Bayesian Constraints on Wilson Coefficients from Radiative and (Semi)leptonic $b \to s$ Decays

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in collaboration with
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based on arxiv:1310.2478

Implications of LHCb measurements and future prospects
October 15th 2013

Theor. Physik 1

UNIVERSITÄT SIEGEN

DFG FOR 1873

Effective Field Theory for $b \to s\ell^+\ell^-$ FCNCs

Flavor Changing Neutral Current (FCNC)

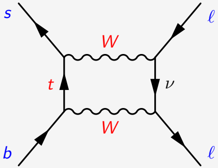
- expand amplitudes in $G_F \sim 1/M_W^2$ (OPE)
- operators (matrix elem. below $\mu_b \simeq m_b$)

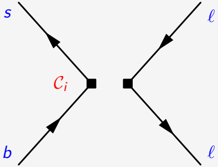
$$\mathcal{O}_i \equiv \left[\bar{s}\Gamma_i b\right]\left[\bar{\ell}\Gamma'_i \ell\right]$$

- Wilson coefficients (above $\mu_b \simeq m_b$)

$$\mathcal{C}_i \equiv \mathcal{C}_i(M_W, M_Z, m_t, \ldots)$$

- use $\mathcal{C}_i = \mathcal{C}_i(\mu_b = 4.2\text{GeV})$





Effective Hamiltonian

$$\mathcal{H} = -\frac{4G_F}{\sqrt{2}}\frac{\alpha_e}{4\pi}\left[V_{tb}V_{ts}^*\sum_i \mathcal{C}_i\mathcal{O}_i + O\left(V_{ub}V_{us}^*\right)\right] + \text{h.c.}$$

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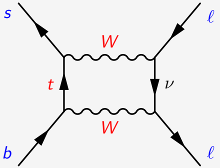

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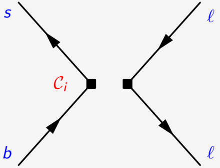

Operators

$$\mathcal{O}_{7(')} = [\bar{s}\sigma^{\mu\nu}P_{R(L)}b]F_{\mu\nu}$$

$$\mathcal{O}_{9(')} = [\bar{s}\gamma^{\mu}P_{L(R)}b][\bar{\ell}\gamma_{\mu}\ell]$$

$$\mathcal{O}_{10(')} = [\bar{s}\gamma^{\mu}P_{L(R)}b][\bar{\ell}\gamma_{\mu}\gamma_5\ell]$$

Effective Field Theory for $b \to s\ell^+\ell^-$ FCNCs

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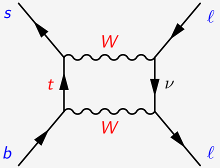

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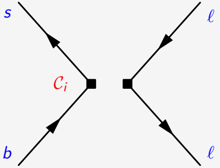

Decay Modes

$B \to K^*\ell^+\ell^-$	$B_s \to \mu^+\mu^-$	$B \to K\ell^+\ell^-$
$B \to K^*\gamma$	$B \to X_s\ell^+\ell^-$	$B \to X_s\gamma$

Model-Independent Framework

Definition of model-independent for the purpose of this work:

Basis of Operators $\mathcal{O}_i$

- include as many $\mathcal{O}_i$ beyond SM as needed/as few as possible
- balancing act, test statistically if choice of basis describes data well!

Wilson Coefficients $\mathcal{C}_i$

- treat $\mathcal{C}_i$ as uncorrelated, generalized couplings
- constrain their values from data
- model builders: confront new physics models with constraints

Fit Scenarios: SM(ν-only)

SM-like Coefficients

- fix $\mathcal{C}_{7,9,10}$ to SM values (NNLL)

Chirality-flipped Coefficients

- fix $\mathcal{C}_{7'} = m_s/m_b \mathcal{C}_7$, fix $\mathcal{C}_{9',10'} = 0$

Nuisance Parameters

- fit nuisance parameters
- informative priors
  - form factors: light-cone sum rules [Khodjamirian/Mannel/Pivovarov/Wang '10]
  - power corrections: power-counting assumptions
  - CKM: tree-level fit [UTfit]
  - quark masses [PDG]

Fit Scenarios: SM Basis

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Fit Scenarios: SM+SM$'$ Basis

SM-like Coefficients

- fit $\mathcal{C}_{7,9,10}$

Chirality-flipped Coefficients

- fit $\mathcal{C}_{7',9',10'}$

Nuisance Parameters

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  - form factors: light-cone sum rules [Khodjamirian/Mannel/Pivovarov/Wang '10]
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  - CKM: tree-level fit [UTfit]
  - quark masses [PDG]

Sensitivity to Fit Parameters

Wilson Coefficients

	$\mathcal{C}_{7(')}$	$\mathcal{C}_{9(')}$	$\mathcal{C}_{10(')}$	
$B_s \to \mu^+\mu^-$	–	–	✓	
$B \to X_s\gamma$	✓	–	–	
$B \to X_s\ell^+\ell^-$	✓	✓	✓	
$B \to K^*\gamma$	✓	–	–	
$B \to K^*\ell^+\ell^-$	✓	✓	✓	12 CP-avg. angular observables
$B \to K\ell^+\ell^-$	✓	✓	✓	3 CP-avg. angular observables

Form Factors

- interplay between $B \to X_s\{\gamma, \ell^+\ell^-\}$ and $B \to K^*\{\gamma, \ell^+\ell^-\}$
- some $B \to K^*\ell^+\ell^-$ obs. form-factor insensitive by construction
- some $B \to K^*\ell^+\ell^-$ obs. dominantly sensitive to form factor ratios

Measurements Entering Analysis

$B \to K^* \ell^+ \ell^-$ $q^2 \in [1, 6]\,\mathbf{GeV}^2$, $q^2 \geq M_{\psi'}^2$

