Bayesian Constraints on Wilson Coefficients from Radiative and (Semi)leptonic $b \to s$ Decays

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in collaboration with
Frederik Beaujean and Christoph Bobeth
based on arxiv:1310.2478

Implications of LHCb measurements and future prospects
October 15th 2013
Effective Field Theory for $b \to s\ell^+\ell^-$ FCNCs

Flavor Changing Neutral Current (FCNC)

- expand amplitudes in $G_F \sim 1/M_W^2$ (OPE)
- operators (matrix elem. below $\mu_b \simeq m_b$)

$$O_i \equiv [\bar{s}\Gamma_i b] [\bar{\ell}\Gamma_i'\ell]$$

- Wilson coefficients (above $\mu_b \simeq m_b$)

$$C_i \equiv C_i(M_W, M_Z, m_t, \ldots)$$

- use $C_i = C_i(\mu_b = 4.2\text{GeV})$

Effective Hamiltonian

$$\mathcal{H} = -\frac{4G_F\alpha_e}{\sqrt{2}\frac{4\pi}{4\pi}} \left[ V_{tb} V_{ts}^* \sum_i C_i O_i + O(V_{ub} V_{us}^*) \right] + \text{h.c.}$$
Effective Field Theory for $b \to s \ell^+ \ell^-$ FCNCs

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Operators

$$O_{7(')} = [\bar{s} \sigma^{\mu\nu} P_{R(L)} b] F_{\mu\nu}$$

$$O_{9(')} = [\bar{s} \gamma^\mu P_{L(R)} b][\ell \gamma_\mu \ell]$$

$$O_{10(')} = [\bar{s} \gamma^\mu P_{L(R)} b][\ell \gamma_\mu \gamma_5 \ell]$$
Effective Field Theory for $b \rightarrow s\ell^+\ell^-$ FCNCs

Flavor Changing Neutral Current (FCNC)

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### Decay Modes

<table>
<thead>
<tr>
<th>$B \rightarrow K^*\ell^+\ell^-$</th>
<th>$B_s \rightarrow \mu^+\mu^-$</th>
<th>$B \rightarrow K\ell^+\ell^-$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B \rightarrow K^*\gamma$</td>
<td>$B \rightarrow X_s\ell^+\ell^-$</td>
<td>$B \rightarrow X_s\gamma$</td>
</tr>
</tbody>
</table>

\[|\Delta B| = |\Delta S| = 1\] Wilson Coefficients
Definition of model-independent for the purpose of this work:

### Basis of Operators $O_i$
- include as many $O_i$ beyond SM as needed/as few as possible
- balancing act, test statistically if choice of basis describes data well!

### Wilson Coefficients $C_i$
- treat $C_i$ as uncorrelated, generalized couplings
- constrain their values from data
- model builders: confront new physics models with constraints
Fit Scenarios: SM(ν-only)

**SM-like Coefficients**

- fix $C_{7,9,10}$ to SM values (NNLL)

**Chirality-flipped Coefficients**

- fix $C_{7'} = m_s/m_b C_7$, fix $C_{9',10'} = 0$

**Nuisance Parameters**

- fit nuisance parameters
- informative priors
  - form factors: light-cone sum rules \cite{KhodjamirianMannelPivovarovWang10}
  - power corrections: power-counting assumptions
  - CKM: tree-level fit \cite{UTfit}
  - quark masses \cite{PDG}
SM-like Coefficients

- fit $C_{7,9,10}$

Chirality-flipped Coefficients

- fix $C_{7'} = m_s/m_b C_7$, fix $C_{9',10'} = 0$

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  - power corrections: power-counting assumptions
  - CKM: tree-level fit [UTfit]
  - quark masses [PDG]
Fit Scenarios: SM + SM' Basis

SM-like Coefficients

- fit $C_{7,9,10}$

Chirality-flipped Coefficients

- fit $C'_{7,9,10}$

Nuisance Parameters

- fit nuisance parameters
- informative priors
  - form factors: light-cone sum rules [Khodjamirian/Mannel/Pivovarov/Wang '10]
  - power corrections: power-counting assumptions
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  - quark masses [PDG]
Sensitivity to Fit Parameters

### Wilson Coefficients

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<thead>
<tr>
<th>Process</th>
<th>$C_7('')$</th>
<th>$C_9('')$</th>
<th>$C_{10}('')$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_s \to \mu^+ \mu^-$</td>
<td>✓</td>
<td></td>
<td>✓</td>
</tr>
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<td>$B \to X_s \gamma$</td>
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<td>✓</td>
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### Form Factors

