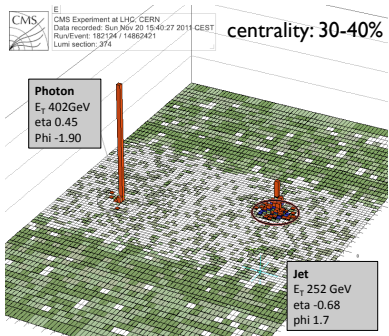


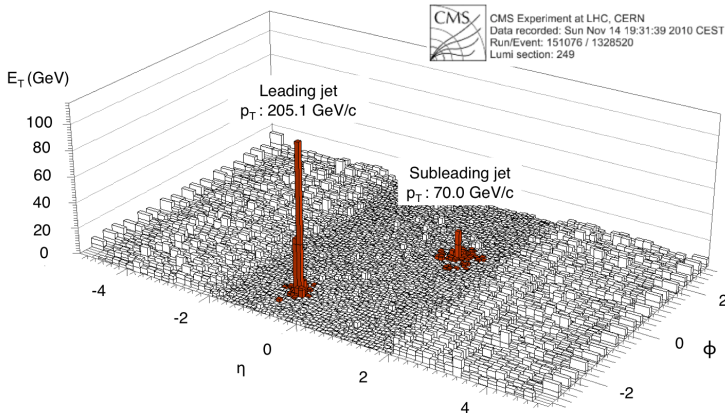
# From Jet Quenching to Turbulence

Edmond Iancu

IPhT Saclay & CNRS

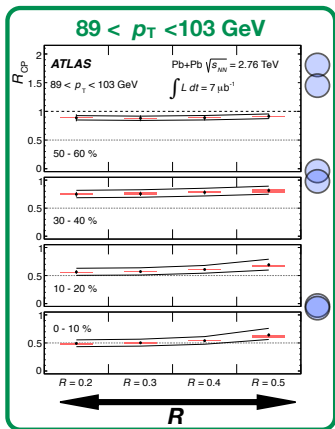


# Motivation: Di-jet asymmetry at the LHC

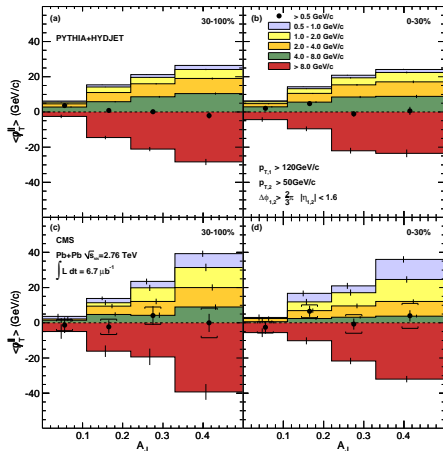


- Additional energy imbalance as compared to p+p : 20 to 30 GeV
- Detailed studies show that the 'missing energy' is associated with the additional radiation of many soft quanta at large angles  
(cf. the talks by Stefan Bathe, Guenther Roland, and Aaron Angerami)

# Motivation: Di-jet asymmetry at the LHC



 COLUMBIA UNIVERSITY  
IN THE CITY OF NEW YORK



- Can we understand that from **first principles** ?
- Is there a natural mechanism in pQCD to radiate **soft quanta at large angles alone** ?

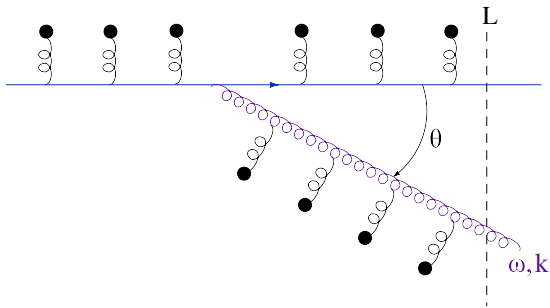
# Medium-induced jet evolution

- We need to understand **medium-induced jet evolution** within pQCD
- Previous studies restricted to a **single**, medium-induced, emission
  - 'BDMPS-Z mechanism', 'opacity expansion', 'higher-twist' ...  
*Baier, Dokshitzer, Mueller, Peigné, and Schiff; Zakharov (96-97); Wiedemann (2000); Guylassy, Levai, Vitev (2000); Arnold, Moore, Yaffe (2002); Wang and Wang (2002) ...*
- Already complicated, due to coherent multiple scattering (**LPM effect**)
- Generalization to **multiple branchings** even more complicated
  - interference between different emitters
  - correlations induced by multiple scattering (many Wilson lines) ...
- **Color decoherence** via medium rescattering simplifies the problem  
*Mehtar-Tani, Salgado, Tywoniuk (10-12); Casalderrey-Solana, E. I. (11)*
- **Factorization** of multiple branchings & **first physical consequences**  
*Blaizot, Dominguez, E.I., Mehtar-Tani (2012-13)*

# pQCD : the BDMPSZ mechanism

- Gluon radiation triggered by interactions in the medium

*Baier, Dokshitzer, Mueller, Peigné, and Schiff; Zakharov (96–97)*



- Gluon emission is linked to **transverse momentum broadening**

$$\Delta k_{\perp}^2 \simeq \hat{q} \Delta t \quad \text{with} \quad \hat{q} \simeq \frac{m_D^2}{\lambda} = \frac{(\text{Debye mass})^2}{\text{mean free path}}$$

- destroys the coherence between the gluon and its parent parton
- increases the emission angle

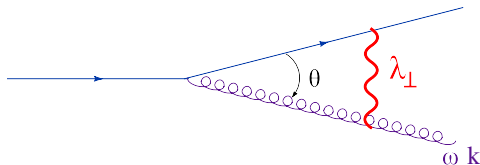
# The formation time

- The gluon must lose **quantum coherence** with respect to its source
- The **quark–gluon transverse separation**  $r_{\perp}$  at the **formation time**  $\tau_f$  should be comparable with the gluon **transverse wavelength**  $\lambda_{\perp}$

$$r_{\perp} \simeq \theta \tau_f \gtrsim \lambda_{\perp} \simeq 1/k_{\perp}$$

$$k_{\perp} \simeq \omega \theta$$

$$\implies \tau_f \simeq \frac{\omega}{k_{\perp}^2} \simeq \frac{1}{\omega \theta^2}$$