- $\mathcal{B}$, A_{FB}, F_{L}, A_{T}^2
- **new**: $A_{\mathrm{T}}^{\mathrm{re}}$, P_4', P_5', P_6'
- ATLAS, BaBar, Belle, CDF, CMS, LHCb

see also talk by N. Mahmoudi

$B \to K^* \gamma$

- $\mathcal{B}$, $S_{K^*\gamma}$, $C_{K^*\gamma}$
- BaBar, Belle, CLEO

$B \to K \ell^+ \ell^-$ $q^2 \in [1, 6]\,\mathbf{GeV}^2$, $q^2 \geq M_{\psi'}^2$

- $\mathcal{B}$
- BaBar, Belle, CDF, LHCb

$B \to X_s \gamma$ $E_{\mathrm{min}}^{\gamma} = 1.8\,\mathbf{GeV}$

- $\mathcal{B}$
- BaBar, Belle, CLEO

$B_s \to \mu^+ \mu^-$

- $\int \mathrm{d}\tau\, \mathcal{B}(\tau)$
- CMS, LHCb

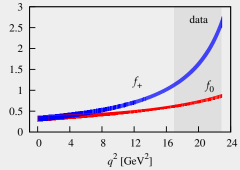
$B \to X_s \ell^+ \ell^-$ $q^2 \in [1, 6]\,\mathbf{GeV}^2$

- $\mathcal{B}$
- BaBar, Belle

Further Theory Constraints

Form Factors from Lattice QCD (LQCD) [HPQCD arxiv:1306.2384]

- $B \to K$ form factors available from LQCD
  - data only at high q^2: 17 – 23 GeV^2
  - no data points given
- reproduce 3 data points from z-parametrization
  - $q^2 \in \{17, 20, 23\}$ GeV^2
  - use as constraint, incl. covariance matrix



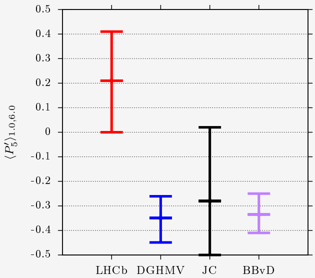
$B \to K^*$ Form Factor (FF) Relation at $q^2 = 0$

- FF $V, A_1 \propto \xi_\perp + \ldots$ [Charles et al. hep-ph/9901378]
  - no α_s corrections [Burdmann/Hiller hep-ph/0011266, Beneke/Feldmann hep-ph/0008255]
  - Large Energy Limit: $V(0) = (1.33 \pm 0.4) \times A_1(0)$
- LCSR constraint: $\xi_{\|}(0) = 0.10^{+0.03}_{-0.02}$, to avoid $\xi_{\|}(q^2) \propto A_0(q^2) < 0$.
- see also FF fits by [Hambrock/Hiller/Schacht/Zwicky 1308.4379]

The $B \to K^* \ell^+ \ell^-$ "Anomaly"

- **LHCb** measurement [1308.1707]
  - deviation from SM prediction in form factor-free obs. $\langle P_5' \rangle_{[1,6]}$
  - LHCb uses one SM prediction (**DGHMV**)

    [Descotes-Genon/Hurth/Matias/Virto 1303.5794]



- however: further SM prediction exist, much larger uncertainty (**JC**)

  [Jäger/Camalich 1212.2263]

- our take on SM prediction $\langle P_5' \rangle_{[1,6]} = -0.34^{+0.09}_{-0.08}$ (**BBvD**)

  see also backups for $P_{4,6}'$ and $[2, 4.3]$ bins

difference: treatment of unknown power corrections
(form factor corrections, $\bar{c}c$ resonances)

Results SM(ν-only)

Pull Values at Best-Fit Point

- largest pulls

-3.4σ	$\langle F_L \rangle_{[1,6]}$, BaBar 2012	$+2.5\sigma$	$\langle \mathcal{B} \rangle_{[16,19.21]}$, Belle 2009
-2.6σ	$\langle F_L \rangle_{[1,6]}$, ATLAS 2013	$+2.2\sigma$	$\langle A_{\text{FB}} \rangle_{[16,19]}$, ATLAS 2013
-2.4σ	$\langle P'_4 \rangle_{[14.18,16]}$, LHCb 2013	$+2.1\sigma$	$\langle P'_5 \rangle_{[1,6]}$, LHCb 2013

p Values

- *p* value 0.15
- taking out ATLAS, BaBar $\langle F_L \rangle_{[1,6]}$: *p* value increases to 0.71

Summary

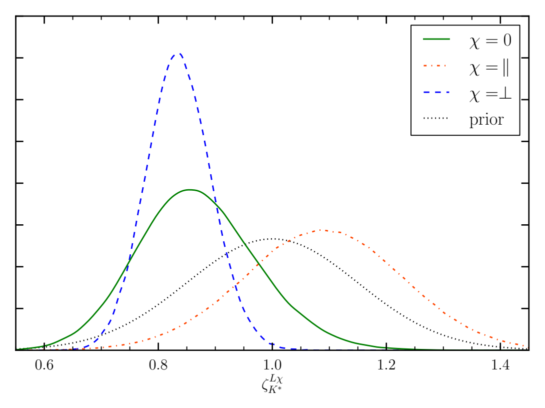
- good fit, no New Physics signal
- we find power corrections on top of QCDF results at large recoil

Parametrization of Power Corrections @ Large Recoil

- six parameters $\zeta_\chi^{L(R)}$ for the $[1, 6]$ bin

$$A_\chi^{L(R)}(q^2) \mapsto \zeta_\chi^{L(R)} A_\chi^{L(R)}(q^2)\,, \quad \chi = \perp, \parallel, 0$$

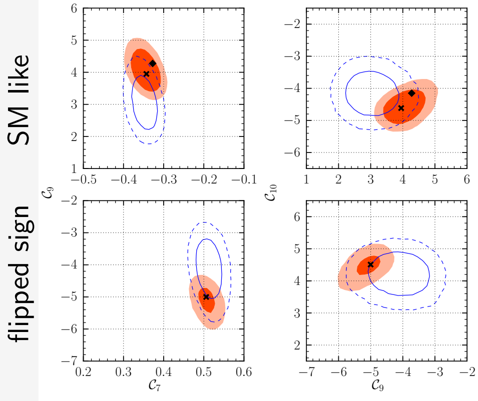
- on top of QCDF correction to transversity amplitudes



- tension diluted by parameters $\zeta_\chi^{L(R)}$
- shift by $\simeq -20\%$ for $\zeta_{\perp,\parallel}^L$
- shift by $\simeq +10\%$ for ζ_0^L
- few percents for ζ_χ^R

improved understanding of power corrections desirable

Results (SM Basis)



♦: Standard Model, ×: best-fit point

(light-) red: 68% CL (95% CL) for full dataset

blue solid (blue dashed): 68% CL (95% CL) for selection

post HEP'13 (selection)

