Second Workshop on Jet Mod., Wayne State, Aug. 2013

Jets in MARTINI

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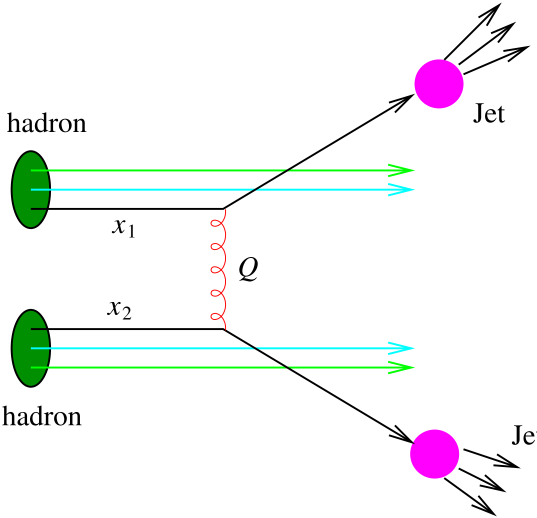
McGill

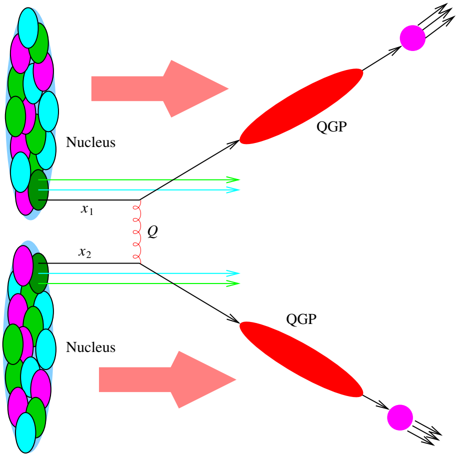
JETS

QCD Jets



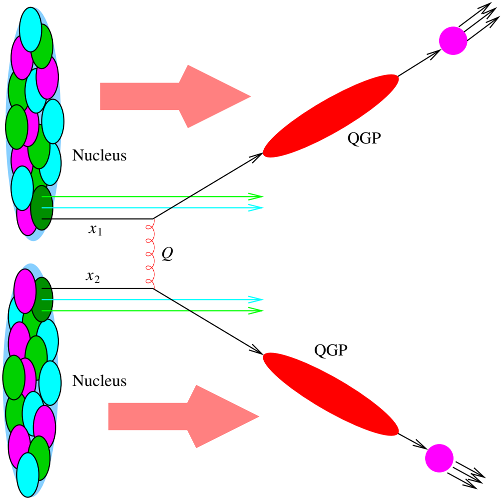
We understand this. More or less.

QCD Jets



Do we understand this?

Schematic Understanding



HIC Jet production scheme:

$$\frac{d\sigma_{AB}}{dt} = \int_{\text{geometry}} \int_{abcdc'}$$
$$\times f_{a/A}(x_a, Q_f) f_{b/B}(x_b, Q_f)$$
$$\times \frac{d\sigma_{ab\to cd}}{dt}$$
$$\times \mathcal{P}(x_c \to x'_c | T, u^\mu)$$
$$\times D(z'_c, Q)$$

$\mathcal{P}(x_c \to x'_c | T, u^\mu)$: Medium modification of high energy parton property

Is this reasonable?

- Separation scale Q_{pdf} in the PDF
  Interpretation: Given the interaction scale Q_s, what part of vacuum fluctuations in hadrons (nuclei) do I regard as "particles"?
  – Not much to do with per-jet energy loss
- Interaction scale Q_s in partonic $\dfrac{d\sigma}{dt}$
  This determines how long the scattering lasts: $\tau_{sc} \sim 1/Q_s$ or the range of the scattering $l_{sc} \sim 1/Q_s$
- So far, pre-QGP. No real sense of "time" evolution.
- Medium plus fragmentation: Needs time evolution.

$$D_{\rm med}(z, Q_f) = \mathcal{P}(x_c \to x'_c | T, u^\mu) \otimes D(z'_c, Q_f)$$

Big Question: Can these two be really separated?

Conceptual problem: $D_{\rm med} \approx \mathcal{P} \otimes D$

Hot medium

- Few approaches possible:
  - Medium increases effective Q_f in the frag. func. (plus drag, YaJEM)
  - Do mixed D and $\mathcal{P}$ (JEWEL, Q-PYTHIA)
  - Do D first then $\mathcal{P}$ (PYQUEN/HYDJET)
  - Do $\mathcal{P}$ first then D (MARTINI)
- None of these are first principle approaches.
- Medium interaction may be formulated in real time, but the splitting scale $1/Q$ is not really “time”.

MARTINI

McGill-AMY-MARTINI Team & Collaborators

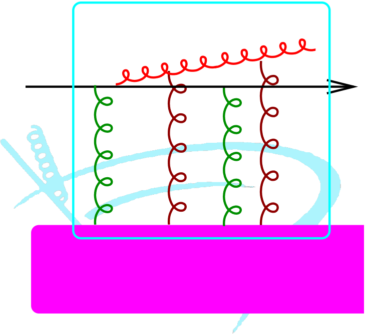
- Charles Gale
- Sangyong Jeon
- *Björn Schenke*
  (Formerly McGill, now BNL)
- *Clint Young*
  (Formerly McGill, Now UMinn)
- *Gabriel Denicol*
- *Matt Luzum*
- Sangwook Ryu
- Gojko Vujanovic
- Jean-Francois Paquet
- Michael Richard
- Igor Kozlov
- Khadija El Berhoumi
- Jean-Bernard Rose

... and many thanks to G. Moore, S. Caron-Huot, U. Heinz, D. Srivastava, S. Bass, C. Nonaka, M. Mustafa, E. Frodermann, R. Fries, A. Majumder, L. Yaffe, P. Arnold, and others ...

