

# The Parton Cascade Model and Jet Quenching in Heavy Ion Physics

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# Outline

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- The Parton Cascade Model
- Early History: The major VNI results
- A brief and incomplete survey of some other models
- Jets Observables with VNI/BMS Box Mode
- What Next?

# The Parton Cascade Model

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- Describes the time evolution of a system of quarks and gluons, a microscopic transport model based upon the Boltzmann equation.

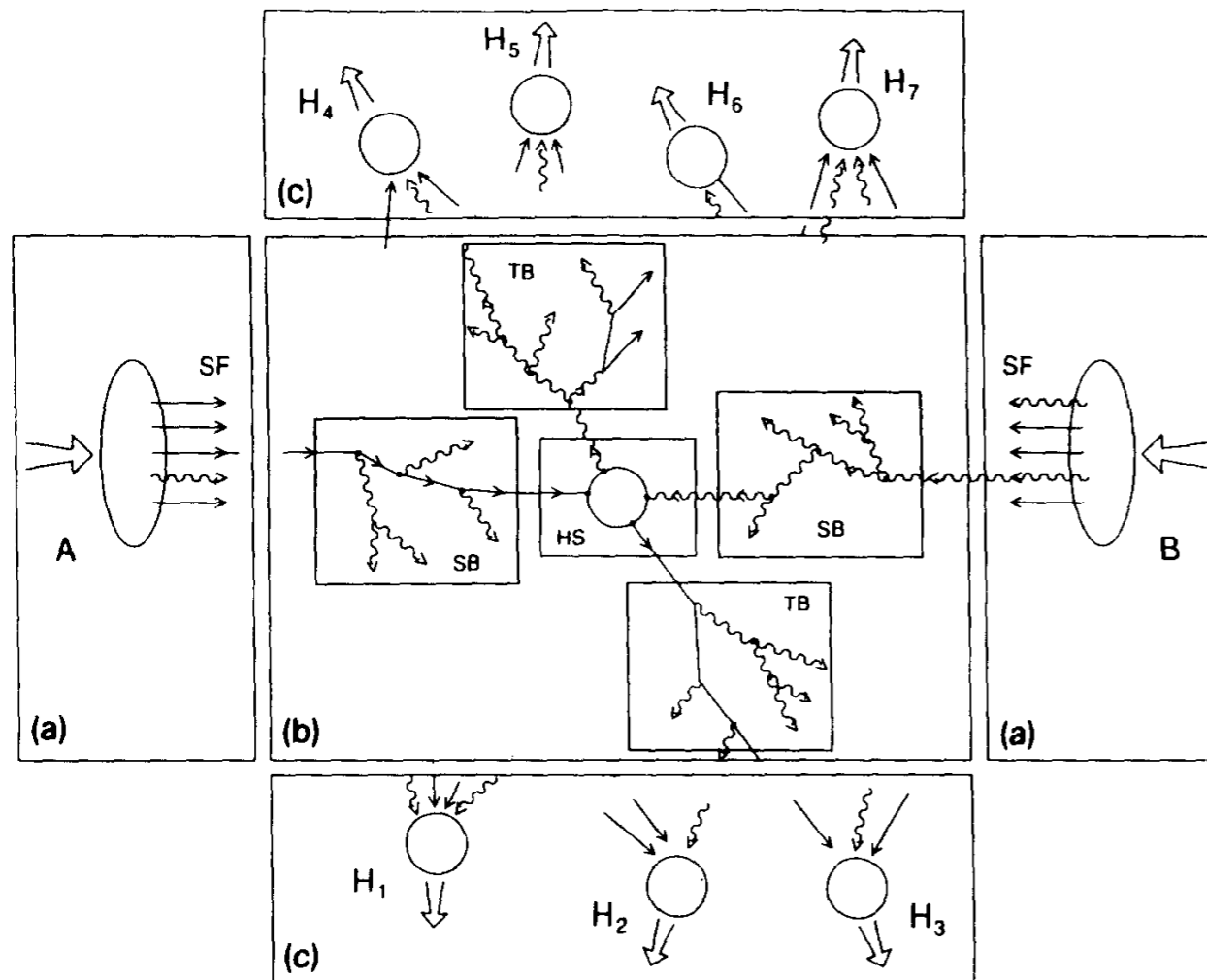
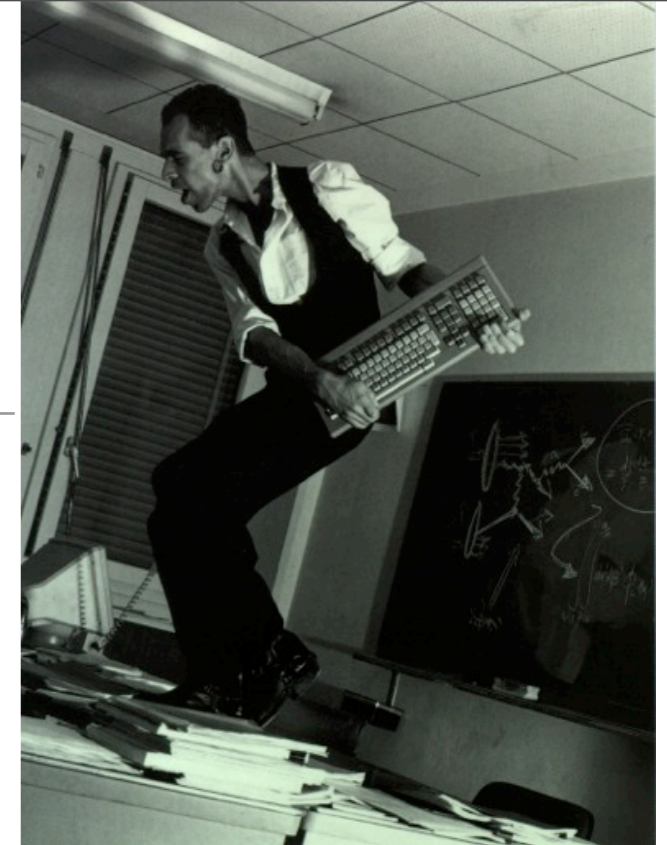
$$p^\mu \frac{\partial}{\partial x^\mu} F_k(x, p) = \sum C_i F(x, p)$$

- Particles follow classical trajectories in phase space. Full relativistic mechanics for scattering kinematics
- A geometric interpretation of the cross-section is used to select interactions
- Collision term is potentially very general. Usually models consider 2->2 processes (binary scatterings) and radiative processes 1->2, 2->N.
- Not clear how to propagate virtual particles.

# Parton Cascades - Past Historical



# VNI - K.Geiger & B.Müller (1991)



- Full set of 2->2 scatterings
- Space-like and time-like branching processes.
- Some off-shell propagation is mooted, not totally clear how it worked in practice
- Global pt cut-off for hard scatterings.
- Partons sampled from PDF's with slightly fuzzy spatial distributions
- Predictions of multiplicity and energy deposition in central region

Parton Cascade Model, designed to describe the energy deposition, thermalization and chemical equilibration of matter in high-energy nuclear collisions

K.Geiger & B.Müller. N Phys B **369** (1992)

# Thermalization - VNI

- Added a lot of soft & phenomenological effects
- Simple LPM Effect
- Smearing soft parton distributions
- Competitive Emission and absorption
- Soft Cross-Section

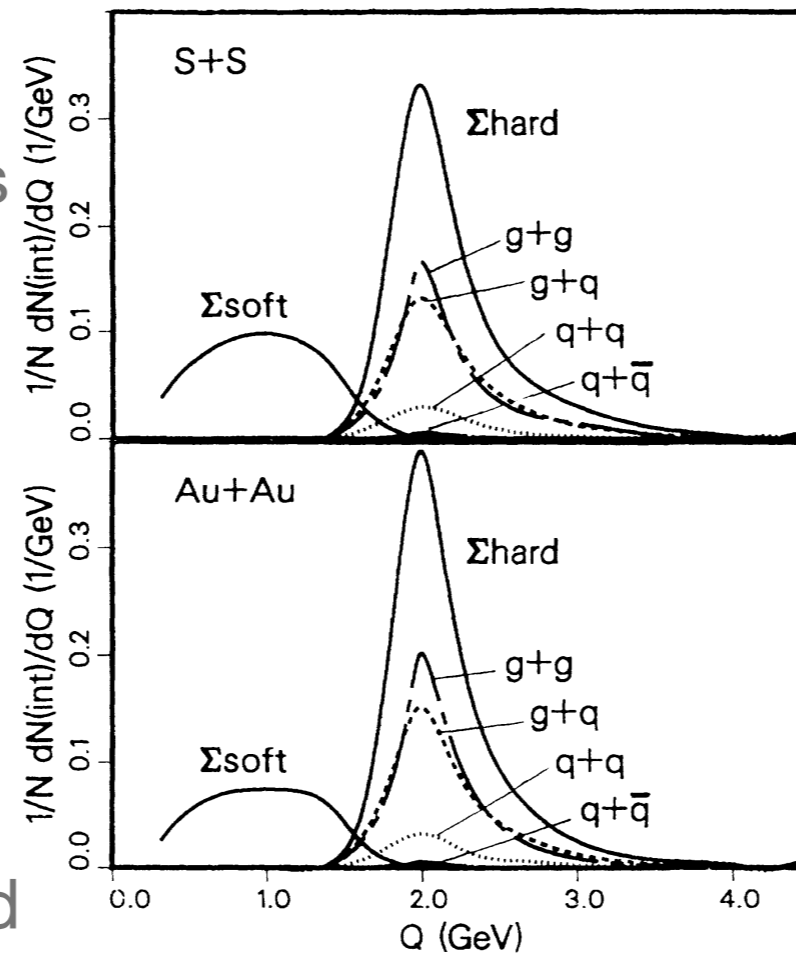


FIG. 5. Relative contributions of (semi)hard and soft parton-parton collisions in  $^{32}\text{S}+^{32}\text{S}$  and  $^{197}\text{Au}+^{197}\text{Au}$  ( $\sqrt{s} = 200A$  GeV). Shown are the differential probabilities  $dP_{\text{int}}/dQ$  as a function of  $Q = p_{\perp}$  (full lines). The (semi)hard collisions are in addition split up into the individual contributions from  $g+g$ ,  $g+q$ ,  $q+q$ , and  $q+\bar{q}$ .

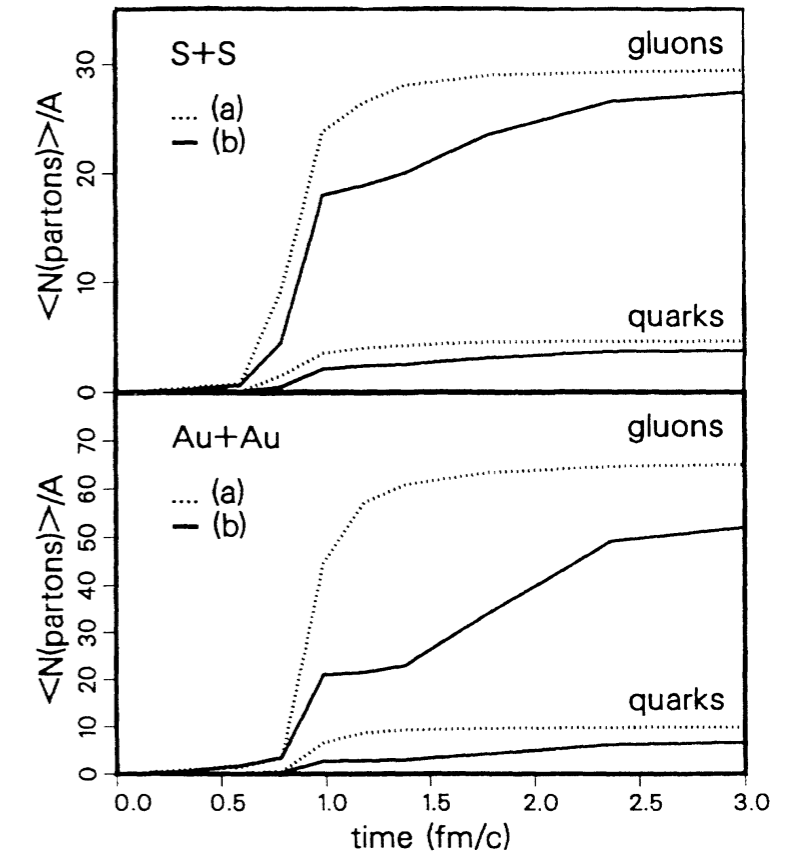


FIG. 3. Average cumulative number of secondary gluons and quarks per nucleon produced during collisions of  $^{32}\text{S}+^{32}\text{S}$  and  $^{197}\text{Au}+^{197}\text{Au}$  at  $\sqrt{s} = 200A$  GeV. The plots (a) and (b) refer to the two space-time evolution scenarios explained in the text.

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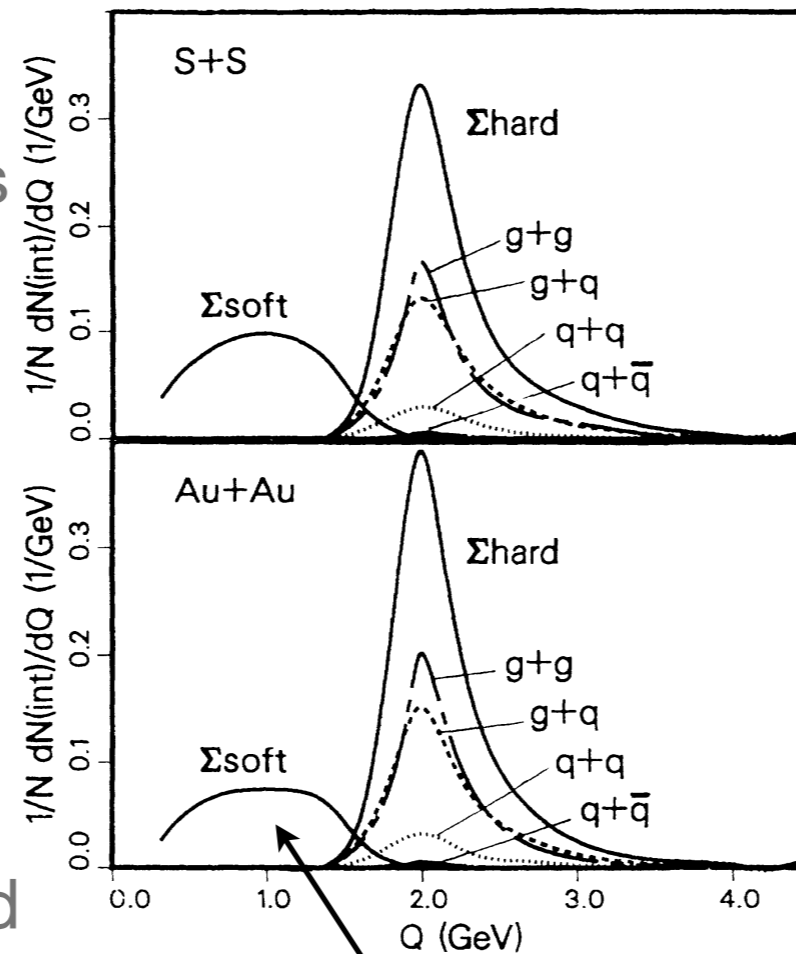


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phenomenological soft cross-section

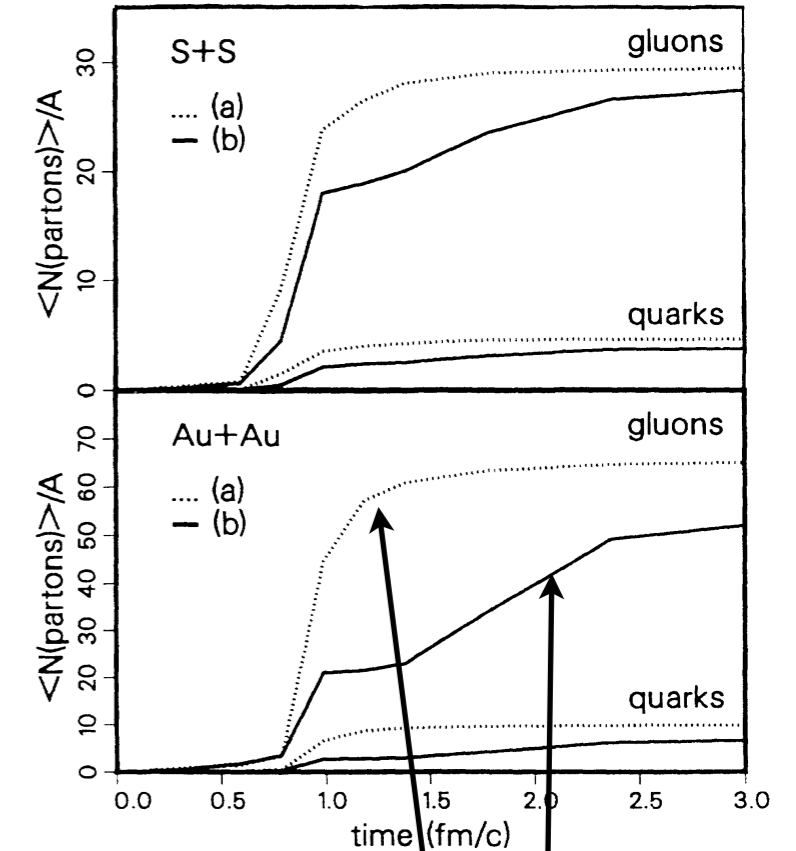


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Multiplicity is dominated by radiated gluons

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(Received 5 May 1992)

# Thermalization - VNI

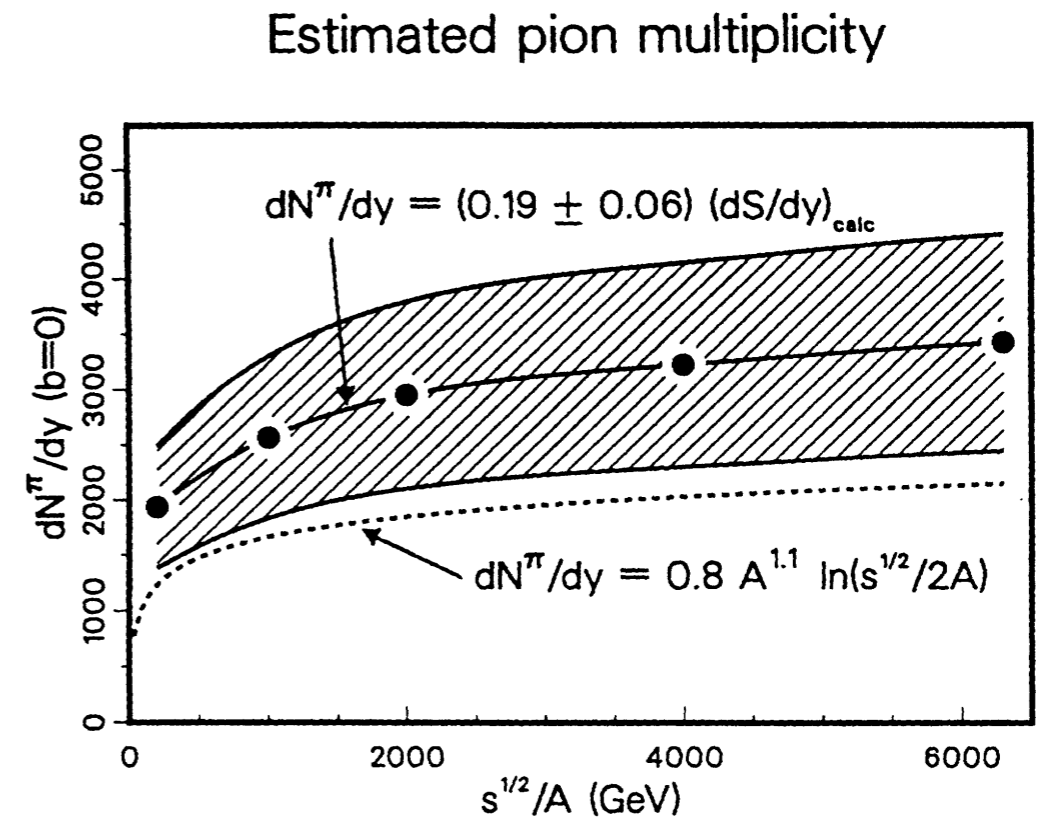
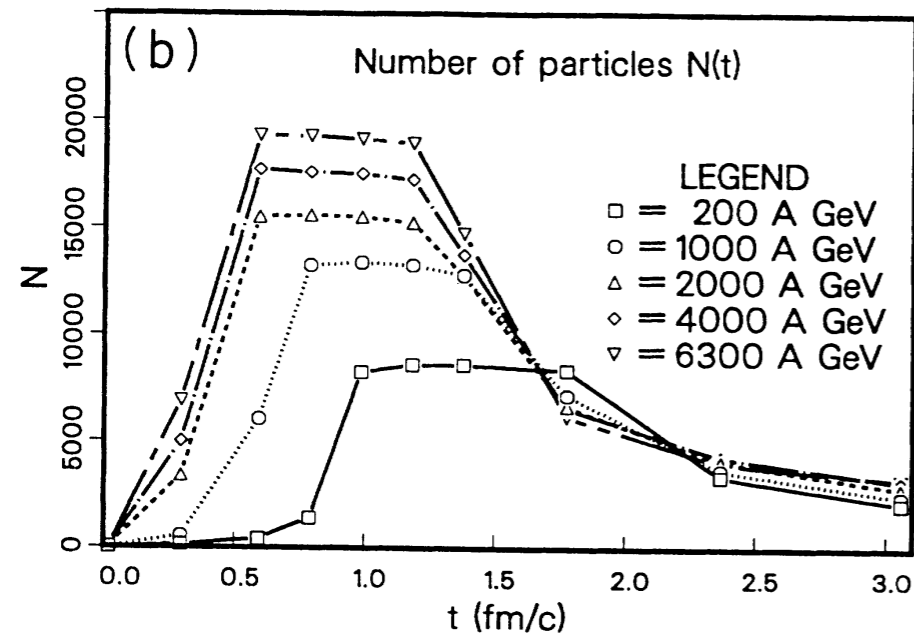
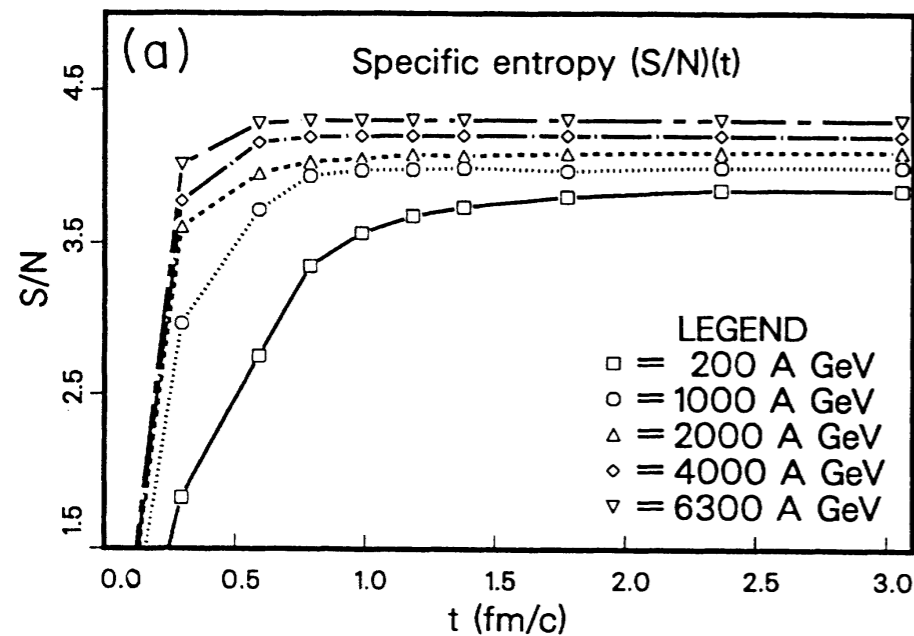


FIG. 14. The pion multiplicities in the central rapidity region around  $y = 0$  estimated from the amount of total entropy in the central unit of rapidity,  $dS/dy$ , produced by the secondary partons during Au + Au collisions with  $\sqrt{s} = 200A$ – $6300A$  GeV. The points result from the computed values of  $dS/dy$  for the beam energies  $\sqrt{s} = 200A$ – $6300A$  GeV. Note that the predictions are based on the estimated ratio  $r = 0.7 \pm 0.2$  of entropy densities between pion gas and quark-gluon plasma, which results in relatively large uncertainties as indicated by the shaded strip between the full curves. For comparison, the dotted curve corresponds to a more moderate empirical estimate for the multiplicity of pions obtained by extrapolation from  $pp$  and  $pA$  data.

