Status of Quadrupole Fringe Fields Study

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Outline

- motivation
- description of the method
- magnetic field map and first considerations
- outlook

Motivation

- HL-LHC project relies on large aperture quadrupoles $\Rightarrow$ the beam is more sensitive to non-linear perturbations like those induced by the fringe fields.
- The spatial extension of these fringe fields increases as well ($\pm$ linearly with coil aperture)

$\Rightarrow$ The total effect on amplitude detuning and chromaticity as been estimated (A. V. Bogomyagkov et al, WEPEA049, IPAC'13).

Although the effect of the fringe fields is small, the effect on the long-term beam dynamics should be evaluated via tracking simulations:

$\Rightarrow$ implementation of fringe fields effect in SixTrack.

Main ingredients of the method

1. Symplectic integrator for a z-dependent Hamiltonian ⇒ to study the long-term stability using SixTrack
2. 3D magnetic field map including fringe fields ⇒ in order to have a detailed description as possible

Several integrators and field map representations can be found in literature. In particular we are considering the method developed by M. Venturini and A. J. Dragt

⇒ it seems to be the most comprehensive, from map computation (MARYLIE's GENMAP routine) to long-term tracking (CTRACK)
⇒ already applied to LHC high-gradient quadrupoles

References:

1. M. Venturini, A.J. Dragt, NIM A 427, p.387,1999
2. M. Venturini, D. Abell and A. Dragt, "Map computation from magnetic field data and application to the LHC high-gradient quadrupoles"
3. M. Venturini, PhD Thesis (1998)
4. A. J. Dragt, www.physics.umd.edu/dsat

Symplectic integrator of z-dependent Hamiltonian

$$K[x, p_x, y, p_y, \delta, l, z, p_z; \sigma] = -\delta + \frac{(p_x - a_x)^2}{2(1+\delta)} + \frac{(p_y - a_y)^2}{2(1+\delta)} - a_z + p_z$$

- $a_x \equiv a_x(x, y, \boldsymbol{z}) = \frac{qA_x(x,y,\boldsymbol{z})}{P_0 c}$; $a_y = a_y(x, y, \boldsymbol{z}) = \frac{qA_y(x,y,\boldsymbol{z})}{P_0 c}$; $a_z = a_z(x, y, \boldsymbol{z}) = \frac{qA_z(x,y,\boldsymbol{z})}{P_0 c}$;
- σ is the independent variable with $d\sigma = dz$
- (***z***,*pz*) is the fourth canonical pairs, needed to have the explicit dependence on ***z***

The solution of the equation of motion (Transfer Map) for this Hamiltonian is obtained by splitting the Hamiltonian into several parts and by using a second order symplectic integrator:

$$\begin{aligned}\mathcal{M}_2(\Delta\sigma) &= exp(: -\tfrac{\Delta\sigma}{2}K_1 :)exp(: -\tfrac{\Delta\sigma}{2}K_2 :)exp(: -\tfrac{\Delta\sigma}{2}K_3 :) \\ &exp(: -\Delta\sigma K_4 :)exp(: -\tfrac{\Delta\sigma}{2}K_3 :)exp(: -\tfrac{\Delta\sigma}{2}K_2 :)exp(: -\tfrac{\Delta\sigma}{2}K_1 :) \\ &= \mathcal{M}(\Delta\sigma) + O((\Delta\sigma)^3)\end{aligned}$$

where $K = K_1 + K_2 + K_3 + K_4$

Reference:

Y. Wu, E. Forest and D. S. Robin, Phys. Rev. E 68, 046502, 2003

Application

If A_x , A_y and A_z are non zero and *K* split as:

- $K_1 = p_z - \delta$
- $K_2 = -a_z$
- $K_3 = \left(\frac{(p_x - a_x)^2}{2(1+\delta)}\right)$
- $K_4 = \left(\frac{(p_y - a_y)^2}{2(1+\delta)}\right)$