- The transverse momentum increases via collisions:  $k_{\perp}^2 \simeq \hat{q} \tau_f$

$$\tau_f \simeq \frac{\omega}{\hat{q} \tau_f} \implies \tau_f \simeq \sqrt{\frac{\omega}{\hat{q}}} \quad \& \quad k_f \simeq (\omega \hat{q})^{1/4}$$

- Actually, this is an **upper limit on**  $\tau_f$  :  $k_{\perp}$  can be larger than  $k_f(\omega)$

# Formation time ( $\tau_f$ ) & angle ( $\theta_f$ )

$$\tau_f(\omega) \simeq \sqrt{\frac{\omega}{\hat{q}}} \quad \& \quad \theta_f(\omega) \gtrsim \frac{k_f}{\omega} \simeq \left(\frac{\hat{q}}{\omega^3}\right)^{1/4}$$

- Soft gluons : short formation times & large emission angles ✓  
... but what does 'soft' really mean ?
  - maximal  $\omega$  for this mechanism :  $\tau_f \simeq L \Rightarrow \omega \lesssim \omega_c \equiv \hat{q}L^2$
  - minimal  $\omega$  :  $\tau_f \simeq \lambda_{\text{mfp}} \Rightarrow \omega \gtrsim \omega_{\text{BH}} \equiv \hat{q}\lambda_{\text{mfp}}^2$  (Bethe-Heitler)
- The gluon can be emitted **anywhere** within the size  $L$  of the medium  
... but its emission takes some time  $\tau_f$ 
  - BDMPSZ gluon spectrum (probability for one gluon emission)

$$\omega \frac{dN}{d\omega} \simeq \alpha_s \frac{L}{\tau_f(\omega)} \simeq \alpha_s \sqrt{\frac{\omega_c}{\omega}}$$

- Bremsstrahlung times the available longitudinal phase

# The BDMPSZ spectrum

$$\omega \frac{dN}{d\omega} \simeq \alpha_s \frac{L}{\tau_f(\omega)} \simeq \alpha_s \sqrt{\frac{\omega_c}{\omega}}$$

- **LPM effect** : the emission rate decreases with increasing  $\omega$ 
  - coherence: many soft collisions contribute to a single, hard, emission
- Relatively hard emissions with  $\omega \sim \omega_c$  :

- rare events : probability of  $\mathcal{O}(\alpha_s)$
- control energy loss by the leading particle (and observables like  $R_{pA}$ )

$$\Delta E = \int^{\omega_c} d\omega \omega \frac{dN}{d\omega} \sim \alpha_s \omega_c$$

- small emission angles  $\theta_c = \theta_f(\omega_c) \Rightarrow$  the energy remains inside the jet
- Arguably, not so important for the **di-jet asymmetry**



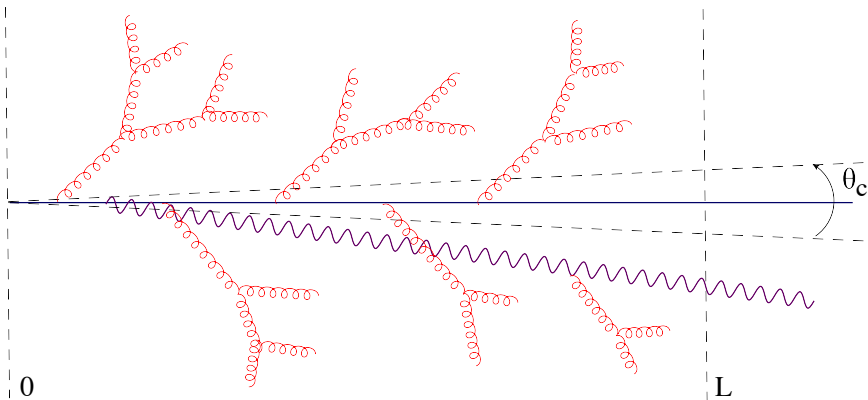
# The BDMPSZ spectrum

$$\omega \frac{dN}{d\omega} \simeq \alpha_s \frac{L}{\tau_f(\omega)} \simeq \alpha_s \sqrt{\frac{\omega_c}{\omega}}$$

- **LPM effect** : the emission rate decreases with increasing  $\omega$ 
  - coherence: many soft collisions contribute to a single, hard, emission
- Relatively soft emissions with  $\omega \ll \omega_c$  :
  - small formation times :  $\tau_f \ll L$
  - quasi-deterministic : probability of  $\mathcal{O}(1)$  for  $\omega \lesssim \alpha_s^2 \omega_c$ 
    - ⇒ they show up in a typical event !
  - relatively small contribution to energy loss :  $\Delta E_{\text{soft}} \sim \alpha_s^2 \omega_c$
  - ... but this can be lost at arbitrarily large angles
- Potentially relevant for the **di-jet asymmetry**

# A typical gluon cascade

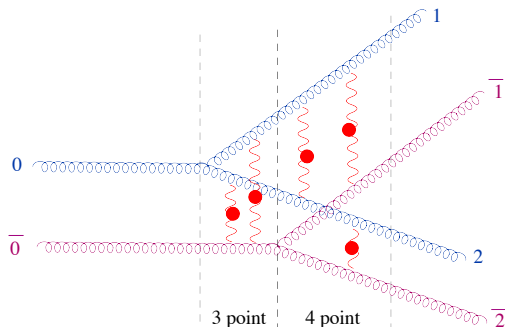
- One needs to understand **multiple medium-induced branchings**



- A rain of soft gluons ( $\omega \ll \omega_c$ ) plus (sometimes) a harder one ( $\omega \sim \omega_c$ )
- From now on, we shall focus on the 'rain' !

