Neutral Meson Mixing

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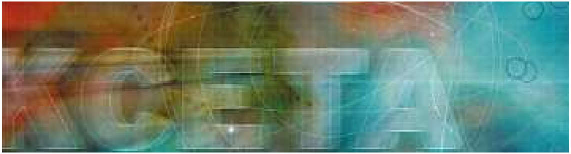

Lattice meets continuum: QCD calculations in flavour physics

Siegen, 29 Sep – 2 Oct 2014

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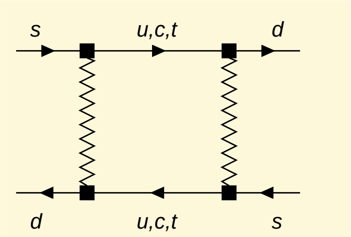
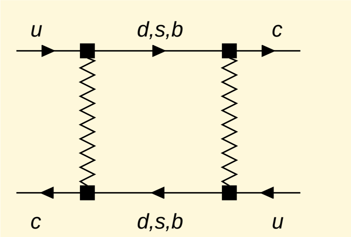
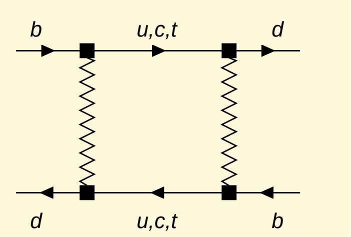
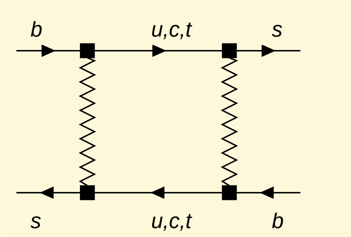
Meson-antimeson mixing

Only K^0,D^0,B_d^0, and B_s^0 mesons mix with their antiparticles:









Effects of meson-antimeson mixing ($\mathrm{M}-\overline{\mathrm{M}}$ mixing, with $M = K, D, B_d$, or B_s):

- The flavour eigenstates $|M\rangle$ and $|\overline{M}\rangle$ are no mass eigenstates.

  This feature is exploited in K physics: The lifetimes of the mass eigenstates K_{long} and K_{short} differ by a factor of 500.

  $\Rightarrow$ Make a K_{long} beam by producing K's and $\overline{K}$'s and wait.

Effects of meson-antimeson mixing ($\mathrm{M}-\overline{\mathrm{M}}$ mixing, with $M = K, D, B_d$, or B_s):

- The flavour eigenstates $|M\rangle$ and $|\overline{M}\rangle$ are no mass eigenstates.
- A meson produced as an $|M\rangle$ at time $t = 0$ oscillates between the states $|M\rangle$ and $|\overline{M}\rangle$.

  This feature is exploited in the study of D, B_d, or B_s mesons.

$B-\bar{B}$ mixing in the Standard Model

$B_q-\bar{B}_q$ mixing with $q = d$ or $q = s$ involves the 2×2 matrices M and Γ.

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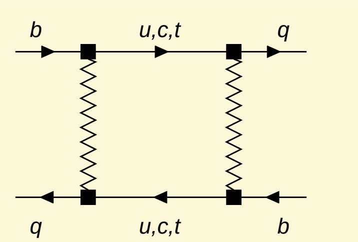
The mass matrix element M_{12}^q stems from the dispersive (real) part of the box diagram, internal t.

The decay matrix element Γ_{12}^q stems from the absorpive (imaginary) part of the box diagram, internal c, u.