The second order integrator writes

$$\mathcal{M}_2(\Delta\sigma) = exp(: -\frac{\Delta\sigma}{2}\,(p_z-\delta)\;:)exp(:\,\frac{\Delta\sigma}{2}a_z\;:)exp(: -\int a_x\,dx\;:)exp(: -\frac{\Delta\sigma}{2}\,\frac{(p_x)^2}{2(1+\delta)}\;:)$$
$$exp(:\int a_x\,dx\;:)exp(: -\int a_y\,dy\;:)exp(: -\Delta\sigma\frac{(p_y)^2}{2(1+\delta)}\;:)exp(:\int a_y\,dy\;:)exp(: -\int a_x\,dx\;:)$$
$$exp(: -\frac{\Delta\sigma}{2}\left(\frac{(p_x)^2}{2(1+\delta)}\right)\;:)exp(:\int a_x\,dx\;:)exp(:\,\frac{\Delta\sigma}{2}a_z\;:)exp(: -\frac{\Delta\sigma}{2}\,(p_z-\delta)\;:)$$

using

$$exp(: -\Delta\sigma K_4 :) = exp(: -\Delta\sigma\left(\frac{(p_y-a_y)^2}{2(1+\delta)}\right)\;:)$$
$$= \exp\left(: -\int a_y\,dy\;:\right) exp\left(: -\Delta\sigma\frac{(p_y)^2}{2(1+\delta)}\;:\right) exp\left(:\int a_y\,dy\;:\right)$$

⇒The number of iterations needed can be reduced choosing a Gauge transformation, so that A_x=0 or A_y=0

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If A_x , A_y and A_z are non zero and *K* split as:

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The second order integrator writes

$$\mathcal{M}_2(\Delta\sigma) = exp(: -\tfrac{\Delta\sigma}{2}\,(p_z - \delta) :) exp(: \tfrac{\Delta\sigma}{2} a_z :) exp(: -\textstyle\int a_x\, dx :) exp(: -\tfrac{\Delta\sigma}{2}\,\frac{(p_x)^2}{2(1+\delta)} :)$$
$$exp(: \textstyle\int a_x\, dx :) exp(: -\textstyle\int a_y\, dy :) exp(: -\Delta\sigma \frac{(p_y)^2}{2(1+\delta)} :) exp(: \textstyle\int a_y\, dy :) exp(: -\textstyle\int a_x\, dx :)$$
$$exp(: -\tfrac{\Delta\sigma}{2} \left(\frac{(p_x)^2}{2(1+\delta)}\right) :) exp(: \textstyle\int a_x\, dx :) exp(: \tfrac{\Delta\sigma}{2} a_z :) exp(: -\tfrac{\Delta\sigma}{2}\,(p_z - \delta) :)$$

Explicit dependence on z

using
$$exp(: -\Delta\sigma K_4 :) = exp(: -\Delta\sigma \left(\frac{(p_y - a_y)^2}{2(1+\delta)}\right) :)$$
$$= \exp\left(: -\textstyle\int a_y\, dy :\right) exp\left(: -\Delta\sigma \frac{(p_y)^2}{2(1+\delta)} :\right) exp\left(: \textstyle\int a_y\, dy :\right)$$

⇒The number of iterations needed can be reduced choosing a Gauge transformation, so that A_x=0 or A_y=0

Transfer map

The seconds half iterations for K_1, K_2 and K_3 are missing in the table.

