Probabilistic Picture for Medium Induced Jet Evolution and Renormalization of the Jet Quenching Parameter

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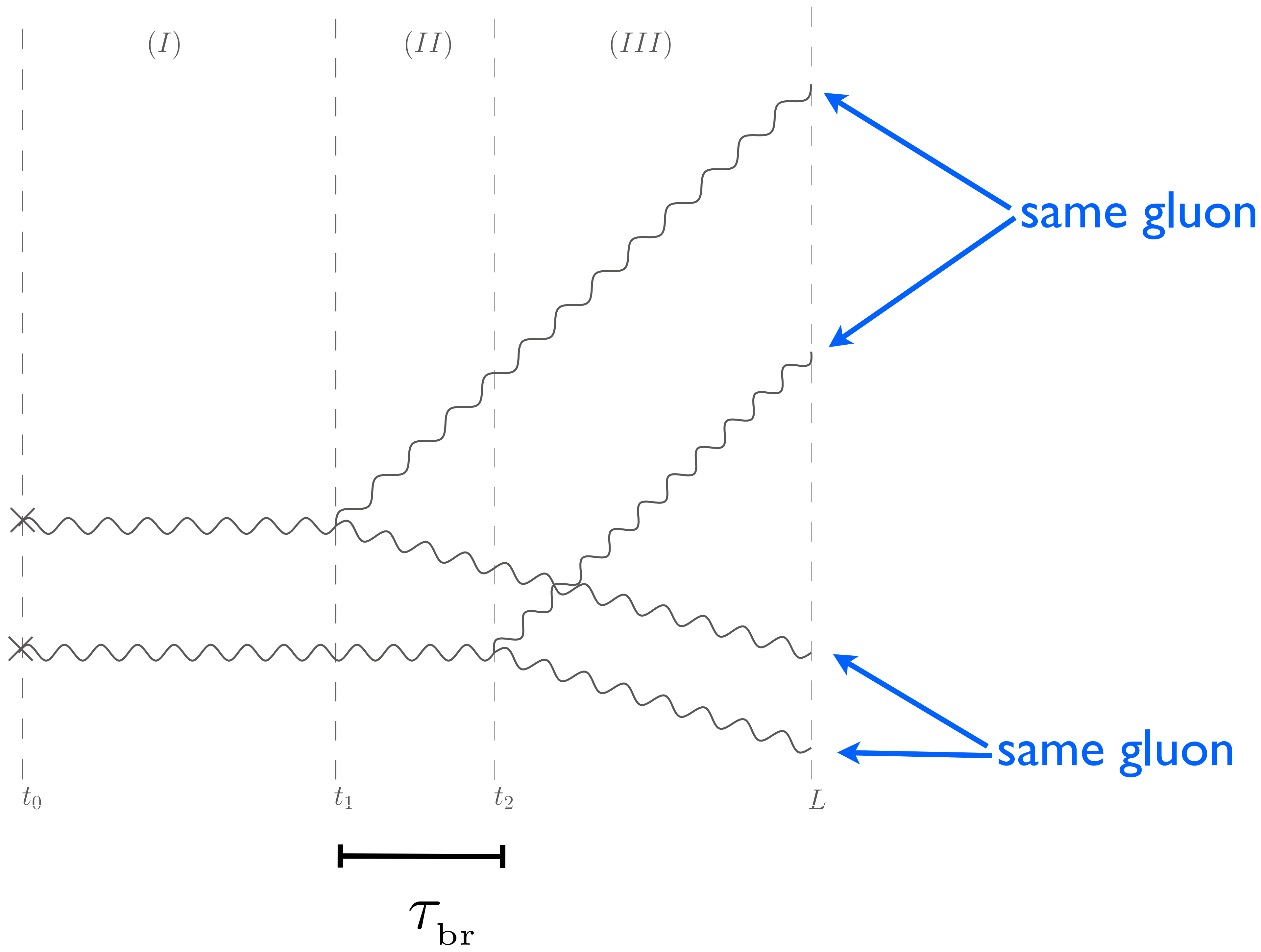
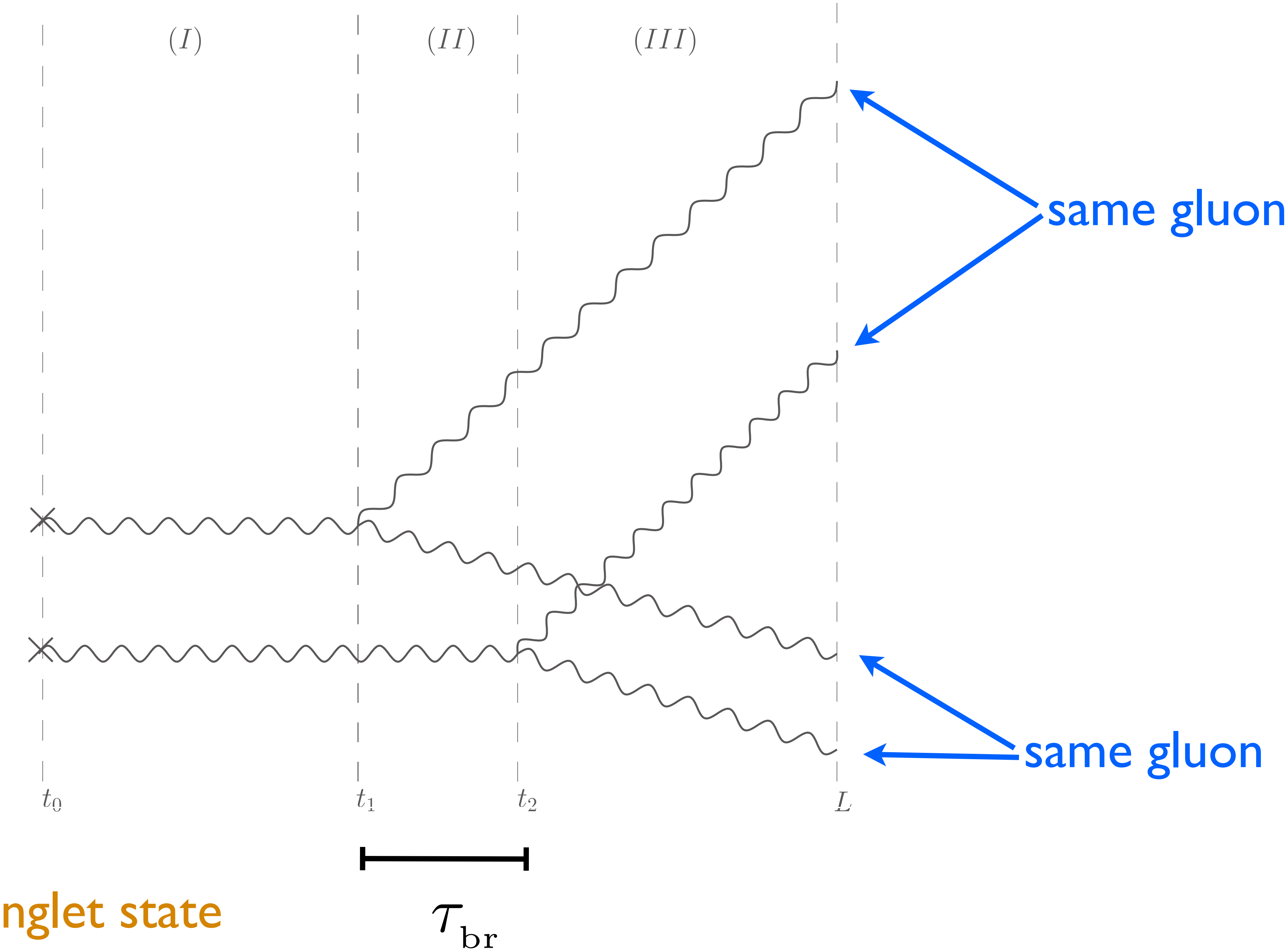
Jet Modification in the RHIC and LHC Era
Wayne State, August 21, 2013