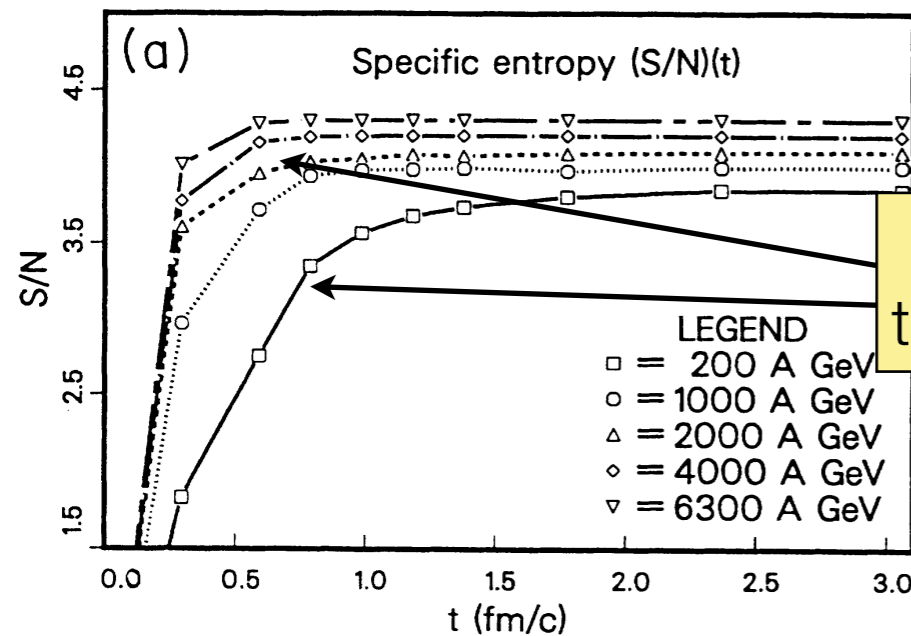
K. Geiger, Phys Rev D **46**, (4965) (1992),  
 K. Geiger, Phys Rev D **46**, (4986) (1992)

Klaus Geiger

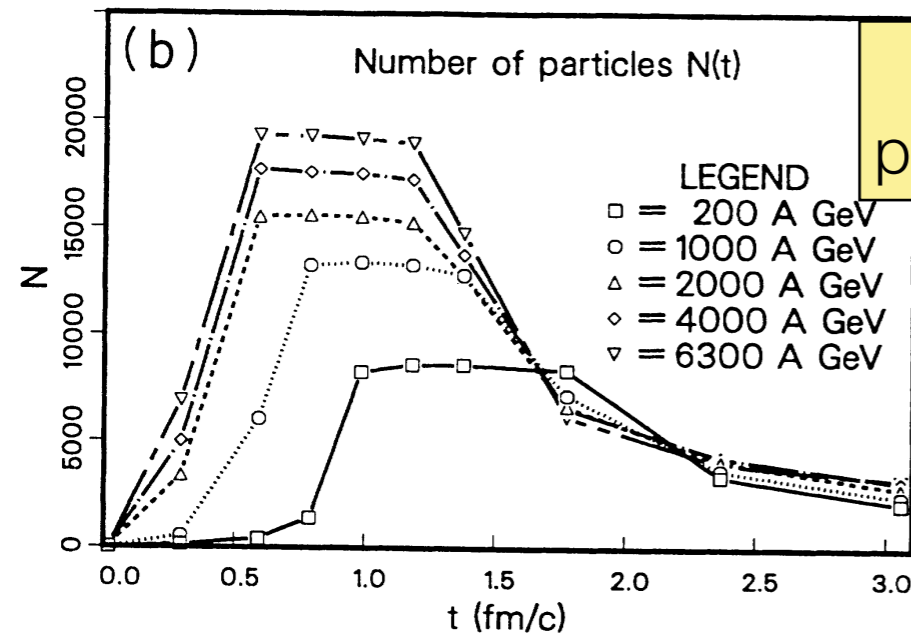
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# Thermalization - VNI



Very rapid thermalization



Huge pion multiplicities

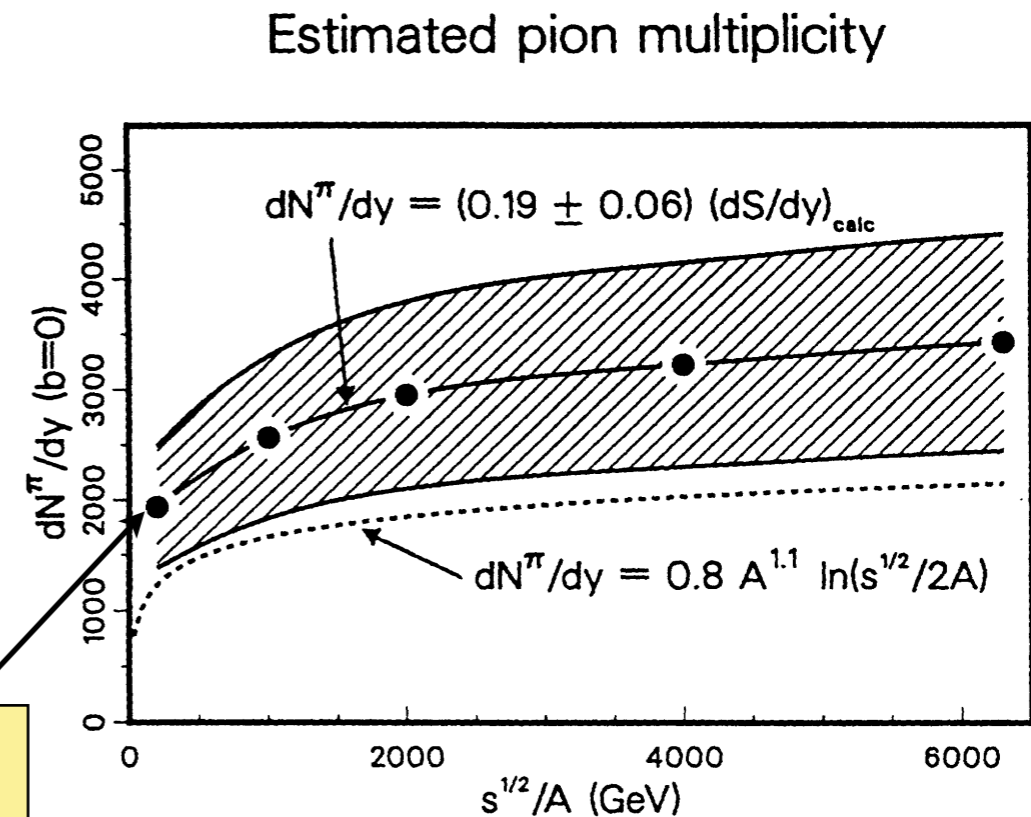


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K. Geiger, Phys Rev D **46**, (4965) (1992),

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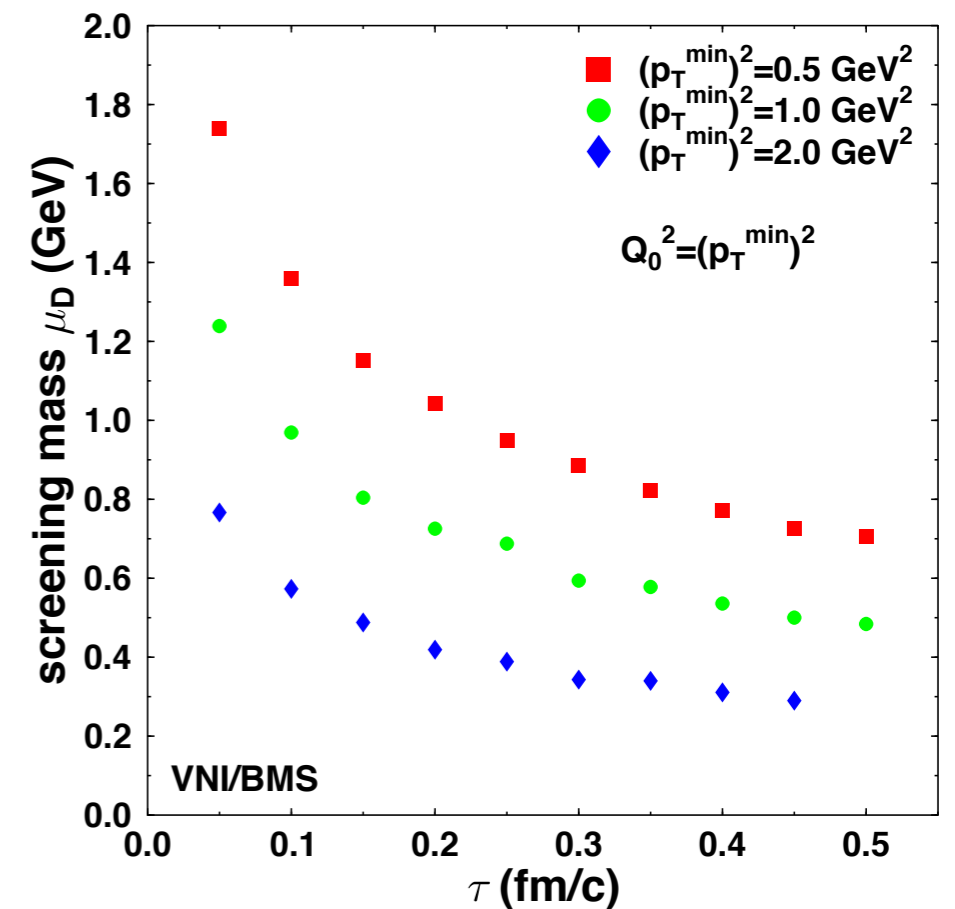
# VNI/BMS - Bass, Müller & Srivastava (2002)

- Hard Pt cutoff is related to the color screening mass instead of being purely phenomenological

$$\mu_D^2 = -\frac{3\alpha_s}{\pi^2} \lim_{|\vec{q}| \rightarrow 0} \int d^3p \frac{|\vec{p}|}{\vec{q} \cdot \vec{p}} \vec{q} \cdot \nabla_{\vec{p}} \left[ F_g(\vec{p}) + \frac{1}{6} \sum_q \{F_q(\vec{p}) + F_{\bar{q}}(\vec{p})\} \right],$$

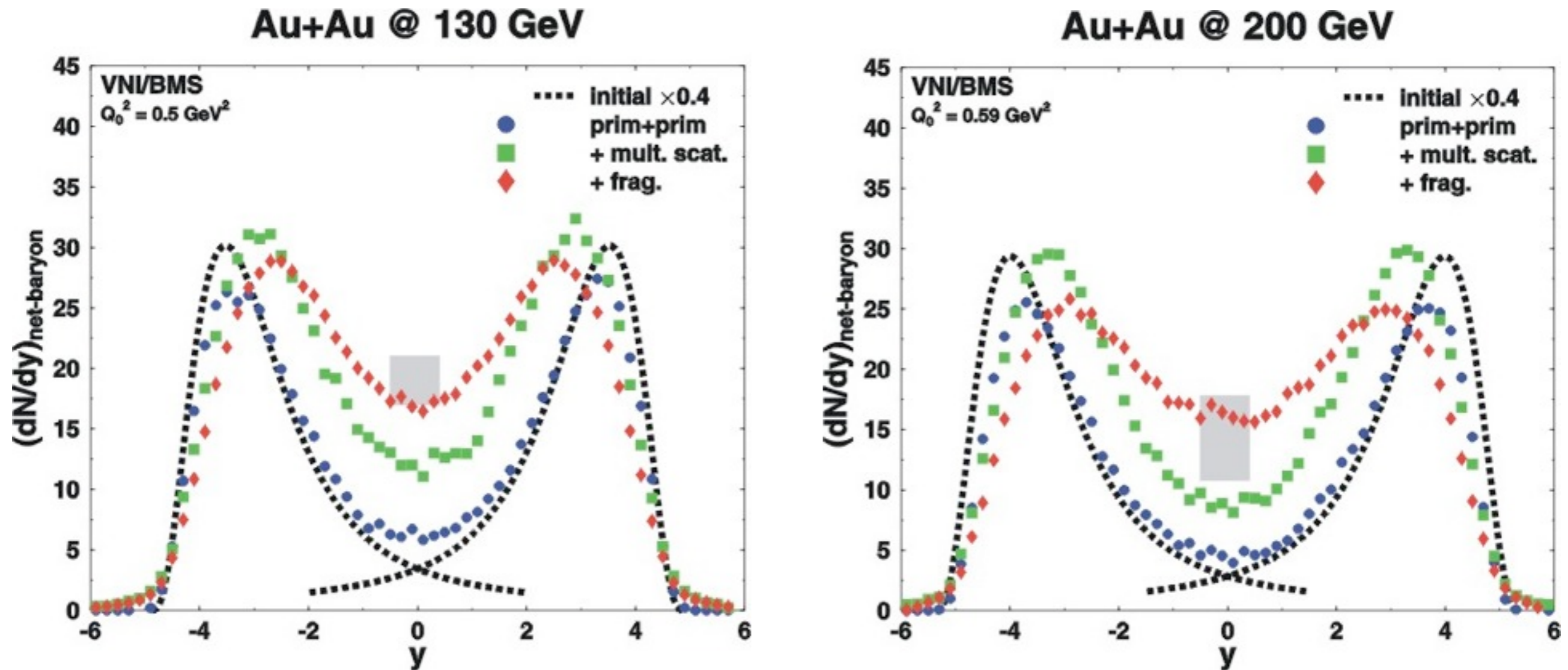
- Color screening mass is **not** dynamically calculated, typical values are used to regulate cross-sections
- Removed all the soft processes
- Replaced uniform time-stepping with collision finding routines (more efficient)
- Added photons to the 2->2 and 2->N processes

Au+Au;  $E_{\text{CM}}=200$  AGeV



S.A.Bass, B.Müller, D.K.Srivastava. Phys Lett B. **551** (2003)

# Stopping at RHIC: VNI/BMS Results



- A lot of effort figuring out a reasonable set of cutoffs for AA
- Multiplicity at mid rapidity works

S.A.Bass, B.Müller, D.K.Srivastava. PRL **91** (2003)

# Many Other Models Arose

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- ZPC (1999). Gluon only, forms the PCM core of AMPT, Parton Subdivision
- MPC (2000). Subdivision of particles, broad set of included processes
- AMPT (2004). Hybrid transport code, includes stringy effects
- BAMPS (2005). Uses subdivision and a grid, rate based approach naturally allows for 2->3 and 3->2. Big problems with GB matrix element
- Andong (2002). Full set of 2->2 processes, 2->3 gluon branching, uses a retarded-potential like approach to address causality issues
- These were all relatively successful (or not) at predicting some bulk properties

ZPC: nucl-th/9709009

MPC: nucl-th/0104018

AMPT: nucl-th/9907017, nucl-th/041110

BAMPS: hep-ph/0406278

Andong: nucl-th/0207041



# MPC

- Subdivide partons to avoid causality problems inherent in geometric interpretation of the cross-section.
- Fully covariant formulation, uses scaling properties of Boltzmann equation.
- Used very large transport cross-sections to obtain thermal equilibrium
- Under current development, now includes fully covariant support for 1->2 and 2->1 interactions.

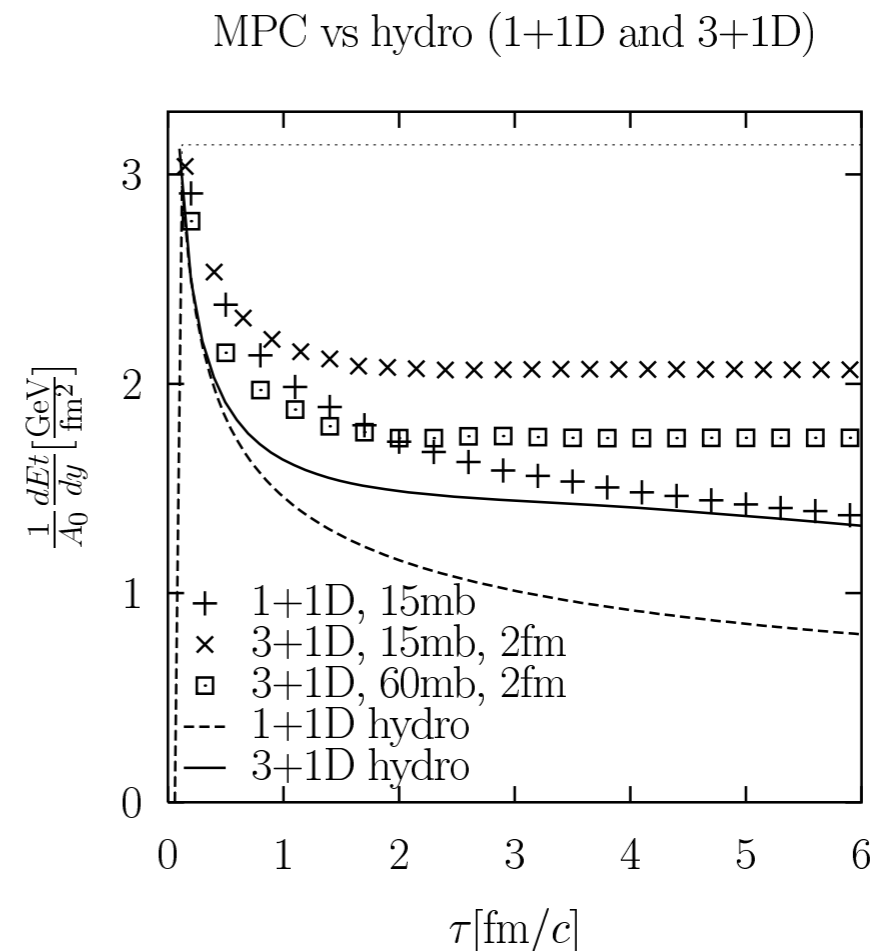


FIG. 1. This figure shows the evolution of the transverse energy  $dE_t/dy$  at midrapidity, normalized by the initial transverse area, from kinetic theory and from hydrodynamics both for 1+1 (transverse periodic) and 3+1 dimensions. The initial distribution was a Bjorken cylinder with a radius  $R_0 = 2$  fm at proper time  $\tau_0 = 0.1$  fm/c in local thermal and chemical equilibrium at  $T_0 = 500$  MeV. The cross sections were  $\sigma = 15$  and 60 mb, with the cutoff

Molnar & Gyulassy nucl-th/0005051 (2000)

# AMPT - A Parton Cascade + ...

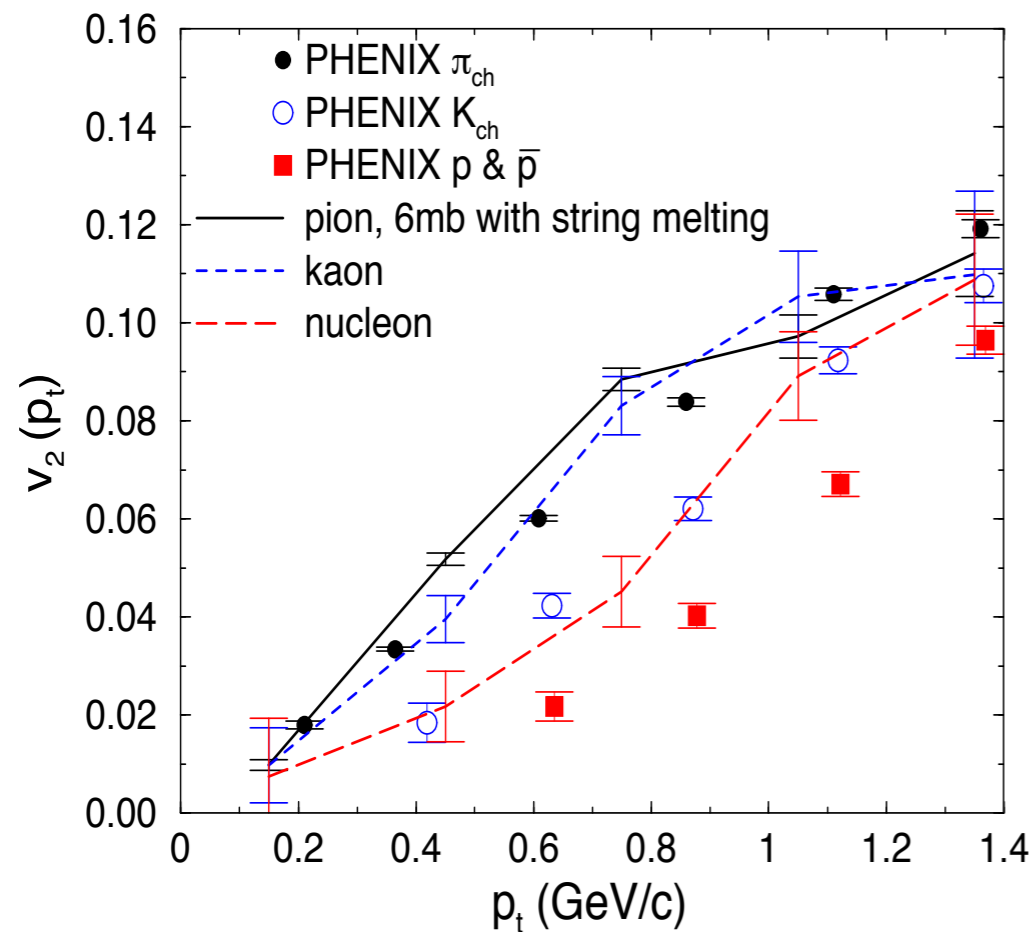


FIG. 29: (Color online) Transverse momentum dependence of elliptic flow of identified hadrons for minimum-bias Au+Au collisions at  $\sqrt{s_{NN}} = 200$  GeV. Circles are data from the PHENIX Collaboration [208] while curves are results from the AMPT model with string melting and using 6 mb for the parton scattering cross section.

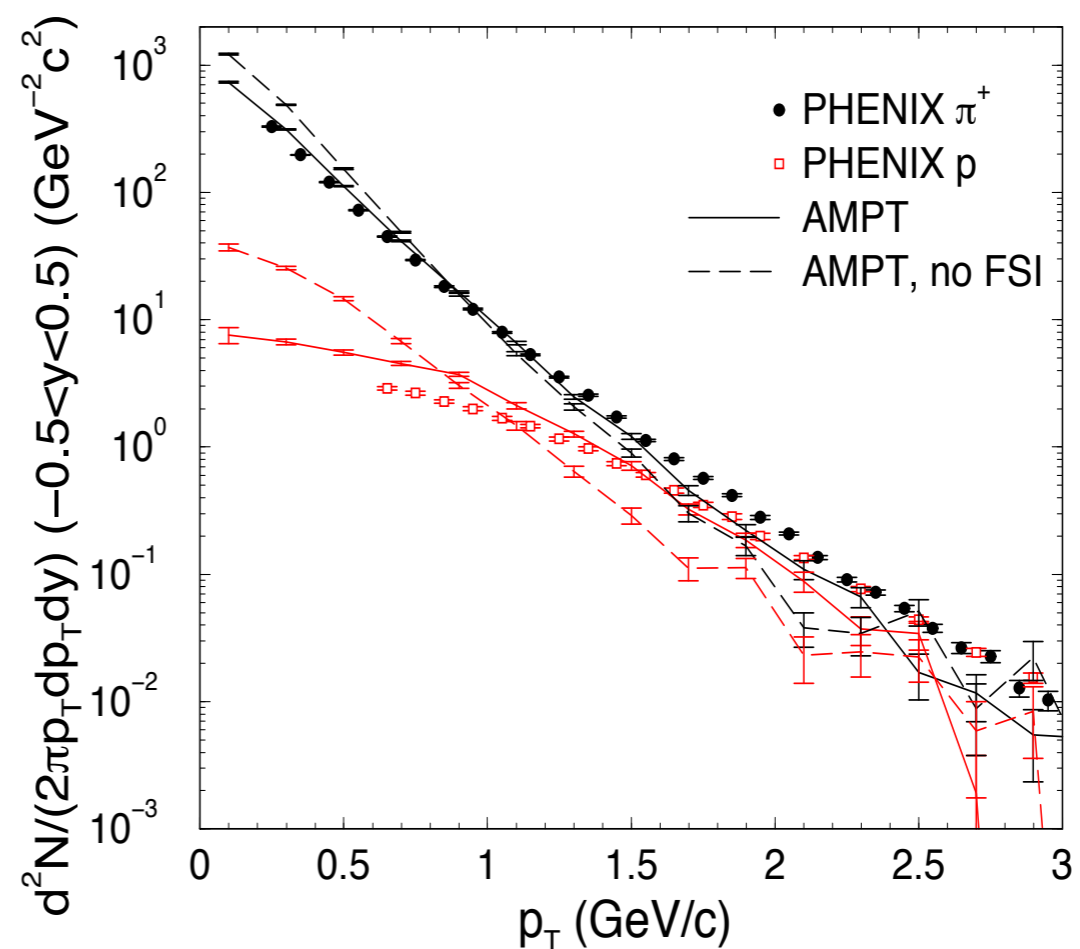


FIG. 25: (Color online) Transverse momentum spectra of mid-rapidity pions and protons from the default AMPT model with (solid curves) and without (dashed curves) final-state interactions in central Au+Au collisions at  $\sqrt{s_{NN}} = 200$  GeV.

String melting fits flow

Regular model fits spectra

# Parton Cascades - Jet Observables

# BAMPS

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- a Boltzmann Approach for MultiParticle Scattering
- Uses sub-division partons and a spatial grid, allows for a rate based interpretation of the collision term.
- Rate based approach naturally allows for 2->3 and 3->2 processes
- Dogged by problems in the 2->3 (Gunion-Bertsch) cross-section.
- Problem was finally tracked down in 2013, small rapidity approximations in GB calculation.