	K_1	K_2	K_3			K_4		
	$-\frac{\Delta\sigma}{2}(p_z-\delta)$	$\frac{\Delta\sigma}{2}a_z$	$-\int a_x dx$	$-\frac{\Delta\sigma}{2}\frac{(\boldsymbol{p}_x)^2}{2(1+\delta)}$	$\int a_x dx$	$-\int a_y dy$	$-\Delta\sigma\frac{(\boldsymbol{p}_y)^2}{2(1+\delta)}$	$\int a_y dy$
x				$+\frac{p_x\Delta\sigma}{2(1+\delta)}$				
p_x		$+\frac{\partial a_z}{\partial x}\frac{\Delta\sigma}{2}$	$-a_x$		$+a_x$	$-\int\frac{\partial a_y}{\partial x}dy$		$+\int\frac{\partial a_y}{\partial x}dy$
y							$+\frac{p_y\Delta\sigma}{(1+\delta)}$	
p_y		$+\frac{\partial a_z}{\partial y}\frac{\Delta\sigma}{2}$	$-\int\frac{\partial a_x}{\partial y}dx$		$+\int\frac{\partial a_x}{\partial y}dx$	$-a_y$		$+a_y$
l	$-\frac{\Delta\sigma}{2}$			$-\frac{(p_x)^2\,\Delta\sigma}{4(1+\delta)^2}$			$-\frac{(p_y)^2\,\Delta\sigma}{2(1+\delta)^2}$	
δ								
z	$+\frac{\Delta\sigma}{2}$							
p_z		$+\frac{\partial a_z}{\partial z}\frac{\Delta\sigma}{2}$	$-\int\frac{\partial a_x}{\partial z}dx$		$+\int\frac{\partial a_x}{\partial z}dx$	$-\int\frac{\partial a_y}{\partial z}dy$		$+\int\frac{\partial a_y}{\partial z}dy$

Particular case: ideal quadrupole

- $a_x = 0$
- $a_y = 0$
- $a_z = \frac{g}{2}(x^2 - y^2)$

Test case only $\Rightarrow$ we found the interleaved drift and kick of the thin lens approx.

	K$_1$	K$_2$	K$_3$	K$_4$	K$_3$	K$_2$	K$_1$
	$-\frac{\Delta\sigma}{2}(p_z - \delta)$	$\frac{\Delta\sigma}{2}\frac{g}{2}(x^2 - y^2)$	$-\frac{\Delta\sigma}{2}\frac{(\boldsymbol{p_x})^2}{2(1+\delta)}$	$-\Delta\sigma\frac{(\boldsymbol{p_y})^2}{2(1+\delta)}$	$-\frac{\Delta\sigma}{2}\frac{(\boldsymbol{p_x})^2}{2(1+\delta)}$	$\frac{\Delta\sigma}{2}\frac{g}{2}(x^2 - y^2)$	$-\frac{\Delta\sigma}{2}(p_z - \delta)$
x			$+\frac{p_x\Delta\sigma}{2(1+\delta)}$		$+\frac{p_x\Delta\sigma}{2(1+\delta)}$		
p$_x$		$-gx\frac{\Delta\sigma}{2}$				$-gx\frac{\Delta\sigma}{2}$	
y				$+\frac{p_y\Delta\sigma}{(1+\delta)}$			
p$_y$		$+gy\frac{\Delta\sigma}{2}$				$+gy\frac{\Delta\sigma}{2}$	
l	$-\frac{\Delta\sigma}{2}$		$-\frac{(p_x)^2\,\Delta\sigma}{4(1+\delta)^2}$	$-\frac{(p_y)^2\,\Delta\sigma}{2(1+\delta)^2}$	$-\frac{(p_x)^2\,\Delta\sigma}{4(1+\delta)^2}$		$-\frac{\Delta\sigma}{2}$
δ							
z	$+\frac{\Delta\sigma}{2}$						$+\frac{\Delta\sigma}{2}$
p$_z$							