Structure of gluon branching

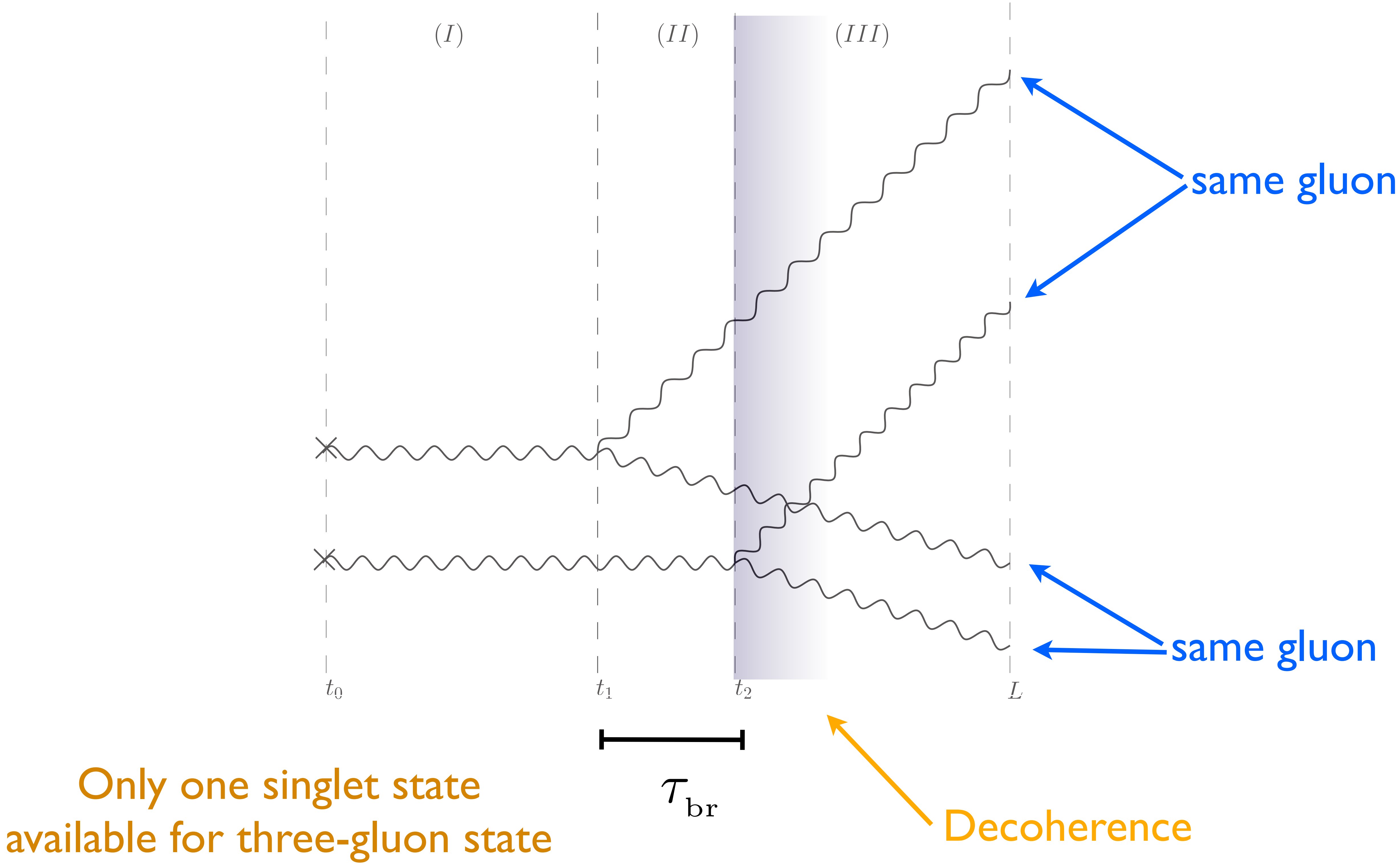


Structure of gluon branching

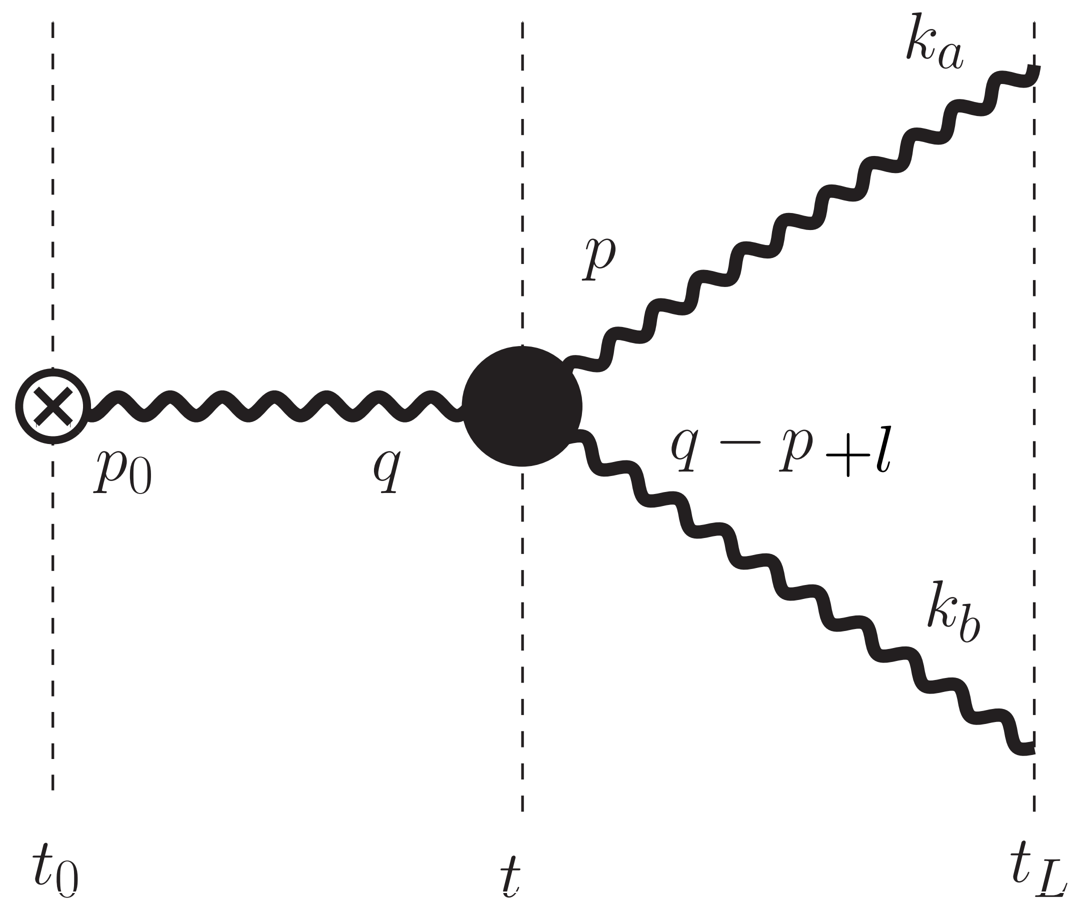


Only one singlet state
available for three-gluon state

Structure of gluon branching



Two-gluon cross section



$$\frac{d^2\sigma_1}{d\Omega_{k_a}d\Omega_{k_b}} = 2g^2 z(1-z)\int_{t_0}^{t_L} dt \int_{\boldsymbol{p}_0,\boldsymbol{q},\boldsymbol{p},\boldsymbol{l}} \mathcal{P}(\boldsymbol{k}_a-\boldsymbol{p};t_L,t)\,\mathcal{P}(\boldsymbol{k}_b-\boldsymbol{q}+\boldsymbol{p}-\boldsymbol{l};t_L,t)$$