## **BAMPS:**

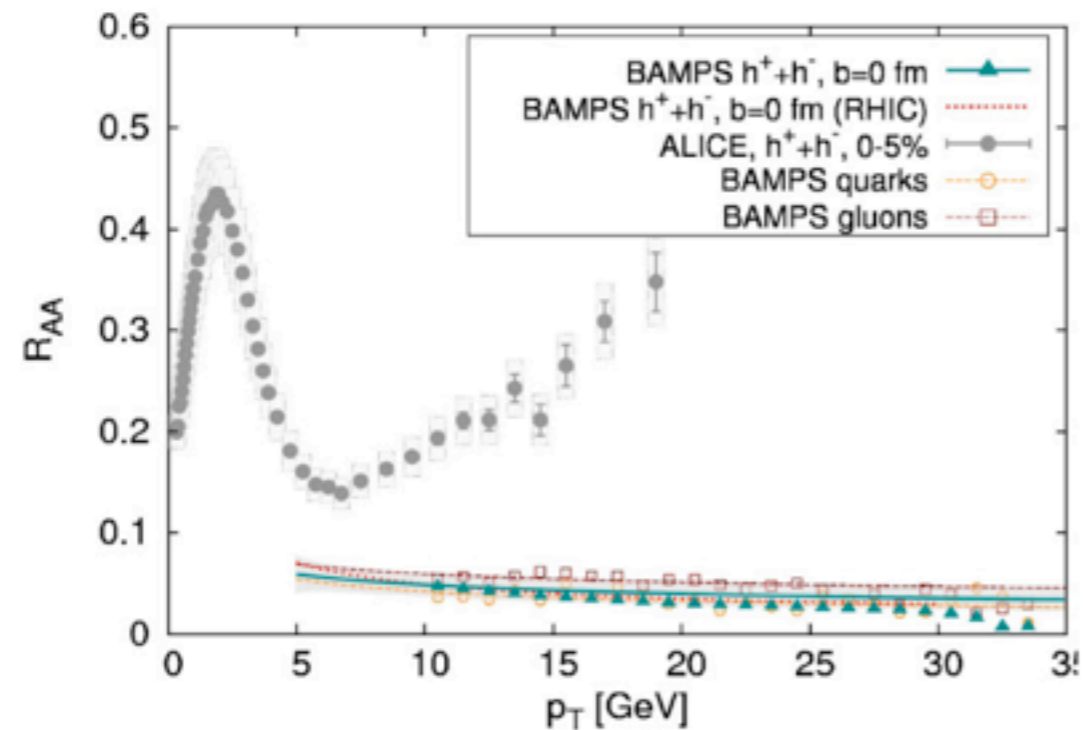
Z. Xu and C. Greiner, PRC 71, 064901 (2005);  
Z. Xu and C. Greiner, PRC 76, 024911 (2007)

## **GB Correction:**

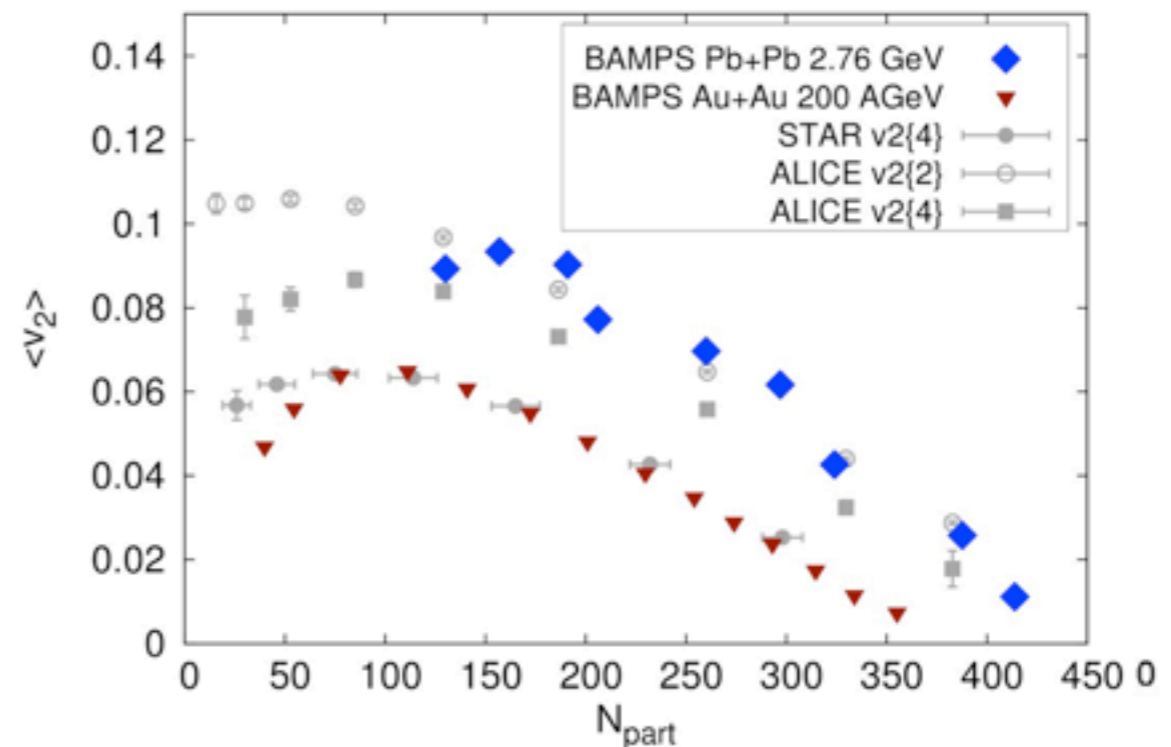
Fochler.O et al, hep-ph/1302.5250

# BAMPS: Raa and Flow at LHC

$R_{AA}$ , Pb + Pb at 2.76 A TeV, 0%–5%



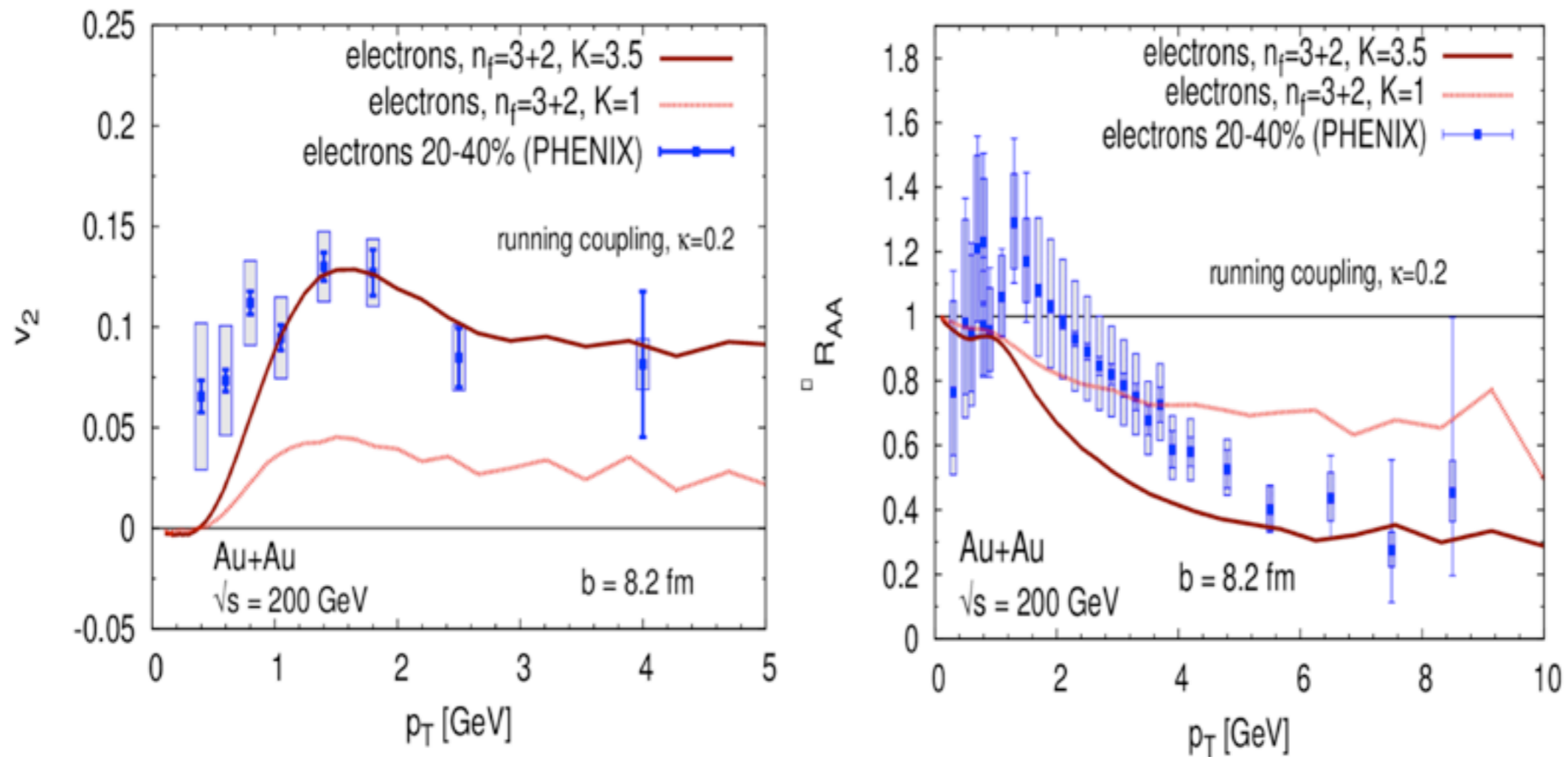
$v_2$ , Pb + Pb at 2.76 A TeV



- Pythia Initial Conditions (Uphoff, Fochler et al, PRC 82 (2010))  $\alpha_s = 0.3$
- Raa similar to RHIC, doesn't rise at high Pt.
- Integrated  $V_2$  shows increase, drops below data at about 50% centrality

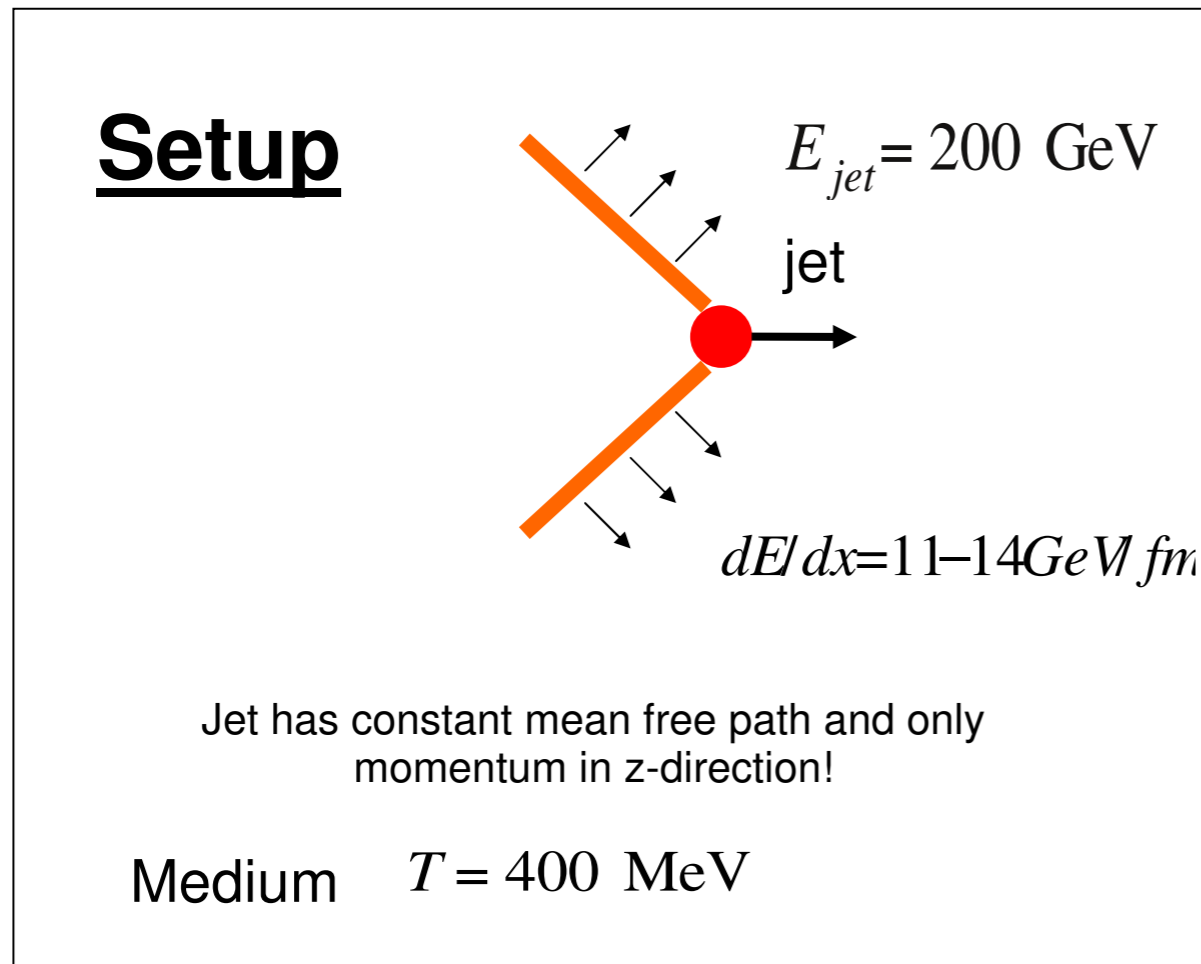
O. Fochler et al, J. Phys. G 38 (2011)

# BAMPS: Heavy Quark $V_2$ and $R_{AA}$ at RHIC



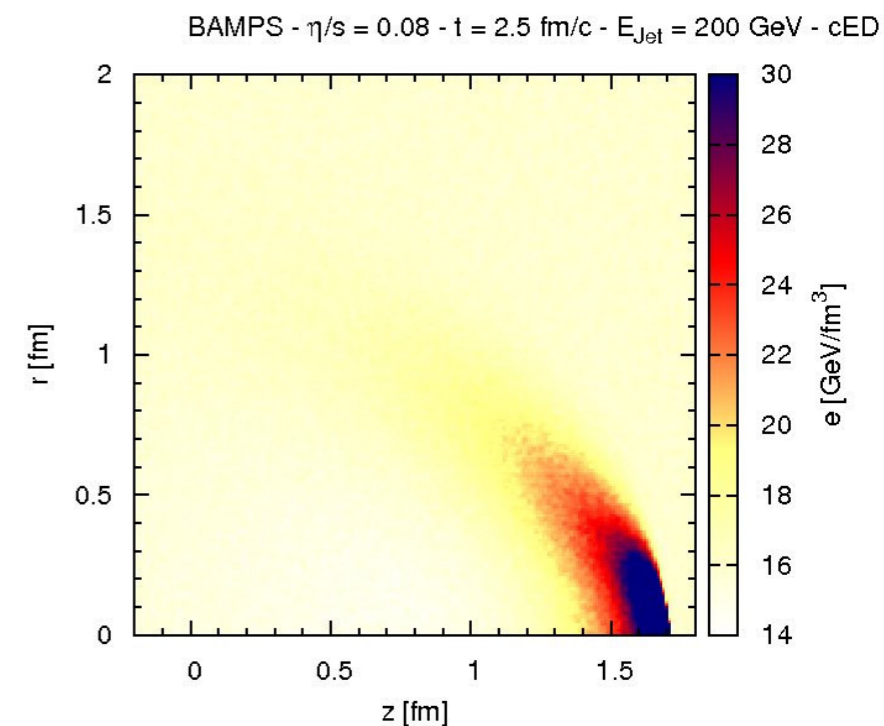
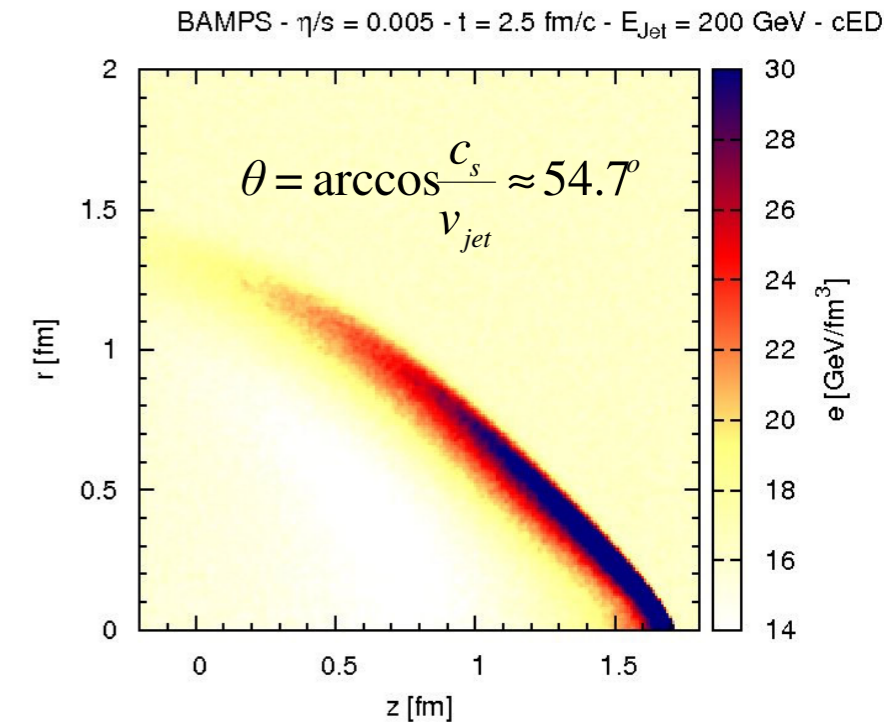
- Needs a large K factor (cross section scaling) to approach data
- Petersen Fragmentation Model: independent fragmentation

# BAMPS: Mach Cones ...



Box scenario, no expansion of the medium, massless Boltzmann gas interactions:  $2 \rightarrow 2$  with isotropic distribution of the collision angle

Looks nice, not a very realistic calculation



# BAMPS: GB Issues

- GB Matrix Element was a de-facto leading order 2->3 calculation (qq->qqg)

$$x = \frac{k_{\perp}}{\sqrt{s}} e^y$$

$$|\mathcal{M}_{qq' \rightarrow qq'g}|^2 \simeq 12g^2 |\mathcal{M}_{qq \rightarrow qq'}|^2 (1-x)^2 \frac{q_{\perp}^2}{k_{\perp}^2 (\vec{q}_{\perp} - \vec{k}_{\perp})^2}$$

- (1-x) dependence is usually dropped, GB interested in mid rapidity.
- Transport calculations need to cover the **full** rapidity range. Calculation becomes rather messy.

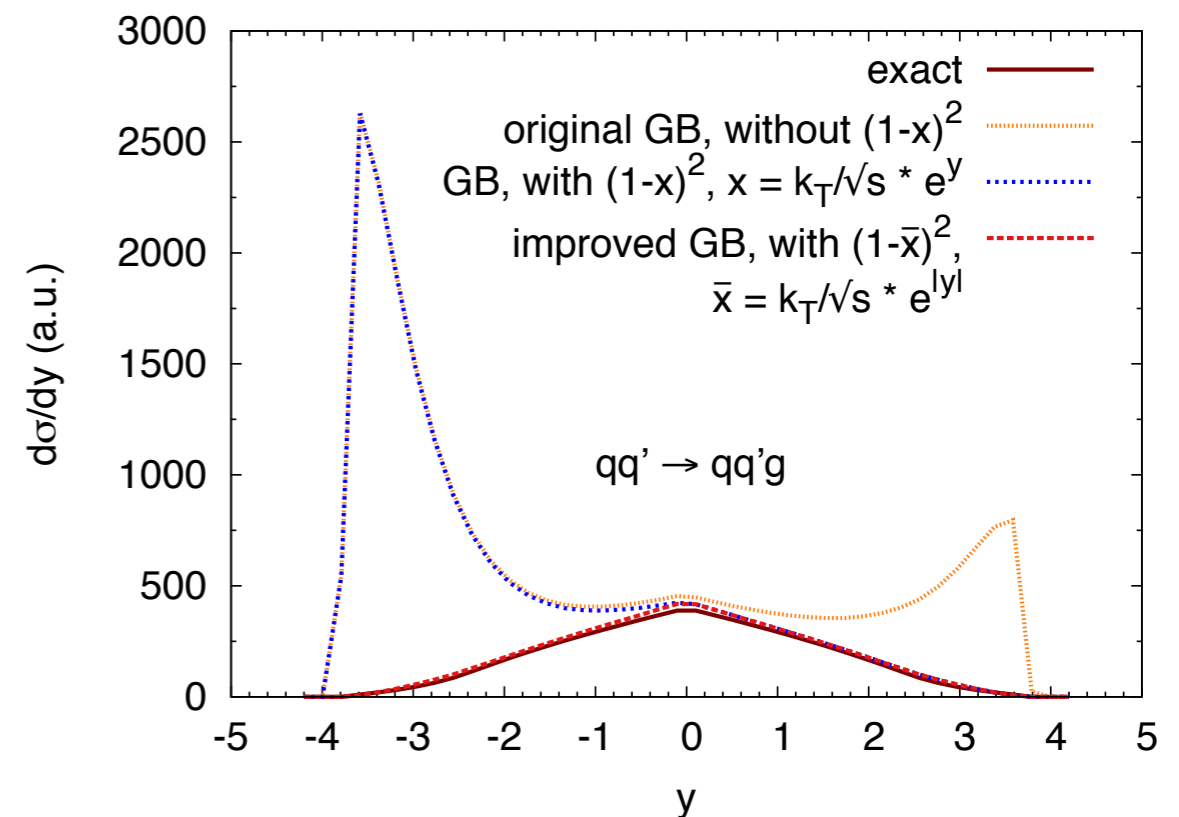
- Awaiting improved results

Gunion.J & Bertsch.G, Phys.Rev.D**25** (1982)

**GB Correction:**

Fochler.O et al, hep-ph/1302.5250

Exact Calculation removes forwards backwards peaks in rapidity (qq)



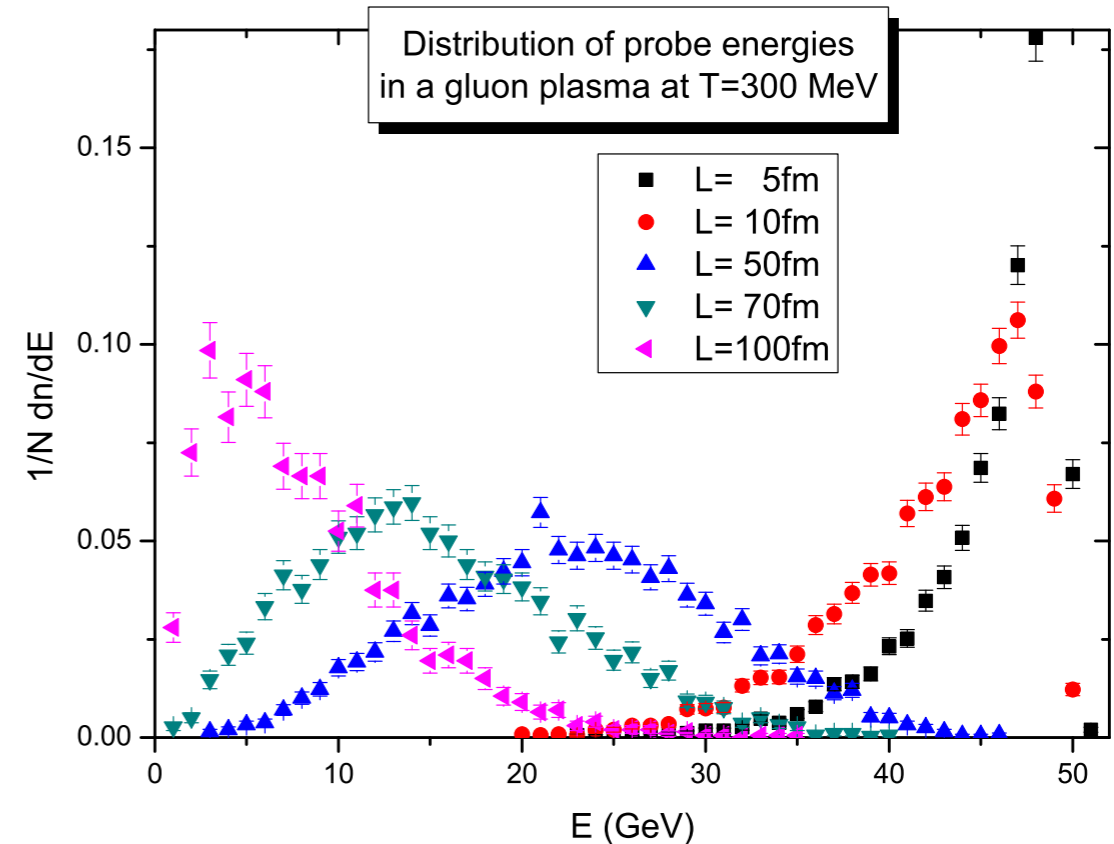
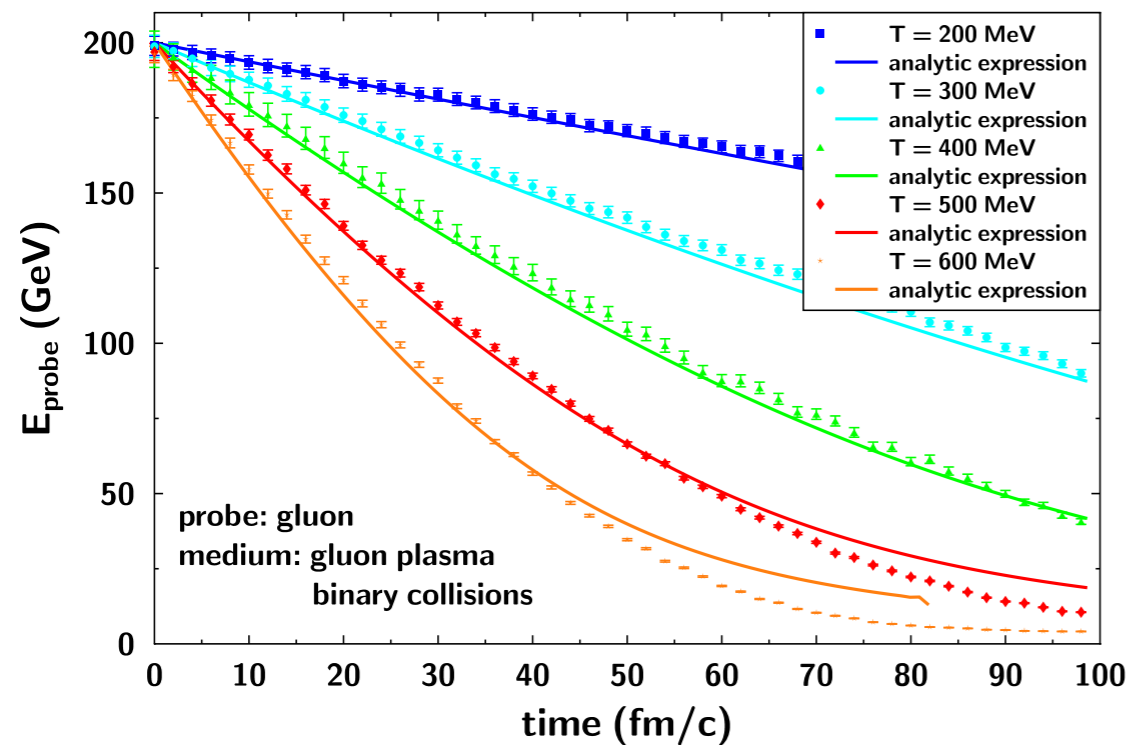
Total cross-section, integrals cutoff at Debye mass



# Parton Cascades - VNI/BMS-2.0

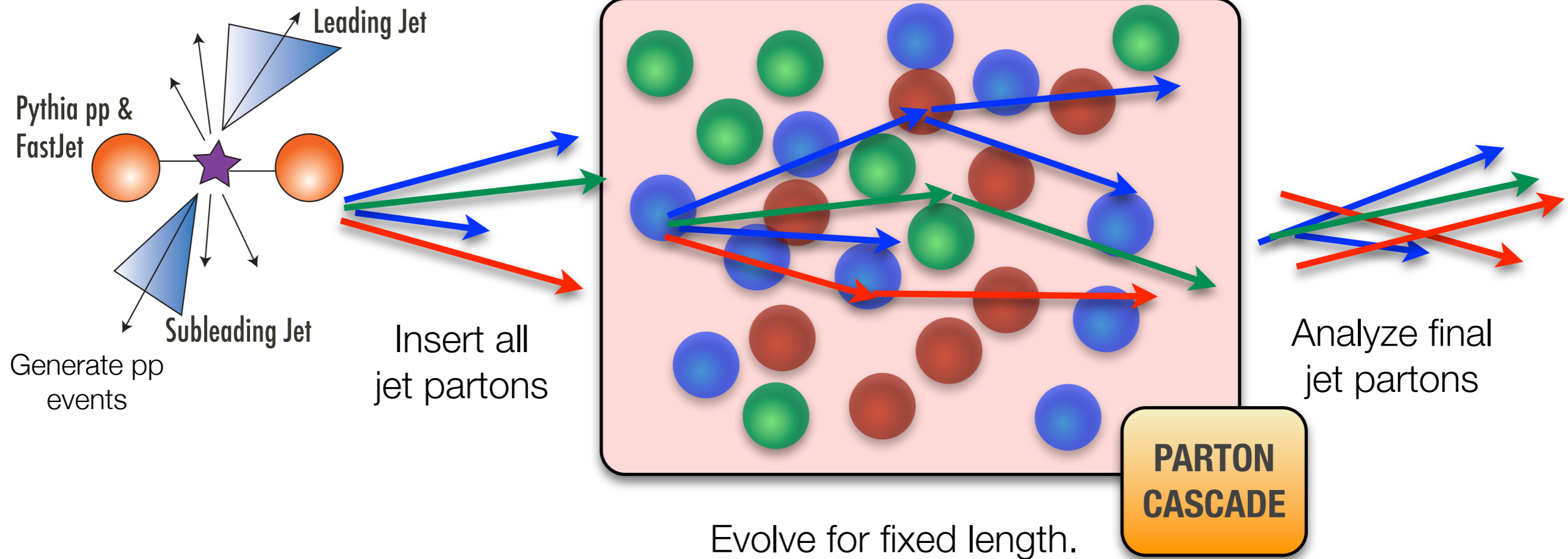
## Box Mode

# Andong & VNI/BMS - Jet Transport



- First introduction of a box mode in VNI/BMS. Equilibrium thermal masses used to regularize cross-sections
- Compared the **elastic** energy loss of single quarks and gluons through a thermal box of ideal pQGP matter

# Jet Simulation Method: Box Mode



- Insert **all partons from each** jet into parton cascade box and **evolve for a fixed path length.**

- Medium is **partonic** and **static**, temperature can be **fixed** for the entire evolution.

