

The CKM unitarity triangle angle γ at LHCb (Experiment)



Paolo Gandini

Syracuse University

on behalf of the LHCb collaboration



Implications of LHCb measurements and future prospects

CERN, 15th October 2013

Outline

As the angle γ is the least experimentally constrained parameter of the UT
The precision measurement of γ is one of the main goals
of the LHCb experiment

- **γ with trees**

- Introduction
- Combination formalism
- Updated Results

$B \rightarrow DK$ & $B \rightarrow D\pi$
Including D mixing
GGSZ with 3fb-1

- **γ with loops**

- Introduction
- Preliminary results

Bayesian approach
Updated with latest
 $B \rightarrow hh$ LHCb results

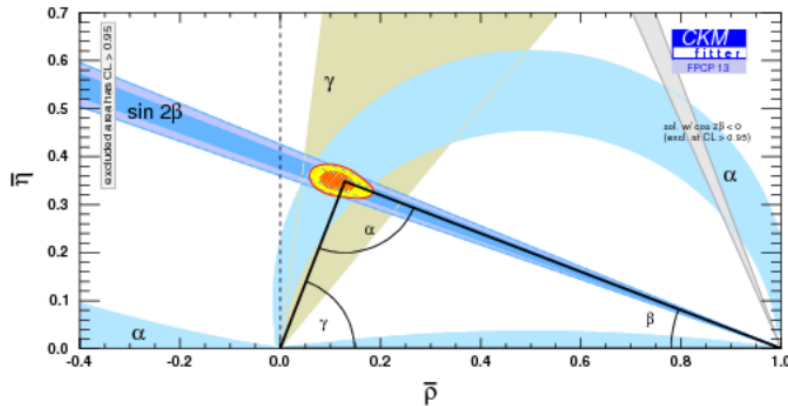
will not cover three-body charmless decays

Most results are preliminary

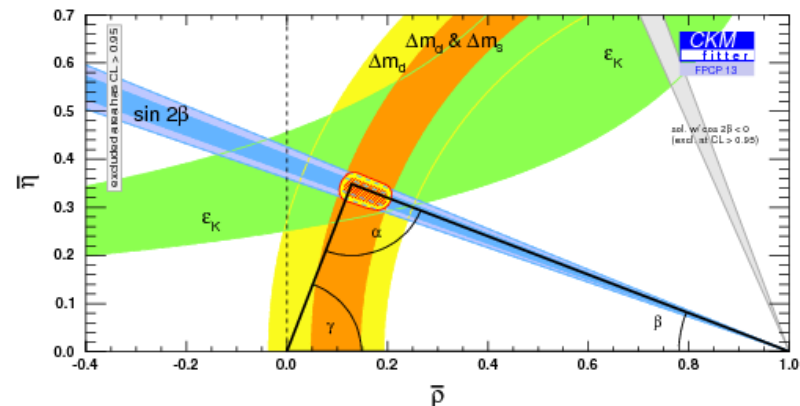
Global Combinations

Courtesy of CKMfitter
(updated August 6, 2013, FPCP)

"Tree" quantities



"Loop" quantities



We present here efforts to extract γ in different ways/many approach
Aim is to generate some discussion
I will not go into many details of the individual analysis

γ from trees

$B \rightarrow DK \text{ \& } B \rightarrow D\pi$

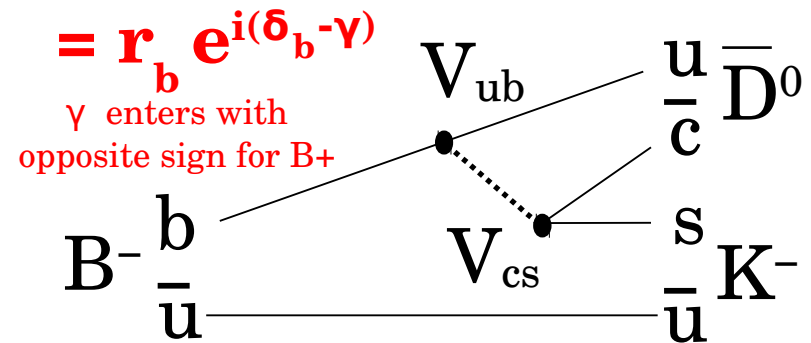
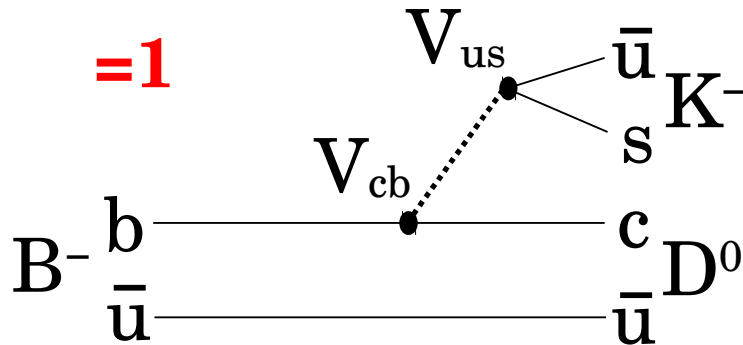
*LHCb has published a combination with a frequentist approach
Work ongoing with a Bayesian approach*

Introduction

$$\gamma = \arg[-V_{ud}V_{ub}^*/(V_{cd}V_{cb}^*)]$$

*From the first and
third column of CKM*

Idea: tree level determination of γ using $B^\pm \rightarrow DK^\pm$ decays
 No contribution from penguins \rightarrow Theoretically clean
 Negligible theoretical uncertainty $\delta\gamma/\gamma \sim O(10^{-7})$



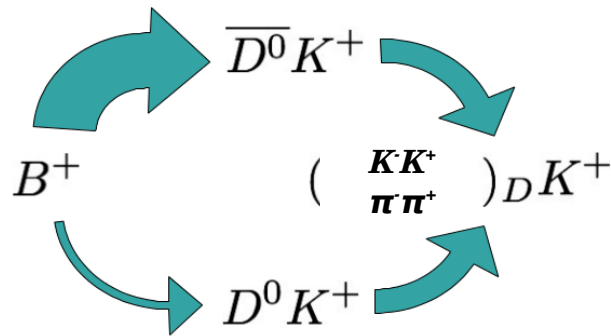
In principle one can use $B \rightarrow D\pi$ as well
 Same formalism \rightarrow add little sensitivity
 Now we fully include $B \rightarrow D\pi$ in the combination

$B^\pm \rightarrow Dh^\pm$ formalism

Exploit interference: D^0 and \bar{D}^0 must decay to the same final state

GLW

Gronau, London, Wyler
Phys. Lett. B 265 17 (1991)

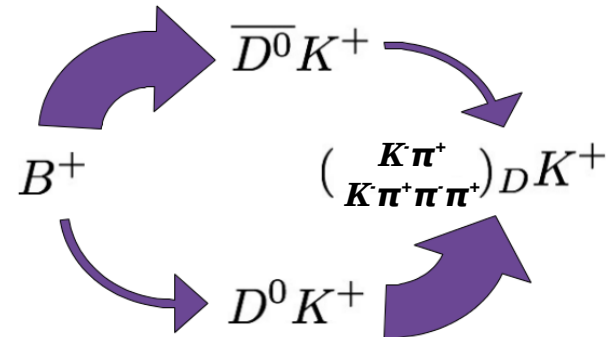


CP eigenstates like K^+K^- and $\pi^+\pi^-$
 Colour FAVOURED } Interference $O(10\%)$
 Colour SUPPRESSED }

$D \rightarrow KK$
 $D \rightarrow \pi\pi$

ADS

Atwood, Dunietz, Soni
Phys.Rev.Lett. 78 (1997)



FAV followed by SUP
 Reverse suppression leads to comparable amplitudes contributing to the decay } Large interference

$D \rightarrow K\pi$
 $D \rightarrow K\pi\pi\pi$

GGSZ

Giri, Grossman,
Soffer, Zupan
Phys. Rev. D68 (2003)
054018

- 3-body decays: $D \rightarrow K_s\pi\pi, K_sKK$
- Interference on the Dalitz plot exploited
- Strong phase δ_D varies on Dalitz space

Other modes can be considered as well, but not included yet in the combination

Strategy and Observables

- General Idea: measure as many γ -related quantities as possible
- Experimental observables are:
- **Charge asymmetries and yield ratios** (many systematics cancel)

$$\text{DK/D}\pi \text{ yield ratio} \quad R_{K/\pi}^f = \frac{\Gamma(B^- \rightarrow D[\rightarrow f]K^-) + \Gamma(B^+ \rightarrow D[\rightarrow f]K^+)}{\Gamma(B^- \rightarrow D[\rightarrow f]\pi^-) + \Gamma(B^+ \rightarrow D[\rightarrow f]\pi^+)}$$

$$\text{Charge asymmetries} \quad A_h^f = \frac{\Gamma(B^- \rightarrow D[\rightarrow f]h^-) - \Gamma(B^+ \rightarrow D[\rightarrow f]h^+)}{\Gamma(B^- \rightarrow D[\rightarrow f]h^-) + \Gamma(B^+ \rightarrow D[\rightarrow f]h^+)}$$

-
- Also one can define **Suppressed/Favoured decay ratio**

$$R_h^\pm = \frac{\Gamma(B^\pm \rightarrow D[\rightarrow f_{\text{sup}}]h^\pm)}{\Gamma(B^\pm \rightarrow D[\rightarrow f]h^\pm)}$$

$$= \frac{r_f^2 + (r_B^h)^2 + 2r_B^h r_f \kappa \cos(\delta_B^h + \delta_f \pm \gamma)}{1 + (r_B^h r_f)^2 + 2r_B^h r_f \kappa \cos(\delta_B^h - \delta_f \pm \gamma)}$$

r, δ, κ

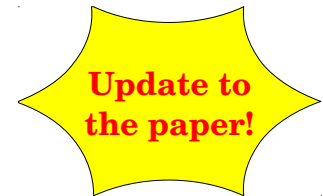
Extra hadronic parameters
depend on the B and D final state

