Energy Loss at "NLO" in Hot QCD

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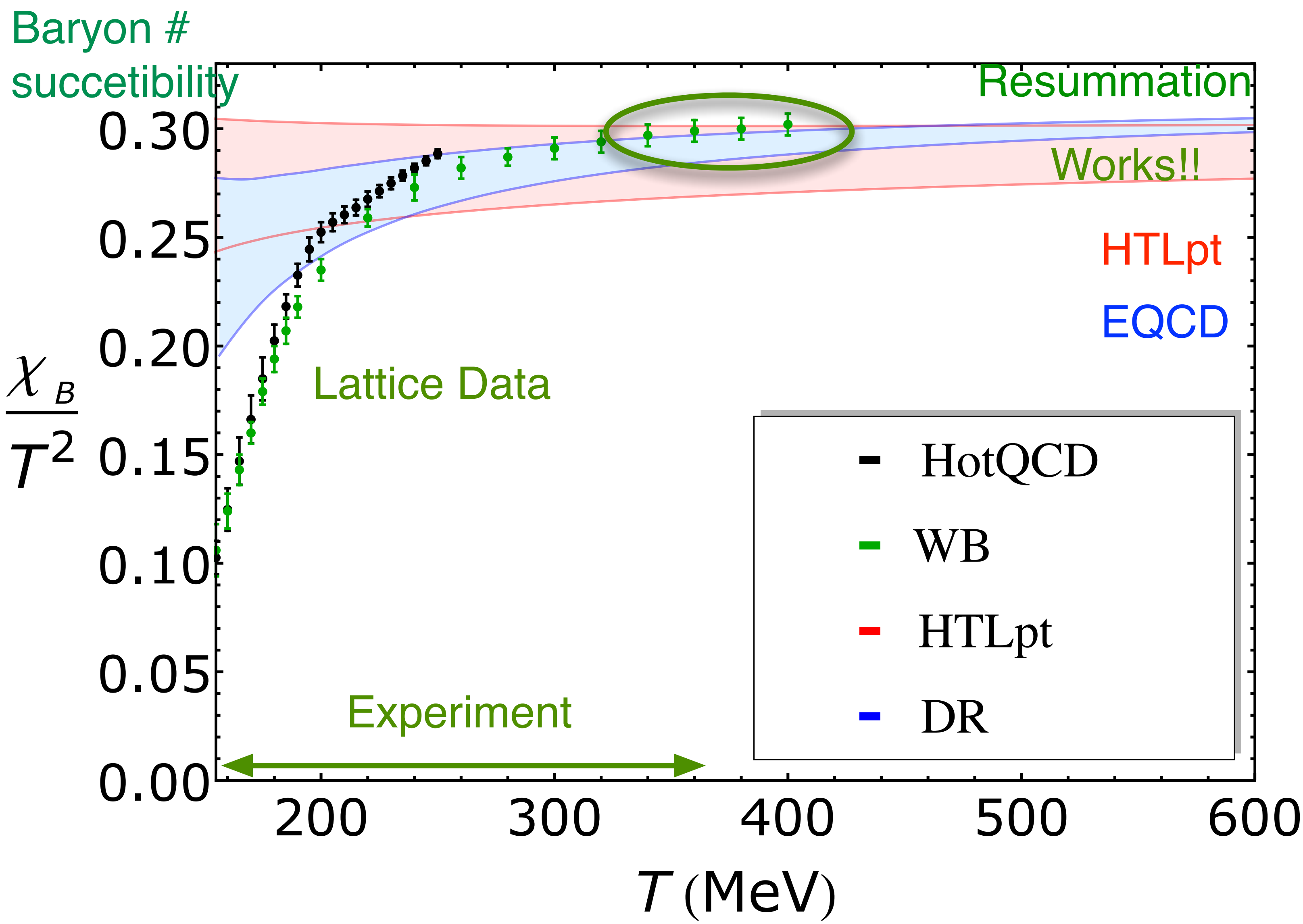
- Jacopo Ghiglieri, J. Hong, A. Kurkela, G. Moore, DT, JHEP
- Jacopo Ghiglieri, H. Gervais, G. Moore, B. Schenke, DT, in progress

Outline: Energy loss and transport in weakly coupled plasmas at "NLO"

1. Philosophy of weakly coupled calculations – there is only one right answer . . .
   (a) Collisional vs. radiative loss
   (b) Corrections to collinear formalism
   (c) Relation between drag (or $\hat{e}$) and radiative loss
2. Heavy Quark Drag (diffusion): S. Caron-Huot and G. Moore, JHEP
3. Energy loss of light quarks and gluons Jacopo Ghiglieri, H. Gervais, G. Moore, DT, in progress
4. Thermal photons Jacopo Ghiglieri, J. Hong, A. Kurkela, G. Moore, DT, JHEP

Perturbation theory can work – from Borsanyi Quark Matter Talk

- HTLpt from Andersen, Su, Strickland. Dimensional Reduction/EQCD – the Finish Group



Want to compute e-loss and viscosity to similar precision at high T!

Heavy Quark Drag at “NLO”

Computing Heavy Quark Diffusion at NLO

- Write down an equation of motion for the heavy quarks.

$$\frac{dx}{dt} = \frac{p}{M}$$

$$\frac{dp}{dt} = -\underbrace{\eta_D v}_{\text{Drag}} + \underbrace{\xi(t)}_{\text{Random Force}}$$

- The drag and the random force are related

$$\langle \xi_i(t)\xi_j(t')\rangle = \frac{\kappa}{3}\delta_{ij}\,\delta(t-t') \qquad\qquad \eta_D = \frac{\kappa}{2T}$$

$\kappa =$ Mean Squared Momentum Transfer per Time

All parameters are related to the heavy quark diffusion coefficient or κ

Giving the diffusion coefficient a rigorous definition

- Compare the Langevin process to the microscopic theory

Langevin

$$\frac{dp}{dt} = -\eta_D v + \xi(t)$$

Microscopic Theory

$$\frac{dp}{dt} = \mathcal{F}(t, \mathbf{x}) = qE(t, \mathbf{x})$$

- Match the Langevin to the Microscopic Theory

Langevin

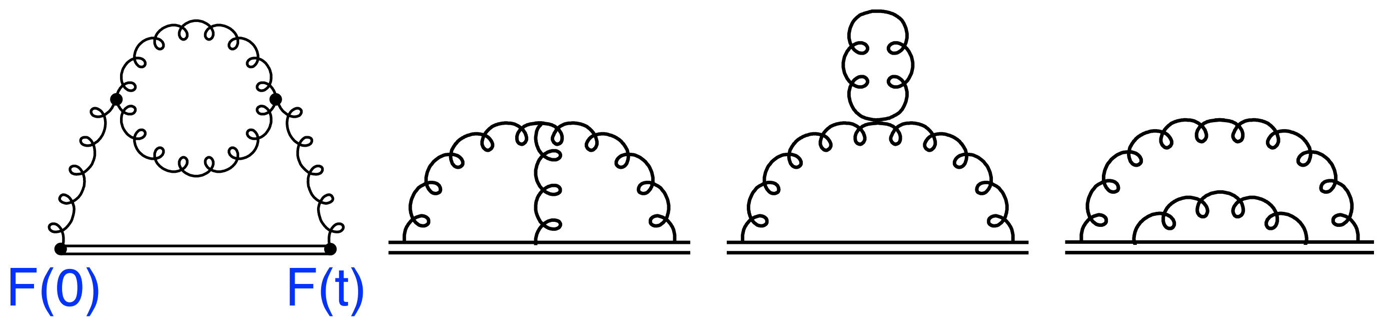
$$\kappa = \int dt \, \langle \xi(t) \, \xi(0) \rangle$$

Microscopic Theory

$$\kappa = \int dt \, \langle \mathcal{F}(t, \mathbf{x}) \mathcal{F}(0, \mathbf{x}) \rangle_{HQ}$$

Momentum Diffusion Coefficient $\leftrightarrow$ Electric Field Correlator

Force-force correlators beyond leading order (Guy D. Moore and Simon-Caron Huot)



Perturbation theory in:

$$g_s \sim \frac{m_D}{T} \qquad \text{NOT} \qquad \alpha_s = \frac{g_s^2}{4\pi}$$

