Neutral Meson Mixing

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Lattice meets continuum: QCD calculations in flavour physics
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Meson-antimeson mixing

Only $K^0, D^0, B^0_d$, and $B^0_s$ mesons mix with their antiparticles:
Effects of meson-antimeson mixing ($M-\bar{M}$ mixing, with $M=K, D, B_d, \text{ or } B_s$):

- The flavour eigenstates $|M\rangle$ and $|\bar{M}\rangle$ are no mass eigenstates.

This feature is exploited in $K$ physics: The lifetimes of the mass eigenstates $K_{\text{long}}$ and $K_{\text{short}}$ differ by a factor of 500.

$\Rightarrow$ Make a $K_{\text{long}}$ beam by producing $K$’s and $\bar{K}$’s and wait.
Effects of meson-antimeson mixing ($\mathcal{M} - \mathcal{M}$ mixing, with $\mathcal{M} = K, D, B_d, \text{ or } B_s$):

- The flavour eigenstates $|\mathcal{M}\rangle$ and $|\mathcal{M}\rangle$ are no mass eigenstates.
- A meson produced as an $|\mathcal{M}\rangle$ at time $t = 0$ oscillates between the states $|\mathcal{M}\rangle$ and $|\mathcal{M}\rangle$.

This feature is exploited in the study of $D, B_d, \text{ or } B_s$ mesons.
$B^-\bar{B}$ mixing in the Standard Model

$B_q^-\bar{B}_q$ mixing with $q = d$ or $q = s$ involves the $2 \times 2$ matrices $M$ and $\Gamma$. 
$B_q - \bar{B}_q$ mixing with $q = d$ or $q = s$ involves the $2 \times 2$ matrices $M$ and $\Gamma$.

The mass matrix element $M_{12}^q$ stems from the dispersive (real) part of the box diagram, internal $t$.

The decay matrix element $\Gamma_{12}^q$ stems from the absorbive (imaginary) part of the box diagram, internal $c, u$. 
$B_q - \bar{B}_q$ mixing with $q = d$ or $q = s$ involves the $2 \times 2$ matrices $M$ and $\Gamma$.

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The decay matrix element $\Gamma_{12}^q$ stems from the absorptive (imaginary) part of the box diagram, internal $c, u$.

3 physical quantities in $B_q - \bar{B}_q$ mixing:

$$|M_{12}^q|, \quad |\Gamma_{12}^q|, \quad \phi_q \equiv \arg\left(-\frac{M_{12}^q}{\Gamma_{12}^q}\right)$$
The two eigenstates found by diagonalising $M - i\Gamma/2$ differ in their masses and widths:

- **mass difference** \( \Delta m_q \approx 2|M_{12}^q| \),
- **width difference** \( \Delta \Gamma_q \approx 2|\Gamma_{12}^q| \cos \phi_q \)
The two eigenstates found by diagonalising $M - i \frac{\Gamma}{2}$ differ in their masses and widths:

- mass difference: $\Delta m_q \approx 2|M_{12}^q|$, 
- width difference: $\Delta \Gamma_q \approx 2|\Gamma_{12}^q| \cos \phi_q$

CP asymmetry in flavor-specific decays (semileptonic CP asymmetry):

$$a_{fs}^q = \frac{|\Gamma_{12}^q|}{|M_{12}^q|} \sin \phi_q$$
**Operator Product Expansion:**

\[ M_{12} = (V^*_{tq} V_{tb})^2 CQ \]

**Local Operator:**

\[ Q = \overline{q}_L \gamma_\nu b_L \overline{q}_L \gamma^\nu b_L \]

Theoretical uncertainty of \( \Delta m_q \) dominated by matrix element:

\[ \langle B_q | Q | \overline{B}_q \rangle = \frac{2}{3} M_{Bq}^2 f_{Bq}^2 B_{Bq} \]