# A few words on the formalism

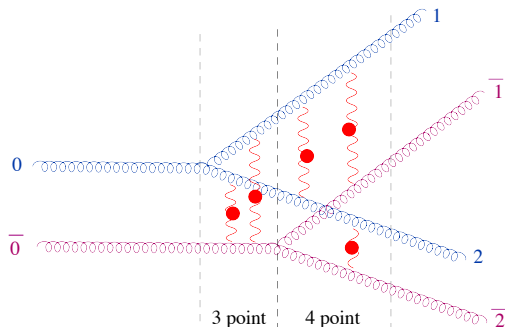
- Quantum emission: amplitude  $\times$  the complex conjugate amplitude



- 'Medium' = randomly distributed scattering centers (Gaussian)
  - Coulomb scattering with Debye screening
  - multiple scattering in eikonal approximation (one Wilson line per gluon)
  - $1 \rightarrow 2$  gluon branching  $\Rightarrow$  3-p and 4-p functions of the Wilson lines
- See also the talk by Fabio Dominguez*

# A few words on the formalism

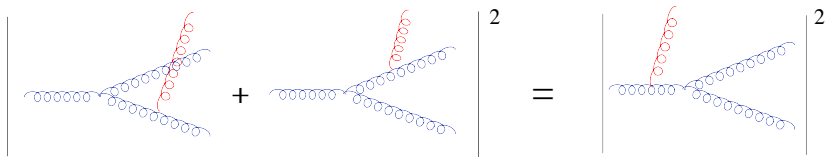
- Quantum emission:  $\text{amplitude} \times \text{the complex conjugate amplitude}$



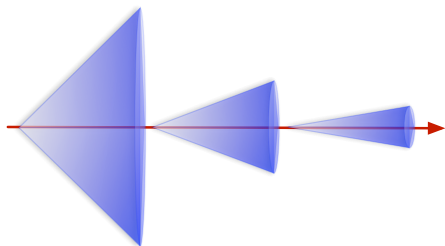
- Contains and extends the original BDMPSTZ/AMI formalisms already at the level of a **single medium-induced emission** ...
  - ▷ transverse momentum dependence for the emission vertex, correct inclusion of single scattering, color (de)coherence after emission ...
- Permits the treatment of **interference & multiple branchings**

# Multiple emissions : vacuum

- After a splitting, the daughter gluons remain in the same overall color state until the next emission : **they are color coherent**
- A subsequent emission **at large angles** sees the **overall** color charge

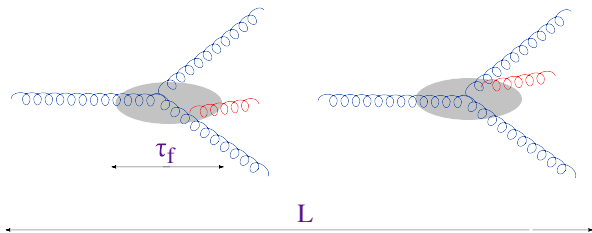


- Equivalent **classical branching process**, but with **angular ordering**



# Multiple emissions : medium

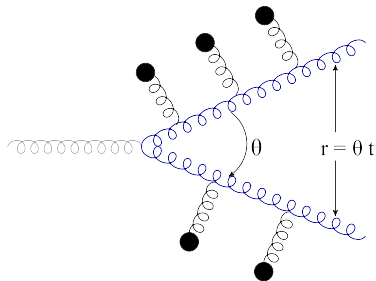
- In the medium, **color coherence** is rapidly lost via rescattering  
*Mehtar-Tani, Salgado, Tywoniuk (arXiv: 1009.2965; 1102.4317);  
Casalderrey-Solana, E. I. (arXiv: 1106.3864)*



- By the time of their emission, the daughter gluons have already lost their color coherence  $\Rightarrow$  **they cannot interfere with each other**  
*Blaizot, Dominguez, E.I., Mehtar-Tani (arXiv: 1209.4585)*
- The interference effects are suppressed by a factor  $\tau_f/L \ll 1$

# Color decoherence in the medium

- A 'color antenna' : two gluons propagating at a fixed angle
  - at  $\tau = 0$ , the two gluons start in a color singlet state ('dipole')
  - how fast do they lose their color coherence via rescattering ?



$$S(\tau, \theta) = \frac{1}{N_g} \left\langle \text{Tr} \left[ U(\tau, \mathbf{v}_1) U^\dagger(\tau, \mathbf{v}_2) \right] \right\rangle$$

$$U^\dagger(\tau, \mathbf{v}) = \text{T exp} \left[ ig \int_0^\tau dx^+ A^-(x^+, \mathbf{v}x^+) \right]$$

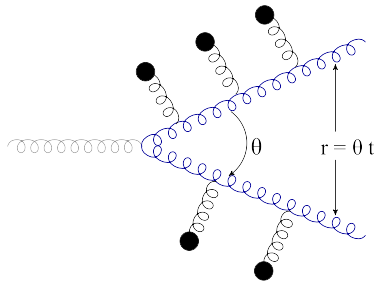
Gaussian average over  $A^- \implies \hat{q}$

$$S(\tau, \theta) \simeq \exp \left\{ -\hat{q} \int_0^\tau dt r_\perp^2(t) \right\} \simeq \exp \left\{ -\hat{q} \theta^2 \tau^3 \right\}$$

- Color coherence is lost for  $\tau \gtrsim \tau_{\text{coh}} \sim \frac{1}{(\hat{q} \theta^2)^{1/3}}$

# Color decoherence in the medium

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Gaussian average over  $A^- \implies \hat{q}$

- If the antenna is generated via gluon splitting:  $\theta = \theta_f(\omega) \sim (\hat{q}/\omega^3)^{1/4}$

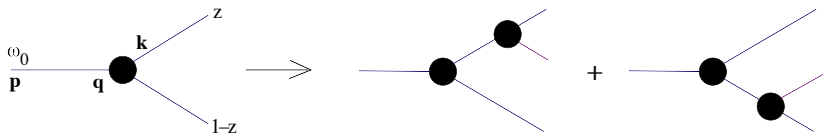
$$\tau_{\text{coh}} \sim \frac{1}{(\hat{q}\theta_f^2)^{1/3}} \simeq \sqrt{\frac{\omega}{\hat{q}}} = \tau_f(\omega)$$

- Successive emissions can be treated as independent



# A classical branching process

- Markovian process in  $D = 3 + 1$  :  $\omega, k_{\perp}$ , time  $t$  (or medium size  $L$ )



- The  $g \rightarrow gg$  splitting vertex (the 'blob') : the BDMPSZ spectrum

$$\mathcal{K}(Q, z, \omega_0) = \frac{2}{\omega_0} \frac{P_{gg}(z)}{z(1-z)} \sin \left[ \frac{Q^2}{2k_f^2} \right] \exp \left[ -\frac{Q^2}{2k_f^2} \right], \quad Q \equiv k - zq$$