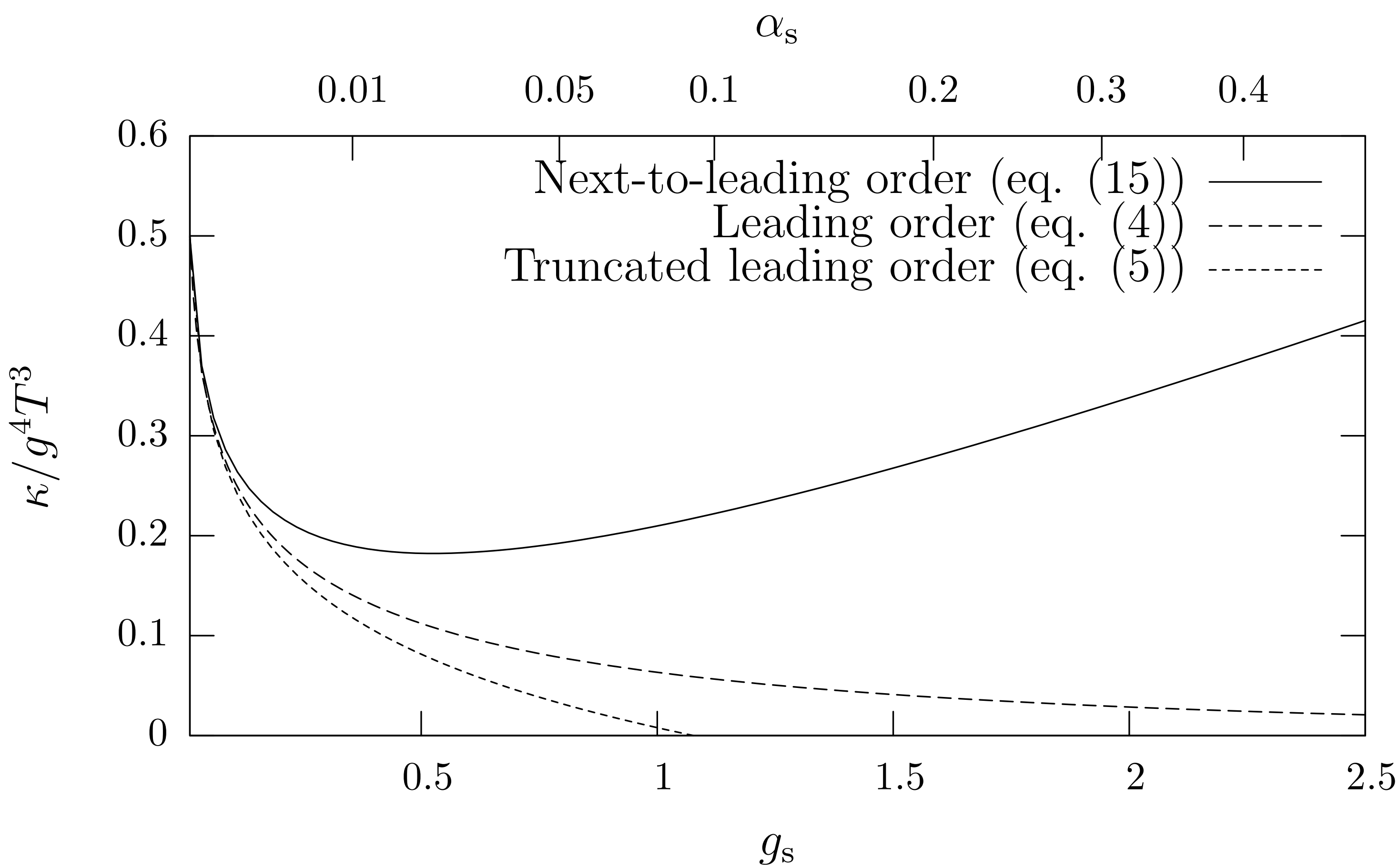
- From from non-linear interactions of thermal classical gauge fields:

$$\text{Force} \;\propto\; n_B \sim \frac{T}{\omega} \sim \frac{1}{g}$$

Schematically we have:

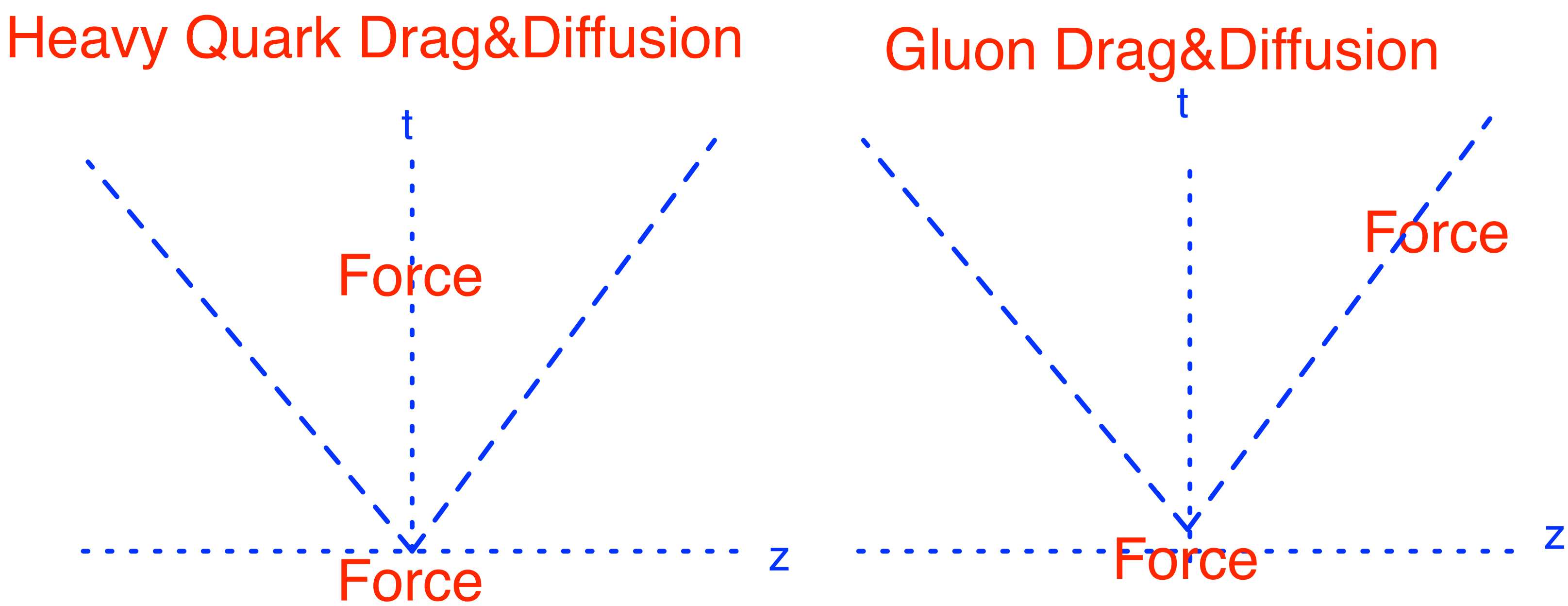
$$\underbrace{\kappa}_{\text{diffusion rate}} = (g^4 T^3)\Big[\underbrace{C_0 \log\left(\frac{T}{m_D}\right) + C_1}_{\text{leading order}} + \underbrace{C_2\, g}_{\text{NLO}}\Big]$$

(Guy D. Moore and Simon Carot-Huot)



Perturbation theory is a disaster for kinetics even for $T = M_Z$.

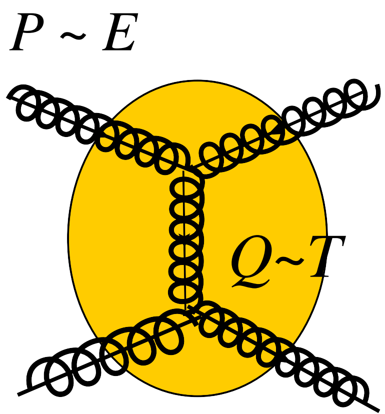
Why?


