

# SU(3) anatomy of hadronic charm decays

Martin Jung



“Implications of LHCb measurements and future prospects”  
CERN  
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Based on works with G.Hiller and St.Schacht,  
Phys.Rev. D87 (2013) 014024 (arXiv:1211.3734), arXiv:131x.xxxx

# Outline

Introduction

Non-leptonic charm decays and SU(3)

SU(3) and traces of the heavy-quark limit

Conclusions

## Why is charm so difficult?

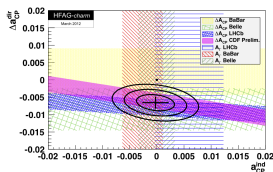
Main problem: missing large hierarchies for  $m_c$

Basically our usual methods don't work here:

- Considering charm as “light” ( $m_c \sim m_{u,d,s}$ ) does not work
- Operator-product expansion (OPE) in  $\Lambda_{\text{QCD}}/m_c$  questionable
- “Energetic” decay products e.g. in  $D \rightarrow PP$  have  $E < 1$  GeV
- Three-body MEs still extremely hard on the lattice
- Also: SU(3) severely broken

➔ Improvement of theoretical description urgently needed!

$\Delta A_{CP}$  has been a large motivator. . .



## Moriond 2013



## Moriond 2013

**Theorist**

$$\Delta A_{\text{CP}}^{\text{D}^*} = -0.34 \pm 0.15 \pm 0.10\%$$

$$\Delta A_{\text{CP}}^{\text{sl}} = +0.49 \pm 0.30 \pm 0.14\%$$

**LHCb****Idea stolen from D. Atwood**

However: WA still about  $3\sigma$  from zero:  $\Delta a_{\text{CP}}^{\text{dir}} = (-0.33 \pm 0.12)\%$   
 Independent interest in understanding dynamics at  $\mu \sim 1 \text{ GeV}$

## Another Outline

Long discussion whether  $\Delta A_{CP}$  is NP or not. . .

↳ We need more information!



What are we aiming at?

- NP or enhanced penguins - other modes should be affected
- Independent of enhancement: SM implies pattern in CPV
- Find a description of the **full**  $D \rightarrow PP$  data, not just  $\Delta A_{CP}$ 
  - ↳ Branching ratios and CP asymmetries,  $\delta_{K\pi}$
- Find (more) discriminants between NP and SM

How are we doing this?

- Exact limits do not work well
  - ↳ Include corrections!



## Amplitudes for non-leptonic decays

1. Use existing hierarchy  $M_W \gg m_c$  to build an **effective theory**  
 ↳ **Local** operators with known coefficients  $C_i$  (box to fish)
2. Classification by level of Cabibbo-suppression:
  - $c \rightarrow s\bar{d}u$ : Cabibbo-favoured (CF),  $V_{cs}^* V_{ud} \approx 1$ ,
  - $c \rightarrow s\bar{s}u(\bar{d}du)$ : Singly-Cabibbo-suppressed (SCS),  
 $V_{cs}^* V_{us} \approx -V_{cd}^* V_{ud} \approx \lambda$ .
  - Additionally in SCS modes:  
 $V_{cb}^* V_{ub} \sim \mathcal{O}(\lambda^5)$ ,  $r_{CKM} = |V_{cb}^* V_{ub} / V_{cs}^* V_{us}| \sim 0.2\%$
3. Find a way to determine **matrix elements** of the operators
4. Calculate via, e.g.,

$$\mathcal{A}^{CF} = V_{cs}^* V_{ud} \sum_i C_i \langle PP | \mathcal{O}_i | D \rangle$$

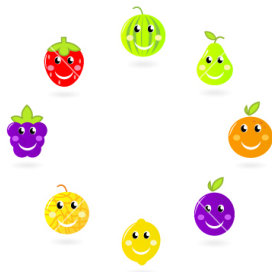
These matrix elements main objects of this talk

# Flavour SU(3) Symmetry

**Wigner-Eckart-Theorem** expresses MEs in terms of fewer **reduced MEs** and Clebsch-Gordan-coefficients

SU(3) flavour symmetry. . .