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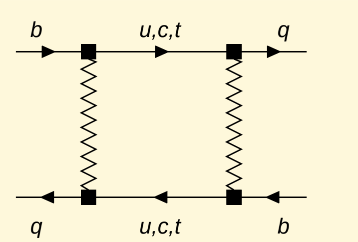

3 physical quantities in $B_q-\bar{B}_q$ mixing:

$$\left|M_{12}^q\right|, \quad \left|\Gamma_{12}^q\right|, \quad \phi_q \equiv \arg\left(-\frac{M_{12}^q}{\Gamma_{12}^q}\right)$$

The two eigenstates found by diagonalising $M - i\,\Gamma/2$ differ in their masses and widths:

$$\begin{aligned} \text{mass difference} \qquad \Delta m_q &\simeq 2|M_{12}^q|, \\ \text{width difference} \qquad \Delta\Gamma_q &\simeq 2|\Gamma_{12}^q|\cos\phi_q \end{aligned}$$

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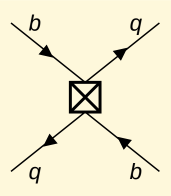
CP asymmetry in flavor-specific decays (semileptonic CP asymmetry):

$$a_{\rm fs}^q = \frac{|\Gamma_{12}^q|}{|M_{12}^q|} \sin\phi_q$$

Δm_s and Δm_d

Operator Product Expansion:



$$M_{12} = (V_{tq}^* V_{tb})^2 \, C Q$$

Local Operator:

$$Q = \overline{q}_L \gamma_\nu b_L \, \overline{q}_L \gamma^\nu b_L$$

Theoretical uncertainty of Δm_q dominated by matrix element:

$$\langle \mathrm{B_q} | Q | \overline{\mathrm{B}}_q \rangle = \frac{2}{3} M_{B_q}^2 \, f_{B_q}^2 \, B_{B_q}$$

Standard Model: $C = C(m_t, \alpha_s)$ is well-known.

$B_s - \bar{B}_s$ mixing: CKM unitarity fixes $|V_{ts}| \simeq |V_{cb}|$. Use lattice results for $f^2_{B_q} B_{B_q}$ to confront $\Delta m_s^{\rm exp}$ with the Standard Model:

$$\Delta m_s = \left(18.8 \pm 0.6_{\,V_{cb}} \pm 0.3_{\,m_t} \pm 0.1_{\,\alpha_s} \right) \text{ps}^{-1} \frac{f^2_{B_s} B_{B_s}}{(220\,\text{MeV})^2}$$

Here $\overline{\rm MS}$-NDR scheme for B_{B_q} at scale m_b.
Often used: scheme-invariant $\widehat{B}_{B_q} = 1.51 B_{B_q}$.

Recall:

$$\Delta m_s = \left(18.8 \pm 0.6_{\,V_{cb}} \pm 0.3_{\,m_t} \pm 0.1_{\,\alpha_s} \right) \text{ps}^{-1} \, \frac{f_{B_s}^2 \, B_{B_s}}{(220\,\text{MeV})^2}$$

CKMfitter lattice averages (Moriond 2014):

$$f_{B_s} = (226.5 \pm 1.1 \pm 5.4)\,\text{MeV}, \qquad B_{B_s} = 0.87 \pm 0.01 \pm 0.02$$

means $f_{B_s}^2 B_{B_s} = [(212 \pm 9)\,\text{MeV}]^2$ and

$$\Delta m_s = (17.4 \pm 1.7)\,\text{ps}^{-1}$$

complying with LHCb/CDF average

$$\Delta m_s^{\text{exp}} = (17.761 \pm 0.022)\,\text{ps}^{-1}.$$

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Few lattice-QCD calculations of $f_{B_s}^2 B_{B_s}$ available!

Prediction of Δm_s largely relies on calculations of f_{B_s} and the prejudice $B_{B_s} \simeq 0.85$.

FLAG recommends to use HPQCD'09 value

$$f_{B_s}\sqrt{B_{B_s}} = (216 \pm 15)\,\mathrm{MeV}$$

giving

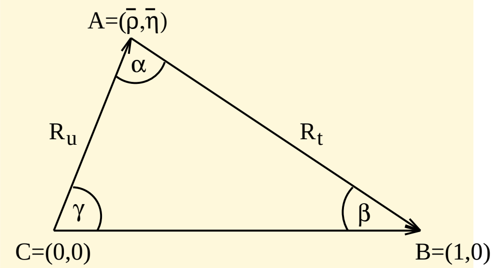
$$\Delta m_s = (18.2 \pm 2.6)\,\mathrm{ps}^{-1}$$

Δm_d

$|V_{cb}|$, short-distance coefficient and some hadronic uncertainties drop out from the ratio $\Delta m_d / \Delta m_s$:

$$\xi^2 = \frac{f_{B_s}^2 B_{B_s}}{f_{B_d}^2 B_{B_d}}$$

$$\frac{\Delta m_d}{\Delta m_s} \propto \frac{|V_{td}|^2}{|V_{ts}|^2} \propto R_t^2$$



Usual way to probe the Standard Model with Δm_d: Global fit to unitarity triangle.

