More Flavor SU(3) Tests for CP Violating B Decays

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Based on: 1308.4143 (Y. Grossman, Z. Ligeti & DR)

Also: 1211.3361 (Y. Grossman & DR)

[Yuval's birthday is today to $\sim \varepsilon^3$, $\varepsilon \equiv f_K/f_\pi - 1$]

Recent Direct CP Results

Some recent LHCb results:

$$A_{\text{CP}}[B_s \to K^- \pi^+] = 0.27 \pm 0.04$$

and

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Combine $B_s \to K\pi$ with previous data, can test U-spin parameter (1304.6173)

$$\Delta \equiv \frac{A_{\rm CP}[B_d \to K^+\pi^-]}{A_{\rm CP}[B_s \to K^-\pi^+]} + \frac{\overline{\Gamma}[B_s \to K^-\pi^+]}{\overline{\Gamma}[B_d \to K^+\pi^-]} = -0.02 \pm 0.05 \pm 0.04 \ .$$

Expect $\Delta = 0$ in U-spin limit.

Questions

Can immediately ask:

- At what scale should U-spin breaking occur in Δ? Are there better defined parameters?

- Are there other sum-rule generated parameters like Δ: What are all the flavor SU(3) CP sum rules for $B \to PP$ and $B \to PV$? Are there any beyond leading order in SU(3) breaking?

- Are QCD or SCET factorization pictures consistent with parameter data?

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  Δ carries arbitrary decay rate and phase space normalizations; we need better-defined parameters.

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  We computed the complete set of SU(3) CP sum rules in flavor and mass bases. There are none beyond LO in SU(3) and isospin breaking in the mass basis.

- Are QCD or SCET factorization pictures consistent with parameter data?

  Yes, QCD (BBNS) factorization and SCET (BPRS) generated relations agree with data for parameters so far.

Sum Rules

- In Wigner-Eckart picture, can factorize amplitudes into the form

$$\text{Amplitudes} = \sum \text{`Invariants'} \times \text{RMEs}$$

'Invariants': Known weak physics: Clebsch-Gordan, Hamiltonian, and symmetry breaking pattern

RMEs: Unknown strong physics

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Unknown strong physics

- Similarly, CP asymmetries in matrix form:

$$\begin{pmatrix} \vdots \\ \delta_{\mathrm{CP}}[I \to f_1 f_2] \\ \vdots \end{pmatrix}_\mu = \begin{pmatrix} \ddots & & \\ & CC^* - \overline{C}\,\overline{C}^* & \\ & & \ddots \end{pmatrix}_\mu^w \begin{pmatrix} \vdots \\ XX^* \\ \vdots \end{pmatrix}_w$$

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Sum Rules $\sum_i c_i \delta_i = 0$

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- NB: δ_{CP} is the splitting of the square amplitudes, not the rates: $\delta_{\mathrm{CP}} \equiv |\overline{\mathcal{A}}|^2 - |\mathcal{A}|^2 = 8\pi \mathcal{P}_{(i;f)} \Delta_{\mathrm{CP}}$

Sum Rules

CP sum rules span the row kernel of the flavor, Hamiltonian matrix, independent of RMEs:

Sum rules factor out (order by order) our ignorance of the reduced matrix elements.

They are 'necessary condition' probes of the SM: failure at larger than expected breaking scale suggest NP effects.

NB: Symmetry breaking can be included via mass spurion insertions in the Hamiltonian

CP Sum Rules

Formalism can be extended to compute sum rules in flavor or mass basis (i.e. including mixing), to arbitrary spurionic order. E.g. for $B \to PP$

Isospin

$$\delta_{\rm CP}[B_s \to \pi^-\pi^+] = 2\delta_{\rm CP}[B_s \to 2\pi^0]\,,$$
$$2\delta_{\rm CP}[B^+ \to \pi^0 K^+] - \delta_{\rm CP}[B^+ \to \pi^+ K^0] = \delta_{\rm CP}[B_d \to \pi^- K^+] - 2\delta_{\rm CP}[B_d \to \pi^0 K^0]$$

