analysis to comply with the standard. However, the portions that estimate the uncertainty of each individual influence quantity (load, displacement, wire diameter, and gauge length) need to be evaluated by the user to determine if they meet the specified uncertainty limits in the standard.



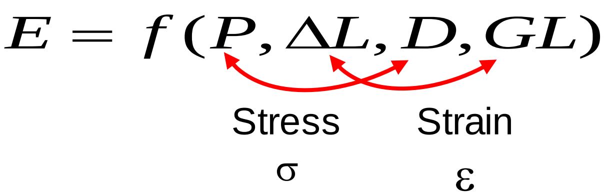
Uncertainty of Yield strength ?



$y = 1320{,}69206x + 0{,}29719$
$R^2 = 0{,}98997$

Uncertainty of Young's modulus ?

IEC Award 2011



$$E = f(P, \Delta L, D, GL)$$

E = Young's modulus, *MPa*
P = load, *N*
ΔL = deflected length of extensometer in zero offset region for the selected load portion, mm
D = diameter of wire, *mm*
GL = Gauge length of extensometer at start of the loading, *mm*

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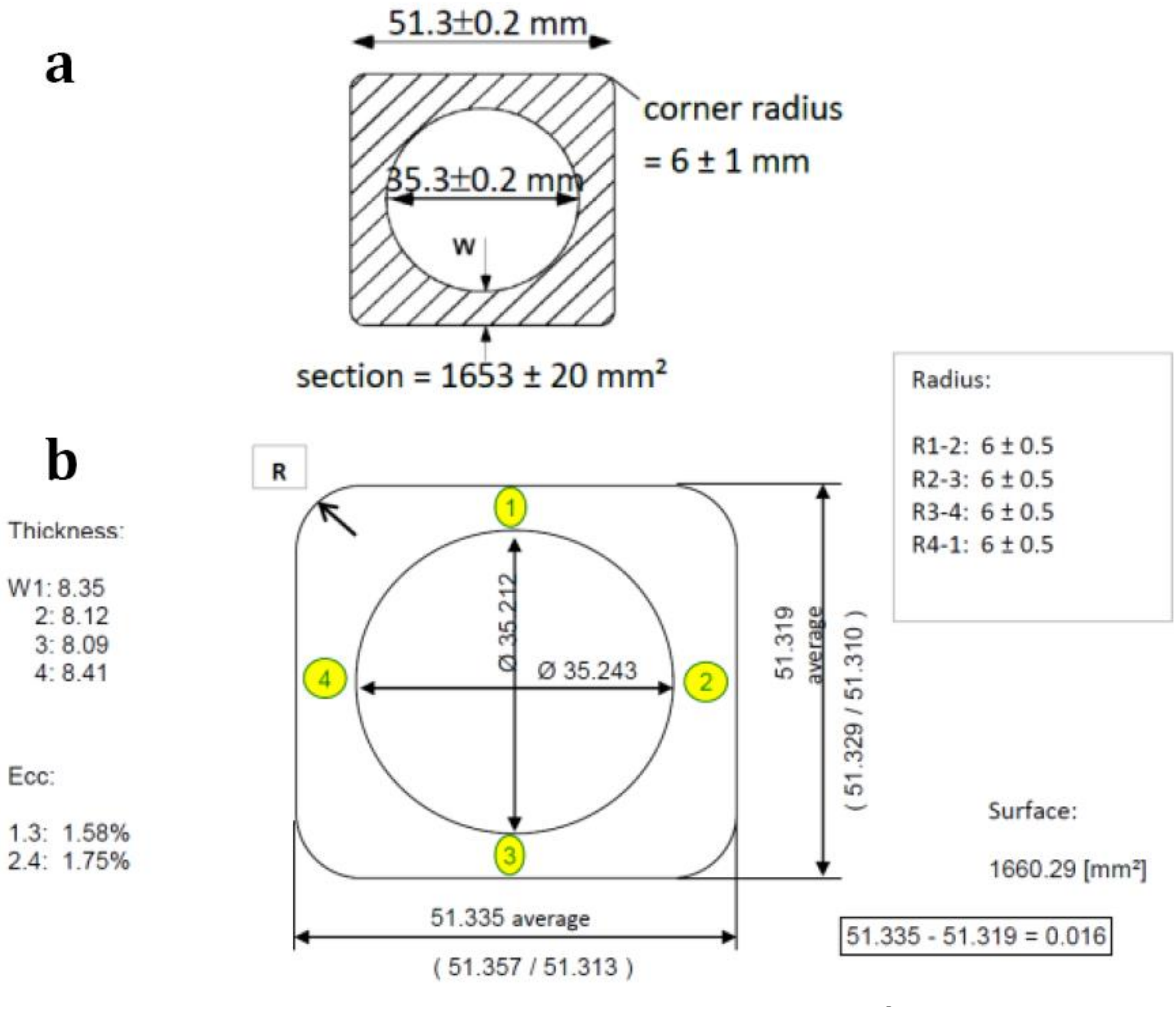
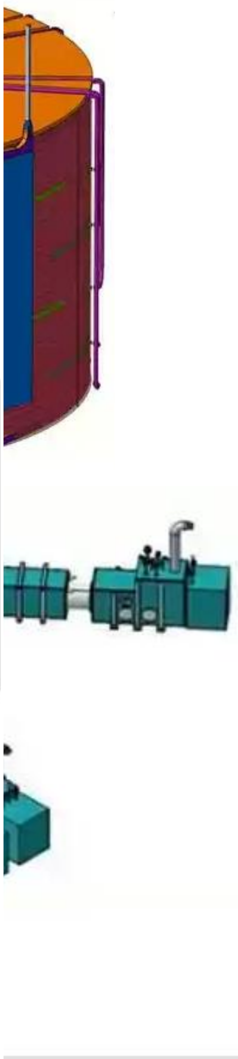
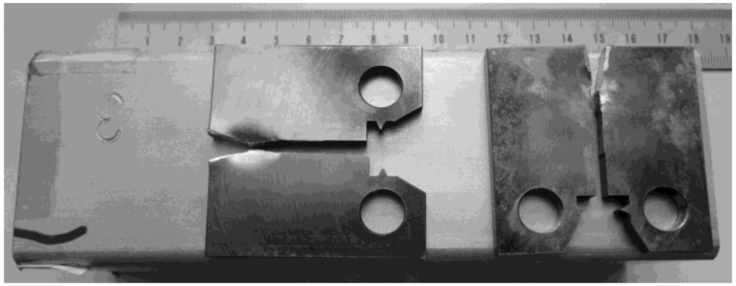


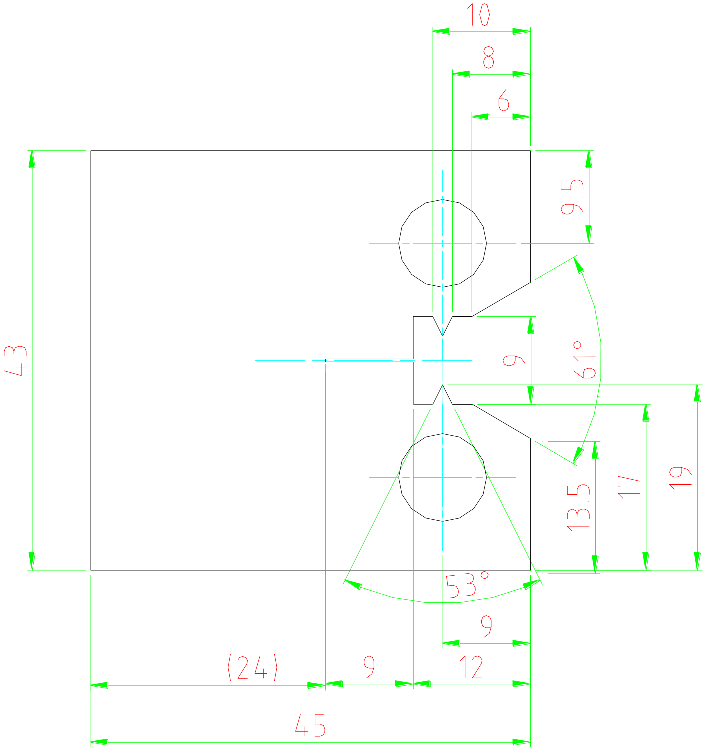


magnet system.