- is approximate, for  $m_u = m_d = m_s$
- does **not** allow to calculate MEs, but relates them
- provides a model-independent approach
- includes FSI



The analysis. . .

- exhibits different structures for SM and NP
- determines reduced MEs from data
  - ↳ improves automatically with coming measurements!



## Breaking SU(3)

Observation:  $BR(D^0 \rightarrow K^+K^-)/BR(D^0 \rightarrow \pi^+\pi^-)|_{\text{exp}} \approx 2.8 \neq 1|_{\text{SU}(3)}$

↳ 30% effect on amplitude level possible explanation! [Savage '91]

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The symmetry-breaking term is known:  $\mathcal{H}_{\text{mass}} = -\sum_q m_q \bar{q}q$

- In principle systematic expansion in  $\epsilon = m_s/\Lambda_{\text{QCD}} \sim 30\%$  possible [Savage'91, Kwong/Rosen'93, Chau/Cheng'94, Gronau et al.'95, Grinstein/Lebed'96, Hinchliffe/Kaeding'96]
- How large is the SU(3)-expansion parameter?
- Is the number of reduced MEs tractable?

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In the remainder of this talk:

- ↳ Check if expansion works for full  $D \rightarrow PP$  data  
(Confirmed for subset  $D \rightarrow P^+P^-$  e.g. in [Feldmann et al., Brod et al.'12] )
- ↳ Number of MEs large, but possible to handle
- ↳ Find ways to reduce number of MEs for sharper predictions  
(See also Yuval's talk for a different idea)
- ↳ This paves the way to address the question of NP

## Some details on the method

Generally:

- Classify initial/final states and Hamiltonian, e.g.  $(D) \sim (\bar{\mathbf{3}})$
- For final states and Hamiltonian:  
tensor products  $\rightarrow$  irreducible representations (CG coeffs)
- Classify reduced matrix elements (MEs)

SU(3) limit for  $D \rightarrow P_8 P_8$ :

- 3 MEs w/o CPV
- 2 more  $\sim r_{\text{CKM}}$
- Problem: fit doesn't work



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SU(3) analysis to first order breaking:

[Pirtskhalava/Uttayarar,Grossman/Robinson,Hiller/MJ/Schacht '12]

- Breaking by quark masses, we leave isospin intact,  $\epsilon \sim (\mathbf{8})$
- $\mathcal{H}_\epsilon \sim \mathcal{H}_0 \otimes (\mathbf{8}) \sim 11$  representations (!)
- Lots of new MEs: ( $\epsilon \times r_{\text{CKM}} \rightarrow 0!$ )  
 $\rightarrow \mathcal{O}(1) + \mathcal{O}(\epsilon) = 3 + 15 \rightarrow 11$ , due to linear dependencies

## What can we do with 11+2 MEs?

Questions we want to address first: [Hiller/MJ/Schacht'12]

- Can the full dataset  $D \rightarrow PP$  be described with reasonable  $SU(3)$  breaking?
- How large the penguin enhancement has to be?
- What are “minimal scenarios” to explain the data?
- Can we differentiate between NP scenarios?

In this process:

- Include all available data on  $D \rightarrow PP$
- Avoid prejudices about representations
- ➡ These goals complicate the analysis a lot

## Quantifying $SU(3)$ breaking

Quantifying  $SU(3)$  breaking non-trivial. Here:

1. Maximum of normalized  $SU(3)$ -breaking ME ( $\delta_X$ )
  - ↳ Ignores suppression by Clebsch-Gordan coefficients
2. Maximum of normalized  $SU(3)$ -breaking amplitude ( $\delta'_X$ )
  - ↳ Ignores possible cancellations

**New:** Include all MEs(!)

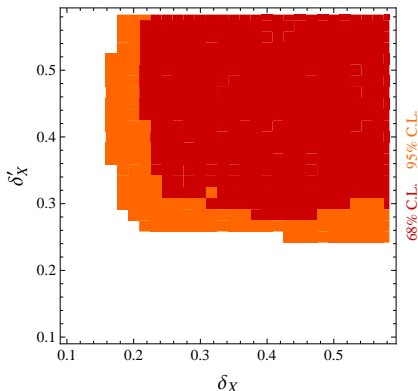
**New:** Include all data

↳ Classify all solutions

↳  $SU(3)$  breaking 25 – 40% ok

“Minimal solutions”:

- need at least two  $\mathcal{O}(\epsilon)$  MEs
- need at least one ME from higher representations



## Quantifying “penguin enhancement” (post-Moriond)

For penguins, analogous to SU(3):

1.  $\delta_3$  max. normalized ME  $\sim r_{\text{CKM}}$
2.  $\delta'_3$  max. amplitude  $\sim r_{\text{CKM}}$

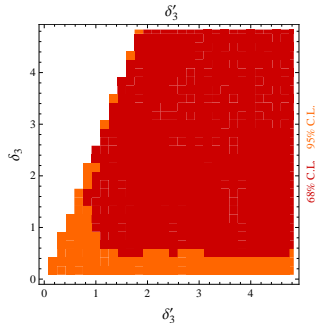
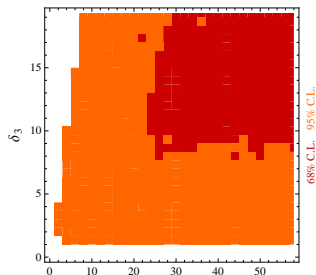
➡  $\delta_3^{(l)}$  remains huge for 68% CL

Reasons?

- Not only  $\Delta a_{\text{CP}}$ !
- Other CPA's with largish c.v.'s ( $D^0 \rightarrow K_S K_S, D_s \rightarrow \pi^+ K_S, K^+ \pi^0$ )  
 ➡ Without these, 'nominal'  $\delta_3$  ok  
 Interest in  $A_{\text{CP}}(D^0 \rightarrow K_S K_S)$ :  
 enhancement  $\sim 1/\epsilon$  expected  
 (see also [Atwood/Soni '12] )

➡ More data necessary (surprise!) to

- check largish asymmetries
- obtain compatible values for  $\Delta A_{\text{CP}}$





## Discriminating NP from SM

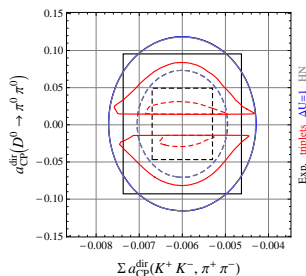
NP sensitivity: **different SU(3) structure**,  $\mathcal{H}^{SCS} = \mathcal{H}_{SM}^{SCS} + \mathcal{H}_{NP}$

- Two options: improve data or theory ( $\rightarrow$  less MEs)
- So far, only  $\Delta A_{CP}$  significant (less so since Moriond)
- $\rightarrow$  2 complex CPV MEs, no predictions with present data

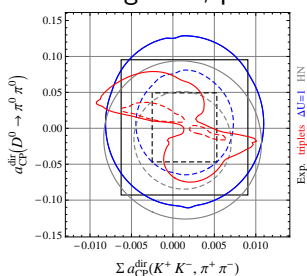
Proof of principle for the future, using **pseudo-data**:

- Assume LHCb projection for  $\Delta a_{CP}$ , similar precisions for 5 more measurements (LHCb+Belle II+BES III)
- All other uncertainties as today

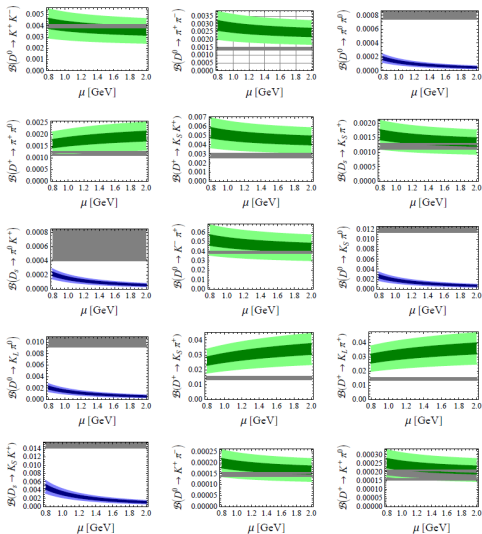
Assume “future” data



Only 3 breaking MEs, present data



# Heavy-quark limit in charm decays!?



## Heavy-quark limit in charm decays!?

Remember: QCD factorization [BBNS'99,'00,BN'03,Grossman et al.'07]