Heavy Quark Drag&Diffusion
t
Force
z
Force
Gluon Drag&Diffusion
t
Force
z
Force

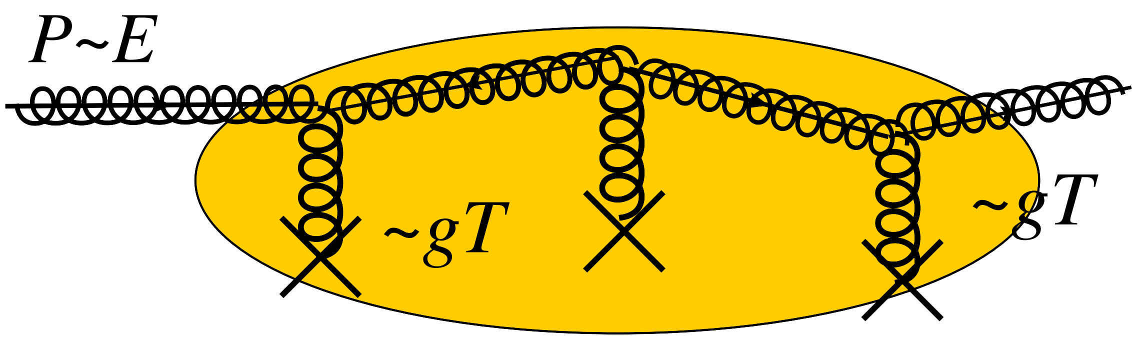
Light quark and gluon e-loss at “NLO”

Three mechanisms for energy loss at LO

1. Hard Collisions: $2 \leftrightarrow 2$



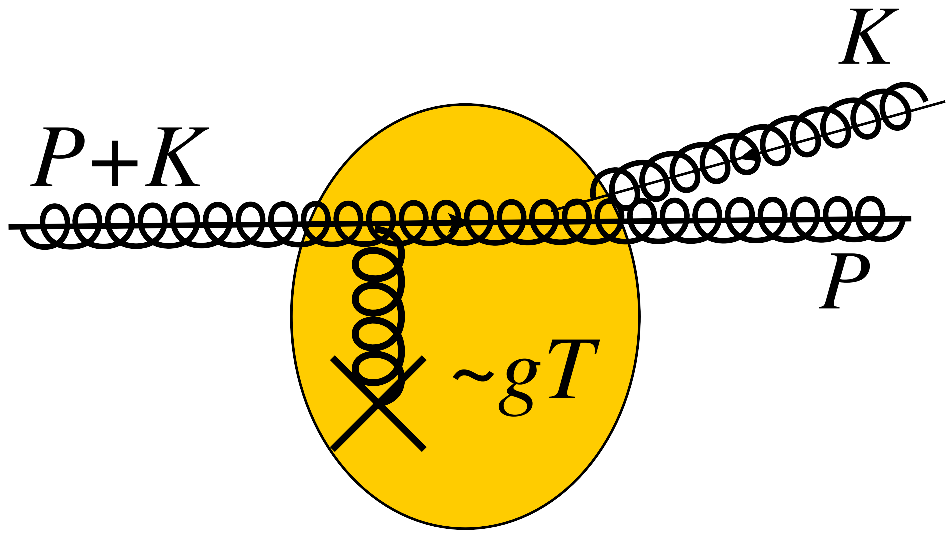
2. Drag: collisions with soft random classical field



$$\frac{dp}{dt} = -\eta(\mu)\hat{\mathbf{v}}$$

3. Brem: $1 \leftrightarrow 2$

- random walk induces collinear bremsstrhalung



- The probability of a transverse kick of momentum $\mathbf{q}_\perp$ from soft fields:

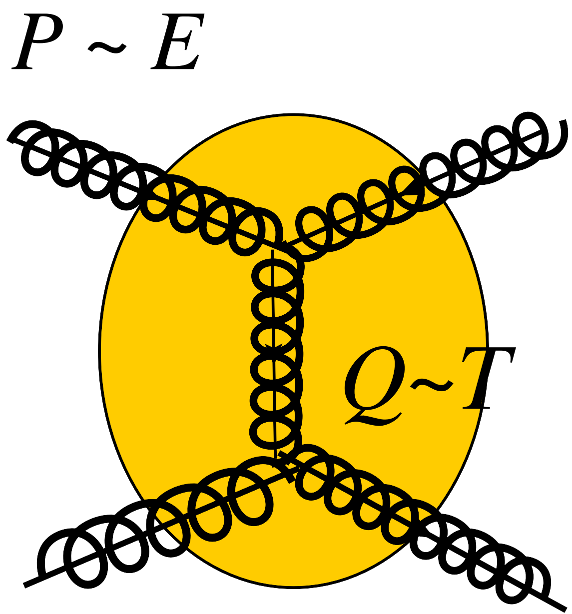
$$C_{LO}[\mathbf{q}_\perp] = \frac{T m_D^2}{q_\perp^2 (q_\perp^2 + m_D^2)}$$

NLO involves corrections to these processes and the relation between them.

Same processes determine the shear viscosity of QCD in high temperature plasma!

Three rates for energy loss at leading order:

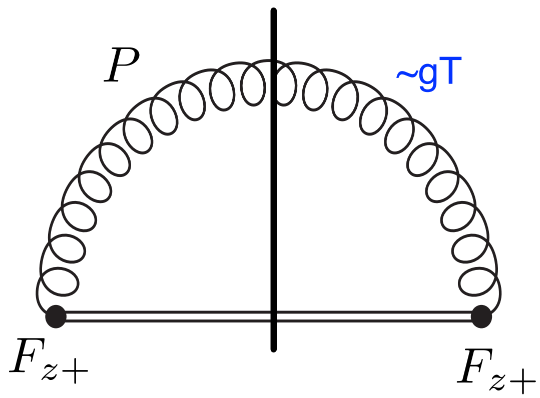
1. Hard Collisions – a $2 \leftrightarrow 2$ processes



$$[\partial_t + v_{\mathbf{k}} \cdot \partial_{\mathbf{x}}]\, f_{\mathbf{k}} = C_{2\leftrightarrow 2}[\mu_\perp]$$

Total $2 \leftrightarrow 2$ scattering rate depends logarithmically on the cutoff

2. Drag: A longitudinal force-force correlator along the light cone



$$[\partial_t + v_{\mathbf{k}} \cdot \partial_{\mathbf{x}}] \, f_{\mathbf{k}} = \eta(\mu) \mathbf{v} \cdot \frac{\partial f_{\mathbf{k}}}{\partial \mathbf{k}}$$

- Evaluate longitudinal force-force with hard thermal loops + sum-rules

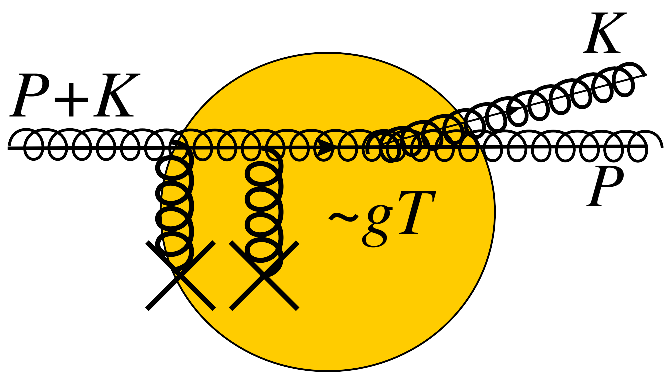
$$\eta(\mu) \propto g^2 C_A \int^{\mu} \frac{d^2 \mathbf{p}_T}{(2\pi)^2} \int \frac{dp_+ dp_0}{(2\pi)^4} \underbrace{\langle F_{z+}(P) F_{z+} \rangle \, 2\pi\delta(p_+)}_{\text{evaluate with sum-rule } p_0 \to \infty}$$

$$\propto g^2 C_A \int^{\mu} \frac{d^2 \mathbf{p}_T}{(2\pi)^2} \frac{m_\infty^2}{p_T^2 + m_\infty^2}$$

$$\propto g^2 C_A \frac{m_\infty^2}{4\pi} \log(\mu^2 / m_\infty^2)$$

The $\mu-$dependence of the drag cancels against μ-dependence of the $2 \to 2$ rate

3. Collinear Bremsstrhalung – a $1 \leftrightarrow 2$ processes



$$[\partial_t + v_{\mathbf{k}} \cdot \partial_x]\, f_{\mathbf{k}} = \underbrace{C_{1\leftrightarrow 2}}$$

LPM + AMY and all that stuff!