Standard Model: \( C = C(m_t, \alpha_s) \) is well-known.
$B_s - \bar{B}_s$ mixing: CKM unitarity fixes $|V_{ts}| \sim |V_{cb}|$. Use lattice results for $f_{Bq}^2 B_{Bq}$ to confront $\Delta m_s^{\text{exp}}$ with the Standard Model:

$$\Delta m_s = \left( 18.8 \pm 0.6 \, v_{cb} \pm 0.3 \, m_t \pm 0.1 \, \alpha_s \right) \, \text{ps}^{-1} \frac{f_{Bq}^2 \, B_{Bs}}{(220 \, \text{MeV})^2}$$

Here $\overline{\text{MS}}$-NDR scheme for $B_{Bq}$ at scale $m_b$.
Often used: scheme-invariant $\hat{B}_{Bq} = 1.51 B_{Bq}$. 
Recall:

\[
\Delta m_s = \left( 18.8 \pm 0.6 \, v_{cb} \pm 0.3 \, m_t \pm 0.1 \, \alpha_s \right) \text{ps}^{-1} \frac{f_{B_s}^2 \, B_{B_s}}{(220 \, \text{MeV})^2}
\]

CKMfitter lattice averages (Moriond 2014):

\[
f_{B_s} = (226.5 \pm 1.1 \pm 5.4) \, \text{MeV}, \quad B_{B_s} = 0.87 \pm 0.01 \pm 0.02
\]

means \( f_{B_s}^2 \, B_{B_s} = [(212 \pm 9) \, \text{MeV}]^2 \) and

\[
\Delta m_s = (17.4 \pm 1.7) \, \text{ps}^{-1}
\]

complying with LHCb/CDF average

\[
\Delta m_s^{\text{exp}} = (17.761 \pm 0.022) \, \text{ps}^{-1}.
\]
$\Delta m_s = (17.4 \pm 1.7) \text{ ps}^{-1}$ versus $\Delta m_s^{\text{exp}} = (17.761 \pm 0.022) \text{ ps}^{-1}$, too good to be true...
\( \Delta m_s = (17.4 \pm 1.7) \text{ ps}^{-1} \) versus
\( \Delta m_s^{\text{exp}} = (17.761 \pm 0.022) \text{ ps}^{-1} \), too good to be true...

Few lattice-QCD calculations of \( f_{B_s}^2 B_{B_s} \) available!

Prediction of \( \Delta m_s \) largely relies on calculations of \( f_{B_s} \) and the prejudice \( B_{B_s} \approx 0.85 \).

FLAG recommends to use HPQCD’09 value

\[
f_{B_s} \sqrt{B_{B_s}} = (216 \pm 15) \text{ MeV}
\]

giving

\( \Delta m_s = (18.2 \pm 2.6) \text{ ps}^{-1} \)
$|V_{cb}|$, short-distance coefficient and some hadronic uncertainties drop out from the ratio $\Delta m_d/\Delta m_s$:

$$\xi^2 = \frac{f_{Bs}^2 B_{Bs}}{f_{Bd}^2 B_{Bd}}$$

$$\frac{\Delta m_d}{\Delta m_s} \propto \frac{|V_{td}|^2}{|V_{ts}|^2} \propto R_t^2$$

Usual way to probe the Standard Model with $\Delta m_d$: Global fit to unitarity triangle.
Easier way:
Determine $R_t$ from $\Delta m_d$:

$$R_t = 0.880 \frac{\xi}{1.16} \sqrt{\frac{\Delta m_d}{0.49 \text{ ps}^{-1}}} \sqrt{\frac{17 \text{ ps}^{-1}}{\Delta m_s}} \frac{0.22}{|V_{us}|} (1 + 0.050 \rho)$$

and compare with indirect determination of $R_t$ from angles:

$$R_t = \frac{\sin \gamma}{\sin \alpha} = \frac{\sin(\alpha + \beta)}{\sin \alpha}$$