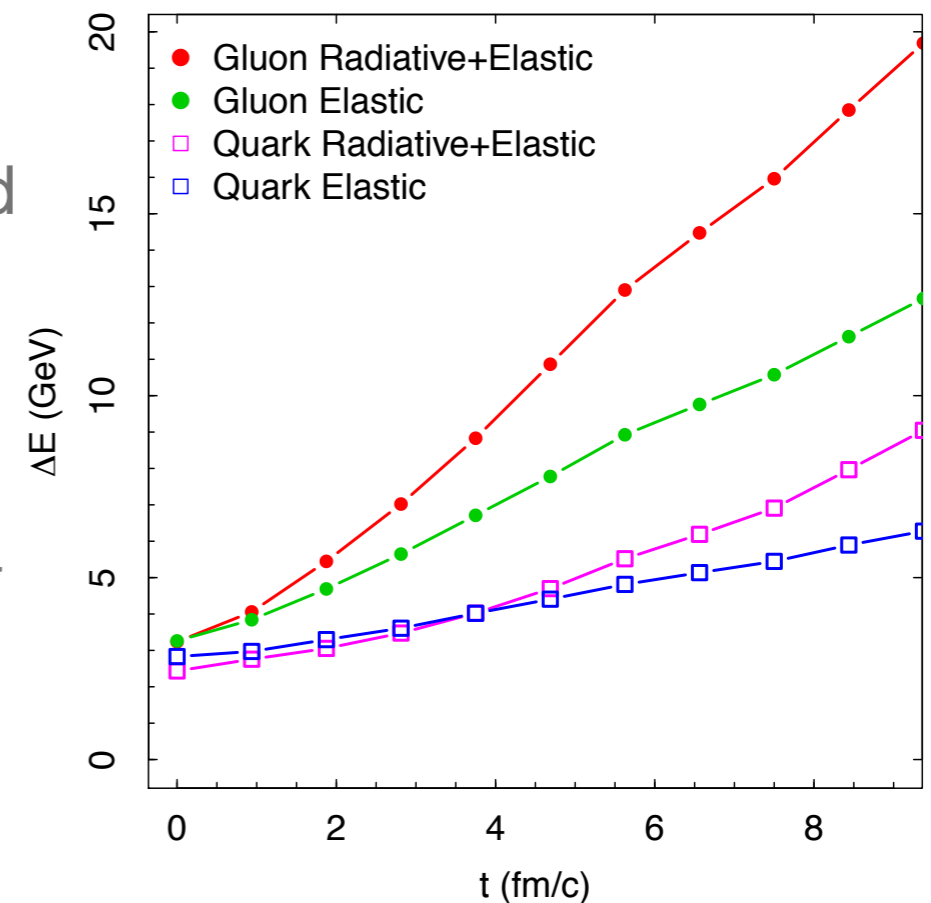
- Medium and jet interact on an **equal footing**, track all resulting partons

A fully controllable brick of QCD matter

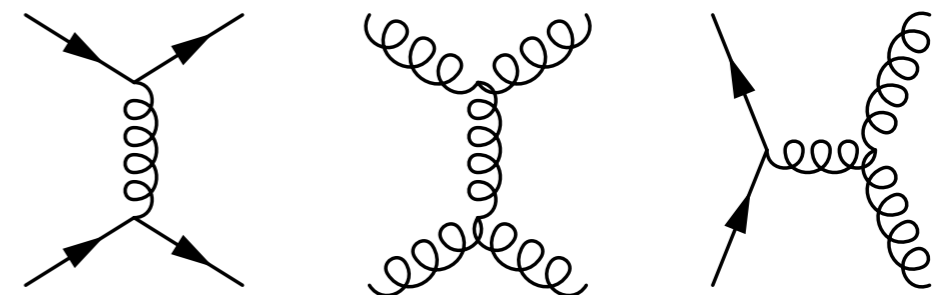
# VNI/BMS - 2.0, a simple JET transport model

- Partonic transport via the Boltzmann equation. Treats medium and jet on an **equal footing**.
- Interactions are tree level 2->2 scatterings and **final-state radiation**. Radiation includes leading order (BDMPS-Z) LPM effect.
- Medium is a box of thermal partonic QGP at a fixed temperature. No expansion!
- Cross sections are screened by Debye mass, computed using the box temperature
- A generated jet is injected, cascade of interacting partons are tracked. **Evolution of entire jet is recorded.**

$$p^\mu \frac{\partial}{\partial x^\mu} F_k(x, \mathbf{p}) = \sum_{\text{processes}} C_i F.$$

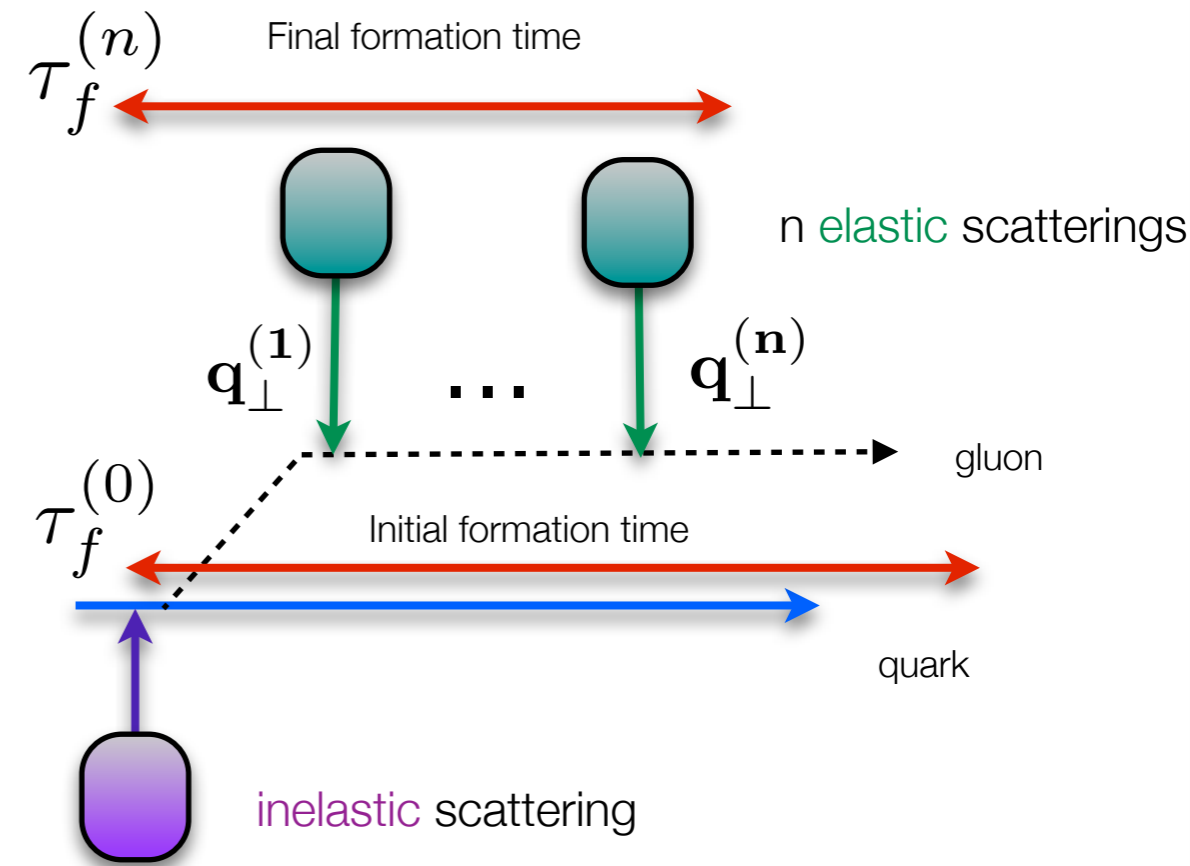


$$-\Delta E_{\text{BDMPS}} = \frac{\alpha_s C_R \mu^2}{8 \lambda_g} L^2 \log \frac{L}{\lambda_g}.$$



# Zapp and Wiedemann, LPM Algorithm

- Probabilistic local implementation of coherence, gives rise to an  $L^2$  energy loss.
- Post **Inelastic** scattering, compute formation time of emitted gluon
  - Emitting parton **does not interact** during this time
  - Radiated gluon **rescatters elastically** off the medium, recompute modified formation time
  - Repeat until **formation time expires**
  - Quark and gluon propagate **freely**

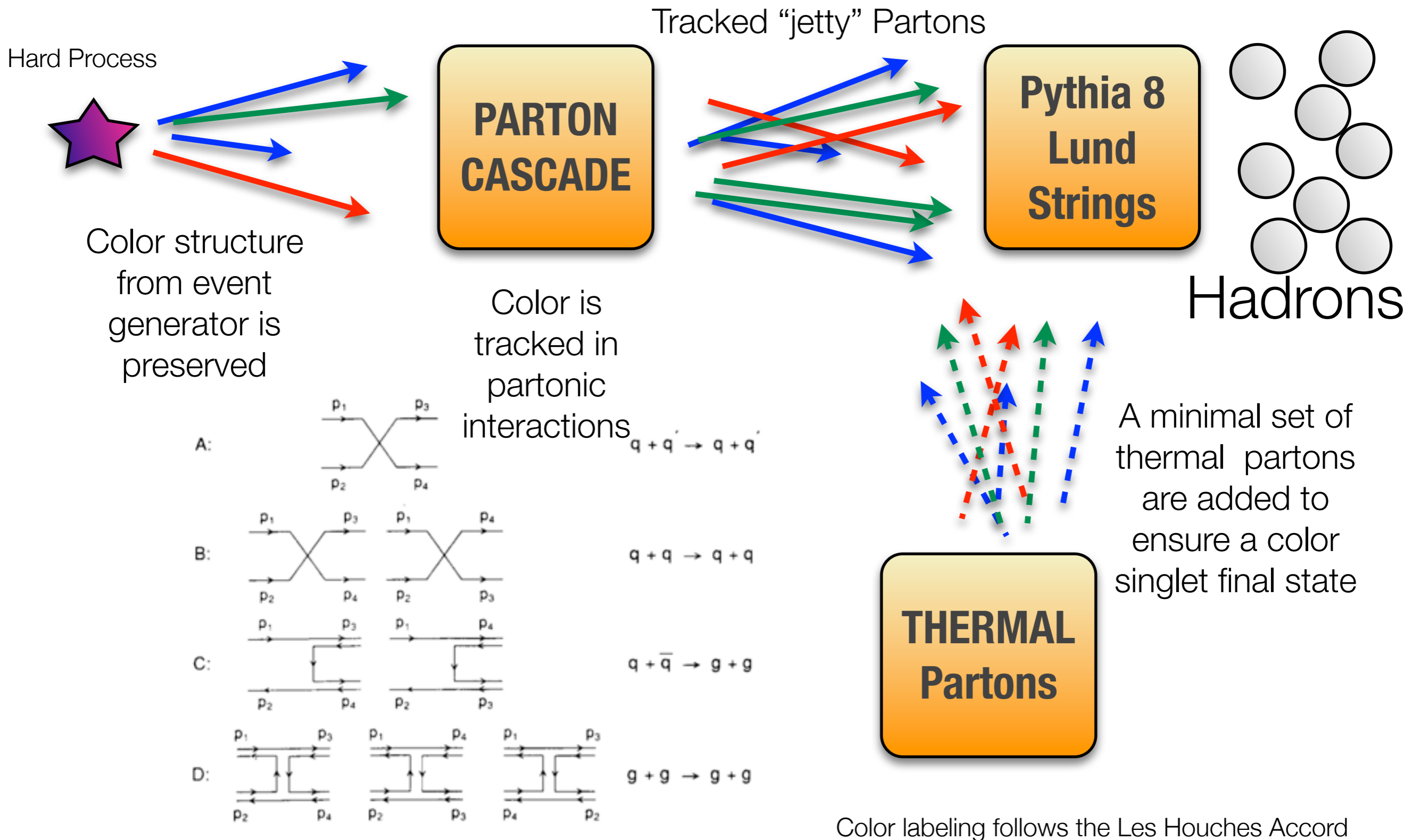


$$\tau_f^{(n)} = \frac{2\omega}{(\mathbf{k}_\perp + \sum_{i=0}^n \mathbf{q}_\perp^{(i)})^2}$$

- Simulates **coherent emission** from multiple centers

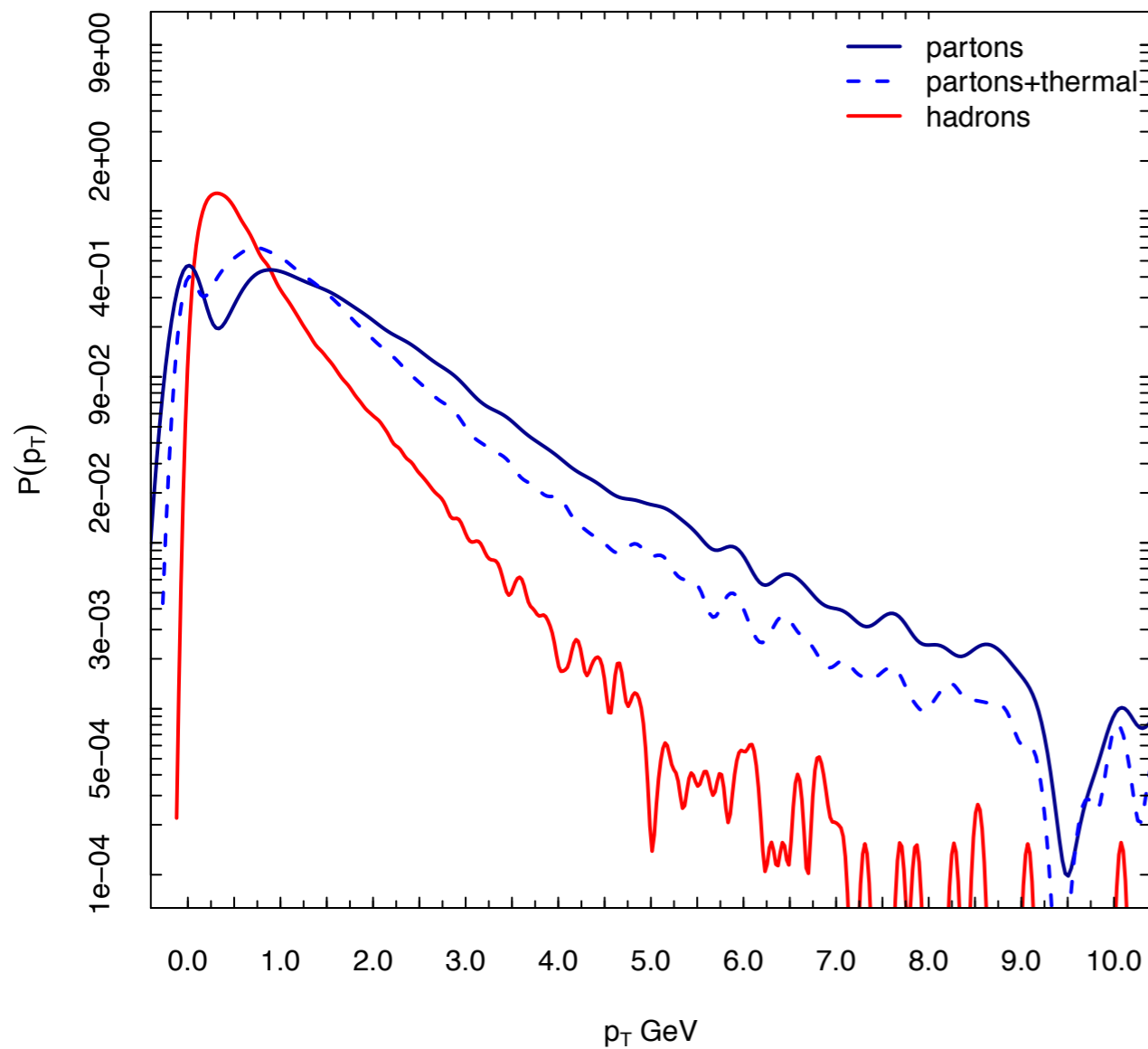
Zapp K, Wiedemann U. *Phys Rev Lett*, 103 (2009) JEWEL  
 CCS, S.A.Bass, D.K.Srivastava, *hep-ph/1101.4895*

# Hadronization In VNI/BMS - 2.0

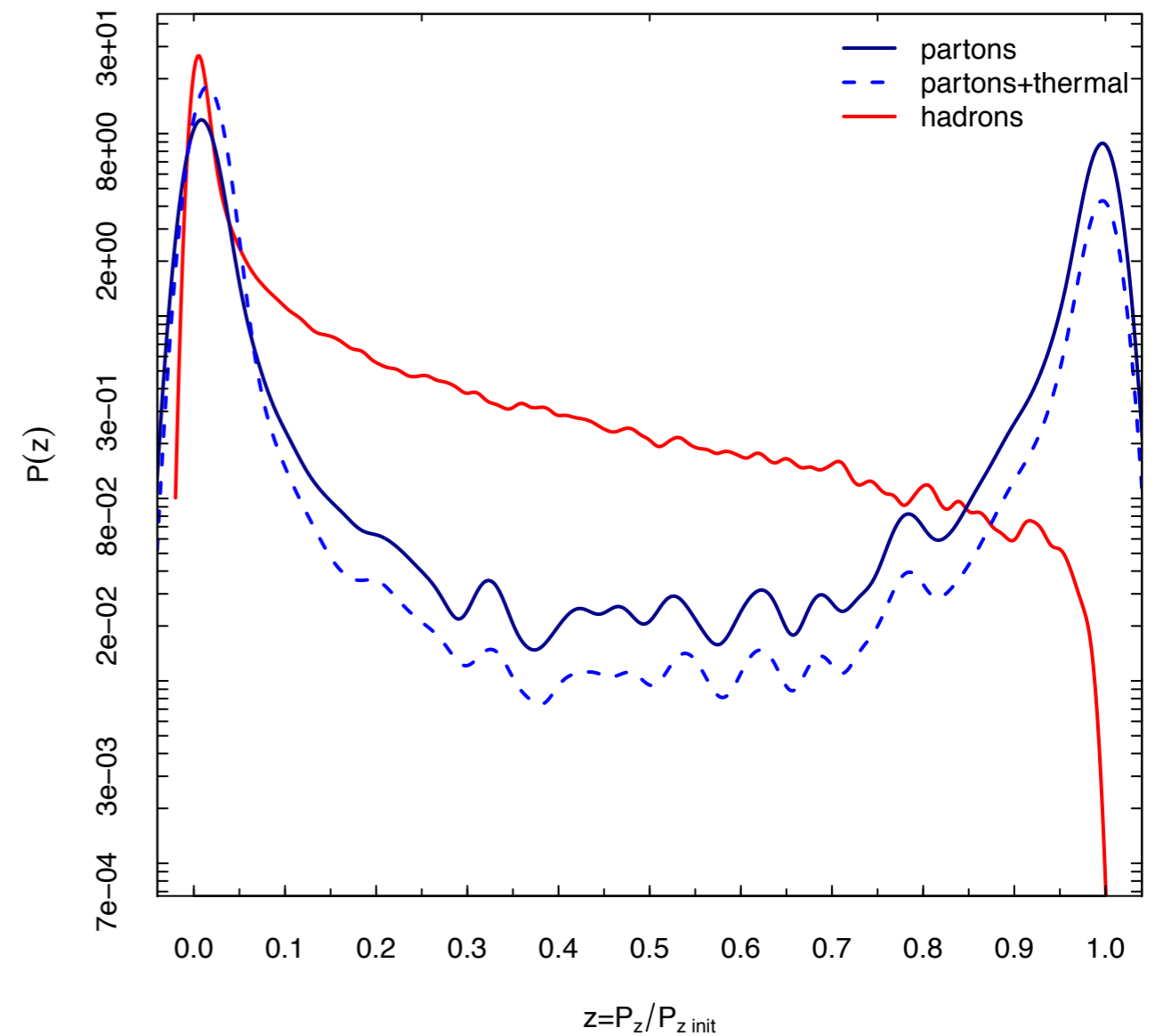


# Hadronization In VNI/BMS

100 GeV quark evolved for 4fm in a box at  $T=350$  MeV



Hadronization contracts the transverse momentum distribution



Hadronization smooths the longitudinal distribution, peak at  $Z=1$  is redistributed

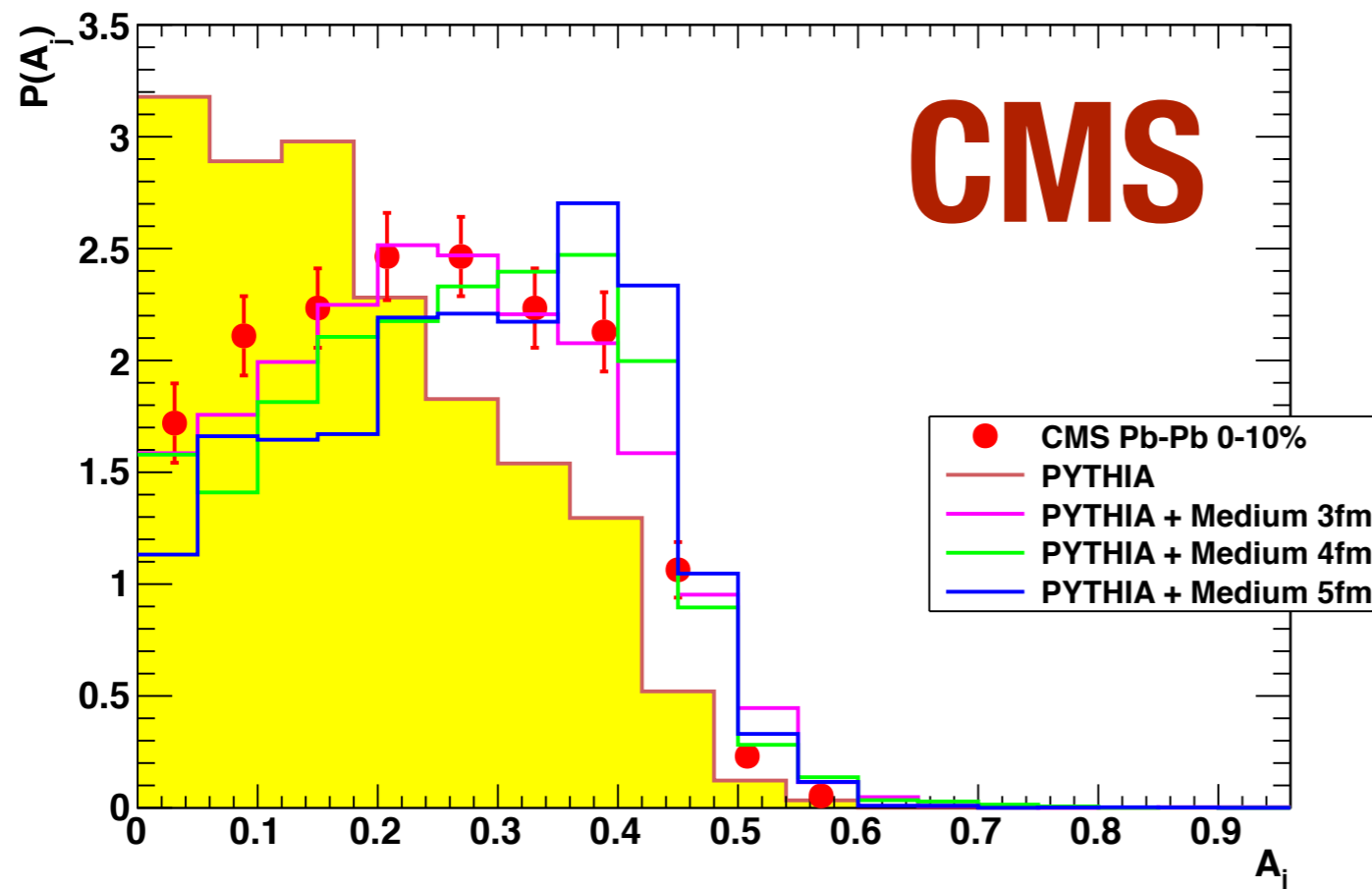
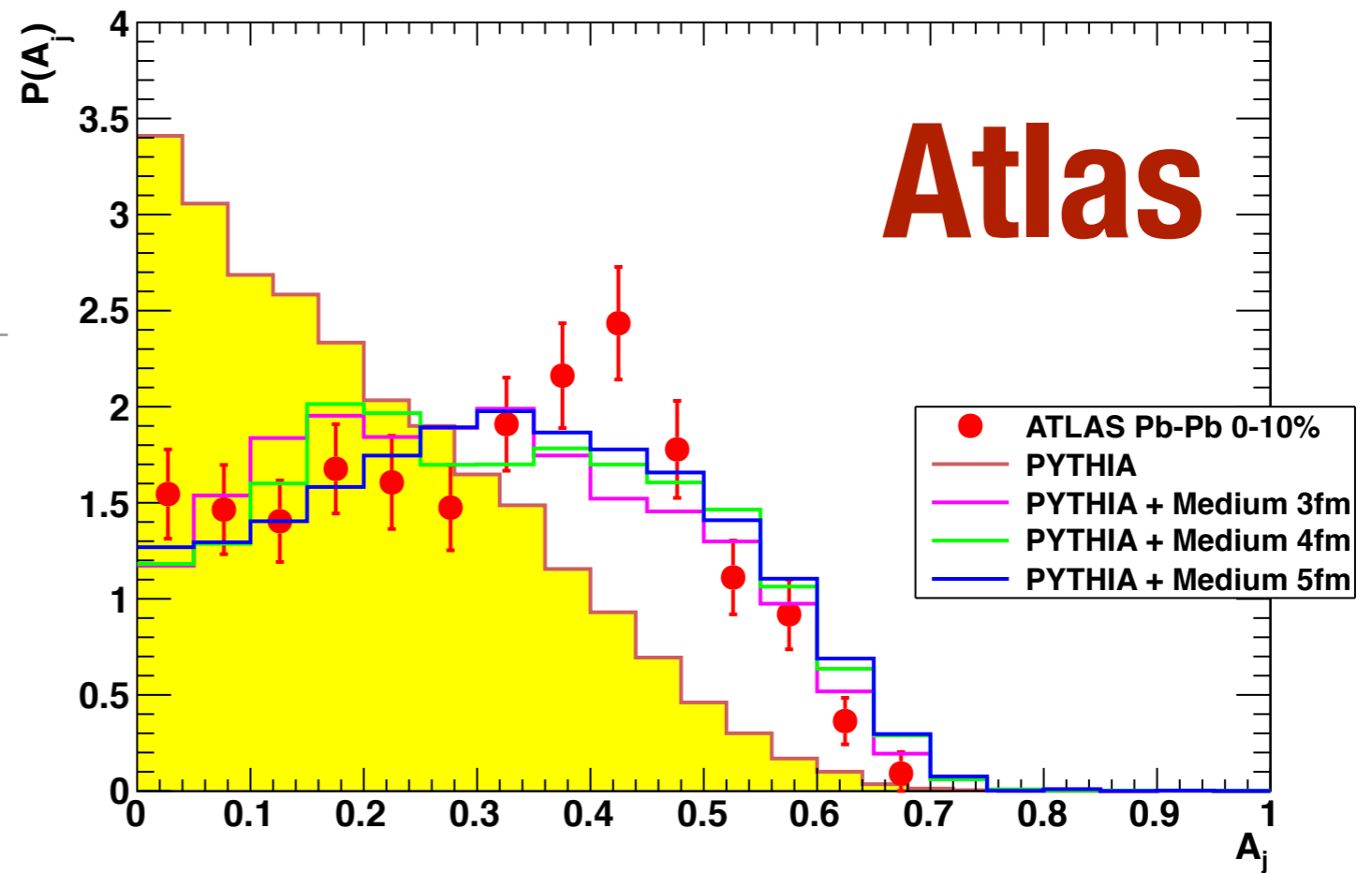
# Jet Observables - VNI/BMS - Box Mode



# LHC Dijet Results

Circle Mode + Glauber Vertices can reproduce LHC data reasonably well.