Frequentist Combination

- Inputs combined to extract γ
- System of equations \rightarrow **solvable if enough equations are considered**
- Measured quantities used in the combination:

B \rightarrow Dh decays	
2body ADS/GLW:	D \rightarrow [Kπ,KK,$\pi\pi$]
4body ADS:	D \rightarrow [K$\pi\pi\pi$]
3body GGSZ:	D \rightarrow [K$_s\pi\pi$,K$_sKK$]

2011 (1 fb⁻¹)
2011 (1 fb⁻¹)
2011 + 2012 (3 fb⁻¹)



- Adopt a frequentist approach (Plugin)
- Assume almost Gaussian observables
- Gaussian systematic uncertainties
- Correct for undercoverage of the method

Phys Lett B 726 (2013) 1-3 151

Strategy

Combine all measurement in a likelihood

$$L(r_B^K, \delta_B^K, \gamma) = \exp \left(-\frac{1}{2} [\vec{x}_{\text{meas}} - \vec{x}_{\text{true}}]^T \mathbf{V}_{\text{TOT}}^{-1} [\vec{x}_{\text{meas}} - \vec{x}_{\text{true}}] \right)$$

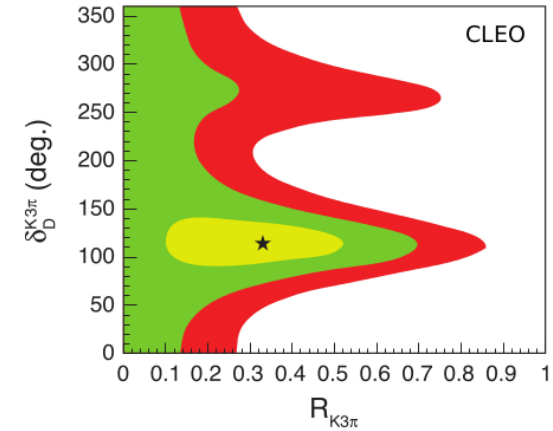
$$\mathbf{V}_{\text{TOT}} = \mathbf{V}_{\text{stat}} + \mathbf{V}_{\text{sys}}$$

Constraints

- Include external constraints in the combination:

CLEO measurements of
 $D \rightarrow hh, K\pi\pi$ systems

Phys. Rev. D80 (2009) 031105



- CP violation in the decays $D \rightarrow KK$ or $D \rightarrow \pi\pi$ affects the GLW equations
- Modify the GLW asymmetries, leave the ratios unchanged

$$\left. \begin{aligned} A_{CP}^{\text{dir}}(KK) &= (-0.31 \pm 0.24) \times 10^{-2} \\ A_{CP}^{\text{dir}}(\pi\pi) &= (+0.36 \pm 0.25) \times 10^{-2} \end{aligned} \right\}$$

HFAG

$$A^{KK} = \frac{2r_B \sin \delta_B \sin \gamma}{1 + (r_B)^2 + 2r_B \cos \delta_B \cos \gamma} + A_{CP}^{\text{dir}}(KK)$$

- D^0 mixing is now fully considered in the B decay (and in the D params)
- Included in the published paper (all analyses with 1fb-1)

Combination rerun with latest results available

Including D^0 mixing in the formalism

More details in
arXiv:1307.4384

- Not going into the details of the calculations
- Same formalism \rightarrow need to add corrections in the equations (extra terms)
- It can be shown that:
 - Effect is negligible for the model-independent GGSZ method
 - Effect is negligible for GLW method (corrections cancel in A_{cp} and are suppressed in double ratio)
 - **It affects the ADS method using $B \rightarrow DK$ at the 10% level**
 - **It affects the ADS method using $B \rightarrow D\pi$ at the 100% level**
- Combination is corrected:
 - Fully considering D mixing in $B \rightarrow DK$ and $B \rightarrow D\pi$ GLW/ADS
 - **Considering the D decay time resolution and acceptance in LHCb**
 - **Using LHCb results on D mixing [Phys. Rev. Lett. 110 (2012) 101802]**

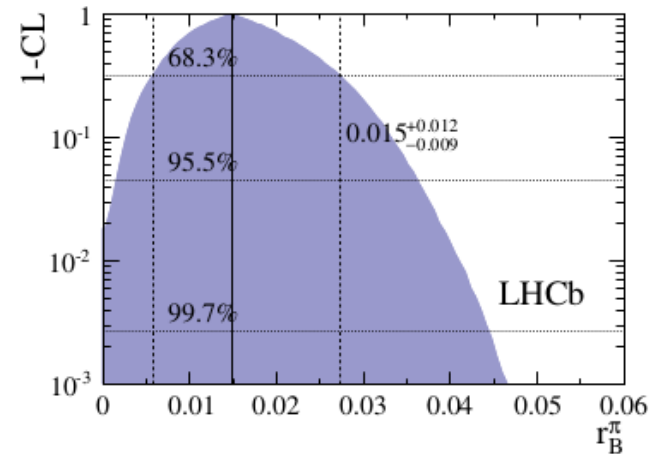
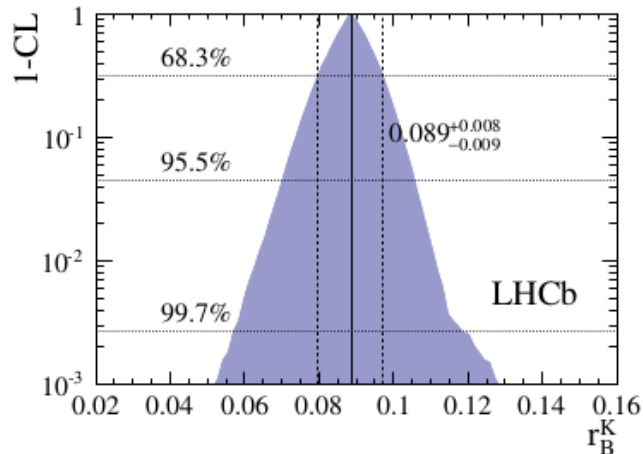
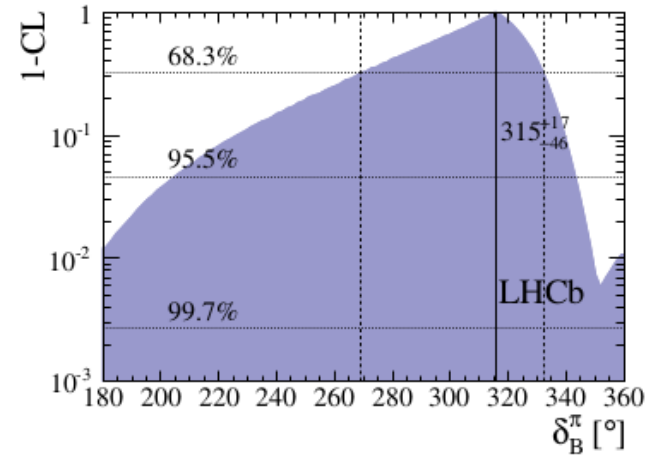
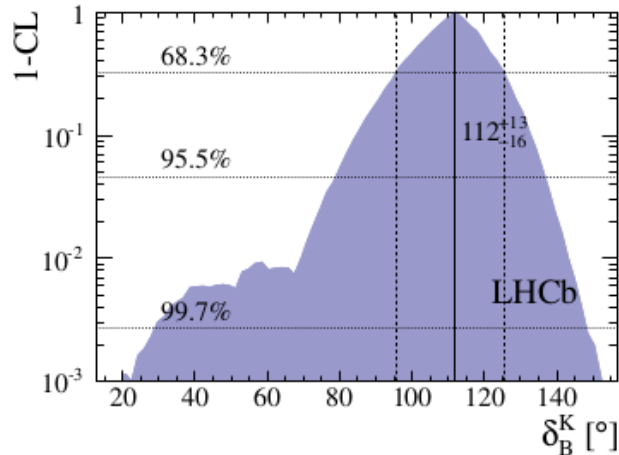
$$R_h^\pm = \frac{\Gamma(B^\pm \rightarrow D[\rightarrow f_{\text{sup}}]h^\pm)}{\Gamma(B^\pm \rightarrow D[\rightarrow f]h^\pm)}$$

$$= \frac{r_f^2 + (r_B^h)^2 + 2r_B^h r_f \kappa \cos(\delta_B^h + \delta_f \pm \gamma) - 2[M_\pm^h]_{\text{sup}}}{1 + (r_B^h r_f)^2 + 2r_B^h r_f \kappa \cos(\delta_B^h - \delta_f \pm \gamma) + 2M_\pm^h}$$