$\beta = 21.5^\circ \pm 0.7^\circ$, $\alpha = 85.4^\circ \pm 4.0^\circ$

$\Rightarrow R_t = 0.960 \pm 0.026$
\[ R_t \text{ from } \Delta m_d: \]

\[
R_t = 0.880 \frac{\xi}{1.16} \sqrt{\frac{\Delta m_d}{0.49 \text{ ps}^{-1}}} \sqrt{\frac{17 \text{ ps}^{-1}}{\Delta m_s}} \frac{0.22}{|V_{us}|} (1 + 0.050\rho)
\]

FLAG recommends Fermilab/MILC (2012): \( \xi = 1.268 \pm 0.063 \)

implying

\[
R_t = 0.942 \pm 0.047 \xi \pm 0.006_{\text{rest}}
\]

agrees well with \( R_t = 0.960 \pm 0.026 \) from angles.

CKMfitter (Moriond 2014) global fit result:

\[
R_t = 0.9171_{-0.0166}^{+0.0082}
\]

QCD sum rule result \( \xi = 1.16 \pm 0.04 \) challenged by data:

\[
R_t = 0.86 \pm 0.03
\]
The calculation $\Gamma^q_{12}$, $q = d, s$, is needed for the width difference $\Delta \Gamma_q \simeq 2|\Gamma^q_{12}| \cos \phi_q$ and the semileptonic CP asymmetry $a_{fs}^q = \frac{|\Gamma^q_{12}|}{|M^q_{12}|} \sin \phi_q$.

In the Standard Model

$\phi_s = 0.22^\circ \pm 0.06^\circ$ and $\phi_d = -4.3^\circ \pm 1.4^\circ$.

Recalling $\phi_q = \text{arg} \left( -\frac{M^q_{12}}{\Gamma^q_{12}} \right)$, a new physics contribution to $\arg M^q_{12}$ may deplete $\Delta \Gamma_q$ and enhance $|a_{fs}^q|$ to a level observable at current experiments.

**But:** Precise data on CP violation in $B_d \to J/\psi K_S$ and $B_s \to J/\psi \phi$ preclude large NP contributions to $\arg \phi_d$ and $\arg \phi_s$. 


Leading contribution to $\Gamma_{12}^s$:

$\Gamma_{12}^s$ stems from Cabibbo-favoured tree-level $b \to c\bar{c}s$ decays, sizable new-physics contributions are impossible.
Updated Standard-Model prediction for $\Delta \Gamma_s / \Delta m_s$ in terms of hadronic parameters:

$$
\frac{\Delta \Gamma_s}{\Delta m_s} \Delta m_s^{\text{exp}} = \left[ 0.082 + 0.019 \frac{\tilde{B}'_{S,Bs}}{B_{BS}} - 0.025 \frac{B_R}{B_{Bs}} \right] \text{ps}^{-1}
$$

Here

$$
\langle B_s | s^\alpha_L b_R^\beta \bar{s}^\beta_L b^\alpha_R | B_s \rangle = \frac{1}{12} M_{Bs}^2 f_{Bs}^2 \tilde{B}'_{S,Bs}
$$

and $B_R = 1 \pm 0.5$ parametrises the size of higher-dimension operators.

$$
\Delta \Gamma_s^{\text{exp}} = (0.091 \pm 0.009) \text{ps}^{-1}
$$
DØ has measured the CP-violating quantity

\[ A_S = \frac{N^{++} - N^{--}}{N^{++} + N^{--}} \]

with \( N^{++} \) and \( N^{--} \) the number of \((\mu^+, \mu^+)\) and \((\mu^-, \mu^-)\) pairs, respectively, resulting from \((b, \bar{b})\) pairs produced in pp collisions.