Easier way:
Determine R_t from Δm_d:

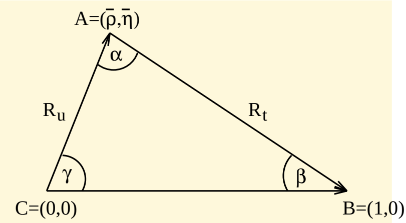
$$R_t = 0.880 \frac{\xi}{1.16} \sqrt{\frac{\Delta m_d}{0.49\,\text{ps}^{-1}}} \sqrt{\frac{17\,\text{ps}^{-1}}{\Delta m_s}} \frac{0.22}{|V_{us}|} (1 + 0.050\bar{\rho})$$

and compare with indirect determination of R_t from angles:

$$R_t = \frac{\sin\gamma}{\sin\alpha} = \frac{\sin(\alpha+\beta)}{\sin\alpha}$$

$$\beta = 21.5^\circ \pm 0.7^\circ, \ \alpha = 85.4^{\circ\,+4.0^\circ}_{\ \,-3.9^\circ}$$

$$\Rightarrow \ R_t = 0.960 \pm 0.026$$



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FLAG recommends Fermilab/MILC (2012): $\xi = 1.268 \pm 0.063$ implying

$$R_t = 0.942 \pm 0.047_{\xi} \pm 0.006_{\text{rest}}$$

agrees well with $R_t = 0.960 \pm 0.026$ from angles.

CKMfitter (Moriond 2014) global fit result:

$$R_t = 0.9171^{+0.0082}_{-0.0166}$$

QCD sum rule result $\xi = 1.16 \pm 0.04$ challenged by data: $R_t = 0.86 \pm 0.03$

Decay matrix

The calculation Γ_{12}^q, $q = d, s$, is needed for

the width difference $\Delta\Gamma_q \simeq 2|\Gamma_{12}^q| \cos\phi_q$

and the semileptonic CP asymmetry $a_{\rm fs}^q = \frac{|\Gamma_{12}^q|}{|M_{12}^q|} \sin\phi_q$

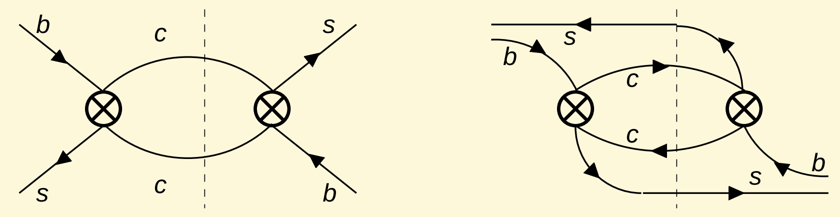
In the Standard Model

$\phi_s = 0.22^\circ \pm 0.06^\circ$ and $\phi_d = -4.3^\circ \pm 1.4^\circ$.

Recalling $\phi_q = \arg\left(-\frac{M_{12}^q}{\Gamma_{12}^q}\right)$, a new physics contribution to $\arg M_{12}^q$ may deplete $\Delta\Gamma_q$ and enhance $|a_{\rm fs}^q|$ to a level observable at current experiments.

But: Precise data on CP violation in $B_d \to J/\psi K_S$ and $B_s \to J/\psi\phi$ preclude large NP contributions to $\arg\phi_d$ and $\arg\phi_s$.

Leading contribution to Γ_{12}^s:



Γ_{12}^s stems from Cabibbo-favoured tree-level $b \to c\bar{c}s$ decays, sizable new-physics contributions are impossible.