- MEs for  $m_c \rightarrow \infty$  in terms of a few **universal, non-perturbative** objects, e.g. decay constants and form factors
- Corrections of higher orders in  $\alpha_s$  are systematically calculable
- **Power corrections  $\mathcal{O}(\Lambda/m_c)$  are the main problem**

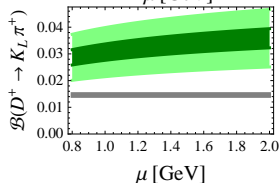
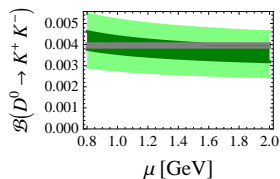
How large are the corrections?

- BR's in heavy-quark limit ( $m_c \rightarrow \infty$ ):
  - colour-allowed-tree decays ✓
  - $\mathcal{O}$ (colour-suppressed decays) ✓
  - annihilation not included

➡ Reasonable starting point

➡ Assume **structure** of QCDF to hold

- Power corrections might be large and flavour-dependent



## QCDF-structure as SU(3) input I [Hiller/MJ/Schacht '13, in prep.]

QCDF in charm decays is known not to work precisely. . .

➡ What do we mean by structural input?

1. No constraint in the SU(3) limit
  2. No explicit computation of  $a_{1,2}^{DM_1M_2}$ ,  $b_{1,2}^{M_1M_2}$
  3. No assumption about strong phases
  4. No simple  $X_{A,H}$  parametrization for annihilation
- ➡ Use relations between amplitudes implied by features of QCDF
- ➡ Match these relations onto SU(3) amplitudes
- ➡ SU(3) constrains general QCDF parametrization

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Starting point **universal**  $a_1$ :

- Pattern of BR's:  $a_2$  has larger corrections than  $a_1$
- SU(3) breaking in  $a_1$ :  $\mathcal{O}(\alpha_s \times \Lambda/m_c \times m_s/\Lambda \times C_2/C_1 \times 1/N_C^2)$
- QCDF: relative influence in  $a_2$   $C_1^2/C_2^2 \sim 10$  stronger
- ➡ Assume  $a_1$  flavour-universal,  $a_2$  process-dependent
- ➡ Yields one relation for MEs in SU(3) approach

## QCDF-structure as SU(3) input II

### Universal structure of annihilation:

- In QCDF, mainly two annihilation coefficients  $b_{1,2}^{M_1 M_2}$

- Both MEs involve identical convolution:

$$b_2^{M_1 M_2} / b_1^{M_1 M_2} = C_2 / C_1$$

- However: holds only in one-gluon approximation

➡ allow for  $b_2^{M_1 M_2} / b_1^{M_1 M_2} = c$

➡ Yields two more relations for MEs in SU(3) approach

➡ Third scenario is w.i.p.

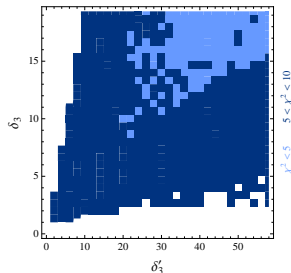
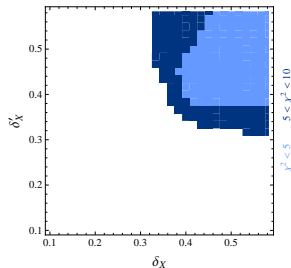
# QCDF and SU(3) - first results I PRELIMINARY

Started analysis in scenarios A and B:

- Sum rules derived ✓
- Minima with  $\chi^2/\text{dof} \sim 1$  ✓
- $\delta_{X^{(\prime)}}$  < 50% ✓

Scenario B ( $\chi^2/\text{dof} \sim 5/5$ ):

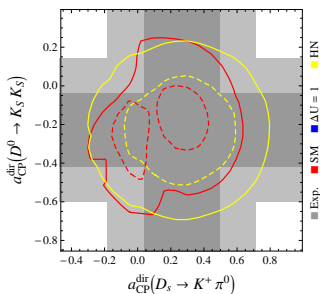
- ➔ Sum rules are restrictive
- ➔ but fit works!
- Somewhat larger SU(3) breaking?
  - plots not directly comparable
  - dof not constant over the plot!
- Little change in penguin enhancement



# QCDF and SU(3) - first results II PRELIMINARY

Scenario B continued:

- Fit with all remaining MEs
- Structures emerge with present data



- ➡ QCDF yields non-trivial constraints for SU(3) analysis
- ➡ We are exploring further consequences, stay tuned. . .