MARTINI

- Modular Alogorithm for Relativistic Treatment of Heavy IoN Interactions
- Hybrid approach
  - Calculate Hydrodynamic evolution of the soft mode
  - Propagate jets in the evolving medium according to McGill-AMY

Parton propagation

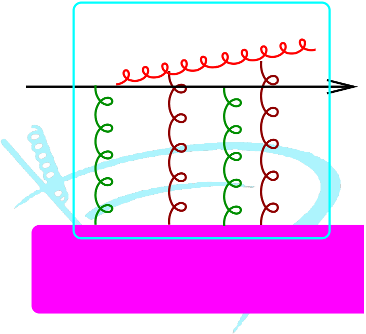


First calculate the *local* radiation rate $\frac{dN_g}{d\omega dt}$

The magenta box:

- QGP medium characterized by T, g_s

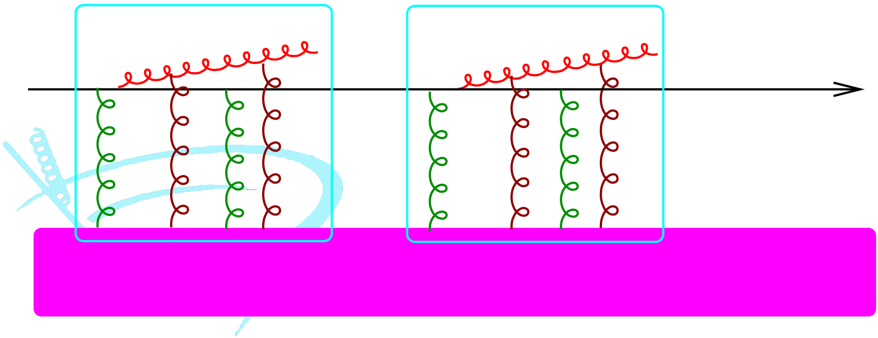
Parton propagation



First calculate the *local* radiation rate $\frac{dN_g}{d\omega dt}$

The cyan box:

- Infinite sum of ladder diagrams in momentum space
- Assumes $L_{\mathrm{medium}} \gg l_{\mathrm{mfp}}$

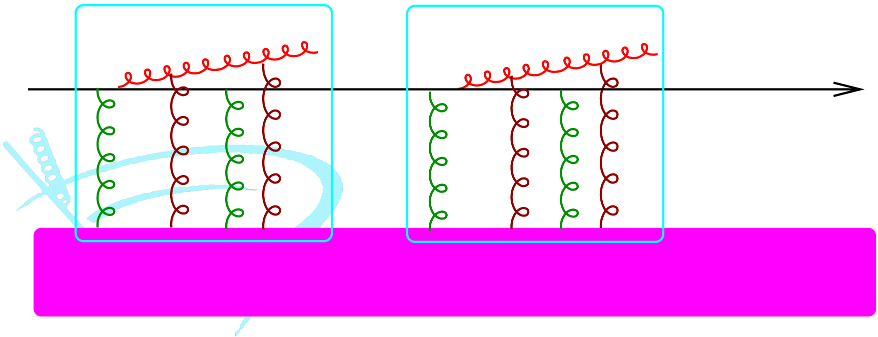


Then string them up $\equiv$ Solve the *rate* equation

$$\frac{dP(p)}{dt} = \int_k \frac{dN(p+k,k)}{dkdt} P(p+k) - P(p) \int_k \frac{dN(p,k)}{dkdt}$$

Coupled equations with separate P_q and P_g.

Parton propagation



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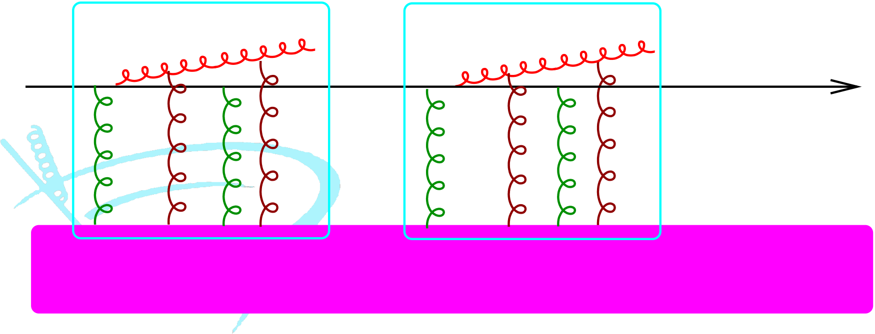
$$\frac{dP(p)}{dt} = \int_k \frac{dN(p+k,k)}{dkdt} P(p+k) - P(p) \int_k \frac{dN(p,k)}{dkdt}$$

The underlying medium evolves: The quantities $T(t,\mathbf{x}), u_\mu(t,\mathbf{x})$ or $\hat{q}(t,\mathbf{x}), l_{\rm mfp}(t,\mathbf{x})$ must be obtained from hydro calc's

Parton propagation



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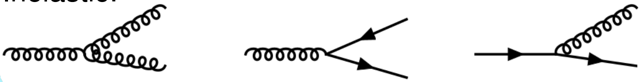
$$\frac{dP(p)}{dt} = \int_k \frac{dN(p+k,k)}{dkdt} P(p+k) - P(p) \int_k \frac{dN(p,k)}{dkdt}$$

Original McGill-AMY: AMY rates + Numerical solution of rate equ's.
MARTINI: AMY rates + Monte-Carlo simulation of rate equ's.

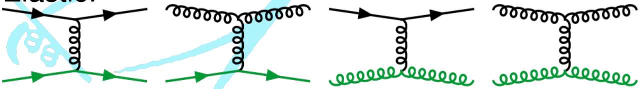
Parton propagation

Process include in MARTINI (all of them can be switched on & off):

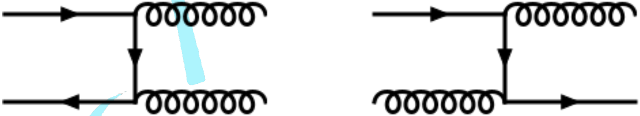
- Inelastic:



- Elastic:

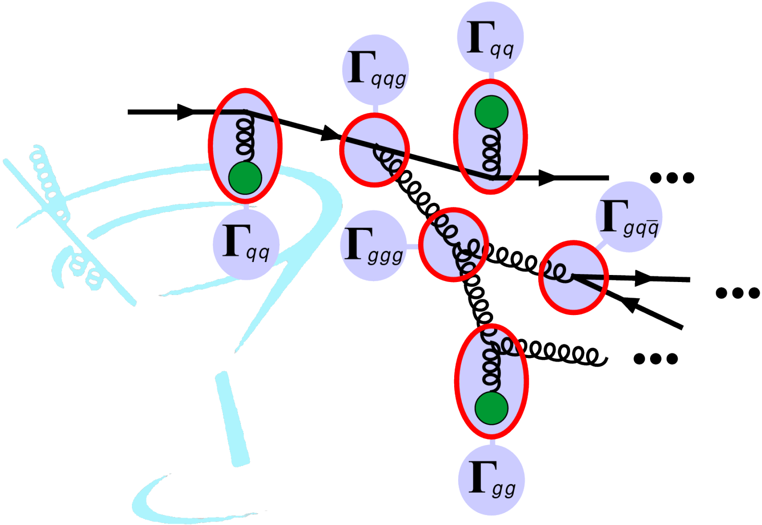


- Conversion:



- Photon: emission & conversion

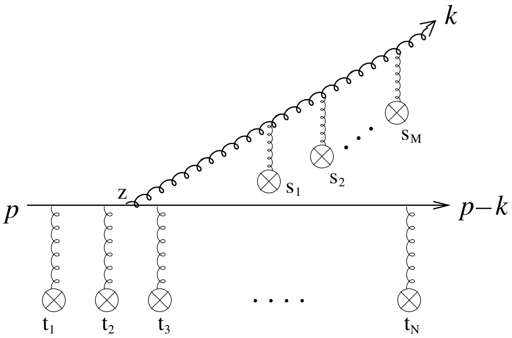
Parton propagation



An example path in MARTINI

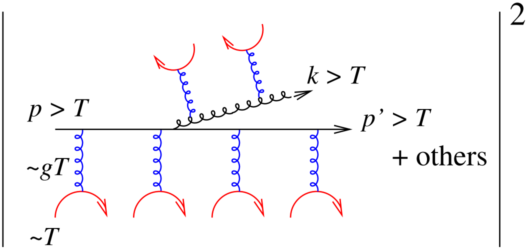
Leading order AMY rates

Main task in Energy Loss Calc's

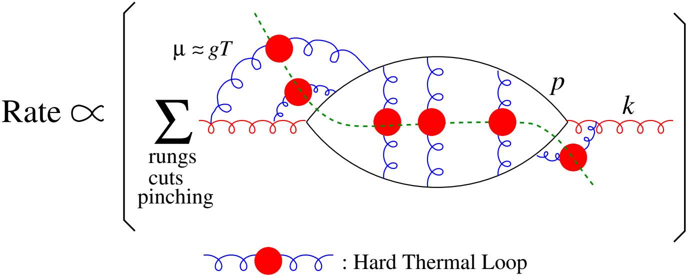


- Main task: To sum over all such diagrams and then square. This gets you the rate.

McGill-AMY



- Medium is weakly coupled QGP with thermal quarks and gluons
- Requires $g \ll 1$, $p > T$, $k > T$
- Sum all interactions with the medium



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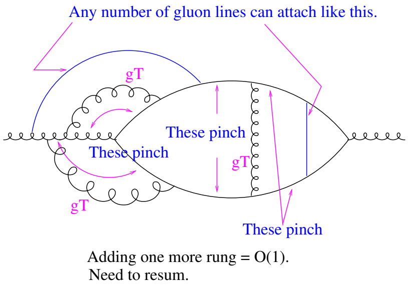


- Medium is weakly coupled QGP with thermal quarks and gluons
- Requires $g \ll 1$, $p > T$, $k > T$
- Sum all interactions with the medium
- Leading order: 3 different kinds of collinear pinching poles

- SD-Eq:

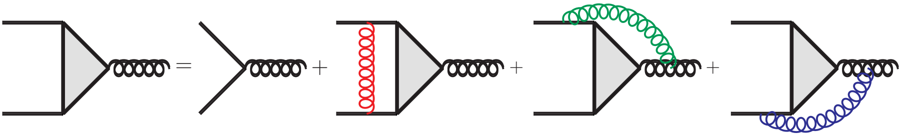


Figure from G. Qin

- Integral Eq:

$$2\mathbf{h} = i\delta E(\mathbf{h},p,k)\mathbf{F}(\mathbf{h}) + g^2\int\frac{d^2q_\perp}{(2\pi)^2}C(q_\perp)\Big\{(C_s - C_a/2)[\mathbf{F}(\mathbf{h}) - \mathbf{F}(\mathbf{h}{-}k\mathbf{q}_\perp)]$$
$$+ (C_a/2)[\mathbf{F}(\mathbf{h}) - \mathbf{F}(\mathbf{h}{+}p\mathbf{q}_\perp)] + (C_a/2)[\mathbf{F}(\mathbf{h}) - \mathbf{F}(\mathbf{h}{-}(p{-}k)\mathbf{q}_\perp)]\Big\}$$
$$\delta E(\mathbf{h},p,k) = \frac{\mathbf{h}^2}{2pk(p{-}k)} + \frac{m_k^2}{2k} + \frac{m_{p-k}^2}{2(p{-}k)} - \frac{m_p^2}{2p}, \quad C(q_\perp) = \frac{m_D^2}{q_\perp^2(q_\perp^2 + m_D^2)}$$

For $g \to q\bar{q}$, $(C_s - C_a/2)$ term is the one with $\mathbf{F}(\mathbf{h} - p\mathbf{q}_\perp)$ rather tan $\mathbf{F}(\mathbf{h} - k\mathbf{q}_\perp)$

AMY Rates

Rate for $p > T, k > T$ (valid for $p \gg T$ and $k \gg T$ as well)

$$\frac{dN_g(p,k)}{dkdt} = \frac{g_s^2}{16\pi p^7} \frac{1}{1 \pm e^{-k/T}} \frac{1}{1 \pm e^{-(p-k)/T}} \times$$
$$\times \left\{ \begin{array}{cc} C_f \frac{1+(1-x)^2}{x^3(1-x)^2} & q \to qg \\ 2N_f T_f \frac{x^2+(1-x)^2}{x^2(1-x)^2} & g \to q\bar{q} \\ C_a \frac{1+x^4+(1-x)^4}{x^3(1-x)^3} & g \to gg \end{array} \right\} \times \int \frac{d^2\mathbf{h}}{(2\pi)^2} 2\mathbf{h} \cdot \mathrm{Re}\, \mathbf{F}(\mathbf{h}, p, k)\,,$$