Non-zero \( A_S \) requires that at least one of the \((b, \bar{b})\) quarks hadronises into a \(B_{d,s}\) which oscillates into \(\bar{B}_{d,s}\). The neutral-\(B\) sample consists of 58% \(B_d\) and 42% \(B_s\) mesons.
If all observed $\mu^\pm$ are from $b, \bar{b}$ decays, $A_S$ is related to the $CP$ asymmetries in flavour-specific decays $a_{fs}^{d,s}$ (a.k.a as semileptonic $CP$ asymmetries) as

$$A_S = 0.58 a_{fs}^d + 0.42 a_{fs}^s.$$ 

SM prediction: $A_S^{SM} = -(2.0 \pm 0.3) \cdot 10^{-4}$

A. Lenz, UN, CKM2010, arXiv:1102.4272

DØ finds $A_S < A_S^{SM}$. Deviations from SM prediction:

<table>
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<th>year</th>
<th>Ref.</th>
<th>deviation</th>
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<tr>
<td>2010</td>
<td>PRL 105, 081801 (2010)</td>
<td>$3.2\sigma$</td>
</tr>
<tr>
<td>2011</td>
<td>PRD 84, 052007 (2011)</td>
<td>$3.9\sigma$</td>
</tr>
<tr>
<td>2013</td>
<td>PRD 89, 012002 (2014)</td>
<td>$3.6\sigma$ (*)</td>
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In (*) mixing-induced CP violation in $b \rightarrow c\bar{c}d$ is included.

→ topic of this talk
Before LHCb

courtesy of M. Vesterinen.
Yesterday M. Vesterinen (LHCb) has presented

$$a_{fs}^d = (-0.2 \pm 1.9 \pm 3.0) \cdot 10^{-3}, \quad \text{LHCb-PAPER-2014-053},$$

newly obtained from 3 fb$^{-1}$ dataset and

$$a_{fs}^s = (-0.6 \pm 5.0 \pm 3.6) \cdot 10^{-3}, \quad \text{PLB 728C 607 (2014)},$$

obtained from 2011 dataset (1 fb$^{-1}$).

Results comply with the SM predictions

$$a_{fs}^{d,SM} = -(4.1 \pm 0.6) \cdot 10^{-4}, \quad a_{fs}^{s,SM} = (1.9 \pm 0.3) \cdot 10^{-5}$$

Lenz,UN, 2006,2011
Breaking news

inofficial average, includes also preliminary
CKM2014 BaBar result $a_{sl}^d = (-3.9 \pm 3.5 \pm 1.9) \cdot 10^{-3}$, courtesy of M. Vesterinen.
Discovery of Guennadi Borissov and Bruce Hoeneisen
(Phys.Rev. D87, 074020 (2013)):

\[ \bar{p} \ p \]
\[ \downarrow \]

\[ \mu^+ X \leftarrow \bar{b} \ b \rightarrow B_d \xrightarrow{\text{mixes}} \frac{p}{q} g_-(t)B_d + g_+(t)\bar{B}_d \rightarrow D^+ D^- \leftarrow \mu^+ X \]

CP violation in the interference of \( B_d - \bar{B}_d \) mixing and \( \bar{B}_d \rightarrow D^+ D^- \) creates an asymmetry w.r.t.

\[ \bar{p} \ p \]
\[ \downarrow \]

\[ \mu^- X \leftarrow b \ \bar{b} \rightarrow B_d \xrightarrow{\text{mixes}} g_+(t)B_d + \frac{q}{p} g_-(t)\bar{B}_d \rightarrow D^+ D^- \leftarrow \mu^- X \]
Discovery of **Guennadi Borissov and Bruce Hoeneisen**
(Phys.Rev. D87, 074020 (2013)):

\[
\begin{align*}
\bar{p} & \quad p \\
\downarrow \\
\mu^+ X & \leftrightarrow \bar{b} \quad b \\
& \rightarrow B_d \xrightarrow{\text{mixes}} \frac{p}{q} g_-(t) B_d + g_+(t) \bar{B}_d \rightarrow D^+ D^- \\
& \leftrightarrow \mu^+ X \\
\end{align*}
\]

CP violation in the interference of $B_d - \bar{B}_d$ mixing and $\bar{B}_d \rightarrow D^+ D^-$ creates an asymmetry w.r.t.