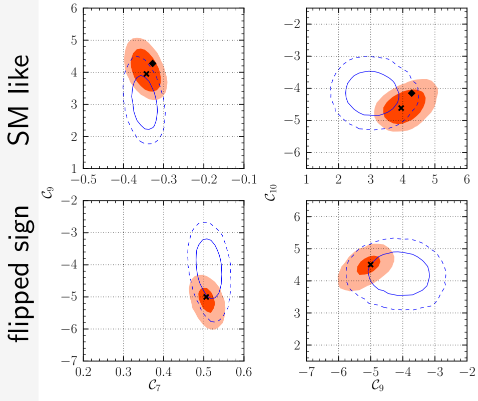
- with $B \to X_s\{\gamma, \ell^+\ell^-\}$
- $B_s \to \mu^+\mu^-$ from LHCb and CMS
- same data as

  [Descotes-Genon/Matias/Virto 1307.5683]

  exclusive decays limited:
  - only $B \to K^*\ell^+\ell^-$!
  - only LHCb data!
  - only $q^2 \in [1, 6]\text{GeV}^2$
- best-fit point similar to

  [Descotes-Genon et al. 1307.5683]
  - less tension, only $\lesssim 2\sigma$
  - $\mathcal{C}_9 - \mathcal{C}_9^{\text{SM}} \simeq -1.3 \pm 0.5$

Results (SM Basis)



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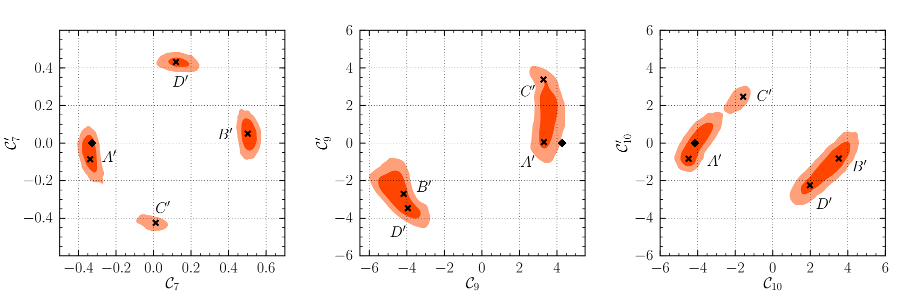
post HEP'13 (full)

- SM-like uncertainty reduced by ~ 2 compared to 2012
- SM at the border of 1σ
- flipped-sign barely allowed at 1σ (26% of evidence)
- cannot confirm NP findings
  - in $(\mathcal{C}_7, \mathcal{C}_9)$

    [Descotes-Genon et al. 1307.5683]
- $\zeta_\chi^{L(R)}$ as in SM(ν-only)
- p value: 0.13 (@SM-like sol.)

Results (SM+SM′ Basis)



♦: Standard Model, ×: best-fit points, (light-) red: 68% CL (95% CL) for full dataset

- four solutions A' through D'
  - $A' =$ SM like, 39% of ev.
  - $B' =$ flipped signs, 41% of ev.
  - C', D' suppressed: 5% and 15% of evidence
- for A' (SM-like sol.)
  - p value 0.17
  - $\mathcal{C}_9 - \mathcal{C}_9^{\mathrm{SM}} = -0.8^{+0.2}_{-0.5}$
  - 2σ deviation from SM
  - $\zeta_\chi^{L(R)}$ decrease wrt. SM(ν-only) and SM basis

(Statistical) Model Comparison

- model comparison using Bayes factor and model priors
- compare scenarios only at SM-like solution $A(')$
- adjust priors to contain only $A(')$
- results
  - SM(ν-only) wins over SM basis: odds of 100:1
  - SM(ν-only) wins over SM+SM$'$ basis: odds of 22:1
  - SM+SM$'$ basis wins over SM basis: odds of 4.5:1

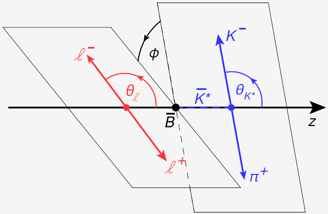
Conclusion

- all three scenarios describe $b \to s(\gamma, \ell^+\ell^-)$ data well
- SM(ν-only) wins comparison with SM and SM+SM′
  - subleading power correction on top of QCDF: 10–20%
- several tensions in all scenarios compare talk by D. Straub
  - $\langle P'_5 \rangle_{[1,6]}$ reduced pull in fit due to power corrections
  - $\langle F_L \rangle_{[1,6]}$ from BaBar, ATLAS (both preliminary) persist
  - $\langle P'_4 \rangle_{[14.18,16]}$ LHCb persists
- new physics signal only for
  - SM+SM′ basis
  - (also: SM basis with "post HEP'13 (selection)" subset of data)
- data also allows inference of form factor parameters
- looking forward to further LHC analyses (2012 datasets) and the prospects of Belle-II

Backup Slides

(Angular) Observables in $B \to K^* \ell^+ \ell^-$

- kinematics
  - dilepton mass squared q^2
  - three angles
- complicated diff. decay width
  - 12(+) angular observables J_n
  - express all observables through J_n
  - compose observ. from J_n with specific benefits



Definitions

$$\Gamma \sim 3J_{1c} + 6J_{1s} - J_{2c} - 2J_{2s} \quad A_{\mathrm{FB}} \sim \frac{J_{6s}}{\Gamma} \quad F_L \sim \frac{3J_{1c} - J_{2c}}{\Gamma}$$

$$P_4' \sim \frac{+J_4}{\sqrt{-J_{2s}J_{2c}}} \quad P_5' \sim \frac{+J_5}{2\sqrt{-J_{2s}J_{2c}}} \quad P_6' \sim \frac{-J_7}{2\sqrt{-J_{2s}J_{2c}}}$$

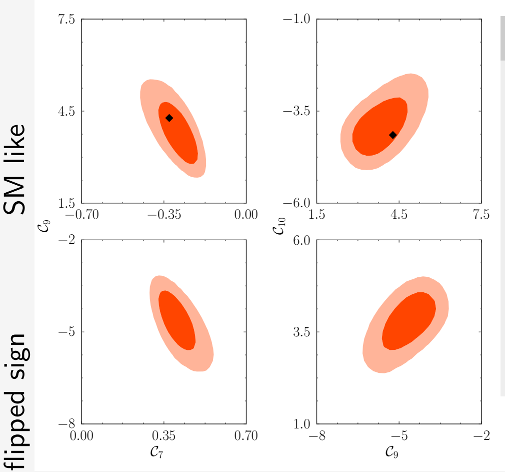
Standard Model Predictions for $P'_{4,5,6}$