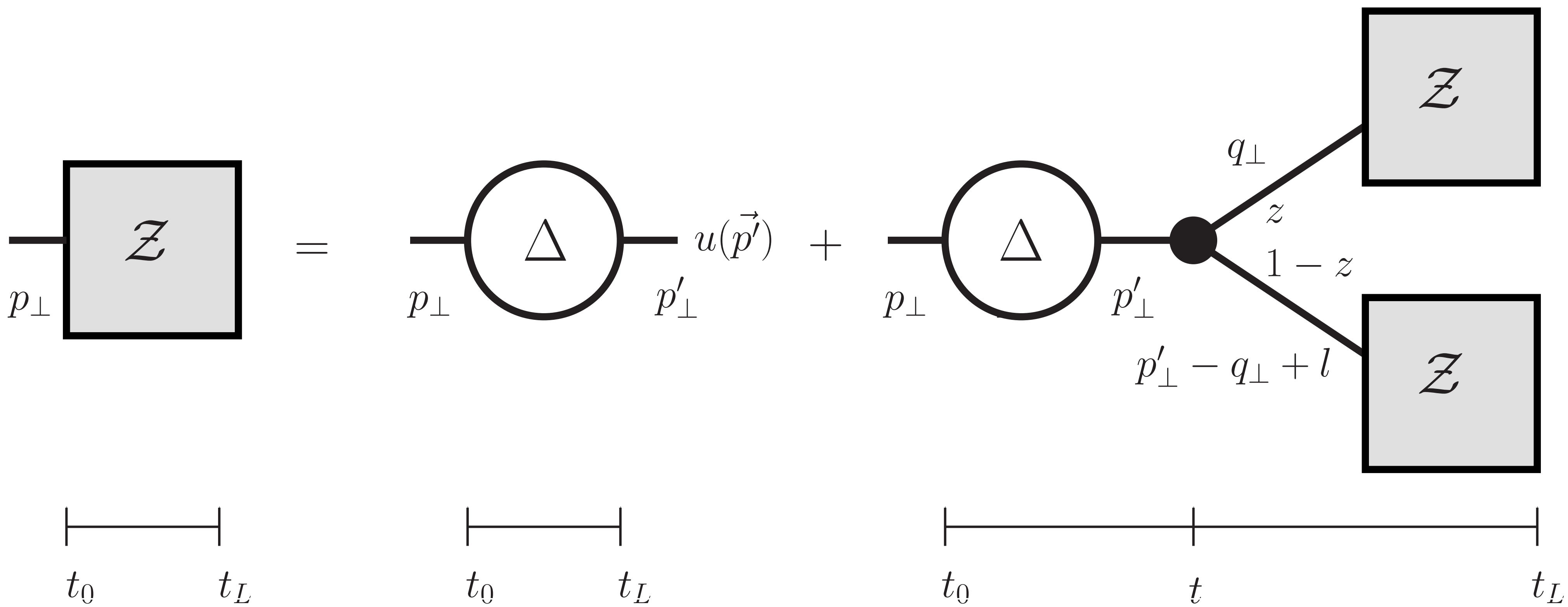
$$\times\, \mathcal{K}(\boldsymbol{Q},\boldsymbol{l},z,p_0^+;t)\,\mathcal{P}(\boldsymbol{q}-\boldsymbol{p}_0;t,t_0)\,\frac{d\sigma_{\rm hard}}{d\Omega_{p_0}}$$

Splitting kernel

Momentum broadening

$$\mathcal{K}(\boldsymbol{Q},\boldsymbol{l},z,p_0^+;t) \equiv \frac{P_{gg}(z)}{[z(1-z)p_0^+]^2}\,\Re e\int_0^\infty d\Delta t\int_{\boldsymbol{P}}(\boldsymbol{P}\cdot\boldsymbol{Q})\,S^{(3)}(\boldsymbol{P},\boldsymbol{Q},\boldsymbol{l},z,p_0^+;t+\Delta t,t)$$