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& \leftrightarrow \mu^- X \\
\end{align*}
\]

This **CP asymmetry** is proportional to $\sin(2\beta)$, with $2\beta$ being the phase of the $B_d - \bar{B}_d$ mixing amplitude $M_{12}$ (in the standard phase convention in which the $b \rightarrow c\bar{c}d$ decay amplitude is (essentially) real).
These $b \to c\bar{c}d$ decays create a contribution $A^\text{int}_S$ to $A_S$. CP-even and CP-odd final state contribute with opposite sign, but:

$$\Gamma(B_{CP+} \to X_{c\bar{c}}) - \Gamma(B_{CP-} \to X_{c\bar{c}}) \simeq \Delta \Gamma$$

Dunietz, Fleischer, UN 2001; Beneke, Buchalla, Lenz, UN 2003

so that

$$A^\text{int}_S = - P_{c \to \mu} \frac{\Delta \Gamma}{\Gamma} \sin(2\beta) \frac{x_d}{1 + x_d^2}$$

Here $x_d = \Delta m/\Gamma$ and $\Gamma$ is the total $B_d$ width.
Within the SM CP violation requires

\[
(m_u^2 - m_c^2)(m_c^2 - m_t^2)(m_u^2 - m_t^2) \times \\
(m_d^2 - m_s^2)(m_s^2 - m_b^2)(m_d^2 - m_b^2) \text{Im} (V_{11} V_{21}^* V_{22} V_{12}^*) \neq 0
\]

\[\Rightarrow\] CP asymmetries vanish for \( m_c = m_u \).
Mass matrix $M$, decay matrix $Γ$:

$$a_{fs}^d = \text{Im} \frac{Γ_{12}}{M_{12}} \propto \frac{m_c^2 - m_u^2}{m_b^2}$$

vanishes for $m_c = m_u$, while

$$ΔΓ = -Δm \text{Re} \frac{Γ_{12}}{M_{12}}$$

and $A_{S}^{\text{int}}$ does not vanish in this limit!
Mass matrix $M$, decay matrix $\Gamma$:

$$a_{fs}^d = \text{Im} \frac{\Gamma_{12}}{M_{12}} \propto \frac{m_c^2 - m_u^2}{m_b^2}$$

vanishes for $m_c = m_u$, while

$$\Delta \Gamma = -\Delta m \text{Re} \frac{\Gamma_{12}}{M_{12}}$$

and $A_{S}^{\text{int}}$ does not vanish in this limit!

⇒ There should be a contribution with up quarks which contributes to $A_{S}^{\text{int}}$ with opposite sign.
\[
\Gamma_{12} = - \left[ \lambda_c^2 \Gamma_{cc}^{12} + 2 \lambda_c \lambda_u \Gamma_{uc}^{12} + \lambda_u^2 \Gamma_{uu}^{12} \right]
\]

with \( \lambda_c = V_{cd}^* V_{cb} \), \( \lambda_u = V_{ud}^* V_{ub} \), and \( \lambda_t = -\lambda_c - \lambda_u = V_{td}^* V_{tb} \).

In the SM the charm-charm contribution dominates

\[
\Delta \Gamma = -\Delta m \text{Re} \frac{\Gamma_{12}}{M_{12}} \approx 2|\lambda_c|^2 \Gamma_{cc}^{12}
\]
\[ |B(t)\rangle = g_+(t) |B\rangle + \frac{q}{p} g_-(t) |\bar{B}\rangle, \]
\[ |\bar{B}(t)\rangle = \frac{p}{q} g_-(t) |B\rangle + g_+(t) |\bar{B}\rangle. \]

Time-dependent decay rate \( \Gamma[B(t) \to f] = N_f |\langle f | B(t) \rangle|^2 \) with phase-space factor \( N_f \).