U-spin

$$\delta_{\rm CP}[B_d \to \pi^- K^+] + \delta_{\rm CP}[B_s \to \pi^+ K^-] = 0\,,$$
$$\delta_{\rm CP}[B_d \to \pi^-\pi^+] + \delta_{\rm CP}[B_s \to K^- K^+] = 0\,,$$
$$\delta_{\rm CP}[B_d \to K^- K^+] + \delta_{\rm CP}[B_s \to \pi^-\pi^+] = 0\,,$$
$$\delta_{\rm CP}[B_s \to \pi^0 \overline{K}^0] + \delta_{\rm CP}[B_d \to \pi^0 K^0] = 0\,,$$
$$\delta_{\rm CP}[B^+ \to K^+ \overline{K}^0] + \delta_{\rm CP}[B^+ \to \pi^+ K^0] = 0\,,$$
$$\delta_{\rm CP}[B_d \to K^0 \overline{K}^0] + \delta_{\rm CP}[B_s \to K^0 \overline{K}^0] = 0\,,$$
$$\delta_{\rm CP}[B_s \to \eta \overline{K}^0] + \delta_{\rm CP}[B_d \to \eta K^0] = 0\,,$$
$$\delta_{\rm CP}[B_s \to \eta' \overline{K}^0] + \delta_{\rm CP}[B_d \to \eta' K^0] = 0\,,$$

Pure SU(3)

$$\delta_{\rm CP}[B^+ \to \pi^+\eta'] + \delta_{\rm CP}[B^+ \to \pi^+\eta] + \delta_{\rm CP}[B^+ \to \eta K^+] + \delta_{\rm CP}[B^+ \to \pi^0 K^+] = 0\,,$$
$$\delta_{\rm CP}[B_d \to 2\eta'] + \delta_{\rm CP}[B_d \to \eta'\eta] + \delta_{\rm CP}[B_d \to \pi^0\eta'] + \delta_{\rm CP}[B_d \to 2\eta]$$
$$+\delta_{\rm CP}[B_d \to \pi^0\eta] + \delta_{\rm CP}[B_d \to 2\pi^0] + \delta_{\rm CP}[B_s \to 2\eta'] + \delta_{\rm CP}[B_s \to \eta'\eta]$$
$$+\delta_{\rm CP}[B_s \to 2\eta] + \delta_{\rm CP}[B_s \to 2\pi^0] = 0\,.$$

CP Sum Rules

No sum rules survive beyond leading order in SU(3) and isospin breaking in mass basis for $B \to PP$ or $B \to PV$. Best we can do are sum rules that should fail at $2\varepsilon \sim \mathcal{O}(40\%)$ (or $\mathcal{O}(2\%)$ for isospin). (Factor of 2 arises from naïve Taylor expansion)

U-spin

- $\Delta = 0$ arises from a U-spin sum rule.
- For charged mesons there are three such sum rules

$$\delta_{\mathrm{CP}}[B_d \to \pi^- K^+] + \delta_{\mathrm{CP}}[B_s \to \pi^+ K^-] = 0\,,$$
$$\delta_{\mathrm{CP}}[B_d \to \pi^- \pi^+] + \delta_{\mathrm{CP}}[B_s \to K^- K^+] = 0\,,$$
$$\delta_{\mathrm{CP}}[B_d \to K^- K^+] + \delta_{\mathrm{CP}}[B_s \to \pi^- \pi^+] = 0\,.$$

- Expect failure at $2\varepsilon \sim \mathcal{O}(40\%)$.

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$$\delta_{\mathrm{CP}}[B_d \to K^- K^+] + \delta_{\mathrm{CP}}[B_s \to \pi^- \pi^+] = 0\,.$$

- Expect failure at $2\varepsilon \sim \mathcal{O}(40\%)$.
- In this language, see Δ carries arbitrary normalizations

$$\Delta \equiv \frac{\overline{\Gamma}[B_s \to \pi^+ K^-]}{\overline{\Gamma}[B_d \to \pi^- K^+]} \left[\frac{\mathcal{P}_{(B_d;\pi^- K^+)}\,\delta_{\mathrm{CP}}[B_d \to \pi^- K^+]}{\mathcal{P}_{(B_s;\pi^+ K^-)}\,\delta_{\mathrm{CP}}[B_s \to \pi^+ K^-]} + 1\right]$$
$$\simeq \frac{2\varepsilon}{4} \sim 10\%.$$

Consistent with data; U-spin breaking scale for Δ is suppressed.