▷ strongly peaked at  $Q^2 = k_f^2 \equiv \sqrt{z(1-z)\omega_0 \hat{q}}$

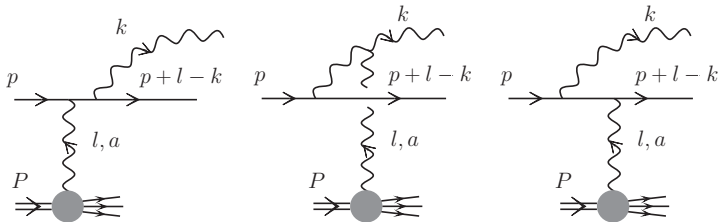
- The propagator (the 'line') : transverse momentum broadening

$$\mathcal{P}(\Delta \mathbf{k}; \Delta t) = \frac{4\pi}{\hat{q}\Delta t} \exp \left[ -\frac{\Delta \mathbf{k}^2}{\hat{q}\Delta t} \right]$$

- Multiple emissions of the BDMPSZ type : multiple soft scattering

# Radiative transverse momentum broadening

- Emissions triggered by a **single scattering** are important as well:
  - unusually hard emissions, with very short formation time  $\omega/k_{\perp}^2$



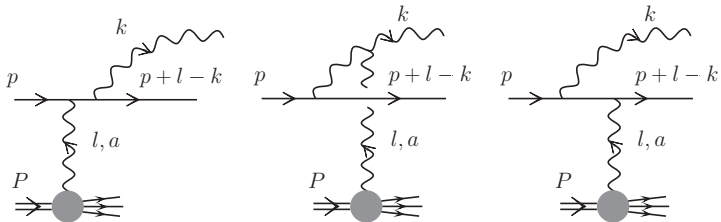
- Gunion–Bertsch spectrum  $\implies$  **power tail at high  $k_{\perp}$**

$$\omega \frac{dN}{d\omega d^2\mathbf{k}} \simeq \frac{\alpha_s N_c}{\pi^2} \frac{\hat{q}L}{k_{\perp}^4} \quad (\text{N.B. : linear in } \hat{q})$$

- Via their recoil, such high- $k_{\perp}$  emissions contribute to the momentum broadening of the emitter  $\implies$  **radiative correction to  $\hat{q}$**

# Radiative transverse momentum broadening

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  - unusually hard emissions, with very short formation time  $\omega/k_{\perp}^2$



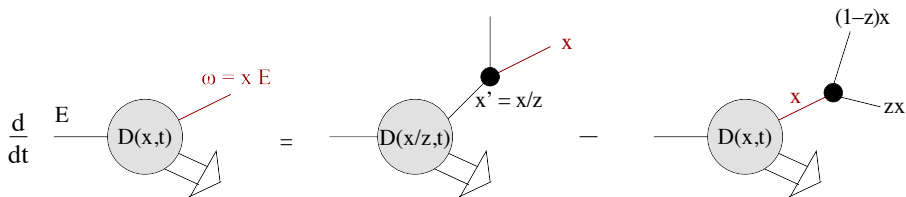
- Formally NLO but **enhanced by a double-log** (*Liou, Mueller, Wu, 13*)

$$\langle p_{\perp}^2 \rangle_{\text{RAD}} = \int_{\omega, \mathbf{k}} k^2 \frac{dN}{d\omega d^2\mathbf{k}} \sim L \alpha_s \hat{q} \int \frac{d\omega}{\omega} \int \frac{d^2 k_{\perp}}{k_{\perp}^4} \equiv L \Delta \hat{q}$$

- Also visible in our formalism: *see next talk by Fabio Dominguez*

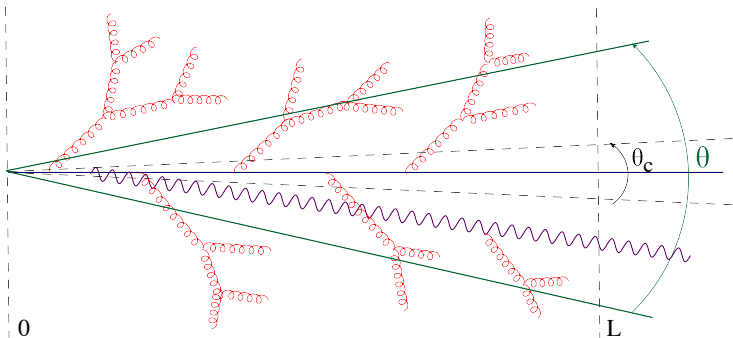
# Energy flow within the cascade

- From now on: focus on the **energy flow** alone (integrate over  $k_{\perp}$ )
- Rate equation for the gluon spectrum:  $D(x, t) \equiv x \frac{dN}{dx}$  ( $x = \omega/E$ )



- $t \rightarrow t + dt$  : one additional branching with **splitting fraction**  $z$
- $\partial D / \partial t = \text{Gain} - \text{Loss}$  (obvious for independent branchings)
  - *Baier, Mueller, Schiff, Son, 2001 : thermalization*
  - *Jeon, Moore, 2003 : jet quenching  $\implies$  MARTINI*
- Numerical solution (*Jeon, Moore*): **energy loss** by the leading particle
- Our focus: energy flow towards **soft quanta** ( $x \ll 1$ ) & **large angles**

# Out-of-cone energy



- How much of the jet energy goes outside a given angle  $\theta$  ?
  - ▷ exploit the correlation between energy ( $x$ ) and the emission angle ( $\theta_f$ )
  - ▷ after emission, the angles are further enhanced by collisions
- A new, efficient, mechanism for angular spreading : wave turbulence

# The rate equation

*J.-P. Blaizot, E. I., Y. Mehtar-Tani, PRL 111, 052001 (2013)*

$$\frac{\partial D(x, t)}{\partial t} = \int dz \left[ 2\mathcal{K} \left( z, \frac{x}{z} \right) D \left( \frac{x}{z}, t \right) - \mathcal{K}(z, x) D(x, t) \right]$$

$$\mathcal{K}(z, x) \equiv \frac{1}{\sqrt{x}} \frac{P_{gg}(z)}{\sqrt{z(1-z)}}, \quad t = \alpha_s \sqrt{\frac{\hat{q}}{E}} L$$

- Compare to DGLAP equation:  $\mathcal{K}_{\text{DGLAP}} = P_{gg}(z) \sim 1/[z(1-z)]$ 
  - emission kernel depends not only upon the splitting fraction  $z$ , but also upon the parent energy  $x$  (LPM effect)
  - stronger singularities at  $z = 0$  and  $z = 1$  (exponent 3/2 vs. 1)
- DGLAP is controlled by **asymmetric splittings** :  $z \ll 1$  or  $1 - z \ll 1$
- For the medium-induced radiation, one would expect this bias to be even stronger (since stronger singularities) ...