*Same Formulas
get extra
correction factors*

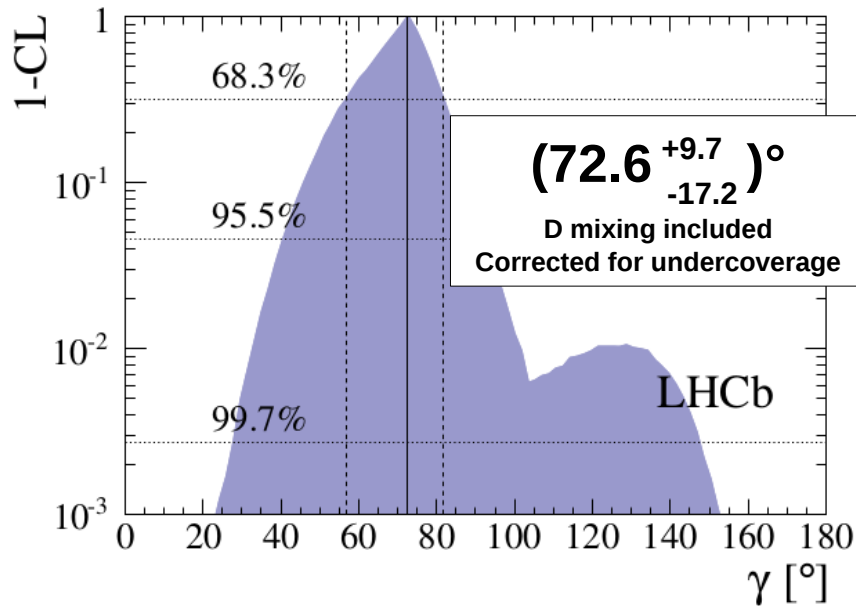
Results: $B \rightarrow DK$ & $B \rightarrow D\pi$

- we include $B \rightarrow D\pi$ into a γ measurement
- First experimental γ combination that takes D mixing into account
- Published 1fb^{-1} result [Phys Lett B 726 \(2013\) 151](#)

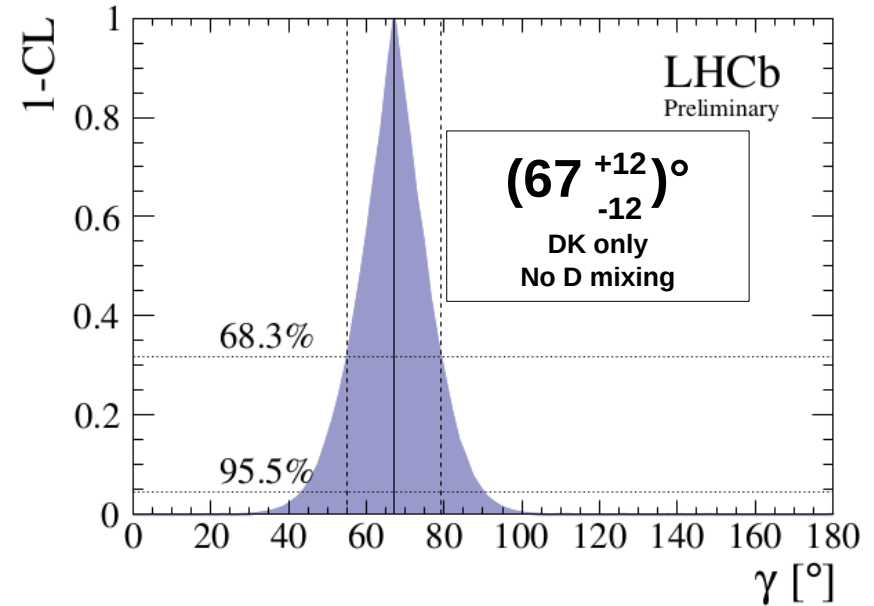


Results: $B \rightarrow DK$ & $B \rightarrow D\pi$

Paper Phys Lett B 726 (2013) 151



LHCb-CONF-2013-006



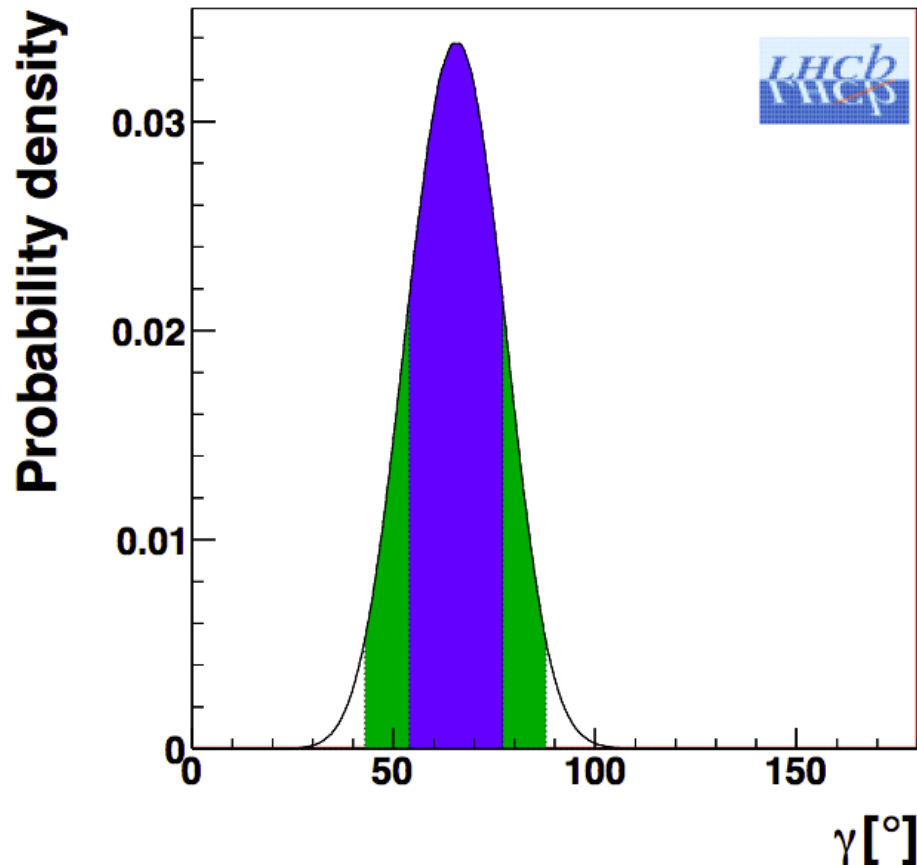
- LHCb has a complete set of 1fb^{-1} results: GLW, ADS, GGSZ
- GGSZ only is updated with 3fb^{-1} dataset (DK combo only)
- D^0 mixing is now included in the full formalism
- Impressive agreement with BaBar and Belle so far
- Expect updates of many analyses with full 3fb^{-1} dataset

BaBar $\gamma = (69-16+17)^\circ$
Phys. Rev. D 87, 052015 (2013)

Belle $\gamma = (68-14+15)^\circ$
arXiv:1301.2033

Bayesian Approach

- Work ongoing on a Bayesian combination for GWT
- To cross-check the combination...
- In general the agreement is encouraging, but some discrepancies must be understood
- In the DK system → very compatible results



**VERY
PRELIMINARY**

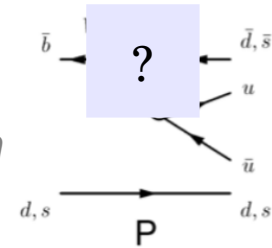
$(66^{+11}_{-12})^\circ$

DK only
No D mixing

γ from loops

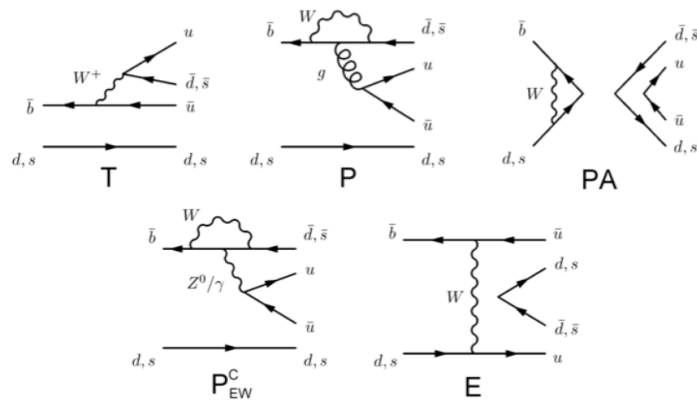
$B \rightarrow hh$

*Work ongoing with a Bayesian approach
Comparison with GWT is important...*



Bayesian Combination: $B \rightarrow hh$

- Work ongoing for a Bayesian combination in LHCb
 - Mainly use charmless two body decays $B \rightarrow h^+h^-$
 - γ can receive contributions from penguin diagrams
 - Use U-spin symmetry to better constraint the system
-
- Several contributions: tree, strong/weak penguins, annihilation, exchange



Decay mode	Contributing diagrams
$B^0 \rightarrow \pi^+\pi^-$	T, P, PA, P_{EW}^C, E
$B^0 \rightarrow K^+\pi^-$	T, P, P_{EW}^C
$B_s^0 \rightarrow \pi^+K^-$	T, P, P_{EW}^C
$B_s^0 \rightarrow K^+K^-$	T, P, PA, P_{EW}^C, E
$B^0 \rightarrow K^+K^-$	PA, E
$B_s^0 \rightarrow \pi^+\pi^-$	PA, E

This talk will not cover time dependent analysis → see Denis' talk
 This talk will not cover three-body charmless decays → see Irina's talk

Methods (1)

• Fleischer method

- Uses $B^0 \rightarrow \pi^+\pi^-$ and $B_s \rightarrow K^+K^-$ decays
- BF and C,S (time dependent parameters)
- 6 observables, 9 unknowns $D, D', d, d', \theta, \theta', \beta, \beta_s, \gamma$
- U-spin \rightarrow reduces number of unknown
- Within factorization

$$d = d', \theta = \theta'$$

$$D'/D = 1.46 \pm 0.15$$

r_{fact}

Phys. Rev. D78 054015 (2008)

Phys. Lett. B459, 306 (1999)
Eur. Phys. J. C52, 267 (2007)

• Gronau-London method

- Uses $B_{u,d} \rightarrow \pi\pi$ (usual determination of $\gamma = \pi-\alpha-\beta$)
- BF and time dependent parameters
- Amplitudes related with isospin
- Can write system of equations as above, etc...