- interplay between $B \to X_s \{\gamma, \ell^+ \ell^-\}$ and $B \to K^* \{\gamma, \ell^+ \ell^-\}$
- some $B \to K^* \ell^+ \ell^-$ obs. form-factor insensitive by construction
- some $B \to K^* \ell^+ \ell^-$ obs. dominantly sensitive to form factor ratios
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<tr>
<td>$B \rightarrow K^* \ell^+ \ell^-$</td>
<td>$q^2 \in [1, 6] \text{GeV}^2$, $q^2 \geq M_{\psi}^2$</td>
<td>- $B$, $A_{FB}$, $F_L$, $A_T^2$</td>
<td>ATLAS, BaBar, Belle, CDF, CMS, LHCb</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>new</strong>: $A_T^{re}$, $P_4'$, $P_5'$, $P_6'$</td>
<td>see also talk by N. Mahmoudi</td>
</tr>
<tr>
<td>$B \rightarrow K^* \gamma$</td>
<td></td>
<td>- $B$, $S_{K^<em>\gamma}$, $C_{K^</em>\gamma}$</td>
<td>BaBar, Belle, CLEO</td>
</tr>
<tr>
<td>$B \rightarrow X_s \gamma$</td>
<td>$E_{\gamma}^{\text{min}} = 1.8 \text{ GeV}$</td>
<td>- $B$</td>
<td>BaBar, Belle, CLEO</td>
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<td>$B_s \rightarrow \mu^+ \mu^-$</td>
<td></td>
<td>- $\int d\tau B(\tau)$</td>
<td>CMS, LHCb</td>
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Form Factors from Lattice QCD (LQCD)

- $B \to K$ form factors available from LQCD
  - data only at high $q^2$: 17 – 23 GeV$^2$
  - no data points given
- reproduce 3 data points from $z$-parametrization
  - $q^2 \in \{17, 20, 23\}$ GeV$^2$
  - use as constraint, incl. covariance matrix

$B \to K^*$ Form Factor (FF) Relation at $q^2 = 0$

- FF $V, A_1 \propto \xi_\perp + \ldots$ [Charles et al. hep-ph/9901378]
  - Large Energy Limit: $V(0) = (1.33 \pm 0.4) \times A_1(0)$
- LCSR constraint: $\xi_\parallel(0) = 0.10^{+0.03}_{-0.02}$, to avoid $\xi_\parallel(q^2) \propto A_0(q^2) < 0$.
- see also FF fits by [Hambrock/Hiller/Schacht/Zwicky 1308.4379]
The $B \to K^* \ell^+ \ell^-$ “Anomaly”

- **LHCb measurement** [1308.1707]
  - deviation from SM prediction in form factor-free obs. $\langle P'_5 \rangle_{[1,6]}$
  - LHCb uses one SM prediction (DGHMV)
    [Descotes-Genon/Hurth/Matias/Virto 1303.5794]

- however: further SM prediction exist, much larger uncertainty (JC) [Jäger/Camalich 1212.2263]

- our take on SM prediction $\langle P'_5 \rangle_{[1,6]} = -0.34^{+0.09}_{-0.08}$ (BBvD)
  see also backups for $P'_{4,6}$ and [2, 4.3] bins

  difference: treatment of unknown power corrections
  (form factor corrections, $\bar{c}c$ resonances)
Pull Values at Best-Fit Point

- largest pulls
  \(-3.4 \sigma \langle F_L \rangle_{[1,6]}, \text{BaBar 2012}\)
  \(+2.5 \sigma \langle B \rangle_{[16,19,21]}, \text{Belle 2009}\)
  \(-2.6 \sigma \langle F_L \rangle_{[1,6]}, \text{ATLAS 2013}\)
  \(+2.2 \sigma \langle A_{FB} \rangle_{[16,19]}, \text{ATLAS 2013}\)
  \(-2.4 \sigma \langle P'_4 \rangle_{[14,18,16]}, \text{LHCb 2013}\)
  \(+2.1 \sigma \langle P'_5 \rangle_{[1,6]}, \text{LHCb 2013}\)

\(p\) Values

- \(p\) value 0.15
- taking out ATLAS, BaBar \(\langle F_L \rangle_{[1,6]}\): \(p\) value increases to 0.71

Summary

- good fit, no New Physics signal
- we find power corrections on top of QCDF results at large recoil
Parametrization of Power Corrections @ Large Recoil

- six parameters $\zeta_L(R)$ for the $[1, 6]$ bin

$$A_L(R)(q^2) \mapsto \zeta_L(R) A_L(R)(q^2), \quad \chi = \perp, \parallel, 0$$

- on top of QCDF correction to transversity amplitudes

- tension diluted by parameters $\zeta_L(R)$

- shift by $\simeq -20\%$ for $\zeta_{\perp, \parallel}^L$

- shift by $\simeq +10\%$ for $\zeta_0^L$

- few percents for $\zeta_R^R$

improved understanding of power corrections desirable
Results (SM Basis)

post HEP’13 (selection)

- with $B \to X_s \{\gamma, \ell^+\ell^-\}$
- $B_s \to \mu^+\mu^-$ from LHCb and CMS
- same data as
  