Computation of the vector potential in cartesian coordinates

- The three components of the quadrupole vector potential can be written as expansions of normal (s) and skew (c) multipoles
- Each of the multipole can be expanded in terms of **homogenous polynomials** in ***x,y*** and ***z*-dependent coefficients** $C_{m,\alpha}{}^{[n]}(z)$ (called **generalized gradients**)

$$A_x = \sum_{m=1}^{\infty} A_x^{m,s} - A_x^{m,c}$$

$$A_y = \sum_{m=1}^{\infty} A_y^{m,s} - A_y^{m,c}$$

$$A_z = \sum_{m=1}^{\infty} A_z^{m,s} - A_z^{m,c}$$

$$\mathbb{C} = (\Re\mathrm{e}, \Im\mathrm{m})$$

if α=s$\rightarrow \Re$e;

if α=c $\rightarrow \mathbb{C}=\Im$m

$$A_x^{m,\alpha} = -\frac{1}{m} x\, \mathbb{C}\left[(x+iy)^m\right] \sum_{l=0}^{\infty} \frac{(-1)^l m!}{2^{2l} l!(l+m)!} C_{m,\alpha}^{[2l+1]}(z)\left(x^2+y^2\right)^l$$

$$A_y^{m,\alpha} = -\frac{1}{m} y\, \mathbb{C}\left[(x+iy)^m\right] \sum_{l=0}^{\infty} \frac{(-1)^l m!}{2^{2l} l!(l+m)!} C_{m,\alpha}^{[2l+1]}(z)\left(x^2+y^2\right)^l$$

$$A_z^{m,\alpha} = \frac{1}{m} \mathbb{C}\left[(x+iy)^m\right] \sum_{l=0}^{\infty} \frac{(-1)^l m!(2l+m)}{2^{2l} l!(l+m)!} C_{m,\alpha}^{[2l]}(z)\left(x^2+y^2\right)^l$$

References:
A. J. Dragt, www.physics.umd.edu/dsat

The generalized gradients

The z-dependent coefficients can be calculated using the multipoles expansion of the magnetic field:

$$C^{[n]}_{m,\alpha=s}(z) = \frac{i^n}{2^m m! \sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{e^{\mathrm{ikz}}\ k^{m+n-1}}{I'_m(kR_{analysis})} \tilde{B}_m(R_{analysis}, k) dk$$

where: $I'_m(kR)$ is the derivative of the modified Bessel function

$$\tilde{B}_m(R,k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\mathrm{ikz}} B_m(R,z) dz$$

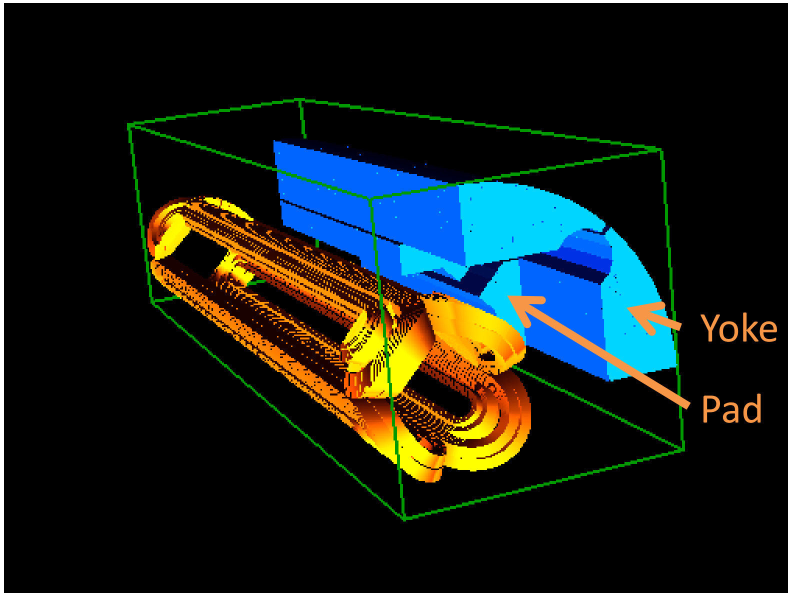
$$B_r(R,\phi,z) = \sum_{m=1}^{\infty} B_m(R,z)\sin(m\phi) + A_m(R,z)\cos(m\phi)$$