Interference term in \( \Gamma[B(t) \to X_{cc}] \):

\[
B_{cc}(t) = 2 \text{Re} \left[ g_+^*(t) \frac{q}{p} g_-(t) \sum_{f \in X_{cc}} N_f \langle B | f \rangle \langle f | \bar{B} \rangle \right] - \chi_c^2 \Gamma_{12}^{cc}
\]
\[ |B(t)\rangle = g_+(t) |B\rangle + \frac{q}{p} g_-(t) |\bar{B}\rangle, \]

\[ |\bar{B}(t)\rangle = \frac{p}{q} g_-(t) |B\rangle + g_+(t) |\bar{B}\rangle. \]

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\]

\[
B_{cc}(t) = \Gamma_{12}^{cc} e^{-\Gamma t} \sin(\Delta mt) \text{Im} \left( \frac{q}{p} \chi_c^2 \right) = \Gamma_{12}^{cc} |\chi_c|^2 e^{-\Gamma t} \sin(\Delta mt) \sin(2\beta)
\]
The interference term in $\Gamma[\bar{B}(t) \rightarrow X_{c\bar{c}}]$ has the opposite sign. Thus the charm-charm contribution to $A_S^{\text{int}}$ is

$$A_{S}^{\text{int},c\bar{c}} = -P_{c \rightarrow \mu} \int_0^{\infty} dt \, 2B_{cc}(t) = -P_{c \rightarrow \mu} \frac{2\Gamma_{cc}^{12}}{\Gamma} |\lambda_c|^2 \sin(2\beta) \frac{x_d}{1 + x_d^2}$$
Add missing up contribution from up quark, taking $m_c = m_u$ here, so that $\Gamma_{12}^{cu} = \Gamma_{12}^{cc}$:

To find $A_{S}^{\text{int},c\bar{c}} + A_{S}^{\text{int},c\bar{u}}$ from $A_{S}^{\text{int},c\bar{c}}$ simply replace

$$\text{Im} \left( \frac{q}{p} \lambda_c^2 \right) \rightarrow \text{Im} \left( \frac{q}{p} \lambda_c (\lambda_c + \lambda_u) \right) = -\text{Im} \left( \frac{q}{p} \lambda_c \lambda_t \right)$$

amounting to

$$|\lambda_c|^2 \sin(2\beta) \rightarrow |\lambda_c \lambda_t| \sin \beta,$$

smaller by factor of 0.49!
To comply with the Jarlskog criterion we need also to add $A_{S}^{\text{int},u\bar{c}} + A_{S}^{\text{int},u\bar{u}}$.

However, in our real world with $m_c \neq m_u$ the probabilities $P_{u \rightarrow \mu}$ and $P_{c \rightarrow \mu}$ are very different. $\mu$’s from the decay chain $b \rightarrow u \rightarrow \mu$ require that e.g. a $K^+$ or $\pi^+$ decays (semi-)muonically before reaching the detector.

In the considered limit $m_c = m_u$:

$$A_{S}^{\text{int}} = -\left( P_{c \rightarrow \mu} - P_{u \rightarrow \mu} \right) \frac{2\Gamma_{12}^{cc}}{\Gamma} |\lambda_c \lambda_t| \sin(\beta) \frac{x_d}{1 + x_d^2}$$
Thus the estimate in *Phys.Rev. D87, 074020 (2013)*

\[ A_S^{\text{int}} = -(4.5 \pm 1.6) \times 10^{-4} \]

gets reduced to

\[ A_S^{\text{int}} > -(2.2 \pm 0.8) \times 10^{-4} \]

and the discrepancy between the DØ dimuon asymmetry and the SM prediction is actually *larger* (by roughly 0.2\(\sigma\)) than the 3.6\(\sigma\) quoted in *Phys. Rev. D 89, 012002 (2014)*.
Important lesson: $A_{S}^{\text{int}}$ depends on the individual components $\Gamma_{12}^{cc}$, $\Gamma_{12}^{cu}$, $\Gamma_{12}^{uc}$, and $\Gamma_{12}^{uu}$ in a different way than $a_{fs}^{d}$ and $\Delta \Gamma$!
Important lesson: $A^\text{int}_S$ depends on the individual components $\Gamma_{12}^{cc}$, $\Gamma_{12}^{cu}$, $\Gamma_{12}^{uc}$, and $\Gamma_{12}^{uu}$ in a different way than $a^d_{fs}$ and $\Delta \Gamma$!