- toy Monte Carlo using priors + theory constraints (FFs)
- calculate observable for 10^5 samples
- find minimal 68% CL intervals

	q^2 [GeV^2]	$\langle P'_4 \rangle$	$\langle P'_5 \rangle$	$10^2 \times \langle P'_6 \rangle$
BBvD	$[1, 6]$	$+0.46\ ^{+0.12}_{-0.11}$	$-0.335\ ^{+0.085}_{-0.075}$	$-6.4\ \pm 1.7$
	$[2, 4.3]$	$+0.48\ ^{+0.11}_{-0.10}$	$-0.315\ ^{+0.074}_{-0.090}$	$-7.2\ ^{+1.5}_{-2.2}$
LHCb†	$[1, 6]$	$+0.58\ ^{+0.33}_{-0.36}$	$+0.21\ ^{+0.20}_{-0.21}$	$+18\ \pm 21$
	$[2, 4.3]$	$+0.74\ ^{+0.11}_{-0.53}$	$+0.29\ ^{+0.40}_{-0.39}$	$+15\ ^{+38}_{-36}$

†: [LHCb 1308.1707], adjusted to theory convention

2012 Results (SM Basis)

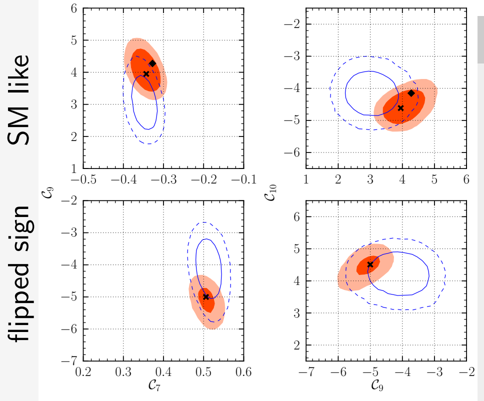


early 2012

- figure from [1205.1838]
- no $B \to X_s\{\gamma, \ell^+\ell^-\}$
- only LHCb bound on $B_s \to \mu^+\mu^-$
- $B \to K^{(*)}\ell^+\ell^-$: $\mathcal{B}, A_{\mathrm{FB}}, F_L, A_T^{(2)}, S_3$
- $B \to K^*\gamma$: $\mathcal{B}, S_{K^*\gamma}, C_{K^*\gamma}$

◆: Standard Model

Results (SM Basis, Selection)



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- same data as [Descotes-Genon/Matias/Virto 1307.5683] exclusive decays limited:
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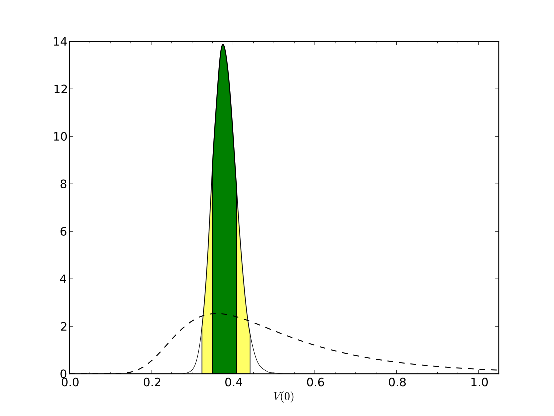
  - only LHCb data!
  - only $q^2 \in [1, 6]\text{GeV}^2$
- best-fit point similar to [Descotes-Genon et al. 1307.5683]
  - less tension, only $\lesssim 2\sigma$
  - $\mathcal{C}_9 - \mathcal{C}_9^{\text{SM}} \simeq -1.3 \pm 0.5$

Results for $B \to K^*$ Form Factors



dashed: prior [Khodjamirian/Mannel/Pivovarov/Wang 1006.4945]

solid: posterior, green: 68% CL, yellow: 95% CL

- more precise than prior
- $B \to K^*$: $\xi_\perp$ from
  - $B \to X_s\gamma$
  - $B \to K^*\gamma$
  - $B \to K^*\ell^+\ell^-$
  - theory input
- results @ 68% CL
  - $V(0) = 0.37^{+0.03}_{-0.02}$

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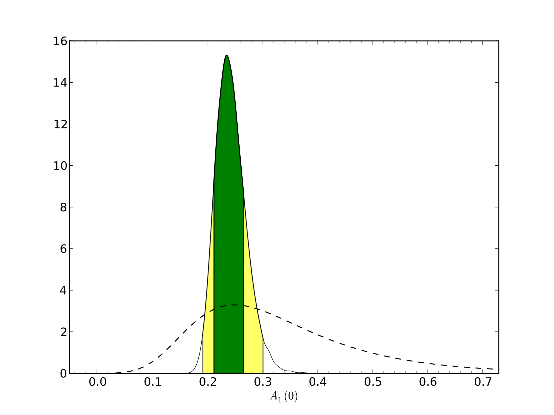

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  - $A_1(0) = 0.24 \pm 0.03$

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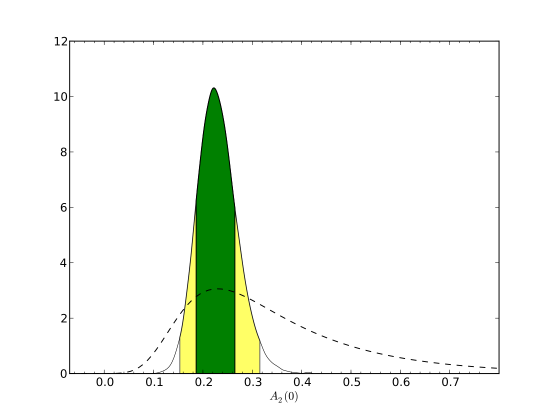

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  - $B \to X_s \gamma$
  - $B \to K^* \gamma$
  - $B \to K^* \ell^+ \ell^-$
  - theory input
- results @ 68% CL
  - $V(0) = 0.37^{+0.03}_{-0.02}$
  - $A_1(0) = 0.24 \pm 0.03$
  - $A_2(0) = 0.22 \pm 0.04$