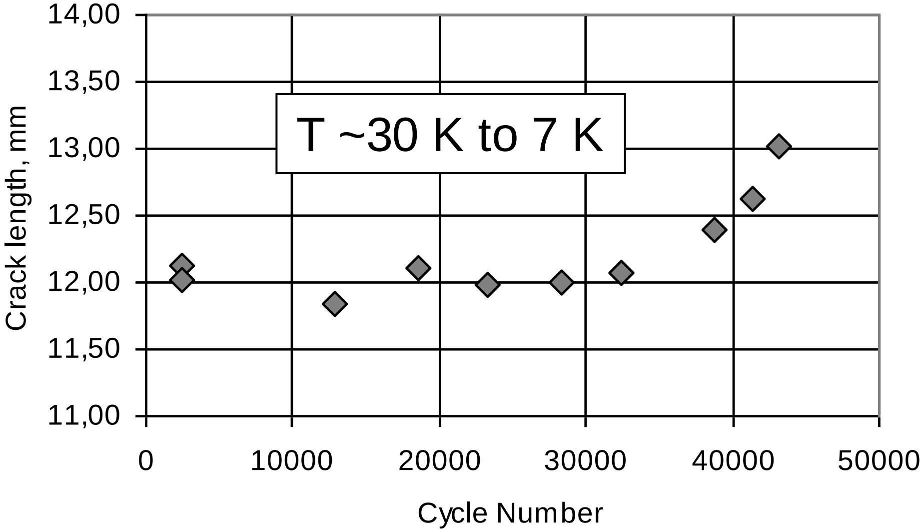
Sample machining & specimen & test details

Longitudinal Transverse





Crack length versus cycle number at 7 K for
46_(3)_CO+ST_2.5%+AG_FCGR+J1C_LD_1 specimen

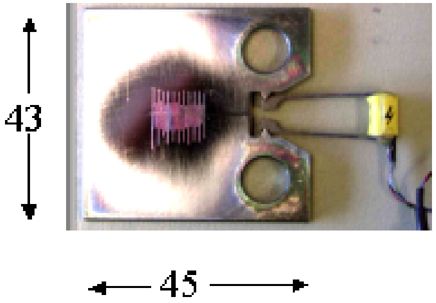


Pre-Cracking Start
at the notch position a/W = 0.33
with ~ ΔK 30 MPa√m

Fatigue Crack Growth Rate (FCGR) Measurement Procedure

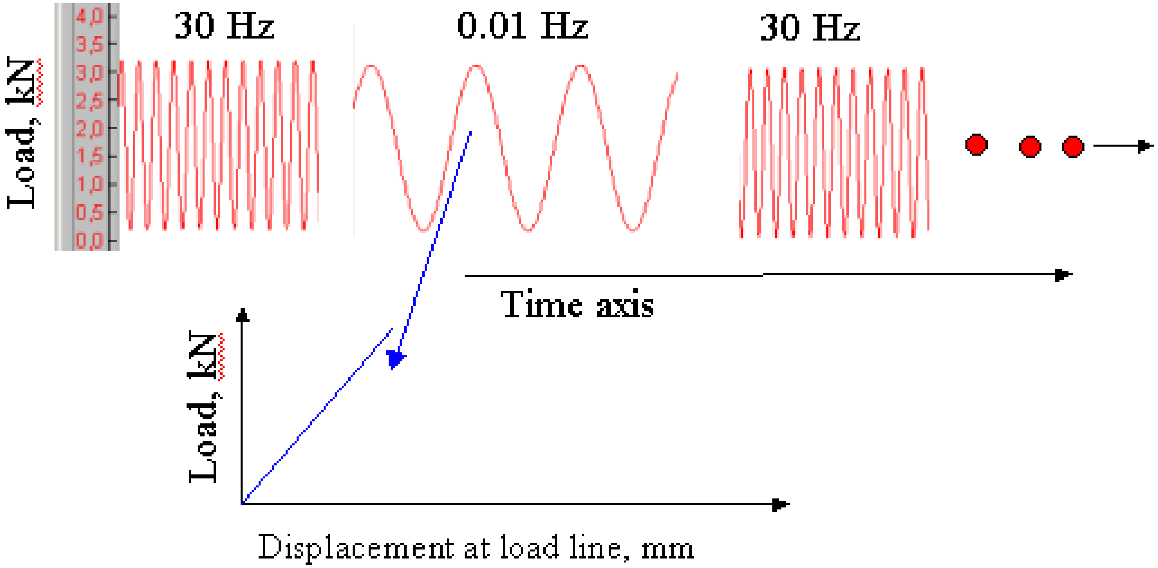
a/W = f(EBC)

$$EBC = Young's\ \ \mathrm{modulus} \cdot Thickness \cdot \frac{1}{Stiffness}$$

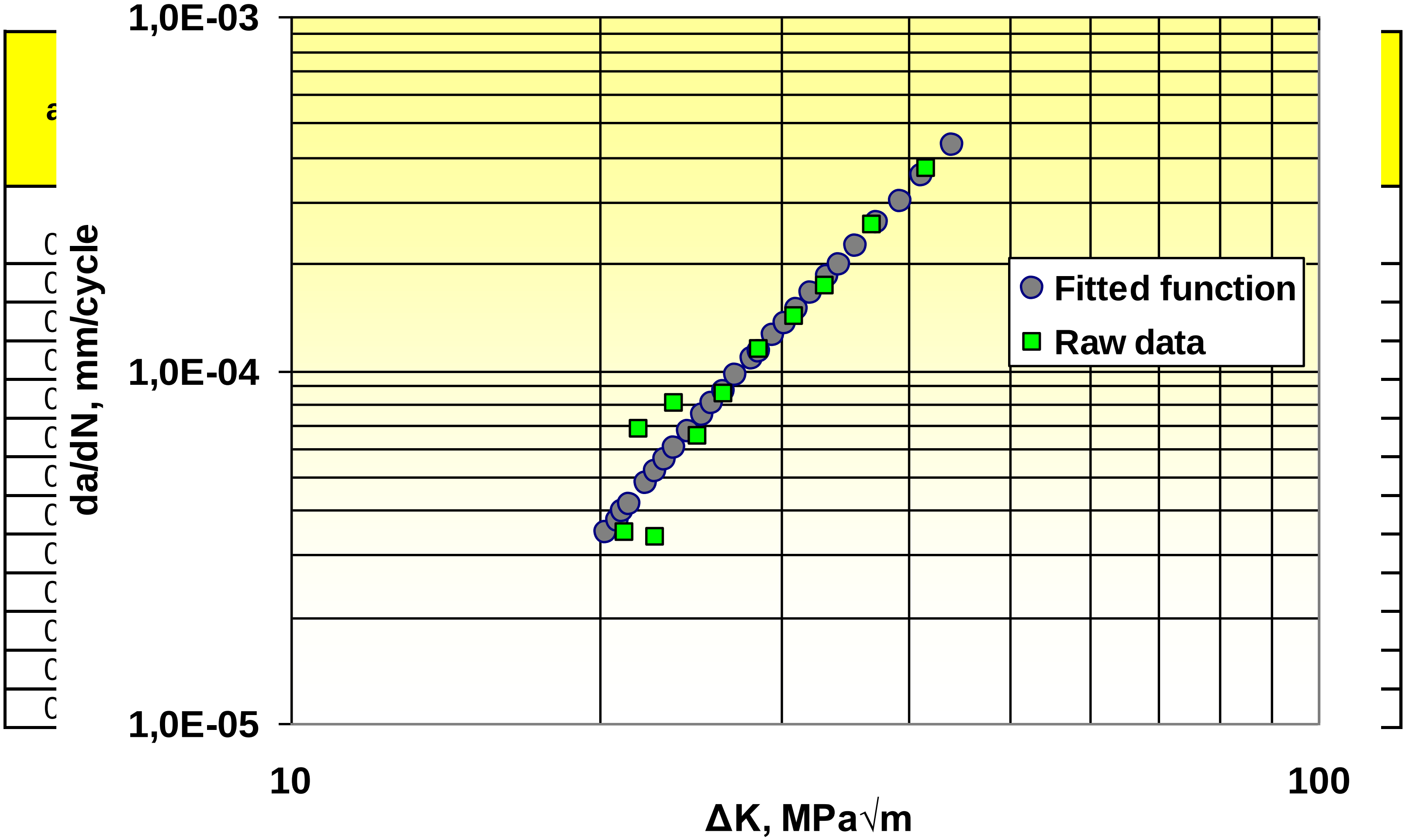


$$a/W = 0.35251 + 0.57213 \cdot \tanh\left(0.85068 \cdot \left(\log(EBC) - 1.26064\right)\right)$$