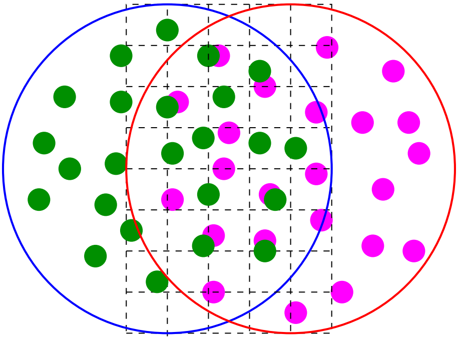
The rate equation to solve

- Rate equation to solve

$$
\begin{aligned}
\frac{dP_{q\bar{q}}(p)}{dt} &= \int_k P_{q\bar{q}}(p+k)\frac{d\Gamma^q_{qg}(p+k,k)}{dkdt} - \int_k P_{q\bar{q}}(p)\frac{d\Gamma^q_{qg}(p,k)}{dkdt} \\
&\quad +2\int_k P_g(p+k)\frac{d\Gamma^g_{q\bar{q}}(p+k,k)}{dkdt}\,, \\
\frac{dP_g(p)}{dt} &= \int_k P_{q\bar{q}}(p+k)\frac{d\Gamma^q_{qg}(p+k,p)}{dkdt} + \int_k P_g(p+k)\frac{d\Gamma^g_{gg}(p+k,k)}{dkdt} \\
&\quad - \int_k P_g(p)\left(\frac{d\Gamma^g_{q\bar{q}}(p,k)}{dkdt} + \frac{d\Gamma^g_{gg}(p,k)}{dkdt}\Theta(k-p/2)\right)
\end{aligned}
$$

- Actual solution by Monte-Carlo

MARTINI Step 1 - Jet positions



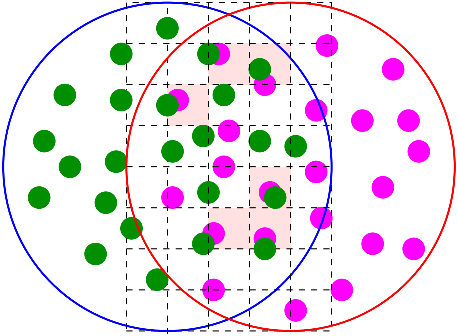
- Choose an impact parameter.
- Sample the projectile and the target nuclei according to the WS distribution.
- In the overlap region, make a transverse-grid of size $\sigma_{\rm inel}$. Determine the number of jet events in a given grid element according to $\sigma_{jet}(p_T^{\rm min})/\sigma_{inel}$:w

- OR: Sample a jet position from

$$P_{AB}(\mathbf{b}, \mathbf{r}_\perp) = \frac{T_A(\mathbf{r}_\perp + \mathbf{b}/2)T_B(\mathbf{r}_\perp - \mathbf{b}/2)}{T_{AB}(b)}$$

and embed one jet per event. Do this many times per event.

MARTINI Step 1 - Jet positions



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MARTINI Step 2 - Initial jet spectrum

- We use **PYTHIA 8.1**
- PDF selection using LHAPDF
- Shadowing through EKS98 or EPS08
- Isospin effect for neutrons taken into account

Unlike PYTHIA 6, PYTHIA 8 parameters are not yet fine-tuned.
MARTINI is usually tuned to reproduce π and γ in *pp*.

MARTINI Step 2 - Showers

- Let **PYTHIA 8.1** do the showers.
- Either
  - Either do the whole shower including the final shower
  - Or stop the shower when the medium evolution starts or when $Q_{\rm min} = \sqrt{\frac{p_T}{\tau_0}}$
- Parton position changes for $\tau < \tau_0$

MARTINI Step 3 - Medium evolution

- First load the hydro evolution history.
- A parton starts at $\tau = 0$ i.e. $z = 0, t = 0$
- Move the position until $\tau = \tau_0$
- Go to the rest frame of cell where the jet parton is currently at.
- According to the local conditions, calculate the total interaction probability within $\Delta t_{\mathrm{local}}$.

$$P = \sum_i \Delta t_{local} \int dk \frac{d\Gamma_i}{dk}$$

- Then generate a random number to decide to have interactions
- If yes, decide which process to do according to the relative weights
- Sample the final state particle momenta according to the chosen rate

MARTINI Step 3 - Medium evolution

- If the energy of the emitted parton is above a threshold, add it to the hard parton list
- Lorentz transform to the lab frame and move the final state particles to the next position
- Repeat

MARTINI Step 4 - Hadronization

- Evolution ends when
  - $E_{jet} < 4T$ in the cell rest frame
  - It enters the hadronic phase
- Hadronization through PYTHIA's lund fragmenation model
  - Keeping track of colors

Ideal MUSIC

- While this is happening in the background ...

MARTINI – The Movie

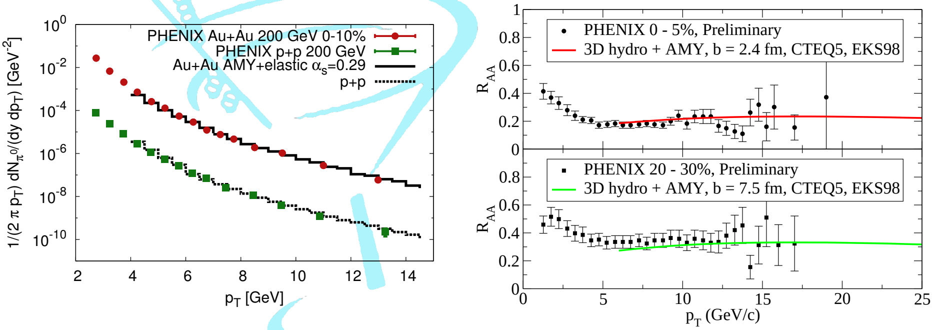
Projection on to the longitudinal plane

MARTINI – The Movie

Projection onto the transverse plane

[Schenke, Jeon and Gale, Phys. Rev. C 80, 054913 (2009)]