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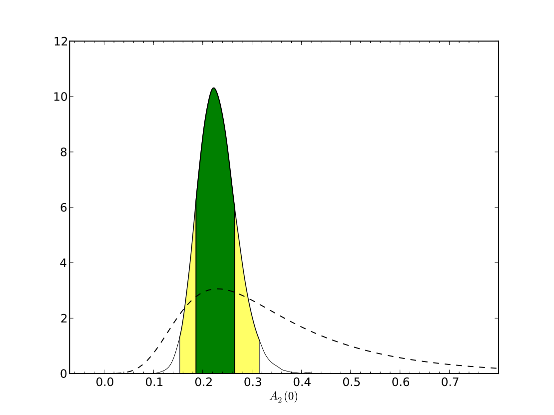

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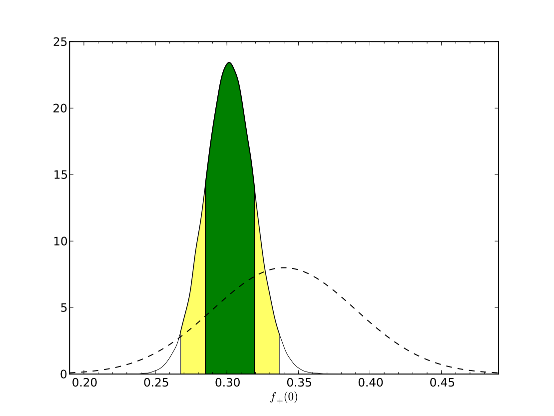
- more precise than prior
- $B \to K^*$: $\xi_\perp$ from
  - $B \to X_s\gamma$
  - $B \to K^*\gamma$
  - $B \to K^*\ell^+\ell^-$
  - theory input
- results @ 68% CL
  - ratio of central values

$$V(0)/A_1(0) \simeq 1.5$$

$$A_2(0)/A_1(0) \simeq 0.9$$

agree w/ (SE2 full) [Hambrock/Hiller/Schacht/Zwicky 1308.4379]

Results for $B \to K$ Form Factors



dashed: prior [Khodjamirian/Mannel/Pivovarov/Wang 1006.4945],
modified to accomodate [Ball/Zwicky hep-ph/0406232]

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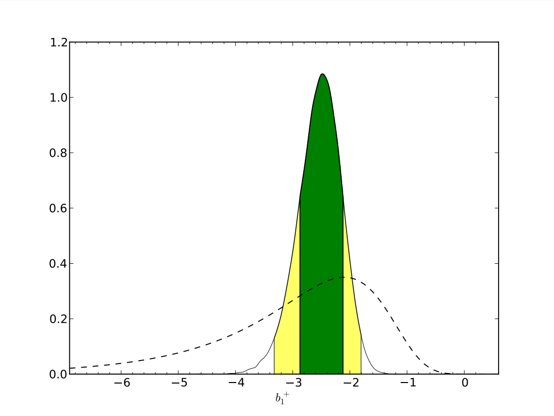
- more precise than prior
- $B \to K$:
  - $B \to K\ell^+\ell^-$
  - Lattice
- results @ 68% CL
  - $f_+(0) = 0.30 \pm 0.02$

Results for $B \to K$ Form Factors



dashed: prior [Khodjamirian/Mannel/Pivovarov/Wang 1006.4945],

solid: posterior, green: 68% CL, yellow: 95% CL

- more precise than prior
- $B \to K$:
  - $B \to K\ell^+\ell^-$
  - Lattice
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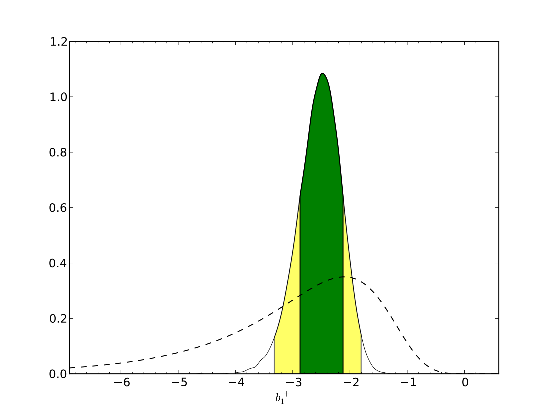
Results for $B \to K$ Form Factors



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- more precise than prior
- $B \to K$:
  - $B \to K\ell^+\ell^-$
  - Lattice
- results @ 68% CL
  - $f_+(0) = 0.30 \pm 0.02$
  - $b_1^+ = -2.5 \pm 0.4$
- small tension
  - LHCb lo q^2: -1.4σ
  - LHCb hi q^2: $+1.1\sigma$
  - Lattice: $+0.5\sigma$

Priors and Parametrizations (I)

Form Factors [Khodjamirian et al. 1006.4945]

- values @ $q^2 = 0$ and slope: two parameters per FF
- z-parametrization
- asymmetric priors, use LogGamma function

CKM [update of hep-ph/0012308]

- Wolfenstein parametrization
- UTfit pre-Moriond2013, tree-level data only

Quark Masses [PDG]

Priors and Parametrizations (I) - Subleading

parametrize unknown subleading contributions

$B \to K^* \ell^+ \ell^-$

- lo q^2: 6 parameters, one scaling factor per amplitude
- hi q^2: 3 parameters

$B \to K \ell^+ \ell^-$

- lo q^2: 1 parameter
- hi q^2: 1 parameter

for all: Gaussian with 1σ interval $\pm\Lambda_{\text{QCD}}/m_b \simeq \pm 0.15$

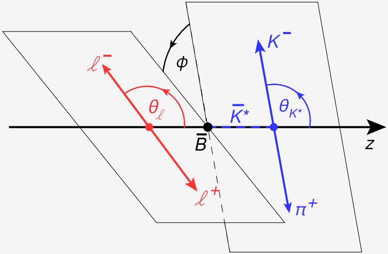
A Note on p Values

- test statistic: function of data and model (parameters) $\chi^2 = \chi^2(D, \vec{\theta})$
- only one data set observed $D_{obs} \Rightarrow \chi^2_{obs}$
- $p \equiv P(\chi^2 > \chi^2_{obs})$

But how to fix $\vec{\theta}$?