Thus the sensitivity to new physics is also different.
Important lesson: $A_{S}^{\text{int}}$ depends on the individual components $\Gamma_{12}^{cc}$, $\Gamma_{12}^{cu}$, $\Gamma_{12}^{uc}$, and $\Gamma_{12}^{uu}$ in a different way than $a_{fs}^{d}$ and $\Delta \Gamma$!

Thus the sensitivity to new physics is also different. Consider a new contribution of the type

\[
\text{real coefficient} \times \lambda_{t} \times \overline{d} b (\overline{u} u + \overline{c} c + \ldots),
\]

i.e. new physics coming with a gluon/photon/Z penguin operator: The interference term with the SM tree amplitude amounts to (for $m_{c} = m_{u}$)

\[
\delta a_{fs}^{d} \propto \text{Im} \frac{\lambda_{t}(\lambda_{u} + \lambda_{c})}{\lambda_{t}^{2}} = -\text{Im} \frac{\lambda_{t}^{2}}{\lambda_{t}^{2}} = 0
\]

while

\[
\delta A_{S}^{\text{int}} \propto \text{Im} \frac{\lambda_{t}(P_{u \rightarrow \mu} \lambda_{u} + P_{c \rightarrow \mu} \lambda_{c})}{\lambda_{t}^{2}} \neq 0.
\]

Also $\Delta \Gamma$ will change from its SM value.
Conclusions

- $B - \bar{B}$ mixing is highly sensitive to new physics and stays interesting.
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- We start to see the chiral logarithms pushing lattice predictions of $\xi$ to $\xi = 1.268 \pm 0.063$ in the data. The old QCD sum rule result $\xi = 1.16 \pm 0.04$ is challenged.
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• The LHCb error on $\Delta \Gamma_s$ is much smaller than the theory error now.

• There is rapid experimental progress on $a_{fs}^d$ and $a_{fs}^s$. 
I agree with Borissov and Hoeneisen that the DØ dimuon asymmetry receives a contribution $A_{S}^{\text{int}}$ from mixing-induced CP violation in decays $B \to X \to X' \mu$. 
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• Final states with all combinations $(c, \bar{c}), (c, \bar{u}), (u, \bar{c}),$ and $(u, \bar{u})$ must be considered.
I agree with Borissov and Hoeneisen that the $D\bar{O}$ dimuon asymmetry receives a contribution $A_{int}^S$ from mixing-induced CP violation in decays $B \rightarrow X \rightarrow X' \mu$.

Final states with all combinations $(c, \bar{c})$, $(c, \bar{u})$, $(u, \bar{c})$, and $(u, \bar{u})$ must be considered.

$$A_{int}^S = -(P_{c \rightarrow \mu} - P_{u \rightarrow \mu}) \frac{\Delta \Gamma}{\Gamma} \frac{|\lambda_t|}{|\lambda_c|} \sin(\beta) \frac{x_d}{1 + x_d^2}$$

is smaller in magnitude by at least a factor of 0.49 compared to the formulae used in the $D\bar{O}$ analysis, so that the discrepancy with the SM is larger than the quoted $3.6\sigma$. 
I agree with Borissov and Hoeneisen that the DØ dimuon asymmetry receives a contribution $A_{S}^{\text{int}}$ from mixing-induced CP violation in decays $B \rightarrow X \rightarrow X' \mu$.