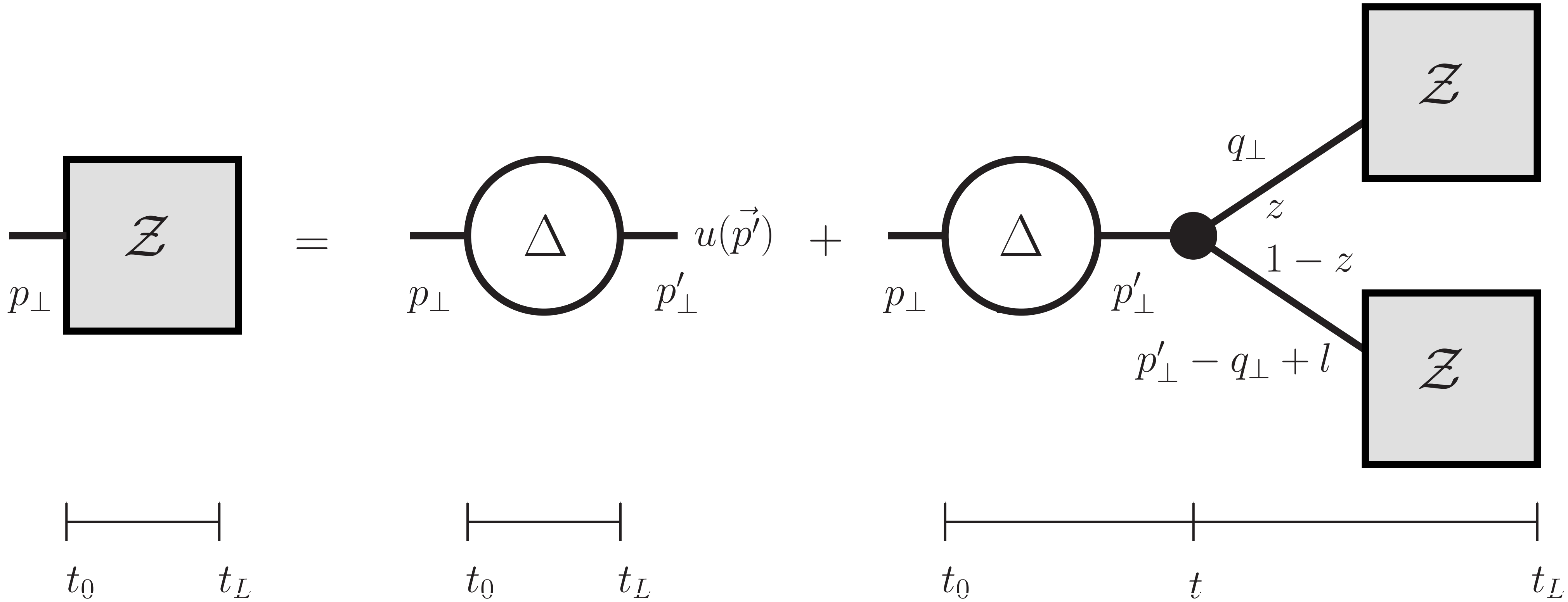
Generating functional



$$\mathcal{Z}(p^+,\boldsymbol{p};t_L,t_0|u) = \int_{\boldsymbol{p}'} \Delta(\boldsymbol{p}'-\boldsymbol{p},p^+;t_L,t_0)\,u(p^+,\boldsymbol{p}') + \alpha_s\int_{t_0}^{t_L} dt \int_{\boldsymbol{p}'} \Delta(\boldsymbol{p}'-\boldsymbol{p},p^+;t,t_0)$$

$$\times \int_0^1 dz \int_{\boldsymbol{q},\boldsymbol{l}} \mathcal{K}(\boldsymbol{q}-z\boldsymbol{p}'-z\boldsymbol{l},\boldsymbol{l},z,p^+,t)\,\mathcal{Z}(zp^+,\boldsymbol{q};t_L,t|u)\,\mathcal{Z}((1-z)p^+,\boldsymbol{p}'+\boldsymbol{l}-\boldsymbol{q};t_L,t|u)$$

Generating functional



Momentum broadening + Sudakov

$$\mathcal{Z}(p^+, \boldsymbol{p}; t_L, t_0|u) = \int_{\boldsymbol{p}'} \Delta(\boldsymbol{p}' - \boldsymbol{p}, p^+; t_L, t_0)\, u(p^+, \boldsymbol{p}') + \alpha_s \int_{t_0}^{t_L} dt \int_{\boldsymbol{p}'} \Delta(\boldsymbol{p}' - \boldsymbol{p}, p^+; t, t_0)$$

$$\times \int_0^1 dz \int_{\boldsymbol{q},\boldsymbol{l}} \mathcal{K}(\boldsymbol{q} - z\boldsymbol{p}' - z\boldsymbol{l}, \boldsymbol{l}, z, p^+, t)\, \mathcal{Z}(zp^+, \boldsymbol{q}; t_L, t|u)\, \mathcal{Z}((1-z)p^+, \boldsymbol{p}' + \boldsymbol{l} - \boldsymbol{q}; t_L, t|u)$$

Generating functional

Get rid of Sudakov by going to differential version

$$-\frac{\partial}{\partial t_0}\mathcal{Z}(p^+,\boldsymbol{p};t_L,t_0|u) = \alpha_s\int_0^1 dz\int_{\boldsymbol{q},\boldsymbol{l}}\mathcal{K}(\boldsymbol{q}-z(\boldsymbol{p}+\boldsymbol{l}),\boldsymbol{l},z,p^+;t_0)$$
$$\times\Big[\mathcal{Z}(zp^+,\boldsymbol{q};t_L,t_0|u)\,\mathcal{Z}((1-z)p^+,\boldsymbol{p}+\boldsymbol{l}-\boldsymbol{q};t_L,t_0|u)-\mathcal{Z}(p^+,\boldsymbol{p}+\boldsymbol{l};t_L,t_0|u)\Big]$$
$$+\int_{\boldsymbol{l}}\mathcal{C}(\boldsymbol{l},t_0)\mathcal{Z}(p^+,\boldsymbol{p}+\boldsymbol{l};t_L,t_0|u)$$

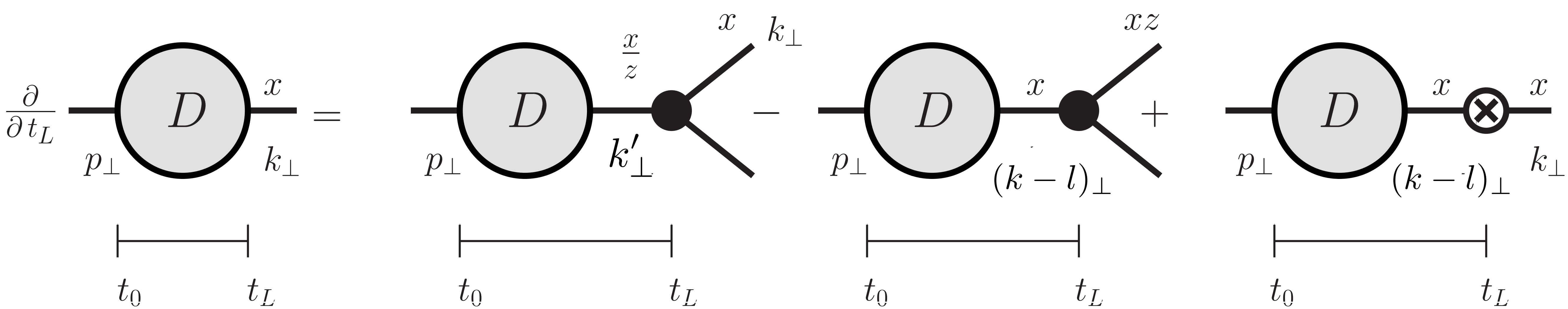
Collision term

$$\frac{\partial}{\partial t}\mathcal{P}(\boldsymbol{k};t,t_0) = \int\frac{d^2\boldsymbol{l}}{(2\pi)^2}\,\mathcal{C}(\boldsymbol{l},t)\,\mathcal{P}(\boldsymbol{k}-\boldsymbol{l};t,t_0)$$