Phys. Rev. Lett. 65 (1990) 3381

Methods (2)

JHEP 10 (2012) 29

- Use approach proposed by Silvestrini et al.
- Combination of Gronau-London isospin analysis & Fleischer U-spin analysis
- Greatly improves sensitivity compared to CL alone

$$C_{\pi^+\pi^-} = -\frac{2d \sin(\vartheta) \sin(\gamma)}{1 - 2d \cos(\vartheta) \cos(\gamma) + d^2},$$

$$S_{\pi^+\pi^-} = -\frac{\sin(2\beta + 2\gamma) + 2d \cos(\vartheta) \sin(2\beta + \gamma) + d^2 \sin(2\beta)}{1 - 2d \cos(\vartheta) \cos(\gamma) + d^2},$$

$$\mathcal{B}_{\pi^+\pi^-} = F(B^0) D^2 (1 - 2d \cos(\vartheta) \cos(\gamma) + d^2),$$

$$C_{\pi^0\pi^0} = -\frac{2dT \sin(\vartheta_T - \vartheta) \sin(\gamma)}{T^2 + 2dT \cos(\vartheta_T - \vartheta) \cos(\gamma) + d^2},$$

$$\mathcal{B}_{\pi^0\pi^0} = F(B^0) \frac{D^2}{2} (T^2 + 2dT \cos(\vartheta_T - \vartheta) \cos(\gamma) + d^2),$$

$$C_{K^+K^-} = \frac{2\tilde{d}' \sin(\vartheta') \sin(\gamma)}{1 + 2\tilde{d}' \cos(\vartheta') \cos(\gamma) + \tilde{d}'^2},$$

$$S_{K^+K^-} = -\frac{\sin(-2\beta_s + 2\gamma) + 2\tilde{d}' \cos(\vartheta') \sin(-2\beta_s + \gamma) + \tilde{d}'^2 \sin(-2\beta_s)}{1 + 2\tilde{d}' \cos(\vartheta') \cos(\gamma) + \tilde{d}'^2}$$

$$\mathcal{B}_{\pi^+\pi^0} = F(B^+) \frac{D^2}{2} (1 + T^2 + 2T \cos(\vartheta_T)),$$

$$\mathcal{B}_{K^+K^-} = F(B_s^0) \frac{\lambda^2}{(1 - \lambda^2/2)^2} D'^2 (1 + 2\tilde{d}' \cos(\vartheta') \cos(\gamma) + \tilde{d}'^2),$$

- 9 observables
 - $C_{\pi^+\pi^-}$, $S_{\pi^+\pi^-}$, $\mathcal{B}_{\pi^+\pi^-}$, $C_{\pi^0\pi^0}$, $\mathcal{B}_{\pi^0\pi^0}$, $\mathcal{B}_{\pi^+\pi^0}$, $C_{K^+K^-}$, $S_{K^+K^-}$, $\mathcal{B}_{K^+K^-}$
- 11 unknowns (to be reduced using U-spin)
 - D , D' , d , d' , T , θ , θ' , θ_T , β , β_s , γ

Combined Fleischer+GL analysis

Inputs

Channel	$\mathcal{B} \times 10^{-6}$	C_f	S_f	$\rho(C_f, S_f)$	Reference
$B^0 \rightarrow \pi^+\pi^-$	—	$-0.25 \pm 0.08 \pm 0.02$	$-0.68 \pm 0.10 \pm 0.03$	-0.06	BaBar
$B^0 \rightarrow \pi^+\pi^-$	—	$-0.33 \pm 0.06 \pm 0.03$	$-0.64 \pm 0.8 \pm 0.03$	-0.10	Belle
$B^0 \rightarrow \pi^+\pi^-$	—	$-0.38 \pm 0.15 \pm 0.02$	$-0.71 \pm 0.13 \pm 0.02$	0.38	LHCb
$B^0 \rightarrow \pi^+\pi^-$	5.10 ± 0.19	—	—	—	HFAG
$B_s^0 \rightarrow K^+K^-$	24.50 ± 1.80	—	—	—	HFAG
$B_s^0 \rightarrow K^+K^-$	—	$0.14 \pm 0.11 \pm 0.03$	$0.30 \pm 0.12 \pm 0.04$	0.02	LHCb

Parameter	Value	Reference
$\sin(2\beta)$	0.682 ± 0.019	HFAG
$2\beta_s$	$-0.01 \pm 0.07 \pm 0.01$	LHCb

Parameters

Parameter	Parameter's range
D	$[0, 10^{-3}]$
d	$[0, 2]$
θ	$[-\pi, \pi]$
T	$[0, 1.5]$
θ_T	$[-\pi, \pi]$
γ	$[-\pi, \pi]$

- Parameterise U-spin breaking
- Use flat priors
- Extra parameter κ

$$D' = D \cdot r_{fact} \cdot r_D$$

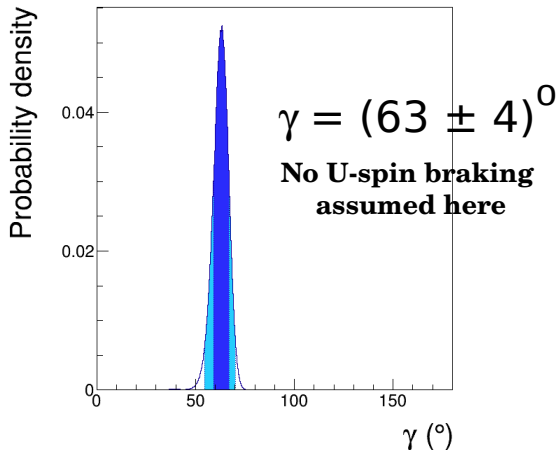
$$d' e^{i\theta'} = d e^{i\theta} (1 + r_d e^{i(r_d - \theta)})$$

Flat priors

$$r_D \in [1 - \kappa, 1 + \kappa]$$

$$r_d \in [0, \kappa]$$

$$r_\theta \in [-\pi, \pi]$$



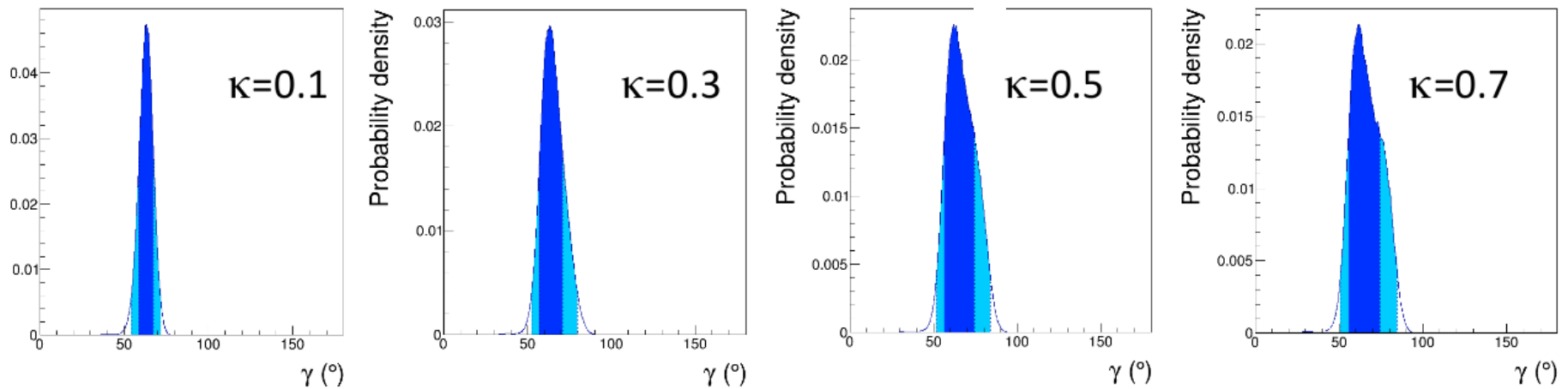
γ is determined with striking precision if no U-spin breaking is assumed

VERY PRELIMINARY

U-Spin Breaking

The parameter κ governs the amount of non-factorizable U-spin breaking allowed

- $\kappa=0$ means no U-spin breaking
- $\kappa=1$ means up to 100% U-spin breaking



Interesting Questions

- What is a reasonable estimate of how much U-spin is broken in these decays?
- Is it possible to give any quantitative prediction?
- Can we expect progress from theory on U-spin breaking in the years to come?