Final states with all combinations $(c, \bar{c})$, $(c, \bar{u})$, $(u, \bar{c})$, and $(u, \bar{u})$ must be considered.

$$A_{S}^{\text{int}} = -(P_{c \rightarrow \mu} - P_{u \rightarrow \mu}) \frac{|\Delta \Gamma|}{\Gamma} \frac{|\lambda_t|}{|\lambda_c|} \sin(\beta) \frac{x_d}{1 + x_d^2}$$

is smaller in magnitude by at least a factor of 0.49 compared to the formulae used in the DØ analysis, so that the discrepancy with the SM is larger than the quoted $3.6\sigma$.

$A_{S}^{\text{int}}$ depends differently on new physics than $a_{fs}^d$. 
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**Slides for discussion**
\[
\frac{\Delta \Gamma_s}{\Delta m_s} \Delta m_s^{\text{exp}} = \left[ 0.082 \pm 0.007 + (0.019 \pm 0.001) \frac{\tilde{B}'_{S,Bs}}{B_{Bs}} \right] \text{ps}^{-1} \\
- (0.027 \pm 0.003) \frac{B_R}{B_{Bs}} 
\]

Leading power in $\Lambda_{QCD}/m_b$: Only two operators:

\[
Q = \bar{s}_\alpha \gamma_\mu (1 - \gamma_5) b_\alpha \bar{s}_\beta \gamma^\mu (1 - \gamma_5) b_\beta \\
\tilde{Q}_S = \bar{s}_\alpha (1 + \gamma_5) b_\beta \bar{s}_\beta (1 + \gamma_5) b_\alpha
\]

with colour indices $\alpha, \beta$.

Can trade $Q_S = \bar{s}_\alpha (1 + \gamma_5) b_\alpha \bar{s}_\beta (1 + \gamma_5) b_\beta$ for the $1/m_b$-suppressed operator

\[
R_0 \equiv Q_S + \tilde{Q}_S + \frac{1}{2} Q
\]

I.e. $R_0$ vanishes identically in HQET.
Most relevant:

\[ \tilde{R}_2 = \frac{1}{m_b^2} s_\beta \bar{D}_\rho \gamma^\mu (1 - \gamma_5) D^\rho b_\alpha s_\alpha \bar{\gamma}_\mu (1 - \gamma_5) b_\beta \]

Any chance to tackle it on the lattice? (In HQET \( D^\rho b \to v^\rho b_v \).)

Second most relevant:

\[ R_0 \equiv Q_S + \tilde{Q}_S + \frac{1}{2} Q \]

Occasionally people take \( \langle B_s| R_0 |B_s \rangle \) from lattice calculations of \( \langle B_s|Q_S|B_s \rangle, \langle B_s|\tilde{Q}_S|B_s \rangle, \) and \( \langle B_s| Q|B_s \rangle \), but to my knowledge the lattice-continuum matching is not done to order \( \alpha_s/m_b \).
$1/m_b$-suppressed operators

\[ \langle B_s | R_0 | \bar{B}_s \rangle = -\frac{4}{3} \left[ \frac{M_{B_s}^2}{m_b \text{pole}^2} \left( 1 + \frac{m_s}{m_b} \right)^2 - 1 \right] M_{B_s}^2 f_{B_s}^2 B_{R_0}, \]

\[ \langle B_s | \tilde{R}_2 | \bar{B}_s \rangle = \frac{2}{3} \left[ \frac{M_{B_s}^2}{m_b \text{pole}^2} - 1 \right] M_{B_s}^2 f_{B_s}^2 B_{\tilde{R}_2}, \]

Note: $\langle B_s | R_2 | \bar{B}_s \rangle = -\langle B_s | \tilde{R}_2 | \bar{B}_s \rangle \left[ 1 + \mathcal{O}(\Lambda_{QCD}/m_b) \right]$.

2007 sum-rule calculation of Mannel, Pecjak, Pivovarov:

$B_{R_2} - 1 = 0.003 \pm 0.003$