# The rate equation

*J.-P. Blaizot, E. I., Y. Mehtar-Tani, PRL 111, 052001 (2013)*

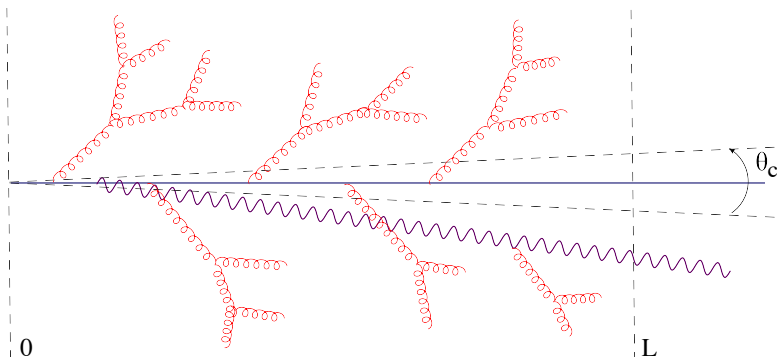
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- DGLAP is controlled by **asymmetric splittings** :  $z \ll 1$  or  $1 - z \ll 1$
- For the medium-induced radiation, one would expect this bias to be even stronger (since stronger singularities) ...
- ... but this is actually **not** the case !

# Quasi-democratic branching

- Medium-induced branchings are quasi-democratic ( $z \sim 1/2$ )



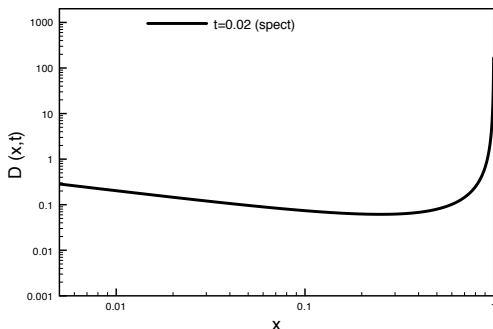
- the leading particle ( $x \sim 1$ ) emits mostly soft gluons ( $z \ll 1$ )
- the subsequent branchings of these soft gluons are quasi-democratic
- When  $x \ll 1$ , the emission probability is of  $\mathcal{O}(1)$  already for  $z \sim 1/2$



# Small time behaviour

- Initial condition:  $D(x, t = 0) = \delta(x - 1)$  (leading particle)
- First iteration  $\implies$  **BDMPSZ spectrum** (by construction)

$$D^{(1)}(x, t) \simeq \frac{t}{\sqrt{x}} = \alpha_s \frac{L}{\tau_f(\omega)} \quad (\text{for } x \ll 1)$$



- The leading particle populates via **direct radiation** all the bins at  $x < 1$ , and **preponderantly those at small  $x \ll 1$**

# Small time behaviour

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- First iteration  $\implies$  **BDMPSZ spectrum** (by construction)

$$D^{(1)}(x, t) \simeq \frac{t}{\sqrt{x}} = \alpha_s \frac{L}{\tau_f(\omega)} \quad (\text{for } x \ll 1)$$

- Energy fraction radiated at large angles  $\theta_f(x) > \theta_0$ , or small  $x < x_0$

$$\mathcal{E}^{(1)}(\theta_f > \theta_0, t) = \int_0^{x_0} dx D^{(1)}(x, t) \simeq 2t\sqrt{x_0}, \quad \theta_0 \propto \frac{1}{x_0^{3/4}}$$

- the larger  $\theta_0 \Rightarrow$  the smaller  $x_0 \Rightarrow$  the smaller the energy loss
- BDMPS radiation by the leading particle is not very efficient in transporting the jet energy towards large angles 😞
- Will the situation improve after including **multiple branching** ?

# A fake scenario inspired by DGLAP

- Via successive branchings, gluons fall at smaller and smaller values of  $x$
- Naively, **energy conservation** seems to imply the sum-rule

$$\int_0^1 dx D(x, t) = 1 \quad \text{for any } t$$

- 'all the energy is stored in the spectrum'
- this is indeed what happens for the DGLAP evolution
- If that was true, then with increasing  $t$  the spectrum should become **steeper and steeper at small  $x$** , but in such a way to remain **integrable**
- The energy fraction contained in the modes with  $x < x_0$ , that is,

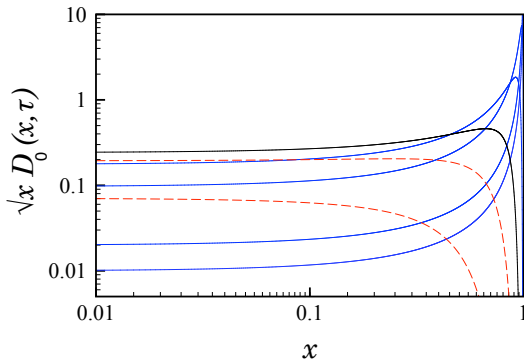
$$\mathcal{E}(x < x_0, t) \equiv \int_0^{x_0} dx D(x, t)$$

- would be dominated by the upper limit  $x_0$
- would rapidly vanish when  $x_0 \rightarrow 0$
- The out-of-cone energy would rapidly decrease with increasing the angle ☹️

# The scaling spectrum

- However, this is **not** what actually happens ! Rather, one finds

$$D(x \ll 1, t) \simeq \frac{t}{\sqrt{x}} e^{-\pi t^2} \quad \& \quad \int_0^1 dx D(x, t) = e^{-\pi t^2}$$



- The spectrum does **not** get steeper at small  $x$  !
- The energy sum-rule is **not** obeyed !