Updated Standard-Model prediction for $\Delta\Gamma_s/\Delta m_s$ in terms of hadronic parameters:

$$\frac{\Delta\Gamma_s}{\Delta m_s}\Delta m_s^{\rm exp} = \left[0.082 + 0.019\frac{\widetilde{B}'_{S,B_s}}{B_{B_S}} - 0.025\frac{B_R}{B_{B_s}}\right]\,\mathrm{ps}^{-1}$$

Here

$$\langle B_s|\overline{s}_L^\alpha b_R^\beta\,\overline{s}_L^\beta b_R^\alpha|\overline{B}_s\rangle = \frac{1}{12}M_{B_s}^2\, f_{B_s}^2\,\widetilde{B}'_{S,B_s}$$

and $B_R = 1 \pm 0.5$ parametrises the size of higher-dimension operators.

$$\Delta\Gamma_s^{\rm exp} = (0.091 \pm 0.009)\,\mathrm{ps}^{-1}$$

Dimuon asymmetry

DØ has measured the CP-violating quantity

$$A_S = \frac{N^{++} - N^{--}}{N^{++} + N^{--}}$$

with N^{++} and N^{--} the number of (μ^+, μ^+) and (μ^-, μ^-) pairs, respectively, resulting from $(b, \bar{b})$ pairs produced in $p\bar{p}$ collisions.
Non-zero A_S requires that at least one of the $(b, \bar{b})$ quarks hadronises into a $B_{d,s}$ which oscillates into $\bar{B}_{d,s}$. The neutral-B sample consists of 58% B_d and 42% B_s mesons.

If all observed $\mu^{\pm}$ are from $b, \bar{b}$ decays, A_S is related to the *CP asymmetries in flavour-specific decays* $a_{\rm fs}^{d,s}$ (a.k.a as *semileptonic CP asymmetries*) as

$$A_S = 0.58 a_{\rm fs}^d + 0.42 a_{\rm fs}^s.$$

SM prediction: $A_S^{\rm SM} = -(2.0 \pm 0.3) \cdot 10^{-4}$

A. Lenz, UN, CKM2010, arXiv:1102.4272

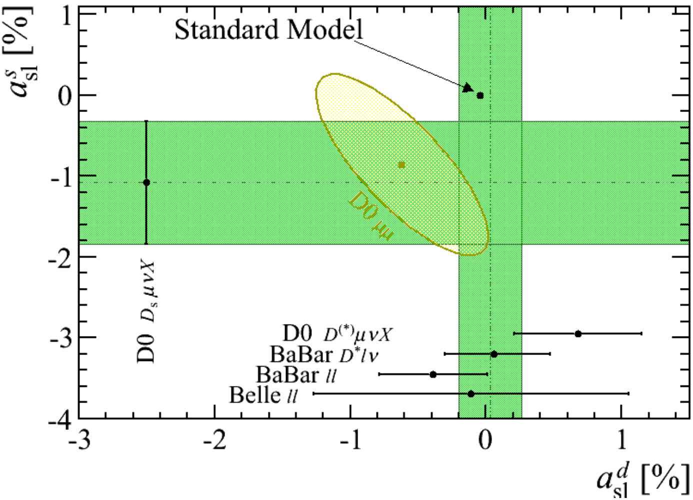
DØ finds $A_S < A_S^{\rm SM}$. Deviations from SM prediction:

year	Ref.	deviation
2010	PRL 105, 081801 (2010)	3.2σ
2011	PRD 84, 052007 (2011)	3.9σ
2013	PRD 89, 012002 (2014)	3.6σ (*)

In (*) mixing-induced CP violation in $b \to c\bar{c}d$ is included.

$\longrightarrow$ topic of this talk

Before LHCb



courtesy of M. Vesterinen.