Pre-Cracking Start
at the notch a/W
with ~ ΔK 30 MPa√m



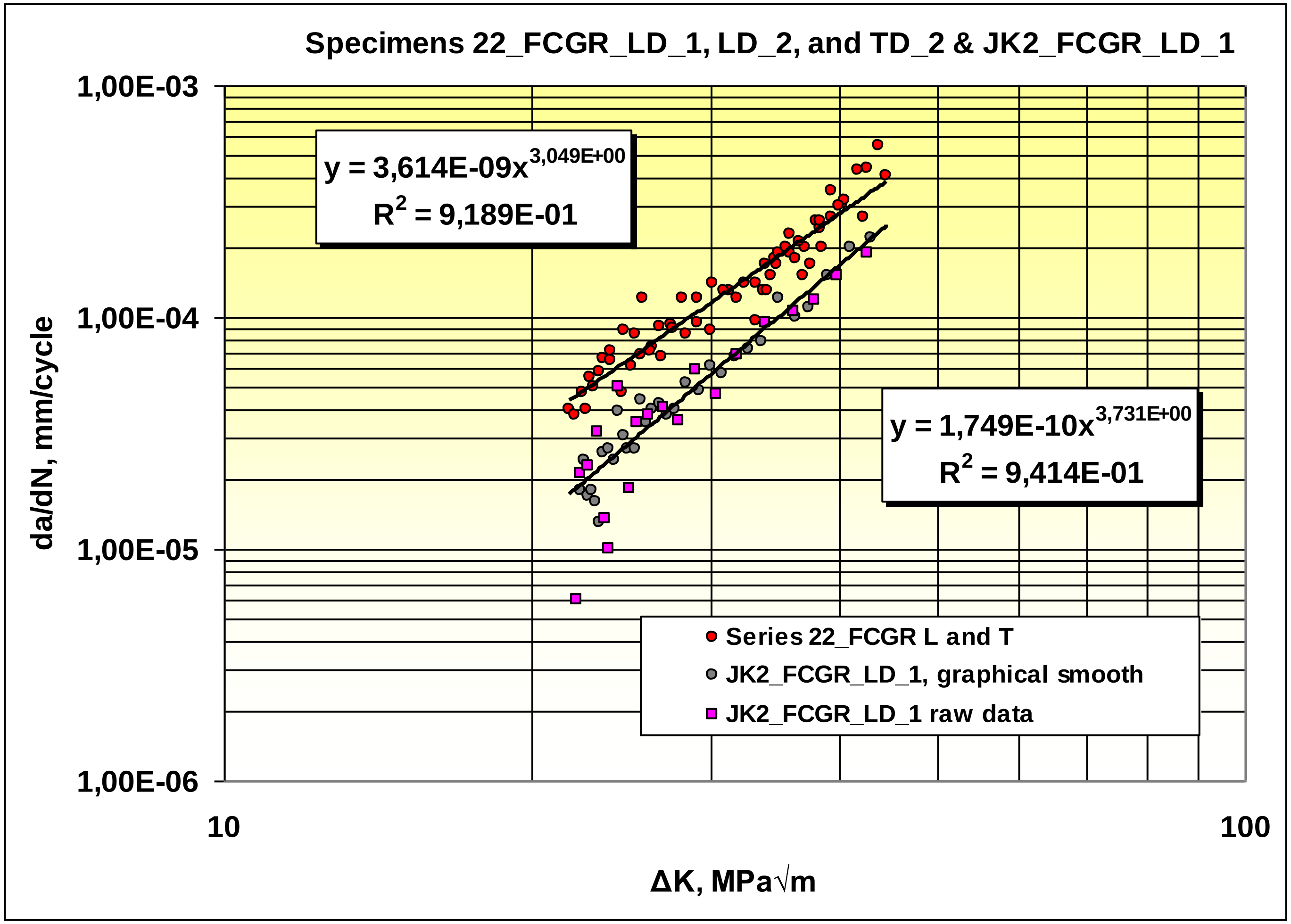
Obtained FCGR raw data & its analysis by curve fitting



Specimen XX_FCGRTD_1

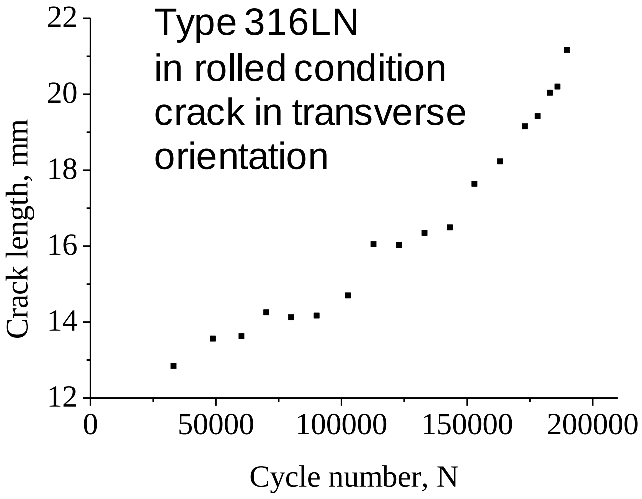
Raw data and smoothing of raw data by using the DataFit Software

Obtained FCGR results, its analysis by mathematical smoothing



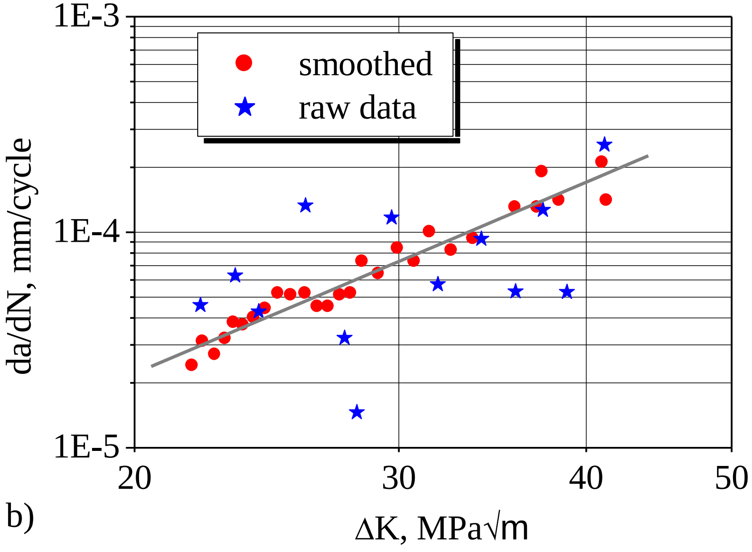
These are 7 K test results of Type 316 LN and JK2 already published

FCGR investigations at 7 K of two different materials

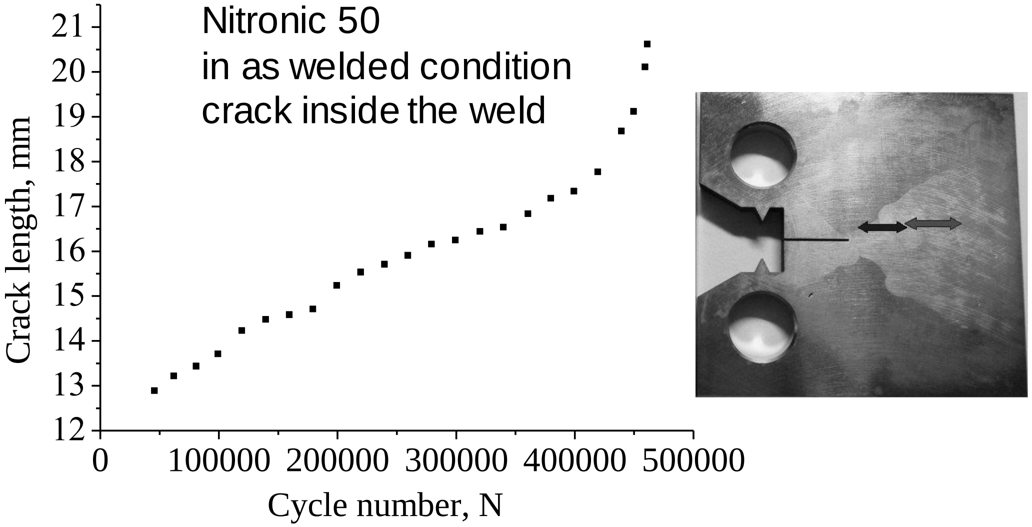


Type 316LN
in rolled condition
crack in transverse
orientation

a)