U-spin Parameters

To avoid arbitrary normalization, define the parameters

$$\widetilde{\Delta} \equiv \frac{\delta_{\mathrm{CP}}[B_d \to K^+\pi^-] + \delta_{\mathrm{CP}}[B_s \to K^-\pi^+]}{\delta_{\mathrm{CP}}[B_d \to K^+\pi^-] - \delta_{\mathrm{CP}}[B_s \to K^-\pi^+]}\,,$$

$$\widetilde{\Delta}' \equiv \frac{\delta_{\mathrm{CP}}[B_s \to K^+K^-] + \delta_{\mathrm{CP}}[B_d \to \pi^+\pi^-]}{\delta_{\mathrm{CP}}[B_s \to K^+K^-] - \delta_{\mathrm{CP}}[B_d \to \pi^+\pi^-]}\,,$$

$$\widetilde{\Xi} \equiv \frac{\delta_{\mathrm{CP}}[B_d \to K^+K^-] + \delta_{\mathrm{CP}}[B_s \to \pi^+\pi^-]}{\delta_{\mathrm{CP}}[B_d \to K^+K^-] - \delta_{\mathrm{CP}}[B_s \to \pi^+\pi^-]}$$

U-spin breaking with its canonical magnitude predicts

$$\widetilde{\Delta} \sim \widetilde{\Delta}' \sim \widetilde{\Xi} \sim \mathcal{O}(\varepsilon)\,.$$

Recent data provides

$$\widetilde{\Delta} = 0.026 \pm 0.106\,, \qquad \text{and} \qquad \widetilde{\Delta}' = 0.40 \pm 0.34\,.$$

Factorization Picture

In QCD (BBNS) factorization obtain relations between CP parameters.

$$\widetilde{\Delta}' \simeq \left[\left(\frac{f_K}{f_\pi}\right)^4 \frac{1-\widetilde{\Delta}}{1+\widetilde{\Delta}} - 1\right] \Big/ \left[\left(\frac{f_K}{f_\pi}\right)^4 \frac{1-\widetilde{\Delta}}{1+\widetilde{\Delta}} + 1\right] .$$

or for SCET (BPRS)

$$\widetilde{\Delta}' \simeq \left[\left(\frac{f_K}{f_\pi}\right)^2 \frac{1-\widetilde{\Delta}}{1+\widetilde{\Delta}} - 1\right] \Big/ \left[\left(\frac{f_K}{f_\pi}\right)^2 \frac{1-\widetilde{\Delta}}{1+\widetilde{\Delta}} + 1\right] .$$

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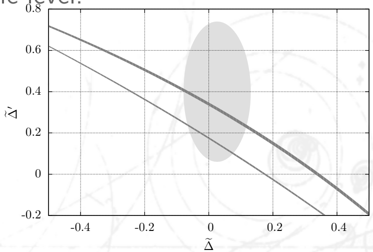
Can test factorization pictures at sum rule level:



Both factorization relations consistent with data, for the moment.

Summary

- Parameters like Δ carry arbitrary decay rate and phase space normalizations, so not ideal choice to parameterize U-spin breaking. We advocate better-defined parameters $\widetilde{\Delta}$, $\widetilde{\Delta}'$ and $\widetilde{\widetilde{\Xi}}$.
- Sum rules probe SM, while factorizing out much of the unknown structure of any single amplitude. We computed the complete set of SU(3) relations at leading and first order in flavor symmetry breaking (also done for charm decays).
- The QCD (BBNS) and SCET (BPRS) factorization pictures are consistent with data: look to more precision and $\widetilde{\widetilde{\Xi}}$ to test this further.

Thank You!