# Where does the energy flow ?

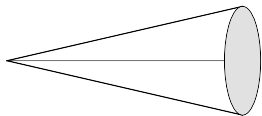
- The 'scaling' spectrum  $\frac{1}{\sqrt{x}}$  : a **fixed point** for the branching process
  - precise cancellation between 'gain' and 'loss' terms at any  $x \ll 1$
- Via successive branchings, the energy **flows** from large  $x$  to small  $x$ , **without accumulating in any bin  $x > 0$** 
  - formally, it accumulates into a 'condensate' at  $x = 0$
  - physically, it goes below  $x_{\text{th}} = T/E \ll 1$ , meaning it **thermalizes**
- In principle, an **arbitrary large fraction of the jet energy** can flow out to **arbitrarily large angles**

$$E_{\text{flow}}(t) \equiv 1 - \int_0^1 dx D(x, t) = 1 - e^{-\pi t^2}$$

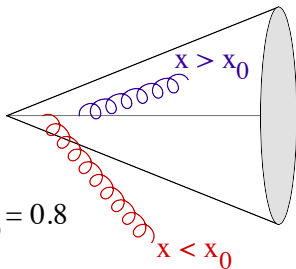
- In practice,  $t = \alpha_s \sqrt{2\omega_c/E} \sim 0.3$  is not that large ( $E = 100$  GeV)
  - $1 - e^{-\pi t^2} \sim 0.25 \Rightarrow$  about 25% of the energy is lost at large angles
  - ... independently of the jet angle & and the medium temperature

# Energy flow at large angles

- The energy inside the jet is **only weakly increasing with the jet angular opening  $R_0$** , within a wide range of values for  $R_0$  😊



$$R_0 = 0.3$$

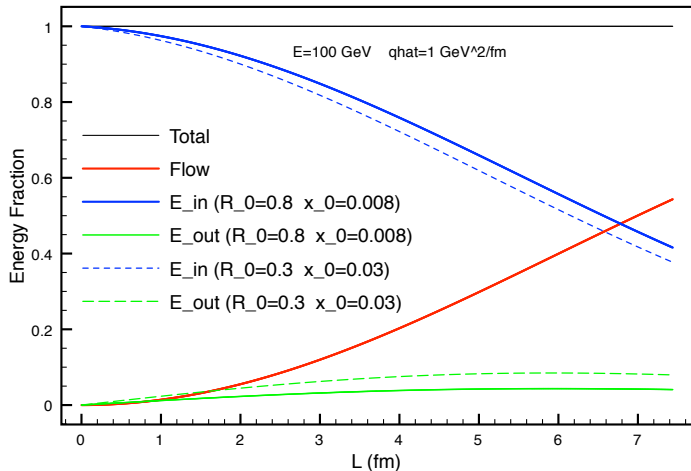


$$R_0 = 0.8$$

- The energy inside the jet  $E_{\text{in}}$  : the energy in the spectrum at  $x > x_0$
- The energy outside the jet :  $E_{\text{out}}(x_{\text{th}} < x < x_0) + E_{\text{flow}} \simeq E_{\text{flow}}$
- $E_{\text{flow}}$  : independent of  $R_0$ ,  $x_{\text{th}}$ , and the original energy  $E$

$$E_{\text{flow}} \simeq v \alpha_s^2 \hat{q} L^2 \quad (\sim 20 \text{ GeV for } L = 5 \text{ fm})$$

# Energy flow at large angles



- Qualitative and semi-quantitative agreement with ATLAS and CMS
- **No fit, rather a first-principle calculation**

# Wave turbulence

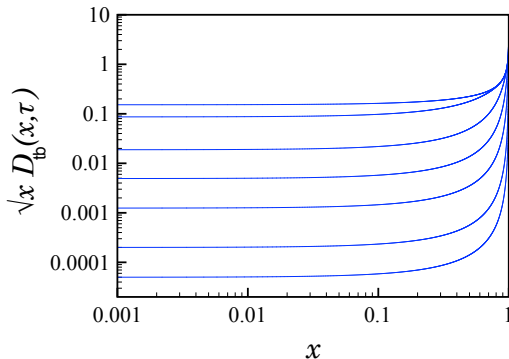
- A branching process in which the **energy flux is independent of  $x$** 
  - the energy **uniformly** flows through the entire spectrum (like water through a pipeline)
- It requires interactions to be **quasi-local in energy**
  - our 'democratic branching'
  - generally associated with scalar field theories, but not with gauge ones
- It implies the existence of a **scaling spectrum  $1/k^\nu$**  where the exponent  $\nu$  is independent of time ('Kolmogorov power')
  - for medium-induced jet evolution:  $\nu = 1/2$
- Generally formulated for steady situations, in which there is a 'source' (at high frequency) and a 'sink' (at low frequency)
  - replace the 'leading particle' by a steady source  $\propto \delta(x - 1) \dots$
  - ... and the medium by a 'sink' at  $x = x_{\text{th}}$



# The usual turbulence set-up

- Steady source at  $x = 1$  and sink at  $x = x_{\text{th}}$  (here  $x_{\text{th}} = 0$ )

$$D_{\text{tb}}(x, t) = \frac{1}{2\pi\sqrt{x(1-x)}} \left( 1 - e^{-\pi\frac{t^2}{1-x}} \right)$$



- The spectrum approaches a steady shape when  $\pi t^2 \gtrsim 1$

# With due respect to Wikipedia ...



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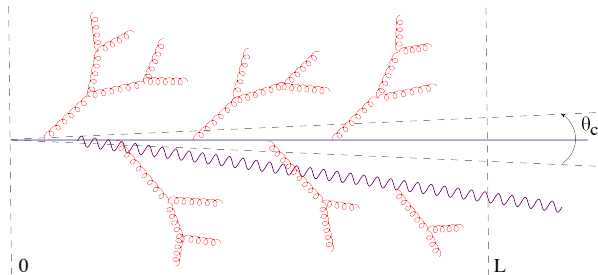
## Wave turbulence

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- *Wave turbulence is a set of waves deviated far from thermal equilibrium. Such state is accompanied by dissipation. It is either decaying turbulence or requires external source of energy to sustain it. Examples are waves on a fluid surface excited by winds or ships, and waves in plasma excited by electromagnetic waves etc. The wave system can be described by kinetic equations and their stationary solutions called Kolmogorov-Zakharov (KZ) energy spectra. They have the form  $1/k^\nu$ , with  $k$  the wavenumber and  $\nu$  a positive constant depending on the specific wave system. The form of KZ-spectra does not depend on the initial magnitude of the total energy or on the details of initial energy distribution over the modes. Only the fact the energy is conserved at some inertial interval is important.*
- *V. Zakharov, V. L'vov, and G. Falkovich, Kolmogorov Spectra of Turbulence, Wave Turbulence (Springer-Verlag, 1992)*

# Conclusions

- 3+1 space-time picture for medium-induced jet evolution in pQCD
  - hard emissions at small angles (energy loss by leading particle,  $R_{AA}$ )
  - multiple soft branchings leading to turbulent flow (di-jet asymmetry)