The bremsstrhalung rate is proportional to the rate of transverse momentum kicks, $C_{LO}[\mathbf{q}_\perp]$:

$C_{LO}[q_\perp] =$ in medium scattering rate with momentum $\mathbf{q}_\perp$

- Need to compute <u>transverse</u> force-force correlators along the light cone.

$$q_\perp^2 C_{LO}[\mathbf{q}_\perp] = \int \frac{dq_+ dq_0}{(2\pi)^2} \underbrace{\langle F_{i+}(Q) F_{i+}\rangle\, 2\pi\delta(q_+)}$$

evaluate with sum rule at $q_0 = 0$

$$= \frac{T m_D^2}{q_\perp^2 + m_D^2}$$

Summary – the full LO Boltzmann equation:

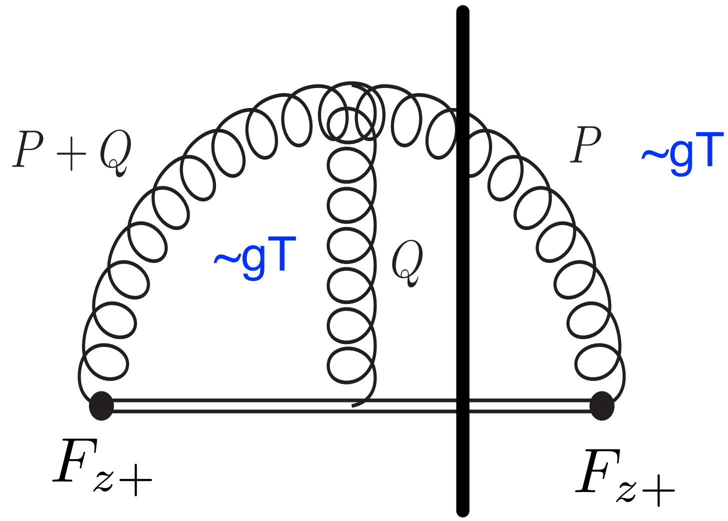
$$[\partial_t + v_{\mathbf{k}} \cdot \partial_{\mathbf{x}}]\, f_{\mathbf{k}} = \eta(\mu)\, \mathbf{v}_{\mathbf{k}} \cdot \frac{\partial f_{\mathbf{k}}}{\partial \mathbf{k}} + C_{2\leftrightarrow 2}[\mu] + C_{1\leftrightarrow 2}$$

The cutoff dependence of the drag cancels against the $2 \to 2$ rate!

$O(g)$ Corrections to Hard Collisions, Drag, Bremm:

1. No corrections to Hard Collisions:

2. Corrections to Drag:

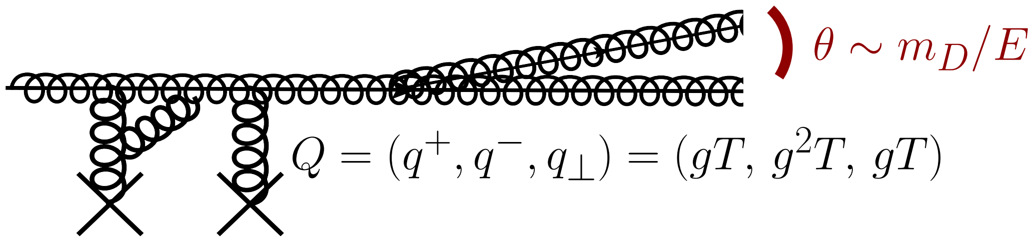


- Nonlinear interactions of soft classical fields changes the force-force correlator
- Doable because of HTL sum rules (light cone causality)

Simon Caron-Huot

3. Corrections to Bremm:

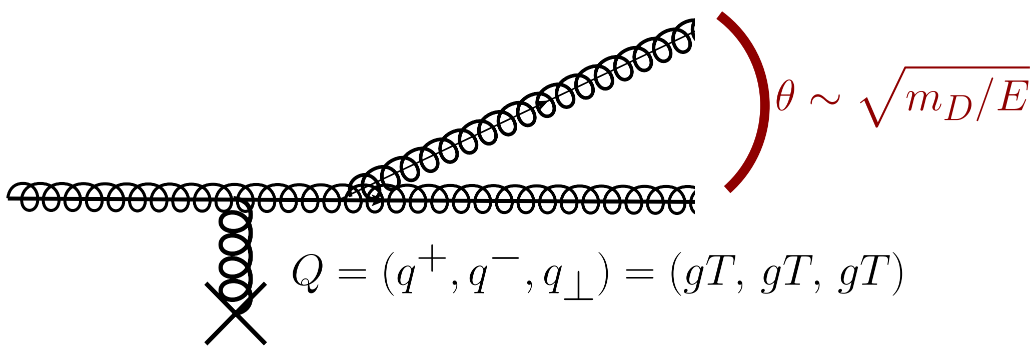
(a) Small angle bremm. Corrections to AMY coll. kernel. (Caron-Huot)



$$C_{LO}[q_\perp] = \frac{Tg^2 m_D^2}{q_\perp^2 (q_\perp^2 + m_D^2)} \to \text{A complicated but analytic formula}$$

(b) Large angle brem and collisions with plasmons.

- Include collisions with energy exchange, $q^- \sim gT$.



The large-angle (semi-collinear radiation) interpolates collisional and rad. loss

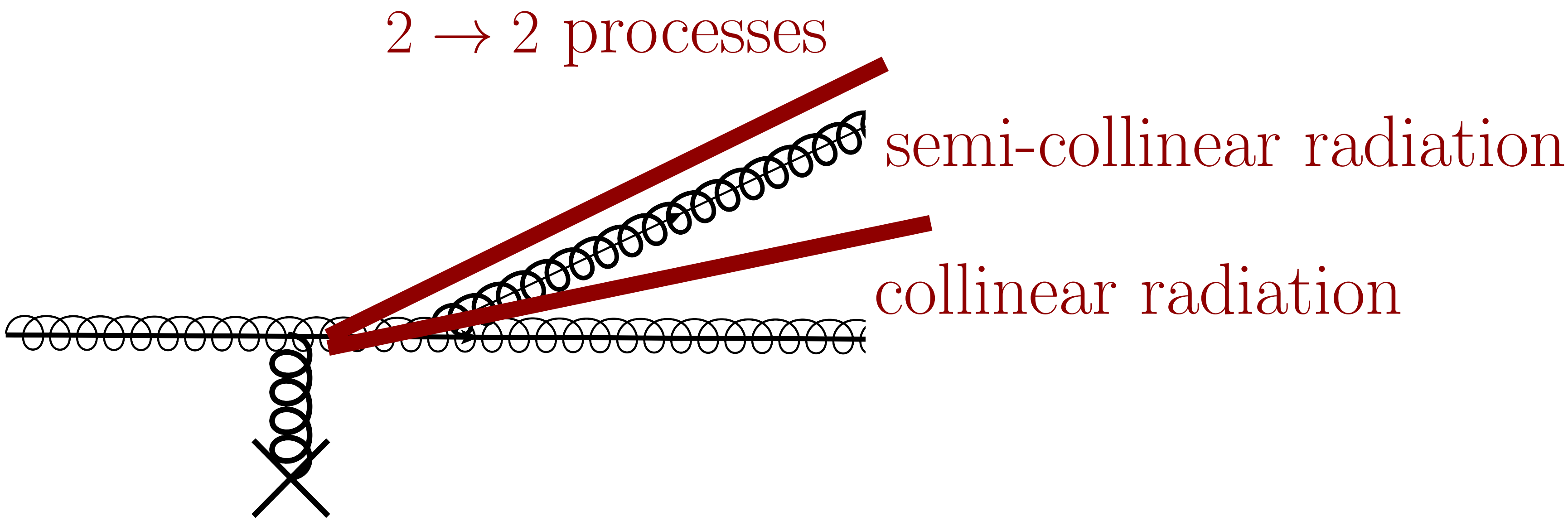
The NLO Boltzmann equation – a preview:

Cutoff dependence cancels

$$\left[\partial_t + v_{\mathbf{k}} \cdot \partial_{\mathbf{x}}\right] f_{\mathbf{k}} = \left(\eta(\mu) + \delta\eta(\mu)\right) \mathbf{v}_{\mathbf{k}} \cdot \frac{\partial f_{\mathbf{k}}}{\partial \mathbf{k}} + C_{2\leftrightarrow 2}[\mu]$$
$$C_{1\leftrightarrow 2} + \delta C_{1\leftrightarrow 2} + C_{\mathrm{semi-coll}}[\mu]$$

The μ-dependence of the drag at NLO cancels the μ-dependence of semi-collinear radiation

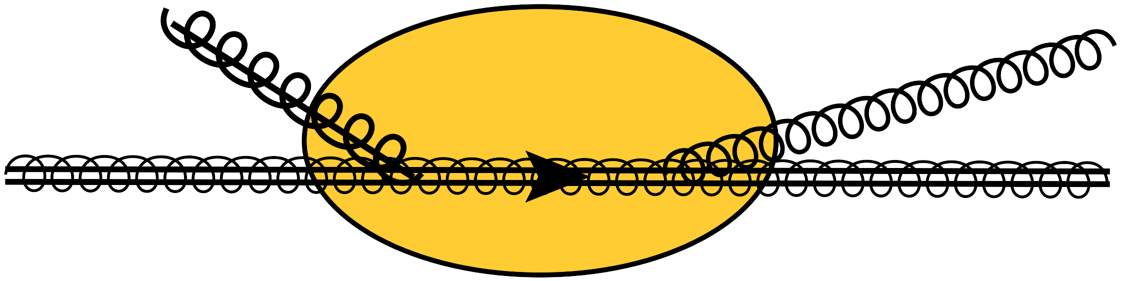
Semi-collinear radiation – a new kinematic window