$$\hat{q}(Q^2) = \int^{Q^2}\frac{d^2\boldsymbol{l}}{(2\pi)^2}\boldsymbol{l}^2\mathcal{C}(\boldsymbol{l}) \approx 4\pi\alpha_s^2 C_A n(t)\ln\frac{Q^2}{m_D^2}$$

One-gluon spectrum

$$k^+ \frac{dN}{dk^+ d^2\boldsymbol{k}}(k^+,\boldsymbol{k},p^+,\boldsymbol{p};t_L,t_0) \equiv D(x,\boldsymbol{k}-x\boldsymbol{p},p^+;t_L,t_0)$$

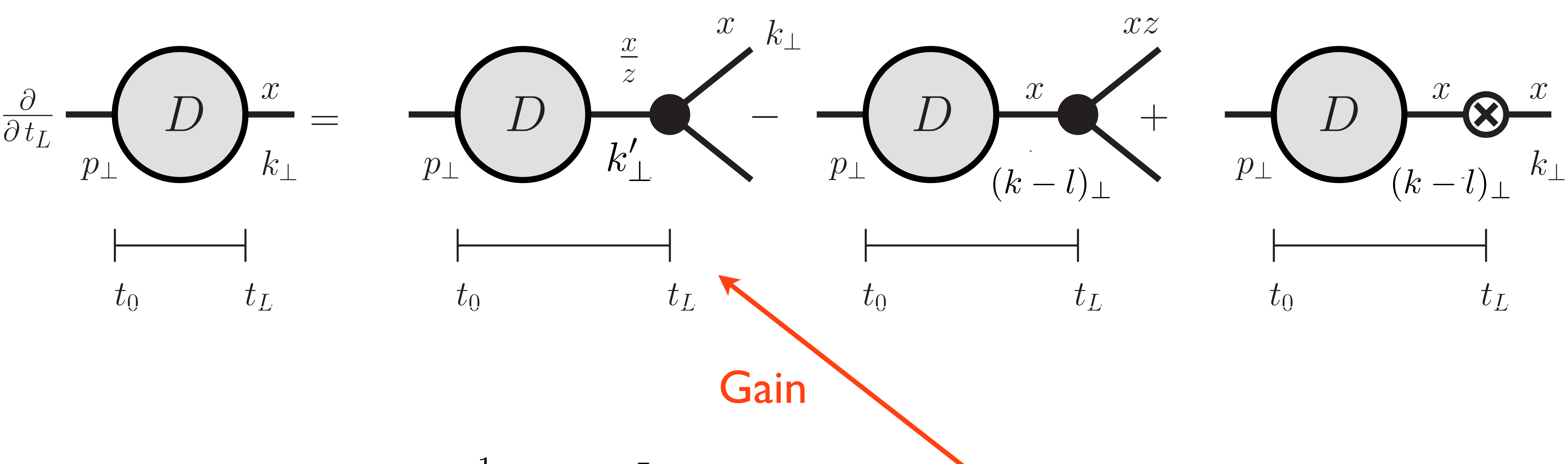


$$\frac{\partial}{\partial t_L} D(x,\boldsymbol{k},t_L) = \alpha_s \int_0^1 dz \int_{\boldsymbol{q},\boldsymbol{l}} \Big[2\mathcal{K}\left(\boldsymbol{k}-z(\boldsymbol{q}+\boldsymbol{l}),\boldsymbol{l},z,\frac{x}{z}p_0^+;t_L\right) D\left(\frac{x}{z},\boldsymbol{q},t_L\right)$$

$$- \mathcal{K}\left(\boldsymbol{q},\boldsymbol{l},z,xp_0^+;t_L\right) D\left(x,\boldsymbol{k}-\boldsymbol{l},t_L\right) \Big] + \int_{\boldsymbol{l}} \mathcal{C}(\boldsymbol{l},t_L) D\left(x,\boldsymbol{k}-\boldsymbol{l},t_L\right)$$

One-gluon spectrum

$$k^+ \frac{dN}{dk^+ d^2\boldsymbol{k}} (k^+, \boldsymbol{k}, p^+, \boldsymbol{p}; t_L, t_0) \equiv D(x, \boldsymbol{k} - x\boldsymbol{p}, p^+; t_L, t_0)$$

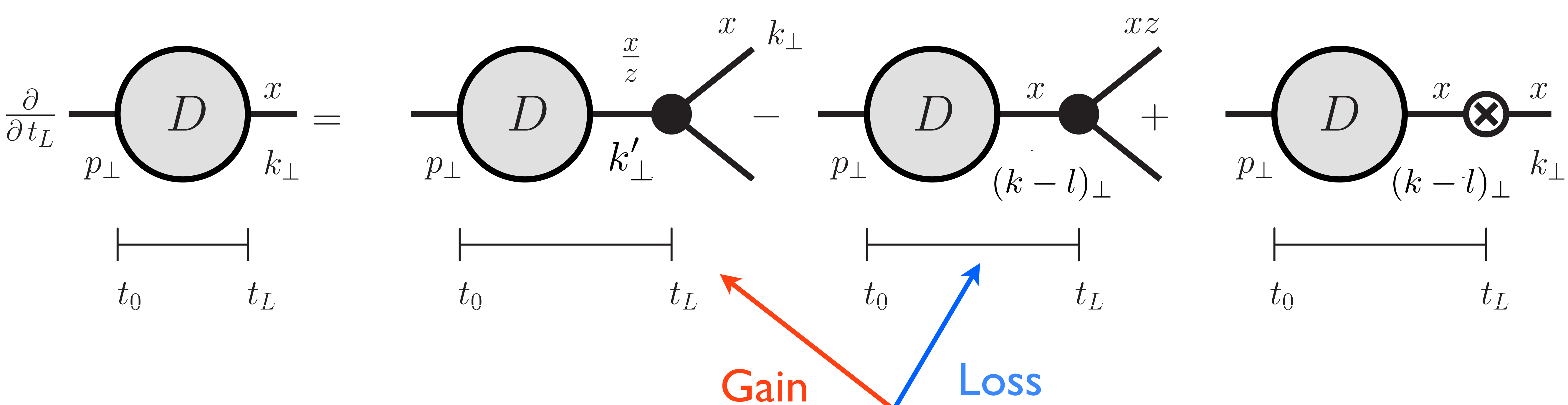


$$\frac{\partial}{\partial t_L} D(x, \boldsymbol{k}, t_L) = \alpha_s \int_0^1 dz \int_{\boldsymbol{q},\boldsymbol{l}} \Big[2\mathcal{K}\left(\boldsymbol{k} - z(\boldsymbol{q}+\boldsymbol{l}), \boldsymbol{l}, z, \frac{x}{z} p_0^+; t_L\right) D\left(\frac{x}{z}, \boldsymbol{q}, t_L\right)$$