Conclusions

γ from trees: $B \rightarrow Dh$

Frequentist paper published using LHCb only
CONF note updated using 3fb-1 GGSZ analysis

Paper

$$(72.6^{+9.7}_{-17.2})^\circ$$

D mixing included
Corrected for undercoverage

CONF

$$(67^{+12}_{-12})^\circ$$

DK only
No D mixing

Impressive agreement with BaBar and Belle
Work ongoing on Bayesian combination \rightarrow comparison

γ from loops: $B \rightarrow hh$

Using a combination of Gronau-London and Fleischer methods, γ can be determined from loop-mediated decays

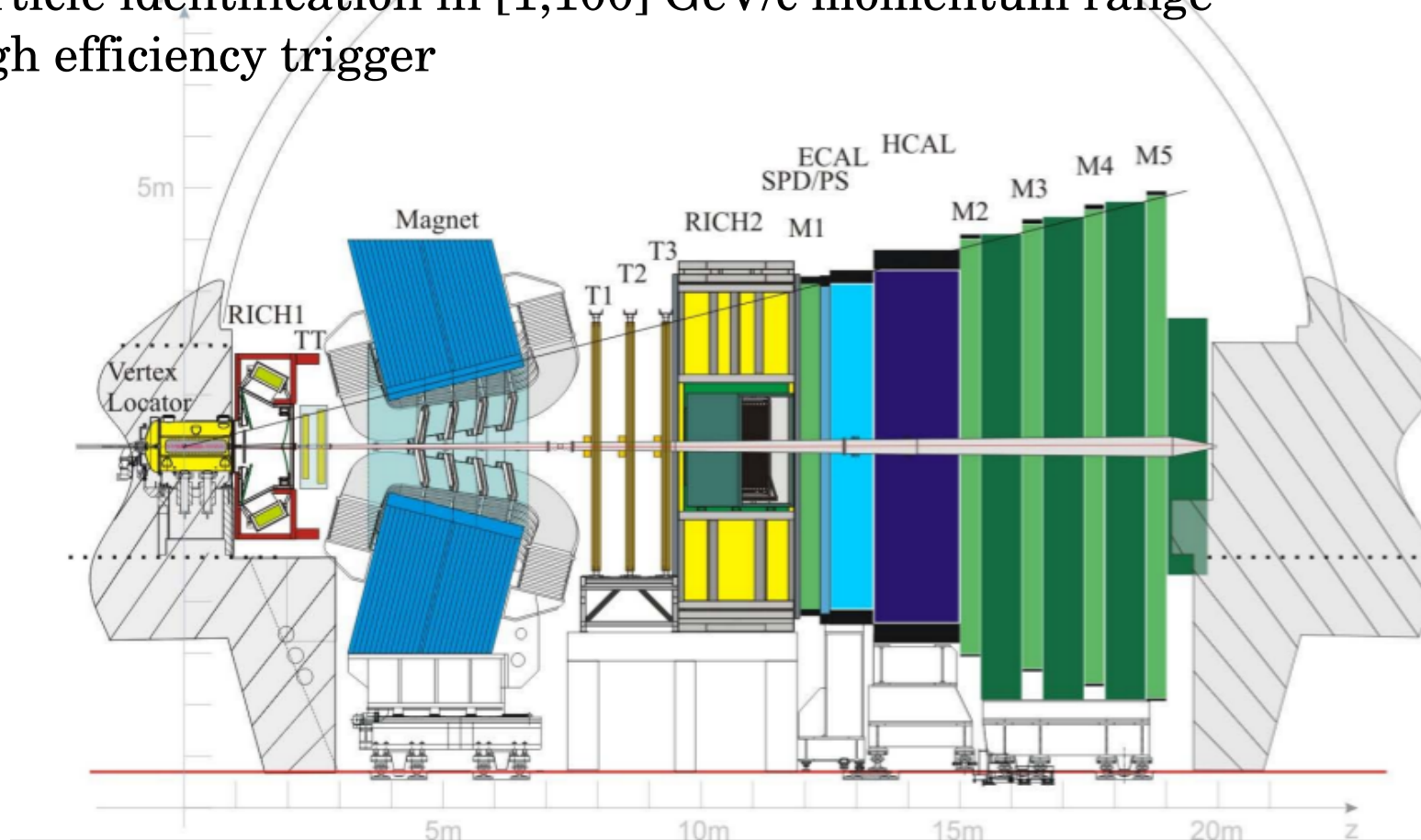
- A sensitivity on γ of about $\pm 10^\circ$ is obtained accounting for large (up to 90%) non-factorizable U-spin breaking
- About $\pm 4^\circ$ in case of no breaking

A vibrant green leaf with water droplets is shown floating on a surface of water. The leaf is reflected in the water below it. The text "Backup Slides" is overlaid on the image, centered between two horizontal lines.

Backup Slides

The LHCb experiment

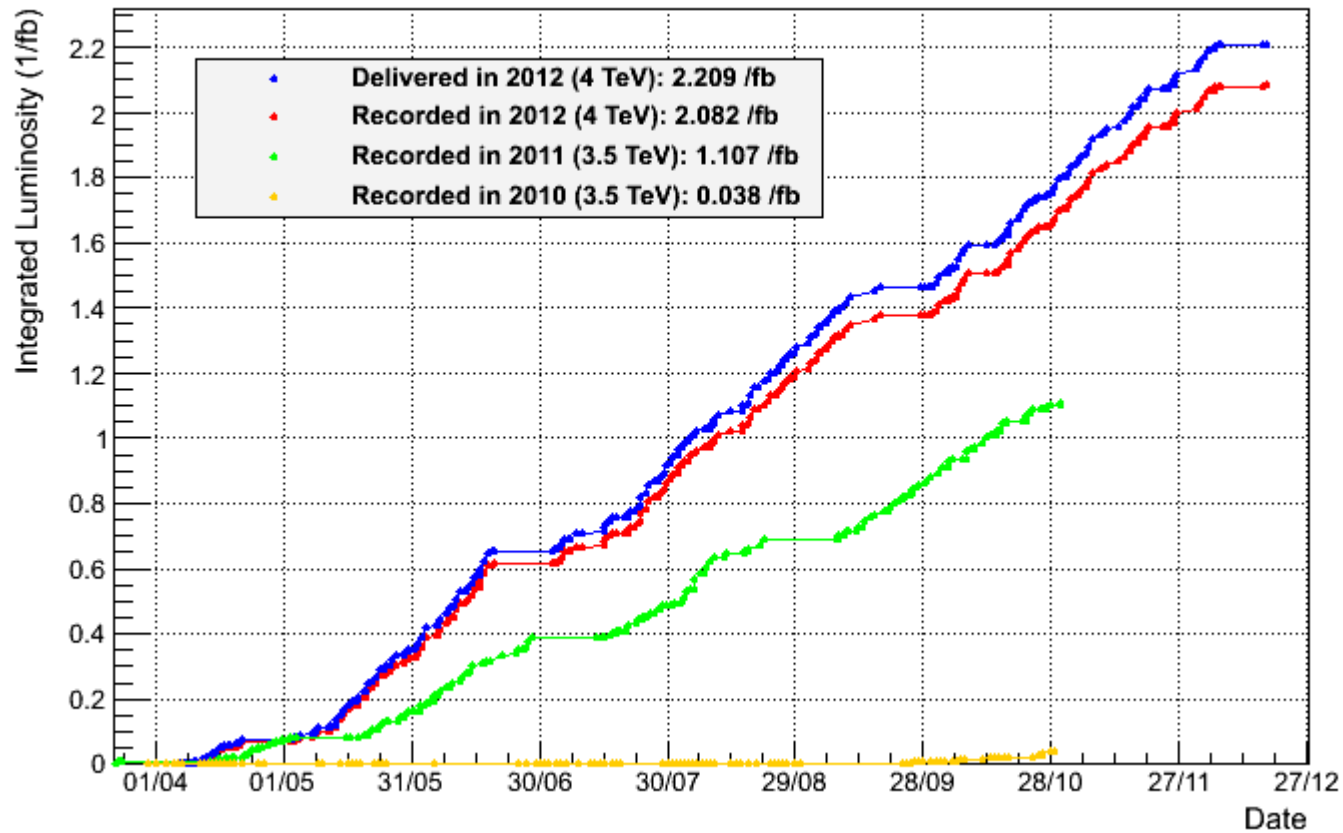
- Single-arm spectrometer (acceptance $1.9 < \eta < 4.9$)
- Precise primary and secondary vertex measurements
- Particle identification in $[1,100]$ GeV/c momentum range
- High efficiency trigger