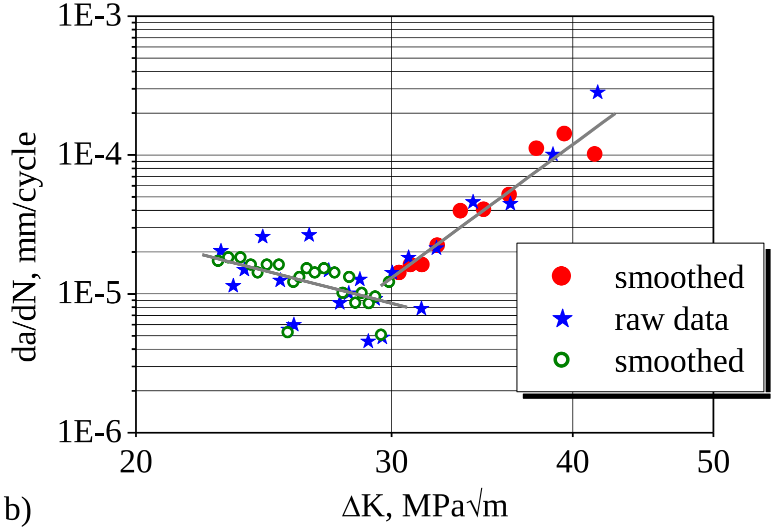


b)



Nitronic 50
in as welded condition
crack inside the weld

a)



b)

Uncertainty as "new" tool since around 1990

Measurement and its reliability

- For reporting the result of a measurement of a physical quantity, it should be obligatory to indicate the quality of the result quantitatively ***so*** that those who use it, can assess its reliability.
- Without such an indication, measurements cannot be compared. It **is** therefore necessary that there be a readily implemented, easily understood, and generally accepted procedure for characterizing the quality of a result of a measurement, that is, for evaluating and expressing its ***uncertainty.***

GUM Method : "The Guide to Expression of Uncertainty in Measurement" was originally published jointly by seven international organizations.
These organizations are BIPM, IEC, IFCC, ISO, IUPAC, IUPAP and OIML.

http://www.gum.dk/e-gumfaq-publishers/GUMpublishers.html

Uncertainty concept a brief description

The result of any physical measurement comprises two parts:
an estimate of the true value of the measurand
and the uncertainty of this “best” estimate.

One can attempt to measure the true value by measuring “**the best estimate**” using uncertainty evaluations such as Type A (repeated measurements in the laboratory in general expressed in form of Gaussian distribution) and Type B (previous experiments, literature data, manufacturer’s information, etc. in general in form of rectangular distribution) uncertainties.

Type A uncertainty

$$u_A = \frac{s}{\sqrt{N}}$$

Type B uncertainty

$$u_B = \sqrt{\left(\frac{1}{3}\right)\cdot M_1^2 + \left(\frac{1}{3}\right)\cdot M_2^2 + \cdots\cdots}$$

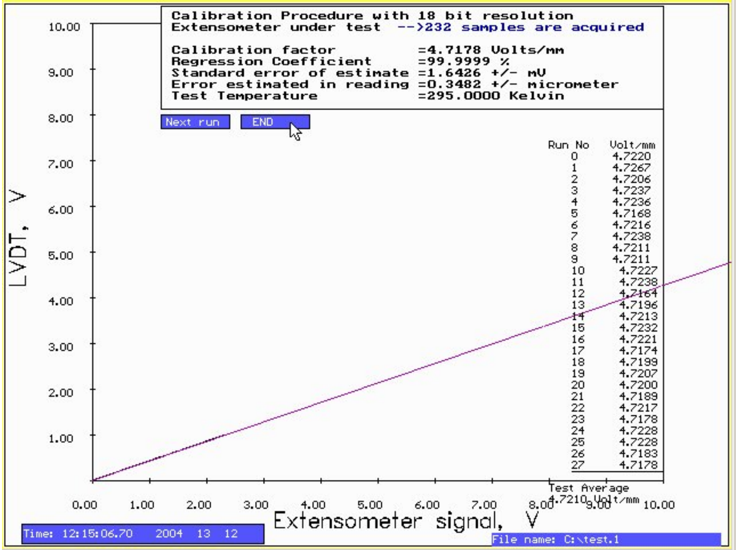
Combined uncertainty

$$u_c = \sqrt{u_A^2 + u_B^2}$$

The GUM, within this context, is a guide for a transparent, standardized documentation of the measurement procedure.

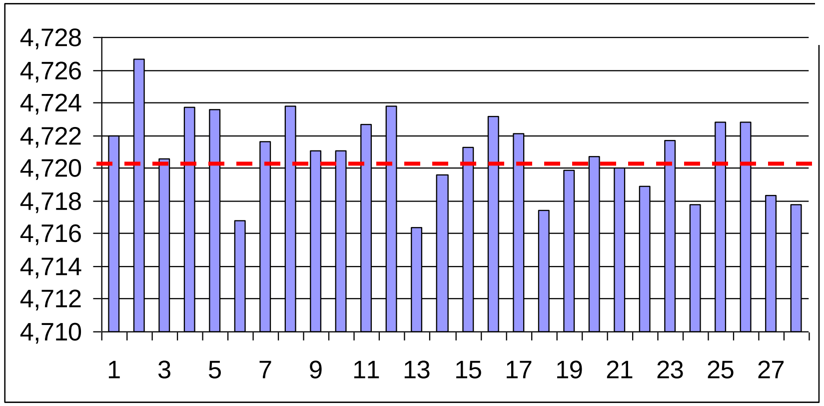
Secret behind of wordings; accuracy and precision, a brief description

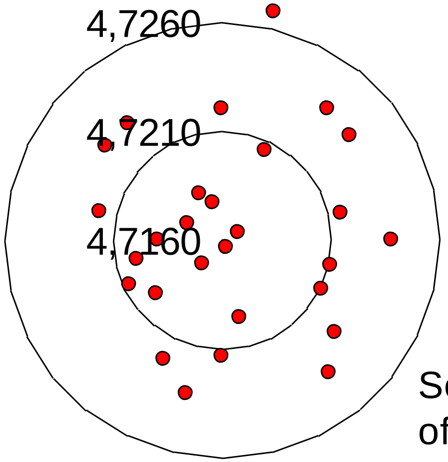
sensitivity, V/mm	
4,7220	
4,7267	
4,7206	
4,7237	
4,7236	
4,7168	
4,7216	
4,7238	
4,7211	
4,7211	
4,7227	
4,7238	
4,7164	
4,7196	
4,7213	
4,7232	
4,7221	
4,7174	
4,7199	
4,7207	
4,7200	
4,7189	
4,7217	
4,7178	
4,7228	
4,7228	
4,7183	
4,7178	
0,002494131	Standard dev
4,7210	Average
0,000471346	Uncertainty



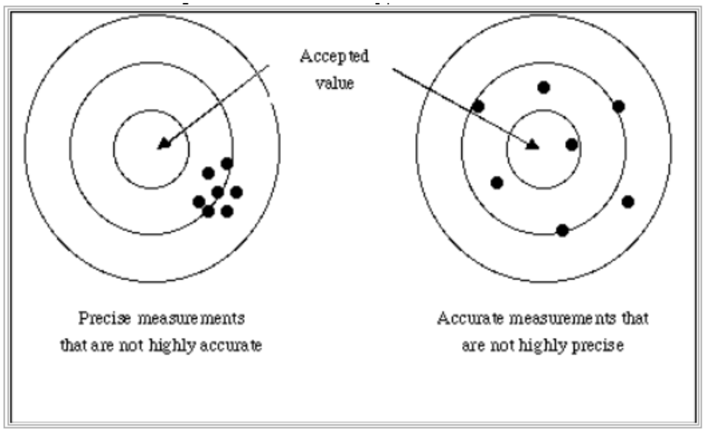
```
Calibration Procedure with 18 bit resolution
Extensometer under test  -->232 samples are acquired

Calibration factor          =4.7178 Volts/mm
Regression Coefficient      =99.9999 %
Standard error of estimate  =1.6426 +/- mV
Error estimated in reading  =0.3482 +/- micrometer
Test Temperature            =295.0000 Kelvin
```

```
Run No   Volt/mm
 0       4.7220
 1       4.7267
 2       4.7206
 3       4.7237
 4       4.7236
 5       4.7168
 6       4.7216
 7       4.7238
 8       4.7211
 9       4.7211
10       4.7227
11       4.7238
12       4.7164
13       4.7196
14       4.7213
15       4.7232
16       4.7221
17       4.7174
18       4.7199
19       4.7207
20       4.7200
21       4.7189
22       4.7217
23       4.7178
24       4.7228
25       4.7228
26       4.7183
27       4.7178
Test Average
4.7210 Volt/mm
```