$$- \mathcal{K}\left(\boldsymbol{q}, \boldsymbol{l}, z, x p_0^+; t_L\right) D\left(x, \boldsymbol{k} - \boldsymbol{l}, t_L\right) \Big] + \int_{\boldsymbol{l}} \mathcal{C}(\boldsymbol{l}, t_L) D\left(x, \boldsymbol{k} - \boldsymbol{l}, t_L\right)$$

One-gluon spectrum

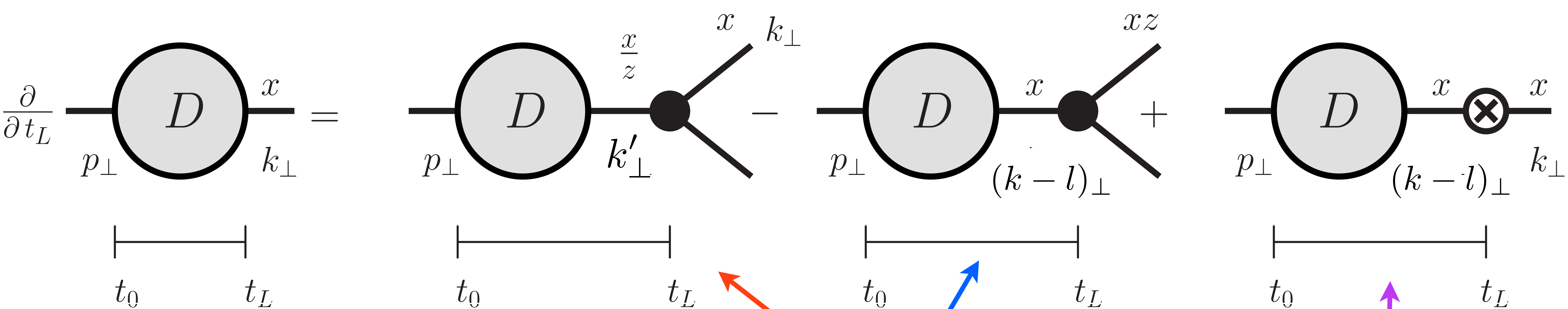
$$k^+ \frac{dN}{dk^+ d^2\boldsymbol{k}} (k^+, \boldsymbol{k}, p^+, \boldsymbol{p}; t_L, t_0) \equiv D(x, \boldsymbol{k} - x\boldsymbol{p}, p^+; t_L, t_0)$$



$$\frac{\partial}{\partial t_L} D(x, \boldsymbol{k}, t_L) = \alpha_s \int_0^1 dz \int_{\boldsymbol{q},\boldsymbol{l}} \Big[2\mathcal{K}\left(\boldsymbol{k} - z(\boldsymbol{q}+\boldsymbol{l}), \boldsymbol{l}, z, \frac{x}{z} p_0^+; t_L\right) D\left(\frac{x}{z}, \boldsymbol{q}, t_L\right) - \mathcal{K}\left(\boldsymbol{q}, \boldsymbol{l}, z, x p_0^+; t_L\right) D\left(x, \boldsymbol{k} - \boldsymbol{l}, t_L\right) \Big] + \int_{\boldsymbol{l}} \mathcal{C}(\boldsymbol{l}, t_L) D\left(x, \boldsymbol{k} - \boldsymbol{l}, t_L\right)$$

One-gluon spectrum

$$k^+ \frac{dN}{dk^+ d^2\boldsymbol{k}}(k^+, \boldsymbol{k}, p^+, \boldsymbol{p}; t_L, t_0) \equiv D(x, \boldsymbol{k} - x\boldsymbol{p}, p^+; t_L, t_0)$$



Gain

Loss

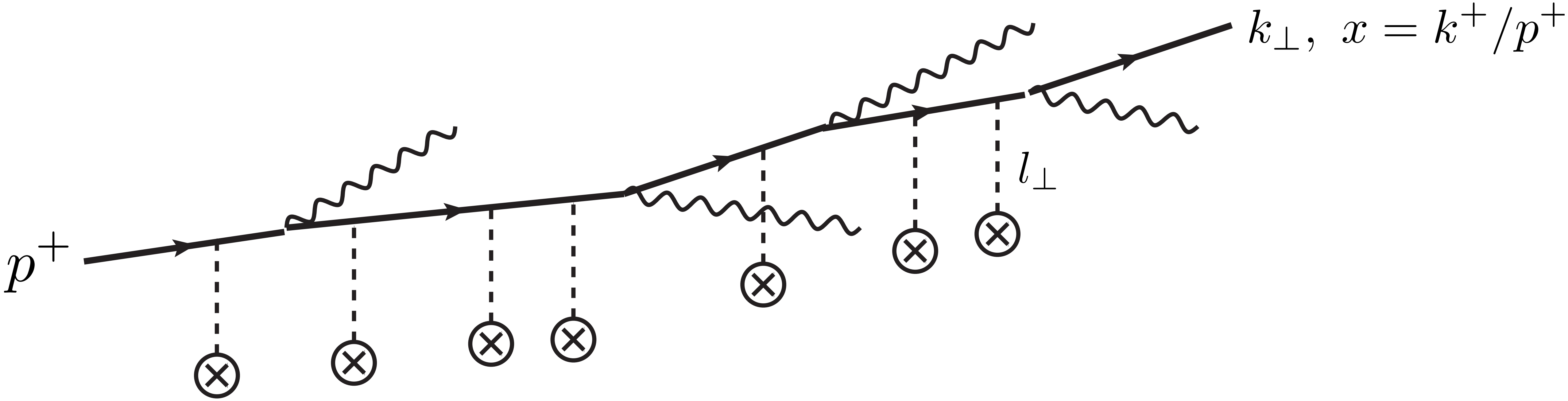
Collision

$$\frac{\partial}{\partial t_L} D(x, \boldsymbol{k}, t_L) = \alpha_s \int_0^1 dz \int_{\boldsymbol{q},\boldsymbol{l}} \Big[2\mathcal{K}\left(\boldsymbol{k} - z(\boldsymbol{q} + \boldsymbol{l}), \boldsymbol{l}, z, \frac{x}{z} p_0^+; t_L\right) D\left(\frac{x}{z}, \boldsymbol{q}, t_L\right)$$

$$- \mathcal{K}\left(\boldsymbol{q}, \boldsymbol{l}, z, x p_0^+; t_L\right) D\left(x, \boldsymbol{k} - \boldsymbol{l}, t_L\right) \Big] + \int_{\boldsymbol{l}} \mathcal{C}(\boldsymbol{l}, t_L) D\left(x, \boldsymbol{k} - \boldsymbol{l}, t_L\right)$$