Luminosity

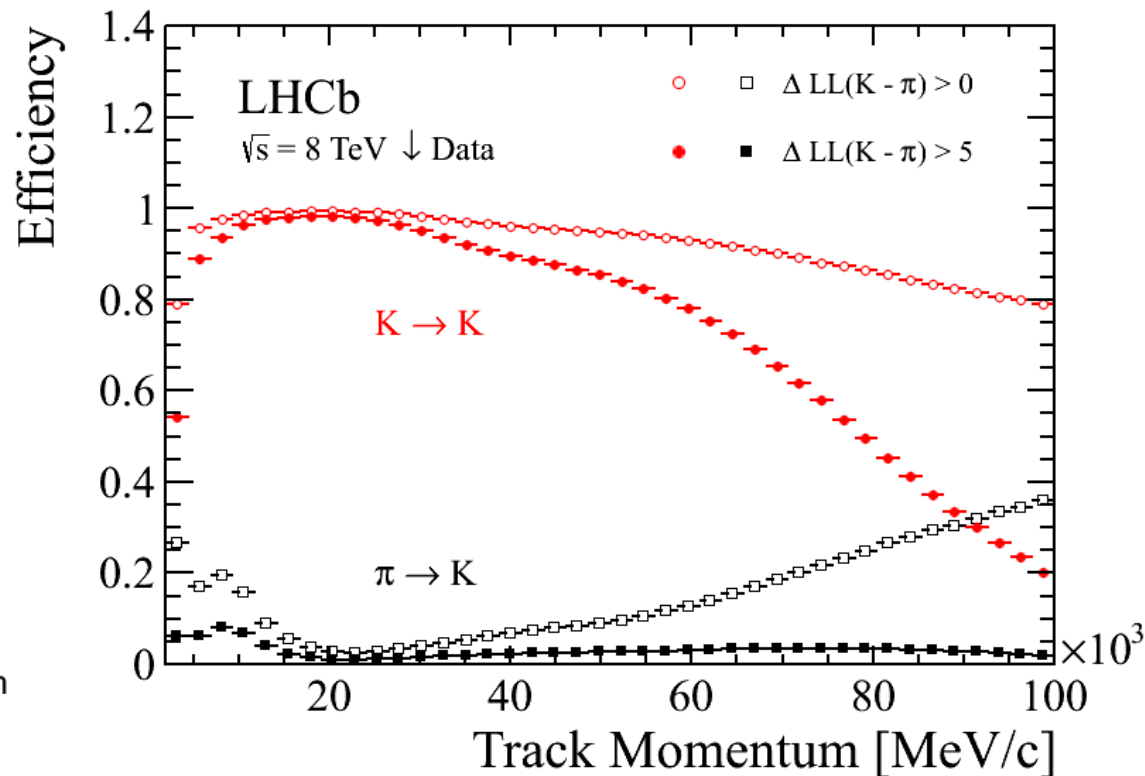
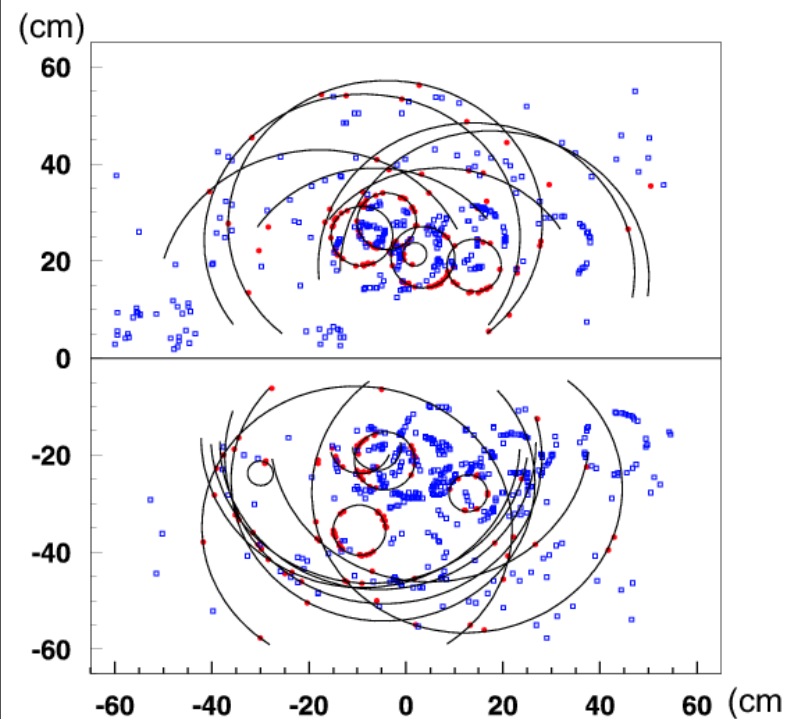
Collected (1.11 + 2.08) fb⁻¹

LHCb Integrated Luminosity pp collisions 2010-2012



Particle Identification

- Fundamental to separate pions and kaons ($B \rightarrow DK$ vs $B \rightarrow D\pi$)
- RICH detectors with 3 separate radiators
- Particle identification up to $p = 100\text{GeV}/c$



Multibody modes: coherence factor

- Multibody modes have the same formalism and equations of 2body decays
- Equations get extra factors due to the integration on the Dalitz plot
- Interference can only occur at same points in phase space
- The hadronic parameters become **functions of the phase space**
- It is common to introduce **effective quantities averaged over phase space**

$$r_{K3\pi}^2 = \frac{\int \bar{A}_D(\vec{m})^2 d\vec{m}}{\int A_D(\vec{m})^2 d\vec{m}}$$

$$\kappa_{K3\pi} e^{i\delta_{K3\pi}} = \frac{\int A_D(\vec{m}) \bar{A}_D(\vec{m}) e^{i\delta(\vec{m})} d\vec{m}}{\sqrt{\int \bar{A}_D(\vec{m})^2 d\vec{m} \times \int A_D(\vec{m})^2 d\vec{m}}}$$

$$R_{\pm} = (r_B^K)^2 + r_{K3\pi}^2 + 2 \kappa_{K3\pi} r_B^K r_{K3\pi} \cos(\pm\gamma + \delta_B^K + \delta_{K3\pi})$$

Coherence factor

Effective phase integrated
on the phase space

Plugin Method

Scan for one specific physics parameter, x : For example γ

1. Find global minimum χ_{\min}^2 and the most probable values for \vec{x} .
2. Fix x to x_0 and minimize with respect to the non-fixed parameters, i.e. obtain \vec{x}' , and $(\chi_{\min}^2)'$. Calculate $\Delta\chi^2 = \chi_{\min}^2 - (\chi_{\min}^2)'$.
3. Generate a Toy MC result for \vec{y} , \vec{y}_{toy} , by interpreting the likelihood as a PDF of \vec{y} .
4. Repeat the first two steps on the toy result, i.e. calculate $\Delta\chi_{\text{toy}}^2$.
5. Calculate $(1 - \text{CL})$ as the fraction

$$1 - \text{CL} = \frac{N(\Delta\chi_{\text{toy}}^2 > \Delta\chi^2)}{N_{\text{toy}}}. \quad (5)$$

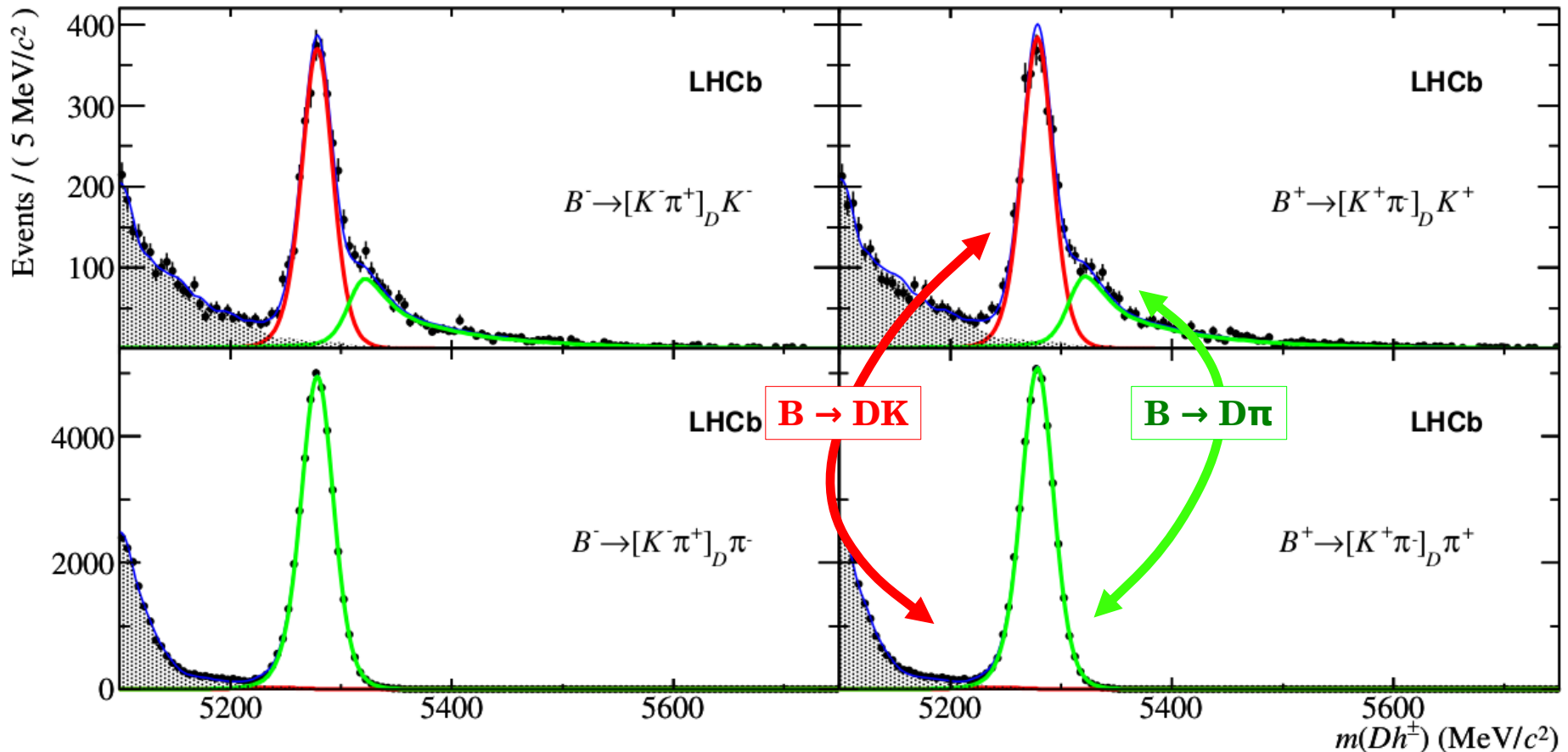
Use the best fit-values
values for the parameters
Can generate problem of
undercoverage

Inputs: $B^\pm \rightarrow [K\pi]_D h^\pm$ Favoured

Simultaneous fit over all modes
Data divided in PASS & FAIL slices

Phys. Lett. B 712 (2012), pp. 203-212

2011 data set (1 fb^{-1})