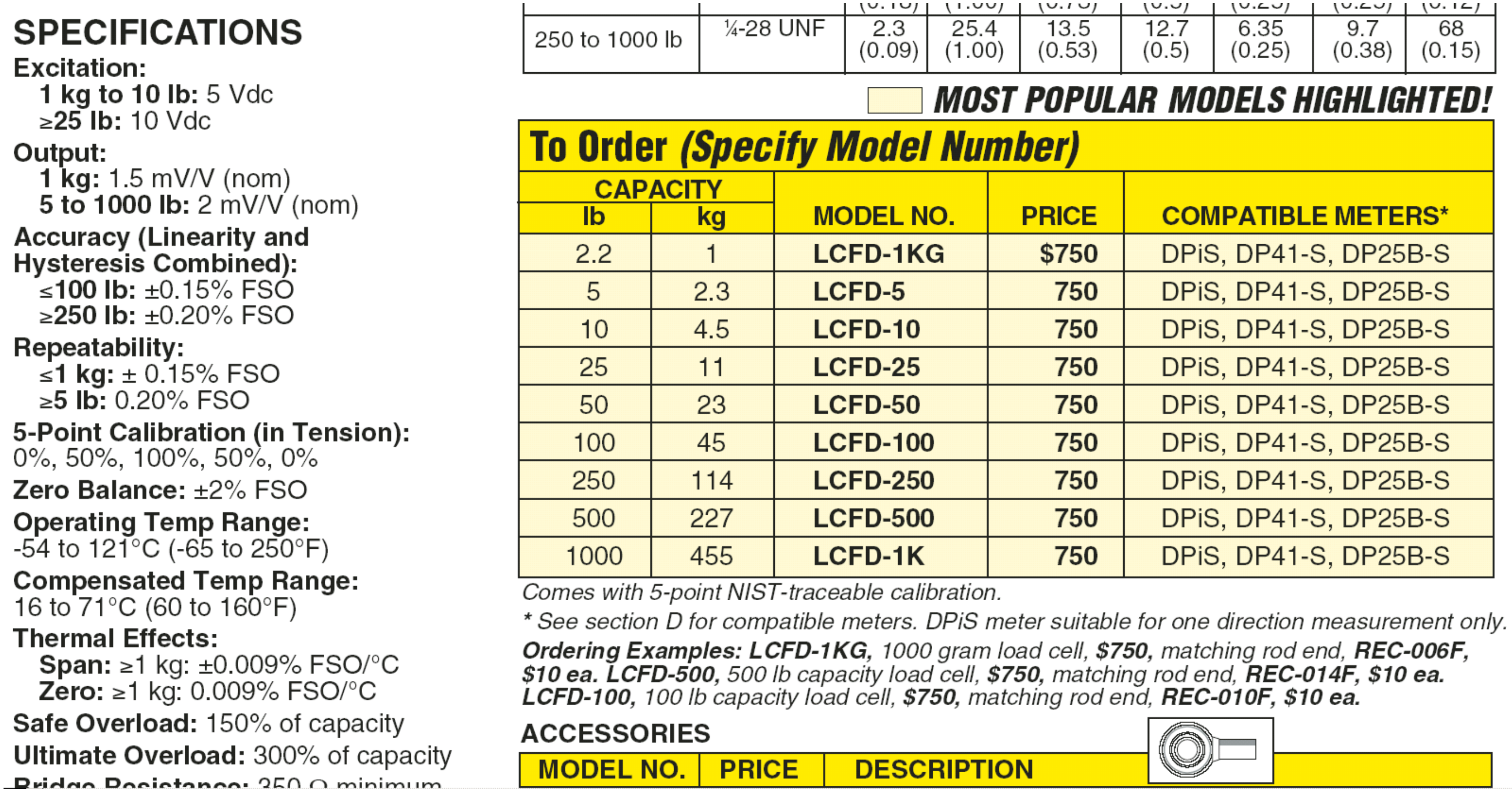




Sensitivity distribution of 28 data points



Present days specification for commonly used transducers



SPECIFICATIONS

Excitation:
1 kg to 10 lb: 5 Vdc
≥25 lb: 10 Vdc

Output:
1 kg: 1.5 mV/V (nom)
5 to 1000 lb: 2 mV/V (nom)

Accuracy (Linearity and Hysteresis Combined):
≤100 lb: ±0.15% FSO
≥250 lb: ±0.20% FSO

Repeatability:
≤1 kg: ± 0.15% FSO
≥5 lb: 0.20% FSO

5-Point Calibration (in Tension):
0%, 50%, 100%, 50%, 0%

Zero Balance: ±2% FSO

Operating Temp Range:
-54 to 121°C (-65 to 250°F)

Compensated Temp Range:
16 to 71°C (60 to 160°F)

Thermal Effects:
Span: ≥1 kg: ±0.009% FSO/°C
Zero: ≥1 kg: 0.009% FSO/°C

Safe Overload: 150% of capacity

Ultimate Overload: 300% of capacity

Bridge Resistance: 350 Ω minimum

250 to 1000 lb	¼-28 UNF	2.3 (0.09)	25.4 (1.00)	13.5 (0.53)	12.7 (0.5)	6.35 (0.25)	9.7 (0.38)	68 (0.15)

🟨 ***MOST POPULAR MODELS HIGHLIGHTED!***

To Order *(Specify Model Number)*

CAPACITY		MODEL NO.	PRICE	COMPATIBLE METERS*
lb	kg			
2.2	1	**LCFD-1KG**	**$750**	DPiS, DP41-S, DP25B-S
5	2.3	**LCFD-5**	**750**	DPiS, DP41-S, DP25B-S
10	4.5	**LCFD-10**	**750**	DPiS, DP41-S, DP25B-S
25	11	**LCFD-25**	**750**	DPiS, DP41-S, DP25B-S
50	23	**LCFD-50**	**750**	DPiS, DP41-S, DP25B-S
100	45	**LCFD-100**	**750**	DPiS, DP41-S, DP25B-S
250	114	**LCFD-250**	**750**	DPiS, DP41-S, DP25B-S
500	227	**LCFD-500**	**750**	DPiS, DP41-S, DP25B-S
1000	455	**LCFD-1K**	**750**	DPiS, DP41-S, DP25B-S

Comes with 5-point NIST-traceable calibration.

** See section D for compatible meters. DPiS meter suitable for one direction measurement only.*

Ordering Examples: LCFD-1KG,** 1000 gram load cell, **$750,** matching rod end, **REC-006F, $10 ea. LCFD-500,** 500 lb capacity load cell, **$750,** matching rod end, **REC-014F, $10 ea. LCFD-100,** 100 lb capacity load cell, **$750,** matching rod end, **REC-010F, $10 ea.

ACCESSORIES

MODEL NO.	PRICE	DESCRIPTION

If accuracy of this load cell is given with +/- 0.15 % FS does it mean that the load cell is 99.70 % inaccurate ???