Diffusion approximation

Take a high energy particle going through the medium, $x \sim 1$



Multiple soft scatterings → $\boldsymbol{l} \ll \boldsymbol{k}$

For collision term: $\displaystyle\int_{\boldsymbol{l}} \mathcal{C}(\boldsymbol{l})\, D\,(x, \boldsymbol{k}-\boldsymbol{l}, t_L) \approx \frac{1}{4}\left(\frac{\partial}{\partial \boldsymbol{k}}\right)^2 \left[\hat{q}_0(\boldsymbol{k}^2)\, D\,(x, \boldsymbol{k}, t_L)\right]$

Diffusion approximation

Expanding the radiation terms also:

$$\frac{\partial}{\partial t_L} D(x, \boldsymbol{k}, t_L) = \alpha_s \int_0^1 dz \int \frac{d^2\boldsymbol{q}}{(2\pi)^2} \left[2\mathcal{K}\left(\boldsymbol{k} - z\boldsymbol{q}, z, \frac{x}{z} p_0^+\right) D\left(\frac{x}{z}, \boldsymbol{q}, t_L\right) - \mathcal{K}\left(\boldsymbol{q}, z, x p_0^+\right) D\left(x, \boldsymbol{k}, t_L\right) \right] + \frac{1}{4} \left(\frac{\partial}{\partial \boldsymbol{k}}\right)^2 \left[\hat{q}_{\text{eff}}(\boldsymbol{k}^2) D\left(x, \boldsymbol{k}, t_L\right)\right]$$

$$\hat{q}_{\text{eff}}(\boldsymbol{k}^2) = \hat{q}_0(\boldsymbol{k}^2) + \hat{q}_1(\boldsymbol{k}^2)$$

$$\hat{q}_1(\boldsymbol{k}^2) = 2\alpha_s \int_x^1 dz \int_{\boldsymbol{Q},\boldsymbol{l}} \left[(\boldsymbol{Q} + \boldsymbol{l})^2 - \boldsymbol{l}^2\right] \mathcal{K}\left(\boldsymbol{Q}, \boldsymbol{l}, z, x p_0^+\right)$$

Double log

$$\hat{q}_1(\boldsymbol{k}^2) = 2\alpha_s \int_x^1 dz \int_{\boldsymbol{Q},\boldsymbol{l}} \left[(\boldsymbol{Q}+\boldsymbol{l})^2 - \boldsymbol{l}^2\right] \mathcal{K}\left(\boldsymbol{Q},\boldsymbol{l},z,xp_0^+\right)$$

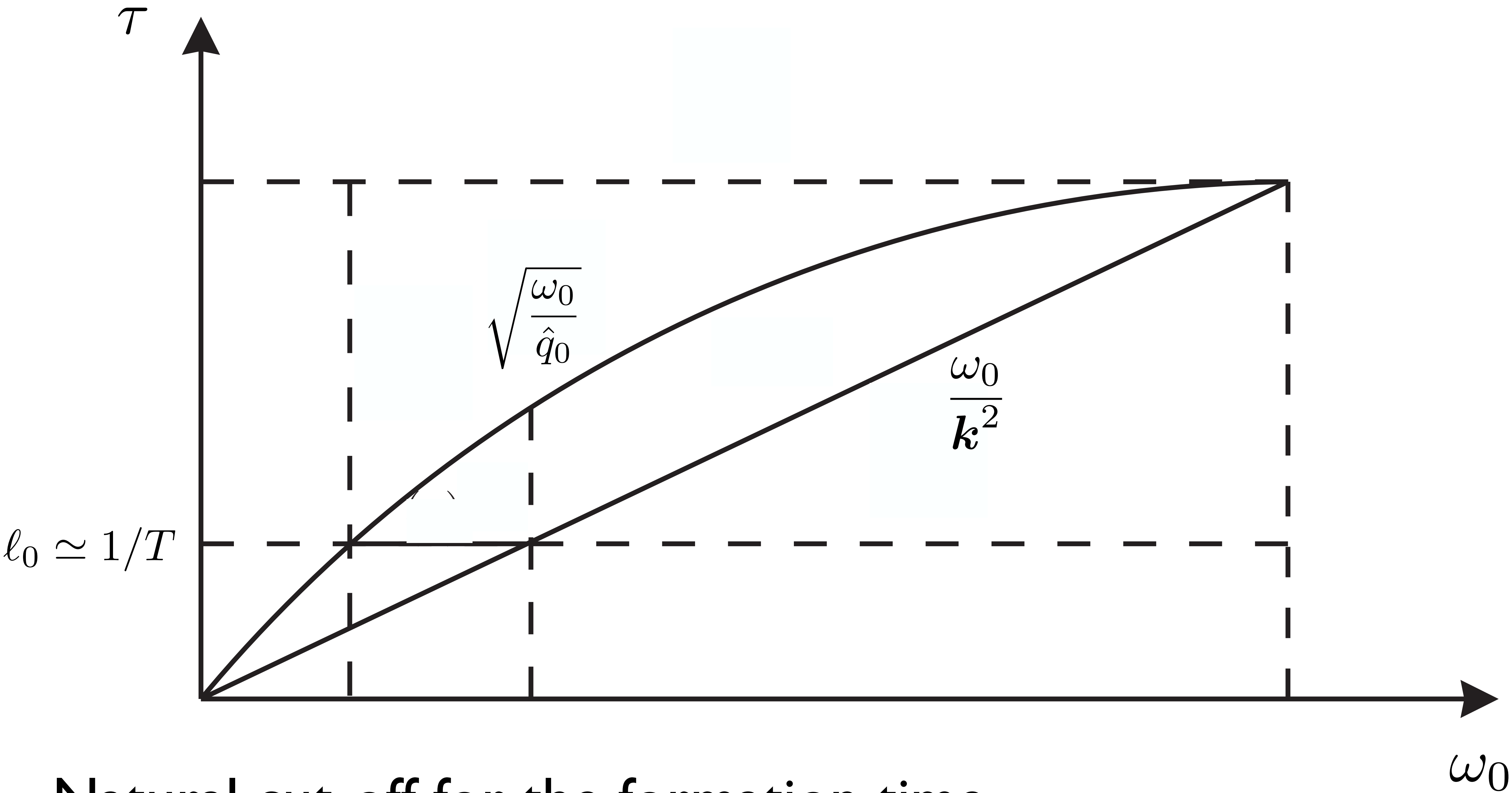
$$\mathcal{K}(\boldsymbol{Q},\boldsymbol{l},z,p_0^+;t) \equiv \frac{P_{gg}(z)}{[z(1-z)p_0^+]^2} \Re e \int_0^\infty d\Delta t \int_{\boldsymbol{P}} (\boldsymbol{P}\cdot\boldsymbol{Q})\, S^{(3)}(\boldsymbol{P},\boldsymbol{Q},\boldsymbol{l},z,p_0^+;t+\Delta t,t)$$