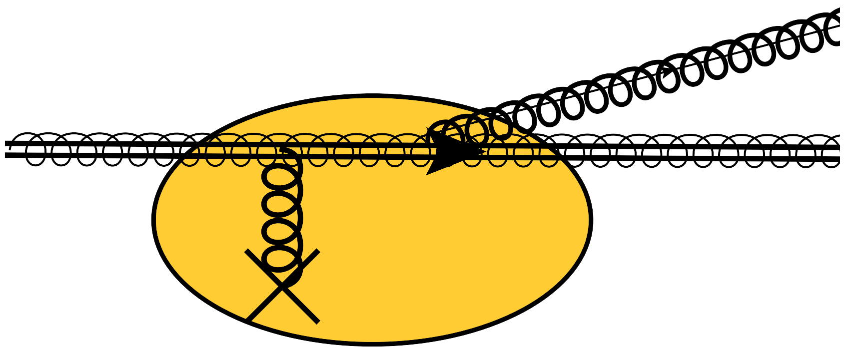
The semi-collinear regime interpolates between brem and collisions

Matching collisions to brem

- When the gluon is hard the $2 \leftrightarrow 2$ collision:

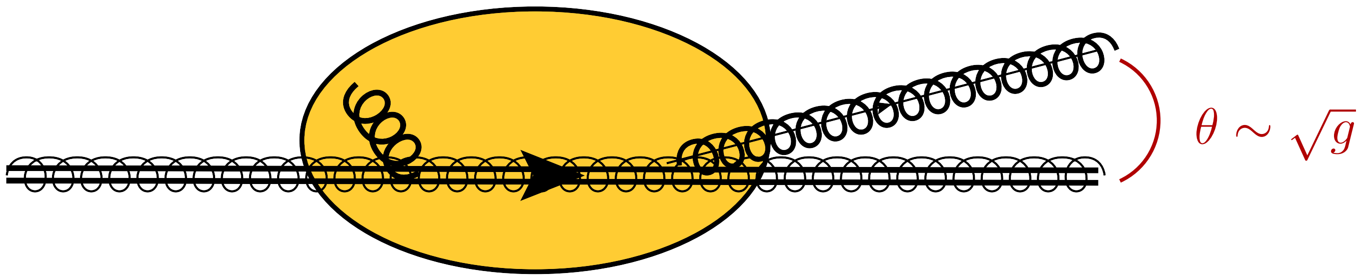


is physically distinct from the wide angle brem

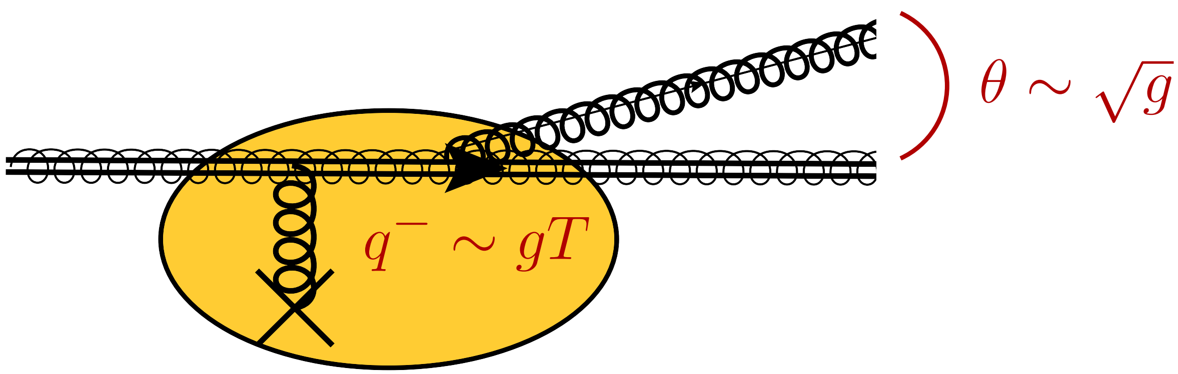


Matching collisions to brem

- When the gluon becomes soft (a plasmon), the $2 \leftrightarrow 2$ collision:



is not physically distinct from the wide angle brem

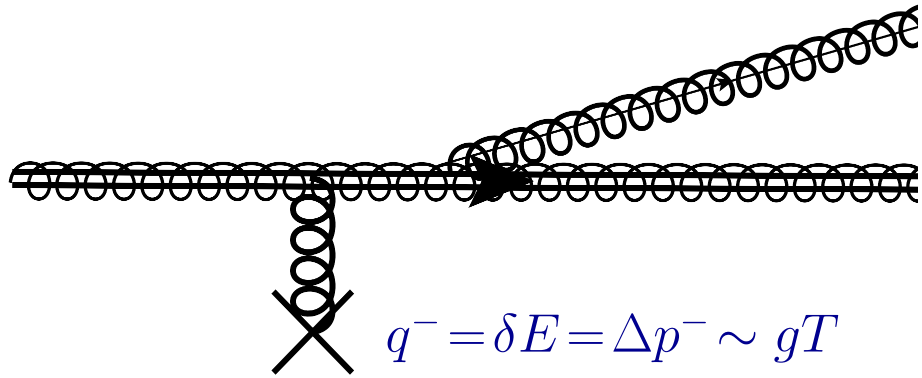


Need both processes

- For harder gluons, $q^- \to T$, this becomes a normal $2 \to 2$ process.
- For softer gluons, $q^- \to g^2T$, this smoothly matches onto AMY.

Brem and collisions at wider angles (but still small!)

- Semi-collinear emission:



- The matrix element is:

$$|\mathcal{M}|^2\,(2\pi)^4\delta^4(P_{\rm tot}) \propto \underbrace{\frac{1+z^2}{z}}_{\text{QCD splitting fcn}} \int_Q \frac{1}{(q^-)^2} \underbrace{\langle F_{i+}(Q)\,F_{i+}\rangle}_{\text{scattering-center}}\; 2\pi\delta(q^- - \delta E)$$

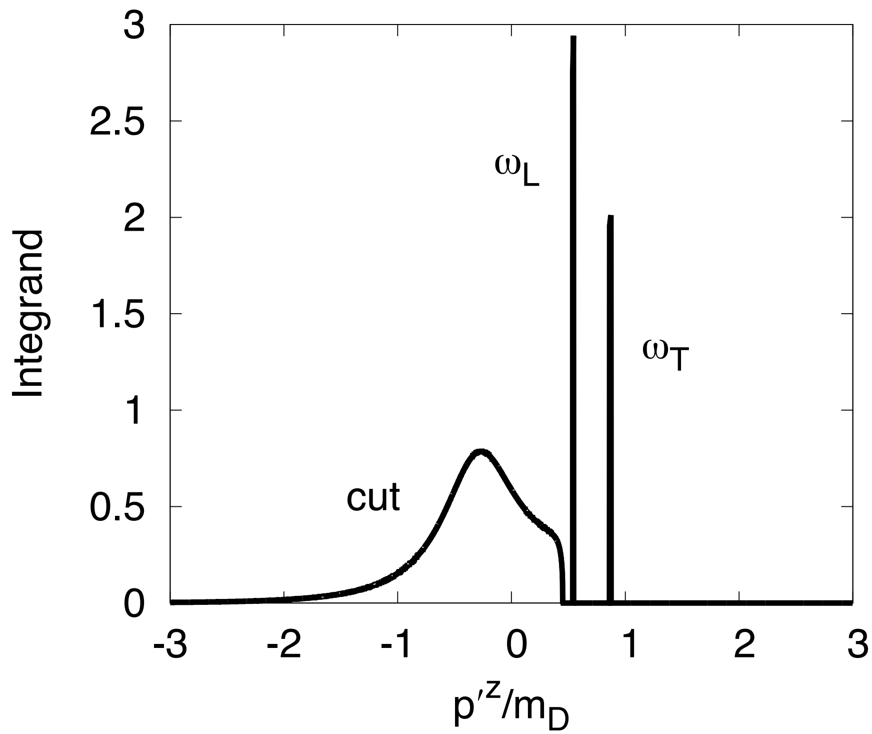
All of the dynamics of the scattering center in a single matrix element $\langle F_{i+}(Q) F_{i+}\rangle$

The scattering center:

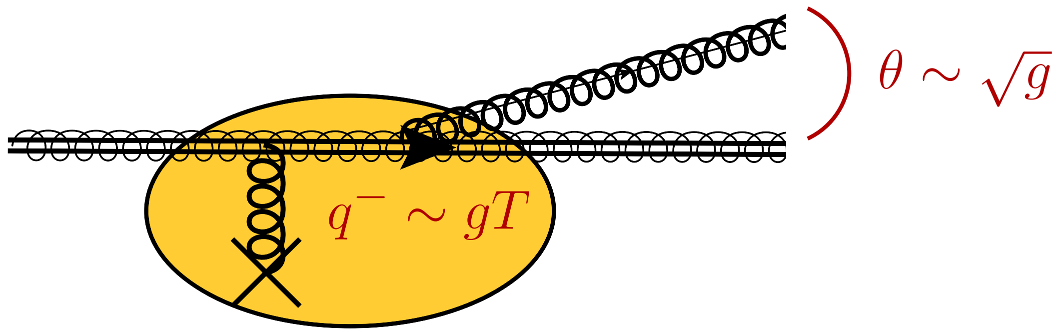
$$C[\mathbf{q}_\perp, \delta E] = \int_Q \frac{1}{(q^-)^2} \langle F_{i+}(Q) F_{i+} \rangle \; 2\pi\delta(q^- - \delta E)$$

1. Soft-correlator has wide angle brem = cut

2. And plasmon scattering = poles



Finite energy transfer sum-rule



- The AMY collision kernel $C[q_\perp]$ involves

Aurenche, Gelis, Zakarat

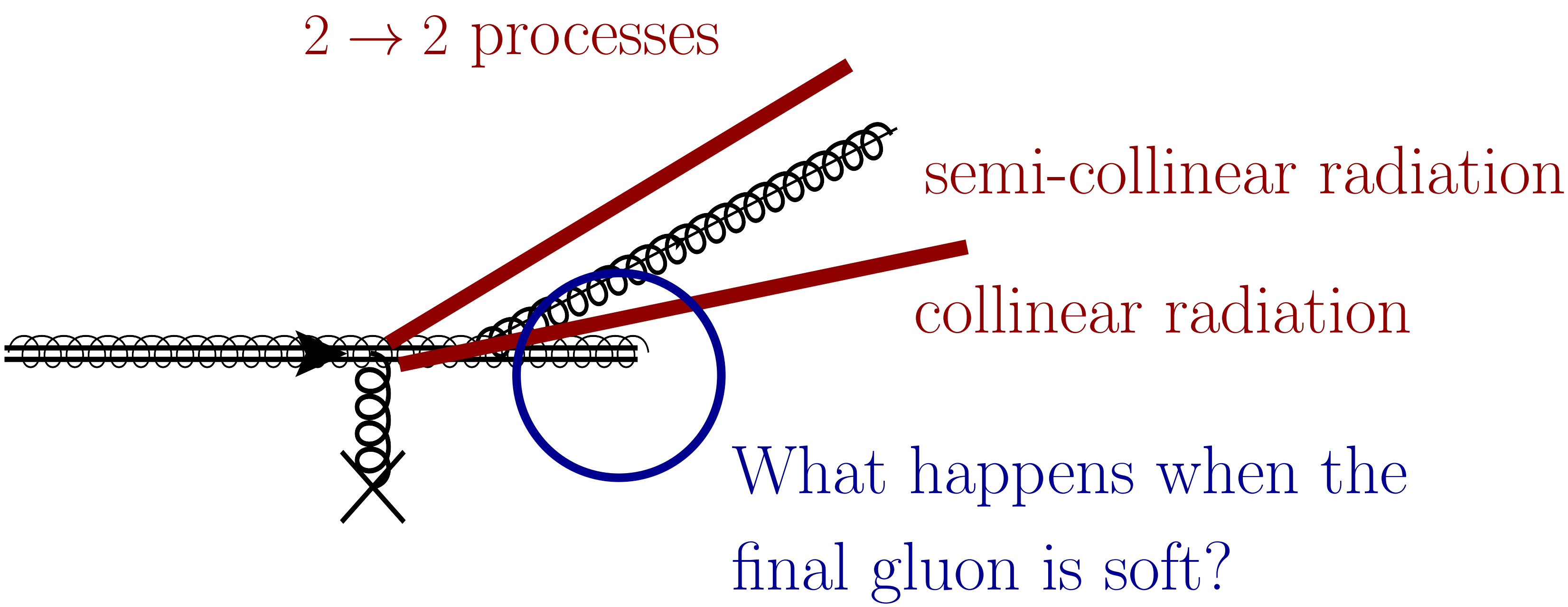
$$\underbrace{q_\perp^2 C[q_\perp] = \int_{-\infty}^{\infty} \frac{\mathrm{d}q_0}{2\pi} \, \langle F_{i+} F_{i+}(Q) \rangle \big|_{q_+=0} = \frac{T m_D^2}{q_T^2 + m_D^2}}_{\text{Rate of transverse kicks of } q_\perp}$$

- We need a finite $q^- = \delta E$ generalization:

$$\underbrace{\int_{-\infty}^{\infty} \frac{\mathrm{d}q_0}{2\pi} \, \langle F_{i+} F_{i+}(Q) \rangle \big|_{q_+=-\delta E} = T \left[\frac{2(\delta E)^2(\delta E^2 + q_\perp^2 + m_D^2) + m_D^2 q_\perp^2}{(\delta E^2 + q_\perp^2 + m_D^2)(\delta E^2 + q_\perp^2)} \right]}_{\text{Rate of transverse kicks of } q_\perp \text{ and energy transfer } \delta E}$$

almost involves the replacement, $q_\perp^2 \to q_\perp^2 + \delta E^2$

Matching between brem and drag



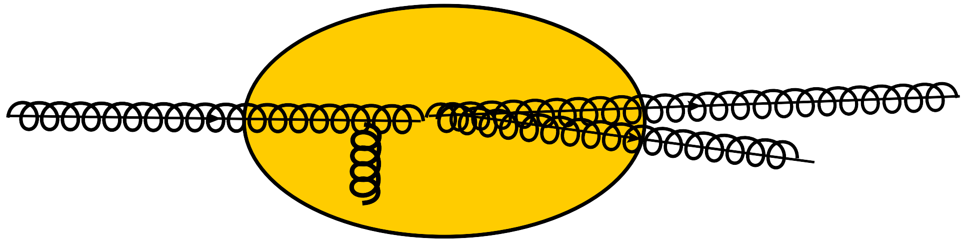
- The semi-collinear emission rate diverges logarithmically when the gluon gets soft

$$\Gamma_{\rm semi-coll} \sim g^2 C_A \quad \overbrace{\frac{\delta m_\infty^2}{4\pi}}^{\sim g^3 T^2} \quad \log\left(\frac{2Tm_D}{\mu}\right)$$

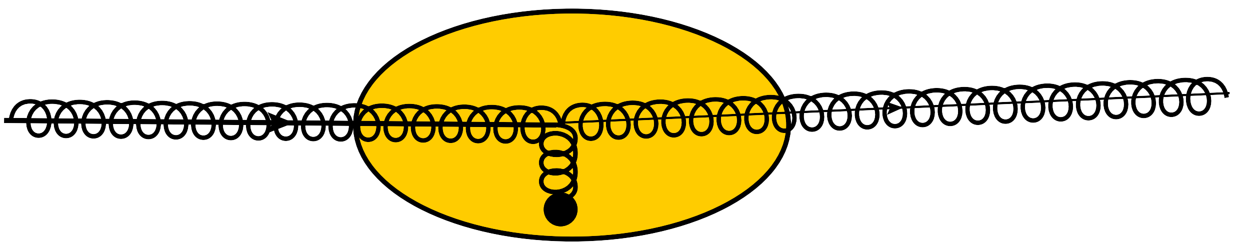
When the gluon becomes soft need to relate radiation and drag.