Central collisions (0-10%)

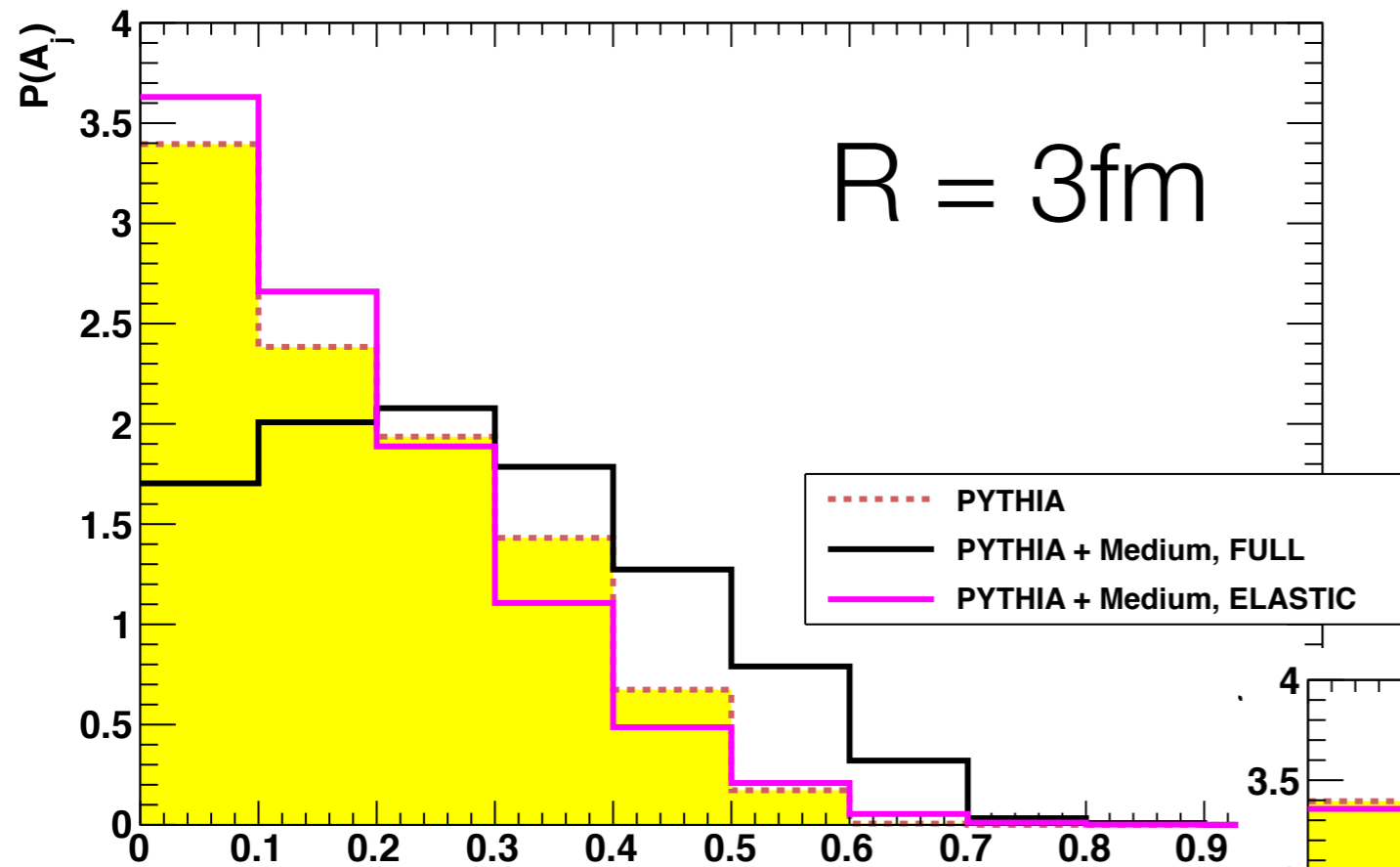


Both results include detector smearing effect

$$E_t^* \sim N(E_t, \alpha \sqrt{E_t}), \quad \alpha \simeq 1.2$$

Presented at QM 2011

# Process Dependence

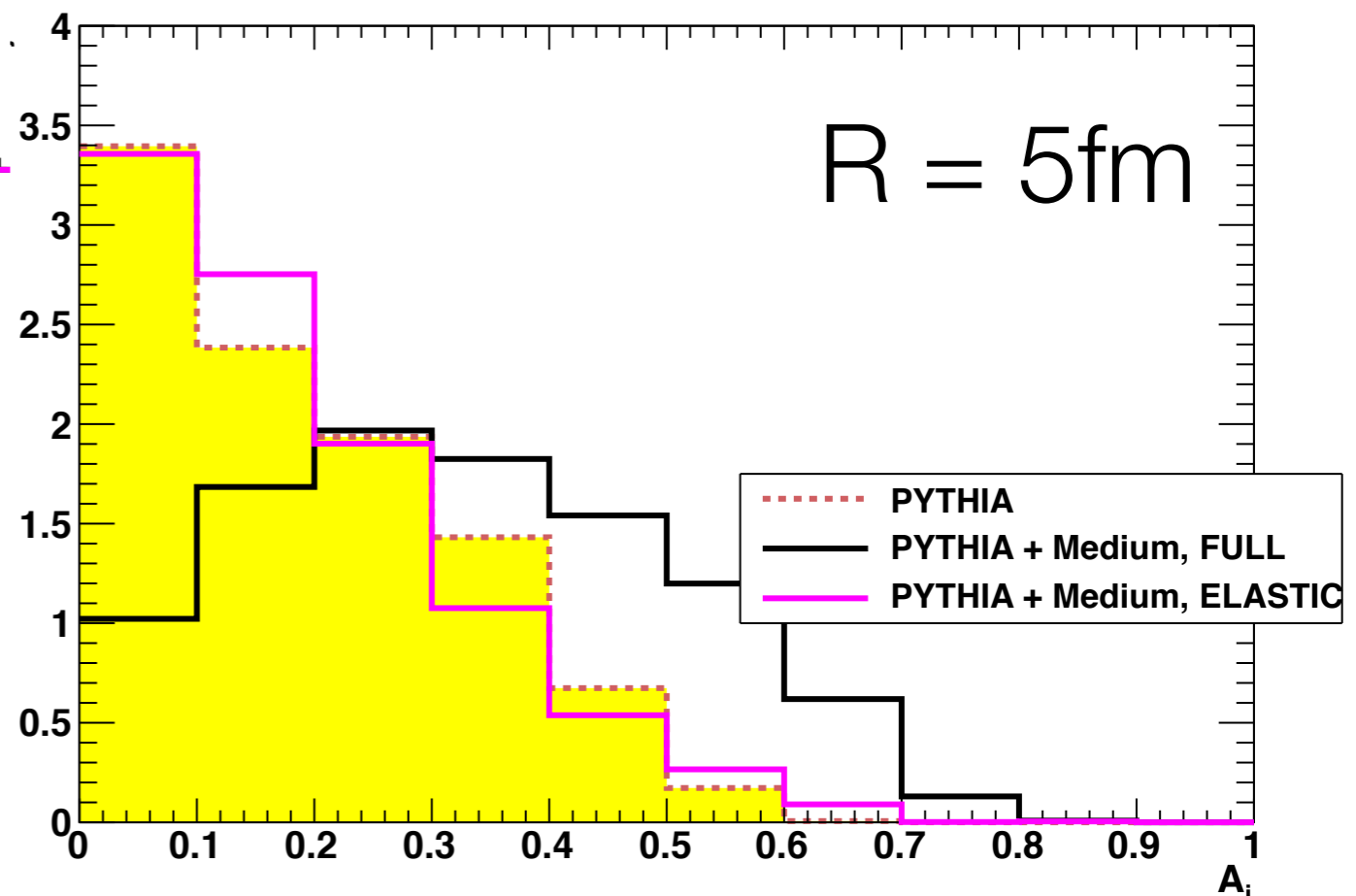


- $T = 0.35\text{ GeV}$
- Elastic only, distribution shifts to the left. **Medium reduces the asymmetry**
- Jet “picks up” medium particles by forward scattering

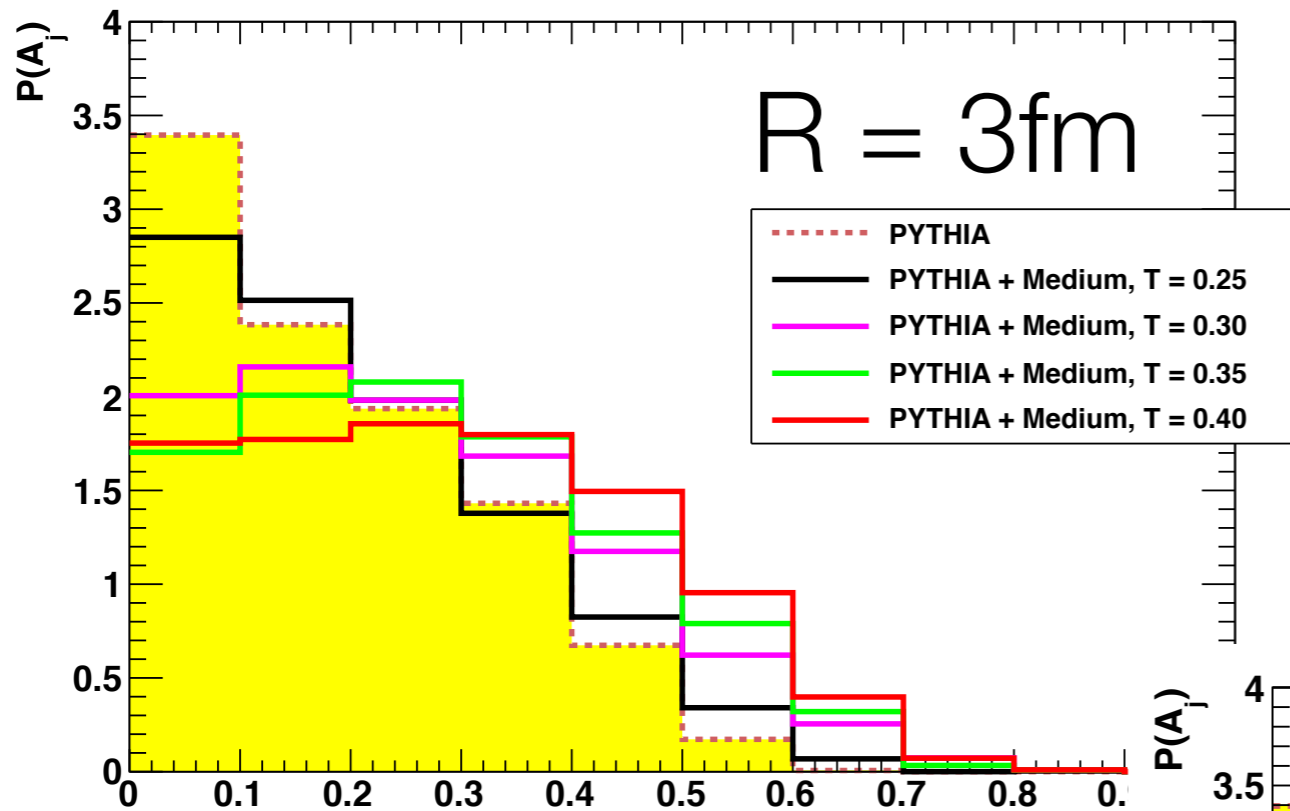
$$E_{T1} > 100\text{GeV} \quad E_{T2} > 25\text{GeV}$$

Radiation plus elastic gives increased asymmetry.

Soft radiated gluons can be **elastically scattered** out of the jet cone



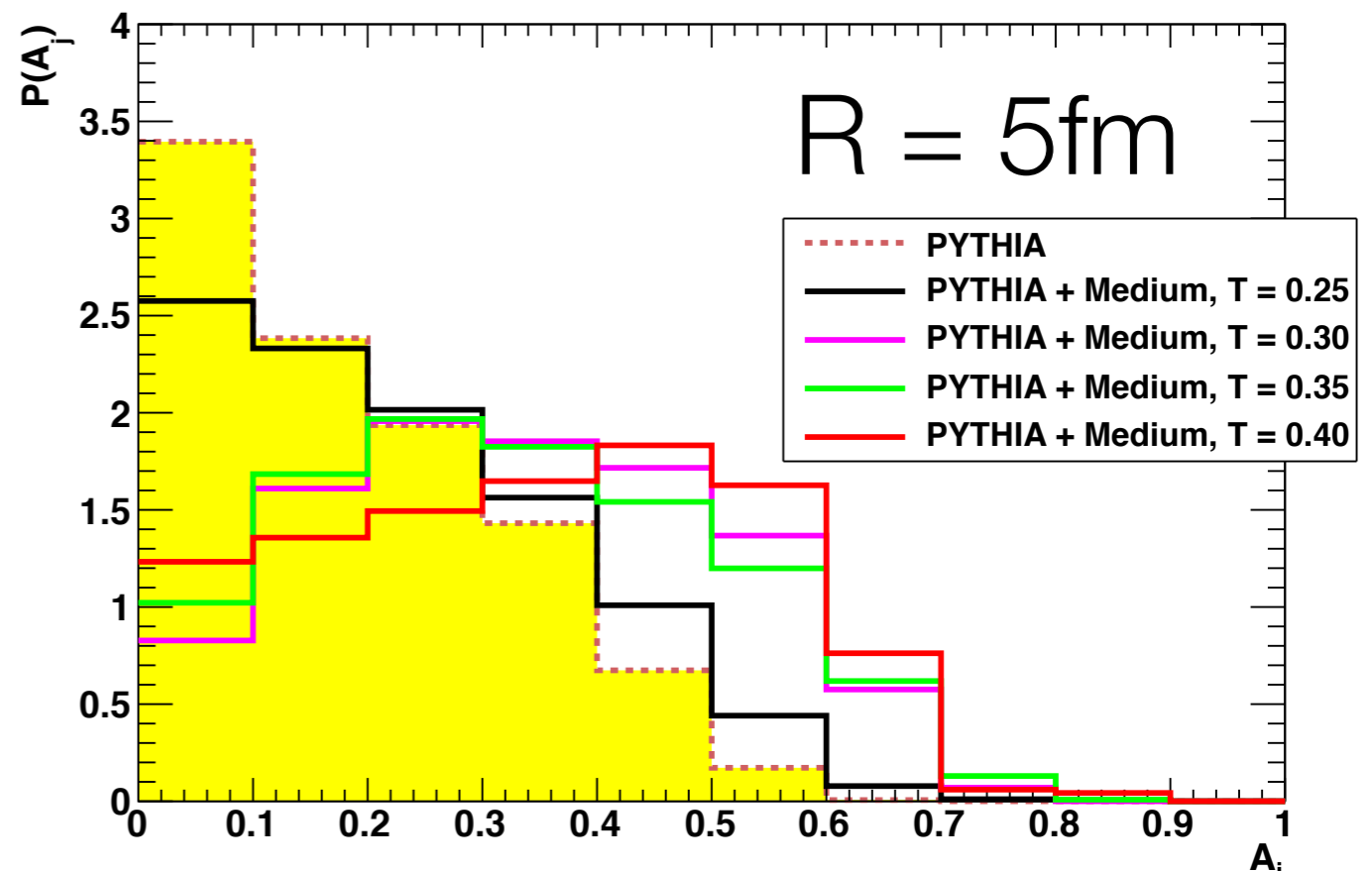
# Temperature Dependence



- Medium temperature controls:
  - Debye screening length
  - Minimum pt cutoff
  - Medium density

$$E_{T1} > 100\text{GeV} \quad E_{T2} > 25\text{GeV}$$

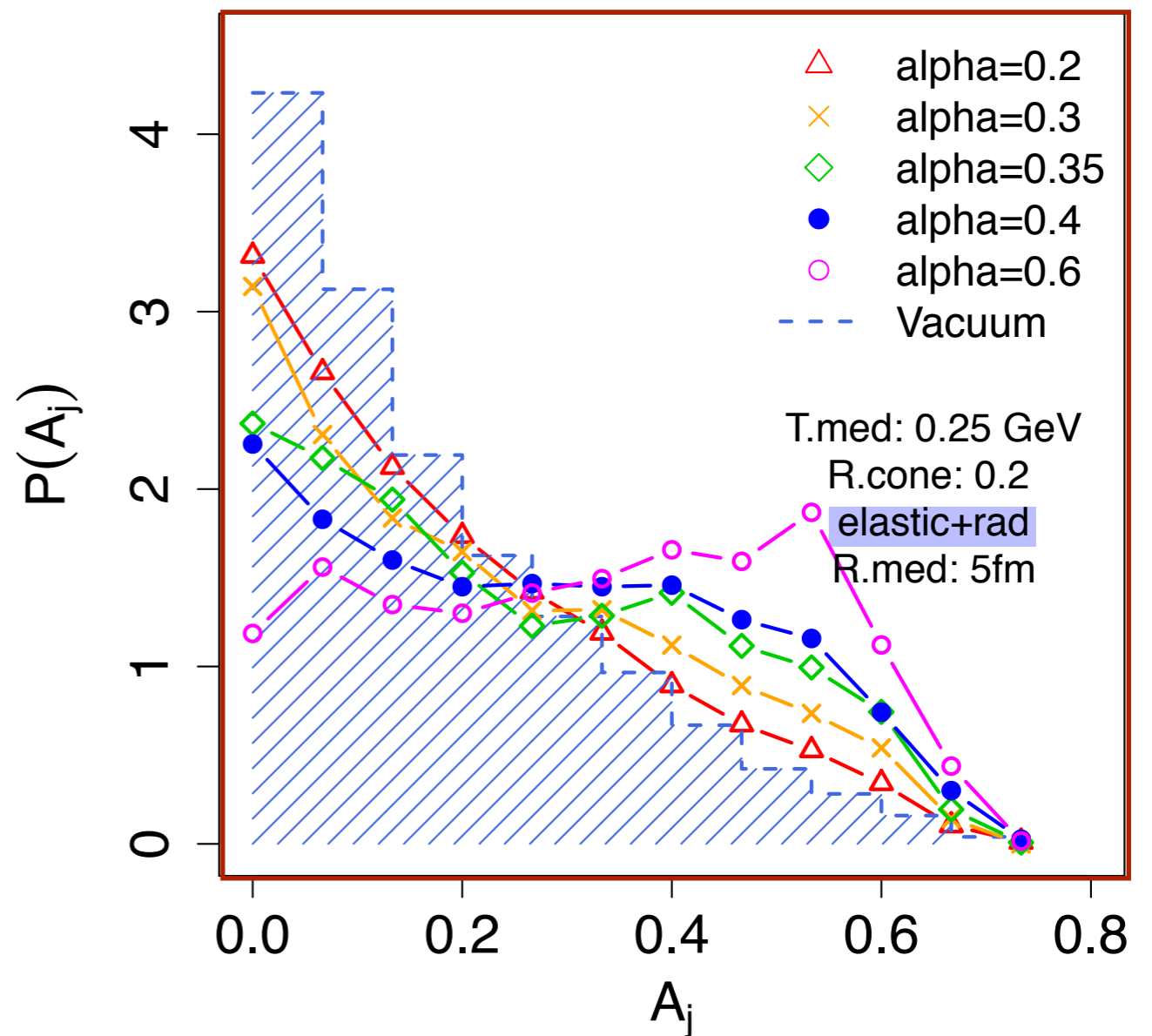
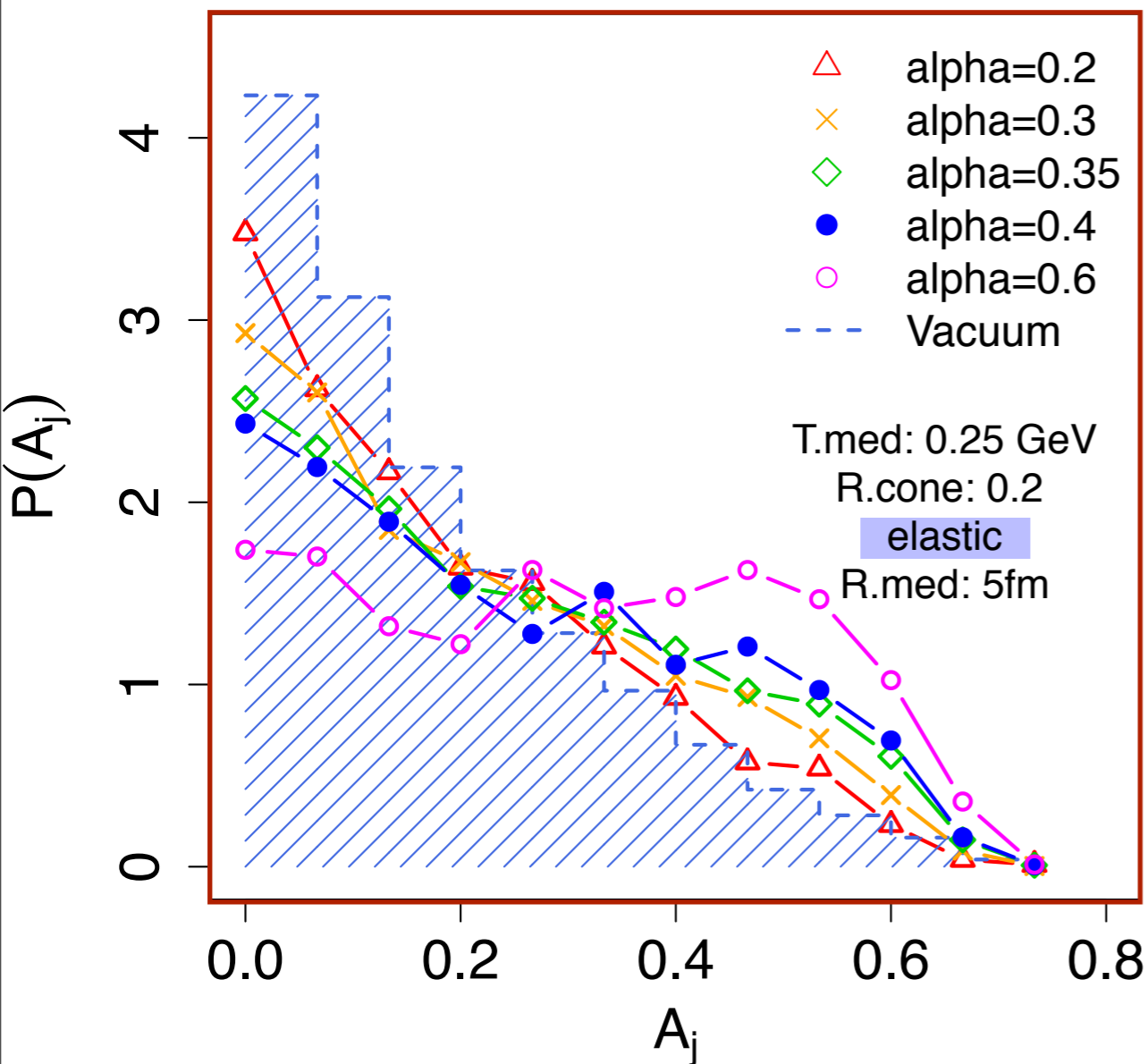
T(MeV)	$\hat{q}$ ( $\text{GeV}^2/\text{fm}$ )
250	1.125
300	1.5
350	2.125
400	3.0