Breaking news

Yesterday M. Vesterinen (LHCb) has presented

$$a_{\rm fs}^d = (-0.2 \pm 1.9 \pm 3.0) \cdot 10^{-3},$$ LHCb-PAPER-2014-053,

newly obtained from 3 fb^{-1} dataset and

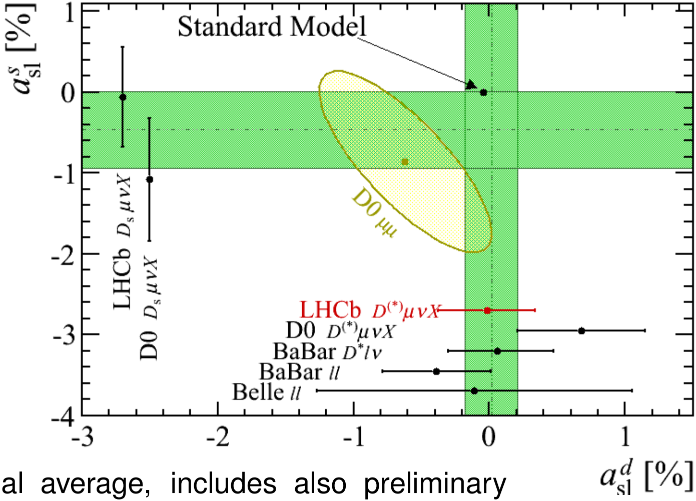
$$a_{\rm fs}^s = (-0.6 \pm 5.0 \pm 3.6) \cdot 10^{-3},$$ PLB 728C 607 (2014),

obtained from 2011 dataset (1 fb^{-1}).
Results comply with the SM predictions

$$a_{\rm fs}^{d,\rm SM} = -(4.1 \pm 0.6) \cdot 10^{-4}, \qquad a_{\rm fs}^{s,\rm SM} = (1.9 \pm 0.3) \cdot 10^{-5}$$

Beneke,Buchalla,Lenz,UN, 2003
Lenz,UN, 2006,2011

Breaking news



inofficial average, includes also preliminary CKM2014 BaBar result $a^d_{\rm sl} = (-3.9 \pm 3.5 \pm 1.9) \cdot 10^{-3}$, courtesy of M. Vesterinen.

Discovery of Guennadi Borissov and Bruce Hoeneisen

(Phys.Rev. D87, 074020 (2013)):

$$\begin{array}{c} \bar{p}\ p \\ \downarrow \\ \mu^+ X \leftarrow \bar{b}\ b \rightarrow \bar{B}_d \xrightarrow{\text{mixes}} \frac{p}{q} g_-(t) B_d + g_+(t) \bar{B}_d \rightarrow D^+ D^- \\ \hookrightarrow \mu^+ X \end{array}$$

CP violation in the interference of B_d–$\bar{B}_d$ mixing and $\overset{(-)}{B}_d \rightarrow D^+ D^-$ creates and asymmetry w.r.t.

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This CP asymmetry is proportional to $\sin(2\beta)$, with 2β being the phase of the B_d–$\bar{B}_d$ mixing amplitude M_{12} (in the standard phase convention in which the $b \rightarrow c\bar{c}d$ decay amplitude is (essentially) real).

These $b \to c\bar{c}d$ decays create a contribution $A_S^{\rm int}$ to A_S.
CP-even and CP-odd final state contribute with opposite sign, but:

$$\Gamma(B_{\rm CP+} \to X_{c\bar{c}}) - \Gamma(B_{\rm CP-} \to X_{c\bar{c}}) \simeq \Delta\Gamma$$

Dunietz,Fleischer,UN 2001; Beneke,Buchalla,Lenz,UN 2003

so that

$$A_S^{\rm int} = -\, P_{c\to\mu}\, \frac{\Delta\Gamma}{\Gamma} \sin(2\beta) \frac{x_d}{1+x_d^2}$$

↗ probability for $c \to \mu$

↑ CP phase

↖ dilution from time integration.

Here $x_d = \Delta m/\Gamma$ and Γ is the total B_d width.

Jarlskog criterion

Within the SM CP violation requires

$$(m_u^2 - m_c^2)(m_c^2 - m_t^2)(m_u^2 - m_t^2)\times$$

$$(m_d^2 - m_s^2)(m_s^2 - m_b^2)(m_d^2 - m_b^2)\,\mathrm{Im}\,(V_{11}V_{21}^*V_{22}V_{12}^*) \neq 0$$

$\Rightarrow$ CP asymmetries vanish for $m_c = m_u$.