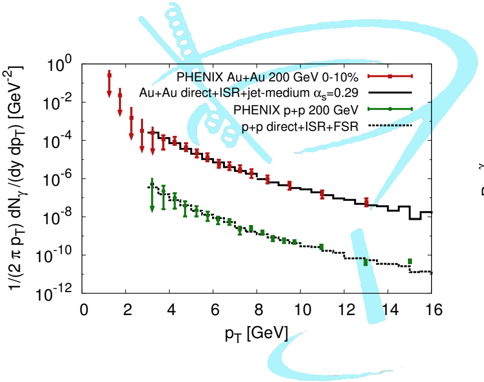
- π^0 spectra and R_{AA}

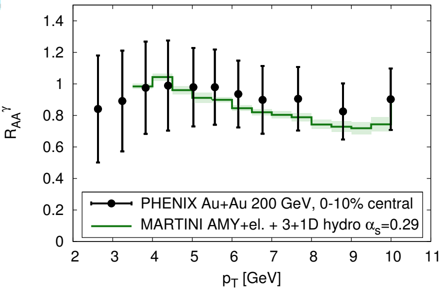


Photon production

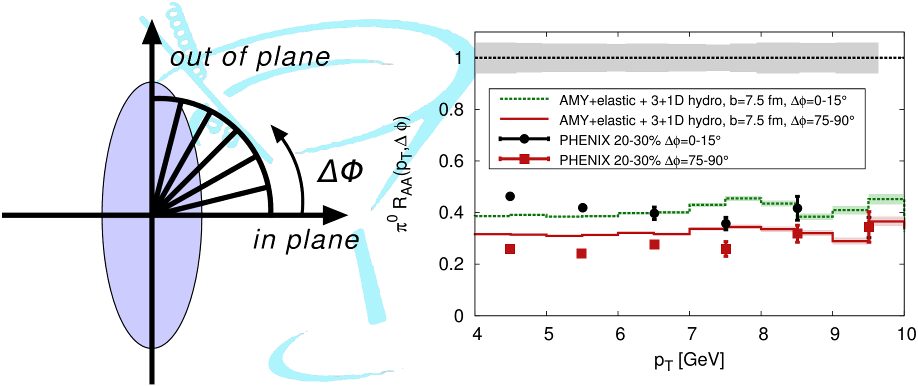
[Schenke, Jeon and Gale, Phys. Rev. C 80, 054913 (2009)]

- Spectra and R_{AA}^{γ}



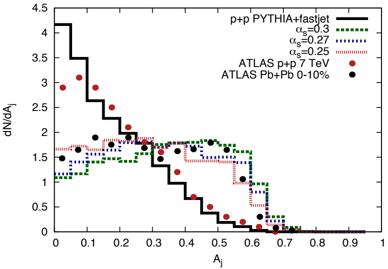


- $R_{AA}(p_T, \Delta\phi)$

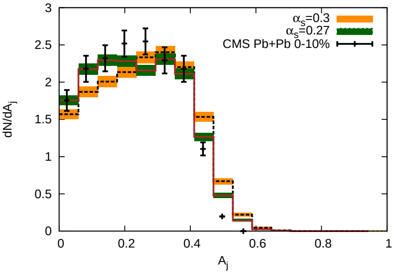


- $A = (E_t - E_a)/(E_t + E_a)$
- Right now ideal hydro with a smooth initial condition
- Full jet reconstruction with FASTJET

[Young, Schenke, Jeon, Gale, arXiv:1103.5769]



ATLAS, QM 2011



CMS, arXiv: 1102.1957 (2011)

Towards NL# MARTINI

Towards NL# MARTINI

- Finite evolution time effects
- Finite medium size effects
- Relaxing soft exchange approximation
- Recoiled thermal partons
- Running coupling

Mainly following:
BDMPS
Salgado & Wiedemann PRD 68 014008 (2003)
Arnold & Dogan, PRD78 065008 (2008)
Arnold & Xiao, PRD78 125008 (2008)
Caron-Huot & Gale PRC 82 064902 (2010)

Local or Holistic – Finite time effects

- In-medium radiation rate

  [Zakharov JETP 65, 615 (1997), Caron-Huot & Gale PRC 82 064902 (2010)]. From now on we follow Caron-Huot and Gale closely.

$$\frac{d\Gamma^a_{bc}}{dk} = \frac{P^{a(0)}_{bc}(x)}{\pi p}\mathrm{Im}\left(\int_0^t dt' \int_{\mathbf{q}_\perp,\mathbf{p}_\perp} \frac{\mathbf{q}_\perp\cdot\mathbf{p}_\perp}{\delta E(\mathbf{q}_\perp)}\mathcal{C}(t)\, K(t,\mathbf{q}_\perp;t',\mathbf{p}_\perp)\right)$$

- $K(t,\mathbf{q}_\perp;t',\mathbf{p}_\perp)$: Retarded propagator of parton a from t' to t with the Hamiltonian $H = \delta E - i\mathcal{C}$

  where

$$\delta E = \frac{p\mathbf{p}_\perp^2}{k(p-k)} + \frac{m_b^2}{2k} + \frac{m_c^2}{2(p-k)} - \frac{m_a^2}{2p} \quad \text{and } C(\mathbf{q}_\perp) = \frac{g_s^2 m_D^2 T}{\mathbf{q}_\perp^2(\mathbf{q}_\perp^2 + m_D^2)}$$

$$\mathcal{C} = \frac{C_b + C_c - C_a}{2}v(\mathbf{x}_\perp) + \frac{C_a + C_c - C_b}{2}v(\frac{k}{p}\mathbf{x}_\perp) + \frac{C_a + C_b - C_c}{2}v(\frac{p-k}{p}\mathbf{x}_\perp)$$

  with

$$v(\mathbf{x}_\perp) = \int_{\mathbf{q}_\perp} C(\mathbf{q}_\perp)(1 - e^{i\mathbf{q}_\perp\cdot\mathbf{x}_\perp}) \quad \text{and } C(\mathbf{q}_\perp) = \frac{g_s^2 m_D^2 T}{\mathbf{q}_\perp^2(\mathbf{q}_\perp^2 + m_D^2)}$$

  The same as the AMY kernel, but for finite t.

Local or Holistic – Finite time effects

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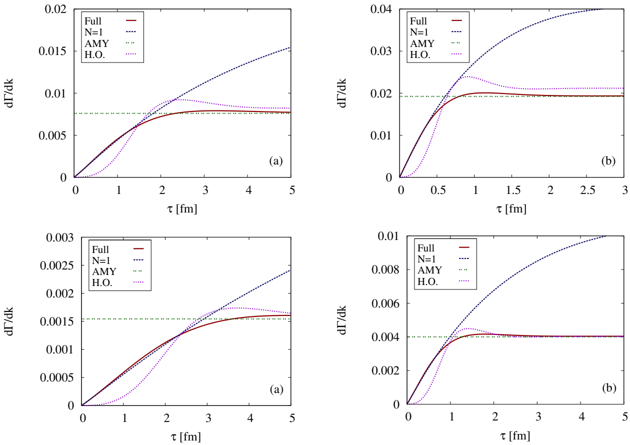
$$\mathcal{C}\psi(\mathbf{p}_\perp) = \int_{\mathbf{q}_\perp} C(\mathbf{q}_\perp)\Big(\frac{C_b + C_c - C_a}{2}[\psi(\mathbf{p}_\perp) - \psi(\mathbf{p}_\perp - \mathbf{q}_\perp)] +$$
$$\frac{C_a + C_c - C_b}{2}[\psi(\mathbf{p}_\perp) - \psi(\mathbf{p}_\perp + (k/p)\mathbf{q}_\perp)] +$$
$$\frac{C_a + C_b - C_c}{2}[\psi(\mathbf{p}_\perp) - \psi(\mathbf{p}_\perp + (1-k/p)\mathbf{q}_\perp)]\Big)$$

  The same as the AMY kernel, but for finite t.