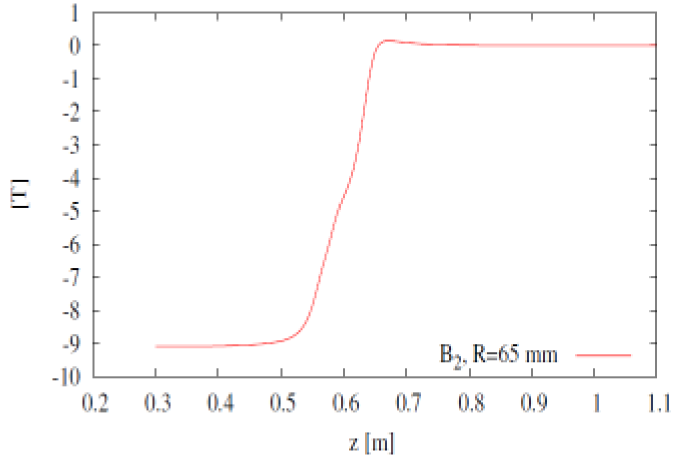
The error in the computation of these coefficients gives the accuracy on the transfer map. Venturini & Dragt have a relative error of 10^{-6} for the coefficients corresponding to the low multipoles (C_2) and of 10^{-4} for the higher ones (C_6).

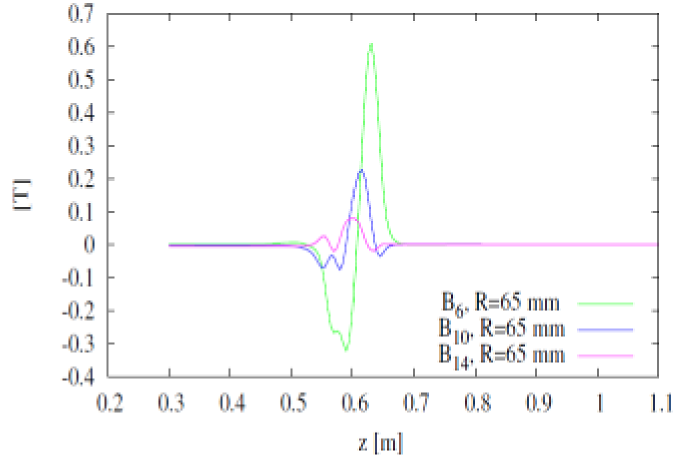
High Luminosity LHC

The harmonic analysis

Susana Izquierdo Bermudez







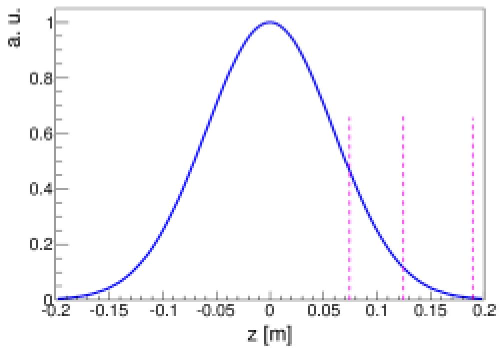
Data provided

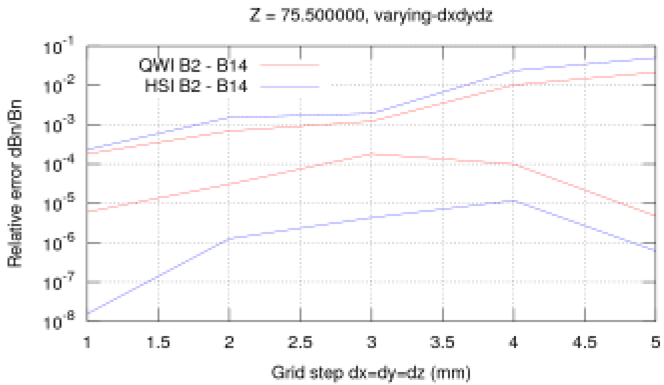
Bx, By, Bz in a Cartesian grid:

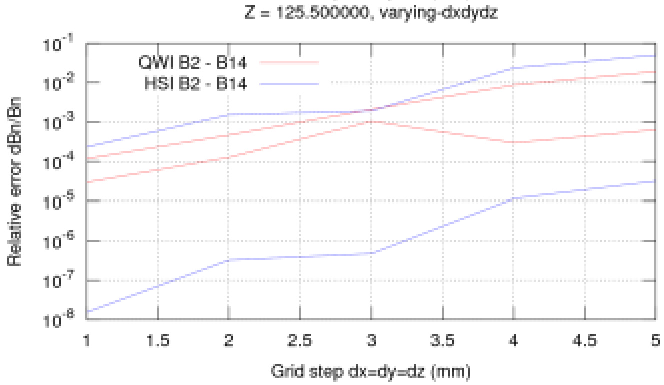
- x = 0:3:75 mm
- y = 0:3:75 mm
- z = 300:5:700 (file z700) and z = 700:1100 (file z1100)

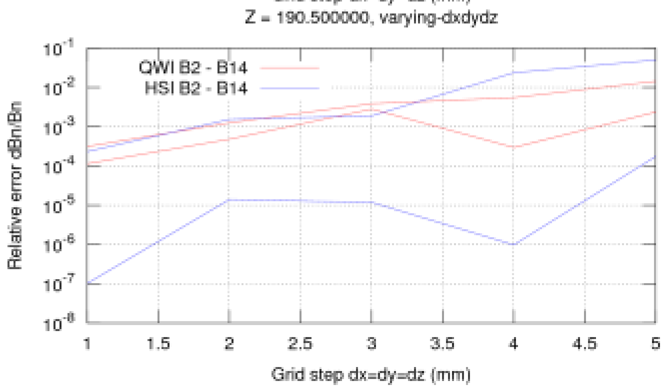
Interpolation methods and errors evaluation

Minimum and maximum relative error in the computation of the harmonics studied using two interpolation methods of an analytical field with a Gaussian z modulation.









- With both interpolators studied the 14th harmonic can be computed with a precision of about 10^{-3} unless we do not consider a grid step of 1 mm.
- The Hermite Spline Interpolator gives better results for the low harmonics.
- The relative error on the 2nd harmonic increases from 10^{-6} to 10^{-5} when the field amplitude decreases $\Rightarrow$ we cannot use very low field values with good precision.

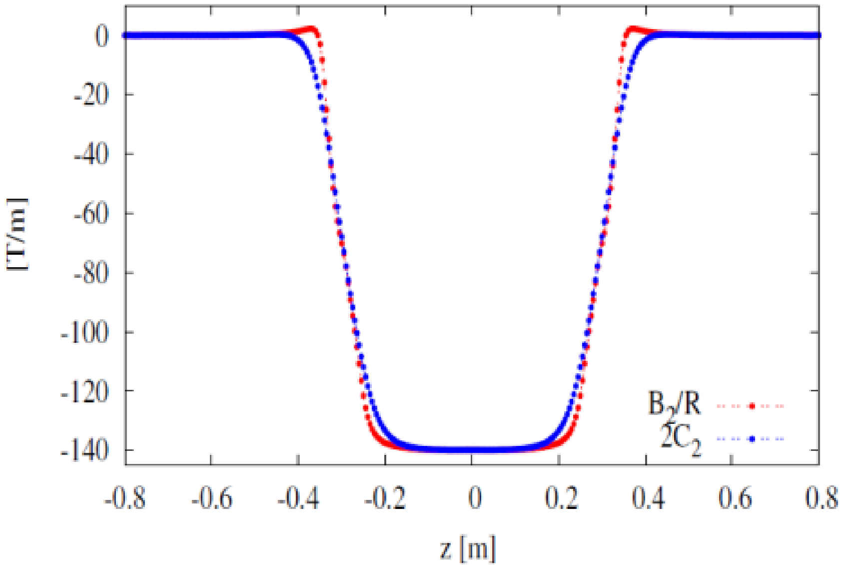
work in progress

Oleg Gabouev, SEN University, Reims

First computation of the gradient C_2

$$C^{[n]}_{m,\alpha=s}(z) = \frac{i^n}{2^m m! \sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{e^{ikz}\ k^{m+n-1}}{I'_m(kR_{analysis})} \tilde{B}_m(R_{analysis}, k) dk$$