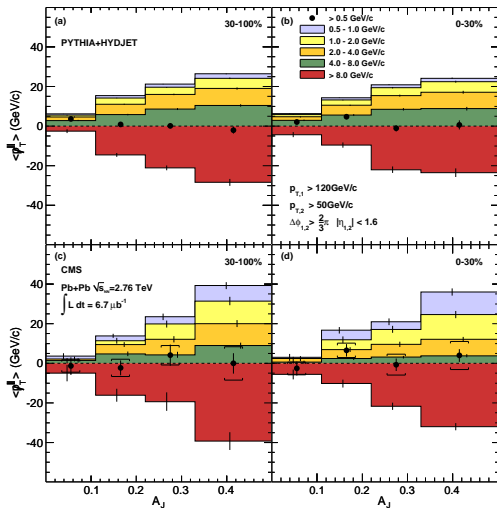


- Anomalously large value for  $\hat{q}$  due to pQCD evolution  
(see the next talk by Fabio Dominguez)
- Encouraging estimates, but still a long way to phenomenology
  - to be implemented in MC event generators like MARTINI

# No missing energy ! *(CMS, arXiv:1102.1957)*

- ... but a pronounced difference in the distribution of the total energy in bins of  $\omega \equiv p_T$  and in the angle w.r.t. the jet axis

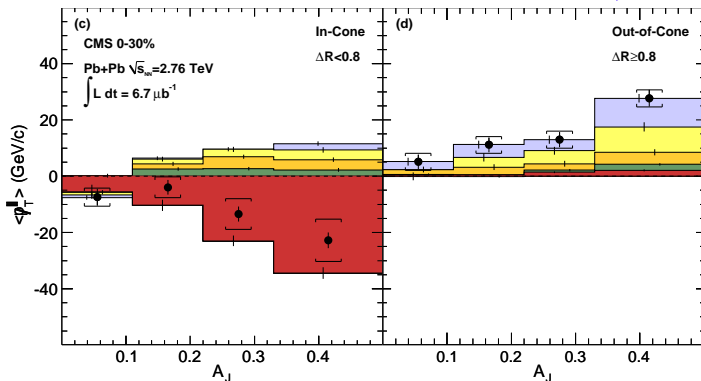
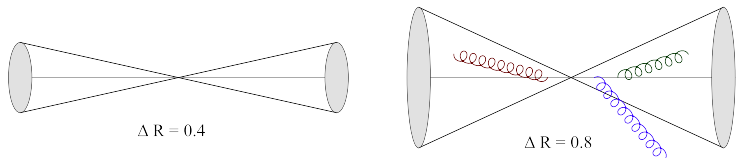
- $p_T^{\parallel}$  : projection of the (transverse) energy along the jet axis
- $p_T^{\parallel} < 0$  : same hemisphere as the trigger jet
- $p_T^{\parallel} > 0$  : same hemisphere as the secondary jet
- all hadrons with  $p_T > 0.5$  GeV are measured



- Excess of soft quanta ( $\leq 4$  GeV) in the hemisphere of secondary jet

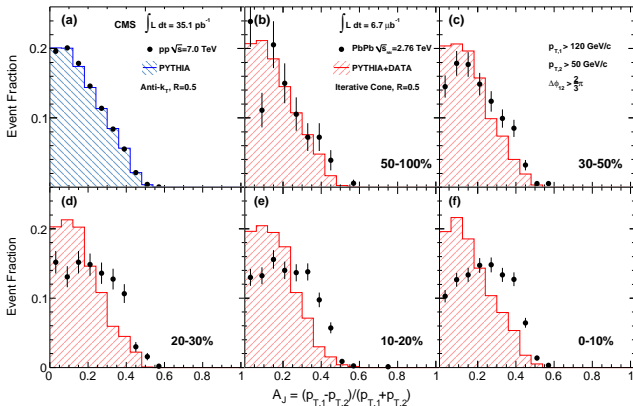
# In-out asymmetry

- Increase the angular opening  $\Delta R$  of the jet



- The soft energy in excess is found at very large angles

# Di-jet asymmetry : $A_J$ (CMS)

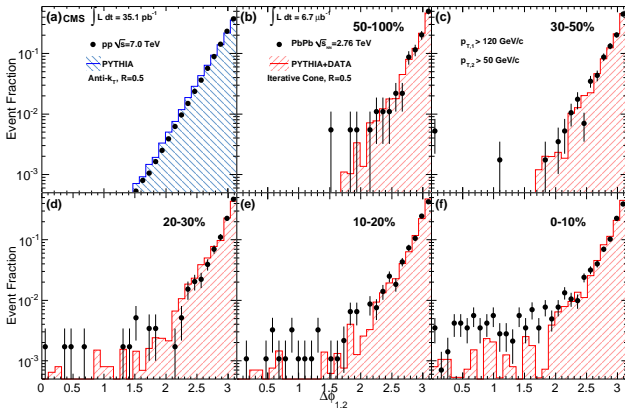


- Event fraction as a function of the di-jet energy imbalance in **p+p (a)** and **Pb+Pb (b-f)** collisions for different bins of centrality

$$A_J = \frac{E_1 - E_2}{E_1 + E_2} \quad (E_i \equiv p_{T,i} = \text{transverse energy})$$

- Additional energy loss of **20 to 30 GeV** due to **the medium**

# Di-jet asymmetry : $\Delta\phi$ (CMS)

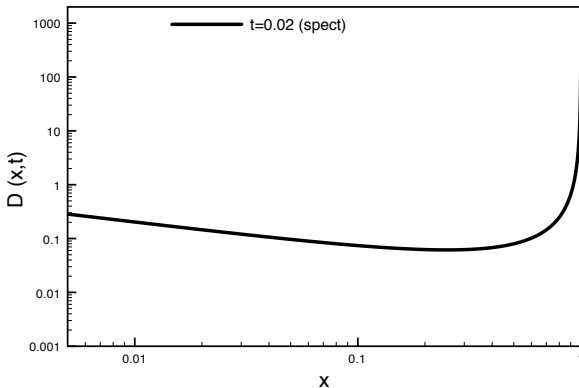


- Event fraction as a function of the azimuthal angle  $\Delta\phi$
- Typical event topology: still a pair of **back-to-back** jets
- The **secondary jet** loses energy without being deflected
- The additional in-medium radiation is **relatively soft**