1. this work
   $\vec{\theta} = (\vec{\mathcal{C}}, \vec{\nu})$ at (local) mode of posterior, $\chi^2 \sim \frac{(x-\mu)^2}{\sigma^2_{exp}}$
2. Descotes-Genon et al. [1307.5683]
   $\vec{\theta} = \vec{\mathcal{C}}$ at (local) mode of likelihood, $\chi^2 \sim \frac{(x-\mu)^2}{\sigma^2_{exp} + \sigma^2_{theo}}$

Kinematics of $\bar{B} \to \bar{K}\pi\ell^+\ell^-$



Kinematic Variables

$$4m_\ell^2 \leq q^2 \leq (M_B - M_{K^*})^2$$

$$-1 \leq \cos\theta_\ell \leq 1$$

$$-1 \leq \cos\theta_{K^*} \leq 1$$

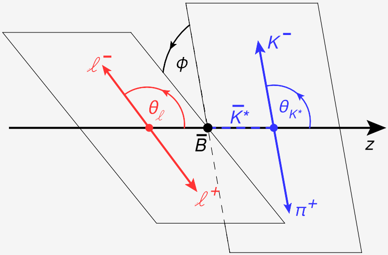
$$0 \leq \phi \leq 2\pi$$

$$\left[(M_K + M_\pi)^2 \leq k^2 \leq (M_B - \sqrt{q^2})^2\right]$$

On-shell and S-Wave

- one usually assumes on-shell decay of P-wave K^* ($\sim \sin\theta_{K^*}, \cos\theta_{K^*}$)
- for high precision: consider width of K^*, and $J = 0$ (S-wave) ($\not\sim \theta_{K^*}$) $K\pi$-final-state from K_0^* and *non-resonant background*

Kinematics of $\bar{B} \to \bar{K}\pi\ell^+\ell^-$



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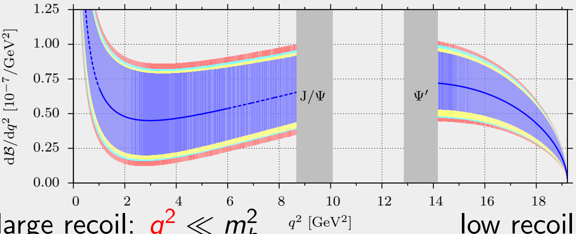
Large vs. Low Recoil (for illustration)



large recoil: $q^2 \ll m_b^2$ low recoil: $q^2 \simeq m_b^2$

Angular Distribution [Krüger/Matias '05, Altmannshofer et al. '08]

Differential Decay Rate for pure P-wave state

$$
\begin{aligned}
\frac{\mathrm{d}^4\Gamma}{\mathrm{d}q^2\mathrm{d}\cos\theta_\ell\mathrm{d}\cos\theta_{K^*}\mathrm{d}\phi} \sim\ & J_{1s}\sin^2\theta_{K^*} + J_{1c}\cos^2\theta_{K^*} \\
&+ (J_{2s}\sin^2\theta_{K^*} + J_{2c}\cos^2\theta_{K^*})\cos 2\theta_\ell \\
&+ (J_3\cos 2\phi + J_9\sin 2\phi)\sin^2\theta_{K^*}\sin^2\theta_\ell \\
&+ (J_4\sin 2\theta_{K^*})\sin 2\theta_\ell\cos\phi \\
&+ (J_5\sin 2\theta_{K^*})\sin\theta_\ell\cos\phi \\
&+ (J_{6s}\sin^2\theta_{K^*} + J_{6c}\cos^2\theta_{K^*})\cos\theta_\ell \\
&+ (J_7\sin 2\theta_{K^*})\sin\theta_\ell\sin\phi \\
&+ (J_8\sin 2\theta_{K^*})\sin 2\theta_\ell\sin\phi\,,
\end{aligned}
$$

$J_i \equiv J_i(q^2)$: 12 angular observables

Angular Distribution [Krüger/Matias '05, Altmannshofer et al. '08, Blake/Egede/Shires '12]

Differential Decay Rate for mixed P- and S-wave state

$$
\begin{aligned}
\frac{d^4\Gamma}{dq^2 d\cos\theta_\ell d\cos\theta_{K^*} d\phi} \sim\ & J_{1s}\sin^2\theta_{K^*} + J_{1c}\cos^2\theta_{K^*} + J_{1i}\cos\theta_{K^*} \\
& + (J_{2s}\sin^2\theta_{K^*} + J_{2c}\cos^2\theta_{K^*} + J_{2i}\cos\theta_{K^*})\cos 2\theta_\ell \\
& + (J_3\cos 2\phi + J_9\sin 2\phi)\sin^2\theta_{K^*}\sin^2\theta_\ell \\
& + (J_4\sin 2\theta_{K^*} + J_{4i}\cos\theta_{K^*})\sin 2\theta_\ell\cos\phi \\
& + (J_5\sin 2\theta_{K^*} + J_{5i}\cos\theta_{K^*})\sin\theta_\ell\cos\phi \\
& + (J_{6s}\sin^2\theta_{K^*} + J_{6c}\cos^2\theta_{K^*})\cos\theta_\ell \\
& + (J_7\sin 2\theta_{K^*} + J_{7i}\cos\theta_{K^*})\sin\theta_\ell\sin\phi \\
& + (J_8\sin 2\theta_{K^*} + J_{8i}\cos\theta_{K^*})\sin 2\theta_\ell\sin\phi\,,
\end{aligned}
$$

$J_i \equiv J_i(q^2, k^2)$: 12 angular observables, no further needed [Bobeth/Hiller/DvD '12]

Conclusion: remove S-wave in exp. analysis

- angular analysis [Egede/Blake/Shires '12]
- side-band analysis (for $J_{1s,1c,2s,2c}$) [Bobeth/Hiller/DvD '12]

Building Blocks of the Angular Observables (I)

Form Factors (P-Wave)

- hadronic matrix elements $\langle \bar{K}^* | \bar{s} \Gamma b | \bar{B} \rangle$ parametrized through 7 form factors:

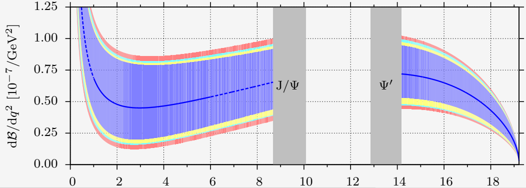
$$\langle \bar{K}^* | \bar{s}\gamma^\mu b | \bar{B} \rangle \sim V \quad \langle \bar{K}^* | \bar{s}\gamma^\mu \gamma_5 b | \bar{B} \rangle \sim A_{0,1,2} \quad \langle \bar{K}^* | \bar{s}\sigma^{\mu\nu} b | \bar{B} \rangle \sim T_{1,2,3}$$