Mass matrix M, decay matrix Γ:

$$a_{\rm fs}^d = \operatorname{Im} \frac{\Gamma_{12}}{M_{12}} \propto \frac{m_c^2 - m_u^2}{m_b^2}$$

vanishes for $m_c = m_u$, while

$$\Delta\Gamma = -\Delta m \operatorname{Re} \frac{\Gamma_{12}}{M_{12}}$$

and $A_S^{\rm int}$ does not vanish in this limit!

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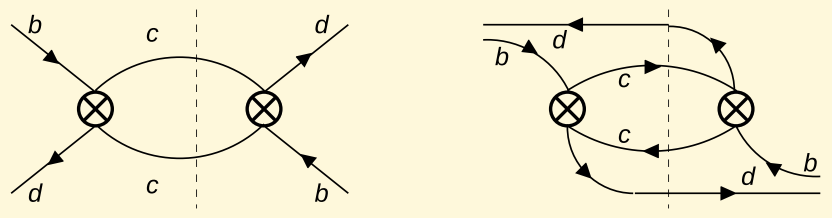
$\Rightarrow$ There should be a contribution with up quarks which contributes to $A_S^{\rm int}$ with opposite sign.

$$\Gamma_{12} = -\left[\lambda_c^2\,\Gamma_{12}^{cc} + 2\,\lambda_c\,\lambda_u\,\Gamma_{12}^{uc} + \lambda_u^2\,\Gamma_{12}^{uu}\right]$$

with $\lambda_c = V_{cd}^* V_{cb}$, $\lambda_u = V_{ud}^* V_{ub}$, and $\lambda_t = -\lambda_c - \lambda_u = V_{td}^* V_{tb}$.

In the SM the charm-charm contribution dominates

$$\Delta\Gamma = -\Delta m\,\mathrm{Re}\,\frac{\Gamma_{12}}{M_{12}} \approx 2|\lambda_c|^2\Gamma_{12}^{cc}$$



$$
\begin{aligned}
|B(t)\rangle &= \quad g_+(t)\,|B\rangle + \frac{q}{p}\,g_-(t)\,|\bar{B}\rangle\,, \\
|\bar{B}(t)\rangle &= \frac{p}{q}\,g_-(t)\,|B\rangle + \quad g_+(t)\,|\bar{B}\rangle.
\end{aligned}
$$

Time-dependent decay rate $\Gamma[B(t) \to f] = N_f |\langle f | B(t) \rangle|^2$ with phase-space factor N_f.
Interference term in $\Gamma[B(t) \to X_{c\bar{c}}]$:

$$
B_{cc}(t) = 2\,\mathrm{Re}\,\left[g_+^*(t) \frac{q}{p} g_-(t) \underbrace{\sum_{f \in X_{c\bar{c}}} N_f \langle B | f \rangle \langle f | \bar{B} \rangle}_{-\lambda_c^2 \Gamma_{12}^{cc}} \right]
$$

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$$

$$
B_{cc}(t) = \Gamma_{12}^{cc} e^{-\Gamma t} \sin(\Delta m t)\,\mathrm{Im}\left(\frac{q}{p}\lambda_c^2\right) = \Gamma_{12}^{cc} |\lambda_c|^2 e^{-\Gamma t} \sin(\Delta m t) \sin(2\beta)
$$

The interference term in $\Gamma[\overline{B}(t) \to X_{c\bar{c}}]$ has the opposite sign. Thus the charm-charm contribution to A_S^{int} is