# Would-be DGLAP

- One branching  $\implies$  **BDMPSZ spectrum** by the leading particle

$$D^{(1)}(x, L) \simeq \alpha_s \frac{L}{\tau_f(\omega)} = \frac{t}{\sqrt{x}} \quad (t = L \text{ in appropriate units})$$



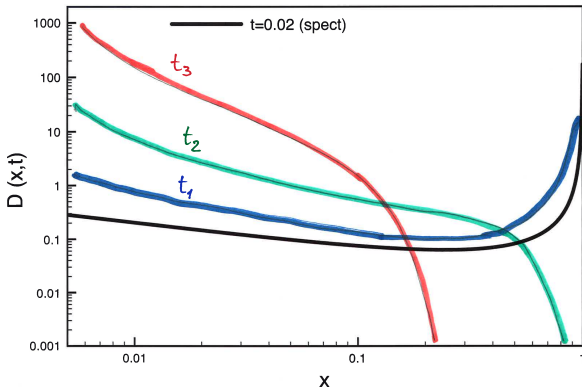
- What happens when including multiple branchings ?



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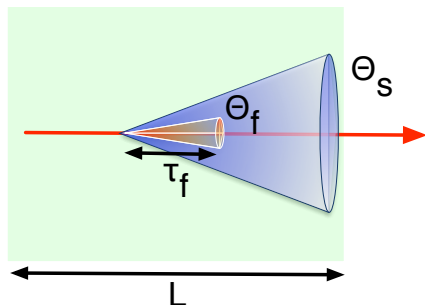
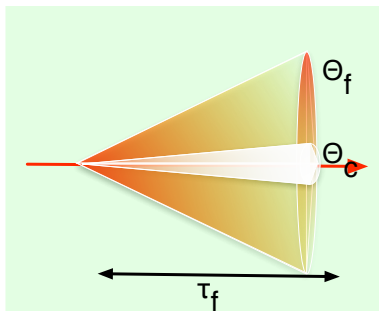


- **DGLAP-like evolution:** the spectrum is depleted at large  $x$  but becomes steeper at small  $x$  :  $\int_0^1 dx D(x, t) = 1$

# Formation time ( $\tau_f$ ) & angle ( $\theta_f$ )

$$\tau_f \simeq \sqrt{\frac{\omega}{\hat{q}}} \quad \theta_f \equiv \frac{\Delta k_{\perp}}{\omega} \simeq \left(\frac{\hat{q}}{\omega^3}\right)^{1/4}$$

- Soft gluons (small  $\omega$ ) : short formation times & large emission angles
- Maximal  $\omega$  for this mechanism :  $\tau_f \simeq L \Rightarrow \omega_c = \hat{q}L^2$

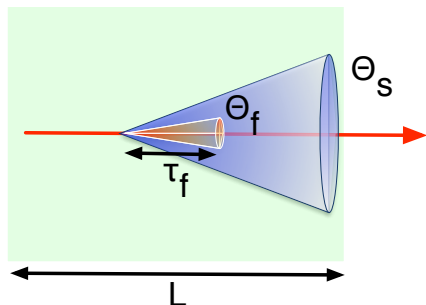
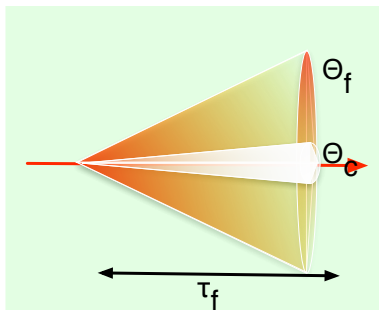


- Soft gluons ( $\omega \ll \omega_c$ ) have  $\tau_f \ll L$  &  $\theta_f \gg \theta_c$

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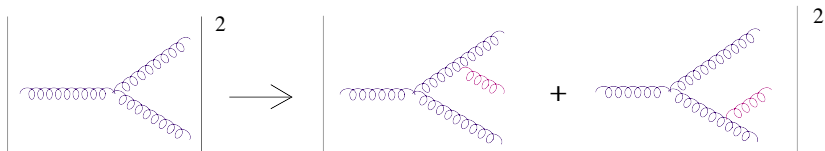
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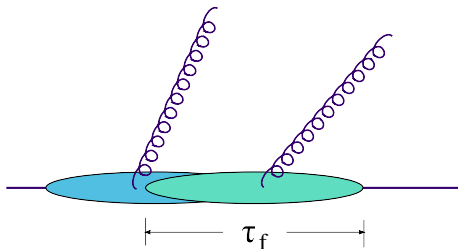
- After emission, the angle can further increase via medium rescattering

# Multiple emissions : difficulties

- New complications specific to multiple emissions:
  - interference effects between different emitters



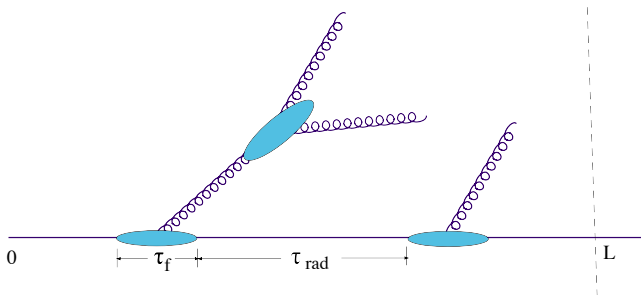
- overlapping between successive emissions



# A classical branching process

- By the time of their emission, the daughter gluons have already lost their color coherence  $\Rightarrow$  they cannot interfere with each other
- Subsequent emissions do typically not overlap with each other :

$$P(\tau) \sim \alpha_s \frac{\tau}{\tau_f} \sim 1 \implies \tau_{\text{rad}} \sim \frac{\tau_f}{\alpha_s} \gg \tau_f$$



- No interference, no overlapping  $\implies$   
successive emissions are effectively independent from each other

# With due respect to Wikipedia ...



Article [Talk](#)

## Jet quenching

From Wikipedia, the free encyclopedia

- *Collisions of ultra-relativistic heavy-ion beams create a hot and dense medium comparable to the conditions in the early universe, and then these jets interact strongly with the medium, leading to a **marked reduction of their energy**. This energy reduction is called **jet quenching**.*



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- A familiar concept, hiding a complex reality



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- **A rather elusive concept, with an unambiguous mathematical signature**

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