- form factors largest source of theory uncertainty
  amplitude $\sim 10\% - 15\% \Rightarrow$ observables: $\sim 20\% - 50\%$
  - available from Light Cone Sum Rules [Ball/Zwicky '04, Khodjamirian et al. '11]
  - Lattice QCD: work in progress [e.g. Liu et al. '11, Wingate '11]
  - extract ratios from low recoil data

    [Hambrock/Hiller '12, Beaujean/Bobeth/DvD/Wacker '12]

blue band:

form factor uncertainty



Building Blocks of the Angular Observables (II)

Transversity amplitudes A_i

- SM-like + chirality flipped: essentially four amplitudes $A_{\perp,\parallel,0,t}$ [Krüger/Matias '05]
- $\mathcal{O}_{S(')}$ give rise to A_S, $\mathcal{O}_{P(')}$ absorbed by A_t [Altmannshofer et al. '08]
- $\mathcal{O}_{T(5)}$ give rise to *6 new* amplitudes A_{ab}, $(ab)=(0t),(\parallel\perp),(0\perp),(t\perp),(0\parallel),(t\parallel)$ [Bobeth/Hiller/DvD '12]
- altogether: 11 complex-valued amplitudes

Angular Observables

- J_i functionals of A_S, A_a, A_{ab}, $a, b = t, 0, \parallel, \perp$ e.g.

$$J_3(q^2) = \frac{3\beta_\ell}{4}\left[|A_\perp|^2 - |A_\parallel|^2 + 16\left(|A_{t\parallel}|^2 + |A_{0\parallel}|^2 - |A_{t\perp}|^2 - |A_{0\perp}|^2\right)\right]$$

β_ℓ: lepton velocity in dilepton rest frame $m_\ell^2/q^2 \to 0 \Rightarrow \beta_\ell \to 1$

"Standard" Observables

considerable theory uncertainty due to form factors

Batch #1, to be extracted from CP average

$$\langle\Gamma\rangle = \langle 2J_{1s} + J_{1c} - \frac{2}{3}J_{2s} - \frac{1}{3}J_2 c\rangle \qquad \langle A_{\mathrm{FB}}\rangle = \frac{\langle 2J_{6s} + J_{6c}\rangle}{2\langle\Gamma\rangle}$$

$$\langle F_L\rangle = \frac{\langle 3J_{1c} - J_2 c\rangle}{\langle 3\Gamma\rangle} \qquad \langle F_T\rangle = \frac{\langle 6J_{1s} - 2J_{2s}\rangle}{\langle 3\Gamma\rangle}$$

Γ: decay width A_{FB}: forward-backward asymm. $F_L = 1 - F_T$: long./trans. pol.

Batch #2, CP (a)symmetries [Bobeth/Hiller/Piranishvili '08, Altmannshofer et al. '08]

$$\langle A_i\rangle \sim \frac{\langle J_i - \overline{J_i}\rangle}{\langle\Gamma + \overline{\Gamma}\rangle} \qquad \langle S_i\rangle \sim \frac{\langle J_i + \overline{J_i}\rangle}{\langle\Gamma + \overline{\Gamma}\rangle}$$

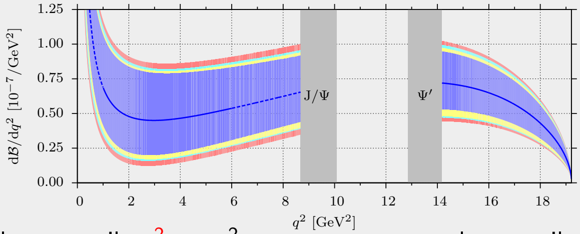
overline: CP conjugated mode, also: mixing-induced CP asymm in $B_s \to \phi\ell^+\ell^-$

$\langle X\rangle \equiv \int \mathrm{d}q^2\, X(q^2)$

Pollution due to Charm Resonances

Narrow Resonances: J/ψ and $\psi(2s)$

- experiments veto q^2-region of narrow charmonia J/ψ and $\psi(2s)$
- however: resonance affects observables outside the veto!



large recoil: $q^2 \ll m_b^2$ low recoil: $q^2 \simeq m_b^2$

Approach by Theorists: Divide and Conquer

- treat region below J/ψ (aka *large recoil*) differently than above $\psi(2s)$
- design combinations of J_i which have reduced theory uncertainty in only one kinematic region

Large Recoil (I)

QCD Factorization (QCDF) + Soft Collinear Effective Theory (SCET)

- calculate $\bar{q}q$ loops perturbatively, expand in $1/m_b$, $1/E_{K^*}$
- relate matrix elements to universal hadronic quantities:
  - Light Cone Distribution Amplitudes (LCDAs)
  - form factors
  - decay constants

[Beneke/Feldmann '00, Beneke/Feldmann/Seidel '01 & '04]

Light Cone Sum Rules (LCSR)

- calculate $\langle \bar{c}c \rangle$, $\langle \bar{c}cG \rangle$ on the light cone for $q^2 \ll 4m_c^2$
- achieves resummation of soft gluon effects
- use analycity of amplitude to relate results to $q^2 < M_{\psi'}^2$
- uses many of the same inputs as QCDF+SCET
- includes parts of QCDF+SCET results

Large Recoil (I)

QCD Factorization (QCDF) + Soft Collinear Effective Theory (SCET)

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  - form factors
  - decay constants

[Beneke/Feldmann '00, Beneke/Feldmann/Seidel '01 & '04]

Combination of QCDF+SCET and LCSR Results

- not yet!
  - no studies yet to find impact on optimized observables at large recoil!
  - LCSR results are not included in following discussion

Large Recoil (II)

SM + chirality flipped

- transversity amplitudes factorize up to power supressed terms

$$A_\perp^{L,R} \sim X_\perp^{L,R} \times \xi_\perp \qquad A_\parallel^{L,R} \sim X_\parallel^{L,R} \times \xi_\perp \qquad A_0^{L,R} \sim X_0^{L,R} \times \xi_\parallel$$

$\xi_{\perp,\parallel}$: soft form factors $\qquad$ $X_i^{L,R}$: combinations of Wilson coefficients