3GeV daughter from 16 GeV parent, $T = 200, 400$ MeV

8GeV daughter from 16 GeV parent, $T = 200, 400$ MeV

$N = 1$: Opacity expansion, H.O.: BDMPS SHO

Full solution: Interpolates between the 1-st order opacity expansion (OE1) and AMY $\Longrightarrow \frac{d\Gamma}{dk} \approx \theta_\Lambda(OE1 < AMY)\frac{d\Gamma_{OE1}}{dk} + \theta_\Lambda(OE1 > AMY)\frac{d\Gamma_{AMY}}{dk}$

For evolving media

[Caron-Huot & Gale PRC 82 064902 (2010)]

$$\frac{d\Gamma^a_{bc}}{dk} = \frac{P^{a(0)}_{bc}(x)}{\pi p} \mathrm{Im}\left(\int_0^t dt' \int_{\mathbf{q}_\perp,\mathbf{p}_\perp} \frac{\mathbf{q}_\perp \cdot \mathbf{p}_\perp}{\overline{\delta E}(t,\mathbf{q}_\perp)} \tilde{\gamma}(t)\mathcal{C}(t)\, K(t,\mathbf{q}_\perp; t',\mathbf{p}_\perp) \right)$$

where

- Energy denominator

$$i/\overline{\delta E}(t,\mathbf{q}_\perp) = \int_t^\infty dt'\, e^{-i\int_t^{t'} dt''\,\delta E(t,\mathbf{q}_\perp)}$$

  Time depenence comes in through the thermal and Debye masses $m \sim gT(t,\mathbf{x})$

- Lorentz γ factor

$$\tilde{\gamma} = \frac{1 - v\cos\theta}{\sqrt{1 - v^2}}$$

Local or Holistic – Finite time effects

- Knowing the rate evolve the density according to

$$
\begin{aligned}
\frac{dP_{q\bar{q}}(p)}{dt} &= \int_k P_{q\bar{q}}(p+k)\frac{d\Gamma^q_{qg}(p+k,k)}{dk} - \int_k P_{q\bar{q}}(p)\frac{d\Gamma^q_{qg}(p,k)}{dk} \\
&\quad +2\int_k P_g(p+k)\frac{d\Gamma^g_{q\bar{q}}(p+k,k)}{dk}\,, \\
\frac{dP_g(p)}{dt} &= \int_k P_{q\bar{q}}(p+k)\frac{d\Gamma^q_{qg}(p+k,p)}{dk} + \int_k P_g(p+k)\frac{d\Gamma^g_{gg}(p+k,k)}{dk} \\
&\quad - \int_k P_g(p)\left(\frac{d\Gamma^g_{q\bar{q}}(p,k)}{dk} + \frac{d\Gamma^g_{gg}(p,k)}{dk}\Theta(k-p/2)\right)
\end{aligned}
$$

- $d\Gamma/dk$: In principle non-local through the trajectory and the finite time dependences

Local or Holistic – Finite time effects

What needs to be done:

- Calculate the Green function $K(t, \mathbf{q}_\perp; t', \mathbf{p}_\perp)$
- Calculate the rate

$$\frac{d\Gamma^a_{bc}(t)}{dk} = \frac{P^{a(0)}_{bc}(x)}{\pi p} \mathrm{Im}\left(\int_0^t dt' \int_{\mathbf{q}_\perp, \mathbf{p}_\perp} \frac{\mathbf{q}_\perp \cdot \mathbf{p}_\perp}{\delta E(\mathbf{q}_\perp)} \mathcal{C}(t)\, K(t, \mathbf{q}_\perp; t', \mathbf{p}_\perp) \right)$$

- Use this rate to do the MC evolution
- Difficulty: Calculate the Green function $K(t, \mathbf{q}_\perp; t', \mathbf{p}_\perp)$

Local or Holistic – Finite time effects

If Brick

- First order opacity expansion is "local" [GLV and also Caron-Huot and Gale]

$$
\begin{aligned}
\frac{d\Gamma^a_{bc}(t)}{dk} &= \frac{P^{a(0)}_{bc}(x)p}{4\pi k^2(p-k)^2} \int_{\mathbf{q}_\perp,\mathbf{p}_\perp} C(\mathbf{q}_\perp) \frac{1-\cos(\delta E(\mathbf{p})t)}{\delta E(\mathbf{p}_\perp)} \\
&\times \left[\frac{C_b + C_c - C_a}{2} \left(\frac{\mathbf{p}_\perp^2}{\delta E(\mathbf{p}_\perp)} - \frac{\mathbf{p}_\perp \cdot (\mathbf{p}_\perp - \mathbf{q}_\perp)}{\delta E(\mathbf{p}_\perp - \mathbf{q}_\perp)} \right) \right. \\
&+ \left. \frac{C_a + C_c - C_b}{2} \left(\frac{\mathbf{p}_\perp^2}{\delta E(\mathbf{p}_\perp)} - \frac{\mathbf{p}_\perp \cdot (\mathbf{p}_\perp + \frac{k}{p}\mathbf{q}_\perp)}{\delta E(\mathbf{p}_\perp + \frac{k}{p}\mathbf{q}_\perp)} \right) \right] \\
&+ \frac{C_a + C_b - C_c}{2} \left(\frac{\mathbf{p}_\perp^2}{\delta E(\mathbf{p}_\perp)} - \frac{\mathbf{p}_\perp \cdot (\mathbf{p}_\perp + \frac{p-k}{p}\mathbf{q}_\perp)}{\delta E(\mathbf{p}_\perp + \frac{p-k}{p}\mathbf{q}_\perp)} \right)
\end{aligned}
$$

T and m_D dependence can be pulled out of the integral except in the cosine.