## Conclusions

- Better understanding necessary, independent of  $\Delta a_{CP}$
- First unbiased, comprehensive analysis of  $D \rightarrow PP$
- Description possible with reasonable SU(3) breaking
- Direct CP violation in charm (all modes!) remains interesting
- Enhanced asymmetry in  $D^0 \rightarrow K_S K_S$  expected
- Generally very hard to make quantitative statements
- ➔ More data will help to improve analysis
- Theory side: idea to use QCDF structure for SU(3) breaking
- Sum rules for decay amplitudes, eliminate SU(3) MEs
- Restricted fits still work

### Outlook:

- Complete analysis for QCDF influence (+NP) w.i.p.
- Interesting times! Measurements to come from LHC(b), Belle II, BES III, ...

# Backup slides

# Available data for $D \rightarrow PP$ I

Observable	Measurement	References
SCS CP asymmetries		
$\Delta a_{CP}^{\text{dir}}(K^+K^-, \pi^+\pi^-)$	$-0.00333 \pm 0.00120$	
$\Sigma a_{CP}^{\text{dir}}(K^+K^-, \pi^+\pi^-)$	$+0.00008 \pm 0.00228$	†
$a_{CP}^{\text{dir}}(D^0 \rightarrow K_S K_S)$	$-0.23 \pm 0.19$	
$a_{CP}^{\text{dir}}(D^0 \rightarrow \pi^0 \pi^0)$	$+0.001 \pm 0.048$	
$a_{CP}^{\text{dir}}(D^+ \rightarrow \pi^0 \pi^+)$	$+0.029 \pm 0.029$	
$a_{CP}^{\text{dir}}(D^+ \rightarrow K_S K^+)$	$+0.0022 \pm 0.0025$	
$a_{CP}^{\text{dir}}(D_s \rightarrow K_S \pi^+)$	$+0.011 \pm 0.007$	†
$a_{CP}^{\text{dir}}(D_s \rightarrow K^+ \pi^0)$	$+0.266 \pm 0.228$	
Indirect CP Violation		
$a_{CP}^{\text{ind}}$	$+0.00015 \pm 0.00052$	
$\delta_L \equiv 2\text{Re}(\epsilon)/(1 +  \epsilon ^2)$	$(3.32 \pm 0.06) \cdot 10^{-3}$	
$K^+ \pi^-$ strong phase difference		
$\delta_{K\pi}$	$(11.7 \pm 10.2)^\circ$	‡

**Table :** The observables and the data on indirect CP violation used in this work. We subtract the contribution from indirect CP violation where appropriate. Note that the BESIII result for  $\delta_{K\pi}$  cannot be taken into account, as it relies on external non-independent input. †Our average with systematic and statistical error being added quadratically. ‡Our symmetrization of uncertainties.