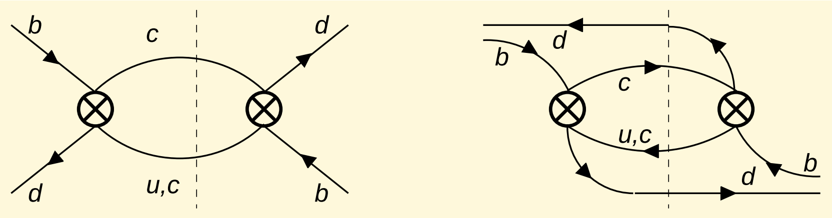
$$A_S^{\text{int},c\bar{c}} = -P_{c\to\mu} \int_0^\infty dt\, 2B_{cc}(t) = -P_{c\to\mu} \frac{2\Gamma_{12}^{cc}}{\Gamma} |\lambda_c|^2 \sin(2\beta) \frac{x_d}{1+x_d^2}$$

Add missing up contribution from up quark, taking $m_c = m_u$ here, so that $\Gamma_{12}^{cu} = \Gamma_{12}^{cc}$:
To find $A_S^{\mathrm{int},c\bar{c}} + A_S^{\mathrm{int},c\bar{u}}$ from $A_S^{\mathrm{int},c\bar{c}}$ simply replace

$$\mathrm{Im}\left(\frac{q}{p}\lambda_c^2\right) \to \mathrm{Im}\left(\frac{q}{p}\lambda_c(\lambda_c + \lambda_u)\right) = -\mathrm{Im}\left(\frac{q}{p}\lambda_c\lambda_t\right)$$

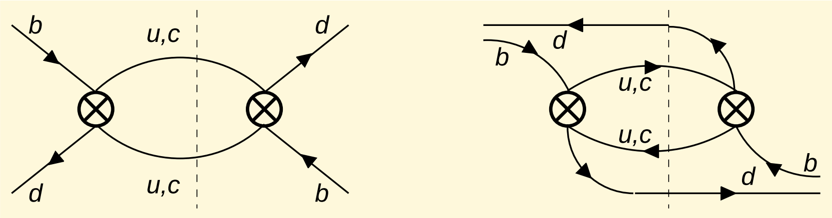
amounting to

$|\lambda_c|^2 \sin(2\beta) \to |\lambda_c\lambda_t| \sin\beta,$ smaller by factor of 0.49!



To comply with the Jarlskog criterion we need also to add $A_S^{\text{int},u\bar{c}} + A_S^{\text{int},u\bar{u}}$.
However, in our real world with $m_c \neq m_u$ the probabilities $P_{u\to\mu}$ and $P_{c\to\mu}$ are very different. μ's from the decay chain $b \to u \to \mu$ require that e.g. a K^+ or π^+ decays (semi-) muonically before reaching the detector.
In the considered limit $m_c = m_u$:

$$A_S^{\text{int}} = -(P_{c\to\mu} - P_{u\to\mu})\,\frac{2\Gamma_{12}^{cc}}{\Gamma}\,|\lambda_c\lambda_t|\,\sin(\beta)\,\frac{x_d}{1+x_d^2}$$



Thus the estimate in Phys.Rev. D87, 074020 (2013)

$$A_S^{\text{int}} = -(4.5 \pm 1.6)10^{-4}$$

gets reduced to

$$A_S^{\text{int}} > -(2.2 \pm 0.8)10^{-4}$$

and the discrepancy between the DØ dimuon asymmetry and the SM prediction is actually *larger* (by roughly 0.2σ) than the 3.6σ quoted in Phys. Rev. D 89, 012002 (2014).

Important lesson: $A_S^{\rm int}$ depends on the individual components Γ_{12}^{cc}, Γ_{12}^{cu}, Γ_{12}^{uc}, and Γ_{12}^{uu} in a different way than $a_{\rm fs}^d$ and $\Delta\Gamma$!

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Thus the sensitivity to new physics is also different. Consider a new contribution of the type

$$\text{real coefficient } \times \lambda_t \times \overline{d}b(\overline{u}u + \overline{c}c + \ldots),$$

i.e. new physics coming with a gluon/photon/Z penguin operator: The interference term with the SM tree amplitude amounts to (for $m_c = m_u$)

$$\delta a_{\rm fs}^d \propto \text{Im}\, \frac{\lambda_t(\lambda_u + \lambda_c)}{\lambda_t^2} = -\text{Im}\, \frac{\lambda_t^2}{\lambda_t^2} = 0$$

while

$$\delta A_s^{\rm int} \propto \text{Im}\, \frac{\lambda_t(P_{u\to\mu}\lambda_u + P_{c\to\mu}\lambda_c)}{\lambda_t^2} \neq 0.$$

Also $\Delta\Gamma$ will change from its SM value.