Matching between semi-collinear brem and drag

- When the final gluon line is hard, the brem process:

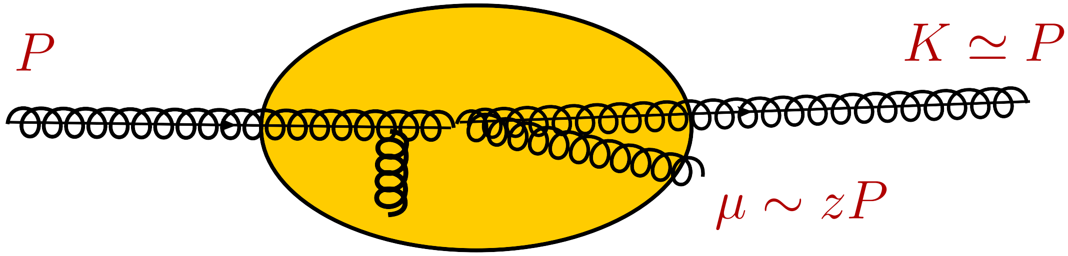


is physically distinct from the drag process:

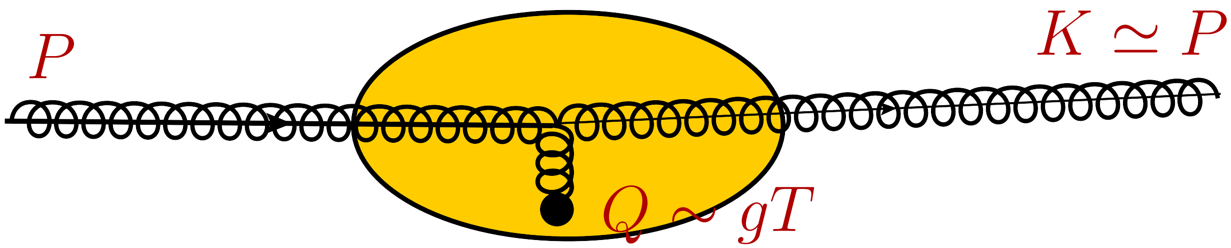


Matching between semi-collinear brem and drag

- When the final gluon line becomes soft, the brem process:



is not physically distinct from the drag process:

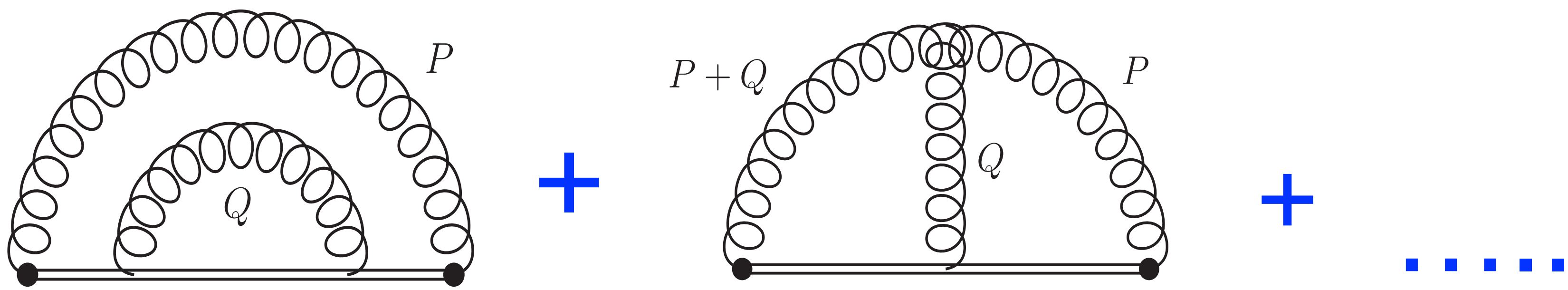


but represents a higher order correction to drag.

Separately both processes depend on the separation scale, $\mu \sim gT$, but ...

the μ dep. cancels when both rates are included

Computing the NLO drag:



- Evaluate NLO longitudinal force-force with hard thermal loops + sum-rules
- Only change relative to LO is the replacement $m_\infty^2 \to m_\infty^2 + \delta m_\infty^2$

$$\eta(\mu) \propto g^2 C_A \int^{\mu} \frac{d^2\mathbf{p}_T}{(2\pi)^2} \frac{m_\infty^2 + \delta m_\infty^2}{p_T^2 + m_\infty^2 + \delta m_\infty^2}$$

$$\propto \text{leading order} + \underbrace{g^2 C_A \frac{\delta m_\infty^2}{4\pi}\left[\log\left(\frac{\mu_\perp^2}{m_\infty^2}\right) - 1\right]}_{\text{NLO correction to drag}}$$

The cutoff dependence of the drag cancels against the semi-collinear emission rate

The NLO Boltzmann equation review:

Cutoff dependence cancels

$$[\partial_t + v_{\mathbf{k}} \cdot \partial_{\mathbf{x}}] f_{\mathbf{k}} = (\eta(\mu) + \delta\eta(\mu))\ \mathbf{v}_{\mathbf{k}} \cdot \frac{\partial f_{\mathbf{k}}}{\partial \mathbf{k}} + C_{2\leftrightarrow 2}[\mu]$$
$$C_{1\leftrightarrow 2} + \delta C_{1\leftrightarrow 2} + C_{\mathrm{semi-coll}}[\mu]$$

Lessons from weak coupling

- Tight relation between drag, wide angle emissions, quasi-particle mass shift.
  - Closely related to dimensional reduction.
- The wide angle emission kernel $C[\mathbf{q}_\perp, \delta E]$ is closely related to $C[\mathbf{q}_\perp]$, almost:

$$\mathbf{q}_\perp^2 \to \mathbf{q}_\perp^2 + \delta E^2$$

  - Closely related to dimensional reduction.
- Understand in detail the transition from radiative to collisional loss

Currently being implemented into MARTINI

$2k(2\pi)^3 \frac{\mathrm{d}\Gamma}{\mathrm{d}^3k} =$ Photon emission rate per phase-space

Same techniques can be used for thermal photon production:

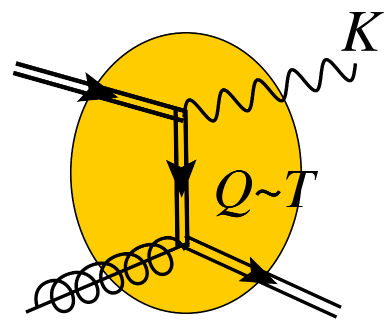
- The rate is function of the coupling coupling constant and k/T:

$$2k(2\pi)^3 \frac{\mathrm{d}\Gamma}{\mathrm{d}^3k} \propto e^2T^2 \Big[\underbrace{O(g^2 \log) + O(g^2)}_{\text{LO AMY}} + \underbrace{O(g^3 \log) + O(g^3)}_{\text{From soft } gT \text{ gluons, } n_B \simeq \frac{T}{\omega} \simeq \frac{1}{g}} + \ldots$$

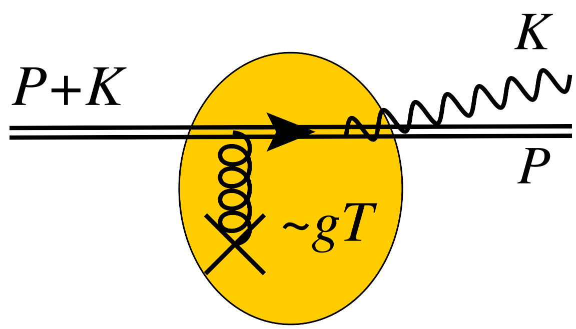
$O(g^3)$ is closely related to open issues in energy loss:

Three rates for photon production at Leading Order

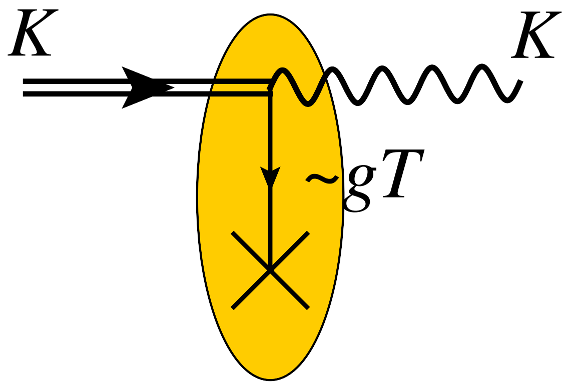
1. Hard Collisions – a $2 \leftrightarrow 2$ processes



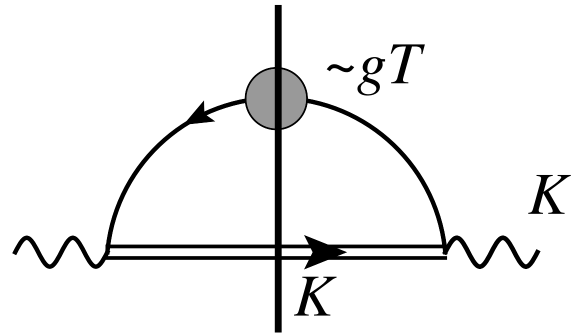
2. Collinear Bremsstrhalung – a $1 \leftrightarrow 2$ processes