# RHIC Dijet Asymmetry - Varying Strong Coupling

medium temperature  $T = 250$  MeV

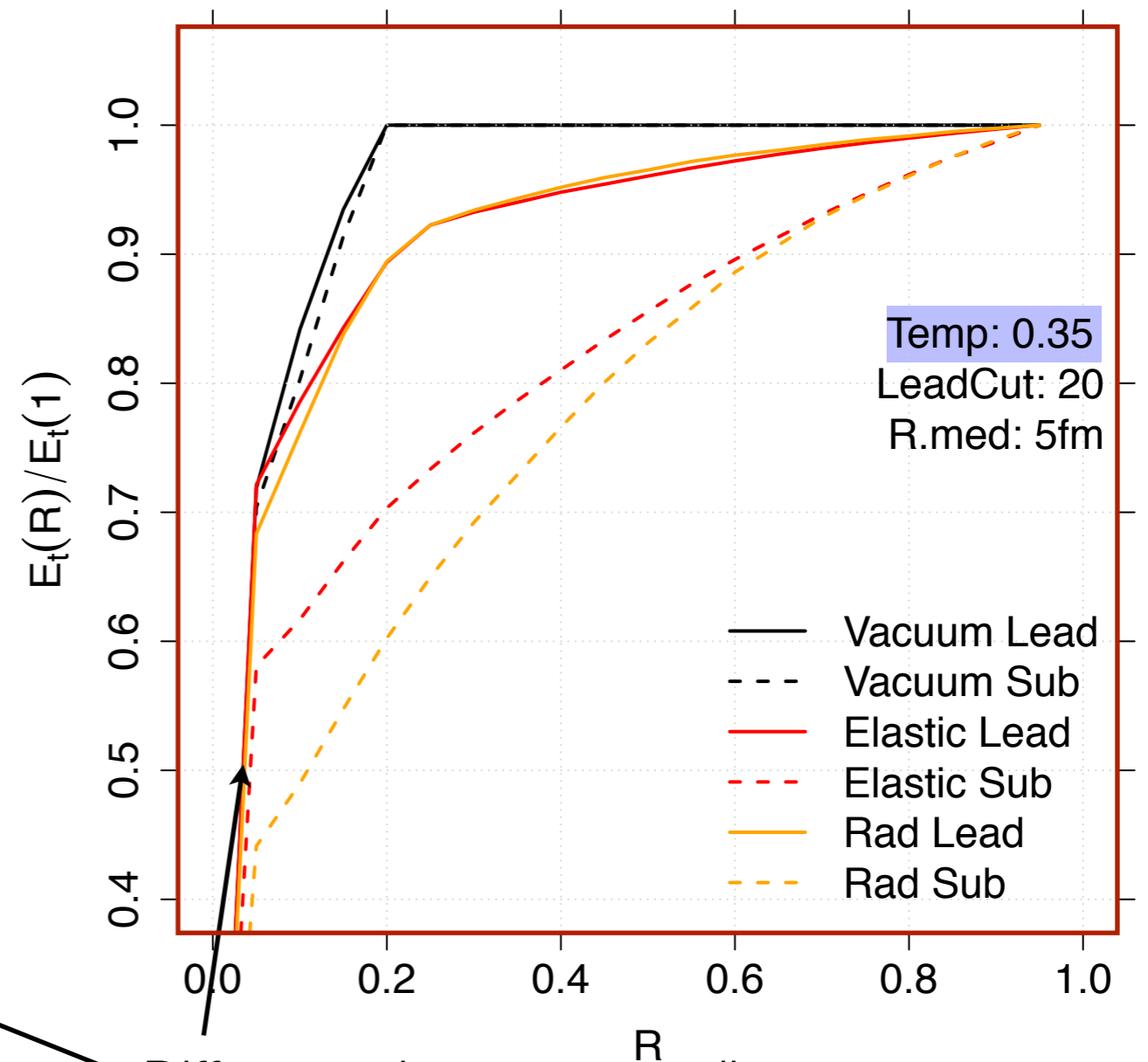
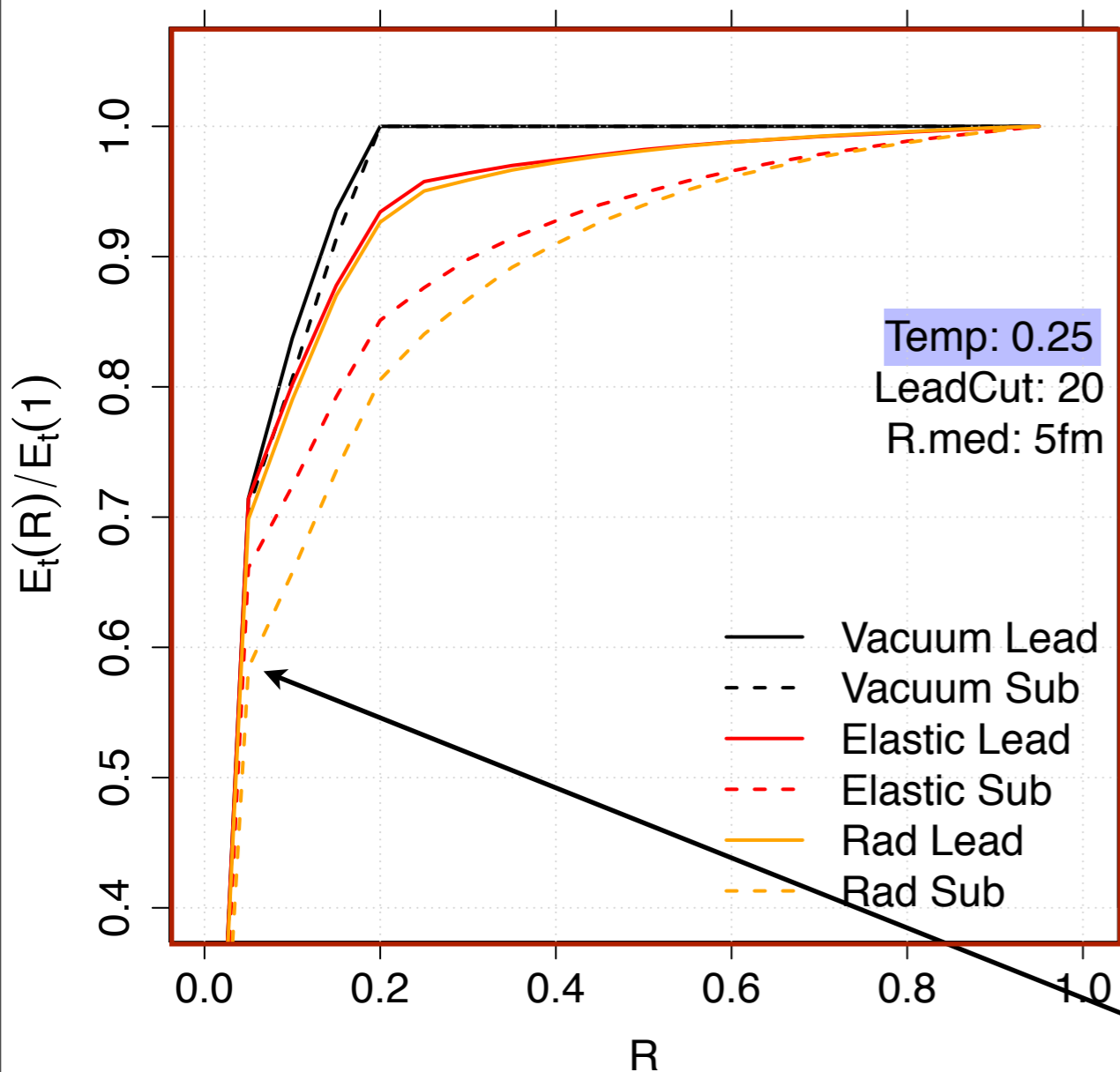
medium radius fixed at 5fm



Increasing Strong Coupling increases asymmetry

# RHIC - Jet Shape

Reconstruct jets with Anti-Kt at successively larger cone radii

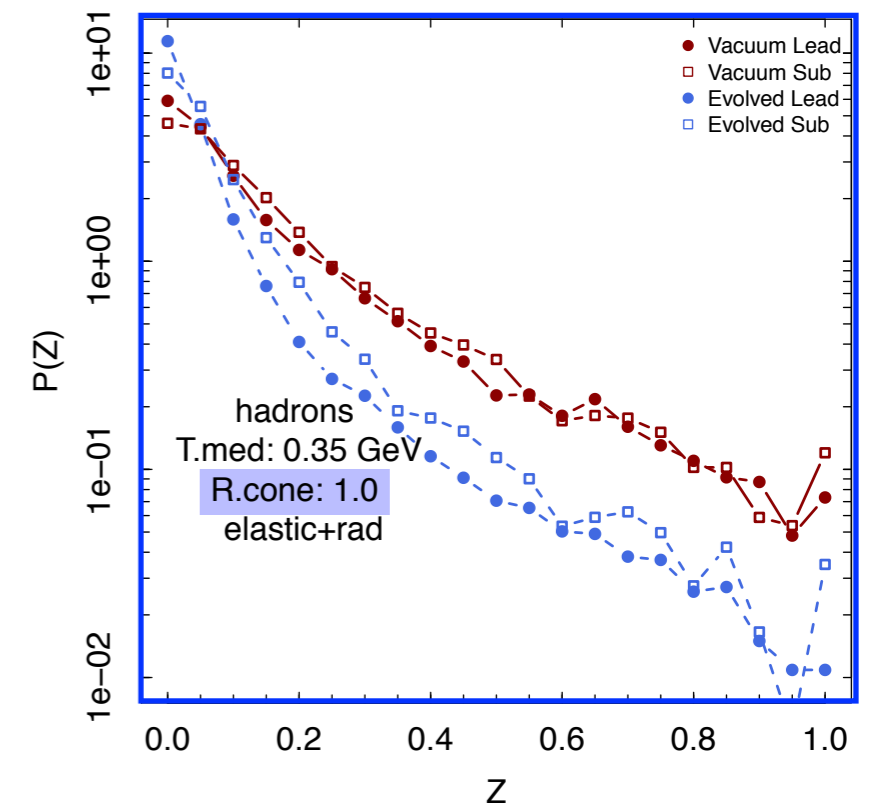
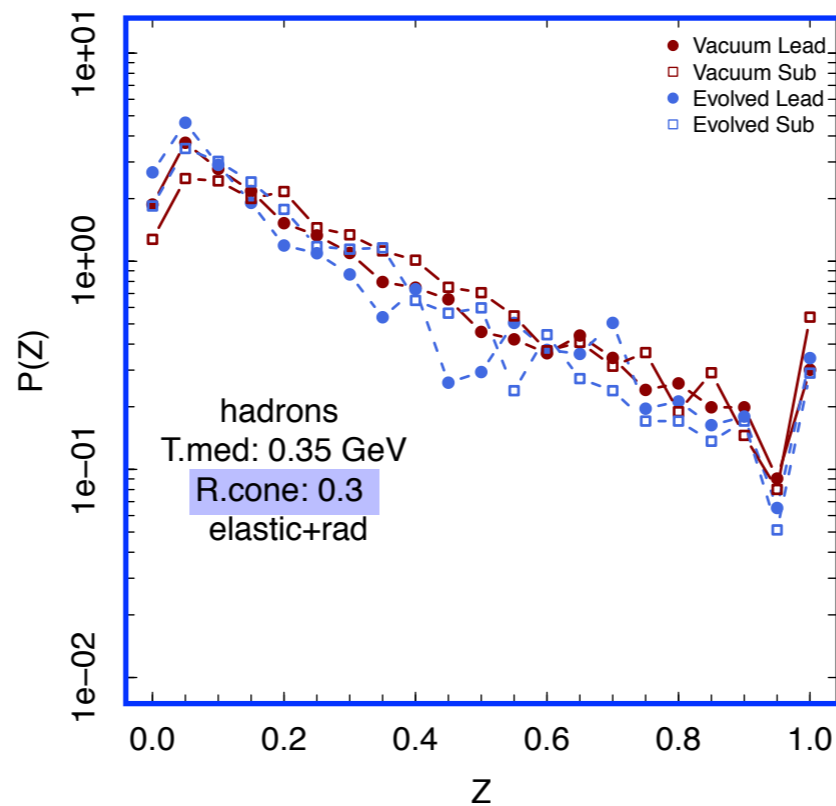
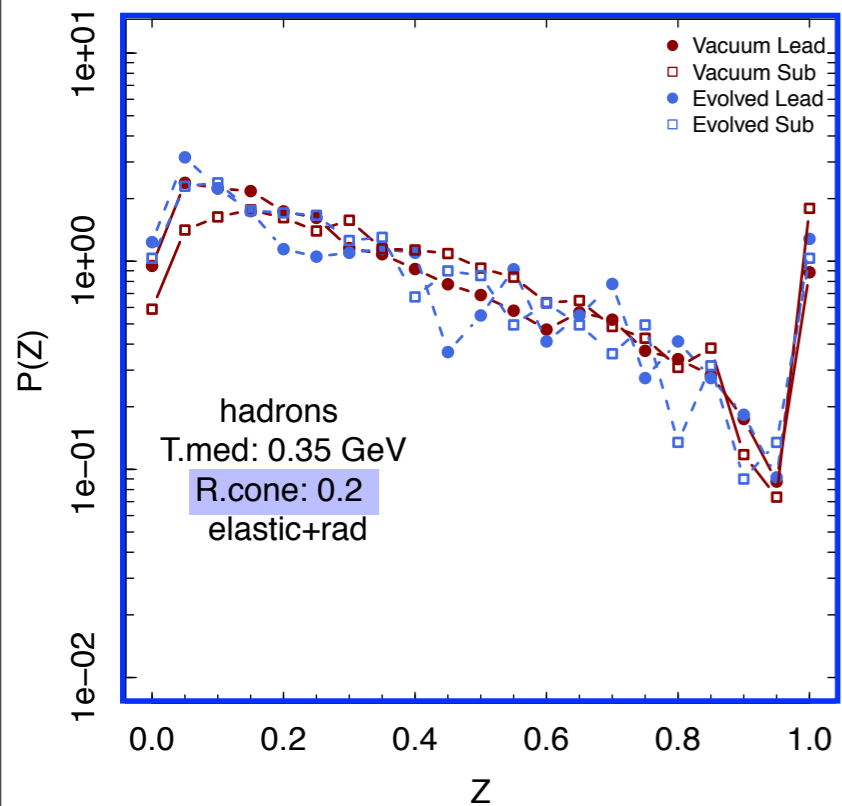
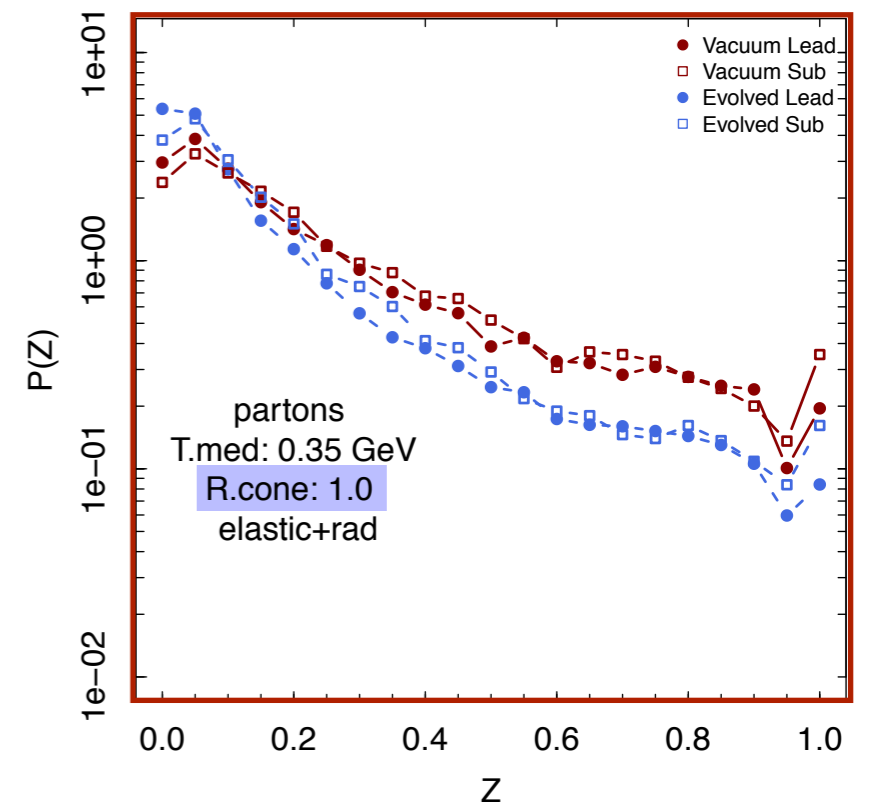
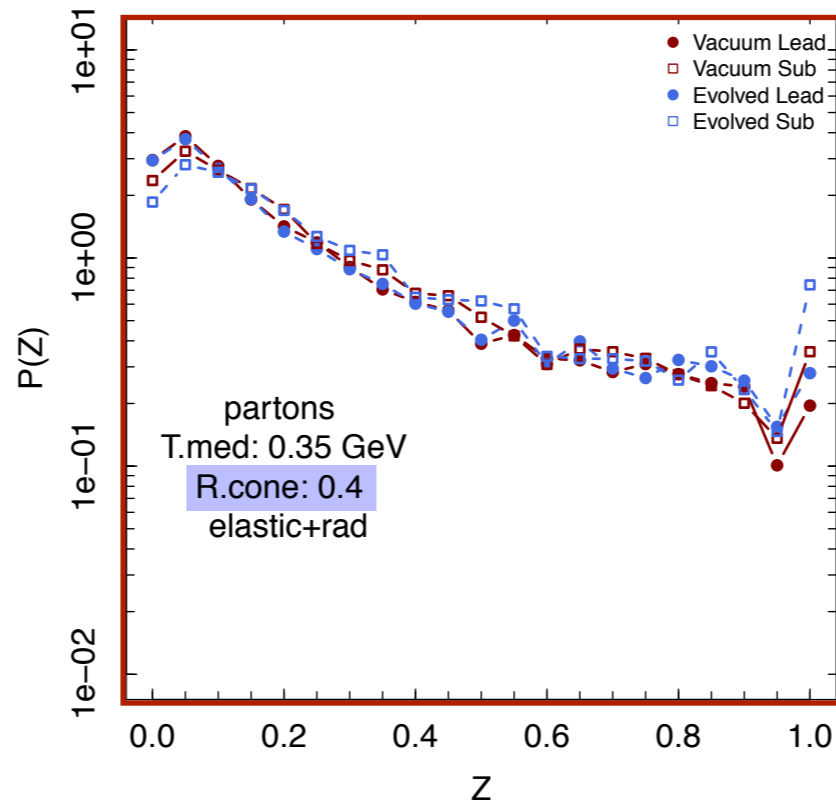
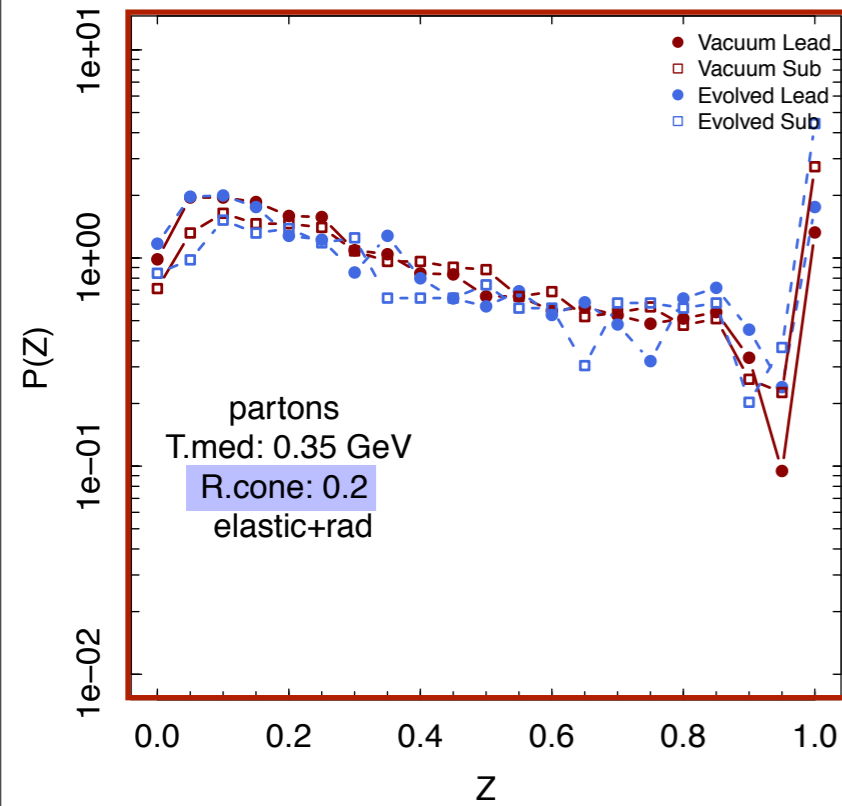


Clear separation between elastic (red) and radiative (orange) modes

Difference between medium temperatures is very strong, note values as  $R \rightarrow 0$

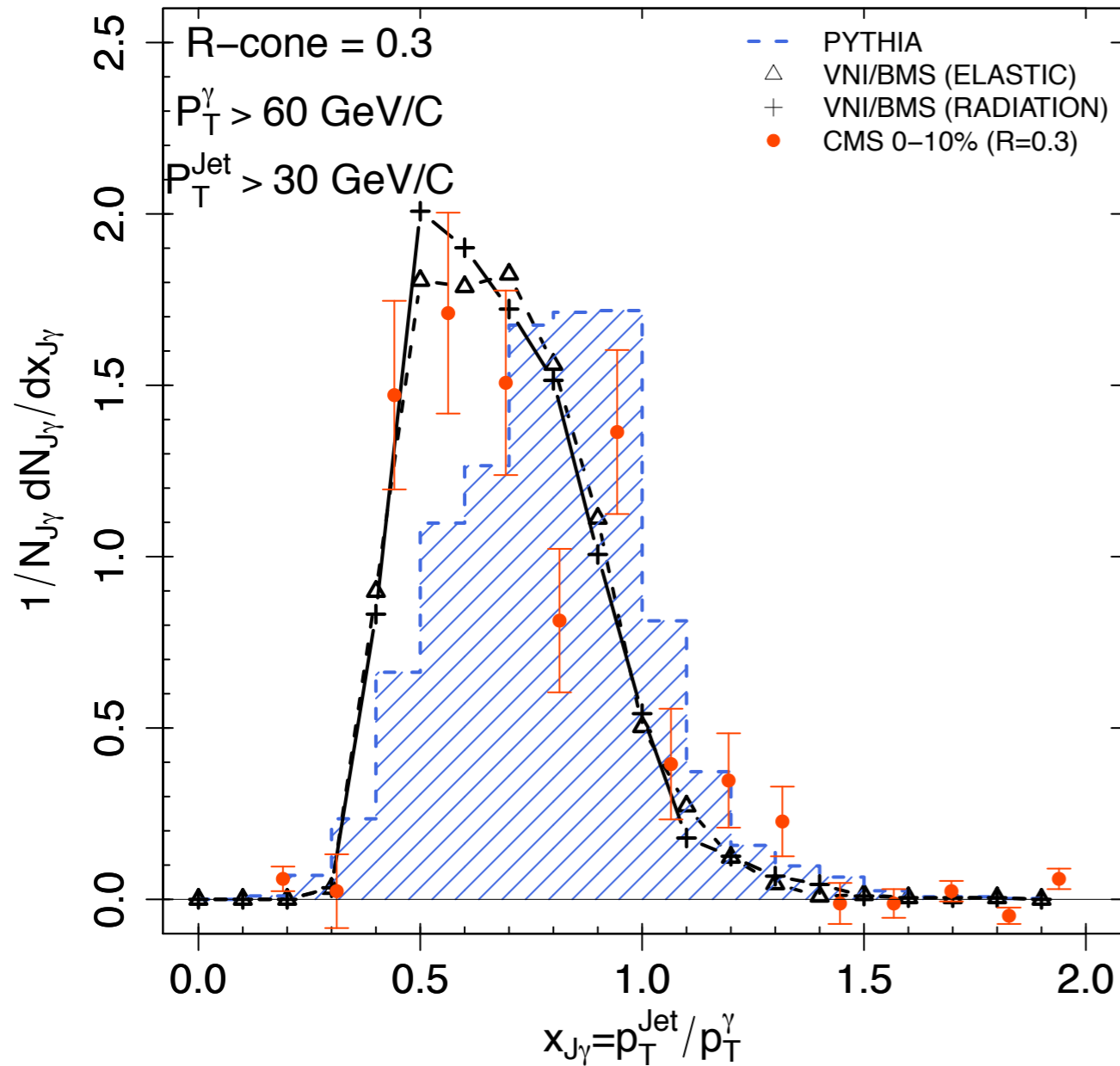
# Jet Fragmentation - Longitudinal

$$z = E_T / E_{T,Jet} \cos \Delta R$$



# CMS Photon - Jet Correlation

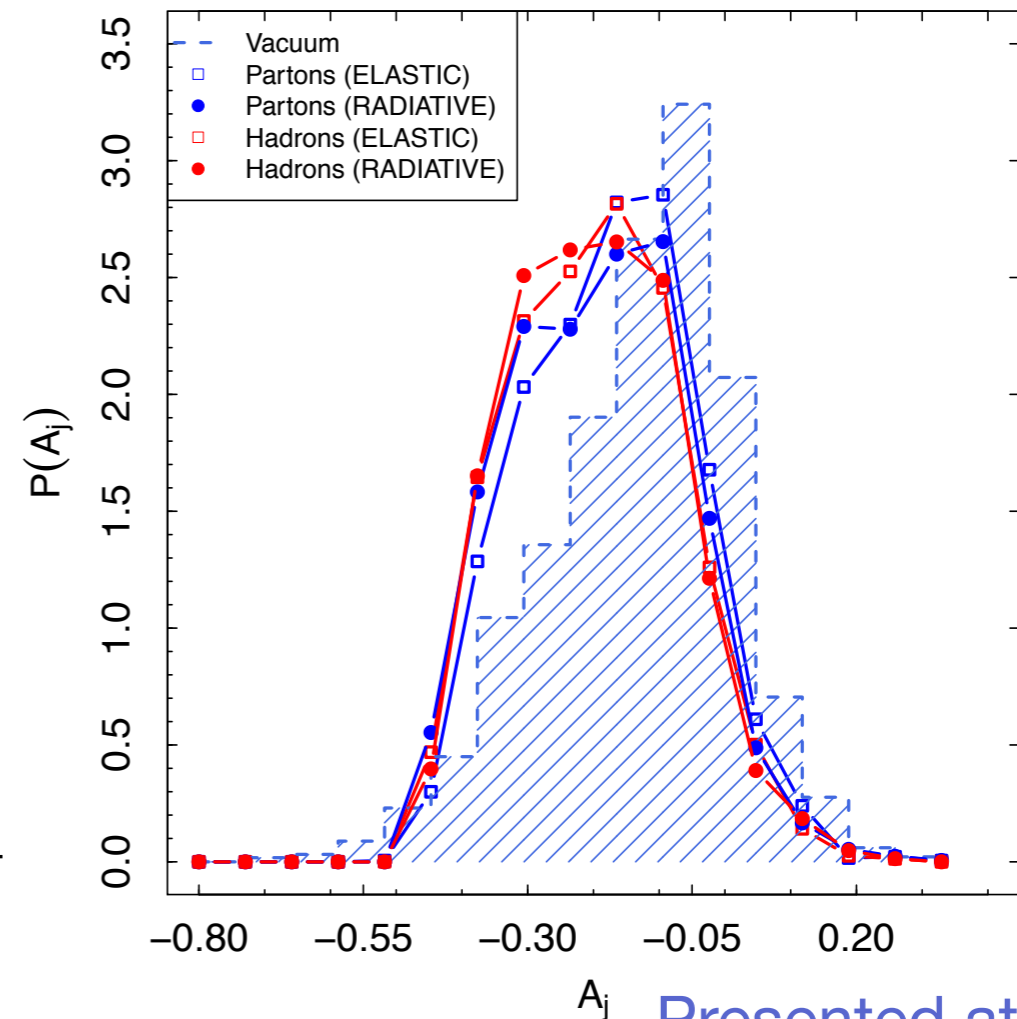
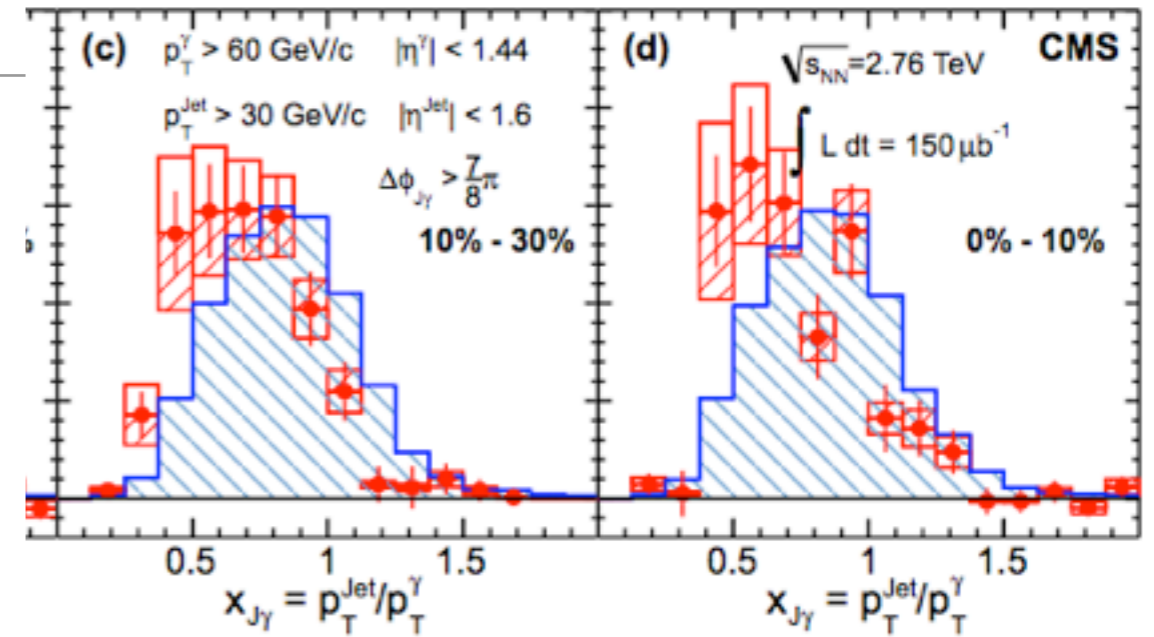
CMS hep-ex/1205.0206



$$E_{T,\gamma} > 60 \text{ GeV}$$

$$E_{T,\text{Jet}} > 30 \text{ GeV}$$

$$A_j = \frac{E_{T,\text{jet}} - E_{T,\gamma}}{E_{T,\text{jet}} + E_{T,\gamma}}$$