  [Descotes-Genon/Matias/Virto 1307.5683]
  exclusive decays limited:
  - only $B \to K^* \ell^+\ell^-$!
  - only LHCb data!
  - only $q^2 \in [1, 6] \text{GeV}^2$

- best-fit point similar to
  
  [Descotes-Genon et al. 1307.5683]
  - less tension, only $\lesssim 2\sigma$
  - $C_9 - C_9^{\text{SM}} \simeq -1.3 \pm 0.5$

---

♦: Standard Model, ×: best-fit point
(light-) red: 68% CL (95% CL) for full dataset
blue solid (blue dashed): 68% CL (95% CL) for selection
Results (SM Basis)

post HEP’13 (full)

- SM-like uncertainty reduced by $\sim 2$ compared to 2012
- SM at the border of 1$\sigma$
- flipped-sign barely allowed at 1$\sigma$ (26% of evidence)
- cannot confirm NP findings
  - in $(C_7, C_9)$
- $\zeta_L(R)$ as in SM($\nu$-only)
- $p$ value: 0.13 (@SM-like sol.)

$\Delta B = |\Delta S| = 1$ Wilson Coefficients
Results (SM+SM′ Basis)

- four solutions $A'$ through $D'$
  - $A'$ = SM like, 39% of ev.
  - $B'$ = flipped signs, 41% of ev.
  - $C'$, $D'$ suppressed: 5% and 15% of evidence

- for $A'$ (SM-like sol.)
  - $p$ value $0.17$
  - $C_9 - C_9^{SM} = -0.8^{+0.2}_{-0.5}$
  - $2\sigma$ deviation from SM
  - $\zeta^L(R)$ decrease wrt. SM($\nu$-only) and SM basis

♦: Standard Model, ×: best-fit points, (light-) red: 68% CL (95% CL) for full dataset
model comparison using Bayes factor and model priors
compare scenarios only at SM-like solution \(A'(\nu)\)
adjust priors to contain only \(A'(\nu)\)
results
- \(\text{SM}(\nu\text{-only})\) wins over SM basis: odds of 100:1
- \(\text{SM}(\nu\text{-only})\) wins over \(\text{SM}+\text{SM}'\) basis: odds of 22:1
- \(\text{SM}+\text{SM}'\) basis wins over SM basis: odds of 4.5:1
• all three scenarios describe $b \to s(\gamma, \ell^+\ell^-)$ data well
• SM(\nu-only) wins comparison with SM and SM+SM'
  ▶ subleading power correction on top of QCDF: 10–20%
• several tensions in all scenarios compare talk by D. Straub
  ▶ $\langle P_5' \rangle_{[1,6]}$ reduced pull in fit due to power corrections
  ▶ $\langle F_L \rangle_{[1,6]}$ from BaBar, ATLAS (both preliminary) persist
  ▶ $\langle P_4' \rangle_{[14,18,16]}$ LHCb persists
• new physics signal only for
  ▶ SM+SM' basis
  ▶ (also: SM basis with “post HEP’13 (selection)” subset of data)
• data also allows inference of form factor parameters
• looking forward to further LHC analyses (2012 datasets) and the prospects of Belle-II
Backup Slides
(Angular) Observables in $B \rightarrow K^* \ell^+ \ell^-$

- kinematics
  - dilepton mass squared $q^2$
  - three angles
- complicated diff. decay width
  - 12(+) angular observables $J_n$
  - express all observables through $J_n$
  - compose observ. from $J_n$ with specific benefits

### Definitions

$$\Gamma \sim 3J_{1c} + 6J_{1s} - J_{2c} - 2J_{2s} \quad A_{\text{FB}} \sim \frac{J_{6s}}{\Gamma}$$

$$P_4' \sim \frac{+J_4}{\sqrt{-J_{2s}J_{2c}}} \quad P_5' \sim \frac{+J_5}{2\sqrt{-J_{2s}J_{2c}}} \quad P_6' \sim \frac{-J_7}{2\sqrt{-J_{2s}J_{2c}}}$$

$$F_L \sim \frac{3J_{1c} - J_{2c}}{\Gamma}$$
Standard Model Predictions for $P'_{4,5,6}$

- toy Monte Carlo using priors + theory constraints (FFs)
- calculate observable for $10^5$ samples
- find minimal 68% CL intervals