[Beneke/Feldmann/Seidel '01 & '04, Bobeth/Hiller/Piranishvili '08, Altmannshofer et al. '08]

Optimized Observables

enhanced sensitivity to right-handed currents, reduced form factor dependence

[Krüger/Matias '05, Egede et al. '08 & '10]

$$A_T^{(2)} = \frac{|A_\perp|^2 - |A_\parallel|^2}{|A_\perp|^2 + |A_\parallel|^2} \sim J_3 \qquad A_T^{(3)} = \frac{|A_0^L A_\parallel^{L*} + A_0^{R*} A_\parallel^R|}{\sqrt{|A_0|^2 |A_\parallel|^2}} \sim J_4, J_7$$

$$A_T^{(4)} = \frac{|A_0^L A_\perp^{L*} - A_0^{R*} A_\perp^R|}{\sqrt{|A_0|^2 |A_\perp|^2}} \sim J_5, J_8 \qquad A_T^{(5)} = \frac{|A_\perp^L A_\parallel^{R*} + A_\perp^{R*} A_\parallel^L|}{|A_\perp|^2 + |A_\parallel|^2}$$

Large Recoil (II)

SM + chirality flipped

- transversity amplitudes factorize up to power supressed terms

$$A_\perp^{L,R} \sim X_\perp^{L,R} \times \xi_\perp \qquad A_\parallel^{L,R} \sim X_\parallel^{L,R} \times \xi_\perp \qquad A_0^{L,R} \sim X_0^{L,R} \times \xi_\parallel$$

$\xi_{\perp,\parallel}$: soft form factors $X_i^{L,R}$: combinations of Wilson coefficients

[Beneke/Feldmann/Seidel '01 & '04, Bobeth/Hiller/Piranishvili '08, Altmannshofer et al. '08]

Further Optimized Observables

enhanced sensitivity to right-handed currents, reduced form factor dependence

[Becirevic/Schneider '11]

$$A_T^{(\mathrm{re})} \propto \frac{J_{6s}}{J_{2s}} \qquad A_T^{(\mathrm{im})} \propto \frac{J_9}{J_{2s}}$$

Low Recoil

SM basis [Bobeth/Hiller/DvD '10] + chirality flipped [Bobeth/Hiller/DvD '12]

- transversity amplitudes factorize

$$A^{L,R}_{\perp,\parallel,0} \sim C^{L,R}_{\pm} \times f_{\perp,\parallel,0} + O\left(\frac{\alpha_s \Lambda}{m_b}, \frac{\mathcal{C}_7 \Lambda}{\mathcal{C}_9 m_b}\right) \quad \text{SM:} \quad C^{L,R}_{+} = C^{L,R}_{-}$$

f_i: helicity form factors $C^{L,R}_{\pm}$: combinations of Wilson coeff.

- 4 combinations of Wilson coefficients enter observables:

$$\rho_1^{\pm} \sim |C^R_{\pm}|^2 + |C^L_{\pm}|^2$$
$$\mathrm{Re}\,(\rho_2) \sim \mathrm{Re}\left(C^R_+ C^{R*}_- - C^L_- C^{L*}_+\right) \quad \text{and } \mathrm{Re}\,(\cdot) \leftrightarrow \mathrm{Im}\,(\cdot)$$

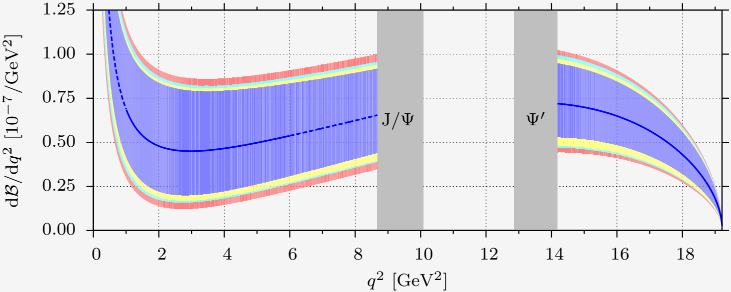
Tensor operators [Bobeth/Hiller/DvD '12]

- 6 new transversity amplitudes, still factorize!

$$A_{ab} \sim \mathcal{C}_{T(T5)} \times f_{\perp,\parallel,0} + O\left(\frac{\Lambda}{m_b}\right)$$

- 3 new combinations of Wilson coefficients

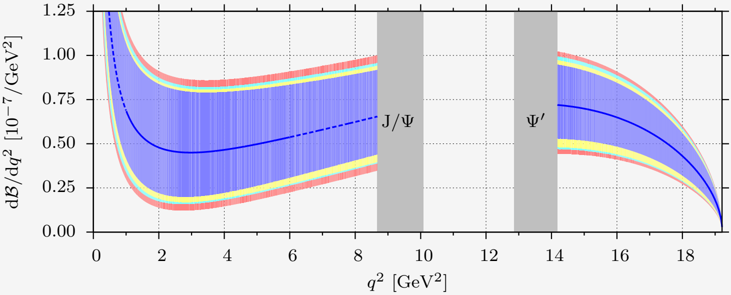
q^2 Spectrum of the Branching Ratio $\mathcal{B} = \tau_B \Gamma$



$\bar{q}q$ Pollution

- 4-quark operators like $\mathcal{O}_{1c,2c}$ induce $b \to s\bar{c}c(\to \ell^+\ell^-)$ via loops
- hadronically $B \to K^* J/\psi(\to \ell^+\ell^-)$ or higher charmonia
- experiment: cut narrow resonances $J/\psi \equiv \psi(1S)$ and $\psi' = \psi(2S)$
- theory: handle non-resonant quark loops/broad resonances $> 2S$

q^2 Spectrum of the Branching Ratio $\mathcal{B} = \tau_B \Gamma$



Large Recoil $E_{K^*} \sim m_b$ QCDF,SCET

- expand in $1/m_b$, $1/E_{K^*}$, α_s
- symmetry: $7 \rightarrow 2$ form factors

[Beneke/Feldmann/Seidel '01 & '04]

[Egede et al. '08 & '10]

Low Recoil $q^2 \sim m_b^2$ OPE,HQET

- expand in $1/m_b$, $1/\sqrt{q^2}$, α_s
- symmetry: $7 \rightarrow 4$ form factors

[Grinstein/Pirjol '04], [Beylich/Buchalla/Feldmann '11]

[Bobeth/Hiller/DvD '10 & '11]