Strongly suppressed for large Δt

Logarithmic divergence comes from emission with very small formation times → Dominated by single scattering region

$$\hat{q}_1(\boldsymbol{k}^2) \approx \frac{\alpha_s N_c}{\pi} \hat{q}_0 \int_{\omega_*}^{\omega} \frac{d\omega_0}{\omega_0} \int_{\tau_*}^{\tau_{\rm br}(\omega_0)} \frac{d\tau}{\tau}$$

Double logs



Natural cut-off for the formation time

$$\hat{q}_1(\boldsymbol{k}^2) \simeq \frac{\alpha_s N_c}{2\pi} \hat{q}_0 \ln^2 \frac{\boldsymbol{k}^2}{\hat{q}_0 \ell_0}$$

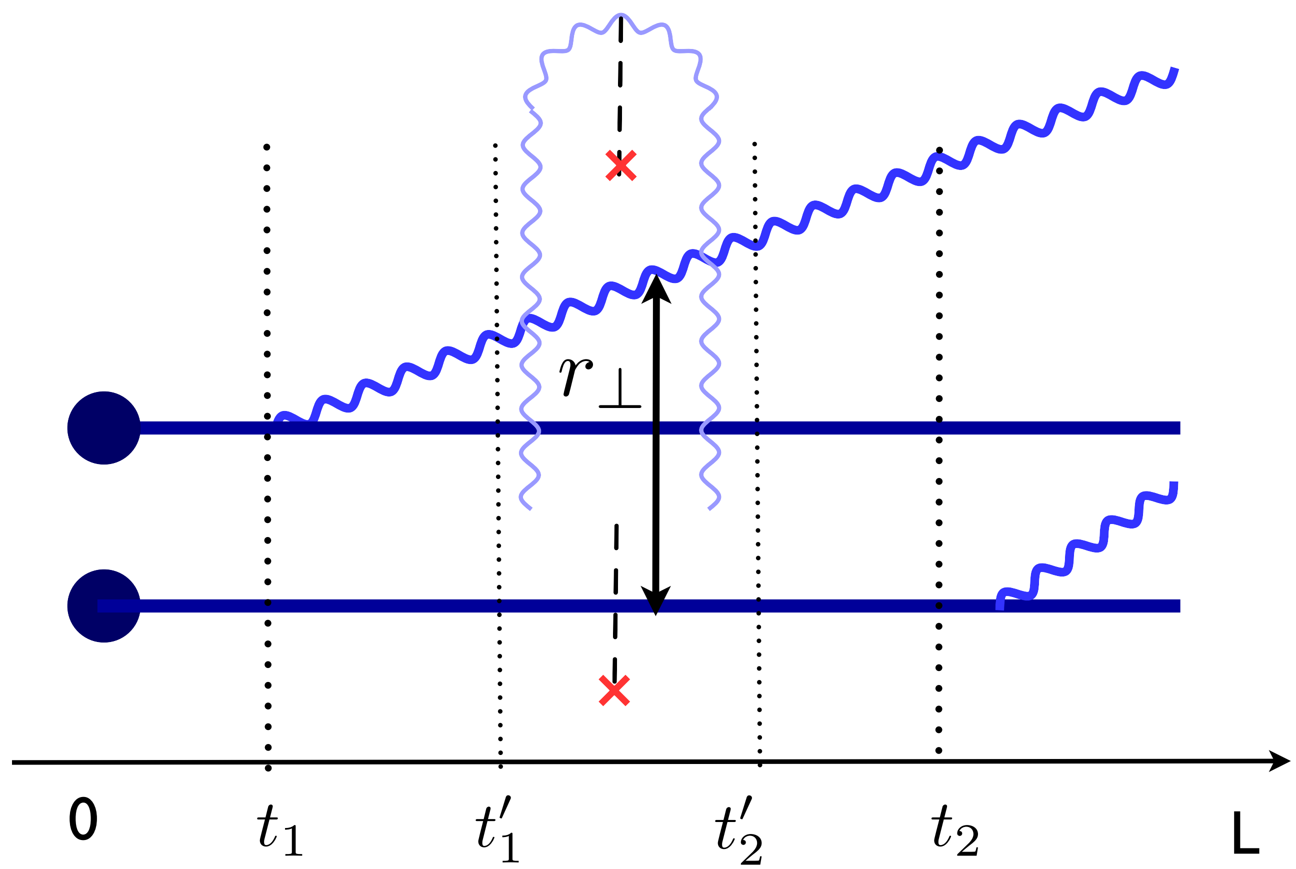
Also found in Liou Mueller Wu 2013
arxiv:1304.7677

Renormalization of the jet quenching parameter

- Go beyond diffusion approximation
- Look at the effect of radiative corrections to the emission kernel

$$\mathcal{K}[\hat{q}_0] \to \mathcal{K}[\hat{q}_0 + \hat{q}_1]$$

Renormalization of the jet quenching parameter



Double log comes from region where the additional fluctuation has a very small formation time and big transverse size

Fluctuation is effectively seen as instantaneous and therefore looks like a correction to the medium interaction

Renormalization of the jet quenching parameter

- Strong ordering in formation time of Gunion-Bertsch-like gluons

- Leading logs can be resumed into an effective $\hat{q}$

$$\hat{q}_{\rm eff}(\boldsymbol{k}^2) \;=\; \hat{q}_0(\boldsymbol{k}^2)\,\frac{1}{\sqrt{\bar{\alpha}}\,\ln(\boldsymbol{k}^2/\hat{q}_0\ell_0)}\,\mathrm{I}_1\left(2\sqrt{\bar{\alpha}}\,\ln\frac{\boldsymbol{k}^2}{\hat{q}_0\ell_0}\right)$$

Summary

- Radiative corrections lead to potentially large contributions through double logs
- These large contributions can be resumed into an effective jet quenching parameter
- Probabilistic picture of incoherent emissions is still valid