# Available data for $D \rightarrow PP$ II

Observable	Measurement	References
SCS branching ratios		
$\mathcal{B}(D^0 \rightarrow K^+ K^-)$	$(3.96 \pm 0.08) \cdot 10^{-3}$	
$\mathcal{B}(D^0 \rightarrow \pi^+ \pi^-)$	$(1.401 \pm 0.027) \cdot 10^{-3}$	
$\mathcal{B}(D^0 \rightarrow K_S K_S)$	$(0.17 \pm 0.04) \cdot 10^{-3}$	
$\mathcal{B}(D^0 \rightarrow \pi^0 \pi^0)$	$(0.80 \pm 0.05) \cdot 10^{-3}$	
$\mathcal{B}(D^+ \rightarrow \pi^0 \pi^+)$	$(1.19 \pm 0.06) \cdot 10^{-3}$	
$\mathcal{B}(D^+ \rightarrow K_S K^+)$	$(2.83 \pm 0.16) \cdot 10^{-3}$	
$\mathcal{B}(D_s \rightarrow K_S \pi^+)$	$(1.21 \pm 0.08) \cdot 10^{-3}$	
$\mathcal{B}(D_s \rightarrow K^+ \pi^0)$	$(0.62 \pm 0.21) \cdot 10^{-3}$	
CF* branching ratios		
$\mathcal{B}(D^0 \rightarrow K^- \pi^+)$	$(3.88 \pm 0.05) \cdot 10^{-2}$	
$\mathcal{B}(D^0 \rightarrow K_S \pi^0)$	$(1.19 \pm 0.04) \cdot 10^{-2}$	
$\mathcal{B}(D^0 \rightarrow K_L \pi^0)$	$(1.00 \pm 0.07) \cdot 10^{-2}$	
$\mathcal{B}(D^+ \rightarrow K_S \pi^+)$	$(1.47 \pm 0.07) \cdot 10^{-2}$	
$\mathcal{B}(D^+ \rightarrow K_L \pi^+)$	$(1.46 \pm 0.05) \cdot 10^{-2}$	
$\mathcal{B}(D_s \rightarrow K_S K^+)$	$(1.48 \pm 0.05) \cdot 10^{-2}$	†
DCS branching ratios		
$\mathcal{B}(D^0 \rightarrow K^+ \pi^-)$	$(1.47 \pm 0.07) \cdot 10^{-4}$	
$\mathcal{B}(D^+ \rightarrow K^+ \pi^0)$	$(1.83 \pm 0.26) \cdot 10^{-4}$	

**Table :** The data on the observables used in this work. †Our average with systematic and statistical error being added quadratically. ‡Our symmetrization of uncertainties. \*Decays with a  $K_{S,L}$  in the final state, *i.e.*, those with a CF and DCS component are assigned to the CF channels.

## Further inputs into the analysis

$m_{D^0}$	$(1864.86 \pm 0.13)$ MeV	
$m_{D_s}$	$(1968.49 \pm 0.32)$ MeV	
$m_{\pi^0}$	$(134.9766 \pm 0.0006)$ MeV	
$m_{K^0}$	$(497.614 \pm 0.024)$ MeV	
$f_D$	$(205.3 \pm 5.2)$ MeV	
$f_{D_s}$	$(257.5 \pm 4.5)$ MeV	
$f_\pi$	$(130.41 \pm 0.03 \pm 0.2)$ MeV	
$f_K$	$(156.1 \pm 0.2 \pm 0.8 \pm 0.2)$ MeV	
$F_0^{DK}(0)$	$0.737 \pm 0.005$	†
$F_0^{D\pi}(0)$	$0.638 \pm 0.012$	†

**Table :** Numerical input for the heavy quark scenarios. †Our average, with systematic and statistical errors being added quadratically.

## Pseudo-data for the future scenario

Observable	“Future” data
SCS CP asymmetries	
$\Delta a_{CP}^{\text{dir}}(K^+ K^-, \pi^+ \pi^-)$	$-0.007 \pm 0.0005$
$\Sigma a_{CP}^{\text{dir}}(K^+ K^-, \pi^+ \pi^-)$	$-0.006 \pm 0.0007$
$a_{CP}^{\text{dir}}(D^+ \rightarrow K_S K^+)$	$-0.003 \pm 0.0005$
$a_{CP}^{\text{dir}}(D_s \rightarrow K_S \pi^+)$	$0.0 \pm 0.0005$
$a_{CP}^{\text{dir}}(D_s \rightarrow K^+ \pi^0)$	$0.05 \pm 0.0005$
$K^+ \pi^-$ strong phase difference	
$\delta_{K\pi}$	$21.4^\circ \pm 3.8^\circ$

**Table :** Future pseudo-data, all other values unchanged. The central values of the single CP asymmetries that correspond to  $\Delta a_{CP}^{\text{dir}}$  and  $\Sigma a_{CP}^{\text{dir}}$  are  $a_{CP}^{\text{dir}}(D^0 \rightarrow K^+ K^-) = -0.0065$  and  $a_{CP}^{\text{dir}}(D^0 \rightarrow \pi^+ \pi^-) = 0.0005$ .