Inputs: $B^\pm \rightarrow [K\pi]_D h^\pm$ Suppressed

Simultaneous fit over all modes

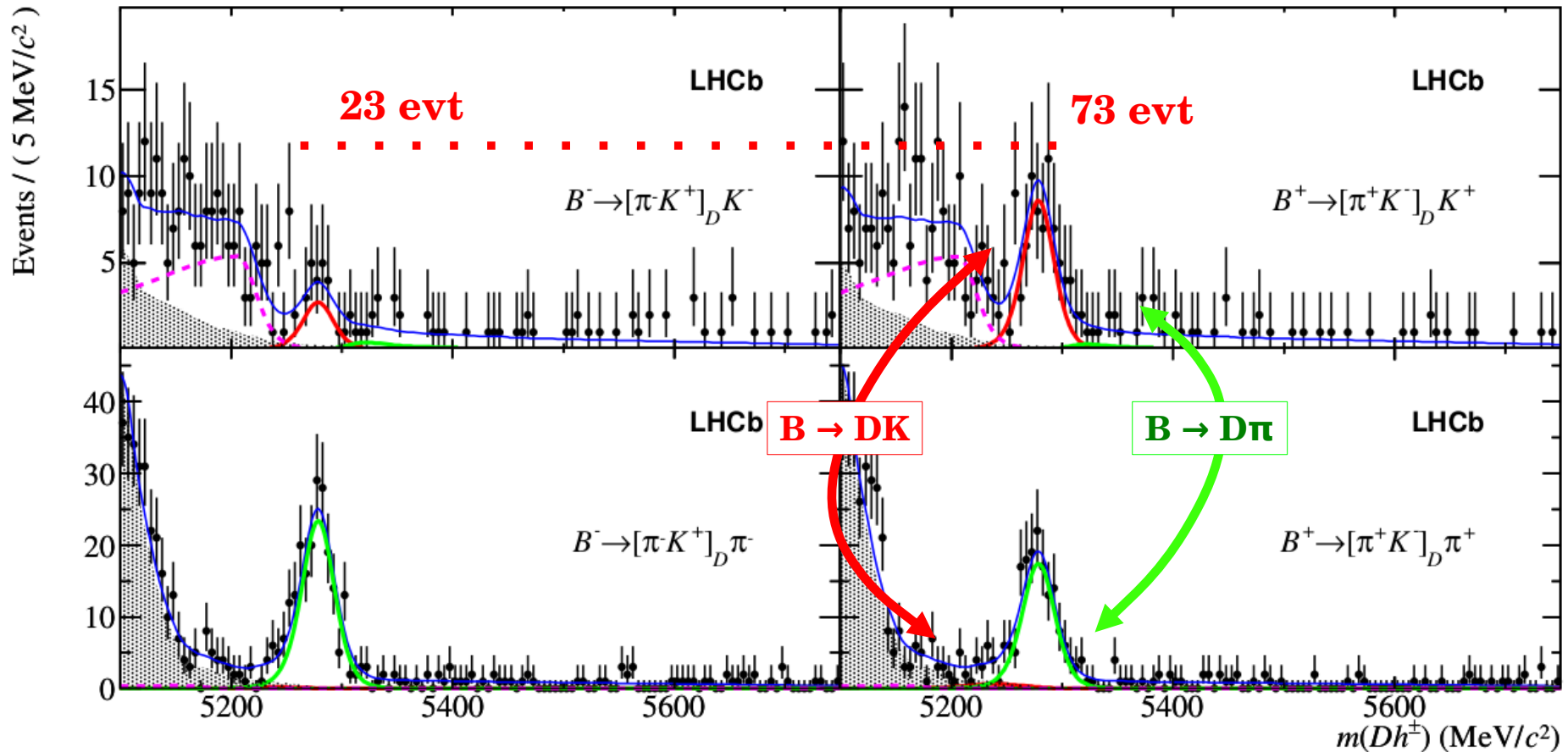
Data divided in PASS & FAIL slices

Here only suppressed $K\pi$ mode is shown

Benefit from the huge Cabibbo favoured mode

Phys. Lett. B 712 (2012), pp. 203-212

2011 data set (1 fb^{-1})

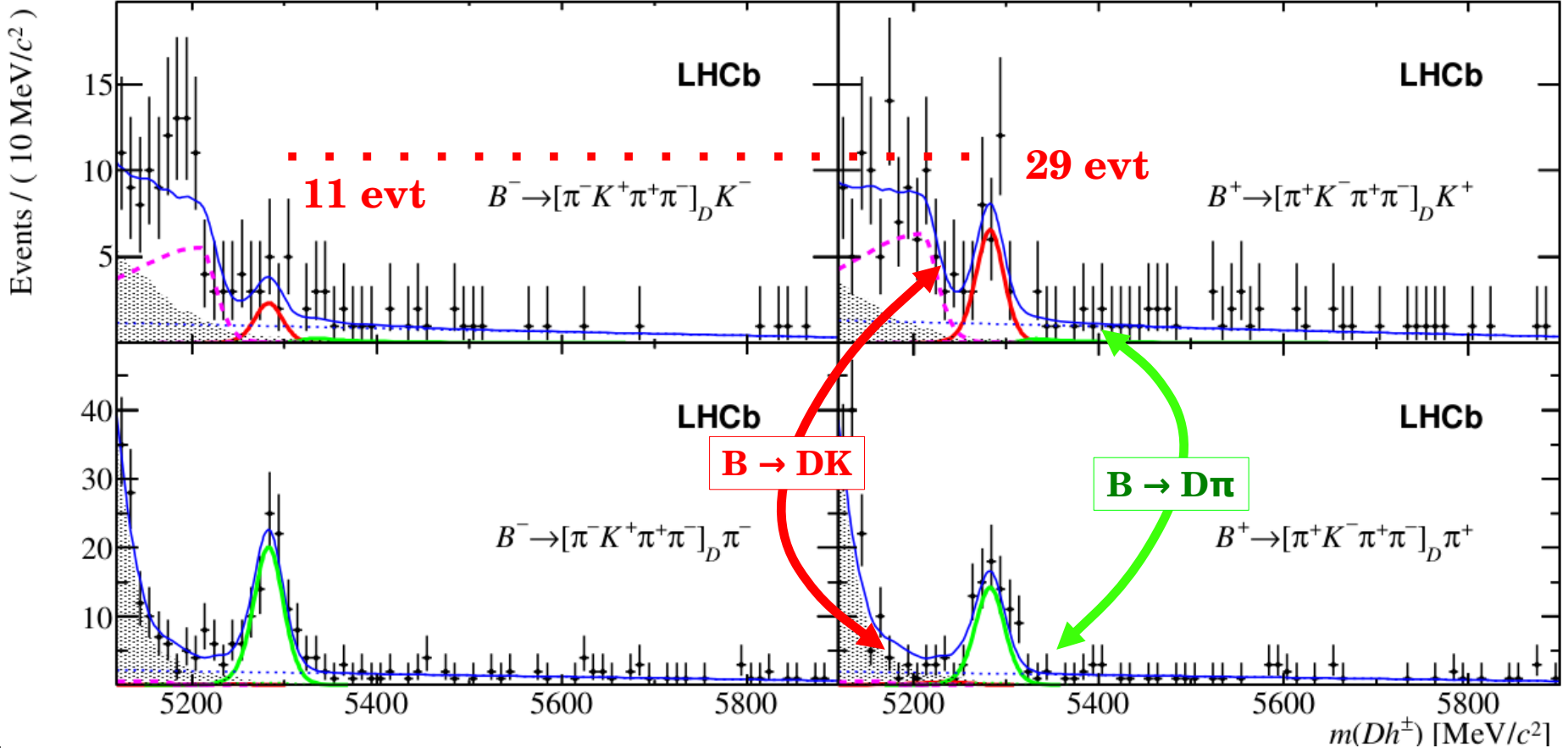


Inputs: $B^\pm \rightarrow [K\pi\pi\pi]_D h^\pm$

Simultaneous fit over all modes
Data divided in PASS & FAIL slices
First Observations

Phys Lett B 723 (2013) 44-53

2011 data set (1 fb⁻¹)



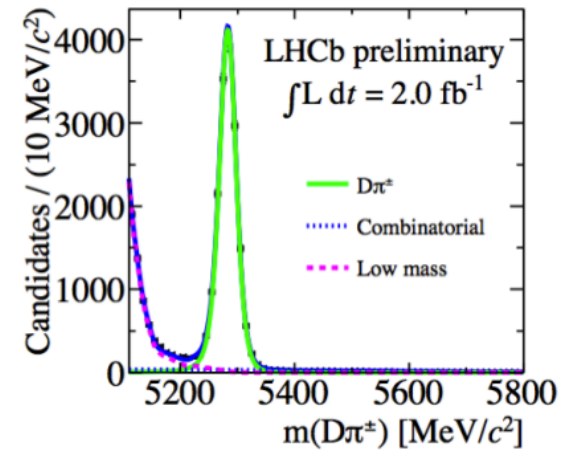
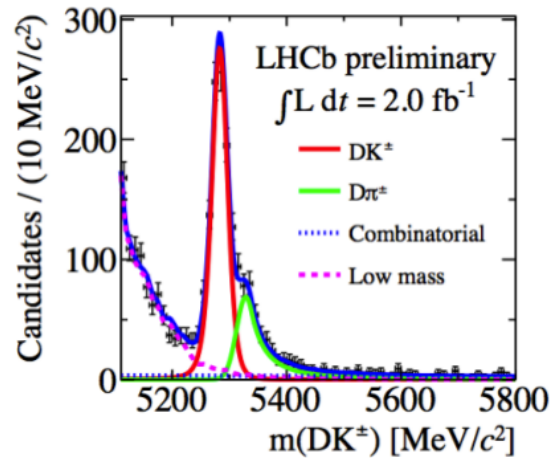
Inputs: $B^\pm \rightarrow [K_s h h]_D K^\pm$