Local or Holistic – Finite time effects

If Brick

Possible approach

- Implement
$$\frac{d\Gamma}{dk} \approx \theta_\Lambda(OE1 < AMY)\frac{d\Gamma_{OE1}}{dk} + \theta_\Lambda(OE1 > AMY)\frac{d\Gamma_{AMY}}{dk}$$
- This requires
  - Either calculate the OE1 rate at every time step until the AMY rate takes over
  - Or calculate a big table of $d\Gamma_{OE1}/dk$ in (p, k, t, T) and interpolate
  - Or use Taylor expansion
    - Typically, $\delta E = O(g^2 T^2/k)$ so $\delta E\, t \lesssim 1$
    - $1 - \cos(\delta E\, t) \approx \frac{(\delta E\, t)^2}{2} - \frac{(\delta E\, t)^4}{24} - \frac{(\delta E\, t)^6}{720}$ may be adequate. This is good up to $\delta E\, t \lesssim 2$
    - Reduces to 3 tables in $(p/T, k/T)$

Local or Holistic – Finite time effects

If Expanding Medium

- Opacity expansion part is short $\sim 1\,\text{fm}$ when temperature is high $\sim 400\,\text{MeV}$
- Two pass approximation may be possible
  - First pass to determine the 0-th order trajectory without any interactions
  - Second pass to evaluate the 1-st order opacity expansion using $K(t, \mathbf{q}_\perp; t', \mathbf{p}_\perp) \approx K_{\text{free}}(t, \mathbf{q}_\perp; t', \mathbf{p}_\perp)$
  - Then implement $\frac{d\Gamma}{dk} \approx \theta_\Lambda(OE1 < AMY)\frac{d\Gamma_{OE1}}{dk} + \theta_\Lambda(OE1 > AMY)\frac{d\Gamma_{AMY}}{dk}$
- It's still going to be computing intensive
- MARTINI currently implements approximate parametrized $d\Gamma/dk$

Local or Holistic – Finite size effects

- Local rate equation approach breaks down when $l_{\rm coh} > L$ where L is the "size of the fireball" and $l_{\rm coh}$ is the "coherence length" (these are not so well defined in an expanding system).
- $l_{\rm coh} \approx l_{\rm mfp}\sqrt{\dfrac{\omega_g}{E_{\rm LPM}}} = \sqrt{\dfrac{\omega_g}{\hat{q}}}$
  where $l_{\rm mfp} \sim 1/g^2 T$, $\mu \sim m_T \sim gT$, $E_{\rm LPM} = \mu^2 l_{\rm mfp}$ and $\hat{q} = \mu^2/l_{\rm mfp}$
- Soft gluon emission, $\omega_g < \mu^2 l_{\rm mfp}$,
  $\Longrightarrow$ Coherence matters not. One BH per $l_{\rm mfp}$. No need to resum.
- Hard gluon emission, $L^2\hat{q} > \omega_g > \mu^2 l_{\rm mfp}$,
  $\Longrightarrow$ Coherence matters. One BH per $l_{\rm coh}$ or more precisely, AMY.
- The rate equation approach is OK for these.

Local or Holistic – Finite size effects

Rough Estimates

- Very Hard gluon emission, $\omega > L^2\hat{q}$
  $\Longrightarrow$ Only one effective scattering possible within L.
  The rate goes back to BH/L.
- $\frac{d\Gamma}{dk}$ should know whether emitted $k = \omega_g > L^2\hat{q}$. But we won't know that until the evolution is done and L determined.

Local or Holistic – Finite size effects

- Conceptually clear. But how to implement it?
- If a brick: With the jet energy $p = E_{\rm jet} > L^2\hat{q}$ and the emitted gluon energy $k = \omega_g$
  - For a given parton, calculate the very hard emission probability

$$P_> = \int_{L^2\hat{q}}^{p} dk \int_0^L dt \, \frac{d\Gamma(p,k;t)}{dk}$$

  - Generate a random number and decide whether to emit very hard gluons
  - If yes, sample

$$\frac{dP_>}{dk} = \int_0^L dt \, \frac{d\Gamma(p,k;t)}{dk}$$

  - If no, use the rate equation but only for $\omega_g < L^2\hat{q}$

Local or Holistic – Finite size effects

- Conceptually clear. But how to implement it?
- If an evolving medium:
  - How to calculate $E_L = L^2\hat{q}$?

$$\hat{q} = \frac{4}{3}\int_{\mathbf{q}_\perp} \mathbf{q}_\perp^2 \frac{g_s^2 m_D^2 T}{\mathbf{q}_\perp^2(\mathbf{q}_\perp^2 + m_D^2)} = \frac{1}{3\pi} g^2 m_D^2 T \ln(1 + q_{\rm max}^2/m_D^2)$$

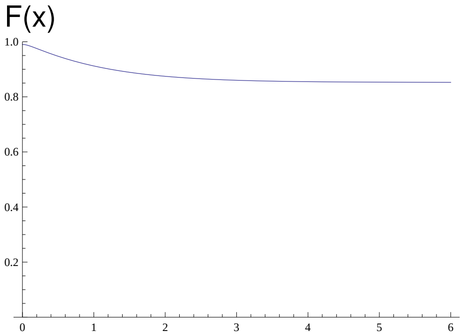
    Local quantity. Changes in time.
    In Phys.Rev. D68 (2003) 014008 by C. Salgado and U. Wiedemann

$$\bar{\hat{q}} = \frac{2}{L^2}\int_{\xi_0}^{L+\xi_0} d\xi\,(\xi - \xi_0)\hat{q}(\xi)$$

    Here ξ is the "time" in the cell frame.
  - Effective $\bar{E}_L = L^2\bar{\hat{q}}$
  - Single pass formalism is not going to work - Iteration needed. Two passes may be enough.

Relaxing the soft exchange approximation

- The elastic scattering kernel $\frac{d\Gamma_{\rm AMY}}{d^2\mathbf{q}_\perp} = \frac{g_s^2 T m_D^2}{\mathbf{q}_\perp^2(\mathbf{q}_\perp^2 + m_D^2)}$ is for soft $\mathbf{q}_\perp$.
- Correction factor valid up to $q_\perp/T = O(1)$ [Arnold and Xiao, Phys.Rev.D78:125008, 2008]



$$\frac{d\Gamma_{\rm el}}{d^2\mathbf{q}_\perp} = \frac{d\Gamma_{\rm AMY}}{d^2\mathbf{q}_\perp} F(q_\perp/T)$$

- Easy to implement and solve the SD-equation for AMY rate – Already in MARTINI.

Recoil

- s-channel elastic processes can convert a thermal parton into a jet parton
- Kinematics is not hard to implement. Lund model hadronization is.