Conclusions

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- The LHCb error on $\Delta\Gamma_s$ is much smaller than the theory error now.
- There is rapid experimental progress on $a_{\rm fs}^d$ and $a_{\rm fs}^s$.

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  is smaller in magnitude by at least a factor of 0.49 compared to the formulae used in the DØ analysis, so that the discrepancy with the SM is larger than the quoted 3.6σ.
- $A_S^{\rm int}$ depends differently on new physics than $a_{\rm fs}^d$.

Slides for discussion

$\Delta\Gamma_s$

$$\frac{\Delta\Gamma_s}{\Delta m_s}\Delta m_s^{\rm exp} = \Big[0.082 \pm 0.007 + (0.019 \pm 0.001)\frac{\widetilde{B}'_{S,B_s}}{B_{B_s}} -(0.027 \pm 0.003)\frac{B_R}{B_{B_s}}\Big]\,\text{ps}^{-1}$$

Leading power in Λ_{QCD}/m_b: Only two operators:

$$Q = \overline{s}_\alpha\gamma_\mu(1-\gamma_5)b_\alpha\,\overline{s}_\beta\gamma^\mu(1-\gamma_5)b_\beta$$

$$\widetilde{Q}_S = \overline{s}_\alpha(1+\gamma_5)b_\beta\,\overline{s}_\beta(1+\gamma_5)b_\alpha$$

with colour indices α, β.
Can trade $Q_S = \overline{s}_\alpha(1+\gamma_5)b_\alpha\,\overline{s}_\beta(1+\gamma_5)b_\beta$ for the $1/m_b$-suppressed operator

$$R_0 \equiv Q_S + \tilde{Q}_S + \frac{1}{2}Q$$

I.e. R_0 vanishes identically in HQET.

$1/m_b$-suppressed operators

Most relevant:

$$\widetilde{R}_2 = \frac{1}{m_b^2}\,\overline{s}_\beta \overleftarrow{D}_\rho \gamma^\mu (1-\gamma_5) D^\rho b_\alpha\, \overline{s}_\alpha \gamma_\mu (1-\gamma_5) b_\beta$$

Any chance to tackle it on the lattice? (In HQET $D^\rho b \to v^\rho b_v$.)
Second most relevant:

$$R_0 \equiv Q_S + \tilde{Q}_S + \frac{1}{2} Q$$

Occasionally people take $\langle B_s | R_0 | \overline{B}_s \rangle$ from lattice calculations of $\langle B_s | Q_S | \overline{B}_s \rangle$, $\langle B_s | \tilde{Q}_S | \overline{B}_s \rangle$, and $\langle B_s | Q | \overline{B}_s \rangle$, but to my knowledge the lattice-continuum matching is not done to order α_s / m_b.

$1/m_b$-suppressed operators

$$\langle B_s|R_0|\overline{B}_s\rangle = -\frac{4}{3}\left[\frac{M_{B_s}^2}{m_b^{\text{pole}\,2}\,(1+\overline{m}_s/\overline{m}_b)^2}-1\right] M_{B_s}^2 f_{B_s}^2 B_{R_0},$$

$$\langle B_s|\widetilde{R}_2|\overline{B}_s\rangle = \frac{2}{3}\left[\frac{M_{B_s}^2}{m_b^{\text{pole}\,2}}-1\right] M_{B_s}^2 f_{B_s}^2 B_{\widetilde{R}_2},$$

Note: $\langle B_s|R_2|\overline{B}_s\rangle = -\langle B_s|\widetilde{R}_2|\overline{B}_s\rangle\left[1+\mathcal{O}(\Lambda_{QCD}/m_b)\right]$

2007 sum-rule calculation of Mannel, Pecjak, Pivovarov:

$$B_{R_2}-1 = 0.003 \pm 0.003$$