<table>
<thead>
<tr>
<th>$q^2$ [GeV^2]</th>
<th>$\langle P'_4 \rangle$</th>
<th>$\langle P'_5 \rangle$</th>
<th>$10^2 \times \langle P'_6 \rangle$</th>
</tr>
</thead>
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<tr>
<td>BBvD</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[1, 6]</td>
<td>+0.46 $^{+0.12}_{-0.11}$</td>
<td>$-0.335$ $^{+0.085}_{-0.075}$</td>
<td>$-6.4$ $^{\pm 1.7}_{-1.7}$</td>
</tr>
<tr>
<td>[2, 4.3]</td>
<td>+0.48 $^{+0.11}_{-0.10}$</td>
<td>$-0.315$ $^{+0.074}_{-0.090}$</td>
<td>$-7.2$ $^{+1.5}_{-2.2}$</td>
</tr>
<tr>
<td>LHCb†</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[1, 6]</td>
<td>+0.58 $^{+0.33}_{-0.36}$</td>
<td>$+0.21$ $^{+0.20}_{-0.21}$</td>
<td>$+18$ $^{\pm 21}_{-21}$</td>
</tr>
<tr>
<td>[2, 4.3]</td>
<td>+0.74 $^{+0.11}_{-0.53}$</td>
<td>$+0.29$ $^{+0.40}_{-0.39}$</td>
<td>$+15$ $^{+38}_{-36}$</td>
</tr>
</tbody>
</table>

†: \[\text{LHCb 1308.1707}\], adjusted to theory convention
2012 Results (SM Basis)

early 2012

- figure from [1205.1838]
- no $B \rightarrow X_s \{\gamma, \ell^+\ell^-\}$
- only LHCb bound on $B_s \rightarrow \mu^+\mu^-$
- $B \rightarrow K^{(*)}\ell^+\ell^-:$ $\mathcal{B}, A_{FB}, F_L, A_{T}^{(2)}, S_3$
- $B \rightarrow K^*\gamma:$ $\mathcal{B}, S_{K^*\gamma}, C_{K^*\gamma}$

$\Delta B = |\Delta S| = 1$ Wilson Coefficients

D. van Dyk (U. Siegen)
Results (SM Basis, Selection)

post HEP’13 (selection)

- with $B \rightarrow X_s \{\gamma, \ell^+\ell^-\}$
- $B_s \rightarrow \mu^+\mu^-$ from LHCb and CMS
- same data as

  [Descotes-Genon/Matias/Virto 1307.5683]

  exclusive decays limited:
  - only $B \rightarrow K^* \ell^+\ell^-$!
  - only LHCb data!
  - only $q^2 \in [1, 6]$ GeV$^2$

- best-fit point similar to

  [Descotes-Genon et al. 1307.5683]

  - less tension, only \( \lesssim 2\sigma \)
  - $C_9 - C_9^{SM} \simeq -1.3 \pm 0.5$

\(\blacklozenge\) Standard Model, \(\times\) best-fit point

(light-) red: 68% CL (95% CL) for full dataset

blue solid (blue dashed): 68% CL (95% CL) for selection
Results for $B \to K^*$ Form Factors

- more precise than prior
- $B \to K^*$: $\xi_\perp$ from
  - $B \to X_s\gamma$
  - $B \to K^*\gamma$
  - $B \to K^*\ell^+\ell^-$
  - theory input
- results @ 68% CL
  - $V(0) = 0.37^{+0.03}_{-0.02}$

dashed: prior [Khodjamirian/Mannel/Pivovarov/Wang 1006.4945]
solid: posterior, green: 68% CL, yellow: 95% CL
Results for $B \to K^*$ Form Factors

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  - $B \to X_s \gamma$
  - $B \to K^* \gamma$
  - $B \to K^* \ell^+ \ell^-$
  - theory input
- results @ 68% CL
  - $V(0) = 0.37^{+0.03}_{-0.02}$
  - $A_1(0) = 0.24 \pm 0.03$

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  - $B \to X_s \gamma$
  - $B \to K^* \gamma$
  - $B \to K^* \ell^+ \ell^-$
  - theory input
- results @ 68% CL
  - $V(0) = 0.37^{+0.03}_{-0.02}$
  - $A_1(0) = 0.24 \pm 0.03$
  - $A_2(0) = 0.22 \pm 0.04$

---

- dashed: prior [Khodjamirian/Mannel/Pivovarov/Wang 1006.4945]
- solid: posterior, green: 68% CL, yellow: 95% CL
Results for $B \to K^*$ Form Factors

- more precise than prior
- $B \to K^*$: $\xi_\perp$ from $B \to X_s\gamma$
  - $B \to K^*\gamma$
  - $B \to K^*\ell^+\ell^-$
  - theory input
- results @ 68% CL
  - ratio of central values
    $V(0)/A_1(0) \approx 1.5$
    $A_2(0)/A_1(0) \approx 0.9$
  - agree w/ (SE2 full)
    [Hambrock/Hiller/Schacht/Zwicky 1308.4379]

---

dashed: prior [Khodjamirian/Mannel/Pivovarov/Wang 1006.4945]
solid: posterior, green: 68% CL, yellow: 95% CL
Results for $B \to K$ Form Factors

- more precise than prior
- $B \to K$:
  - $B \to K \ell^+ \ell^-$
  - Lattice
- results @ 68% CL
  - $f_+(0) = 0.30 \pm 0.02$

dashed: prior [Khodjamirian/Mannel/Pivovarov/Wang 1006.4945], modified to accomodate [Ball/Zwicky hep-ph/0406232]
solid: posterior, green: 68% CL, yellow: 95% CL
Results for $B \rightarrow K$ Form Factors