Presented at QM 2012

# Thermal Masses for medium partons

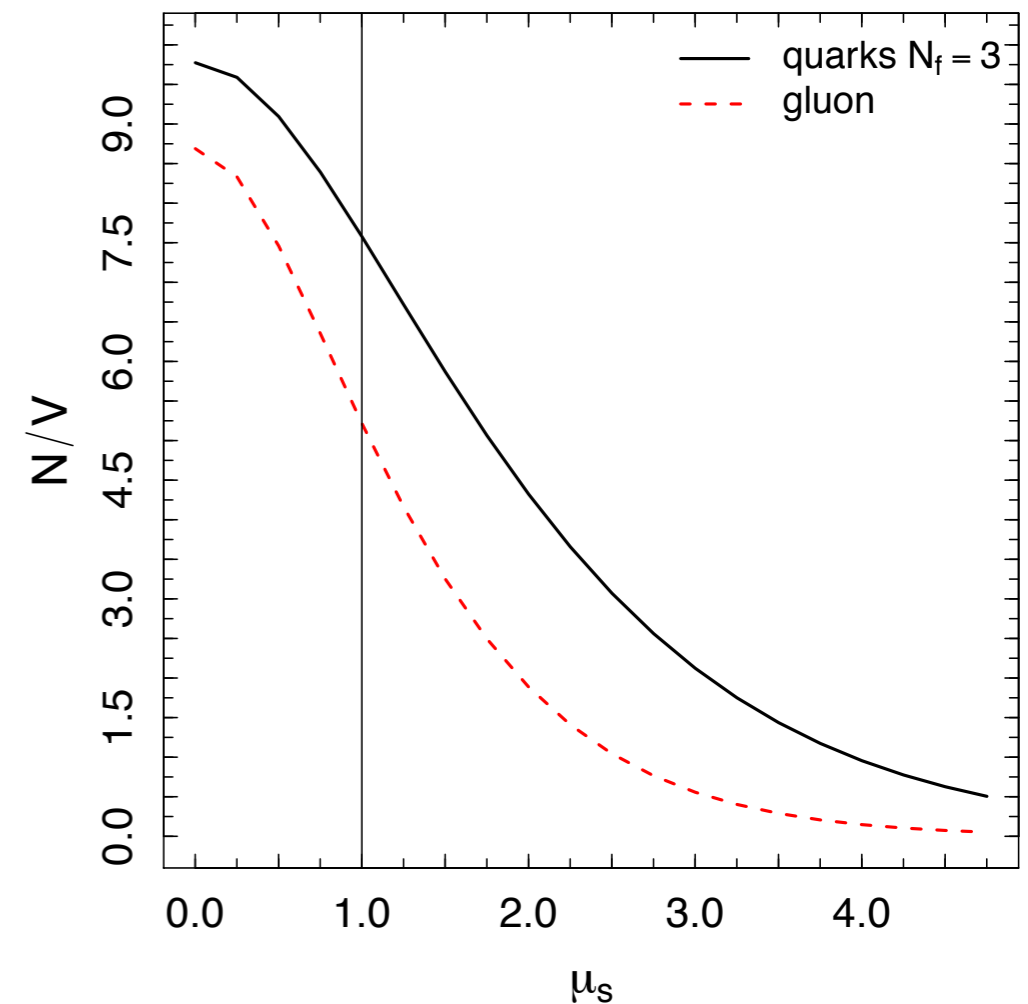
- Previous results from VNI/BMS derived from a medium of **massless** partons
- Interaction cross-sections are always screened by the Debye mass  $m_D^2 = \frac{1}{6}(2N_c + N_f)g^2T^2$
- Introduce **asymptotic HTL** masses for medium partons

$$m^2 = kg^2T^2, \quad k = \begin{cases} \frac{1}{6}N_c + \frac{1}{12}N_f & \text{gluon} \\ \frac{1}{4}C_F & \text{quark} \end{cases}$$

- Introduce a dimensionless scaling parameter  $\mu_s$  to 'dial' medium masses

$$m^2 = k\mu_s^2g^2T^2.$$

- NB: Medium number density now scales with masses



$$\frac{N}{V} = \frac{gTm^2}{2\pi^2} K_2\left(\frac{m}{T}\right)$$



# Measuring Transport Coefficients in VNI/BMS

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- Fix medium temperature  $T=350$  MeV, run code **without radiation** or **hadronization**.
- Run events with quark probes at fixed energies
- Extract  $\hat{q}$  and  $\hat{e}$  from transverse momentum and energy loss accumulated by the probe

$$\hat{q} = \frac{1}{L} \sum_{i=1}^{N_{\text{coll}}} \Delta p_{T,i}^2.$$

$$\hat{e} = \frac{1}{L} \sum_{i=1}^{N_{\text{coll}}} \Delta E_i,$$

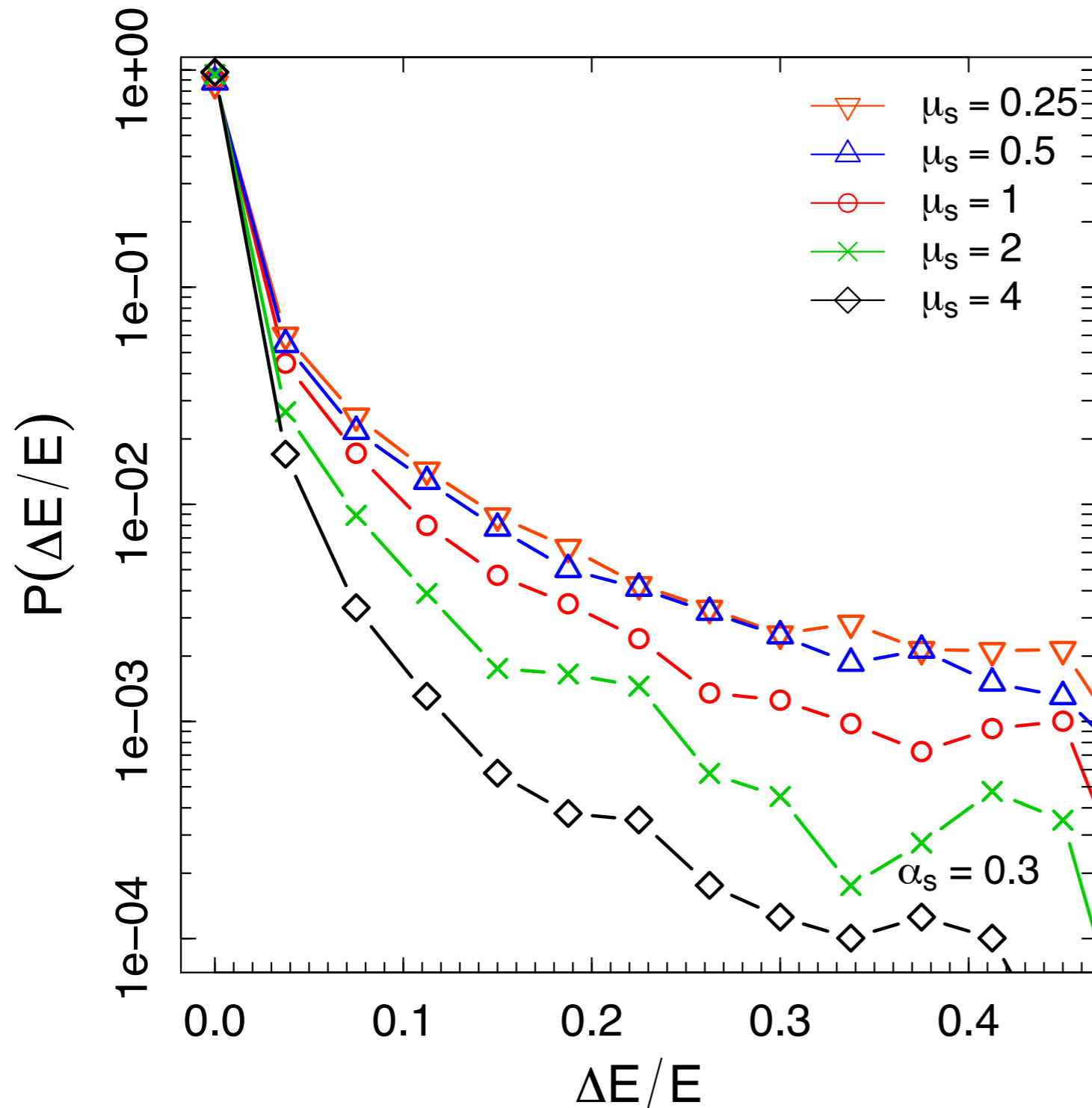
- For light probes in a massless medium we expect from pQCD calculations

$$\hat{q}(T) = 4\pi C_R \alpha_s^2 \mathcal{N}(T) \ln \left( \frac{q_{\text{max}}^2}{m_D^2} + 1 \right),$$

$$\mathcal{N}(T) = \frac{\zeta(3)}{\pi^2} \left( 2N_c + \frac{3}{2}N_f \right) T^3,$$

$$\hat{e}(T) = \frac{4\pi\alpha_s^2 T^2}{3} \left( 1 + \frac{N_f}{6} \right) \ln \left( \frac{ET}{m_D^2} \right)$$

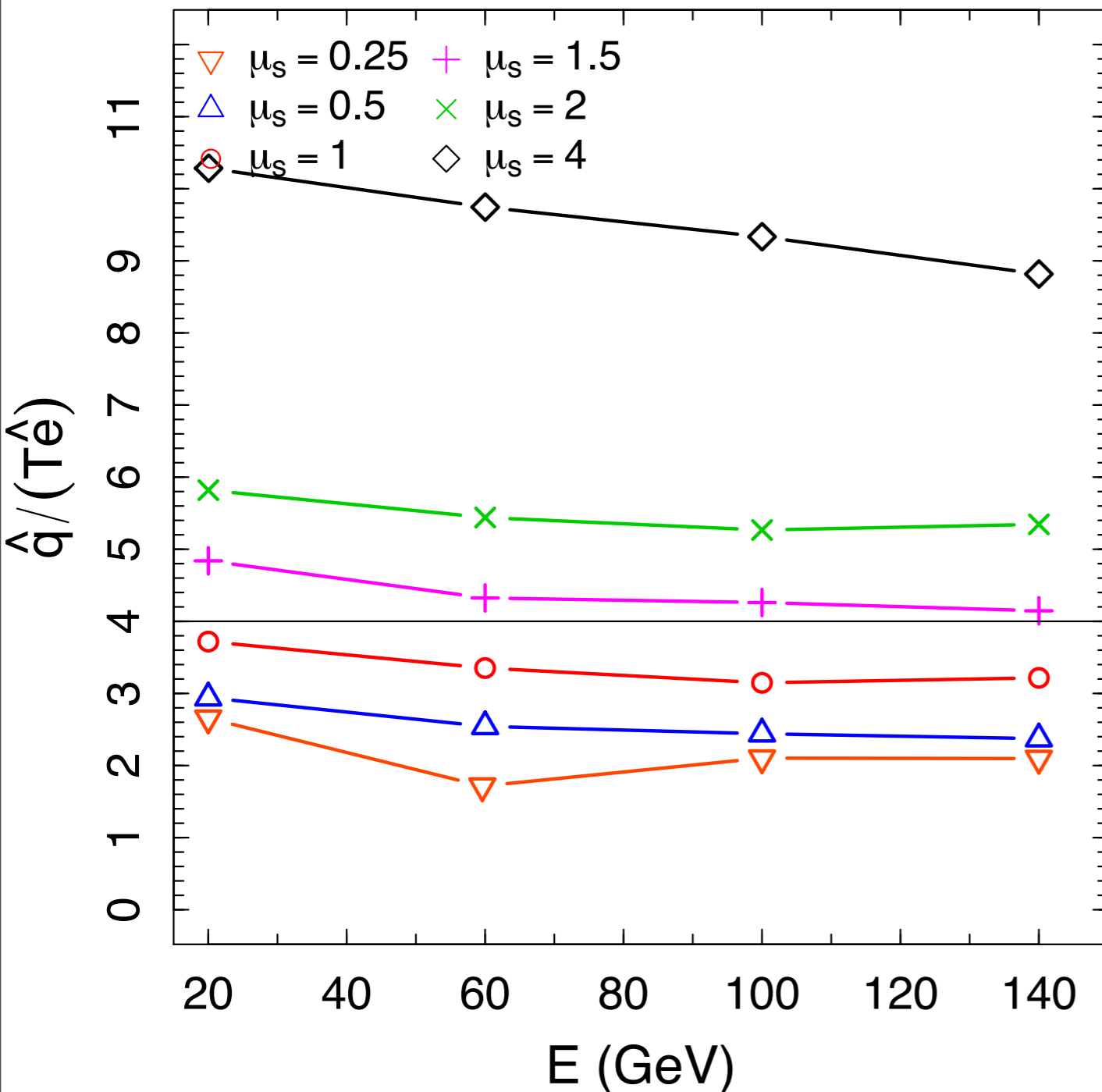
# Elastic Energy Loss



Confirms kinematic intuition, higher medium constituent masses recoil less resulting in a lower energy loss

$T=350$  MeV,  
 $\text{Alpha} = 0.3$

# Q-hat / E-hat Ratio

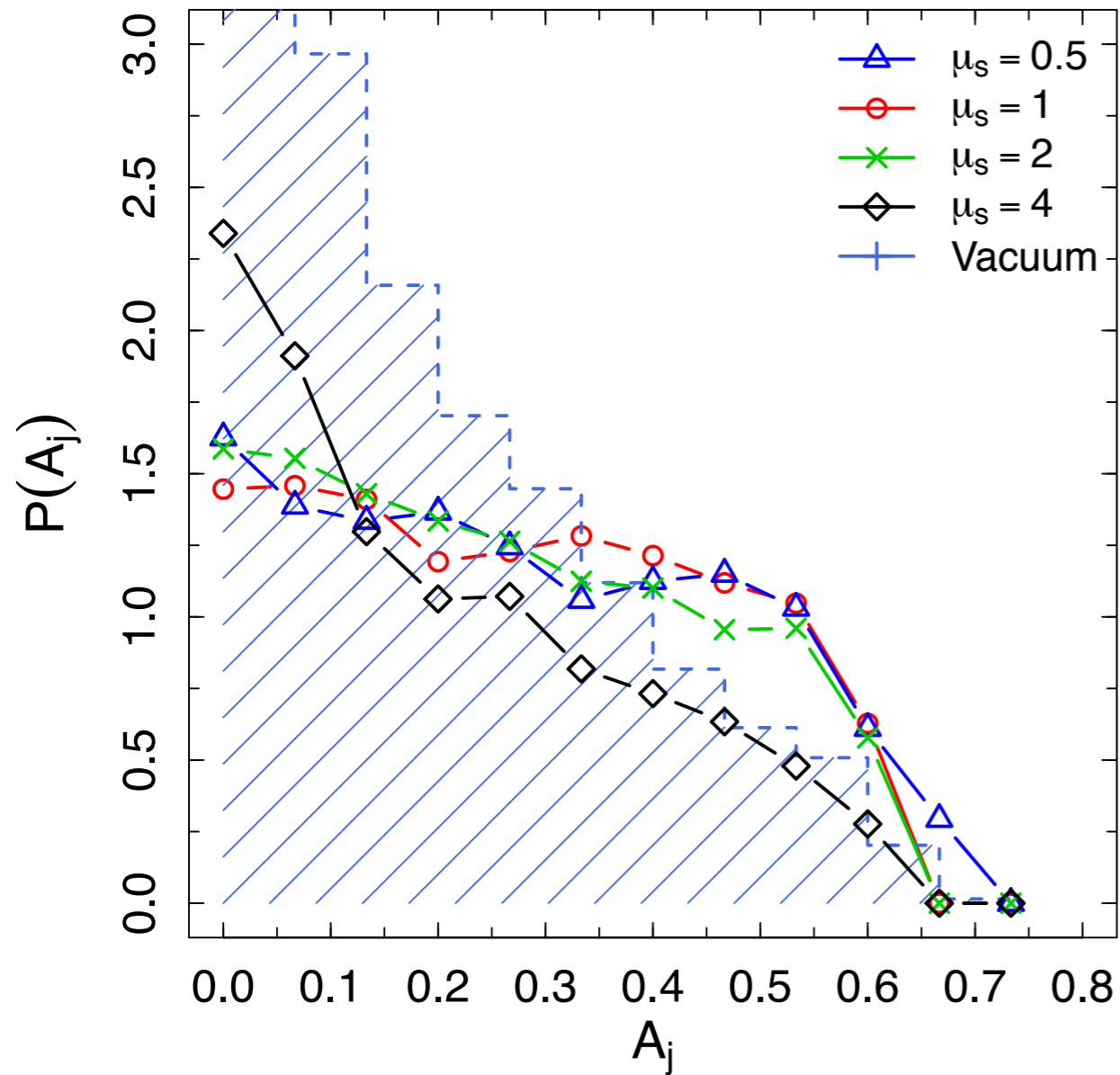


- Ratio scales **linearly** with the medium mass scale  $\mu_s$ .
- **Experimental** measurements of q-hat and e-hat could provide insight into the nature of the QGP as seen by jets.
- Measurements made at different jet scales may reveal structure in quasi-particle mass spectrum.
- A possible **precision measurement** of hard probes and the QGP