Model equation and the procedure of uncertainty determination

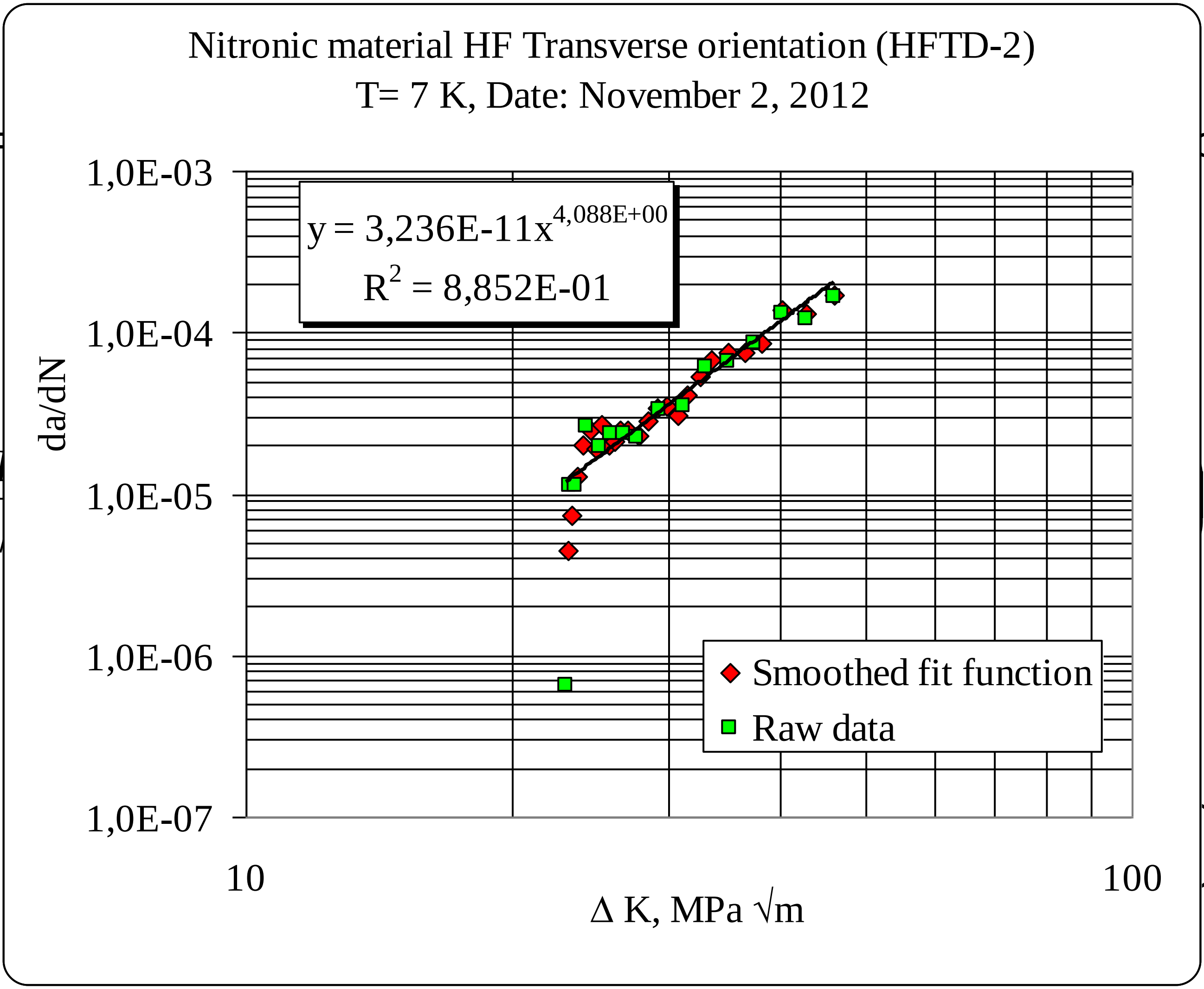
$$\frac{\partial a}{\partial N} =$$

ouble log graph

$$\Delta K = \frac{\Delta}{B \cdot \sqrt{}}$$

$$\Big)^{3} - 5{,}6 \cdot \left(\frac{a}{W} \right)^{4} \Big)$$

$$\Big)$$



Combined standard uncertainty and its treatment

Model equation $\Longrightarrow$ $\frac{\partial a}{\partial N} = C_0 \cdot \Delta K^m$ $\Longrightarrow$ $\frac{\partial a}{\partial N} = R = f(C_0, m, \Delta P, W, B, a)$

Combined standard uncertainty using the Gaussian distribution law

$$u_c = \sqrt{\left(\frac{\partial R}{\partial C_0}\right)^2 u_1^2 + \left(\frac{\partial R}{\partial m}\right)^2 u_2^2 + \left(\frac{\partial R}{\partial \Delta P}\right)^2 u_3^2 + \left(\frac{\partial R}{\partial W}\right)^2 u_4^2 + \left(\frac{\partial R}{\partial B}\right)^2 u_5^2 + \left(\frac{\partial R}{\partial a}\right)^2 u_6^2}$$

$u_1 \cdots$ efficients,

whils ensitivity coefficients

Partial differentiation & det coefficients

$$\frac{\delta R}{\delta C_0} = \Delta K^m$$

& $\Delta K^m \cdot \ln \Delta K$

$$\frac{\partial R}{\partial \Delta P} = C_0 \cdot \Delta K^m \cdot$$

$\cdot \Delta K^m \cdot \frac{m}{B}$

Partial differentiation of the model equation with respect to width "W", the final equation

$$\frac{d}{dW}R \rightarrow C0\cdot\left[\frac{\Delta P}{B\cdot W^{\frac{1}{2}}}\cdot\left(2+\frac{a}{W}\right)\cdot\frac{\left(.886+4.64\cdot\frac{a}{W}-13.32\cdot\frac{a^{2}}{W^{2}}+14.72\cdot\frac{a^{3}}{W^{3}}-5.6\cdot\frac{a^{4}}{W^{4}}\right)}{\left(1-\frac{a}{W}\right)^{1.5}}\right]^{m}\cdot m\cdot$$

$$\left[\frac{\frac{-1}{2}\cdot\frac{\Delta P}{B\cdot W^{\frac{3}{2}}}\cdot\left(2+\frac{a}{W}\right)\cdot\frac{\left(.886+4.64\cdot\frac{a}{W}-13.32\cdot\frac{a^{2}}{W^{2}}+14.72\cdot\frac{a^{3}}{W^{3}}-5.6\cdot\frac{a^{4}}{W^{4}}\right)}{\left(1-\frac{a}{W}\right)^{1.5}}-\frac{\Delta P}{B\cdot W^{\frac{5}{2}}}\cdot a\cdot\frac{\left(.886+4.64\cdot\frac{a}{W}-13.32\cdot\frac{a^{2}}{W^{2}}+14.72\cdot\frac{a^{3}}{W^{3}}-5.6\cdot\frac{a^{4}}{W^{4}}\right)}{\left(1-\frac{a}{W}\right)^{1.5}}+}{\Delta P}\right.$$

$$\left.\frac{\frac{\Delta P}{B\cdot W^{\frac{1}{2}}}\cdot\left(2+\frac{a}{W}\right)\cdot\frac{\left(-4.64\cdot\frac{a}{W^{2}}+26.64\cdot\frac{a^{2}}{W^{3}}-44.16\cdot\frac{a^{3}}{W^{4}}+22.4\cdot\frac{a^{4}}{W^{5}}\right)}{\left(1-\frac{a}{W}\right)^{1.5}}-1.5\cdot\frac{\Delta P}{B\cdot W^{\frac{5}{2}}}\cdot\left(2+\frac{a}{W}\right)\cdot\frac{\left(.886+4.64\cdot\frac{a}{W}-13.32\cdot\frac{a^{2}}{W^{2}}+14.72\cdot\frac{a^{3}}{W^{3}}-5.6\cdot\frac{a^{4}}{W^{4}}\right)}{\left(1-\frac{a}{W}\right)^{2.5}}\cdot a}{}\right]\cdot$$

$$B\cdot\frac{W^{\frac{1}{2}}}{\left(2+\frac{a}{W}\right)\cdot\left(.886+4.64\cdot\frac{a}{W}-13.32\cdot\frac{a^{2}}{W^{2}}+14.72\cdot\frac{a^{3}}{W^{3}}-5.6\cdot\frac{a^{4}}{W^{4}}\right)}\cdot\left(1-\frac{a}{W}\right)^{1.5}$$

Partial differentials for width “W” and crack length “a”

$$\frac{\partial R}{\partial W} = C_0 \cdot \Delta K^m \cdot m \cdot \frac{\left[F_A + F_B + F_C + F_D\right]}{\Delta P} \cdot B \cdot F_E$$

$$\frac{\partial R}{\partial a} = C_0 \cdot \Delta K^m \cdot m \cdot \frac{\left[-\frac{2 \cdot F_A}{\left(2 + \frac{a}{W}\right)} + F_F - \frac{F_D \cdot \sqrt[5]{W}}{\sqrt[3]{W}} \right]}{\Delta P} \cdot B \cdot F_E$$