LHCb-CONF-2013-004

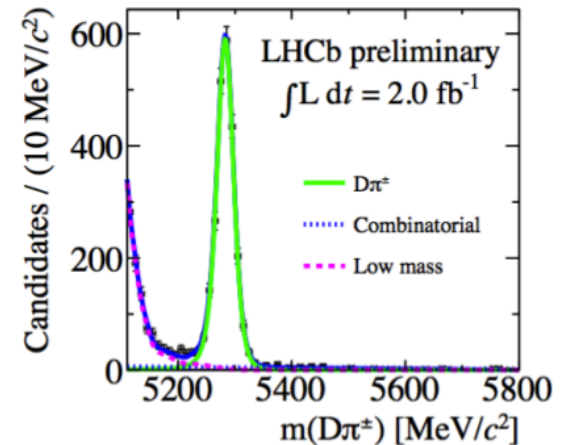
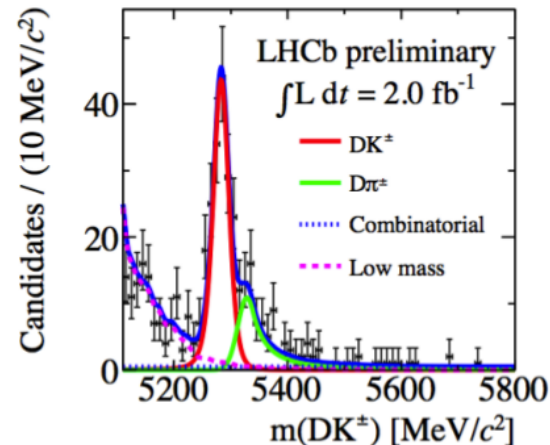
- Consider self-conjugate 3-body final states of the D
- Dependence of the strong phase on the Dalitz plot
- Phase variation measured by CLEO (used as input)

2011 + 2012
data set (3 fb^{-1})

$B^\pm \rightarrow [K_s \pi \pi]_D h^\pm$
2012 only shown

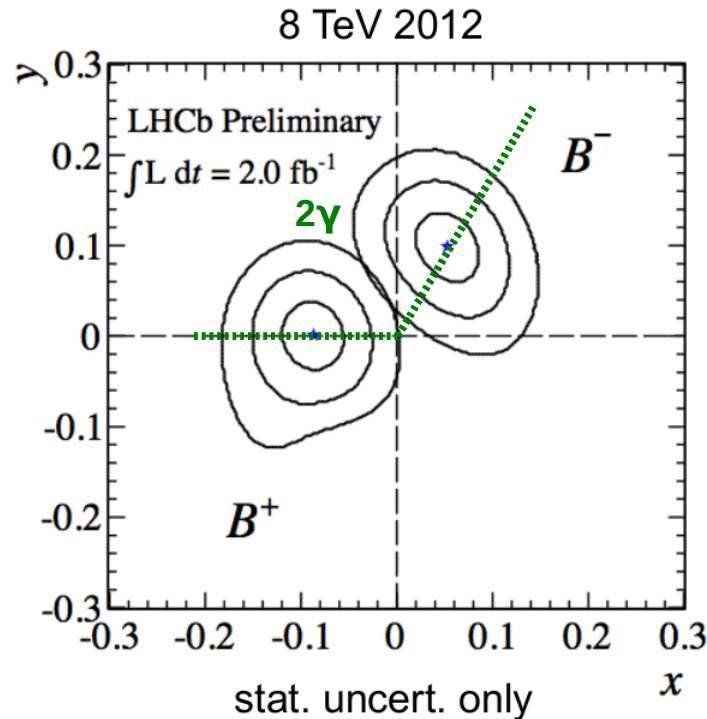


$B^\pm \rightarrow [K_s \text{KK}]_D h^\pm$
2012 only shown



Inputs: $B^\pm \rightarrow [K_s h h]_D K^\pm$

- Efficiency on the Dalitz determined comp the D yield to the model prediction in $B \rightarrow D\pi$
- $B \rightarrow D\pi$ used as a control sample: assume no CPV (systematic assigned)
- Density information \rightarrow used to estimate reco efficiency
- The efficiency is fed into the equations that determine CP quantities (part of the fit)



LHCb-CONF-2013-004

$$x_{\pm} = r_B \cos(\delta_B \pm \gamma)$$

$$y_{\pm} = r_B \sin(\delta_B \pm \gamma)$$

independent measurement of $\gamma = (57 \pm 16)^\circ$

Including D^0 mixing in the formalism

- Amplitudes now consider D mixing:

$$A(D^0(t) \rightarrow f) = g_+(t)A_f + q/p g_-(t)\bar{A}_f$$

$$A(\bar{D}^0(t) \rightarrow f) = g_+(t)\bar{A}_f + p/q g_-(t)\bar{A}_f$$

$$g_{\pm} = \frac{1}{2} \left(e^{-im_1 t - \frac{1}{2}\Gamma_1 t} \pm e^{-im_2 t - \frac{1}{2}\Gamma_2 t} \right)$$

- Neglecting terms of $O(x^2, y^2)$, assuming $p/q = 1$ and integrating for $(t=0, \infty)$, Amplitudes are:

$$\int_0^{\infty} |A(B^- \rightarrow D(\rightarrow f, t)h^-)|^2 dt = |A_D|^2 |\bar{A}_f|^2 \times \left[\begin{array}{l} \boxed{\left((r_B^h)^2 + r_f^2 + 2r_B^h \kappa_f r_f \cos(\delta_B^h + \delta_f - \gamma) \right) I_+} \\ \text{zero at } O(x^2, y^2) + \left(1 + (r_B^h)^2 r_f^2 + 2r_B^h \kappa_f r_f \cos(\delta_B^h + \delta_f - \gamma) \right) I_- \\ \text{Corrections } O(x, y) + 2 \left(\kappa_f r_f ((r_B^h)^2 + 1) \cos \delta_f + r_B^h (1 + r_f^2) \cos(\delta_B^h - \gamma) \right) I_y \\ - 2 \left(\kappa_f r_f ((r_B^h)^2 - 1) \sin \delta_f + r_B^h (1 - r_f^2) \sin(\delta_B^h - \gamma) \right) I_x \end{array} \right] \left. \vphantom{\int_0^{\infty}} \right\} \begin{array}{l} I_+ \equiv 1 \\ I_y = -M_{xy} x_D / \Gamma \\ I_- = 0, \\ I_x = M_{xy} y_D / \Gamma \end{array}$$

Gronau-London analysis

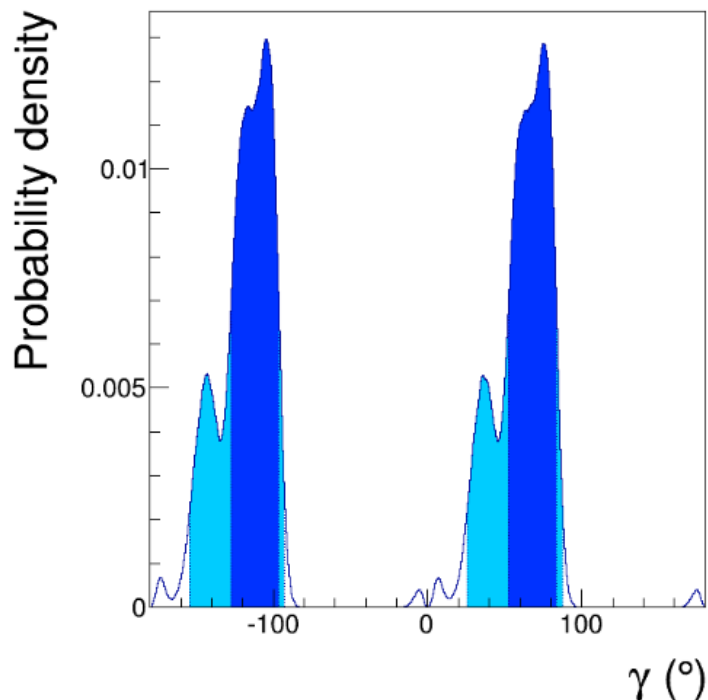
Inputs

Channel	$\mathcal{B} \times 10^{-6}$	C_f	S_f	$\rho(C_f, S_f)$	Reference
$B^0 \rightarrow \pi^+\pi^-$	—	$-0.25 \pm 0.08 \pm 0.02$	$-0.68 \pm 0.10 \pm 0.03$	-0.06	BaBar
$B^0 \rightarrow \pi^+\pi^-$	—	$-0.33 \pm 0.06 \pm 0.03$	$-0.64 \pm 0.8 \pm 0.03$	-0.10	Belle
$B^0 \rightarrow \pi^+\pi^-$	—	$-0.38 \pm 0.15 \pm 0.02$	$-0.71 \pm 0.13 \pm 0.02$	0.38	LHCb
$B^0 \rightarrow \pi^+\pi^-$	5.10 ± 0.19	—	—	—	HFAG
$B^0 \rightarrow \pi^0\pi^0$	1.91 ± 0.23	-0.43 ± 0.24	—	—	HFAG
$B^+ \rightarrow \pi^+\pi^0$	5.48 ± 0.35	—	—	—	HFAG

Parameter	Value	Reference
$\sin(2\beta)$	0.682 ± 0.019	HFAG

Parameters

Parameter	Parameter's range
D	$[0, 10^{-3}]$
d	$[0, 2]$
θ	$[-\pi, \pi]$
T	$[0, 1.5]$
θ_T	$[-\pi, \pi]$
γ	$[-\pi, \pi]$



$\gamma = [52.5^\circ, 83.6^\circ]$ at 68% probability
 $\gamma = [25.9^\circ, 87.6^\circ]$ at 95% probability

**VERY
PRELIMINARY**