- more precise than prior
- $B \rightarrow K$:
  - $B \rightarrow K \ell^+ \ell^-$
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  - $f_+(0) = 0.30 \pm 0.02$
  - $b_1^+ = -2.5 \pm 0.4$

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dashed: prior [Khodjamirian/Mannel/Pivovarov/Wang 1006.4945],

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Results for $B \rightarrow K$ Form Factors

- more precise than prior
- $B \rightarrow K$:
  - $B \rightarrow K \ell^+ \ell^-$
  - Lattice
- results @ 68% CL
  - $f_+(0) = 0.30 \pm 0.02$
  - $b_1^+ = -2.5 \pm 0.4$
- small tension
  - LHCb lo $q^2$: $-1.4\sigma$
  - LHCb hi $q^2$: $+1.1\sigma$
  - Lattice: $+0.5\sigma$

dashed: prior [Khodjamirian/Mannel/Pivovarov/Wang 1006.4945],
solid: posterior, green: 68% CL, yellow: 95% CL
Form Factors [Khodjamirian et al. 1006.4945]

- values @ $q^2 = 0$ and slope: two parameters per FF
- $z$-parametrization
- asymmetric priors, use LogGamma function

CKM [update of hep-ph/0012308]

- Wolfenstein parametrization
- UTfit pre-Moriond2013, tree-level data only

Quark Masses [PDG]
parametrize unknown subleading contributions

\( B \rightarrow K^* \ell^+ \ell^- \)
- \( \text{lo } q^2 \): 6 parameters, one scaling factor per amplitude
- \( \text{hi } q^2 \): 3 parameters

\( B \rightarrow K \ell^+ \ell^- \)
- \( \text{lo } q^2 \): 1 parameter
- \( \text{hi } q^2 \): 1 parameter

for all: Gaussian with \( 1\sigma \) interval \( \pm \Lambda_{QCD}/m_b \simeq \pm 0.15 \)
test statistic: function of data and model (parameters) $\chi^2 = \chi^2(D, \vec{\theta})$
only one data set observed $D_{obs} \Rightarrow \chi^2_{obs}$
$p \equiv P(\chi^2 > \chi^2_{obs})$

But how to fix $\vec{\theta}$?

1. this work
   $\vec{\theta} = (\vec{C}, \vec{\nu})$ at (local) mode of posterior, $\chi^2 \sim \frac{(x-\mu)^2}{\sigma^2_{exp}}$

2. Descotes-Genon et al. [1307.5683]
   $\vec{\theta} = \vec{C}$ at (local) mode of likelihood, $\chi^2 \sim \frac{(x-\mu)^2}{\sigma^2_{exp} + \sigma^2_{theo}}$
**Kinematics of $\bar{B} \rightarrow \bar{K} \pi \ell^+ \ell^-$**

**Kinematic Variables**

\[
4m_\ell^2 \leq q^2 \leq (M_B - M_{K^*})^2
\]

\[-1 \leq \cos \theta_\ell \leq 1\]

\[-1 \leq \cos \theta_{K^*} \leq 1\]

\[0 \leq \phi \leq 2\pi\]

\[
[(M_K + M_\pi)^2 \leq k^2 \leq (M_B - \sqrt{q^2})^2]
\]

---

**On-shell and S-Wave**

- one usually assumes on-shell decay of P-wave $K^*$ ($\sim \sin \theta_{K^*}, \cos \theta_{K^*}$)
- for high precision: consider width of $K^*$, and $J = 0$ (S-wave) ($\sim \theta_{K^*}$)

$K\pi$-final-state from $K_0^*$ and *non-resonant background*
Kinematics of $\bar{B} \rightarrow \bar{K} \pi \ell^+ \ell^-$

**Kinematic Variables**

\[
4m_{\ell}^2 \leq q^2 \leq (M_B - M_{K^*})^2 \\
-1 \leq \cos \theta_{\ell} \leq 1 \\
-1 \leq \cos \theta_{K^*} \leq 1 \\
0 \leq \phi \leq 2\pi \\
[(M_K + M_\pi)^2 \leq k^2 \leq (M_B - \sqrt{q^2})^2]
\]

**Large vs. Low Recoil (for illustration)**

- **Large Recoil:** $q^2 \ll m_b^2$
- **Low Recoil:** $q^2 \simeq m_b^2$
Angular Distribution [Krüger/Matias '05, Altmannshofer et al. '08]