Functions F_A ….. F_F of the partial derivative equation

$$F_A = -\Delta P \cdot \left(2+\frac{a}{W}\right) \cdot \frac{\left(8.886+4.64\cdot\left(\frac{a}{W}\right)-13.32\cdot\left(\frac{a}{W}\right)^2+14.72\cdot\left(\frac{a}{W}\right)^3-5.6\cdot\left(\frac{a}{W}\right)^4\right)}{\left(1-\frac{a}{W}\right)^{1.5}\cdot 2\cdot B\cdot\sqrt[3]{W}}$$

$$F_B = -\Delta P \cdot a \cdot \frac{\left(8.886+4.64\cdot\left(\frac{a}{W}\right)-13.32\cdot\left(\frac{a}{W}\right)^2+14.72\cdot\left(\frac{a}{W}\right)^3-5.6\cdot\left(\frac{a}{W}\right)^4\right)}{\left(1-\frac{a}{W}\right)^{1.5}\cdot B\cdot\sqrt[5]{W}}$$

$$F_C = \Delta P \cdot \frac{\left(2+\frac{a}{W}\right)\cdot\left(-4.64\cdot\frac{a}{W^2}+26.64\cdot\frac{a^2}{W^3}-44.16\cdot\frac{a^3}{W^4}+22.4\cdot\frac{a^4}{W^5}\right)}{\left(1-\frac{a}{W}\right)^{1.5}\cdot B\cdot\sqrt{W}}$$

$$F_D = -1.5\cdot\Delta P \cdot \frac{\left(2+\frac{a}{W}\right)\cdot\left(8.886+4.64\cdot\left(\frac{a}{W}\right)-13.32\cdot\left(\frac{a}{W}\right)^2+14.72\cdot\left(\frac{a}{W}\right)^3-5.6\cdot\left(\frac{a}{W}\right)^4\right)}{\left(1-\frac{a}{W}\right)^{2.5}\cdot B\cdot\sqrt[5]{W}}$$

$$F_E = \frac{\sqrt{W}}{\left(8.886+4.64\cdot\frac{a}{W}-13.32\cdot\left(\frac{a}{W}\right)^2+14.72\cdot\left(\frac{a}{W}\right)^3-5.6\cdot\left(\frac{a}{W}\right)^4\right)}\cdot\frac{\left(1-\frac{a}{W}\right)^{1.5}}{\left(2+\frac{a}{W}\right)}$$

$$F_F = \Delta P \cdot \frac{\left(2+\frac{a}{W}\right)\cdot\left(4.64\cdot\frac{1}{W}-26.64\cdot\frac{a}{W^2}+44.16\cdot\frac{a^2}{W^3}-22.4\cdot\frac{a^3}{W^4}\right)}{\left(1-\frac{a}{W}\right)^{1.5}\cdot B\cdot\sqrt{W}}$$

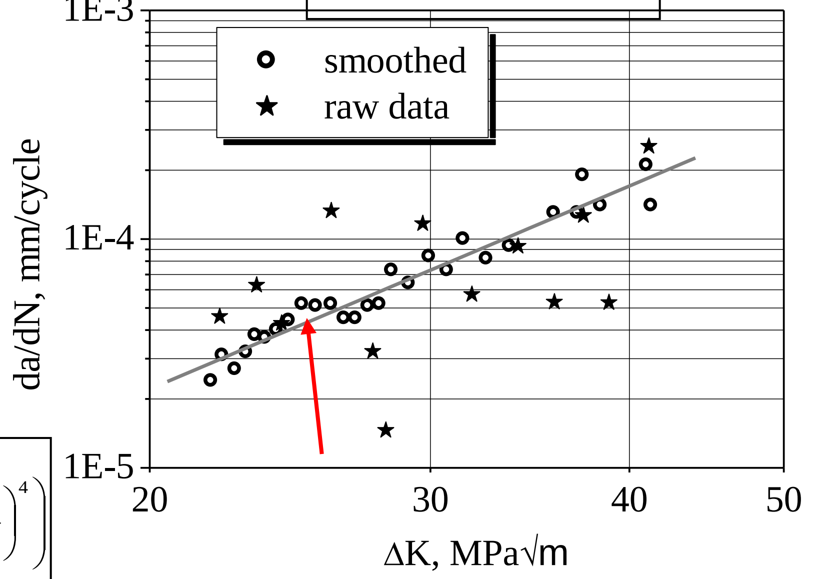
Computation of sensitivity values

Computed Paris law constants
C_0= 3.226.10^{-9} & m = 2.949

with
a = 1.5 cm, W = 3.6 cm, B = 0.4 cm, ΔP = 2.5 kN

$$\Delta K = \frac{\Delta P}{B \cdot \sqrt{W}} \cdot \frac{\left(2 + \frac{a}{W}\right)}{\left(1 - \frac{a}{W}\right)^{1.5}} \cdot \left(0{,}886 + 4{,}64 \cdot \frac{a}{W} - 13{,}32 \cdot \left(\frac{a}{W}\right)^2 + 14{,}72 \cdot \left(\frac{a}{W}\right)^3 - 5{,}6 \cdot \left(\frac{a}{W}\right)^4\right)$$

ΔK is ~ 25 MPa√m



Type 316LN
in rolled condition
crack in transverse
orientation

Results of the computed partial differentials (sensitivity coefficients)

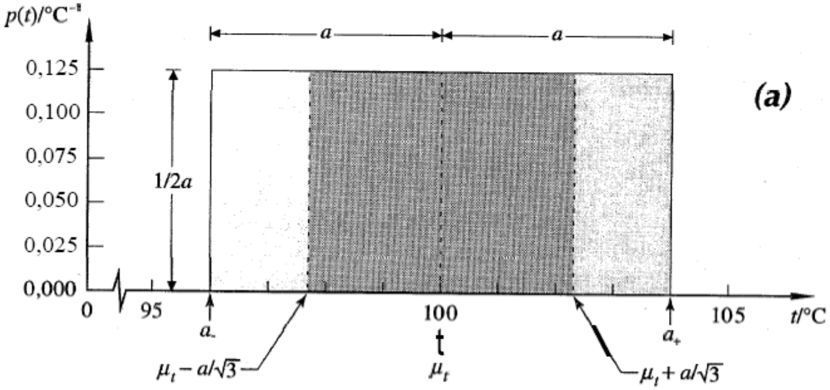
$\frac{da}{dN}$	ΔK	$\frac{\partial R}{\partial C_0}$	$\frac{\partial R}{\partial m}$	$\frac{\partial R}{\partial \Delta P}$	$\frac{\partial R}{\partial W}$	$\frac{\partial R}{\partial B}$	$\frac{\partial R}{\partial a}$
$4.311 \cdot 10^{-5}$	25.066	$1.336 \cdot 10^{4}$	$1.918 \cdot 10^{-4}$	$5.085 \cdot 10^{-5}$	$-5.724 \cdot 10^{-5}$	$-3.178 \cdot 10^{-4}$	$9.50 \cdot 10^{-5}$

Uncertainty terms for ΔP, W, a, B and its treatment

There are two types, Type A and Type B:

Type A obtained by repeated observations. Standard deviation divided by square root of number for repeated measurements.
For W and B treatment according to Type A

Type B obtained not by repeated observations



Manufacturer data with respect to used load cell

Load cell capacity, N	Hysteresis % / full scale	Temperature coeff. on zero % / K	Temperature coeff. on sensitivity % / K
25000	0.05	0.002	0.002

$$P = hysteresis + T_{CoeffonZero} + T_{CoeffonSens}$$

$$u_P = \sqrt{\left(\frac{hysteresis \cdot 2500}{100 \cdot \sqrt{3}}\right)^2 + \left(\frac{T_{CoeffonZero} \cdot 2500}{100 \cdot \sqrt{3}}\right)^2 + \left(\frac{T_{CoeffonSens} \cdot 2500}{100 \cdot \sqrt{3}}\right)^2} = 0.776 \quad N$$

$$u_3 = 0.776 \quad N \quad or \quad 0.0008 \quad kN$$

Uncertainty terms for m and C_0 and its treatment

Assumption: experimental errors are identical for all four tests as the uncertainties resulting from the geometrical constraints has been considered elsewhere. Therefore, m and C_0 are determined using the experimental results.