- The kernel $\frac{k^{m+n-1}}{I'_m(kR_{analysis})}$ acts as a low pass filter $\Rightarrow$ insensitivity to noise
- integration not trivial
- need to study a method for it (Venturini used a Filon-type integral, a clear reference does not exist)



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Conclusion & Outlook

- To study the effect of the fringe fields on the long-term beam dynamics via tracking simulations $\Rightarrow$ a model of the fringe fields region in SixTrack is needed.

- A second order symplectic integrator for z-dependent Hamiltonian has been studied $\Rightarrow$ details of the SixTrack dimensions still need to be fixed

- The calculation of the transfer maps from 3D magnetic field data follows the method proposed by Venturini & Dragt:

  $\Rightarrow$ study of harmonics analysis accuracy (almost done)
  new magnetic field may be map required

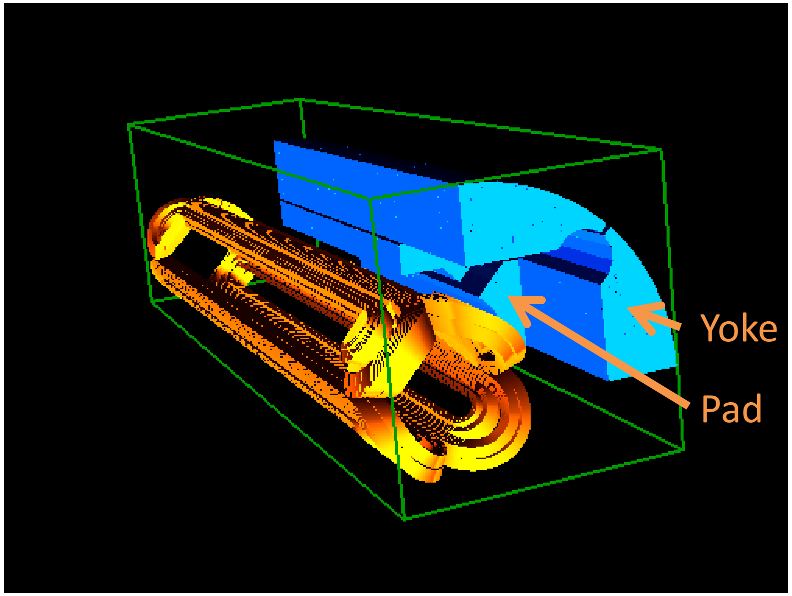
  $\Rightarrow$ generalized gradients computation and errors evaluation

  $\Rightarrow$ tracking simulations

CERN

HL-LHC prototype

Susana Izquierdo Bermudez



QXF: Return end (Symmetric)

z=[0,500] mm: Magnetic yoke and pad
z=[500,680] mm: Magnetic yoke, non-magnetic pad

Data provided

Bx, By, Bz in a Cartesian grid:

- x = 0:3:75 mm
- y = 0:3:75 mm
- z = 300:5:700 (file z700) and z = 700:1100 (file z1100)

I	J	K	X	Y	Z	BX	BY	BZ
1	1	1	0.000000	0.000000	700.000000	0.000000	-0.000000	0.000000
1	1	2	0.000000	0.000000	705.000000	0.000000	0.000000	0.000000
1	1	3	0.000000	0.000000	710.000000	0.000000	0.000000	-0.000000
1	1	4	0.000000	0.000000	715.000000	0.000000	0.000000	0.000000
1	1	5	0.000000	0.000000	720.000000	0.000000	-0.000000	-0.000000
1	1	6	0.000000	0.000000	725.000000	0.000000	-0.000000	0.000000

MQXFC_3D_fringefield_z700_150213.matrf

MQXFC_3D_fringefield_z1100_150213.matrf

Example…