Differential Decay Rate for pure P-wave state

\[
\frac{d^4 \Gamma}{dq^2 d \cos \theta_\ell d \cos \theta_{K^*} d \phi} \sim J_{1s} \sin^2 \theta_{K^*} + J_{1c} \cos^2 \theta_{K^*} \\
+ (J_{2s} \sin^2 \theta_{K^*} + J_{2c} \cos^2 \theta_{K^*}) \cos 2\theta_\ell \\
+ (J_3 \cos 2\phi + J_9 \sin 2\phi) \sin^2 \theta_{K^*} \sin^2 \theta_\ell \\
+ (J_4 \sin 2\theta_{K^*}) \sin 2\theta_\ell \cos \phi \\
+ (J_5 \sin 2\theta_{K^*}) \sin \theta_\ell \cos \phi \\
+ (J_{6s} \sin^2 \theta_{K^*} + J_{6c} \cos^2 \theta_{K^*}) \cos \theta_\ell \\
+ (J_7 \sin 2\theta_{K^*}) \sin \theta_\ell \sin \phi \\
+ (J_8 \sin 2\theta_{K^*}) \sin 2\theta_\ell \sin \phi,
\]

\[J_i \equiv J_i(q^2)\): 12 angular observables
### Angular Distribution

[Krüger/Matias '05, Altmannshofer et al. '08, Blake/Egede/Shires '12]

#### Differential Decay Rate for mixed P- and S-wave state

\[
\frac{d^4 \Gamma}{dq^2 d \cos \theta \ell d \cos \theta_{K^*} d \phi} \sim J_{1s} \sin^2 \theta_{K^*} + J_{1c} \cos^2 \theta_{K^*} + J_{1i} \cos \theta_{K^*} \\
+ (J_{2s} \sin^2 \theta_{K^*} + J_{2c} \cos^2 \theta_{K^*} + J_{2i} \cos \theta_{K^*}) \cos 2 \theta \ell \\
+ (J_{3} \cos 2 \phi + J_{9} \sin 2 \phi) \sin^2 \theta_{K^*} \sin^2 \theta \ell \\
+ (J_{4} \sin 2 \theta_{K^*} + J_{4i} \cos \theta_{K^*}) \sin 2 \theta \ell \cos \phi \\
+ (J_{5} \sin 2 \theta_{K^*} + J_{5i} \cos \theta_{K^*}) \sin \theta \ell \cos \phi \\
+ (J_{6s} \sin^2 \theta_{K^*} + J_{6c} \cos^2 \theta_{K^*}) \cos \theta \ell \\
+ (J_{7} \sin 2 \theta_{K^*} + J_{7i} \cos \theta_{K^*}) \sin \theta \ell \sin \phi \\
+ (J_{8} \sin 2 \theta_{K^*} + J_{8i} \cos \theta_{K^*}) \sin 2 \theta \ell \sin \phi,
\]

\[J_i \equiv J_i(q^2, k^2): 12 \text{ angular observables, no further needed} \quad [\text{Bobeth/Hiller/DvD '12}]
\]

### Conclusion: remove S-wave in exp. analysis

- angular analysis [Egede/Blake/Shires '12]
- side-band analysis (for \( J_{1s,1c,2s,2c} \)) [Bobeth/Hiller/DvD '12]
Form Factors (P-Wave)

- hadronic matrix elements $\langle \bar{K}^* | \bar{s} \gamma b | \bar{B} \rangle$ parametrized through 7 form factors:

  $$\langle \bar{K}^* | \bar{s} \gamma^{\mu} b | \bar{B} \rangle \sim V \quad \langle \bar{K}^* | \bar{s} \gamma^{\mu} \gamma_5 b | \bar{B} \rangle \sim A_{0,1,2} \quad \langle \bar{K}^* | \bar{s} \sigma^{\mu \nu} b | \bar{B} \rangle \sim T_{1,2,3}$$

- form factors largest source of theory uncertainty amplitude $\sim 10\% - 15\% \Rightarrow$ observables: $\sim 20\% - 50\%$

  - available from Light Cone Sum Rules [Ball/Zwicky ’04, Khodjamirian et al. ’11]
  - Lattice QCD: work in progress [e.g. Liu et al. ’11, Wingate ’11]
  - extract ratios from low recoil data [Hambrock/Hiller ’12, Beaujean/Bobeth/DvD/Wacker ’12]

blue band:
form factor uncertainty

\[ \frac{dB}{dq^2} \left[ 10^{-7} / \text{GeV}^2 \right] \]
Building Blocks of the Angular Observables (II)

### Transversity amplitudes $A_i$

- SM-like + chirality flipped: essentially four amplitudes $A_{\perp,\parallel,0,t}$ [Krüger/Matias '05]
- $O_S(\prime)$ give rise to $A_S$, $O_P(\prime)$ absorbed by $A_t$ [Altmannshofer et al. '08]
- $O_T(5)$ give rise to 6 new amplitudes $A_{ab}$,
  
  $$(ab)=(0t),(\|\perp),(0\perp),(t\perp),(0\parallel),(t\parallel)$$  [Bobeth/Hiller/DvD '12]
- altogether: 11 complex-valued amplitudes

### Angular Observables

- $J_i$ functionals of $A_S, A_a, A_{ab}$, $a, b = t, 0, \|, \perp$ e.g.