Specimen, condition, & code	C_0, specimen # 1	C_0, specimen # 2	absolute difference	m, specimen # 1	m, specimen # 2	absolute difference
316LN rolled plate, longitudinal	$1.79 \cdot 10^{9}$	$2.65 \cdot 10^{9}$	$8.64 \cdot 10^{10}$	3.092	3.012	0.080
316LN rolled plate, transversal	$3.23 \cdot 10^{9}$	$1.37 \cdot 10^{9}$	$1.86 \cdot 10^{9}$	2.949	3.193	0.244

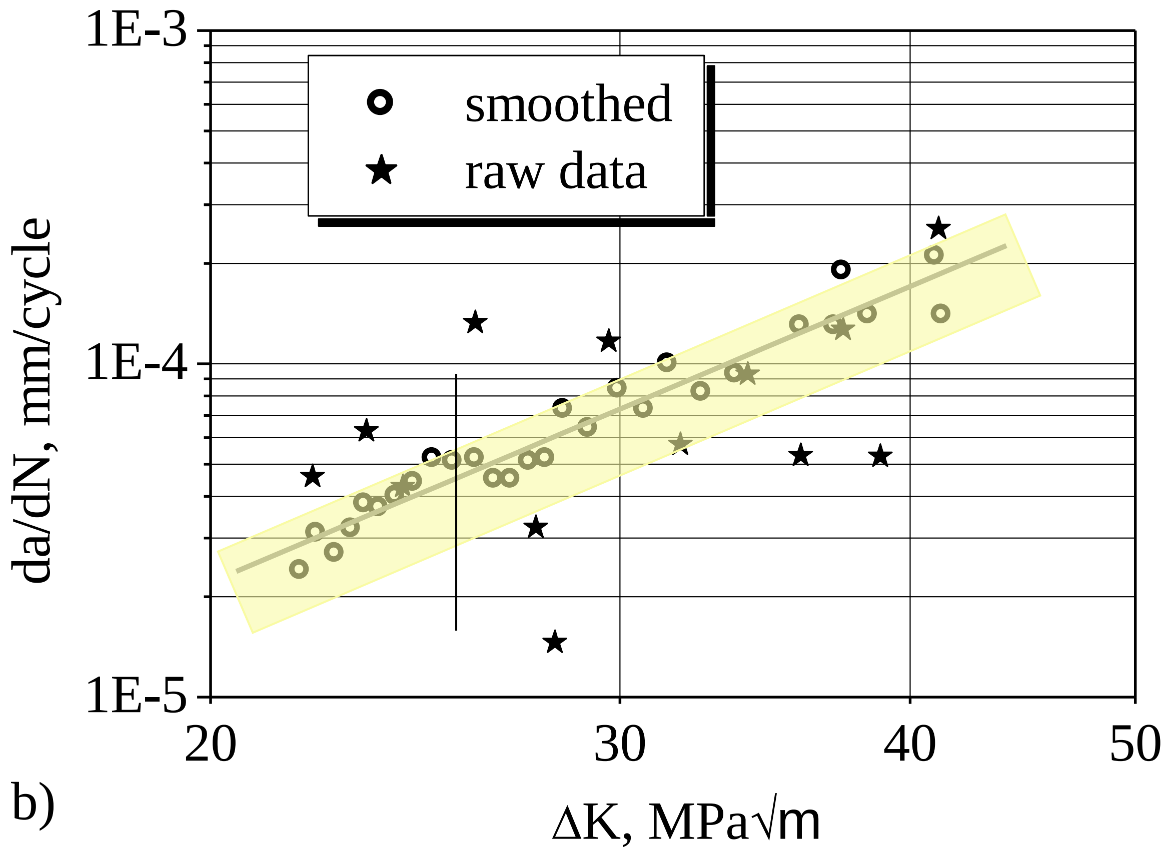
For m, a, C_0, and, ΔP treatment according to Type B. W and B according to Type A

$$u_c = \sqrt{\left(1.336 \cdot 10^{4}\right)^2 \cdot \left(3.93 \cdot 10^{-10}\right)^2 + \left(1.918 \cdot 10^{-4}\right)^2 \cdot (0.047)^2 + \left(5.085 \cdot 10^{-5}\right)^2 \cdot (0.0008)^2 + \left(-5.724 \cdot 10^{-5}\right)^2 \cdot (0.002)^2 + \left(-3.178 \cdot 10^{-4}\right)^2 \cdot (0.002)^2 + \left(9.5 \cdot 10^{-5}\right)^2 \cdot (0.115)^2} \quad \frac{mm}{cycle}$$

$$\boxed{u_C = 1.51 \cdot 10^{-5} \quad \frac{mm}{cycle}}$$

$$\boxed{\frac{da}{dN} = R = 4.311 \cdot 10^{-5} \quad \pm \quad 1.51 \cdot 10^{-5} \quad \frac{mm}{Cycles} \quad \textit{for} \quad \Delta K = 25.07 \quad MPa\sqrt{m}}$$

Best estimate and the uncertainty of the measurement

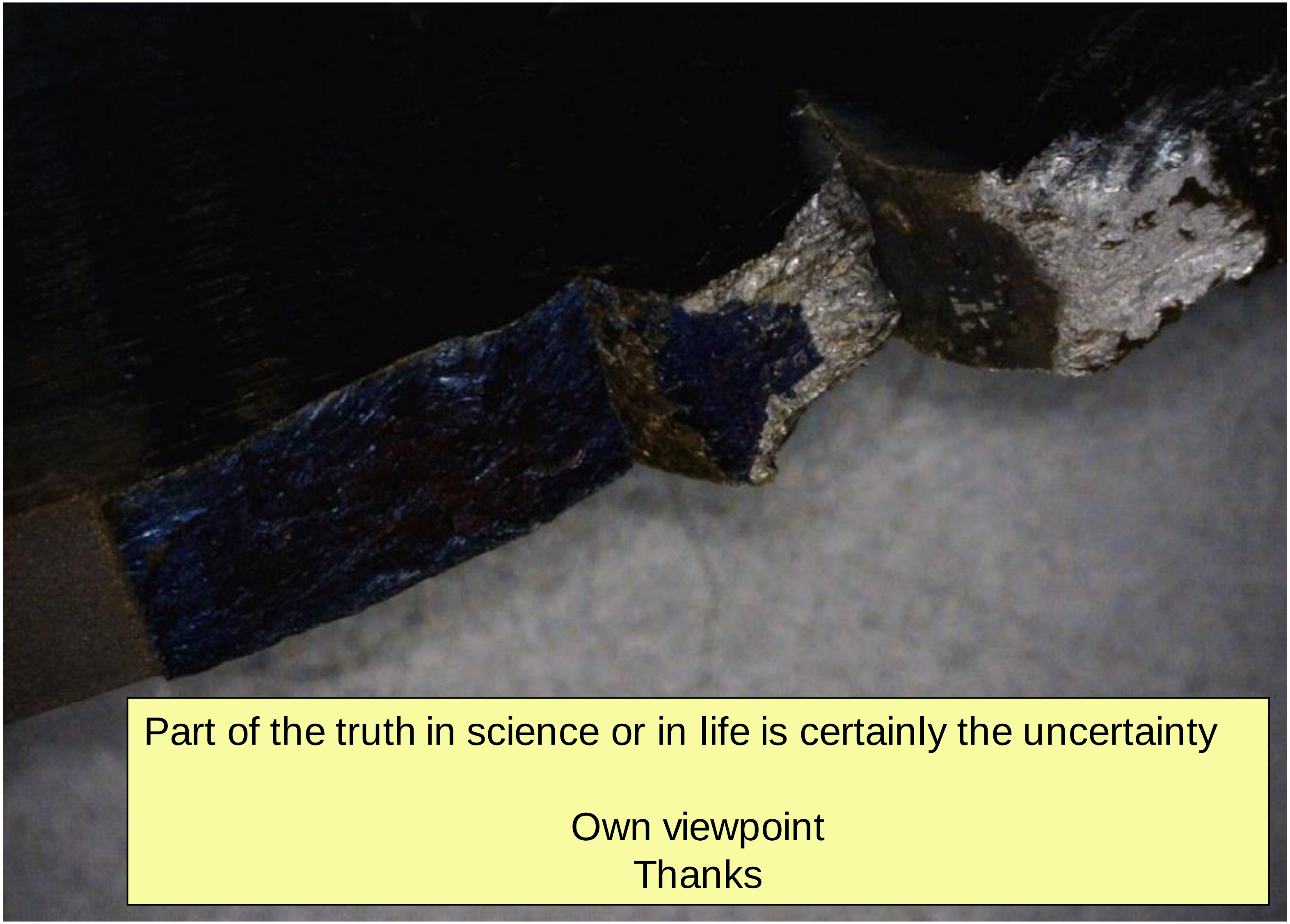


b)

Type 316LN
in rolled condition
crack in transverse
orientation

Conclusions

- Concept of uncertainty as given by GUM could be successfully applied on FCGR measurements, which were performed at 7 K
- Partial derivates for all variables resulting from Paris law and stress intensity factor equation has been analyzed and rigorously computed
- Uncertainty terms obtained by repetitive measurements, Type A and uncertainty terms related to manufacturer data, Type B has been used
- In addition, for the determination of exponent m variation, four individual tests of the same material has been applied to obtain the uncertainty term
- The final result show the best estimate of the rate and the uncertainty with a +/- bandwidth which should be considered during any design



Part of the truth in science or in life is certainly the uncertainty

Own viewpoint
Thanks