3. Quark Conversions – $1 \leftrightarrow 1$ processes (analogous to drag)



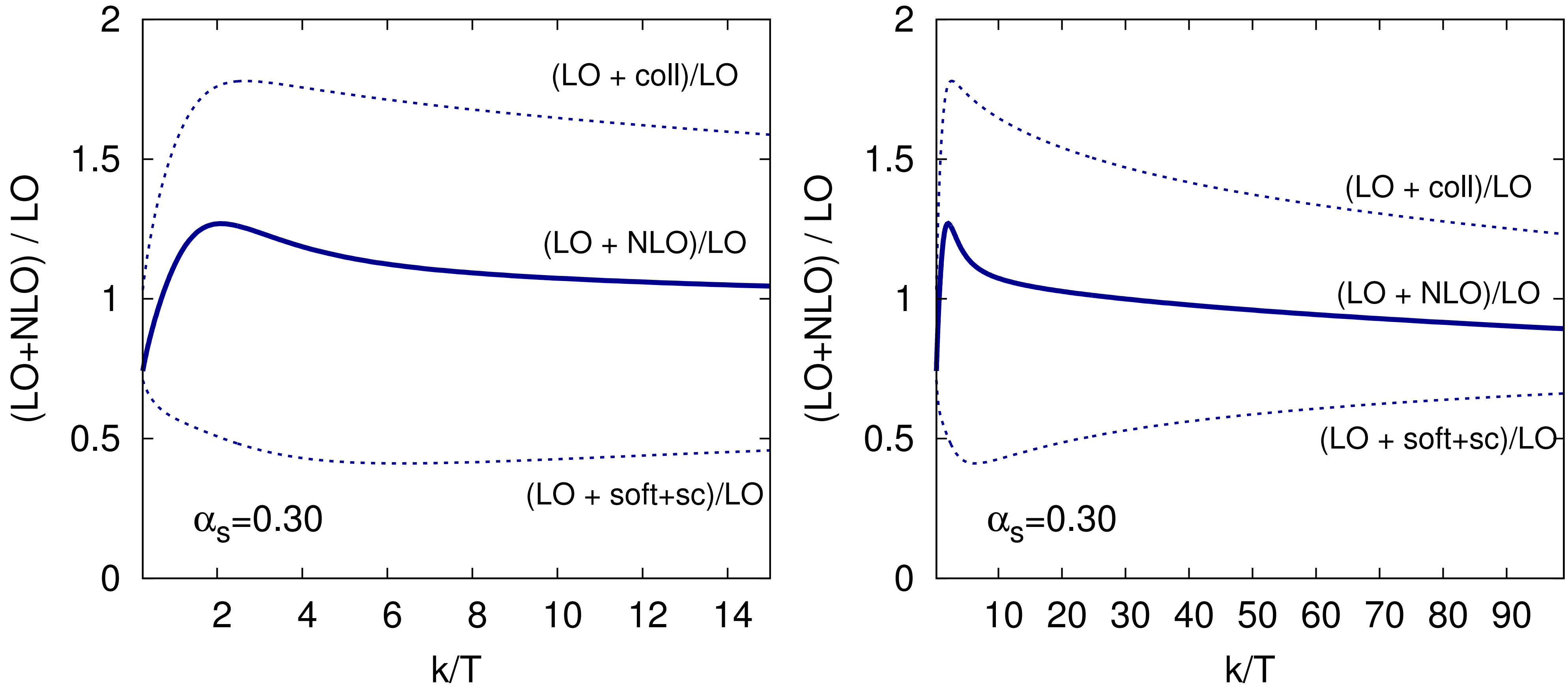
or



NLO involves corrections to these three processes.

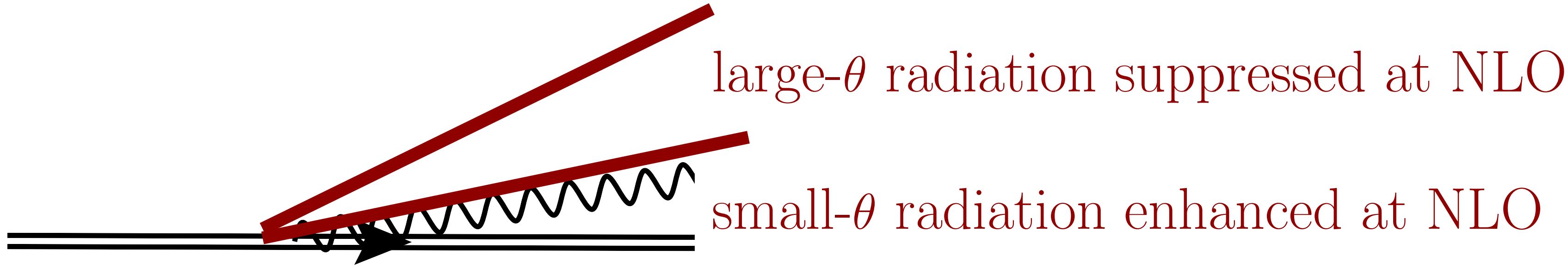
Full rate is independent of scale $\mu_\perp$.

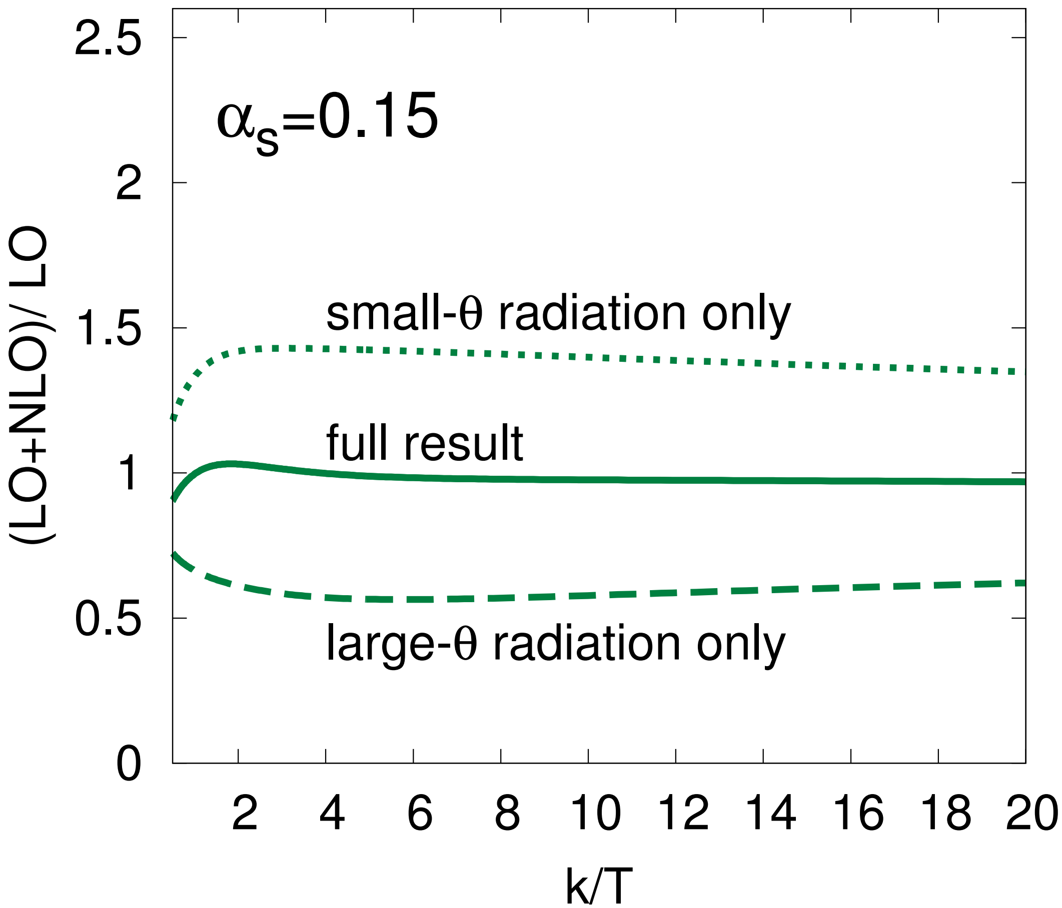
NLO Results: $\Gamma_{NLO} \sim \mathrm{LO} + g^3 \log(1/g) + g^3$



NLO corrections are modest and roughly k independent

The different contributions at NLO (conversions are not numerically important)





Conclusion:

- The size of NLO corrections is much larger for heavy quarks than light quarks
- NLO corrections to collinear processes seem to be modest.
- All of the soft sector buried into a few coefficients, $C[q_\perp, \delta E]$, $\hat{q}_{\rm cnvrt}$, δm_∞^2
  - Can we compute these non-perturbatively with dimensional reduction?
  - Use these non-perturbative parameters to compute η/s

Can imagine computing all of energy loss perturbatively rather precisely for

$T \sim 800\,{\rm MeV}$!

Let's get to it!