Running coupling

- Where g appears: AMY Integral equation

$$
\begin{aligned}
2\mathbf{h} &= i\delta E(\mathbf{h},p,k)\mathbf{F}(\mathbf{h}) + g^2 \int \frac{d^2 q_\perp}{(2\pi)^2} C(q_\perp) \times \\
&\times \Big\{ (C_s - C_a/2)[\mathbf{F}(\mathbf{h}) - \mathbf{F}(\mathbf{h}{-}k\mathbf{q}_\perp)] \\
&+(C_a/2)[\mathbf{F}(\mathbf{h}) - \mathbf{F}(\mathbf{h}{+}p\mathbf{q}_\perp)] \\
&+(C_a/2)[\mathbf{F}(\mathbf{h}) - \mathbf{F}(\mathbf{h}{-}(p{-}k)\mathbf{q}_\perp)] \Big\} \\
\delta E(\mathbf{h},p,k) &= \frac{\mathbf{h}^2}{2pk(p{-}k)} + \frac{m_k^2}{2k} + \frac{m_{p-k}^2}{2(p{-}k)} - \frac{m_p^2}{2p} . \\
C(q_\perp) &= \frac{m_D^2}{q_\perp^2 (q_\perp^2 + m_D^2)}
\end{aligned}
$$

- Scaling $\mathbf{h} \to g\tilde{\mathbf{h}}$, $\mathbf{q}_\perp \to g\tilde{\mathbf{q}}_\perp$, $m \to g\tilde{m}$ and $\mathbf{F}(\mathbf{h}) \to \tilde{\mathbf{F}}(\tilde{\mathbf{h}})/g$ renders this equation independent of g

Running coupling

- Two different origin of g in the rate

$$\frac{dN_g(p,k)}{dkdt} = \frac{g^2}{16\pi p^7}\frac{1}{1\pm e^{-k/T}}\frac{1}{1\pm e^{-(p-k)/T}}$$
$$\times \left\{ \begin{array}{cc} C_f \frac{1+(1-x)^2}{x^3(1-x)^2} & q\to qg \\ 2N_f T_f \frac{x^2+(1-x)^2}{x^2(1-x)^2} & g\to q\bar{q} \\ C_a \frac{1+x^4+(1-x)^4}{x^3(1-x)^3} & g\to gg \end{array} \right\}$$
$$\times g^2 \int \frac{d^2\tilde{\mathbf{h}}}{(2\pi)^2} 2\tilde{\mathbf{h}}\cdot \mathrm{Re}\tilde{\mathbf{F}}(\tilde{\mathbf{h}},p,k)\,,$$

- g^2: Gluon radiation vertex
- g^2: Elastic scattering rate

Running coupling

Radiation:

- $g^2/4\pi \to \alpha_s(Q)$ where Q is the hard scale
- Implemented in MARTINI

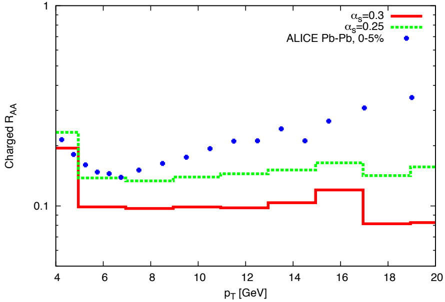
Elastic:

- The elastic scattering kernel $\frac{d\Gamma_{\rm AMY}}{d^2\mathbf{q}_\perp} = \frac{g_s^2 T m_D^2}{\mathbf{q}_\perp^2(\mathbf{q}_\perp^2 + m_D^2)}$ is for soft $\mathbf{q}_\perp$.
- To include large $q_\perp$ [Arnold and Dogan, Phys.Rev.D78:065008,2008]

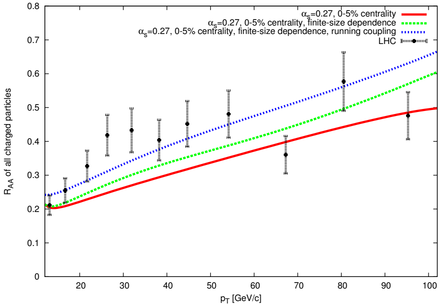
$$\frac{g_s^2 m_D^2}{\mathbf{q}_\perp^2(\mathbf{q}_\perp^2 + m_D^2)} \to 4\pi \frac{\alpha_S(q_\perp) m_D^2}{\mathbf{q}_\perp^2(\mathbf{q}_\perp^2 + m_D^2)}$$

- Not yet in MARTINI, but easy enough to implement

MARTINI with finite time and running coupling corrections



Before

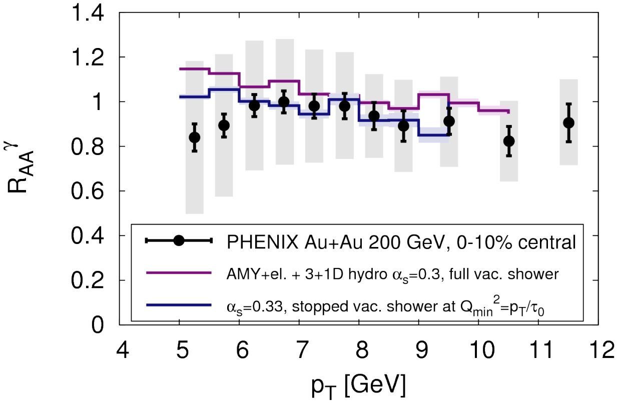


After
Young, Schenke, Jeon, Gale, arXiv:1209.5679

- Slope change (flattening) at high p_T may be due to the finite size effect

Effect of different vacuum-medium transition

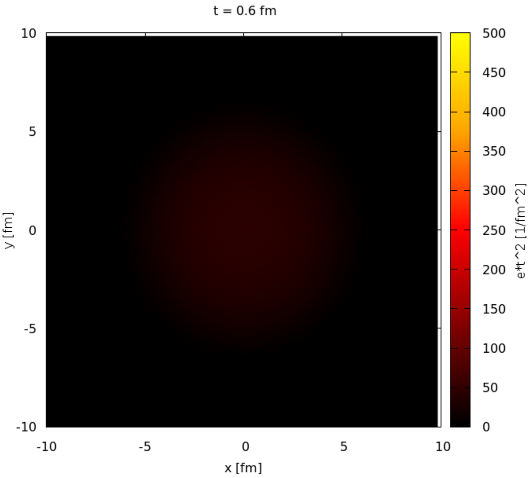
- Photons: Full PYTHIA shower vs. $Q^2_{\mathrm{min}} = p_T^2/\tau_0^2$



PRELIMINARY

Conclusions/Outlook

- Biggest conceptual problem in MC for HIC: Formulation of time evolution.
- NL# MARTINI
  - Finite time
  - Finite size
  - Correction to the elastic kernel
  - Running coupling
- MUSIC+MARTINI+UrQMD is in the work.


t = 0.6 fm
10
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-5
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y [fm]
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e*t^2 [1/fm^2]