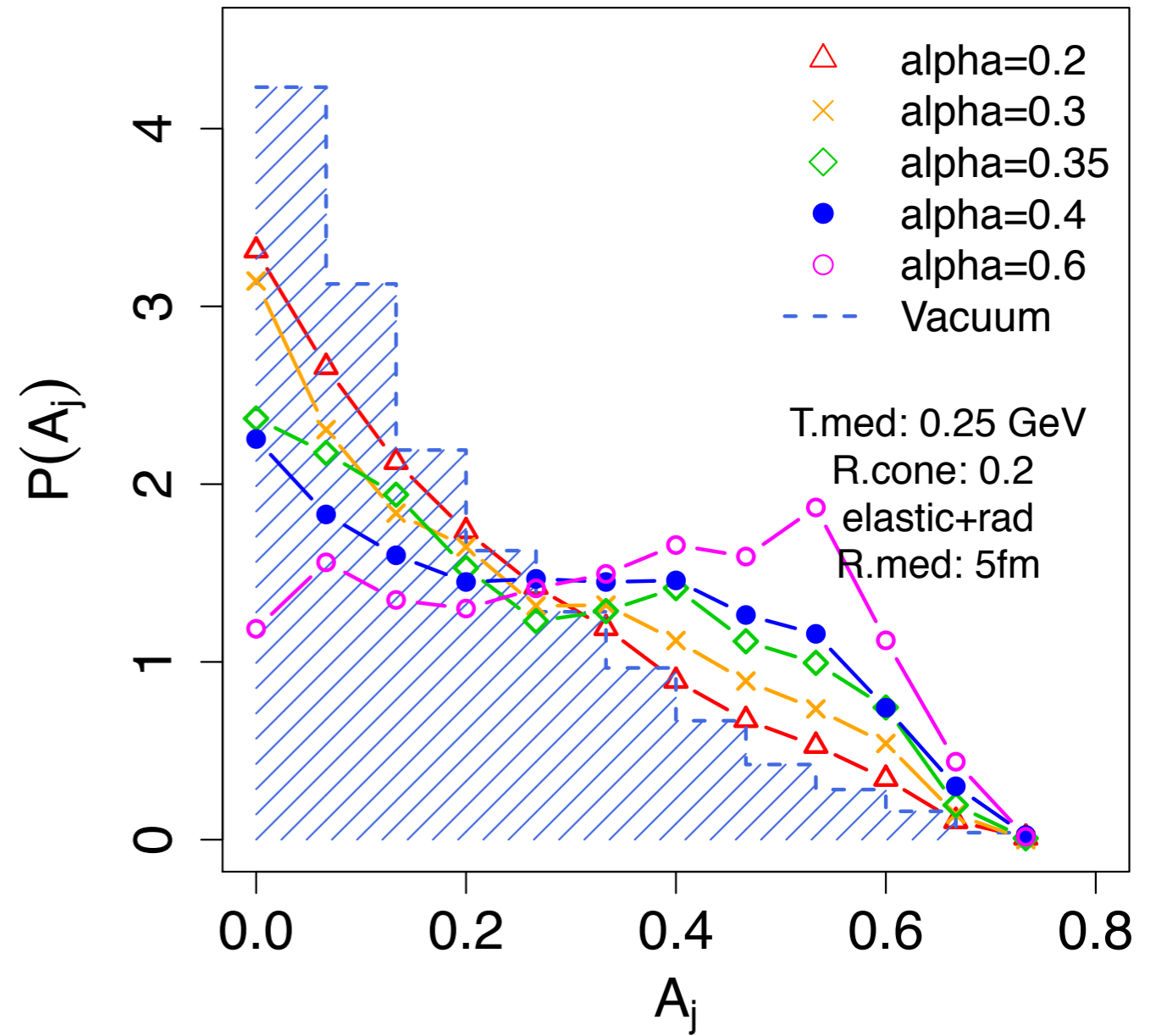
# Dijet Response at RHIC - $A_j$

$T=350$  MeV,  $E_t > 20$  GeV

Varying  $\mu_s$



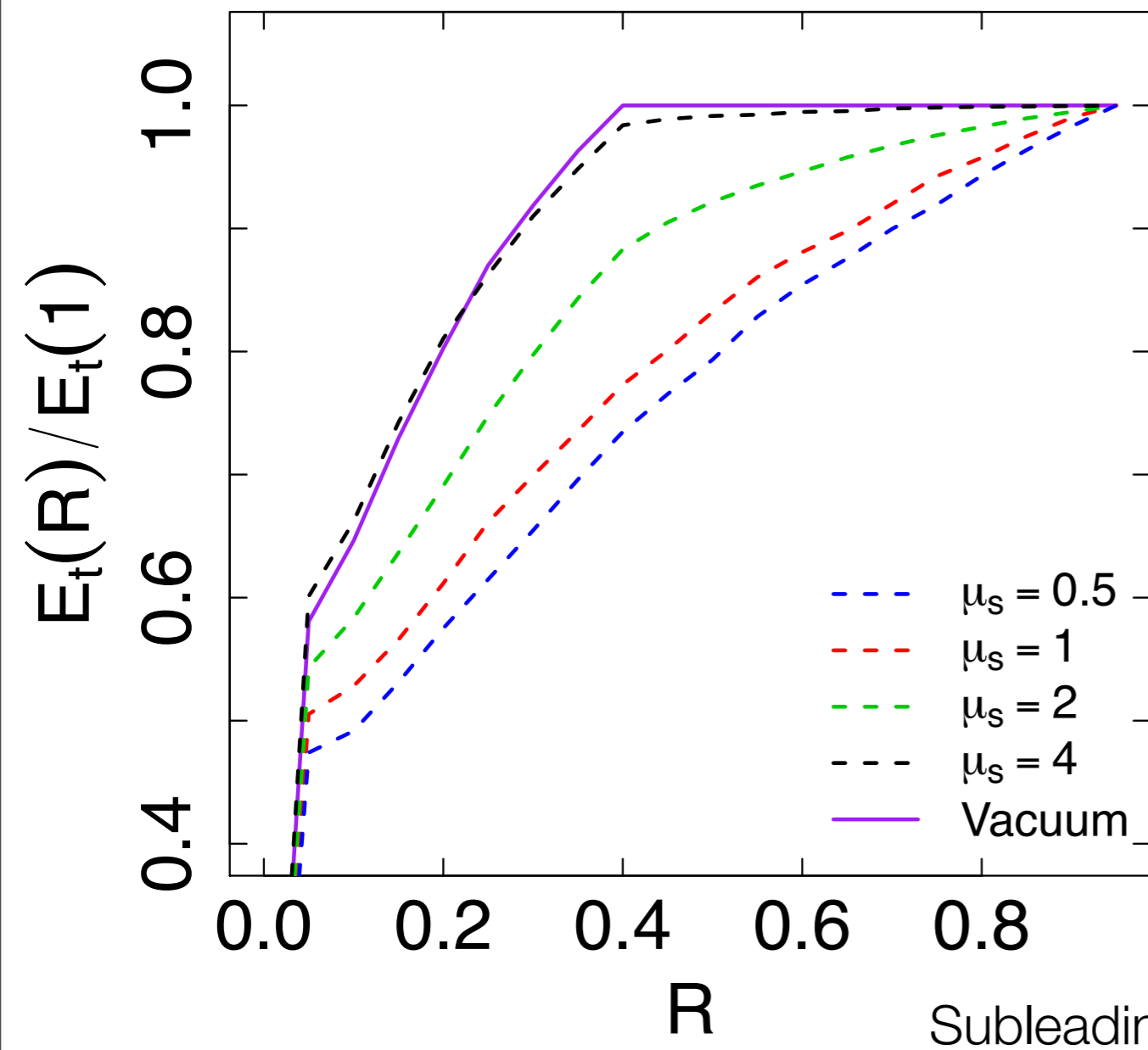
Varying  $\alpha_s$



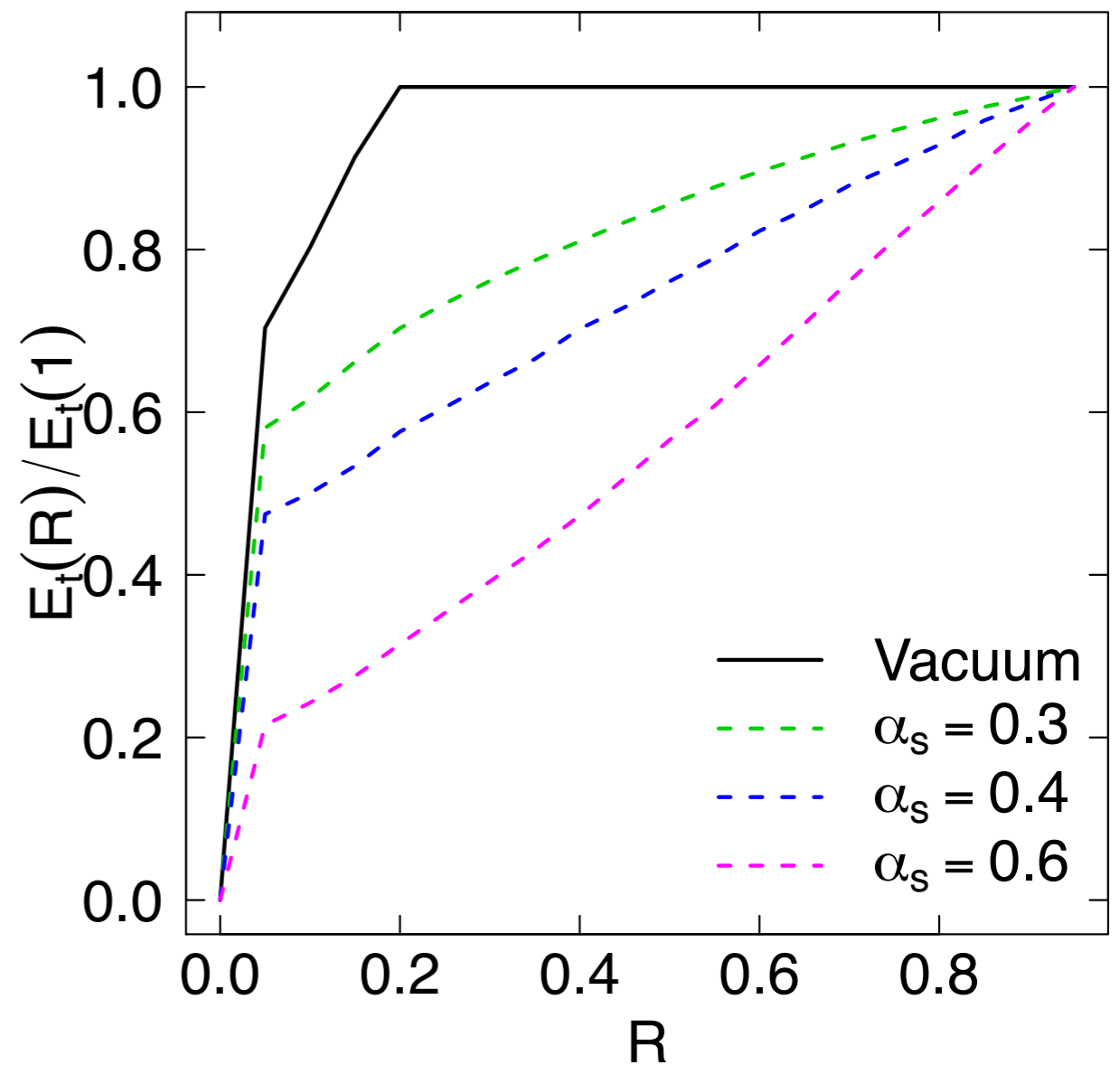
Elastic Interactions Only

# Dijet Response at RHIC - Subleading Jet Shape

Varying  $\mu_s$



Varying  $\alpha_s$

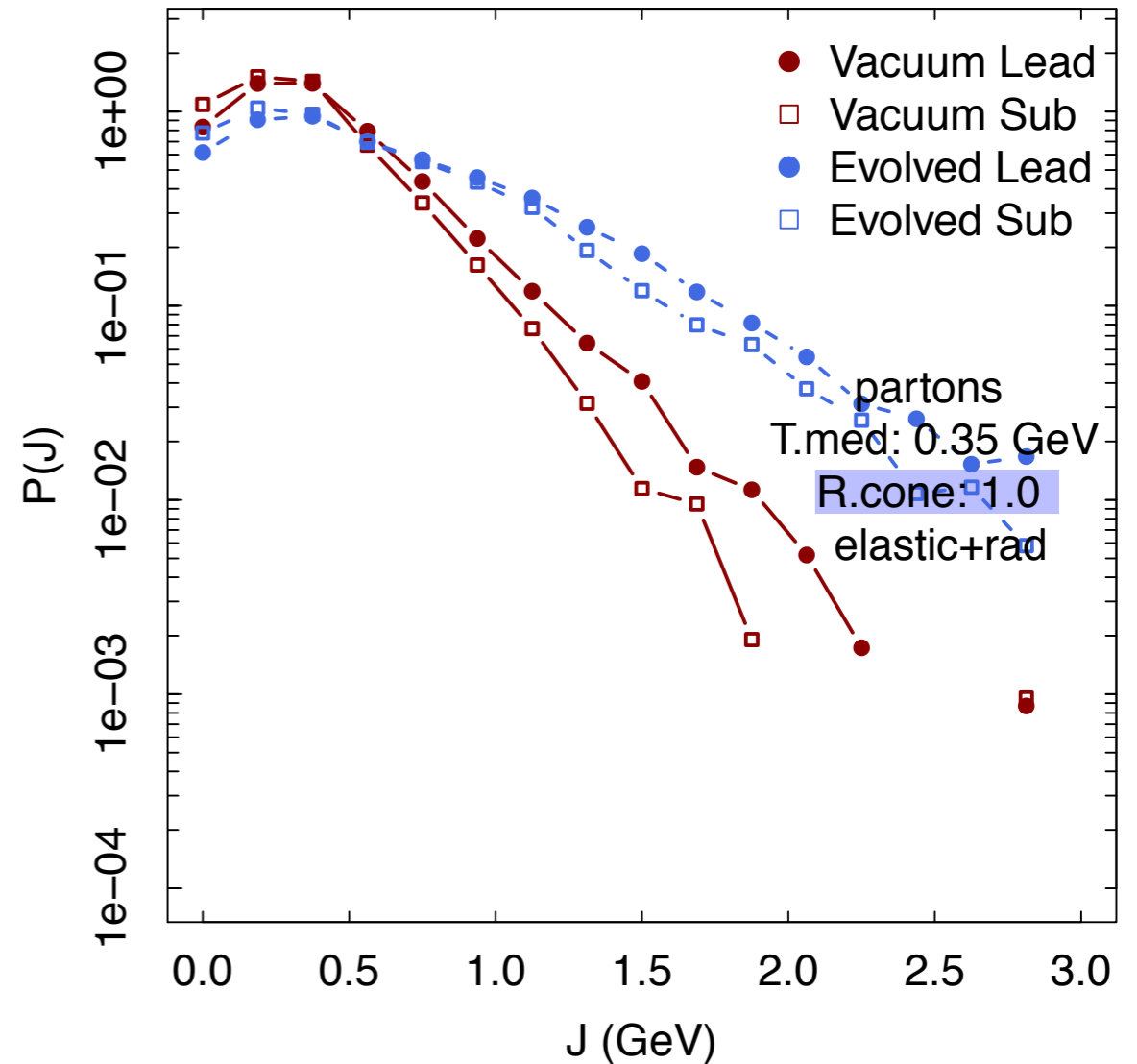
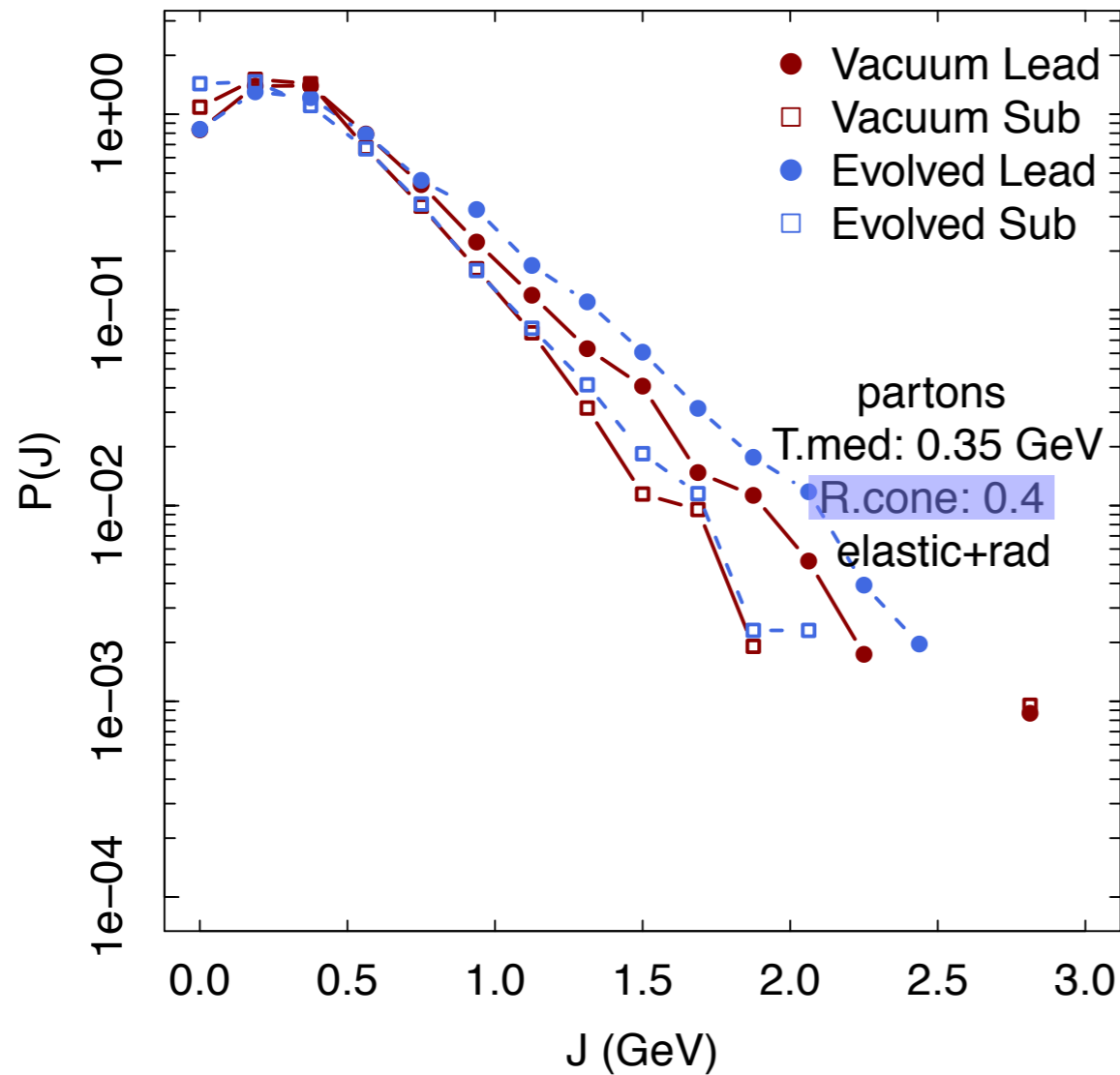


Subleading Jet Shape

Jet shape is sensitive to elastic energy loss

# RHIC: Other Sensitive Observables?

$$J_T = E_T \sin \Delta R$$



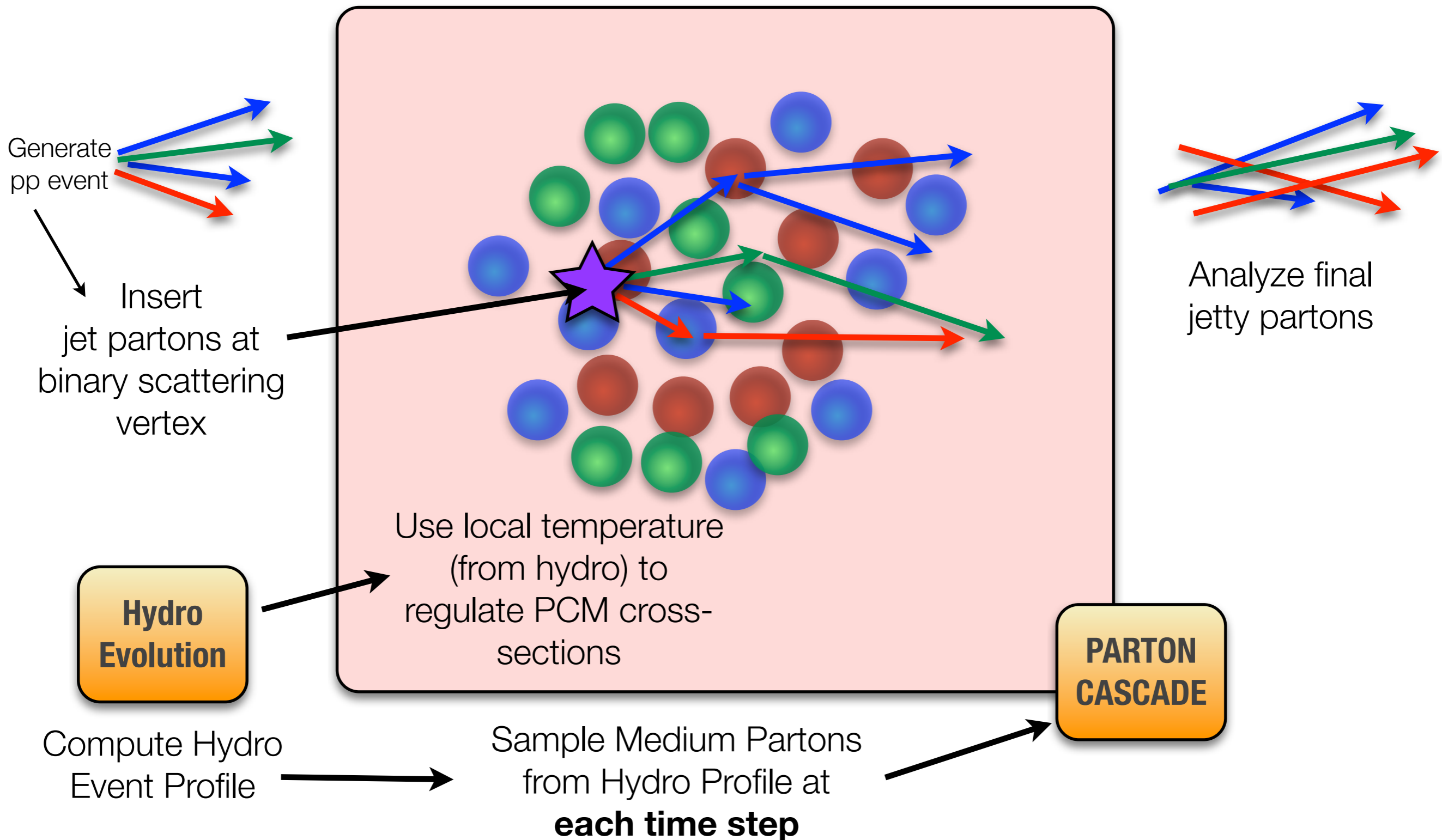
Narrow jets fragment similarly to vacuum

Extract  $\hat{Q}$  from transverse fragmentation broadening?

$R=1.0$  jets show strong transverse broadening

# VNI/BMS - 2.0 + Hydro

Keep jetty partons at each time step. Discard 'old' medium partons



# VNI/BMS - 2.0 Current + Future Features

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- Elastic and Radiative energy loss models, with BDMPS-Z LPM effect
- Lund Stringy Hadronization with full color tracking from the initial generator.
- Fixed medium temperature in simple box mode.
- Event by event hydro background.
- Variable medium constituent masses give control of  $q_{\text{hat}}/e_{\text{hat}}$  ratio.
- Integration with Pythia for hard process generation
- Jet level data analysis built in (and single hard probe)
- Relatively simple user-options
- Modern build system (CMAKE)



# Conclusions

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- Is there a place for Parton Cascade Models in the future?
- Tracking full shower evolution seems to be key to understanding full jet observables. Do we need to use a full Parton Cascade to do this? Lattice Boltzmann
- Hydro is a very good description of the bulk dynamics. Can ‘fake’ this by ramping up cross-sections, at the cost of understanding the physics.
- Boltzmann equation allows treatment of non-equilibrium systems. Perhaps returning to the basic ideas would be interesting, PCM + CGC as an IC generator for hydro?

Extras

# VNI/BMS Cross-Sections

$g g \rightarrow g g$	$\frac{9}{2} \left( 3 - \frac{tu}{s^2} - \frac{su}{t^2} - \frac{st}{u^2} \right)$	$q q' \rightarrow q q'$	$\frac{4}{9} \frac{s^2 + u^2}{t^2}$
$q g \rightarrow q g$	$-\frac{4}{9} \left( \frac{s}{u} + \frac{u}{s} \right) + \frac{s^2 + u^2}{t^2}$	$q qbar \rightarrow q' qbar'$	$\frac{4}{9} \frac{t^2 + u^2}{s^2}$
$g g \rightarrow q qbar$	$\frac{1}{6} \left( \frac{t}{u} + \frac{u}{t} \right) - \frac{3}{8} \frac{t^2 + u^2}{s^2}$	$q g \rightarrow q \gamma$	$-\frac{e_q^2}{3} \left( \frac{u}{s} + \frac{s}{u} \right)$
$q q \rightarrow q q$	$\frac{4}{9} \left( \frac{s^2 + u^2}{t^2} + \frac{s^2 + t^2}{u^2} \right) - \frac{8}{27} \frac{s^2}{tu}$	$q qbar \rightarrow g \gamma$	$\frac{8}{9} e_q^2 \left( \frac{u}{t} + \frac{t}{u} \right)$
$q qbar \rightarrow q qbar$	$\frac{4}{9} \left( \frac{s^2 + u^2}{t^2} + \frac{u^2 + t^2}{s^2} \right) - \frac{8}{27} \frac{u^2}{st}$	$q qbar \rightarrow \gamma \gamma$	$\frac{2}{3} e_q^4 \left( \frac{u}{t} + \frac{t}{u} \right)$
$q qbar \rightarrow g g$	$\frac{32}{27} \left( \frac{t}{u} + \frac{u}{t} \right) - \frac{8}{3} \frac{t^2 + u^2}{s^2}$		

- Dominant elastic interactions including screening mass

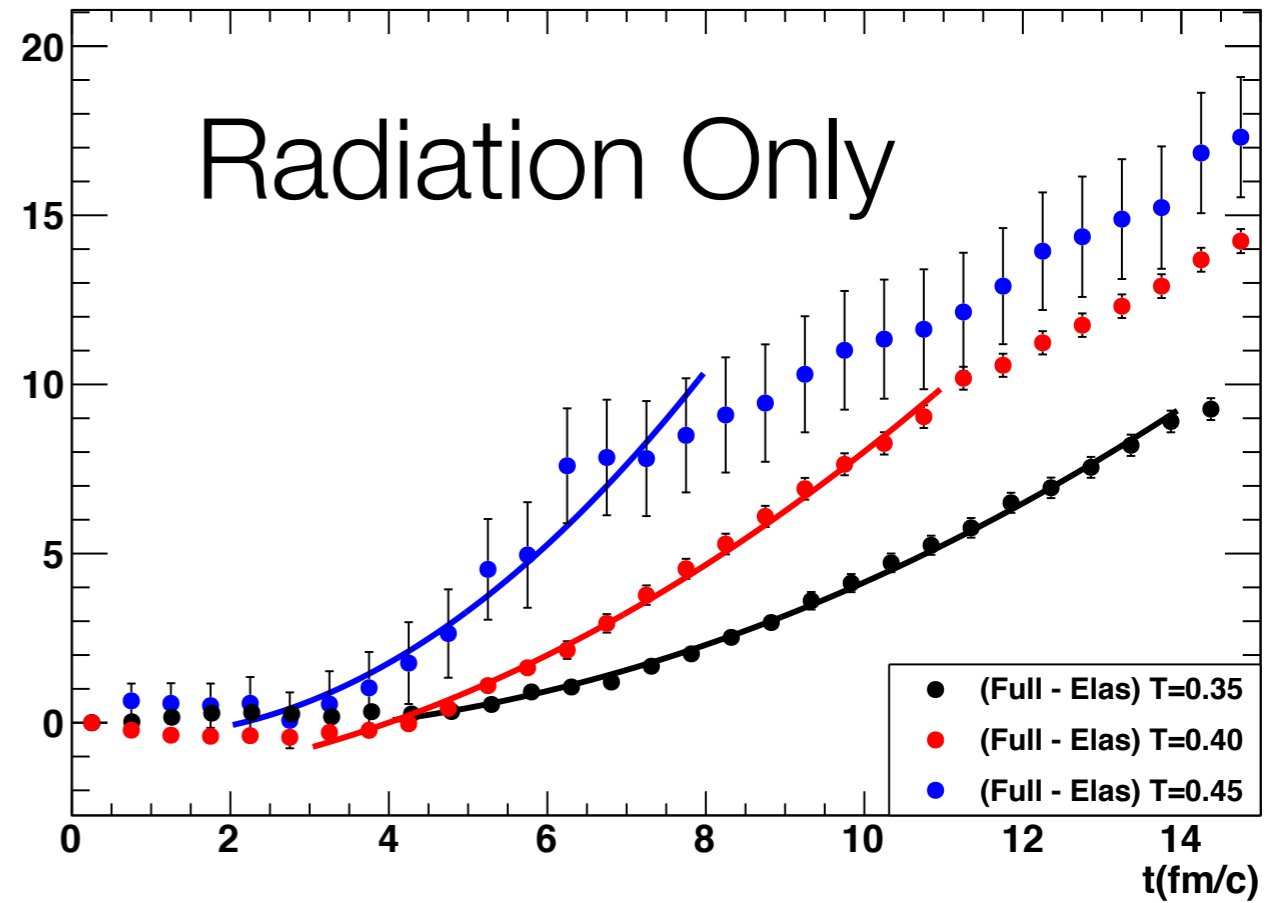
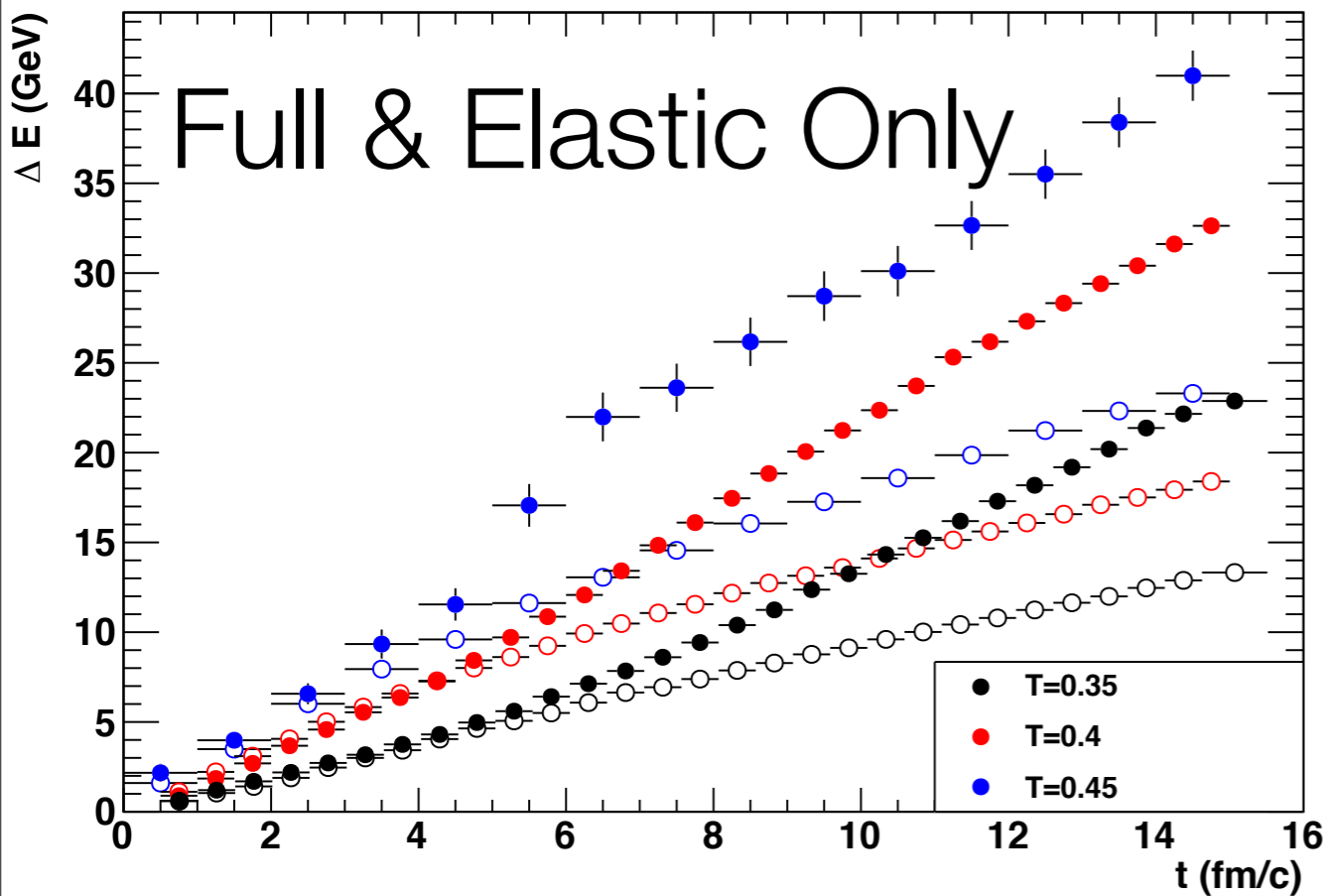
$$\mu_D = \sqrt{(2N_c + N_f)/6gT}$$

$$\frac{d\sigma^{gg \rightarrow gg}}{dq_{\perp}^2} = 2\pi\alpha_s^2 \frac{9}{4} \frac{1}{(q_{\perp}^2 + \mu_D^2)^2},$$

$$\frac{d\sigma^{gq \rightarrow gq}}{dq_{\perp}^2} = 2\pi\alpha_s^2 \frac{1}{(q_{\perp}^2 + \mu_D^2)^2},$$

$$\frac{d\sigma^{qq \rightarrow qq}}{dq_{\perp}^2} = 2\pi\alpha_s^2 \frac{4}{9} \frac{1}{(q_{\perp}^2 + \mu_D^2)^2},$$

# Leading Parton Energy Loss



Radiation reproduces BDMPS-Z coefficient to 20%.

Elastic (open circles) well described by pQCD results.

$$\Delta E_{elas} \propto T^2 L$$

$$-\Delta E_{\text{BDMPS}} = \frac{\alpha_s C_R \mu^2}{8} \frac{L^2}{\lambda_g} \log \frac{L}{\lambda_g}.$$

$$\hat{q} = \frac{1}{L} \sum_{i=1}^{N_{coll}} (\Delta p_{\perp,i})^2 = 2.2 \text{ (GeV}^2/\text{fm)}$$

T=0.35 GeV

Calibration of PCM elastic processes: Bass.S.A et al *J.Phys.G37105112* (2010)  
 Baier et al (BDMPS) *Nucl Phys.B478* (1996), B.Zakharov, *JETP Lett.63* (1996),