  $$J_3(q^2) = \frac{3\beta_\ell}{4} \left[ |A_{\perp}|^2 - |A_{\parallel}|^2 + 16(|A_{t\parallel}|^2 + |A_{0\parallel}|^2 - |A_{t\perp}|^2 - |A_{0\perp}|^2) \right]$$

  $\beta_\ell$: lepton velocity in dilepton rest frame

  $$m_\ell^2/q^2 \to 0 \Rightarrow \beta_\ell \to 1$$
“Standard” Observables

considerable theory uncertainty due to form factors

Batch #1, to be extracted from CP average

\[
\langle \Gamma \rangle = \langle 2J_{1s} + J_{1c} - \frac{2}{3}J_{2s} - \frac{1}{3}J_{2c} \rangle
\]

\[
\langle A_{FB} \rangle = \frac{\langle 2J_{6s} + J_{6c} \rangle}{2\langle \Gamma \rangle}
\]

\[
\langle F_L \rangle = \frac{\langle 3J_{1c} - J_{2c} \rangle}{3\langle \Gamma \rangle}
\]

\[
\langle F_T \rangle = \frac{\langle 6J_{1s} - 2J_{2s} \rangle}{3\langle \Gamma \rangle}
\]

\(\Gamma\): decay width  \(A_{FB}\): forward-backward asymm.  \(F_L = 1 - F_T\): long./trans. pol.

Batch #2, CP (a)symmetries [Bobeth/Hiller/Piranishvili '08, Altmannshofer et al. '08]

\[
\langle A_i \rangle \sim \frac{\langle J_i - \bar{J}_i \rangle}{\langle \Gamma + \bar{\Gamma} \rangle}
\]

\[
\langle S_i \rangle \sim \frac{\langle J_i + \bar{J}_i \rangle}{\langle \Gamma + \bar{\Gamma} \rangle}
\]

overline: CP conjugated mode, also: mixing-induced CP asymm in \(B_s \to \phi \ell^+ \ell^-\)

\[
\langle X \rangle \equiv \int dq^2 X(q^2)
\]
Pollution due to Charm Resonances

Narrow Resonances: $J/\psi$ and $\psi(2s)$

- experiments veto $q^2$-region of narrow charmonia $J/\psi$ and $\psi(2s)$
- however: resonance affects observables outside the veto!

![Graph showing $d\mathcal{B}/dq^2$ vs. $q^2$ for $J/\psi$ and $\psi(2s)$](image)

large recoil: $q^2 \ll m_b^2$
low recoil: $q^2 \simeq m_b^2$

Approach by Theorists: Divide and Conquer

- treat region below $J/\psi$ (aka large recoil) differently than above $\psi(2s)$
- design combinations of $J_i$ which have reduced theory uncertainty in only one kinematic region
Large Recoil (I)

**QCD Factorization (QCDF) + Soft Collinear Effective Theory (SCET)**

- calculate $\bar{q}q$ loops perturbatively, expand in $1/m_b, 1/E_{K^*}$
- relate matrix elements to universal hadronic quantities:
  - Light Cone Distribution Amplitudes (LCDAs)
  - form factors
  - decay constants

[Beneke/Feldmann '00, Beneke/Feldmann/Seidel '01 & '04]

**Light Cone Sum Rules (LCSR)**

- calculate $\langle \bar{c}c \rangle$, $\langle \bar{c}cG \rangle$ on the light cone for $q^2 \ll 4m_c^2$
- achieves resummation of soft gluon effects
- use analyticity of amplitude to relate results to $q^2 < M_{\psi'}^2$
- uses many of the same inputs as QCDF+SCET
- includes parts of QCDF+SCET results

[D. van Dyk (U. Siegen) | $|\Delta B| = |\Delta S| = 1$ Wilson Coefficients | 15.10.2013 31 / 14]
QCD Factorization (QCDF) + Soft Collinear Effective Theory (SCET)

- calculate $\bar{q}q$ loops perturbatively, expand in $1/m_b, 1/E_{K^*}$
- relate matrix elements to universal hadronic quantities:
  - Light Cone Distribution Amplitudes (LCDAs)
  - form factors
  - decay constants

[Beneke/Feldmann '00, Beneke/Feldmann/Seidel '01 & '04]

Combination of QCDF+SCET and LCSR Results

- not yet!
  - no studies yet to find impact on optimized observables at large recoil!
  - LCSR results are not included in following discussion
Large Recoil (II)

SM + chirality flipped

- transversity amplitudes factorize up to power suppressed terms
  \[ A_{\perp}^{L,R} \sim X_{\perp}^{L,R} \times \xi_{\perp} \quad A_{\parallel}^{L,R} \sim X_{\parallel}^{L,R} \times \xi_{\perp} \quad A_{0}^{L,R} \sim X_{0}^{L,R} \times \xi_{\parallel} \]

  \[ \xi_{\perp,\parallel} : \text{soft form factors} \quad X_{i}^{L,R} : \text{combinations of Wilson coefficients} \]

**Optimized Observables**

enhanced sensitivity to right-handed currents, reduced form factor dependence

\[ A_{T}^{(2)} = \frac{|A_{\perp}|^2 - |A_{\parallel}|^2}{|A_{\perp}|^2 + |A_{\parallel}|^2} \sim J_3 \]

\[ A_{T}^{(3)} = \frac{|A_{0}^{L}A_{\perp}^{L*} + A_{0}^{R*}A_{\parallel}^{R}|}{\sqrt{|A_{0}|^2|A_{\parallel}|^2}} \sim J_4, J_7 \]

\[ A_{T}^{(4)} = \frac{|A_{0}^{L}A_{\perp}^{L*} - A_{0}^{R*}A_{\perp}^{R}|}{\sqrt{|A_{0}|^2|A_{\perp}|^2}} \sim J_5, J_8 \]

\[ A_{T}^{(5)} = \frac{|A_{\perp}^{L}A_{\parallel}^{R*} + A_{\perp}^{R}A_{\parallel}^{L*}|}{|A_{\perp}|^2 + |A_{\parallel}|^2} \]
SM + chirality flipped

- transversity amplitudes factorize up to power suppressed terms

\[
A_{\perp}^{L,R} \sim X_{\perp}^{L,R} \times \xi_{\perp} \quad A_{\parallel}^{L,R} \sim X_{\parallel}^{L,R} \times \xi_{\perp} \quad A_{0}^{L,R} \sim X_{0}^{L,R} \times \xi_{\parallel}
\]

\[\xi_{\perp,\parallel}: \text{soft form factors} \quad X_{i}^{L,R}: \text{combinations of Wilson coefficients}\]

[Beneke/Feldmann/Seidel '01 & '04, Bobeth/Hiller/Piranishvili '08, Altmannshofer et al. '08]

Further Optimized Observables

enhanced sensitivity to right-handed currents, reduced form factor dependence

[Becirevic/Schneider '11]

\[
A_{T}^{(re)} \propto \frac{J_{6s}}{J_{2s}} \quad A_{T}^{(im)} \propto \frac{J_{9}}{J_{2s}}
\]

- Transversity amplitudes factorize

\[ A_{\perp,||,0}^{L,R} \sim C_{\pm}^{L,R} \times f_{\perp,||,0} + O \left( \frac{\alpha_s \Lambda}{m_b}, \frac{C_7 \Lambda}{C_9 m_b} \right) \]

\( f_i \): helicity form factors \( C_{\pm}^{L,R} \): combinations of Wilson coeff.

- 4 combinations of Wilson coefficients enter observables:

\[ \rho_{\pm} \sim |C_{\pm}^{R}|^2 + |C_{\pm}^{L}|^2 \]

\[ \text{Re} (\rho_2) \sim \text{Re} \left( C_+^{R} C_{-}^{R*} - C_{-}^{L} C_{+}^{L*} \right) \quad \text{and } \text{Re} (\cdot) \leftrightarrow \text{Im} (\cdot) \]

Tensor operators [Bobeth/Hiller/DvD ’12]

- 6 new transversity amplitudes, still factorize!

\[ A_{ab} \sim C_{T(T5)} \times f_{\perp,||,0} + O \left( \frac{\Lambda}{m_b} \right) \]

- 3 new combinations of Wilson coefficients
\( q^2 \) Spectrum of the Branching Ratio \( B = \tau_B \Gamma \)

\[ \frac{d\mathcal{B}}{dq^2} \left[ 10^{-7} / \text{GeV}^2 \right] \]

\[ q^2 \left[ \text{GeV}^2 \right] \]

\[ J/\Psi, \Psi' \]

\( \bar{q}q \) Pollution

- 4-quark operators like \( \mathcal{O}_{1c,2c} \) induce \( b \rightarrow s\bar{c}c (\rightarrow \ell^+\ell^-) \) via loops
- hadronically \( B \rightarrow K^* J/\psi (\rightarrow \ell^+\ell^-) \) or higher charmonia
- experiment: cut narrow resonances \( J/\psi \equiv \psi(1S) \) and \( \psi' = \psi(2S) \)
- theory: handle non-resonant quark loops/broad resonances \( > 2S \)
$q^2$ Spectrum of the Branching Ratio $\mathcal{B} = \tau_B \Gamma$

Large Recoil $E_{K^*} \sim m_b$ QCD, SCET

- expand in $1/m_b$, $1/E_{K^*}$, $\alpha_s$
- symmetry: $7 \rightarrow 2$ form factors

Low Recoil $q^2 \sim m_b^2$ OPE, HQET

- expand in $1/m_b$, $1/\sqrt{q^2}$, $\alpha_s$
- symmetry: $7 \rightarrow 4$ form factors

[Grinstein/Pirjol '04], [Beylich/Buchalla/Feldmann '11]
[Bobeth/Hiller/DvD '10 & '11]